Hydraulic control of Shatt Al-Hilla within Hilla City

A Thesis

Submitted to the College of Engineering of the University of Babylon in Partial Fulfillment of the requirements for the degree of Master of Science in Water Resources Engineering

By

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CHAPTER ONE

INTRODUCTION

١.١ *General*

Civilization has always developed along rivers whose presence guaranteed access to and from the seacoast, irrigation of crops, water supplies for urban communities, and latterly power development and industrial water supply. The many advantages have always been counterbalanced by the danger of floods or dryness. Therefore, channels, whether natural or artificial, should be under control.

For a channel to satisfy irrigation demands, the channel discharge should not be less than the design discharge and the water level should not be lower than the one which provides the necessary hydraulic command. However, for flood – control purposes, the channel water – level (and consequently, the respective discharge) should not be higher than a certain maximum value.

Usually, the control may aim at maintaining nominated discharges or water levels at certain locations. For a channel that conveys water for irrigation and passes adjacent to locations that should not be endangered by drowning, special hydraulic measures should be available permanently or on call to control the water level and regulate the discharge at the critical location; in other words, by providing a control section which has a unique relationship between head and discharge of the channel. Such a control may be provided by altering the width – depth ratio of the flow, altering the bed slop, and / or controlling the hydraulic gradient along the critical reach of the channel by means of a structure across the channel.

Shatt Al-Hilla within Hilla City is a typical example for the aforementioned case, particularly with its proposed development plan, which involves increasing its normal discharge from (٢٠٠ – ٢٣٠ $m^{٣}$/sec) to (٣٠٣ $m^{٣}$/sec) [BWRD, (١٩٩٨)].

١.٢ *Objectives of the research*

The objectives of the research are:

١. Developing a mathematical model to simulate gradually varied flow in Shatt Al-Hilla within Hilla City.
٢. Using the developed model to evaluate the performance of certain feasible hydraulic measures in providing the control of Shatt Al-Hilla aimed at therein.

١.٣ *Methodology of the research*

The research adopts the following methodology:

١. Under the assumption that Manning's roughness coefficient remains constant with time and distance along the considered reach, the Saint – Venant equations are to be solved by the four – point implicit scheme.
٢. For stability and computational efficiency of the numerical solution, the numerical solution is to be tested for a natural channel to find suitable values of the computational and hydraulic parameters.

٣. Using the tested mathematical model to find the influences of the followings:

A. An upstream control by the proposed cross – regulator of [Al-Msaudi, (٢٠٠١)].

B. A downstream control by the existing Dora cross – regulator.

C. Changing the appropriate parameters of the flow in Shatt Al-Hilla, which are:

I. The flow depth.

II. The side slope.

III. The bed width.

IV. The bed slope.

V. Manning's (n).

VI. Some appropriate combinations.

CHAPTER TWO

REVIEW OF LITERATURE

٢.١ *The unsteady flow equations*

Estimates of the flow rate or water level at important locations in a channel system can be obtained through using a distributed flow routing model or hydraulic routing model, [Langendoen, (٢٠٠٤)].

Unsteady or transient flow in rivers may be simulated by two partial differential equations expressing the conservation of mass and momentum; these are the famous Saint – Venant equations (for one – dimensional flow) that allow the flow rate and water level or depth to be computed as functions of space and time. The full Saint – Venant equations are too complicated to be solved analytically. However, they may be solved by numerical techniques which use algebraic finite – difference equations to approximate the partial differential equations.

The derivation of these equations, the continuity equation (conservation of mass) and momentum equation (conservation of momentum) are available in most text books of open channel hydraulics, e.g., [Wiley et al., (١٩٩٨)], [DWOPER, (٢٠٠٢)], [Wasantha Lal, (٢٠٠٤a)], and [Langendoen, (٢٠٠٤)]. The expanded forms of these equations, respectively, are:

$$\frac{\partial \mathbf{Q}}{\partial \mathbf{x}} + \frac{\partial (\mathbf{A} + \mathbf{A}_0)}{\partial \mathbf{t}} \pm \mathbf{q} = \mathbf{0} \qquad \textbf{[2.1]}$$

$$\frac{\partial \mathbf{Q}}{\partial \mathbf{t}} + \frac{\partial (\mathbf{Q}^2 / \mathbf{A})}{\partial \mathbf{x}} + \mathbf{gA}\left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}} + \mathbf{S_f} + \mathbf{S_e}\right] - \mathbf{qV_x} + \mathbf{W_f T} = \mathbf{0} \qquad \textbf{[2.2]}$$

where:

Q: discharge, ($L^{٣}$ / T);

A: cross – sectional area of flow, ($L^{٢}$);

A_o: off–channel cross–sectional area where inflow velocity is considaered negligible, ($L^{٢}$) [and in this research (A_o) is considered negligible];

h: water surface elevation, (L);

$$\mathbf{h = y + z} \qquad [٢.٣]$$

y: water depth, (L);

z: channel bed elevation, (L);

q: lateral inflow (positive) or outflow (negative), ($L^{٣}$ / T / L);

x: distance along the channel, (L);

t: time, (T);

g: gravity – acceleration constant, (L / $T^{٢}$);

V_x: velocity of lateral inflow or outflow in the x-direction which can be calculated by:

$$\mathbf{V_x = Q / A} \qquad [٢.٤]$$

W_f: wind shear force, (F), [which is considered negligible in this research because there is no information or field data about it];

T: channel top width, (L);

S_f: friction slope (dimensionless), defined as:

$$\mathbf{S_f = \frac{n^2 Q^2}{A^2 R^{4/3}}} \qquad [٢.٥]$$

in which:

n: Manning’s roughness coefficient;

R: hydraulic radius, (L);

S_e: eddy loss slope. When the flood plain characteristics are not specified and the total cross – section is treated as a composite section, the term (S_e) is identified as follows [Fread and Lewis, (١٩٩٨)]:

$$S_e = \frac{K_e}{2g}\frac{\delta(Q/A)^2}{\delta x} \qquad [٢.٦]$$

K_e: expansion or contraction coefficient, (dimensionless).

S_e is considered negligible because the cross – section used in the research is a simple section.

The Hydraulic state of unsteady flow in natural rivers is too complicated to be amenable for a direct solution. Consequently, simplifying assumptions are almost always indispensable.

The basic assumptions in this respect are [Chow et al., (١٩٨٨), Wiley et al., (١٩٩٨), and Wasantha Lal, (٢٠٠٤b)]:

١. The flow is one – dimensional (i.e., the velocity is uniform over the cross – section and the water level across the section is horizontal).
٢. The streamline curvature is small and vertical accelerations are negligible, hence the pressure is hydrostatic.
٣. The effects of boundary friction and turbulence can be accounted for through resistance laws analogous to those used with the steady flow.
٤. The channel bed slope is small, so that the cosine of the angle it makes with the horizontal may be replaced by unity.

٢.٢ *Dynamic routing*

Alternative distributed flow routing models can be produced by using the full continuity equation while eliminating some terms of the momentum equation [Wasantha Lal, (٢٠٠٤b)]. These models are kinematic – wave routing, diffusion – wave routing and dynamic – wave routing.

As stated by [Govindaraju et al., (١٩٨٨)]: "Ponce et al., (١٩٧٨) considered the determination of criteria for applicability of the kinematic and diffusion wave models as approximations to the full Saint – Venant (dynamic) equations. They concluded that the dynamic equations have strongly dissipative tendencies and that the diffusion wave model is applicable for a wider range of slopes than the kinematic wave model. The diffusion wave model has the added advantage over the kinematic wave model in that it allows for physical attenuation".

In this research, the dynamic wave routing based on the complete one – dimensional Saint – Venant unsteady flow equations, shall be used as the basic hydraulic routing algorithm for the model. Such a use is first practiced by Stoker, (١٩٥٣) and Isaacson, et al., (١٩٥٦) in their pioneering investigation of flood routing for the Ohio River [Quoted in: Aral et al., (١٩٩٨)].

This choice is based on the ability of the algorithm to provide more accuracy in simulating the unsteady flood wave than the other less complex hydraulic methods, namely, the kinematic – wave method and diffusion – wave method, as the dynamic – wave method is the only method which accounts for the acceleration effects associated with the dam – break wave and the influence of downstream unsteady backwater effects produced by channel constrictions, dams, and tributary inflows or outflows [Fread and Lewis, (١٩٩٨)].

٢.٣ *Numerical solution of Saint – Venant equations*

There are many numerical methods to solve the Saint – Venant equations to give approximate solutions with satisfactory accuracy. The most common ones are: the finite – elements method, the spectral method, the method of characteristics, and the method of finite – differences (explicit or implicit).

Wiley et al., (١٩٩٨) stated that the characteristics method and various finite – difference methods have been used more extensively than other methods to solve the full Saint – Venant equations. Hence, the method of characteristics and finite – difference methods are briefly reviewed hereinafter.

As mentioned in Govindaraju et al., (١٩٨٨), Iwajaki, (١٩٥٥) developed an approximate method for calculation of unsteady flow in open channels using the method of characteristic. He considered abrupt changes in lateral inflows to the streams and studied the resulting effects in stream water depth and discharge. His analysis is, however, restricted to rivers with steep slopes.

Amein, (١٩٦٦) presented the exact numerical solution of the stream flow routing by the method of characteristics using a digital computer. Fletcher and Hamilton (١٩٦٧) used the characteristic method for flood routing within irregular channel. Aral et al., (١٩٩٨) states that: “Bowers, (١٩٧١) applied an explicit scheme to characteristic equations with computational accuracy in first order”.

To solve the Saint – Venant equations by the characteristic method, first Eqs. (٢.١) and (٢.٢) must be written in the characteristic form (Ordinary differential equations):

$$\frac{dQ}{dt} - \left(\frac{Q}{A} \mp \sqrt{gy}\right)\frac{dA}{dt} = gA(S_\circ - S_f) - q * \left(\frac{Q}{A} \mp \sqrt{gy}\right) \quad [٢.٧]$$

With $\frac{dx}{dt} = \frac{Q}{A} \pm \sqrt{gy}$ [٢.٨]

where:

S_o: channel bed slope, (dimensionless).

The method involves the integration of Eq. (٢.٧) along the two sets of characteristic curves given by Eq. (٢.٨). A network of points in the (x, t) plane is then located by the intersection of the forward and backward characteristic curves. This variable mesh characteristic method has the major disadvantages of having to interpolate, to calculate the relevant channel data, as well as to provide results at regular time intervals. The latter difficulty can be overcame by adapting the principles behind the variable mesh characteristic method for a fixed mesh, though suitable interpolation procedures still have to be used to calculate values of the dependent variables on the characteristics curves at the old time level [Price, (١٩٧٤)]. Because of all the above disadvantages, the method of characteristic is not used in this research.

In the finite – differences method, the schemes employ a rectangular grid on the (x – t) plane and may be explicit or implicit, depending on how the time and space derivatives are expressed in terms of the flow variables. A general form can be seen in Fig. (٢.١) which shows the (x – t) plane for the finite – differences method.

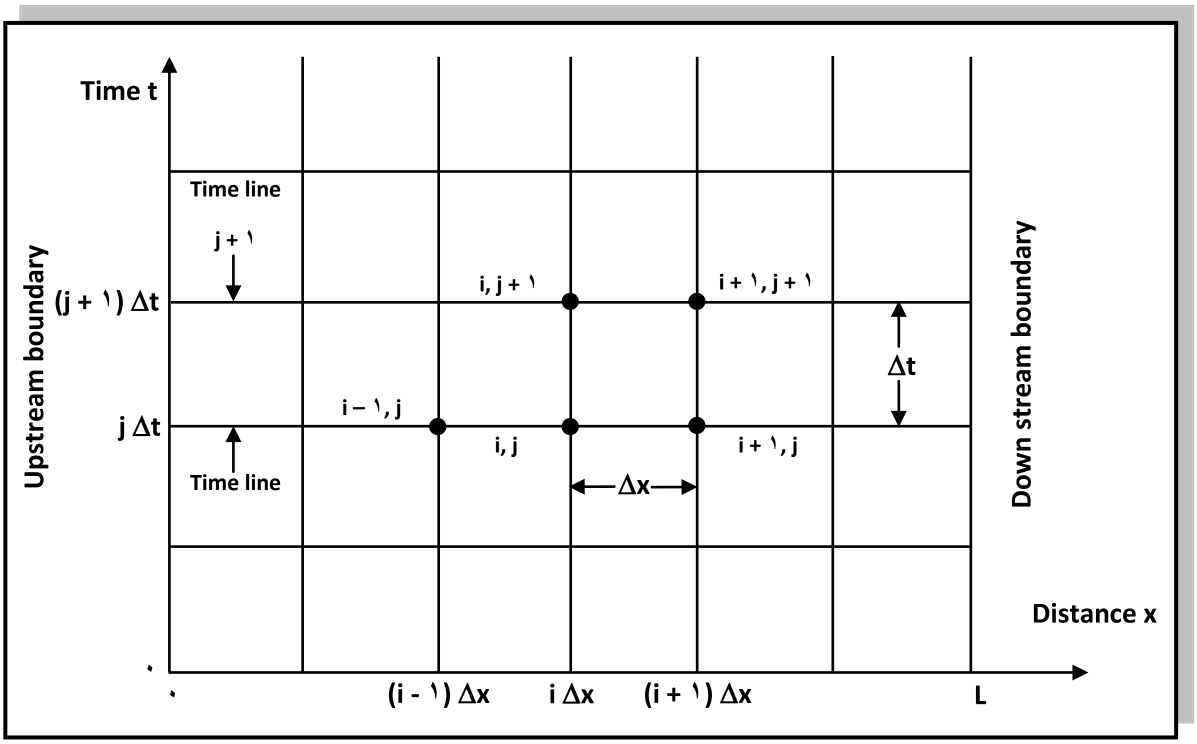

Fig. (٢.١): Network of points on (x – t) plane.

[After: Chow et al., (١٩٨٨)]

Explicit finite – difference schemes express one unknown nodal value at the (j + ١) time line directly in terms of known nodal values along the previous, or jth, time line. The solution proceeds from one unknown point to the next until all values have been found along the current (j + ١ th) time line, then advances to the next time line.

The application of explicit models to flood flows is mostly the outcome of the pioneering work of Stoker, (١٩٥٣). More details and description of this type of models are found in Liggett and Woolhiser, (١٩٦٧).

The analysis of several finite – difference explicit models as applied for routing of rivers and reservoirs are found in Amein and Fang, (١٩٦٩), Garrison et al., (١٩٦٩), Johanson, (١٩٧٤) and Liggett and Cunge, (١٩٧٥).

Navarro and Zorraquino, (١٩٩٣) used the explicit method to solve the Saint – Venant equations for flood propagation through a system of reservoirs.

Tucciarelli, (٢٠٠٣) mentioned that Toro, (١٩٩٢) and Zoppou and Roberts, (١٩٩٩ a,b) used the explicit method to compute the element variables, one element after the other, with the solution of one homogenous and one ordinary differential equations.

The major weakness of the explicit method in its capacity of dealing with "critical" conditions (where the solution easily becomes unstable), is that the time step size is conditioned to the Courant number or, more generally, to the velocity and the wave propagation celerity [Tucciarelli, (٢٠٠٣)].

The Courant stability condition is given by the following inequality [Fread, (١٩٨٢)]:

$$\Delta t \le \frac{\Delta x}{\left| v + \sqrt{gy} \right|}$$ [٢.٩]

The implicit method is very popular because it is unconditionally stable and allows the use of large time steps, although the solution procedure is rather tedious [Huang and Song, (١٩٨٥)].

The basic principle of the fixed mesh implicit model in the numerical solution of the unsteady flow equations is advancing the solution from one time line to the next for all points along the time line. The relationship among the variables result from the equations of unsteady flow after the time and the space partial derivatives of these equations have been replaced by their finite – difference approximations. The finite – difference equations will then constitute a system of nonlinear simultaneous algebraic equations that generally must be solved by iteration.

Aral et al., (١٩٩٨) stated that: "When one reviews numerical methods used in the solution of open – channel network problems, one may see that the implicit finite difference schemes have been utilized extensively by [Liggett and Cunge (١٩٧٥); Terzidis and Strelkoff, (١٩٧٩); Abbott, (١٩٧٩); Cunge et al., (١٩٨٠); Schaffranek, Baltze and Goldberg, (١٩٨١); Fennema and Chaudhry, (١٩٨٧); Choi and Molinas, (١٩٩٣)]".

The implicit schemes may differ according to the way the equations are discretized.

Preissmann, (١٩٦١) introduced the four – point implicit scheme, which sets the time derivative by the central difference and the distance derivative by the weighted average.

Vasiliev, (١٩٦٥) proposed the fully implicit scheme in which both dependent variables are computed at all grid points.

Abbott and Ionescue, (١٩٦٧) proposed a four – point implicit scheme in which the two dependent variables are computed at alternate grid points.

Amein and Fang, (١٩٧٠) applied the box – scheme (which is the four – point central difference implicit scheme) to a natural channel and also compared the results with those obtained by the explicit scheme and the characteristics solution, concluding that their implicit solution has substantial advantages over the other two schemes. Fread, (١٩٧١) presented useful discussion on the previous work of Amein and Fang, (١٩٧٠).

Fread and Harbaurgh, (١٩٧١) used the Newton's iteration technique for computation of surface profiles for steady gradually varied flow.

Fread, (١٩٧٣a) proved that the implicit method appears to be best suited for modeling transient flows with durations in the order of days or weeks such as the natural floods occurring in large river systems. He found that the implicit methods, unlike the other methods, theoretically do not restrict the size of time step because of the numerical stability characteristics of the finite difference equations.

Fread, (١٩٧٣ b) used the weighted four – point implicit scheme to simulate a natural channel with a tributary.

Price, (١٩٧٤) made a comparison between four numerical methods for flood routing and he discovered that the four – point implicit was the most efficient and maintained stability under sever test conditions.

Shayo, (١٩٨٤) proposed the four – point scheme in Rufiji River by using finite – difference technique and the fourth order Runge – Kutta method for solving the system of the resulting equations.

Strelkoff, (١٩٨٥) proposed the full implicit scheme for Chuquatonchec Creek River with simple prismatic geometry. Koehler, (١٩٨٨) followed the same procedure with Columbia River but this time for nonprismatic channel and with lateral inflow.

Johanson, (١٩٩٠) proposed branched full implicit river model in Kentucky Lake.

As stated in [Al-Eoubaidy, (١٩٩٣)]: “Dortoh, Schnrider, Martin, Zimmerrman and Griffin followed the same procedure of Shayo (١٩٨٤) in different streams in Nov. (١٩٩٠) but this time using Newton – Raphson procedure to solve the nonlinear Saint–Venant equations”.

Fread and Lewis, (١٩٩٣) presented a theoretical explanation for the bases of the empirical distance step (Δx) and time step (Δt) selection criteria which is used in the application of unsteady flow models based on the (four – point implicit, nonlinear finite – difference) solution of the complete one – dimensional Saint – Venant equations. The suitability of the selection criteria is demonstrated by using a numerical convergence testing technique for a wide spectrum of unsteady flow applications ranging from rapidly to slowly rising hydrographs in very flat to very steep sloping channels.

Fread and Lewis, (١٩٩٨) developed for the National Weather Service (NWS) the Flood Wave routing model (FLDWAV) which is suitable for efficient operational use in a wide variety of applications involving the prediction of unsteady flows in rivers, reservoirs and estuaries which is based on an implicit (four – point, nonlinear finite – difference) solution of the complete one – dimentional Saint – Venant equations of unsteady flow.

Karim, (١٩٩٨) made a comparison between hydrologic and hydraulic flood routing methods. He used the weighted four – implicit scheme to simulate the flood routing in the Tigris River between Mosul and Baiji. He found that the hydraulic flood routing method gives the most accurate results.

Al-Msaudi, (٢٠٠١) used the weighted four – point implicit scheme to route Shatt Al-Hilla upstream Hilla City.

DWOPER, (٢٠٠٢) introduced DWOPER routing model which is a dynamic wave flood routing model that routes an inflow hydrograph to a point downstream which is applicable to a single river or a system of rivers. DWOPER uses the weighted four – point nonlinear implicit finite – difference scheme with Newton – Raphson iterative technique to obtain solutions for the Saint – Venant equations.

Tucciarelli, (٢٠٠٣) mentioned that Katopodes, (١٩٨٤), Gracia – Navarro et al., (١٩٩٤), and Delis et al., (٢٠٠٠) solved the nodal variables (water levels and flow rates) at the unknown time stage by using the implicit schemes.

Among the several possible formulations of the implicit scheme, the four – point difference scheme, whereby two equations of unsteady flow are applied to the flow occurring between two adjacent cross sections, is the most viable.

The weighted four – point schemes allow a convenient flexibility in embracing large changes in channel geometry, large fluctuations in discharge ranging from abrupt to gradually varied, and a variety of boundary conditions. Application to field problems demonstrates the versatility of the implicit method [Chow et al., (١٩٨٨); Choi and Molinas, (١٩٩٣); Langendoen, (٢٠٠٤)].

Because of its stability and accuracy, the four – point implicit scheme has been chosen for use in this research.

٢.٤. *Hydraulic control of open – channel flow*

In general, control of open channel flow could be an upstream control, a downstream control, some control measures within the considered channel reach, or a combination of the aforementioned ones. Hydraulic structures are extensively used as controls or transitions, changing local rating curves (Langendoen, ٢٠٠٤).

Cunge and Woolhiser, (١٩٧٥) used the movable gates to make control upstream and downstream an open channel. They established an efficient mathematical model to simulate the flow under the gate (submerged or free).

Fread and Lewis, (١٩٩٨) introduced a mathematical model for many of the hydraulic structures like dams, spillways, gates, etc…, as internal boundaries in which the Saint – Venant equations are not applicable.

Wei et al., (١٩٩٩) presented a computer simulation model for unsteady flow in canal system and the hydraulic structures constructed on it for operation purposes, where they treated the hydraulic structures as two types; division structures, including sluice gate, weir and pump, and control structures, including sluice gate and weir.

Al-Msaudi, (٢٠٠١) made a hydraulic control on Shatt Al-Hilla upstream Hilla City by using a cross – regulator with (٦) gates downstream the reach to raise the water elevation for irrigation purposes.

Langendoen, (٢٠٠٤) gave very efficient mathematical models to simulate culverts, bridge crossing, gates, drop structures and generic structures (which can be any structure for which a rating curve is available).

CHAPTER THREE

FOMULATION OF THE MATHEMATICAL MODEL

٣.١. *The implicit four – point method*

As mentioned in chapter two, the four point implicit finite-difference scheme has been adopted for use in this research.

In the implicit method for solving partial differential equations, the calculations are performed on a grid placed over the (x – t) plane. The (x – t) grid is a network of points defined by taking distance increments of length (Δx) and time increments of duration (Δt). As shown in Fig. (٣ – ١), the distance points are denoted by index (i) and the time points by index (j). A time line is a line parallel to the x-axis through all the distance points at a given value of time.

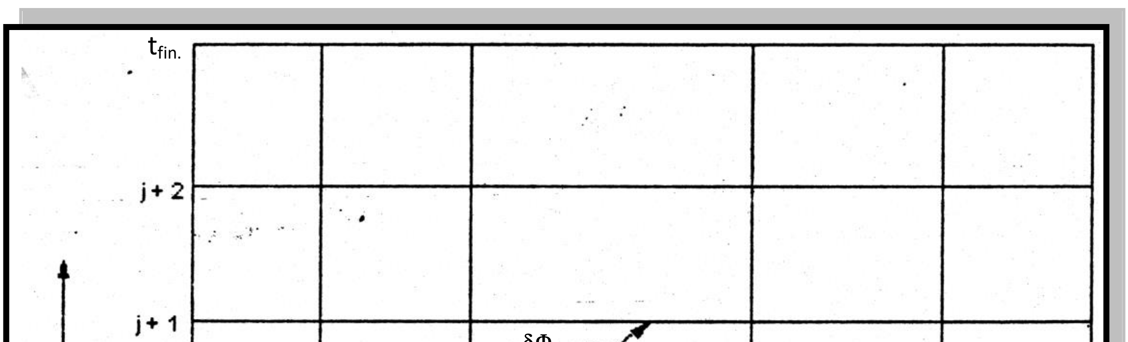

Fig. (٣.١): The (x – t) solution plane,

[After DWOPER, (٢٠٠٢)]

The t – axis, where (x = ٠) may be used as the upstream channel boundary location. The last line drawn parallel to the t – axis, denoted the Nth – line, can be used to represent the downstream boundary location (N is the total number of cross – sections).

The unsteady flow equations may be solved numerically by performing two basic steps. First, the partial differential equations are represented by a corresponding set of approximate finite-difference algebraic equation; second, the system of algebraic equations is solved in conformance with prescribed initial and boundary conditions. [Fread and Lewis (١٩٩٨)].

A function (Φ) [where Φ represents any variable (Q, h, A, etc.)] in the intervals (i, i+١) and (j, j +١) may be replaced by its weighted average between these intervals [Chow et al., (١٩٨٨); Fread and Lewis, (١٩٩٨) and DWOPER, (٢٠٠٢)].

For the time interval $\left(t_j \leq t \leq t_{j+1}\right)$:

$$\Phi(x_i,t) = \theta\Phi_i^{j+1} + (1-\theta)\Phi_i^j \qquad [٣.١]$$

$$\Phi(x_{i+1},t) = \theta\Phi_{i+1}^{j+1} + (1-\theta)\Phi_{i+1}^j \qquad [٣.٢]$$

i = ١, ٢, ... , N; j =١, ٢, ..., $t_{fin.}$.

where:

$t_{fin.}$= final time step of a certain prescribed time interval.

θ = weighting factor for the two time positions of an x – location.

and for the space interval $(x_i \le x \le x_{i+1})$:

$$\Phi(x,t_j) = \psi\Phi_{i+1}^j + (1-\psi)\,\Phi_i^j \qquad [٣.٣]$$

$$\Phi(x,t_j+1) = \psi\Phi_{i+1}^{j+1} + (1-\psi)\,\Phi_i^{j+1} \qquad [٣.٤]$$

where: ψ = weighting factor of two x – locations along a single time line.

The equations (٣.١) and (٣.٢) indicate that the value of the variable (Φ) at any (x) location is the weighted average of its value at two consecutive times (j) and (j + ١) at the particular location. Similarly, equations (٣.٣) and (٣.٤) indicate that the value of (Φ) at any time is the weighted average of its value at two neighboring locations (i) and (i+١).

The time and space derivatives of Φ become:

$$\frac{\partial \Phi}{\partial t} = \frac{\Phi(x,t^{j+1}) - \Phi(x,t)^{j}}{\Delta t}$$

$$= \frac{\psi(\Phi_{i+1}^{j+1} - \Phi_{i+1}^{j}) + (1-\psi)(\Phi_{i}^{j+1} - \Phi_{i}^{j})}{\Delta t}$$ [٣.٥]

$$\frac{\partial \Phi}{\partial x} = \frac{\Phi(x_{i+1},t) - \Phi(x_{i},t)}{\Delta x}$$

$$= \frac{\theta(\Phi_{i+1}^{j+1} - \Phi_{i}^{j+1}) + (1-\theta)(\Phi_{i+1}^{j} - \Phi_{i}^{j})}{\Delta x}$$ [٣.٦]

When ($\psi = \frac{1}{2}$), the system of equations (٣.١) to (٣.٦) becomes the Preissman four – point scheme whereby the time derivative is [Chow et al., (١٩٨٨); Fread and Lewis, (١٩٩٨) and DWOPER, (٢٠٠٢)]:

$$\frac{\partial \Phi}{\partial t} = \frac{\Phi_{i+1}^{j+1} + \Phi_{i}^{j+1} - \Phi_{i+1}^{j} - \Phi_{i}^{j}}{2\Delta t}$$ [٣.٧]

The value of (Φ) is given by:

$$\Phi = \frac{\theta}{2}(\Phi_{i+1}^{j+1} + \Phi_{i}^{j+1}) + \frac{(1-\theta)}{2}(\Phi_{i+1}^{j} + \Phi_{i}^{j})$$ [٣.٨]

The weighting factor (θ) is usually given a value between (٠.٥) and (١.٠) [Amein and Fang, (١٩٧٠)]. A (θ) weighting factor of (١.٠) yields the fully implicit or backward difference scheme used by Baltzer and Lai, (١٩٦٨). A weighting factor of (٠.٥) yields the box scheme used by Amein and Fang, (١٩٧٠). The influence of the (θ) weighting factor on the accuracy of the computations was examined by Fread, (١٩٧٤), who concluded that the accuracy tends to somewhat decrease as (θ) departs from (٠.٥) and approaches (١.٠).

Al-Eoubaidy (١٩٩٣) used (θ) as (٠.٥), (٠.٧٥) and (١.٠) in his mathematical model for the whole length (١٠١ km) of Shatt Al-Hilla; he found that (θ = ١.٠) gives the suitable accuracy. However, Al-Msaudi, (٢٠٠١) considered Shatt Al-Hilla upstream of Hilla City; on examining (θ) values in the range (٠.٤٩ – ١.٠), found that (θ = ٠.٩٥) was more accurate. The choosing of the value of (θ) will be discussed later in chapter four.

٣.٢. *The Saint–Venant equations in finite–difference forms*

On using the area of flow, (A), neglecting the off – channel cross – sectional area, (A_o), and replacing, ($\frac{\delta A}{\delta t}$) by ($T\frac{\delta y}{\delta t}$), Eq. (٢.١) will read as:

$$\frac{\partial Q}{\partial x}+T\frac{\partial y}{\partial t}\mp q=0 \qquad [٣.٩]$$

Moreover, by neglecting eddy losses and wind shear effect, Eq. (٢.٢) will become:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\left(\frac{\partial(y+z)}{\partial x}+S_f\right)\pm q\frac{Q}{A}=0 \qquad [٣.١٠]$$

When the finite-difference operators defined by Eqs. (٣.٥ – ٣.٨) are used to replace the derivatives and other variables in Eqs. (٣.٩) and (٣.١٠), the following weighted, four – point implicit, finite – difference equations are obtained:

Multiplying by ($\frac{\Delta t}{T_i^{j+1}}$), the continuity equation becomes, [Mohammed, (١٩٩٣); Al-Eoubaidy, (١٩٩٣) and Nagar, (١٩٩٩)]:

$$C_i = \frac{\Delta t}{T_i^{j+1}\,\Delta x}\left[\theta\left(Q_{i+1}^{j+1} - Q_i^{j+1}\right) + (1-\theta)\left(Q_{i+1}^{j} - Q_i^{j}\right)\right]$$

$$+\frac{1}{2}\left[\frac{T_{i+1}^{j+1}}{T_i^{j+1}}\left(y_{i+1}^{j+1} - y_{i+1}^{j}\right) + \left(y_i^{j+1} - y_i^{j}\right)\right] \pm q\,\frac{\Delta t}{T_i^{j+1}} = 0 \qquad [٣.١١]$$

Multiplying by $(\frac{\Delta x}{g})$, the momentum equation becomes, [Mohammed, (١٩٩٣); Al-Eoubaidy, (١٩٩٣) and Nagar, (١٩٩٩)]:

$$M_i = 0.5\,\frac{\Delta x}{\Delta t\,g}\left[\frac{Q_{i+1}^{j+1}}{A_{i+1}^{j+1}} + \frac{Q_i^{j+1}}{A_i^{j+1}} - \frac{Q_{i+1}^{j}}{A_{i+1}^{j}} - \frac{Q_i^{j}}{A_i^{j}}\right]$$

$$+\frac{1}{2g}\left[\theta\left(\left(\frac{Q_{i+1}^{j+1}}{A_{i+1}^{j+1}}\right)^2 - \left(\frac{Q_i^{j+1}}{A_i^{j+1}}\right)^2\right) + (1-\theta)\left(\left(\frac{Q_{i+1}^{j}}{A_{i+1}^{j}}\right)^2 - \left(\frac{Q_i^{j}}{A_i^{j}}\right)^2\right)\right]$$

$$+q\,\frac{\Delta x}{2g}\left[\theta\left(\frac{Q_{i+1}^{j+1}}{(A_{i+1}^{j+1})^2} - \frac{Q_i^{j+1}}{(A_i^{j+1})^2}\right) + (1-\theta)\left(\frac{Q_{i+1}^{j}}{(A_{i+1}^{j})^2} - \frac{Q_i^{j}}{(A_i^{j})^2}\right)\right] \qquad [٣.١٢]$$

$$+\Delta x\left[\begin{array}{l}\theta\left(y_{i+1}^{j+1} + z_{i+1}^{j+1} - y_i^{j+1} - z_i^{j+1} + \overline{S}_f^{j+1}\right) + \\ (1-\theta)\left(y_{i+1}^{j} + z_{i+1}^{j} - y_i^{j} - z_i^{j} + \overline{S}_f^{j}\right)\end{array}\right]$$

where:

$$\overline{S}_f = \frac{n^2\,\overline{Q}_i^2}{\overline{A}_i^2\,\overline{R}_i^{4/3}}$$

[٣.١٣] $$\overline{A}_i = \frac{A_i + A_{i+1}}{2}$$

[٣.١٤] $$\overline{Q}_i = \frac{Q_i + Q_{i+1}}{2}$$

[٣.١٥] $$\overline{P}_i = \frac{P_i + P_{i+1}}{2}$$

[٣.١٦] $$\overline{R}_i = \frac{\overline{A}_i}{\overline{P}_i}$$

[٣.١٧]

P_i = wetted perimeter, (L).

Equations (٣.١١) and (٣.١٢) constitute a system of nonlinear partial differential equations with two independent variables, x and t, and two dependent variables, y and Q; the remaining terms are either functions of x, t, y, and / or Q, or they are constants.

The terms associated with the j^{th} time line are known from either the initial conditions or previous computations from a solution of the Saint – Venant equations.

Equations (٣.١١) and (٣.١٢) cannot be solved in an explicit or direct manner for the unknowns since there are four unknowns, $\mathbf{Q_i^{j+1}}, \mathbf{y_i^{j+1}}, \mathbf{Q_{i+1}^{j+1}}$ and $\mathbf{y_{i+1}^{j+1}}$, whereas only two equations are available. However, if Eqs. (٣.١١) and (٣.١٢) are applied to each of the (N – ١) rectangular grids shown in Fig. (٣.١) between the upstream and downstream boundaries, a total of (٢N – ٢) equations with ٢N unknowns can be formulated. (N denotes the total number of nodes or cross – sections). Then, prescribed boundary conditions for subcritical flows, one at the upstream boundary and one at the downstream boundary, provide the necessary two additional equations required for the system to be determinate (same number of equations and number of unknowns).

The upstream boundary condition can be specified as one of the following (DWOPER, ٢٠٠٢):

١. Known stage (water surface elevation) hydrograph,

$$\mathbf{h_i^{j+1} = h(t)} \quad [٣.١٨]$$

in which h(t) represents a time series of water – surface elevation at each time (t).

٢. Known discharge hydrograph:

$$Q_i^{j+1} = Q(t)$$ [٣.١٩] in which Q(t) represents a time series of discharge (flow) at each time (t).

The feasible options for the downstream boundary condition are [Fread and Lewis, (١٩٩٨)]:

١. Single – value rating:

$$Q_N^{j+1} = Q(h)$$ [٣.٢٠] in which Q(h) represents a tabular relation of Q and h (rating curve).

٢. Generated dynamic loop-rating using the Manning equation with a dynamic energy slope term (S) computed by one of two options:

$$Q_N^{j+1} = \frac{1}{n} A_N^{j+1} R_N^{j+12/3} S_{f\,N-1}^{1/2} = K_N^{j+1} S_{f\,N-1}^{1/2}$$ [٣.٢١] where:

$$S_{f\,N-1} = \frac{h_{N-1} - h_N}{\Delta x_{N-1}} + \frac{(Q'_N - Q_N)}{0.5g(A_N + A_{N-1})\Delta t} + \frac{(Q_{N-1}^2 / A_{N-1} - Q_N^2 / A_N)}{0.5g(A_N + A_{N-1})\Delta x_{N-1}}$$ [٣.٢٢] or:

$$S_{f\,N-1} = \frac{n^2 \overline{Q}^2}{\overline{A}\,\overline{R}^{4/3}}$$ [٣.٢٣] in which $Q'N$ is the discharge at time $(t_j + 1)$ whereas all other terms in the equations are the jth time, and $\overline{A}, \overline{Q}, \overline{R}$ are reach average values for the N – ١ reach according to Eqs. (٣.١٣ – ٣.١٧).

٣. Generated single – value rating in which Eq. (٣.٢١) is used, but S is used as the channel bottom slope in the vicinity of the Nth cross section.

٤. Critical flow rating that occurs at a waterfall or beginning of short, steep rapids:

$$Q_N^{j+1} = \left[g (A_N^{j+1})^3 / T_N^{j+1} \right]^{0.5} \quad [٣.٢٤]$$

٥. Water – surface elevation time series:

$$h_N^{j+1} = h(t) \quad [٣.٢٥]$$ in which h (t) represents a time series of water – surface elevation at each time (t) at the N^{th} cross section.

٦. Discharge time series:

$$Q_N^{j+1} = Q(t) \quad [٣.٢٦]$$ in which Q (t) represents a time series of discharge at each time (t) at the N^{th} cross section.

If a channel control exists, i.e., the flow at section (N) is controlled by the channel properties, then either Eq. (٣.٢٠) or (٣.٢١) can be selected. Equation (٣.٢٠) is useful if an empirical Q(h) relation is available which is essentially single – valued, i.e., for each water–surface elevation there is only one discharge. When a known Q(h) relation does not exist, option (٣) can be used; however, when the relation is not single – valued, then dynamic loop – rating, Eq. (٣.٢١), may be used.

The loop – rating allows two water-surface elevations to exist for each discharge value. On the rising limb of the hydrograph, the water-surface elevation is usually less than that which occurs for the same discharge on the recession limb [Fread and Lewis, (١٩٩٨)].

Water–surface elevation time series may be used when the downstream boundary is located in a wide estuary or bay where the water–surface elevation is controlled only by the tidal fluctuation and not by the flow emanating from the upstream routing reach [Fread and Lewis, (١٩٩٨)].

When flow is supercritical throughout the entire routing reach of channel, the solution technique previously described can be somewhat simplified. Instead of a solution involving (٢N × ٢N) equations, supercritical flow can be solved via a system of only (٢ × ٢) equations. The unknown y and Q at the upstream section are determined from the two boundary equations. Then progressing from upstream to downstream in a cascading manner, Eqs. (٣.١١) and (٣.١٢), being nonlinear with respect to $\mathbf{y_{i+1}}$ and $\mathbf{Q_{i+1}}$, are solved by the Newton – Raphson iterative technique applied to a system of two equations with two unknowns. For supercritical flow, this technique provides a somewhat more stable solution than one involving (٢N × ٢N) equations [Fread and Lewis, (١٩٩٨)]. Therefore, Froude number must be calculated to know the type of flow (subcritical or supercritical) for the selection of the boundary conditions.

٣.٣. *Solution of the unsteady flow finite-difference equations*

The resulting system of ٢N nonlinear equations from the application of the continuity equation (C_i) and the momentum equation (M_i) to the flow to (N – ١) cross sections with the unknowns $\mathbf{y_i^{j+1}}, \mathbf{y_{i+1}^{j+1}}, \mathbf{Q_i^{j+1}}$ and $\mathbf{Q_{i+1}^{j+1}}$ can be expressed as:

$$
\begin{array}{ll}
UB\,(y_1, Q_1) = 0 & \text{Upstream boundary condition} \\
C_1\,(y_1, Q_1, y_2, Q_2) = 0 & \text{Continuity for grid 1} \\
M_1\,(y_1, Q_1, y_2, Q_2) = 0 & \text{Momentum for grid 1} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\
C_i\,(y_i, Q_i, y_{i+1}, Q_{i+1}) = 0 & \text{Continuity for grid i} \\
M_i\,(y_i, Q_i, y_{i+1}, Q_{i+1}) = 0 & \text{Momentum for grid i} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\
C_{N-1}\,(y_{N-1}, Q_{N-1}, y_N, Q_N) = 0 & \text{Continuity for grid } N-1 \\
M_{N-1}\,(y_{N-1}, Q_{N-1}, y_N, Q_N) = 0 & \text{Momentum for grid } N-1 \\
DB\,(y_N, Q_N) = 0 & \text{Downstream boundary condition}
\end{array}
\qquad [٣.٢٧]
$$

In this research, this system of ٢N nonlinear equations in ٢N unknowns is solved for each time step by the Newton – Raphson iterative method which was first applied by Amein and Fang, (١٩٧٠) to an implicit nonlinear formulation of the Saint – Venant equations. This procedure was used too by [Chow et al., (١٩٨٨); Al-Eoubaidy, (١٩٩٣); Mohammed, (١٩٩٣); Fread and Lewis, (١٩٩٨); Al-Msaudi, (٢٠٠١) and DWOPER, (٢٠٠٢)].

The computational procedure for each time (j + l) starts by assigning trial values to the ٢N unknowns at that time. These trial values of Q and y can be the values known at time j (from the initial condition if j = ١) or from calculations during the previous time step. Using the trial values given by Eqs. (٣.٢٧) result in ٢N residuals. For the k^{th} iteration, the respective residuals concerning the upstream boundary condition, RUB, and the downstream boundary condition, RDB, can be expressed as:

$$\mathbf{UB\,(y_1^k, Q_1^k) = RUB^k}$$ **residual for upstream boundary condition**

$$\mathbf{C_1\,(y_1^k, Q_1^k, y_2^k, Q_2^k) = RC_1^k}$$ **residual for continuity at grid 1**

$$\mathbf{M_1\,(y_1^k, Q_1^k, y_2^k, Q_2^k) = RM_1^k}$$ **residual for momentum at grid 1**

…………………………………

…………………………………

$$\mathbf{C_i\,(y_i^k, Q_i^k, y_{i+1}^k, Q_{i+1}^k) = RC_i^k}$$ **residual for continuity at grid i** **[3.28]**

$$\mathbf{M_i\,(y_i^k, Q_i^k, y_{i+1}^k, Q_{i+1}^k) = RM_i^k}$$ **residual for momentum at grid i**

……………………………………

……………………………………

$$\mathbf{C_{N-1}\,(y_{N-1}^k, Q_{N-1}^k, y_N^k, Q_N^k) = RC_{N-1}^k}$$ **residual for continuity at grid (N – 1)**

$$\mathbf{M_{N-1}\,(y_{N-1}^k, Q_{N-1}^k, y_N^k, Q_{iN}^k) = RM_{N-1}^k}$$ **residual for momentum at grid (N – 1)**

$$\mathbf{DB\,(y_N^k, Q_N^k) = RDB^k}$$ **residual for downstream boundary condition**

The solution is approached by finding values of the unknowns Q and y so that the residuals are forced to zero or very close to zero (tolerable limits).

Taking the partial derivatives of C_i and M_i with respect to the four unknowns (y and Q at (i) and (i + ١) sections) in (j + ١) time, the result would be [Nagar, (١٩٩٩)]:

$$\frac{\partial C_i}{\partial y_i^{j+1}} = 0.5 - 0.5\,\frac{T_{i+1}^{j+1}}{(T_i^{j+1})^2}\,(y_{i+1}^{j+1} - y_{i+1}^{j})\left(\frac{\partial T}{\partial y}\right)_i^{j+1} - \frac{\Delta t}{\Delta x (T_i^{j+1})^2}\left[\theta (Q_{i+1}^{j+1} - Q_i^{j+1}) + (1-\theta)\,(Q_{i+1}^{j} - Q_i^{j})\right] \pm q\,\frac{\Delta t}{\Delta x (T_i^{j+1})^2}\left(\frac{\partial T}{\partial y}\right)_i^{j+1} \qquad [٣.٢٩]$$

$$\frac{\partial C_i}{\partial y_{i+1}^{j+1}} = 0.5\,\frac{T_{i+1}^{j+1}}{T_i^{j+1}} + \frac{0.5}{T_i^{j+1}}\,(y_{i+1}^{j+1} - y_{i+1}^{j})\left(\frac{\partial T}{\partial y}\right)_{i+1}^{j+1} \qquad [٣.٣٠]$$

$$\frac{\partial C_i}{\partial Q_i^{j+1}} = -\frac{\Delta t}{\Delta x}\left(\frac{\theta}{T_i^{j+1}}\right) \qquad [٣.٣١]$$

$$\frac{\partial C_i}{\partial Q_{i+1}^{j+1}} = \frac{\Delta t}{\Delta x}\left(\frac{\theta}{T_i^{j+1}}\right) \qquad [٣.٣٢]$$

$$\frac{\partial M_i}{\partial y_i^{j+1}} = -0.5\,\frac{\Delta x}{g.\Delta t}\left(\frac{Q_i^{j+1}}{(A_i^{j+1})^2}\right)\left(\frac{\partial A}{\partial y}\right)_i^{j+1} - \frac{\theta}{g}\left(\frac{(Q_i^{j+1})^2}{(A_i^{j+1})^3}\right)\left(\frac{\partial A}{\partial y}\right)_i^{j+1} + \theta + \Delta x \left(\frac{S_{fi}^{j+1}}{A_i^{j+1}}\right)\left(\frac{\partial A}{\partial y}\right)_i^{j+1} + \frac{2}{3}\Delta x\left(\frac{S_f}{R}\right)_i^{j+1}\left(\frac{\partial R}{\partial y}\right)_i^{j+1} \mp q\; T_i^{j+1}\left(\frac{\Delta x.Q_i^{j+1}}{g(A_i^{j+1})^3}\right) \qquad [٣.٣٣]$$

$$\frac{\partial M_i}{\partial y_{i+1}^{j+1}} = -0.5\,\frac{\Delta x}{g.\Delta t}\left(\frac{Q_{i+1}^{j+1}}{(A_{i+1}^{j+1})^2}\right)\left(\frac{\partial A}{\partial y}\right)_{i+1}^{j+1} - \frac{\theta}{g}\left(\frac{(Q_{i+1}^{j+1})^2}{(A_{i+1}^{j+1})^3}\right)\left(\frac{\partial A}{\partial y}\right)_{i+1}^{j+1} + \theta + \Delta x \left(\frac{S_{f\,i+1}^{j+1}}{A_{i+1}^{j+1}}\right)\left(\frac{\partial A}{\partial y}\right)_{i+1}^{j+1} + \frac{2}{3}\Delta x\left(\frac{S_f}{R}\right)_{i+1}^{j+1}\left(\frac{\partial R}{\partial y}\right)_{i+1}^{j+1} \mp q\; T_{i+1}^{j+1}\left(\frac{\Delta x.Q_{i+1}^{j+1}}{g(A_{i+1}^{j+1})^3}\right) \qquad [٣.٣٤]$$

$$\frac{\partial M_i}{\partial Q_i^{j+1}} = 0.5\frac{\Delta x}{\Delta t}\left(\frac{1}{g\,A_{i+1}^{j+1}}\right) + \frac{Q_{i+1}^{j+1}.\theta}{g\left(A_{i+1}^{j+1}\right)^2} - 2\left(\frac{S_{f\,i}^{j+1}}{Q_i^{j+1}}\right) \pm \frac{1}{2}\,q\left(\frac{\Delta x}{g\left(A_i^{j+1}\right)^2}\right) \qquad [٣.٣٥]$$

$$\frac{\partial M_i}{\partial Q_{i+1}^{j+1}} = 0.5\frac{\Delta x}{\Delta t}\left(\frac{1}{g\,A_{i+1}^{j+1}}\right) + \frac{Q_{i+1}^{j+1}.\theta}{g\left(A_{i+1}^{j+1}\right)^2} - 2\left(\frac{S_{f\,i+1}^{j+1}}{Q_{i+1}^{j+1}}\right) \pm \frac{1}{2}\,q\left(\frac{\Delta x}{g\left(A_i^{j+1}\right)^2}\right) \qquad [٣.٣٦]$$

where:

$$\frac{\partial A}{\partial y} = T(y) \qquad [٣.٣٧]$$

If the values of A(y) and T(y) are obtained by field measurements, the measurements errors may cause T(y) to be different from the values obtained by differentiating the area A(y) with respect to depth. For numerical stability, it is important that A(y) and T(y) should be compatible. Therefore, if either A(y) or T(y) is obtained by measurement, the other should be determined by calculus [Amein and Fang, (١٩٧٠)].

The partial derivatives of (S_f) would be:

$$\frac{\partial \mathbf{S_f}}{\partial \mathbf{y}} = -2\mathbf{S_f}\left[\frac{(2/3)}{\mathbf{R}}\left(\frac{\partial \mathbf{R}}{\partial \mathbf{y}}\right) + \frac{\mathbf{T}}{\mathbf{A}}\right] \qquad [٣.٣٨]$$

$$\frac{\partial \mathbf{S_f}}{\partial \mathbf{Q}} = \frac{2\mathbf{S_f}}{\mathbf{Q}} \qquad [٣.٣٩]$$

$$\mathbf{R} = \frac{\mathbf{A}}{\mathbf{P}} \qquad [٣.٤٠]$$

$$\frac{\partial \mathbf{R}}{\partial \mathbf{y}} = \left(\frac{\mathbf{T}}{\mathbf{P}} - \frac{\mathbf{A}}{\mathbf{P}^2}\left(\frac{\partial \mathbf{P}}{\partial \mathbf{y}}\right)\right) \qquad [٣.٤١]$$

Solving by the Newton – Raphson method necessitates an iterative process. The partial derivatives of the system, [Eqs. (٣.٢٧)], are related to the residuals of Eqs. (٣.٢٨) by the following relationships [Nagar, (١٩٩٩)]:

$$
\left.
\begin{aligned}
&\frac{\partial UB}{\partial y_1}\ dy_1 + \frac{\partial UB}{\partial Q_1}\ dQ_1 = -RUB^k \\
&\frac{\partial C_1}{\partial y_1}\ dy_1 + \frac{\partial C_1}{\partial Q_1}\ dQ_1 + \frac{\partial C_1}{\partial y_2}\ dy_2 + \frac{\partial C_1}{\partial Q_2}\ dQ_2 = -RC_1^k \\
&\frac{\partial M_1}{\partial y_1}\ dy_1 + \frac{\partial M_1}{\partial Q_1}\ dQ_1 + \frac{\partial M_1}{\partial y_2}\ dy_2 + \frac{\partial M_1}{\partial Q_2}\ dQ_2 = -RM_1^k \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&\frac{\partial C_i}{\partial y_i}\ dy_i + \frac{\partial C_i}{\partial Q_i}\ dQ_i + \frac{\partial C_i}{\partial y_{i+1}}\ dy_{i+1} + \frac{\partial C_i}{\partial Q_{i+1}}\ dQ_{i+1} = -RC_i^k \\
&\frac{\partial M_i}{\partial y_i}\ dy_i + \frac{\partial M_i}{\partial Q_i}\ dQ_i + \frac{\partial M_i}{\partial y_{i+1}}\ dy_{i+1} + \frac{\partial M_i}{\partial Q_{i+1}}\ dQ_{i+1} = -RM_i^k \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&\frac{\partial C_{N-1}}{\partial y_{N-1}}\ dy_{N-1} + \frac{\partial C_{N-1}}{\partial Q_{N-1}}\ dQ_{N-1} + \frac{\partial C_{N-1}}{\partial y_N}\ dy_N + \frac{\partial C_{N-1}}{\partial Q_N}\ dQ_N = -RC_{N-1}^k \\
&\frac{\partial M_{N-1}}{\partial y_{N-1}}\ dy_{N-1} + \frac{\partial M_{N-1}}{\partial Q_{N-1}}\ dQ_{N-1} + \frac{\partial M_{N-1}}{\partial h_N}\ dy_N + \frac{\partial M_{N-1}}{\partial Q_N}\ dQ_N = -RM_{N-1}^k \\
&\frac{\partial DB}{\partial h_N}\ dy_N + \frac{\partial DB}{\partial Q_N}\ Qy_N = -RDB^k
\end{aligned}
\right]
\qquad [3.42]
$$

For the k^th iteration cycle (k = ١, ٢, ..., K'); K' is the iteration cycle at which the residuals become zero or very close to zero (tolerable limits), the partial derivatives are evaluated as follows:

$$
\left.
\begin{aligned}
&dy_1 = y_1^{k+1} - y_1^k\ ,\ dQ_1 = Q_1^{k+1} - Q_1^k \\
&dy_i = y_i^{k+1} - y_i^k\ ,\ dQ_i = Q_i^{k+1} - Q_i^k \\
&dy_N = y_N^{k+1} - y_N^k\ ,\ dQ_N = Q_N^{k+1} - Q_N^k
\end{aligned}
\right]
\qquad [٣.٤٣]
$$

This could be expressed in matrix notation as:

$$
\mathbf{J\ X = R} \qquad [٣.٤٤]
$$

or:

$$
\mathbf{J}=\begin{bmatrix}
\frac{\partial \mathbf{UB}}{\partial \mathbf{y}_1} & \frac{\partial \mathbf{UB}}{\partial \mathbf{y}_1} & & & & & & & \\
 & \frac{\partial \mathbf{C}_1}{\partial \mathbf{y}_1} & \frac{\partial \mathbf{C}_1}{\partial \mathbf{Q}_1} & \frac{\partial \mathbf{C}_1}{\partial \mathbf{y}_2} & \frac{\partial \mathbf{C}_1}{\partial \mathbf{Q}_2} & & & & \\
 & \frac{\partial \mathbf{M}_1}{\partial \mathbf{y}_1} & \frac{\partial \mathbf{M}_1}{\partial \mathbf{Q}_1} & \frac{\partial \mathbf{M}_1}{\partial \mathbf{y}_2} & \frac{\partial \mathbf{M}_1}{\partial \mathbf{Q}_2} & & & & \\
 & & & \frac{\partial \mathbf{C}_2}{\partial \mathbf{y}_2} & \frac{\partial \mathbf{C}_2}{\partial \mathbf{Q}_2} & \frac{\partial \mathbf{C}_2}{\partial \mathbf{y}_3} & \frac{\partial \mathbf{C}_2}{\partial \mathbf{Q}_3} & & \\
 & & & \frac{\partial \mathbf{M}_2}{\partial \mathbf{y}_2} & \frac{\partial \mathbf{M}_2}{\partial \mathbf{Q}_2} & \frac{\partial \mathbf{M}_2}{\partial \mathbf{y}_3} & \frac{\partial \mathbf{M}_2}{\partial \mathbf{Q}_3} & & \\
 & & & & & * & * & * & * \\
 & & & & & * & * & * & * \\
 & & & & & & & \frac{\partial \mathbf{DB}}{\partial \mathbf{y}_N} & \frac{\partial \mathbf{DB}}{\partial \mathbf{Q}_N}
\end{bmatrix}
$$

$$
\mathbf{X}=\begin{bmatrix}
\mathbf{dy}_1 \\ \mathbf{dQ}_1 \\ \mathbf{dy}_2 \\ \mathbf{dQ}_2 \\ . \\ . \\ . \\ \mathbf{dy}_N \\ \mathbf{dQ}_N
\end{bmatrix}
\quad ,\mathbf{R}=\begin{bmatrix}
-\mathbf{RUB} \\ -\mathbf{RC}_1 \\ -\mathbf{RM}_1 \\ -\mathbf{RC}_2 \\ -\mathbf{RM}_2 \\ . \\ . \\ . \\ -\mathbf{RC}_{2-1} \\ -\mathbf{RM}_{N-1} \\ -\mathbf{RDB}
\end{bmatrix}
$$

where:

J: The Jacobian (a coefficient matrix whose elements are the partial derivatives evaluated at the k^{th} iteration cycle).

X: A column vector of the (٢N) unknowns: $dy_{١}$, $dQ_{١}$, $dy_{٢}$, $dQ_{٢}$, …, dy_N, dQ_N.

R: Vector of residuals.

All the partial derivatives are evaluated at the k^{th} iteration cycle. The system of Eqs. (٣.٤٢) is a system of ٢N linear equations in ٢N unknowns and they can be solved by Gaussian elimination or the matrix inversion method or any other standard method. The solution of the system provides the values of the variables (y_i) and (Q_i) at the $(k + ١)^{th}$ iteration cycle.

The Jacobin (coefficient) matrix, J, of the linear system has a banded structure which allows the system to be solved by a compact, quad diagonal, Gaussian elimination algorithm (Fread, ١٩٧١) which is very efficient with respect to computing time and storage. The required matrix's storage is reduced from ٢N × ٢N to ٢N × ٤ and the required number of computational steps is greatly reduced (from $^{١٦}/_{٣}N^{٣} + ٨N^{٢} + ^{١٤}/_{٣}N$ to approximately ٣٨N) [Fread and Lewis, (١٩٩٨)].

٣.٤. The elements of the model

The river system is schematized as reaches connecting cross – sections. A reach is a stream segment that transfers information between two cross – sections. Some of these cross – sections considered nodes that hold hydraulic information. The information transfer between nodes tells how the flow at one cross–section relates to the flow at its upstream and downstream cross – sections, and these are the elements of the model:

٣.٤.١. *Reaches*

For the purposes of the research, the river system is divided into a number of reaches (which may be of unequal lengths). A reach contains a minimum of three cross – sections. It begins and ends with a node. The reaches are governed by the continuity and momentum equations. The initial conditions of discharge (Q) and depth (y) for each cross section must be defined along the reach.

٣.٤.٢. *Channel geometry*

The geometry of a reach is described by cross – sections. The cross – section can be defined by one of the two following procedures:

I. A polynomial giving the bottom elevation as a function of the lateral distance [which is not considered in this research].
II. A simple or composite geometric cross – section. In either case, provision must be made to calculate the relevant hydraulic parameters of the cross – section as a function of depth (y) or stage (h). These parameters include the area, top width, wetted perimeter, conveyance and various other differentiable and non-differentiable terms of the cross – sectional geometry.

٣.٤.٣. Nodes

There are two types of nodes:

I. External nodes: They are the nodes at the external boundaries of the river system (model) [the upstream and downstream boundaries] where only certain types of nodes where stage or discharge hydrographs or rating curves are used [as mentioned in section (٣.٣)], with partial derivatives as follows:

I.A. For the upstream boundary condition, node (١):

I.A.i. If a discharge hydrograph is used, then:

$$UB(h_1^{j+1}, Q_1^{j+1}) = Q_1^{j+1} - Q'(t^{j+1}) = 0$$ [٣.٤٥] with partial derivatives:

$$\frac{\partial UB}{\partial h_1^{j+1}} = \in$$ [٣.٤٦] $$\frac{\partial UB}{\partial Q_1^{j+1}} = 1$$

[٣.٤٧]

where:

$\in$

: tolerance limit close to zero.

I.A.ii. If a stage hydrograph is used, then:

$$UB(h_1^{j+1}, Q_1^{j+1}) = h_1^{j+1} - h^{j+1}(t) = 0$$ [٣.٤٨] with partial derivatives:

$$\frac{\partial UB}{\partial h_1^{j+1}} = 1$$ [٣.٤٩] $$\frac{\partial US}{\partial Q_1^{j+1}} = \varepsilon$$

[٣.٥٠]

$\in$

I.B. For the downstream boundary condition, node (N):

I.B.i. If a water – surface elevation time series is used, then:

$$\mathbf{DB(h_N^{j+1},Q_N^{j+1}) = h_N^{j+1} - h^{j+1}(t) = 0}$$ [٣.٥١] with partial derivatives:

$$\frac{\partial \mathbf{DB}}{\partial \mathbf{h_N^{j+1}}} = \mathbf{1}$$ [٣.٥٢] $$\frac{\partial \mathbf{DB}}{\partial \mathbf{Q_N^{j+1}}} = \boldsymbol{\varepsilon}$$

[٣.٥٣]

∈

I.B.ii. If a discharge time series is used, then:

∈

$$\mathbf{DB(h_N^{j+1},Q_N^{j+1}) = Q_N^{j+1} - Q^{j+1}(t) = 0}$$ [٣.٥٤] with partial derivatives:

$$\frac{\partial \mathbf{DB}}{\partial \mathbf{h_N^{j+1}}} = \in$$ [٣.٥٥] $$\frac{\partial \mathbf{DB}}{\partial \mathbf{Q_N^{j+1}}} = \mathbf{1}$$

[٣.٥٦]

I.B.iii. If one of equations [٣.٢٠ – ٣.٢٤] is used, then:

$$\mathbf{DB(h_N^{j+1},Q_N^{j+1}) = Q_N^{j+1} - DB'(h_N^{j+1}) = 0}$$ [٣.٥٧] with partial derivatives:

$$\frac{\partial \mathbf{DB}}{\partial \mathbf{h_N^{j+1}}} = \mathbf{DB'(h_N^{j+1})}$$ [٣.٥٨] $$\frac{\partial \mathbf{DB}}{\partial \mathbf{Q_N^{j+1}}} = \mathbf{1}$$

[٣.٥٩]

The derivative $\mathbf{DB'(h)}$ is the derivative of discharge with respect to stage depending on the type of function relating the stage and discharge at the downstream boundary (Nth node).

II. Internal nodes: The internal nodes are the interior boundary conditions for the river system (model) and they are all similar as they are conditions of compatibility. Thus, there is always a continuity condition for the discharge and another condition usually concerning the water stage or the energy line compatibility. The internal nodes may be a junction or a special structure type, so that the interior nodes used in the model are:

A. The Junction

In the junction of a river and a distributary, as shown in Fig. (٣.٢), two equations are supplied by the energy principle and a third is supplied by the continuity principle.

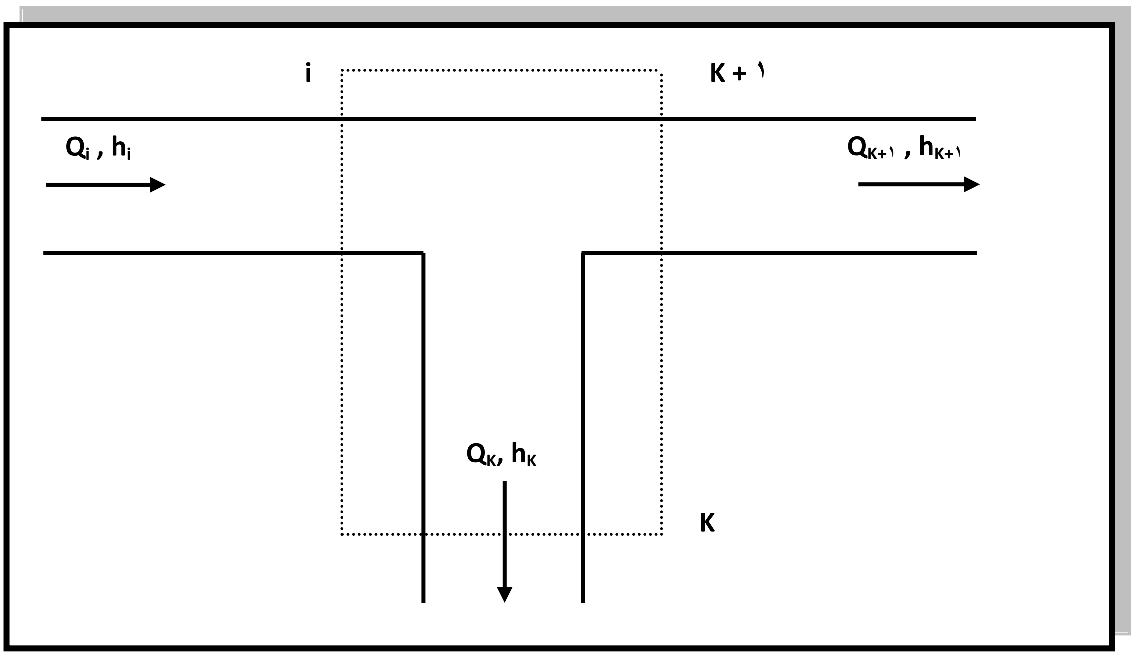

Fig. (٣.٢): Scheme of junction treatment

Applying the energy principle to the junction produces three equal energy levels at the three branches of the junction and by neglecting the velocity head and other losses through the junction, the energy relation reduces to equality of the three depths:

$$\mathbf{h_i = h_K = h_{K+1}}$$ [٣.٦٠] and the first equation of these two equations becomes:

$$\mathbf{E_i(h_i^{j+1}, h_{K+1}^{j+1}) = h_i^{j+1} - h_{K+1}^{j+1} = 0}$$ [٣.٦١] with partial derivatives:

$$\frac{\partial \mathbf{E_i}}{\partial \mathbf{h_i^{j+1}}} = \mathbf{1}$$ [٣.٦٢] $$\frac{\partial \mathbf{E_i}}{\partial \mathbf{Q_i^{j+1}}} = \varepsilon$$

[٣.٦٣] and the ∈cond equation becomes:

$$\mathbf{E_K(h_K^{j+1}, h_{K+1}^{j+1}) = h_K^{j+1} - h_{K+1}^{j+1} = 0}$$ [٣.٦٤]

with partial derivatives:

$$\frac{\partial \mathbf{E_K}}{\partial \mathbf{h_K^{j+1}}} = \mathbf{1}$$ [٣.٦٥] $$\frac{\partial \mathbf{E_K}}{\partial \mathbf{Q_K^{j+1}}} = \varepsilon$$

[٣.٦٦] and th∈ continuity equation would be:

$$\mathbf{C_{K+1}(Q_{K+1}^{j+1}, Q_K^{j+1}, Q_i^{j+1}) = Q_{K+1}^{j+1} + Q_K^{j+1} - Q_i^{j+1} = 0}$$ [٣.٦٧] where:

$\mathbf{Q_i^{j+1}}$: discharge just before confluence position.

$\mathbf{Q_{K+1}^{j+1}}$: discharge just after confluence position.

$\mathbf{Q_{K}^{j+1}}$: discharge at the distributary position.

with partial derivatives:

$$\frac{\partial \mathbf{C_{K+1}}}{\partial \mathbf{y_{K+1}}} = \in \qquad [٣.٦٨]$$

$$\frac{\partial \mathbf{C_{K+1}}}{\partial \mathbf{Q_{K+1}^{j+1}}} = \mathbf{1} \qquad [٣.٦٩]$$

In the junction, the distributary's flow can be a lateral inflow to the river or a lateral outflow from the river. The distributary's lateral inflow (q) can be introduced and defined together with the other cross – sectional data. The distributary's lateral inflow (q) may vary from section to another. This feature can be used to represent tributary's lateral inflow when the hydraulics in the main river is only of concern [Mohammed, (١٩٩٣)].

The distributary's lateral outflow (q) is calculated by considering the distributary's flow as a flow over a side weir or a sill and found by using the sharp-crested weir formula [Cunge and Woolhiser, (١٩٧٥)]:

$$\mathbf{q} = \frac{\mathbf{2}}{\mathbf{3}} \times \mathbf{Cd} \times \sqrt{\mathbf{2g}} \times \mathbf{H^{1.5}} \qquad [٣.٧٠]$$

$$\mathbf{H} = \mathbf{y} - \mathbf{c} \qquad [٣.٧١]$$

where:

y = the water depth above the sill, (L);

c = the sill height, (L);

Cd = the discharge coefficient, (dimensionless).

B. Special structures

The special structure is used to define nodes were hydraulic structures are constructed on the channel system for operation purposes. The hydraulic structures are classified as control structures, normally including sluice gates and a weir.

Whenever there is a hydraulic structure constructed on the canal system then the Saint – Venant equations are not applicable for flow through that hydraulic structure due to the sudden change of water surface between its upstream side and downstream side. Therefore, the equations for flow through a hydraulic structure can be obtained according to discharge continuity and hydraulic dynamics of the structure.

The equation of discharge continuity between section (i) upstream the gate and section (i + ١) downstream the gate would be:

$$\mathbf{C_i(Q_i^{j+1}, Q_{i+1}^{j+1}) = Q_{i+1}^{j+1} - Q_i^{j+1} = 0}$$

[٣.٧٢] with partial derivatives:

$$\frac{\partial \mathbf{C_i}}{\partial \mathbf{y_i^{j+1}}} = \in$$

[٣.٧٣] $$\frac{\partial \mathbf{C_i}}{\partial \mathbf{Q_i^{j+1}}} = \mathbf{1}$$

[٣.٧٤] $$\frac{\partial \mathbf{C_i}}{\partial \mathbf{y_{i+1}^{j+1}}} = \boldsymbol{\varepsilon}$$

[٣.٧٥] $$\frac{\partial \mathbf{C_i}}{\partial \mathbf{Q_{i+1}^{j+1}}} = \mathbf{1}$$

[٣.٧٦]

The second equation is the energy compatibility equation which is represented by the discharge formula through the regulator:

$$Q = f(C_g, d, Hg, hg)$$ [٣.٧٧] where:

Q: discharge upstream the gate, ($L^{٣}/T$);

d: gate opening, which is variable, being prescribed either as a function of time or the water stage immediately downstream from the regulator;

Hg: upstream gate flow depth, (L);

hg: downstream gate flow depth, (L);

$$C_g = b_g \sqrt{2g}$$ [٣.٧٨] b_g: width of the gate opening, (L);

g: acceleration due to gravity, ($L/T^{٢}$).

From Fig. (٣.٣), there are four cases for the flow under a sluice gate [Cunge and Woolhiser, (١٩٧٥)]:

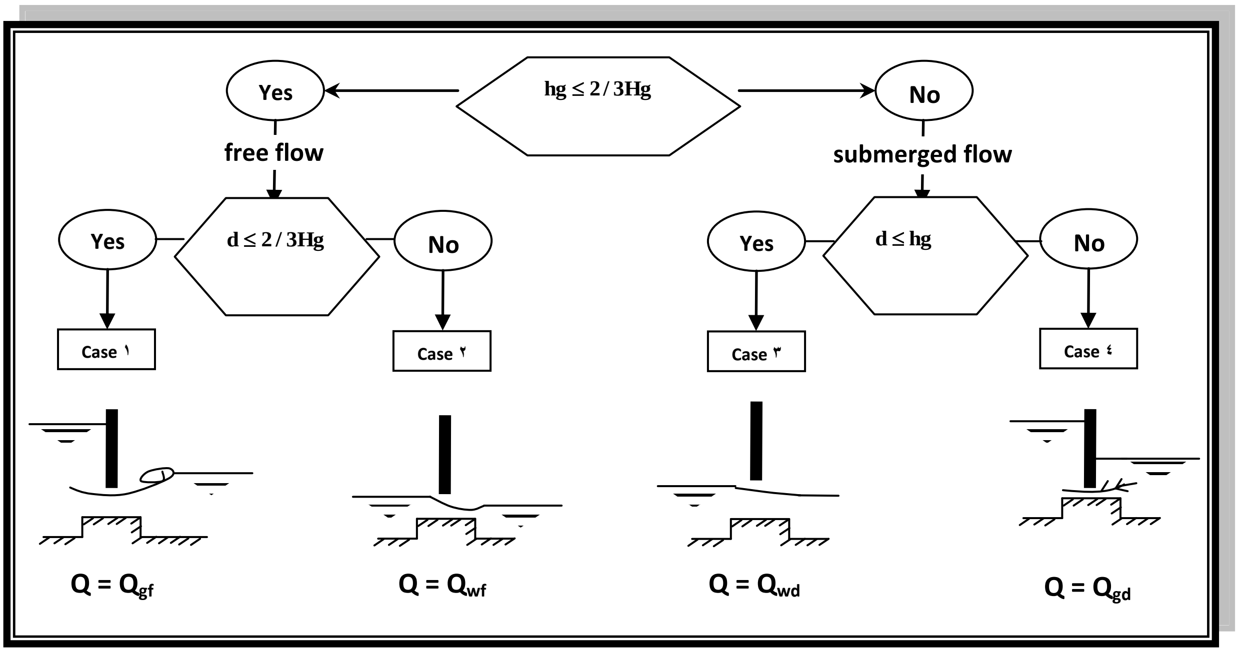

Fig. (٣.٣): Flowchart of logical concept of the gate simulation [Adopted from Cunge and Woolhiser, (١٩٧٥) with modification]

Case (١): When the flow is free, but influenced by the gate:

$$Q_i^{j+1} = Q_{gf} = C_g\, d^{j+1}\sqrt{Hg - d^{j+1}} \qquad [٣.٧٩]$$

with partial derivatives:

$$\frac{\partial E_i}{\partial y_i^{j+1}} = -0.5\, C_g\, d^{j+1}(Hg - d^{j+1}) \qquad [٣.٨٠]$$

$$\frac{\partial E_i}{\partial Q_i^{j+1}} = \in \qquad [٣.٨١] \qquad \frac{\partial E_i}{\partial y_{i+1}^{j+1}} = \varepsilon$$

$$[٣.٨٢] \qquad \frac{\partial E_i}{\partial Q_{i+1}^{j+1}} = \in \qquad [٣.٨٣]$$

where:

$$d^{j+1} = d^{j} \overset{\in}{\mp} \Delta d \qquad [٣.٨٤]$$

Δd: change in gate opening according to the operation rule which closes or opens the gate by the amount (Δd) during one time step (Δt).

Case (٢): When flow is free over the weir [the gates are fully open]:

$$Q_i^{j+1} = Q_{wf} = \frac{2}{3\sqrt{3}} C_g Hg^{1.5}$$ [٣.٨٥] with partial derivatives:

$$\frac{\partial E_i}{\partial y_i^{j+1}} = -1.5\left(\frac{2}{3\sqrt{3}}\right) C_g\, Hg^{0.5}$$ [٣.٨٦] $$\frac{\partial E_i}{\partial Q_i^{j+1}} = 1$$ [٣.٨٧]

$$\frac{\partial E_i}{\partial y_{i+1}^{j+1}} = \in$$ [٣.٨٨]

$$\frac{\partial E_i}{\partial Q_{i+1}^{j+1}} = \in$$ [٣.٨٩]

Case (٣): When the gates are completely open with submerged weir flow:

$$Q_i^{j+1} = Q_{wd} = C_g\, hg\sqrt{Hg - hg}$$ [٣.٩٠] with partial derivatives:

$$\frac{\partial E_i}{\partial y_i^{j+1}} = -\,0.5 C_g\, hg\,(Hg - hg)^{-0.5}$$ [٣.٩١]

$$\frac{\partial E_i}{\partial Q_i^{j+1}} = 1$$ [٣.٩٢]

$$\frac{\partial E_i}{\partial y_{i+1}^{j+1}} = -\,C_g\left[(-\,0.5\, hg\,(Hg - hg)^{-0.5} + (Hg - hg)^{-0.5})\right]$$ [٣.٩٣]

$$\frac{\partial E_i}{\partial Q_{i+1}^{j+1}} = \in$$ [٣.٩٤]

Case (٤): When the flow is influenced by gates opening:

$$Q_i^{j+1} = Q_{gd} = C_g\, d^{j+1} \sqrt{Hg - hg} \qquad [٣.٩٥]$$

with partial derivatives:

$$\frac{\partial E_i}{\partial y_i^{j+1}} = 0.5 C_g\, d^{j+1} (Hg - hg)^{-0.5} \qquad [٣.٩٦]$$

$$\frac{\partial E_i}{\partial Q_i^{j+1}} = 1 \qquad [٣.٩٧]$$

$$\frac{\partial E_i}{\partial y_{i+1}^{j+1}} = -0.5 C_g\, d^{j+1} (Hg - hg)^{-0.5} \qquad [٣.٩٨]$$

$$\frac{\partial E_i}{\partial Q_{i+1}^{j+1}} = \in \qquad [٣.٩٩]$$

٣.٥ *Initial conditions*

In the model the river system is divided into number of river reaches or segments which are joined by nodes. The unsteady flow equations (continuity and momentum) are written for different sections of each reach. These equations are then solved simultaneously, subject to the boundary and initial conditions and the special conditions imposed by nodes. The solution gives the water level or depth and discharge for each cross – section along the river at the end of the time step. For the next time step the last values are used as initial conditions. Then the equations are solved repeatedly until the time period of interest is completely covered.

In order to solve the Saint – Venant unsteady flow equations, the state of the flow [h_i (or y_i) and Q_i] must be known at all cross – sections at the beginning of the simulation (t = ٠). This is known as the initial conditions of the flow. The initial condition may be either a steady or unsteady flow condition.

In the unsteady state condition, the [h_i (or y_i) and Q_i] at each (ith) cross – section can be estimated values, or computed values saved from a previous unsteady flow simulation. In the steady state condition, which is the state considered in this research, the flow is assumed to be steady and nonuniform with the flow at each cross – section initially computed as [Fread and Lewis, (١٩٩٨)]:

$$\mathbf{Q_{i+1} = Q_i - q_i \ \Delta x_i} \qquad [٣.١٠٠]$$

where:

Q_i = the known steady discharge at (t = ٠) at the upstream boundary, ($L^٣$ / T);

The water – surface elevations (h_i) associated with the steady flow also must be determined at (t = ٠). For the subcritical flow, this is accomplished by using the iterative Newton – Raphson method to solve the following equation for (h_i) [Fread and Lewis, (١٩٩٨)]:

$$\left(\frac{\mathbf{Q}^2}{\mathbf{A}}\right)_{\mathbf{i}} - \left(\frac{\mathbf{Q}^2}{\mathbf{A}}\right)_{\mathbf{i+1}} + \mathbf{g\overline{A}_i (h_i - h_{i+1} + \Delta x_i \ \overline{S}_f) = 0} \qquad [٣.١٠١]$$

٣.٦. *Stability and convergence*

The stability of the finite – difference solutions means that the numerical errors of truncation and round – off are small and not amplified during the iterations of the solution [Strelkoff, (١٩٧٠) and Fread, (١٩٧٤)].

It is found that the stability of the solution depends on the value of the weighting factor (θ). Fread and Lewis, (١٩٩٨) stated that: "The influence of the (θ) weighting factor on the stability was examined by (Fread, (١٩٧٤)), who concluded that the stability tends to somewhat decrease as (θ) departs from ٠.٥ and approaches ١.٠. This effect becomes more pronounced as the magnitude of the computational time step increases".

The convergence means that the finite – difference equations solution for a finite grid size is almost near to the analytical solution of the partial differential equations.

The computational control parameters which are distance step (Δx), time step (Δt) and weighting factor (θ) are the parameters that affect the accuracy, convergence and stability of the numerical solution.

The first computational control parameter is the subreach length or distance step (Δx) and in many practical problems it is dictated by considerations of data availability [Fread and Lewis, (١٩٩٣)].

The values of the time step (Δt) are selected according to the physical requirements rather than the numerical stability requirements. Fread, (١٩٧٣a) proved that the implicit finite – difference technique appears to be best suited for modeling transient flows with durations of days or weeks as time step (Δt) without restrictions by the numerical stability.

By using the Newton – Raphson iterative method, it is found that the stability of the numerical solution is depending on the selection of the weighting factor (θ). This factor determines the position at which the Saint – Venant equations are evaluated on the (x – t) plane.

Mohammed, (١٩٩٣) states that: "The accuracy of the initial conditions and the values of the tolerance limits are also important".

Since the solution algorithm involves an iteration process, it is necessary to specify tolerance limits for the value of the variables (stages and discharges) within which the iteration process is terminated. These limits must necessarily reflect the accuracy of model input data and the cost of computation [Al-Eoubaidy, (١٩٩٣)].

Usually, the iteration is terminated when the change in stage and discharge values in two consecutive iterations are within certain limits at the same location.

It must be mentioned that the selection of the Manning's roughness coefficient (n) has a pronounced effect on the computed results where the Manning’s roughness coefficient (n) in the unsteady flow equations is used to describe the resistance to flow due to channel roughness caused by sand / gravel bed forms, bank vegetation and obstructions, bed effects, and circulation – eddy losses [Fread and Lewis, (١٩٩٨)].

There are two ways to evaluate the roughness coefficient. The first way (which is the more accurate way) is from actual field measurement in the river under different flow conditions; however, this way requires an extensive, long and costly measurement program which may render the simulation model unfeasible. The second way (which most modelers usually use) is by depending on a limited program to evaluate (n) within a certain range of flow conditions for use as starting value in model calibration [Chow, (١٩٥٩) and Cunge et al., (١٩٨٠)].

The method used in the research to select the Manning’s (n) value to include it as input data to the model is the trial– and– error method [Fread and Lewis, (١٩٩٨)]. This method proceeds as follows:

١. Using an observed flow hydrograph as the upstream boundary condition and an observed stage hydrograph at the downstream boundary;
٢. Estimating the values of Manning’s (n) throughout the routing reach;
٣. Obtaining computed h and Q from the solution of the Saint – Venant equations;
٤. Comparing the computed values with the observed values at the upstream and downstream boundary;

٥. If the computed values are not approximately equal to the observed values then repeat steps (٣) and (٤) until the computed and observed values are approximately the same.

The solution algorithm requires calibration in the form of choice of appropriate values of the weighting factor (θ) and the computational time step (Δt) to insure stability, convergence and accuracy of the solution. The calibration process then involves successive adjustment of the convergence factors until satisfactory agreement between the computed and measured flow data is achieved. The model must then be verified by comparative testing with other independent sets of observed data. If agreement is not good, model calibration parameters must be adjusted further until agreement is achieved or the cause of deviation is identified. Details of the calibration and verification process are given in Chapter four.