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بسم الله الرحمن الرحيم

﴿اللهُ نُورُ ٱلسَّمَاوَاتِ وَٱلأَرْضِ مَثَلُ نُورِهِ كَمِشْكَاةٍ فِيهَا
مِصْبَاحٌ ٱلْمِصْبَاحُ فِي زُجَاجَةٍ ٱلزُّجَاجَةُ كَأَنَّهَا كَوْكَبٌ دُرِّيٌّ
يُوقَدُ مِن شَجَرَةٍ مُّبَارَكَةٍ زَيْتُونَةٍ لا شَرْقِيَّةٍ وَلا غَرْبِيَّةٍ
يَكَادُ زَيْتُهَا يُضِيءُ وَلَوْ لَمْ تَمْسَسْهُ نَارٌ نُّورٌ عَلَى نُورٍ
يَهْدِي اللهُ لِنُورِهِ مَن يَشَآءُ وَيَضْرِبُ اللهُ ٱلأَمْثَالَ لِلنَّاسِ
وَاللهُ بِكُلِّ شَيْءٍ عَلِيمٌ﴾

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NON-EQUILIBRIUM HEAT CONDUCTION IN FINITE MEDIUM SUBJECTED TO CONSTANT HEAT FLUX

A THESIS SUBMITTED TO THE COLLEGE OF ENGINEERING OF THE UNIVERSITY OF BABYLON IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF MASTER OF

SCIENCE IN MECHANICAL

ENGINEERING

By

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B.Sc., ٢٠٠١

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الخلاصة

منذ أكثر من خمسين عاما ،حاول الباحثون تكرار قانون فورير(Fourier's law) للحرارة لموديل انتقال حرارة في هندسة المواد. المعادلة لا تستطيع توقع درجات الحرارة في بعض التطبيقات ، مثلا أثناء الانتقال العابر في الأوساط الصغيرة. على أية حال، حتى في الحالات حيث الزمن كبير نسبيا، فأن قانون (فورير) قد يفشل بتوقع سلوك لا فورير (non-Fourier). لذا، نموذج لا فورير(non-Fourier model) يجب أن يطبق في بعض التطبيقات الهندسية، الذي فيه الدقة ضرورية في عرض درجة الحرارة لأغراض التصميم. تعتمد الدراسة الحالية على قانون لا فورير للتوصيل الحراري(non-Fourier heat conduction) الذي يعرف في بعض الأحيان بالتوصيل الحراري اللامتوازن (non-equilibrium heat conduction) و على ديناميك الحرارة اللا رجوعي الموسع(extended irreversible thermodynamic)(عدم الاتزان في اتجاه واحد)(non-equilibrium thermodynamic). تم في هذا البحث دراسة انتقال الحرارة اللامتوازن خلال وسط محدد معرض لفيض حراري ثابت في أحدى جهاته ومعزول من الجهة الثانية. وقد تم التحليل باستخدام التوصيل الحراري اللابعدي في معادلة ألقطع الزائد(non-dimensional hyperbolic heat conduction equation) ومقارنة نتائجها مع نتائج التوصيل الحراري اللابعدي في معادلة ألقطع المكافئ(non-dimensional parabolic heat conduction equation) . كذلك تم حساب التغير في العشوائية(entropy change) باستخدام معادلة كتانو-فيرنوت (Cattaneo-Vernotte). هذه المعادلات مُثلت بتقنية الفرو قات المحددة (Finite Difference Technique). وباستخدام طريقة (explicit). كذلك تم تحليل عملية انتقال الحرارة باتجاه واحد في شريحة من الألمنيوم (كحالة دراسية) وحل معادلة ألقطع الزائد للتوصيل الحراري، ومعادلة (Cattaneo-Vernotte)، وحساب التغير في العشوائية. ومعرفة قيم الفيض الحراري التي تحول المسألة من حل لافورير (non-Fourier solution) إلى لا فورير لتغير الطور (non-Fourier phase change). أجريت الدراسة لثلاث قيم لا بعديه للفيض الحراري هما (٠.٥,١.٠,١٠) وقيمتان بعدية للفيض الحراري هما (١Mw,١Gw). تم بناء برنامج باستخدام لغة (QuickBasic) لإنجاز الحل العددي. في الدراسة الحالية وجد ان توزيع درجات، الفيض الحراري، والتغير في العشوائية تمتلك خواصا موجية. وكذلك وجد ان هناك زيادة ملحوظة في درجات الحرارة والفيض الحراري السبب هو ظاهرة الانعكاس التي حدثت.

الإهداء

إلى ......رسول الله وأهل بيته وصحبه(ص).

إلى ...... الحبيب الخالد الحاج هادي جعفر.

إلى ...... أساتذتي الأفاضل.

إلى ...... أمي وأبي وأخواني وأخواتي.

إلى ...... أصدقائي خصوصا علي صفاء وموكب وحسينية أحباب الحسين(ع).

EXAMINING COMMITTEES CERTIFICATE

We certify that we have read this thesis entitled ***"Non-Equilibrium Heat Conduction in Finite Medium Subjected To Constant Heat Flux"*** *and as an examining committee, examined the student,* ***"Hassanain Jabbar Abd"****, in its contents and that in our opinion it meets standard of a thesis for the degree of Master of Science in Mechanical Engineering.*

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We certify that this thesis entitled **"Non-Equilibrium Heat Conduction in Finite Medium Subjected To Constant Heat Flux"** was prepared by **"Hassanain Jabbar Abd"** under our supervision at **Babylon University**, College of Engineering in partial fulfilment of the requirements for the degree of Master of Science in Mechanical Engineering (Power Engineering).

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Hassanein Jabbar

٢٠٠٦

ABSTRACT

For over fifty years, researchers have attempted to refine the Fourier heat equation to model heat transfer in engineering materials. The equation cannot accurately predict temperatures in some applications, such as during transients in microscale ($<10^{-12}s$**) situations. However, even in situations where the time duration is relatively large, the Fourier heat equation might fail to predict observed non-Fourier behavior. Therefore, non-Fourier models must be created for certain engineering applications, in which accurate temperature modeling is necessary for design purposes. The**

present study depends on non-Fourier heat conduction which is known as (non-equilibrium in heat conduction) and on extended irreversible thermodynamics (non-equilibrium in thermodynamics). in the present work a finite medium subjected to a constant heat flux at one end and isolated at the other end has been selected as a case study. The study solved the non-dimensional hyperbolic heat conduction equation and compared the results with the results of non-dimensional parabolic heat conduction equation. The change in entropy during the process is also calculated using Cattaneo-Vernotte equation. These equations have been represented in a finite difference technique. So, in this study a finite medium of Aluminum (case study) has been solved the hyperbolic heat conduction, the change in entropy by using the Cattaneo-Vernotte equation. And known the amount of heat flux which convert the problem from non-Fourier solution to non-Fourier phase change solution. The study was made for three values of non-dimensional heat flux (٠.٥,١.٠,١٠) and two values of dimensional heat flux (١٠٠Mw,١Gw). A computer program in QuickBasic Language was built to perform the numerical solution for

constant heat flux. In this work, it is observed that temperature distribution, heat flux, and entropy change exhibit wave. Also, observed that the temperature distributions, heat flux increasing the reason of this is the reflection phenomena.

CONTENTS

NOMENCLATURE

The following symbols are used generally throughout the text. Other are defined as when used

Symbol	Definition	Unit
c	Thermal Wave Speed	$m/\sec$
c_p	Specific Heat at Constant Pressure	$J/kg.C^\circ$
J	Flux	w/m^2
J_s	Entropy Flux	$w/m^2.C^\circ$
k	Thermal Conductivity	$w/m.C^\circ$
L	Thickness of Slab	m
M	Number of Time Step	——
N	Total Number of Spatial Nodes	——
P	Pressure	N/m^2
Pico	10^{-12} sec	sec
Femto	10^{-15} sec	sec
q	Heat Flux	w/m^2
q^*	Non-Dimensional Heat Flux	——
Q	Constant Heat Flux	w/m^2
Q^*	Constant Non-Dimensional Heat Flux	——
s	Entropy	$J/kg.C^\circ$
s_{le}	Local Equilibrium Entropy	$J/kg.C^\circ$

s^*	Non-Dimensional Entropy	——
T	Temperature	C°
T_{ne}	Non-Equilibrium Temperature	C°
t	Time	*sec*
$t_{\max}$	Maximum Time	*sec*
Δt	Size of Time Step	*sec*
u	Internal Energy	J/kg
v	Specific Volume	m^3/kg
X	Generalized Thermodynamic Force	C°/m
x	The Distance	*m*
Δx	The Distance Between Two Nodal Points	*m*

Greek Symbols

Symbol	Definition	Unit
α	Thermal Diffusivity	$m^{٢}/s$
β	Non-Dimensional Time	——
$\Delta\beta$	Non-Dimensional Size of Time Step	——
δ	Non-Dimensional Distance	——
$\Delta\delta$	Non-Dimensional Distance Between Two Nodal	——
θ	Non-Dimensional Temperature	——
ρ	Density	$Kg/m^{٣}$
σ^s	Entropy Production per Unit Volume	$w/m^3.C^\circ$
δ	Hydrodynamic Boundary Layer Thickness	*m*
τ	Relaxation Time	*sec*
τ_q	Relaxation Time of Heat Flux	*sec*
τ_T	Relaxation Time of Temperature	*sec*

Superscripts

Symbol	Definition	Unit
j, i	Denotes Nodal Positions in Numerical Solution	——

Subscripts

Symbol	Definition	Unit
o	Initial	——
w	Refers to Wall	——

Abbreviation

Symbol	Definition	Unit

CIT	Classical Irreversible Thermodynamics	———
C-V	Cattaneo-Vernotte Equation	———
DPL	Dual Phase Lag	———
EIT	Extended Irreversible Thermodynamics	———
HHCE	Hyperbolic Heat Conduction Equation	———
PHCE	Parabolic Heat Conduction Equation	———
QHT	Quantum Hyperbolic Heat Transfer Equation	———
RT	Rational Thermodynamics	———
SSP	Second Sound Phenomena	———

APPENDIX A

SIMPLIFING OF EXTENDED IRREVERSIBLE THERMODYNAMICS EQUATION

To simplify the modified EIT equation , equation (٣.٤) can be written in the form,

$$ds = \theta^{-1} du - \alpha(u,v) q.dq \qquad \text{....(A.١)}$$

Where:

$$\alpha(u,v) = \tau v (k T_{ne}{}^{2})^{-1} \qquad \text{....(A.٢)}$$

Now, equation (A.١) can be re-written by using equation (A-٢) as,

$$ds = \theta^{-1} du - \frac{\tau v}{k T_{ne}{}^{2}} q.dq \qquad \text{....(A.٣)}$$

By integration of this equation yields the most important results, i.e., the generalized non-equilibrium entropy of non-Fourier heat conduction in the following form:

$$s(T,q) = s_{eq}(T) - \frac{\tau}{2\rho k T_{ne}{}^{2}} q^2 \qquad \text{....(A.٤)}$$

Where s_{eq} and T_{ne} are the local-equilibrium entropy and non-equilibrium temperature respectively. Equation (A.٤) can be written as follows:

$$s(T,q) - s_{eq}(T) = \Delta s = -\frac{\tau}{2\rho k T_{ne}{}^{2}} q^2 \qquad \text{....(A.٥)}$$

The non-dimensional form of (A.٥) can be written as:

$$\Delta s^* = \frac{q^{*2}}{2\theta^2} \qquad \text{....(A.٦)}$$

CHAPTER ONE 1

INTRODUCTION

١.١ General

The most widely used equation governing heat propagation in isotropic media is Fourier's law of heat conduction:

$$q = -k\nabla T \qquad \text{....(١.١)}$$

Where q is the local heat flux across a surface, k is the local thermal conductivity of the media, and ∇T is the local gradient of temperature T in the media. The energy equation for a rigid material with no volumetric energy supply is:

$$\rho c_p \partial_t T = -\nabla \cdot q \qquad \ldots.(١.٢)$$

Where ρ is the material density, c_p is the specific heat capacity, ∂_t is the partial differential operator with respect to time t, and $\nabla \cdot q$ is the divergence of the heat flux. When Fourier's law **(١.١)** is substituted into the energy equation **(١.٢)** for a homogeneous material with constant material properties, Fourier's heat equation is obtained:

$$\partial_t T = \alpha \nabla^2 T \qquad \ldots.(١.٣)$$

$$\alpha = \frac{k}{\rho c_p} \qquad \ldots.(١.٤)$$

Where α is the thermal diffusivity of the material and ∇^2 is the spatial Laplacian operator.

Even though Fourier's heat equation **(١.٣)** is an excellent model for many heat transfer problems, the model is not physically realistic. Equation **(١.٣)** is a parabolic equation. As a result, any temperature disturbance will propagate at an infinite speed through the media **[١]**. Thus, using equation **(١.١)** results in thermal waves traveling at an infinite speed, which is physically unrealizable**[٢].** Fourier's law does not predict finite wave speeds, the law does not accurately approximate the heat transfer in certain cases. The assumption of instantaneous energy transmission fails during a short duration of an initial transient, or when the thermal propagation speed is not high, such as in low temperature cases **[١].** In other words, Fourier's law breaks down at temperatures near absolute zero or when the observation time is extremely small during a transient. For these cases, the fact that thermal disturbances travel with finite speeds of propagation must invalidate Fourier's law **(١.١)**. Thus, the wave nature of thermal transport becomes dominant, rendering Fourier's law to be invalid as an approximation for these cases **[٣].**

From an engineering standpoint, it may be essential to model the heat transfer for the cases where Fourier's law fails. For example, Fourier's law does not accurately predict the transient temperature during microscale **(**$<10^{-12}s$**)** laser heating of thin metal films **(**$\leq 10^{-6}m$**)** **[٤].** Therefore, Non-Fourier behaviors may be significant for such microscale applications as pulsed-laser processing of metal and semiconductors, thin-film applications, and even laser surgery **[٥]**. In fact, laser and microwave heating with extremely short durations or very high frequencies have been used for numerous purposes like surface melting of metal and sintering of ceramics **[٦].** Also, Fourier's law may fail to approximate transient temperatures for micro scale situations, where time scales are relatively large compared to those for microscale conditions **[٤].** Because Non-Fourier behavior exists in engineering applications, Fourier's law should be modified for such applications.

Non-Fourier heat conduction models have been proposed to replace models based on Fourier's law. Specifically, non-Fourier models have been designed to predict ***second sound*** in solids, which is the finite wave speed of heat propagation **[٧].** One Non-Fourier model is based on the modified flux law:

$$q+\tau\frac{\partial q}{\partial t}=-k\nabla T \quad ....(١.٥)$$

where τ is called the *relaxation time* [١]. The heat flux vector now has a memory that keeps track of the time-history of the temperature gradient [٨]. Equation (١.٥) was first proposed independently by Cattaneo [٩] and Vernotte [١٠].

When equation **(١.٥)** is combined with the energy equation **(١.٢)**, the hyperbolic heat conduction equation is obtained:

$$\tau\partial_t^2 T + \partial_t T = \alpha\nabla^2 T \qquad \text{....(١.٦)}$$

Equation **(١.٦)** is known as a hyperbolic heat equation **(or a telegraph equation)** because of the additional term that modifies the parabolic Fourier heat equation **(١.٣)** **[**٦]. The hyperbolic heat equation has two double-derivative terms in **(١.٦).** which are called the wave terms. Unlike Fourier's law **(١.١),** the modified heat flux equation**(١.٥)** predicts a finite speed of heat propagation because of the relaxation time **τ** associated with heat transfer **[٧].** In fact, it can be shown that the speed C of heat propagation is :

$$C=\sqrt{\alpha/\tau} \qquad \text{....(١.٧)}$$

As τ decreases, the thermal wave speed C increases. Discontinuities of temperature and temperature gradient at the wave front and thermal shocks around a moving point source are results of the finite speed C of propagation **[٦].** Typical wave speeds in metals are on the order of $10^5\ m/s$ **[١١].**

The hyperbolic equation **(١.٦)** of heat transfer has been used to model heat transfer in the cases where Fourier's heat equation fails to predict accurate temperatures. For instance, researchers like Glass et al. **[٣]** have investigated the discontinuity of temperature gradient at the thermal wave front. Others like Yang **[١]** have applied the hyperbolic heat model to thermal shocks around a fast-moving heat source, and some have even applied the hyperbolic heat equation to study heat transfer near a rapidly propagating crack tip. Various solutions of the hyperbolic model for finite mediums under different initial and boundary conditions can also be found in literature. Most solutions were attained for a pulse heat flux or a sudden temperature change **[٦].** Tang and Araki **[٦]** have even solved equation **(١.٦)** in a finite medium with a periodic surface heat flux, finding that the wave partially reflects off the boundaries while dissipating until the response becomes periodic.

While the hyperbolic model was created to deal with the problems associated with the Fourier model, the hyperbolic heat equation **(١.٦)** is still in question for multiple reasons. First, it is not based on the details of energy transport in the material, such as the interaction of electrons and phonons(vibrations of the metal lattice) **[٧].** Second, material properties may not be able to be regarded as constant. The relaxation time τ is generally temperature-dependent[٨], and the thermal diffusivity α depends on processing parameters, such as the laser pulse duration and intensity, during short-pulse laser heating **[٧].**

These facts provide impetus for deriving an equation that is grounded more on a physical basis than the hyperbolic heat equation **(١.٦).**

١.٢ Objective and Scope

The objective of the present work is to predict the temperature field in a finite medium subjected to constant heat flux of one end and insulated at the other end using the hyperbolic conduction model. The other objective is to

calculate the entropy change during the process using the extended irreversible thermodynamics theory. Consequently, a computer program has been developed for this purpose employing the finite differences technique. The temperature distribution, heat flux, and entropy change are obtained. Results of non-equilibrium temperature distribution are to be compared with equilibrium temperature distribution to be predicated by Fourier diffusion theory .

This thesis is organized in six chapters:

Chapter One: gives a brief introduction to the problem, and describes the aim and scope of the present study.

Chapter Two: contains a brief review of previous studies related to the subject under consideration.

Chapter Three: presents the formulation of the numerical analysis and the computer program.

Chapter Four: presents the results obtained and the discussion of these results.

Chapter Five: gives a summary of the conclusions drawn from this study and suggestions for further works.

CHAPTER TWO 2

LITERATURE REVIEW

A great deal of effort was invested in the development of theories dealing with non-equilibrium thermodynamics and non-Fourier heat conduction. These efforts improve our insight to the study of connection between these two different subjects and help to redirect the path of research for new solution procedure.

In the following sections, the theories of non-equilibrium thermodynamics and non-Fourier heat conduction are reviewed.

٢.١ Theories ٠f Non-Equilibrium Thermodynamics

Non-equilibrium thermodynamics presents several faces: the most popular theory, referred to as thermodynamics of the first type, is the Classical Irreversible Thermodynamics (CIT). Besides Classical Irreversible Thermodynamics, other theories are the Extended Irreversible Thermodynamics (EIT) and the rational thermodynamic (RT).

٢.١.١ Classical Irreversible

De**Groot and Mazur [١٢],** emphasized that CIT is based on the concept of local equilibrium. The fundamental hypothesis assumes that the system can be split mentally into cells which are sufficiently large to be treated as macroscopic thermodynamic subsystems but, at the same time, sufficiently small that equilibrium is very close to being realized in each cell. The hypothesis postulates that the local and instantaneous relations between the thermal and mechanical properties of a physical system are the same as for uniform system at equilibrium. This implies that all the variables of equilibrium thermodynamics remain significant and that all the relationships of classical thermodynamics between variables remain valid outside equilibrium provided that they are stated locally at each instant of time. This means particularly that entropy outside equilibrium depends on the same variables as at equilibrium.

Prigogine [١٣], proposed that the classical irreversible thermodynamics is grounded on the local equilibrium and the rate at which entropy is produced during irreversible process. Examples of these relation of CIT are Fourier's law in heat conduction, Ohm's law in electricity and Newton's law of fluid mechanics. The rate of local entropy production (σ)follows from the general balance equation of entropy,

$$\frac{\partial(\rho s)}{\partial t} = -\nabla.(J_s + \rho s V) + \sigma \qquad ....(٢.١)$$

with

$$\sigma \geq 0 \qquad ....(٢.٢)$$

Here ρ is the mass density, s is the local specific entropy, J_s is the local entropy flux and V is the velocity field. ∇ means the nabla operator and it is worth noticing that the flux J_s contains two terms: the first is connected with heat conduction and the second arises from the diffusion. Relation (٢.٢) is in agreement with the second law of thermodynamics.

Hutter [١٤], explained that the local rate of entropy in CIT is calculated in terms of the "force" that drives an irreversible process and the response of this force, the "flux". Gibbs equation has been written in the form:

$$T\dot{s} = \dot{u} + p\dot{v} - \sum_k \mu_k \dot{c}_k \qquad \text{....(٢.٣)}$$

Where the lower case letters $s, u,$ and v indicate extensive variables per unit mass, μ_k is the chemical potential of the constitutive k and c_k the mass fraction of k.

DeGroot and Mazur [١٥], presented in their text book a comprehensive and insightful survey of the foundations of the (CIT). They provided a complete discussion of the linear theory of irreversible thermodynamics. The applications of this theory were found in diffusion, heat conduction, fluid dynamics, relaxation phenomena, and the behavior of systems in an electromagnetic field. They treated in detail the statistical foundations of non-equilibrium thermodynamics and classified the various irreversible phenomena according to their 'tensorial character' as follows:

١. Scalar phenomena: which are related to chemical reactions.
٢. Vectorial phenomena: which are related to diffusion and heat conduction.
٣. Tensorial phenomena: which are related to the tensorial quantities occurring in the thermodynamics of fluid systems such as the pressure tensor and the velocity gradient.

Lebon [١٦], proposed that the entropy in CIT is a function of conserved variables and the entropy flux is simply equal to the heat flux divided by the temperature. In CIT the phenomenological laws are steady and the wave propagate with infinite speed. Gibbs equation may be written as:

$$ds = \frac{1}{T}(du + \frac{p}{\rho^2} d\rho) \qquad ....(٢.٤)$$

Jou, Casas and Lebon [١٧], emphasized that the basic formula for the rate of entropy production have the characteristic sum of the products of thermodynamic fluxes (for example heat flux, diffusion, chemical reaction) and of conjugated generalized forces (gradients of temperature, gradients of chemical potential). The basic formula for the rate of entropy production of irreversible process is in the form:

$$\sigma = \sum_k J_k X_k \qquad ....(٢.٥)$$

Where:

J_k : thermodynamic fluxes.

X_k : generalized thermodynamic forces.

Equation (٢.٥) means that the rate of production of entropy is the sum of products of each flux with the associated force. Evidently, all $J_K and X_K$ vanish at equilibrium.

The existence of a non-negative entropy production is one of the main points underlying CIT. They postulated that the fluxes J_i and the generalized forces X_j are related linearly:

$$J_i = \sum_j L_{ij} X_j \qquad \text{....(٢.٦)}$$

Equation (٢.٦) is called a phenomenological equation and the coefficients L_{ij} a phenomenological (kinetic) coefficients. Experimental evidence and theoretical considerations in statistical mechanics have confirmed that a wide class of CIT processes can be described by means of linear relations between fluxes and forces. This is true in particular for transport processes. The phenomenological coefficients are subjected to the rule of selectivity limiting the possibility of interference between irreversible processes of different tensorial character and they are dominated by the **Onsager** reciprocal relations which is state as:

$$L_{ij} = L_{ji} \qquad \text{....(٢.٧)}$$

It is, when the flux J_i corresponding to the irreversible process i is influenced by the force X_j of the irreversible process j , then the flux J_j is also influenced by the force X_i through the same coefficient.

Nettleton and Sobolev [١٨], proposed that classical irreversible thermodynamics of multi component systems is formulated upon the assumption of local equilibrium thermodynamics, which states that the entropy per unit mass of the mixture depends on the internal energy, the specific volume, and the mass fraction c_γ. The local Gibbs equation has been written as:

$$ds_{le} = \frac{1}{T}du + \frac{P}{T}dv - \sum_{\gamma}\frac{\mu_\gamma}{T}dc_\gamma \qquad \text{....(٢.٨)}$$

Where:

s_{le} : local equilibrium entropy.

μ_γ : chemical potential.

٢.١.٢ Rational Thermodynamics

Trusedell [١٩], presented lectures about rational thermodynamics. He referred that the (RT) presents a very mathematical formalism and it abandons the hypothesis of local equilibrium. It's constitutive equations take the form of time –functional. Finally, he pointed out that the cornerstone of this theory was its assumption that the internal energy must depend on the physical fluxes in addition to its classical variables.

Liu [٢٠], studied the restrictions of rational thermodynamics imposed on the processes by the balance equation of mass, momentum and energy by means of Lagrange multipliers. In case of heat conduction, he assumed that there exists an entropy that obeys a balance law with a non-negative production. The entropy production:

$$\rho\frac{ds}{dt} + \nabla\cdot J_s - \Lambda_\circ(\rho\frac{du}{dt} + \nabla\cdot q) \geq 0 \qquad \text{....(٢.٩)}$$

Where:

$$\Lambda_{\circ} = \frac{1}{T} \qquad ....(٢.١٠)$$

$$J_s = \Lambda_{\circ} q = T^{-1} q \qquad ....(٢.١١)$$

Bataille and Kestin [٢١], explained that the concept of local equilibrium in Rational thermodynamics(RT) was abandoned and for the characterization of a system the new concept of memory is introduced. The behavior of the system was thus determined not only by the present value of the variables but also by the hole history of their past value. The main objective of the RT was to provide a method for deriving constitutive equation which serves for a most faithful description of actual physical process.

Lebon, Jou and Vazquez [٢٢], explained that in rational thermodynamics the physical fluxes are viewed as dependent quantities. In RT the temperature and entropy are introduced as primitive concepts of general validity without a sound physical interpretation. Also, in RT the second law of thermodynamics takes the form of the so-called Clausius-Duhem inequality, which locally in the absence of the heat supply is written as:

$$\rho \dot{s} \geq \nabla \cdot (q/T) \qquad ....(٢.١٢)$$

Jou, Vazquez and Lebon [٢٣], studied the rational thermodynamics (RT) theory and they summarized the main principles of rational thermodynamics as follows:

١- Selection of quantities describing the system (primitive concepts and concepts defined in terms of them).

٢- General laws or balances are valid for all the sphere under investigation (e.g. the first and the second law, balance laws of mass, momentum and energy).

٣- Construction of constitutive equations(it is a sort of generalized equations of state formulated in an abstract form).

٤- Application of constitutive postulates, which are used for a general formulation of constitutive relations.

٢.١.٣ Extended Irreversible

Jou, Casas, Lebon [٢٤], explained that EIT provides a link between thermodynamics and dynamics of fluxes, and it is especially useful in describing systems with relatively long relaxation times, e.g. solids at low temperature, super fluids, some visco elastic fluids. The question is what were the reasons for choosing the fluxes rather than the gradients of the classical variables as new independent quantities. The answer of this question is as follows? The fluxes were associated with well defined microscopic operators and they are advantageous for slow and steady state phenomena. By expressing entropy in terms of the fluxes the classical theory of fluctuations can be easily generalized. The formalization of EIT is the subject of some criticism. In particular:

١. Every dissipative flux is considered as quantity characterized by a single evolution equation. However, this is not always the case in practice.

٢. There exist still other "additional" variables and variety of evolution equations for the fluxes as well.

٣. Entropy is regarded as an analytical function of the fluxes, but this is not an essential assumption and non-analytical developments have been proposed.

Garcia-Colin and Selva [٢٥], proposed that in EIT the additional macroscopic quantities (e.g., heat flux in conductive systems) play the role of independent variables. This means that the whole set of space variables describing the system is formed by the classical (conserved) variables plus the (non-conserved) fluxes presented in the system. They found that the modified Gibbs equation can been written as:

$$ds = \frac{1}{T} du + \frac{p}{T} dv - \sum_{\gamma} \frac{\mu_{\gamma}}{T} dc_{\gamma} - \frac{\tau}{\rho} J \cdot dJ \qquad ....(٢.١٣)$$

Lebon [١٦], pointed out that the extended irreversible thermodynamics used to modify the classical irreversible thermodynamics. The thermal heat wave propagates with the finite speed in EIT. The entropy in EIT does not depend only on the classical variables like CIT but, it depends on an addition variables (e.g. heat flux). The extended irreversible thermodynamic is applicable to describe a large class of systems taking place far from equilibrium like non-Fourier heat conduction and non-Newtonian fluids.

Lebon, Jou and Vazquez [٢٣], pointed out that (EIT) was originally born out to generalize the local equilibrium assumption and to extend the Fick, Fourier and Newton laws outside the linear and steady domain covered by (CIT). They summarized the (EIT) as follows:

١- the physical fluxes in EIT are considered as independent variables.

٢- In EIT, a generalized Clausius-Duhem relation is used. This relation states that the entropy flux is not simply (qT^{-1}) for all cases but may contain supplementary terms; moreover the entropy is not a function of the classical variables but depends on the physical fluxes.

٣- The temperature and entropy in EIT assigned a physical interpretation.

Hussein [٢٦], studied the CIT and EIT. He found that the entropy in the framework of classical irreversible thermodynamics is not compatible with the non-Fourier heat conduction when the approach to equilibrium is described by this model. In contrast, the generalized non-equilibrium entropy of extended irreversible thermodynamics is better than the classical one for description of non-Fourier heat conduction.

Fort, Vazquez and Mendez [٢٧], studied an incompressible, multi component fluid in which diffusion and other transport processes were absent and assuming for simplicity that the heat of reaction may be neglected. They found that the evolution equation for the specific entropy in non equilibrium processes may be written in the form:

$$\rho \frac{ds}{dt} = \sigma^s = J\left(\frac{A}{T} - \frac{\alpha\rho}{T}\frac{dJ}{dt}\right) \qquad \text{....(٢.١٤)}$$

Where:

A: affinity of the chemical reaction= $-\sum_{\gamma} v_{\gamma}\mu_{\gamma}$

The second law of thermodynamics required that $\sigma^S \geq 0$. The simplest way to assure this was to assume an evolution equation of the form:

$$\frac{A}{T}-\frac{\alpha\rho}{T}\frac{dJ}{dt}=\beta J \qquad ....(٢.١٥)$$

with $\beta \geq 0$. Equation(٢.١٥) analogue to Maxwell-Cattaneo equation for transport processes.

Bhalekar [٢٨], demonstrated that the temperature and pressure concept retains the same physical meaning whether the system is in equilibrium or non-equilibrium. Extended Gibbs equation of EIT has been written as:

$$d_t s = \theta^{-1} d_t u + \theta^{-1} \pi d_t v - \alpha_q q \cdot d_t q - \alpha_\Pi \Pi : d_t \Pi \qquad ....(٢.١٦)$$

Where:

s : entropy.

θ: non-equilibrium temperature.

π : non-equilibrium pressure.

Π : dissipative stress tensor

α_q : coefficient depend on q .

α_Π : coefficient depend on Π.

Bourhaleb, Sahoumi, Boughaleb and Fliyou [٢٩], studied the heat transport in semiconductors within EIT theory. The starting point of their study was the evolution equations of EIT compatible with the second law of thermodynamics, namely generalized Maxwell-Cattaneo equation of the particle diffusion flux J. The equation appeared the divergence of the second order flux $J^{(2)}$ as:

$$\tau \frac{\partial J}{\partial t} + J = -D\nabla n - \nabla \cdot J^{(2)} \qquad \text{....(٢.١٧)}$$

Where D is the diffusion coefficient and n is the concentration of particles, $J^{(2)}$ represents the flux of J. when $J^{(2)}$=٠ equation (٢.١٧) reduces to Maxwell-Cattaneo equation. From equation (٢.١٧) they derived the entropy production in the form :

$$\sigma^s = J \otimes \left[-\alpha_1 \frac{\partial J}{\partial t} - \nabla \cdot \vartheta^{-1}\mu + \beta_1 \nabla \cdot J^{(2)} \right] + \sum_{n=2}^{\infty} J^{(n)} \otimes \left[-\alpha_n \frac{\partial J^{(n)}}{\partial t} - \beta_{(n-1)} \nabla \cdot J^{(n-1)} + \beta_n \nabla \cdot J^{(n+1)} \right] \qquad \text{...(٢.١٨)}$$

where β_n are coefficients which depend only on u and n.

Vavruch [٣٠], summarized Lebon method of **EIT** and the **CIT** in two points:

١-The theory introduces besides the classical thermodynamic variables, such as density, concentration, temperature, as new independent variables some non equilibrium quantities taking the form of the heat flux, the viscous pressure tensor, the flux of matter, the flux of electric current, etc. These "complementary" variables are then treated on the same level as the usual classical variables.

٢-To compensate for the lack of evolution equations, supplementary rate equations for the dissipative fluxes are introduced, in addition to the usual balance equations of mass, momentum and energy. These rate equations are compatible with the second law of thermodynamics, whereas the evolution equations for the classical variables are given by the usual balance laws. The method was summarized as that the central role of the method is the entropy and the system depends locally not only on the classical variables, but also on the dissipative fluxes. It is worth noticing that the fluxes, and also generalized forces in equations of EIT are not necessarily scalar quantities, but they represent vectorial and tensorial quantities. The generalized Gibbs equation for astokesian fluid as:

$$ds = T^{-1}du + T^{-1}pdv - T^{-1}v(\alpha_{10}q \cdot dq + \alpha_{00}p^{v}dp^{v}) \qquad ....(٢.١٩)$$

Where:

$\alpha_{10} and \alpha_{00}$: coefficient depend on q and p^{v} respectively.

Sieniutycz and Berry[٣١], discussed the description of macroscopic representations of thermal fields with a finite signal speed by composite variational principles involving suitably constructed potentials along with original physical variables. They wrote Cattaneo equation of heat transfer in the form:

$$\frac{\partial q}{c_{\circ}^{2}\partial t} + \frac{q}{c_{\circ}^{2}\tau} + \nabla\rho_{e} = 0 \qquad ....(٢.٢٠)$$

Where :

ρ_{e} : are the density of thermal energy.

The entropy production for equation(٢.٢٠) is:

$$\frac{\partial s}{\partial t} + \nabla \cdot (\frac{q}{T}) = T^{-1}(\frac{\partial E}{\partial t} + \nabla \cdot q) - \frac{\tau}{kT^{2}} q \cdot dq + q \cdot \nabla T^{-1} \qquad ....(٢.٢١)$$

Where:

E : the total energy.

Lebon, Gremla and Lhuillier [٣٢], studied non-Fickean thermo diffusion in binary fluid within the framework non equilibrium thermodynamics EIT, They found that the modified equation of EIT for Non-Fickean law is written in the form:

$$ds = \frac{1}{T}du - \frac{p}{\rho^2 T}d\rho - \frac{\tau}{kT}q \cdot dq \qquad ....(٢.٢٢)$$

He concluded from the study that the heat is not conducted via a Fourier law but rather by means of Cattaneo relation exhibiting the existence of temperature waves and known as the second sound . In EIT, the non-equilibrium entropy depends on the union of the classical variables and their corresponding fluxes.

Jou and Vazquez [٣٣], proposed that the non-equilibrium entropy in extended irreversible thermodynamics also depends upon the quantities which vanish in thermodynamic equilibrium (e.g. heat flux) and the heat flux at a certain time depends on the temperature not only at the present time but also at an earlier time. They found that the generalized Gibbs equation can been written as:

$$d\eta = \frac{1}{T}(d\varepsilon - \frac{p}{\rho^2}d\rho - q \cdot dq) \qquad ....(٢.٢٣)$$

Where:

η :entropy production.

ε : internal energy.

٢.٢ Non-Fourier Heat

One of the first studies to resolve the paradox of the infinite speed of heat propagation was performed by Morse and Feshbash [٣٤], **they postulated that the actual transient heat conduction process must depend on the speed of heat propagation, which should be finite. They assumed that the damped wave (hyperbolic) equation is in the form:**

$$\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\alpha}\frac{\partial T}{\partial t}=\nabla^2 T \qquad ....(٢.٢٤)$$

Where c **is thermal wave speed.**

Maurer and Thompson [٣٥], **found that when the surface heat flux is greater than** $10^7\, w/cm^2$ **, the classical Fourier thermal conduction model equation(**١.١**), fails. The anomaly of the classical theory is the assumption that the heat flux and the temperature gradient across a material volume occur instantaneously. Such an immediate response results in an infinite heat propagation speed. They proposed a modification of Fourier law equation (**١.٥**).**

Sadd and Didlake [٣٦], **proposed the first study of non-Fourier phase change problem. They investigated the melting of semi-infinite**

solid subjected to a step change in temperature and compared the results with the parabolic solutions. They concluded that hyperbolic heat condution equations are more accurate than parabolic heat conduction equation for many physical situations.

Wiggert[٣٧], proposed that the equations describing early-time one-dimensional heat transfer are hyperbolic with temperature equation (٢.٢٤) and heat flux equation (١.٥). He solved the hyperbolic heat conduction equation for a semi-infinite medium analytically by using Laplace transform. He concluded that HHCE has become a widely acceptable and versatile procedure for analyzing solid problems

Glass and Ozisik [٣٨], **solved the non-Fourier heat conduction equation for a semi-infinite medium with periodic surface heat flux numerically by Mac-Cormack's predictor-corrector scheme. They concluded that the temperature are much higher for hyperbolic solution than the parabolic solution, the hyperbolic solution and the parabolic solution converge in the interior region both with increasing time.**

Cheng [٣٩] **, studied the wave properties of heat conduction and the propagation of temperature pulses using a discrete velocity microscopic model. In this model, molecules move with two possible speeds along one of six**

allowable directions. Macroscopic quantities such as temperature and density were extracted from the distribution of molecules among various possible states. The results explained that heat-conduction indeed had wave like properties, and a characteristic finite propagation speed, without restoring to the hyperbolic heat equation or any macroscopic equation. This was achieved by solving the Boltzmann's transport equation, which has the following form:

$$\frac{\partial f}{\partial t} + v.\nabla f = C(f), \qquad \text{....(٢.٢٥)}$$

Where:

f **: the distribution function.**

v **: the velocity.**

$C(f)$**: the collision integral.**

Qiu and Tien [٤٠], **proposed a two-step model to describe the electron temperature and the lattice temperature. They derived the hyperbolic two-step model from the Boltzmann transport with the following assumptions:**

١- **electron-phonon interaction was the dominant scattering process for electron.**

٢- **Conduction of heat by phonon was negligible.**

٣- **Phonon and electron have temperature T_l and T_e, respectively.**

The hyperbolic heat conduction equation for metal lattice:

$$\frac{1}{C_E^2}\frac{\partial^2 T_l}{\partial t^2}+\frac{1}{\alpha_E}\frac{\partial T_l}{\partial t}-\frac{\alpha_l}{C_E^2}\frac{\partial(\nabla^2 T_l)}{\partial t}=\nabla^2 T_l \qquad \text{....(٢.٢٦)}$$

Similarly, the hyperbolic heat conduction equation for electron,

$$\frac{1}{C_E^2}\frac{\partial^2 Te}{\partial t^2}+\frac{1}{\alpha_E}\frac{\partial T_e}{\partial t}-\frac{\alpha e}{C_E^2}\frac{\partial(\nabla^2 Te)}{\partial t}=\nabla^2 Te \qquad \text{....(٢.٢٧)}$$

Where:

$$\alpha_e=\frac{k}{c_e+c_l} \qquad \text{....(٢.٢٨)}$$

$$C_E=\sqrt{\frac{kG}{c_e c_l}} \qquad \text{....(٢.٢٩)}$$

$$G=\frac{\pi^4(n_e v k_\beta)^2}{k} \qquad \text{....(٢.٣٠)}$$

Joshi **and** Majumdar [٤١], **solved the Cattaneo-Vernotte equation and then they compared the solution of Cattaneo-Vernotte with the solution of the transient Boltzmann equation, the Fourier law for phonon heat conduction perpendicular to a thin film plane. They concluded that neither the Fourier nor the Cattaneo equation can represent well the heat**

conduction processes in small scale and/or fast transient. The Boltzmann equation, even in it’s simplest from, however, is difficult to solve since it involves variables in both real and momentum spaces, as well as time.

Cheng and Liu [٤٢], **solved one-dimensional hyperbolic heat conduction equation using Laplace transform and a finite volume technique. They concluded that the hyperbolic equation is more suitable for high fluxes and very short time.**

Liu and Tan [٤٣], **studied the non-Fourier effects on transient coupled radiative-conductive heat transfer in one-dimensional semi transport medium subjected to a periodic irradiation. The modified HHCE for the coupled radiative-conductive, which has the following non-dimensional form:**

$$\frac{\partial Q(\tau,\xi)}{\partial \xi} + \frac{N}{\xi_{tr}}\frac{\partial Q(\tau,\xi)}{\partial \tau} + \frac{1}{\xi_{tr}}Q(\tau,\xi) - \frac{\partial Q^{r}(\tau,\xi)}{\partial \xi} - \frac{1}{\xi_{tr}}Q^{r}(\tau,\xi) = 0 \quad \text{....(٢.٣١)}$$

Where:

N: conduction-radiation parameter = $\frac{kB}{4n^{-2}\sigma T_r^4}$ **.**

$Q(\tau,\xi)$**: non-dimensional total heat flux =** $\frac{q^c + q^r}{4n^{-2}\sigma T_r^4}$ **.**

$Q^r(\tau,\xi)$**: non-dimensional radiative heat flux =** $\frac{q^r}{4n^{-2}\sigma T_r^4}$ **.**

$Q^c(\tau,\xi)$**: non-dimensional conductive heat flux =**

$= \dfrac{q^c}{4n^{-2}\sigma T_r^4}$ **.**

ξ**: non-dimensional time =** $\alpha\beta^2 t$ **.**

ξ_{tr}**: non-dimensional relaxation time of conduction =** $\alpha\beta^2 t_{tr}$**.**

τ**: optical variable =** βx**.**

σ**: Stephan-Boltzmann constant .**

β**: extinction coefficient .**

n^{-2}**: refractive index of medium .**

The HHCE solved by the flux-splitting method, and the radiative transfer equation solved by the discrete ordinate method. The transient responses obtained from HHCE compared with those obtained from classical parabolic heat conduction equation. The results show that non-Fourier effect can be important when the conduction to radiation parameter and the thermal relaxation time of heat conduction are larger.

Lu, Zhang and Zhou [٤٤], **stated that in non-stationary heat conduction problems, the Fourier's law should be replaced by the equation:**

$$q(z,t)+\tau(z)\frac{\partial q(z,t)}{\partial t}=-k(z)\frac{\partial T(z,t)}{\partial z} \qquad \ldots.(٢.٣٢)$$

Where τ **and** k **are a function of depth** z**. They found that the measurements of the relaxation**

time at room temperature on some materials differs from the experimental values of relaxation time for these materials. Then, they discussed the theory of reconstruction of the relaxation time with depth in homogenous materials to solve the differs values of relaxation time.

Cheng [٤٥], **presented the derivation of a new type of heat conduction equations, named Ballistic-Diffusive equations, which were derived from Boltzmann equation under the relaxation time approximation. The distribution function is divided into two parts. One represents the ballistic transport originating from the boundaries and the other is the transport of the scattered and excited carriers. The latter is further approximated as diffusive process. The ballistic-diffusive heat conduction equation has been written in the form:**

$$c(\tau \frac{\partial^2 T_m}{\partial^2 t} + \frac{\partial T_m}{\partial t}) = \nabla(k \nabla T_m) - \nabla \cdot q_b + (\mathring{q}_e + \tau \frac{\partial \mathring{q}_e}{\partial t}) \quad \text{....(٢.٣٣)}$$

The subscripts m **and** b **represent diffusive and ballistic component respectively and** $\mathring{q}_e$ **is the volumetric heat generation. The major difference of equation(٢.٣٣) compared to the hyperbolic heat conduction equation is the**

additional term $\nabla \cdot q_b$. The non-dimensional form of ballistic-diffusive heat conduction equation has been written in the form:

$$\frac{\partial^2 \theta_m}{\partial t^{*2}} + \frac{\partial \theta_m}{\partial t^*} = \frac{K_n{}^2}{3} \frac{\partial^2 \theta_m}{\partial \eta^2} - K_n \frac{\partial q_b^*}{\partial \eta} \quad ....(٢.٣٤)$$

Where

θ_m**: non-dimensional temperature =**$\frac{T - T_\circ}{\Delta T}$**.**

q_b^***: non-dimensional heat flux =** $\frac{q_b - q_{b\circ}}{C_v \Delta T}$**.**

t^* **: non-dimensional time =**t/τ**.**

η **: non-dimensional distance =** x/L**.**

K_n**: phonon Knudsen number =** $\frac{\Lambda}{L}$**.**

Λ **: heat carrier mean free path at frequency(**ω**)=** $|v| T_\omega$**.**

v **: the carrier group velocity.**

The Ballistic-Diffusive equation was solved and the results compared to the solutions for the same problem based on the Boltzmann equation, Fourier law and the Cattaneo equation. He concluded that the Ballistic-Diffusive heat equation is much more better alternative to the Fourier and the Cattaneo-Vernotte equation and much simpler to solve than Boltzmann equation.

Deng and Liu [٤٦], **found that the phase change behavior and the thermal stress inside**

the skin tissue subjected to freezing may be numerically investigated considering the non-Fourier effect. They studied the influence of non-Fourier effect on the process of skin freezing. The thermal and stress analyses were performed independently; and the mathematical model was based on three principle assumptions: i- the frozen tissue is treated as an elastic solid; ii- the problem is one-dimensional since the skin is very thin layer; iii-the skin is frozen symmetrically from it's two sides.

The governing equation for the frozen phase can be written as:

$$\tau_s \frac{\partial^2 T_s}{\partial t^2} + \frac{\partial T_s}{\partial t} = \alpha_s \frac{\partial^2 T_s}{\partial x^2} \qquad t > 0, 0 < x < s(t) \qquad \text{....(٢.٣٥)}$$

α_s **and** τ_s **is the thermal diffusivity and relaxation time of frozen skin respectively, similarly the governing equation for the unfrozen phase is:**

$$\tau_l \frac{\partial^2 T_l}{\partial t^2} + \frac{\partial T_l}{\partial t} = \alpha_l \frac{\partial^2 T_l}{\partial x^2} \qquad t > 0, s(t) < x < d \qquad \text{....(٢.٣٦)}$$

α_l **and** τ_l **the thermal diffusivity and relaxation time of unfrozen skin respectively. They solved the equations by using the finite differences method and compared the results obtained from the hyperbolic heat conduction with those**

obtained from the classical parabolic heat conduction equation. The results show that at relaxation time increases non-Fourier effect becomes stronger

Kozlowski [٤٧], **studied the transfer process of the quantum particles in the context of thermal energy transport in a highly excited matter. It is shown that when matter is excited with short thermal perturbation the response of the matter can be described by quantum hyperbolic heat transfer equation (QHT) which is the generalization of the parabolic quantum heat transfer equation has the form:**

$$\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\tau c^2}\frac{\partial T}{\partial t}=\frac{\alpha_i^2}{3}\nabla^2 T \qquad ....(٢.٣٧)$$

Kunadian [٤٨], **presented Ultra-fast laser heating of nano-filmes using one-dimensional and three-dimensional DPL heat transport equations with laser heating at different locations on the metals film. A numerical solution based on explicit finite difference method to solve the problems. They compared the results with those obtained from classical diffusion and hyperbolic heat conduction equation is developed. The three-dimensional DPL equation has been written in the form:**

$$\frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\alpha}\frac{\partial T}{\partial t}-\tau_T\frac{\partial(\nabla^2 T)}{\partial t}=\nabla^2 T+\frac{1}{k}(s+\tau_q\frac{\partial s}{\partial t}) \quad ....(٢.٣٨)$$

Where:

s: heat source.

Haiquan, Yanhua and Haiyan [٤٩], **analyzed non-Fourier heat conduction induced by ultra fast heating of metal with high-energy intensity beam. The non-Fourier effects during high heat flux heating were illustrated by comparing the transient temperature response to different heat flux and material relaxation times. They solved hyperbolic heat conduction equation by using hybrid method combining an analytical solution and numerical inversion of the Laplace transforms of a semi infinite body with heat flux.**

Gembarovic [٥٠], **calculated the temperature distribution in a finite medium in case of non-Fourier heat conduction numerically by using simple iterative algorithm based on dumped heat wave. In this algorithm the temperature is calculated explicitly in one simple calculation that was repeated for each time step as the heat wave marches through the medium with constant speed. Then they compared the results with exact analytical solution.**

Szekers[٥١], **proposed a short summary on the hyperbolic heat conduction equation which is named as second sound phenomenon (**SSP**). He started from Fourier's equation, neglecting the mechanical interactions, which leads to parabolic differential equation on the temperature. Then he explained how the experts solved the contradiction of parabolic heat equation by developed the** Cattaneo-Vernotte **modified law of heat conduction, as**

$$\tau h_t + h = -kT_x \qquad ....(٢.٣٩)$$

Where h **is the heat flux,** k **is the conductivity and τ is the so called relaxation time. Replacing the Fourier law by** Cattaneo-Vernotte **laws,**

$$dT_{xx} + T_t + \tau T_{tt} = 0 \qquad ....(٢.٤٠)$$

Where:

d **: thermal diffusivity.**

After the summary he pointed to the coupling problem of the thermo elasticity, he derived the generalized equations of motion and the heat conduction based on the basic equations of the systems:

$$u_{tt} - \frac{E}{\rho} u_{xx} + \frac{E\alpha}{\rho} T_x = 0 \qquad ....(٢.٤١)$$

$$dT_{xx} + \tau T_{tt} + T_t - \frac{\tau E}{\rho c}(u_x - \alpha T)u_{xtt} - \frac{E}{\rho c}(u_x - \alpha T)u_{xt} = 0 \qquad ....(٢.٤٢)$$

Where u **is the displacement,** E **is the Young modules and** α **is the coefficient of thermal expansion.**

Pakdemirli and Sahin [٥٢], **presented the hyperbolic heat conduction equation with temperature dependent properties. The thermal conductivity, specific heat and density were assumed to be function of temperature as:**

$$\tau \frac{\partial}{\partial t}\left[\rho(T)cp(T)\frac{\partial T}{\partial t}\right] + \rho(T)cp(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] \qquad ....(٢.٤٣)$$

Then they cast the equation into non-dimensional form suitable for perturbation analysis:

$$\frac{\partial^2 \theta_o}{\partial \xi^2} + 2\frac{\partial \theta_o}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2} \qquad ....(٢.٤٤)$$

Where:

ξ**: non-dimensional time** $= \frac{c_o^2 t}{2\alpha_o}$ **.**

η**: non-dimensional distance** $= \frac{c_o x}{2\alpha_o}$

θ**: non-dimensional temperature** $= \frac{T - T_f}{T_i - T_f}$

They solved equation(٢.٤٤) by employing a newly developed approximate theory with various similarity solutions corresponding to the symmetries. They concluded that this model

was better in representing situations involving very low temperature, very high temperature gradients or extremely short times.

Shiomi and Maruyama [٥٣], **proposed the non-Fourier heat conduction in single-walled carbon nanotubes (SWNT) subjected to a local heat pulse with time duration of femto-picoseconds. Molecular dynamics simulations used to solve the HHCE. They demonstrated that the distinct heat is conducted as a wave.**

Ai and Li[٥٤], **proposed a universal model for heat conduction to cover the fundamental behaviors of diffusion, thermal wave, where the phonon-electron can not be adequately described by the classical Fourier's law of heat conduction. The model was generalized from the so-called dual-phase lag concept by introducing two phase lags (i.e. that is two relaxation times) to both the heat flux vector and temperature gradient. The suggested equation was:**

$$q(r,t)+\tau_q\frac{\partial q}{\partial t}(r,t)=-k\nabla T(r,t)-k\tau_T\frac{\partial}{\partial t}\nabla T(r,t) \qquad ....(٢.٤٥)$$

Where :

r **: space coordinate.**

Equation (٢.٤٥) is combined with the energy equation (١.٢) to obtain:

$$\frac{1}{\alpha}\partial_t T + \frac{\tau_q}{\alpha}\partial_t^2 T = \nabla^2 T + \tau_T \partial_t \nabla^2 T \qquad ....(٢.٤٦)$$

Where $\tau_q and \tau_T$ **are the characteristic relaxation times of heat flux and temperature gradient, respectively. Equation (٢.٤٥) reduces to Cattaneo-Vernotte by setting** $\tau_T = 0$ **and reduces to Fourier's heat conduction by setting** $\tau_T = \tau_q = 0$**.**

Ishikawa [٥٥], **proposed the temperature control problem for stochastic hyperbolic heat conduction equation model as:**

$$\rho\tau c \frac{\partial^2 T(t,x)}{\partial t^2} + \rho c \frac{\partial T(t,x)}{\partial t} - k \frac{\partial^2 T(t,x)}{\partial x^2} = (f(t,x) + \tau \frac{\partial f(t,x)}{\partial t}) \qquad ....(٢.٤٧)$$

Where

$f(x,t)$**: heat energy entering the system per unit time.**

The stochastic hyperbolic heat conduction equation was solved depending on two aspect. First, by taking the randomness in the input signal into consideration. Secondly, the free boundary problem(Stephan's problem).

Banerjee, Ogale, Das, Mitra and Subrmanin [٥٦], **presented the analysis of the heat affected zone in materials such as meat samples, araldite resin-simulating tissue phantoms, and fiber composites irradiated using mode locked short pulse laser and compared it with that of a**

continuous wave laser of the same average power. There are many applications of this analysis such as in a number of high-precision medical procedures like neuro surgery, ophthalmology, cornel surgery. They found that the thermal analysis of laser-material interaction in various application is usually conducted via the tradition parabolic Fourier conduction model. The hyperbolic model accounts for the time required for the heat flux to relaxe or adjust to a change in the temperature gradients if the speed of propagation of the thermal signals is considered finite. To compare the experimentally measured temperature, a non-Fourier damped wave model for the case of laser penetration and absorption of the intensity within the material was considered :

$$q(r,z,t)+\tau\frac{\partial q(r,z,t)}{\partial t}=-k\nabla T(r,z,t) \qquad \ldots.(٢.٤٨)$$

Where r, z coordinates. After they solved the HHCE and the PHCE and compared the results they concluded from the comparison that HHCE was more accurate than PHCE.

Mirzaei, Nayeeny and Makaremi [٥٧], **studied several test cases of HHCE including one-dimensional and two-dimensional**

problems. The first test case was a one-dimensional finite slab with no heat source. The second case was one-dimensional finite slab with pulsed heat source. The third case was two-dimensional heat conduction problem. The method of solution was hybrid schemes. A modified hyperbolic heat conduction for one-dimensional has been written in the form:

$$\tau\frac{\partial^2 T}{\partial t^2}+\frac{\partial T}{\partial t}=\alpha\nabla^2 T+\frac{\alpha\tau}{k}\frac{\partial g}{\partial t}+\frac{\alpha}{k}g \quad \text{....(٢.٤٩)}$$

for two-dimensional:

$$\tau\frac{\partial^2 T}{\partial t^2}+\frac{\partial T}{\partial t}=\alpha(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})+\frac{\alpha\tau}{k}\frac{\partial g}{\partial t}+\frac{\alpha}{k}g \quad \text{....(٢.٥٠)}$$

the dimensionless form of equation(٢.٤٩)

$$\frac{\partial^2\theta}{\partial A^2}+2\frac{\partial\theta}{\partial A}=\frac{\partial^2\theta}{\partial\eta^2}+\frac{1}{2}\frac{\partial G}{\partial A}+G \quad \text{....(٢.٥١)}$$

the dimensionless of equation (٢.٥٠),

$$\frac{\partial^2\theta}{\partial A^2}+2\frac{\partial\theta}{\partial A}=\frac{\partial^2\theta}{\partial\eta^2}+\frac{\partial^2\theta}{\partial\xi^2}+\frac{1}{2}\frac{\partial G}{\partial A}+G \quad \text{....(٢.٥٢)}$$

Where:

η**: non-dimensional coordinate in the x direction=**$\frac{cx}{2\alpha}$

ξ**: non-dimensional coordinate in the y direction=**$\frac{cy}{2\alpha}$

A: non-dimensional time = $\frac{t}{2\tau}$

θ**: non-dimensional temperature=** $\frac{Tkc}{\alpha}$

G: non-dimensional heat source = $\frac{4\alpha g}{c}$

The results of each case are compared with the results which were obtained from parabolic heat conduction equation. The results showed that the present method can be applied for non-Fourier heat conduction with high degree of accuracy.

Volz and Carminati [٥٨], **studied the thermal response of silicon crystal to pico-femto second heat pulse. They concluded that the response implies disagreement with classical predictions. They made reformulation of Fourier's law by adding an inertia term which allows to derive the well known hyperbolic heat condution equation(٢.٢١).**

٢.٦ Literature Conclusions

From the preceding review of literatures, it is clear that most of literatures are based on the following:

- The governing equations were solved either analytically by Laplace transform or numerically by finite differences or finite elements.

- ❖ All studies of non-Fourier heat conduction equation depends on non-equilibrium thermodynamics which using the extended irreversible thermodynamics (EIT).
- ❖ A finite medium or semi finite medium.
- ❖ Heat flux or unit step temperature.
- ❖ A uniform base temperature.
- ❖ The compared of the solution of hyperbolic heat conduction equation with the solution of parabolic heat conduction equation.
- ❖ The properties such as thermal conductivity, specific heat, and density assumed to be dependent or independent of temperature.

Table (٢-١) shows the summary of literature review work.

In the present work, an attempt based on the determination of temperature distribution for non-Fourier heat conduction equation, Fourier heat conduction equation for a finite slab subjected to a constant heat flux then compared the results of non-Fourier heat conduction with those obtained from Fourier heat conduction equation then find heat flux distribution and entropy change distribution. The method of solution is finite difference method.

Table (٢-١) Literature summary.

١- Classical Irreversible Thermodynamics:

Author	Analysis	Conclusion
DeGroot and Mazur, [١٢]	CIT	CIT based on the concept of equilibrium
Prigogine, [١٣]	CIT	CIT is grounded on the local equilibrium, and found the entropy production
Hutter, [١٤]	CIT	Explained the local rate of entropy in CIT
DeGroot and	CIT	Presented survey of the foundations of CIT

Mazur, [١٥]		
Lebon, [١٦]	CIT	Proposed the entropy in CIT as a function of conserved variables.
Jou, Casas and Lebon, [١٧]	CIT	Found the basic formula for the rate of entropy production
Nettleton and Sobolev, [١٨]	(١-D) and (٢-D) Steady State	Proposed the CIT of multi component systems

٢-Rational Thermodynamics:

Author	Analysis	Conclusion
Trusedell, [١٩]	RT	Lectures about RT.
Liu, [٢٠]	RT	The RT and found the entropy production equation.
Bataille and Kestin, [٢١]	RT	The local equilibrium in RT.
Lebon, Jou and Vazquez, [٢٢]	RT	Methods of RT.
Lebon, Jou and Vazquez, [٢٣]	RT	main principles of RT.

٣-Extended Irreversible Thermodynamics:

Author	Analysis	Conclusion
Jou, Casas and Lebon, [٢٤]	EIT	That EIT provides a link between thermodynamics and dynamics of fluxes.
Garcia-Colin and Selva, [٢٥]	EIT	That EIT contains conserved variables plus non-conserved variables.
Lebon, [١٦]	EIT	EIT used to modify CIT and the wave propagate with finite speed.
Jou, Casas and Lebon, [٢٣]	EIT	Method of the EIT

Hussein, [٢٦]	CIT and EIT	The entropy change in semi-finite medium.
Fort, Vazquez, and Mendez, [٢٧]	EIT	EIT equation in an incompressible multi component fluid.
Bhalekar, [٢٨]	EIT	Modified Gibbs equation.
Bourhaleb, Sahoumi, Boughaleb, and Fliyou, [٢٩]	EIT	EIT equation in semiconductors.
Vavruch, [٣٠]	EIT	Summarized EIT and CIT.
Sieniutycz and Berry, [٣١]	EIT	The entropy production equation.
Lebon, Gremla and Lhuilier, [٣٢]	EIT	Studied EIT in non-Fickean thermo diffusion.
Jou and Vazquez, [٣٣]	EIT	Non-equilibrium entropy

٤- Non-Fourier Heat Conduction :

Author	Analysis	Case of study	Method of solution
Morse and Feshbash, [٣٤]	Non-Fourier heat conduction equation	----	----
Maurer and Thompson, [٣٥]	Non-Fourier heat conduction equation	----	----
Sadd and Didlake, [٣٦]	Non-Fourier phase change heat conduction equation	Semi-finite solid subjected to a step change in temperature.	Analytical solution.
Wiggert, [٣٧]	(١-D) HHCE	Semi-finite slab.	Laplace transform.
Glass and Ozisik, [٣٨]	(١-D) HHCE	Semi-finite medium with heat flux.	Predictor-corrector scheme.
Cheng, [٣٩]	Boltzmann equation.	Thin film	Analytical solution.
Qiu and Tien, [٤٠]	Two-step heat conduction equation	Semi-finite.	Finite difference method.

Joshi and Majumdar, [٤١]	HHCE, Boltzmann equation,	Thin film	Analytical solution.
Cheng and Liu, [٤٢]	(١-D) HHCE	Finite medium	Laplace transform and finite volume technique.
Liu and Tan, [٤٣]	HHCE for the coupled radative-conductive	Finite medium	Finite difference
cheng, [٤٥]	Ballistic-diffusive equation	Thin film	Analytical solution.
Deng and Liu, [٤٦]	Non-Fourier phase change	Finite medium	Finite difference solution.
kozlowski, [٤٧]	Quantum HHCE	----.	----
kunadian, [٤٨]	DPL	Semi-finite	Finite difference solution.
Haiquan, Yanhue and Haiyan, [٤٩]	Non-Fourier heat conduction	Semi-finite	Laplace transform and analytical solution.
Gembarovic, [٥٠]	Non-Fourier heat conduction.	Finite medium	Finite difference solution
Szekers, [٥١]	Coupling problem of the thermo elasticity.	----	----
Pakdemirli and Sahin, [٥٢]	HHCE with temperature dependent .	Semi-finite	Similarity solution.
Shiomi and Maruyama, [٥٣]	Non-Fourier heat conduction	Single-walled carbon nano tubes	Molecular dynamics.
Ai and Li, [٥٤]	DPL	----	----
Ishikawa, [٥٥]	Stochastic HHCE	Finite medium	Finite difference solution
Banerjee, Ogale, Das, Mitra and Subramanin , [٥٦]	HHCE	Finite medium	Analytical solution.
Mirzaei, Nayeeny and Makaremi, [٥٧]	(١-D) and (٢-D) HHCE	Finite slab	Hybrid schemes.
Volz and Carminati, [٥٨]	HHCE	----	----

CHAPTER THREE

3

NUMERICAL ANALYSIS AND COMPUTER PROGRAM

٣.١ General

Mostly heat conduction problems involving simple geometries with simple boundary conditions can be solved analytically. But many problems encountered in practice involving complicated geometry with complex boundary conditions or variable properties cannot be solved analytically. In such case, sufficiently

accurate approximate solutions can be obtained by computer using a numerical method.

Hyperbolic heat conduction equation can be solved analytically by using Laplace transformation or numerically by using finite differences methods, finite elements methods, boundary elements methods, finite strips methods, and collocations methods. In this work the finite differences method is used to solve the governing equations of non-Fourier heat conduction in finite medium with constant heat flux.

٣.٢ Assumptions

The hyperbolic heat conduction equation is solved under the following assumptions:

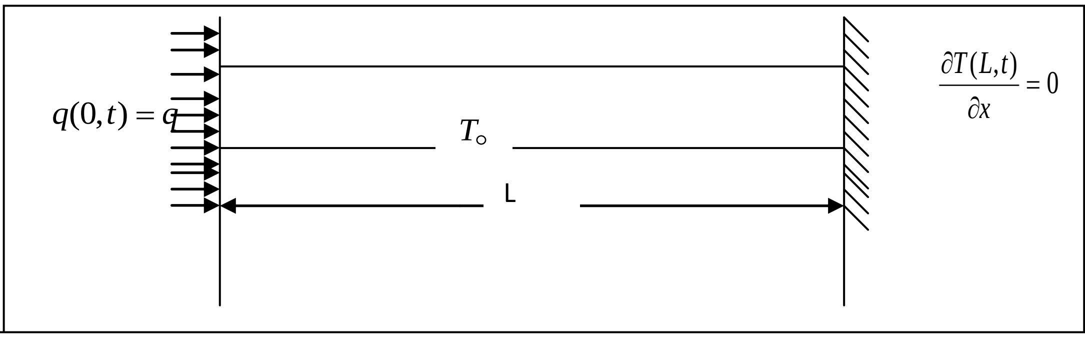
١. Non-equilibrium convection and radiation are assumed negligible, i.e. conduction is only considered.

٢. The physical properties are independent of temperature variation and are assumed constant.

٣. The slab is treated as a one-dimensional solid.

٣.٣ Geometry

In order to solve the hyperbolic heat conduction equation, consider a finite slap of length L subjected to a constant heat flux of magnitude q at the left boundary and thermally insulated at the other boundary which means that q equal zero at the other boundary. Fig.(٣.١) and Fig.(٣-٢) show a schematic representation of the case of study of parabolic heat conduction and hyperbolic heat conduction equation respectively..



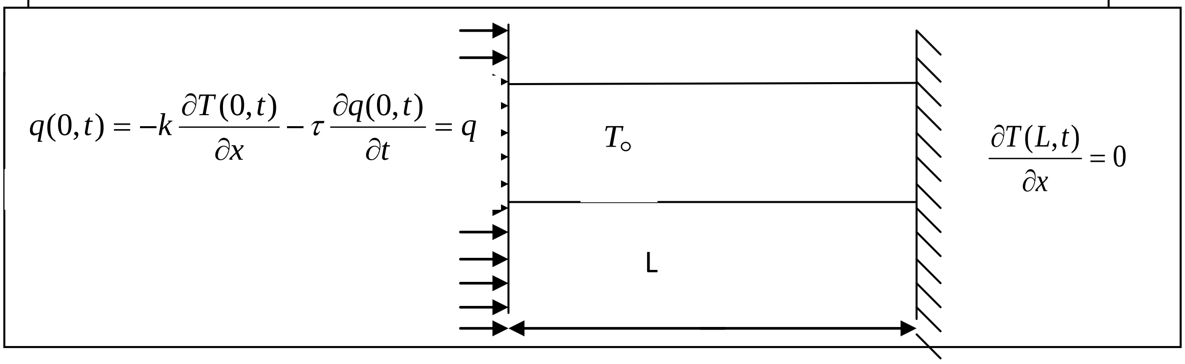


Fig. (٣.٢) Finite Medium Subjected to a Constant Heat Flux for Hyperbolic Equation.

٣.٤ Governing Differential Equations

The governing differential equation is represented by the hyperbolic heat conduction equation and the parabolic heat conduction

equation for temperature distribution, non-Fourier equation for heat flux distribution, and a modified extended irreversible thermodynamics for entropy change.

٣.٤.١ Hyperbolic Heat Conduction Equation

This equation is represented by the equation which is derived from the non-Fourier heat conduction equation which has the form of hyperbolic equation,

$$\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2} \qquad ....(٣.١)$$

٣.٤.٢ Parabolic Heat Conduction Equation

This equation represents the equation which is derived from Fourier's law of heat conduction. The results of this equation will be compared with the results of Non-Fourier heat conduction (HHCE). The equation can be written as,

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2} \qquad ....(٣.٢)$$

٣.٤.٣ Heat Flux Equation

This equation represents the Cattaneo-Vernotte equation which is the modified equation of Fourier's law. The equation is written as,

$$q+\tau\frac{\partial q}{\partial t}=-k\nabla T \qquad ....(٣.٣)$$

٣.٤.٤ Entropy Generation Equation

The entropy change is solved by using the modified equation which depends on the heat flux which is written as,

$$\rho\frac{ds}{dt}=\frac{1}{T}\nabla\cdot q-\frac{\tau}{kT^2}q\cdot\frac{dq}{dt} \quad ....(٣.٤)$$

٣.٥ Boundary Conditions

The requirement that the dependent variable or its derivative must be satisfied on the boundary of the partial differential equation is called the boundary condition. The boundary conditions represent the statements of physical facts at specified values of the independent variable. The initial and the boundary conditions used in the solution of the present problem which is a finite medium subjected to a constant heat flux at one end and insulated at the other end are[٢٦]:

$$\left.\begin{array}{l} T=T_\circ \\ \frac{\partial T}{\partial t}=0 \\ q(x,0)=0 \end{array}\right\} \textbf{at } t=0, x>0 \quad ....(٣.٥)$$

And

$$\left.\begin{array}{l} \left.\begin{array}{l} q(0,t)=Q \\ -k\frac{\partial T(0,t)}{\partial x}=\tau\frac{\partial q(0,t)}{\partial t}+q(0,t) \end{array}\right\} HHCE \\ \left.\begin{array}{l} q(0,t)=Q \\ -k\frac{\partial T(0,t)}{\partial x}=q(0,t) \end{array}\right\} PHCE \end{array}\right\} \quad ....(٣.٦)$$

$$\left.\begin{aligned} \frac{\partial T(L,t)}{\partial x} &= 0 \\ q(L,t) &= 0 \end{aligned}\right\} \text{ for HHCE} \qquad ....(٣.٧)$$

٣.٦ The Non-Dimensional Quantities

The dimensionless form is often more convenient to express the quantities in a form where each term is dimensionless. It is developed to simplify the solution of many engineering problems and to avoid larger quantities in calculation. The non-dimensional groups in this work will be smaller in number than the original number of variables and parameters. This simplifies the mathematics and presentation of results. These are:

$$\left.\begin{aligned} q^* &= \frac{\alpha q}{ck(T_w - T_\circ)} \\ \theta &= \frac{T - T_\circ}{T_w - T_\circ} \\ \delta &= \frac{cx}{2\alpha} \\ \beta &= \frac{c^2 t}{2\alpha} \\ s^* &= \frac{s}{c_p} \end{aligned}\right\} \qquad ....(٣.٨)$$

٣.٧٧ Non-Dimensional Boundary Conditions

Depending on the non-dimensional quantities mentioned above the non-dimensional boundary conditions are :

$$\left.\begin{aligned}&\theta(\delta,0)=0\\&\frac{\partial\theta}{\partial\beta}=0\\&q^*(\delta,0)=0\end{aligned}\right\}\text{at }\beta=0,\delta>0 \qquad ....(٣.٩)$$

$$\left.\begin{aligned}&\left.\begin{aligned}&q^*(0,\beta)=Q^*\\&-\frac{\partial\theta(0,\beta)}{\partial\delta}=\frac{\partial q^*}{\partial\beta}+2q^*\end{aligned}\right\}HHCE\\&\left.\begin{aligned}&-\frac{\partial\theta(0,\beta)}{\partial\delta}=2q^*\\&q^*(0,\beta)=Q^*\end{aligned}\right\}PHCE\end{aligned}\right\} \qquad ....(٣.١٠)$$

$$\left.\begin{aligned}&\frac{\partial\theta(1,\beta)}{\partial\delta}=0\\&q^*(1,\beta)=0\end{aligned}\right\} \qquad ....(٣.١١)$$

٣.٨ Non-Dimensional Governing Equations

The non-dimensional form of the governing equations are:

٣.٨.١ Non-Dimensional Hyperbolic Heat Conduction Equation

The hyperbolic heat conduction equation(٣.١) may be made non-dimensional form by choice of the non-dimensional quantities in (٣.٨):

$$\frac{\partial^2\theta}{\partial\beta^2}+2\frac{\partial\theta}{\partial\beta}=\frac{\partial^2\theta}{\partial\delta^2} \qquad ....(٣.١٢)$$

٣.٨.٢ Non-Dimensional Parabolic Heat Conduction Equation

The parabolic heat conduction equation(٣.٢) may be made non-dimensional form by choice of the non-dimensional quantities in (٣.٨):

$$\frac{\partial\theta}{\partial\beta}=\frac{1}{2}\frac{\partial^2\theta}{\partial\delta^2} \qquad ....(٣.١٣)$$

٣.٨.٣ Non-Dimensional Heat Flux Equation

The heat flux equation (٣.٣) may be made non-dimensional form by choice of the non-dimensional quantities in (٣.٨):

$$2q^* + \frac{\partial q^*}{\partial \beta} = -\nabla \theta \qquad ....(٣.١٤)$$

٣.٨.٤ Non-Dimensional Entropy Generation Equation

The entropy generation equation (٣.٤) may be made non-dimensional form by choice of the non-dimensional quantities in (٣.٨):

$$\frac{ds^*}{d\beta} = \frac{\nabla \cdot q^*}{\theta} - \frac{1}{\theta^2} q^* \cdot \frac{dq^*}{d\beta} \qquad ....(٣.١٥)$$

٣.٩ Finite Difference Solutions

The finite difference method begins with the discretization of space and time such that there is an integer number of points in space and an integer number of times which are called mesh generation.

٣.٩.١ Mesh Generation

The mesh is the set of locations where the discrete solution is computed. These points are called nodes, and if one were to draw lines between adjacent nodes in the domain the resulting image would resemble a net or mesh. Two key parameters of the mesh are Δx, the local distance between adjacent points in space, and Δt, the local distance between adjacent time steps. For the codes developed in this work the discrete x are uniformly spaced in the interval $0 \le x \le L$ **such that:**

....(٣.١٦)

$$x_{(j+1)} = x_{(j)} + \Delta x_{(j)} \qquad j = 1,2,......N$$

Where N is the total number of spatial nodes, including those on the boundary. Given L and N. $\Delta x_{(1)}$ calculated as follows:

$$\left.\begin{aligned}\Delta x_{(j)} &= \Delta x_{(j+1)} = \Delta x_{(1)} \\ \Delta x_{(1)} &= \frac{L}{N}\end{aligned}\right\} \qquad j = 1,\ldots\ldots,N \qquad \text{....(٣.١٧)}$$

Fig.(٣.٣) shows the mesh generation.

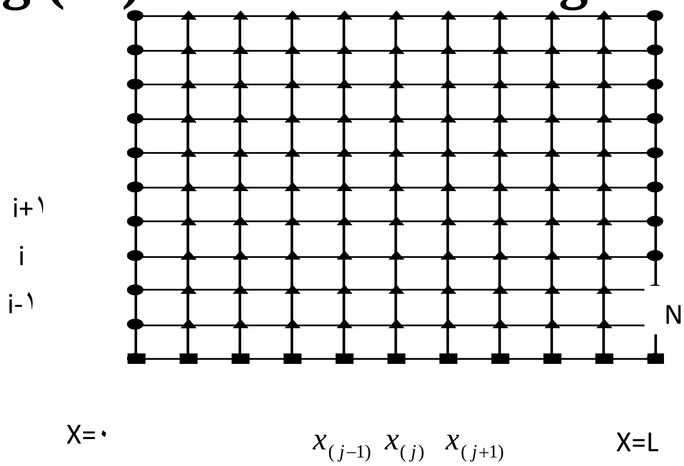


Fig.(٣.٣): Mesh of Uniform Grids.

Similarly, the discrete t is uniformly spaced in $0 \le t \le t_{\max}$:

$$t_{i+1} = t_i + \Delta t\,, \qquad i = 1,2,\ldots.M \qquad \text{....(٣.١٨)}$$

Where M is the number of time steps and Δt is the size of time step calculated as follows:

$$\Delta t = \frac{t_{\max}}{M} \qquad \text{....(٣.١٩)}$$

٣.٩.٢ Explicit Finite Difference Method

The forward Euler algorithm, also called explicit time stepping, uses the field values of only the previous time step to calculate those of the next. This means that the spatial derivatives

will be evaluated at the time step n and the time derivatives n+١. this algorithm is very simple, in that each new temperature, heat flux at time step n+١ is calculated independently.

٣.٩.٢.١ Non-Dimensional Temperature Distributio

Explicit finite difference method will be used to solve the non-dimensional hyperbolic heat conduction equation (٣.١٢) as:

$$\frac{\theta_j^{i+1}+\theta_j^{i-1}-2\theta_j^i}{\Delta\beta^2}+\frac{\theta_j^{i+1}-\theta_j^i}{\Delta\beta}=\frac{\theta_{j+1}^i-(\gamma+1)\theta_j^i+\gamma\theta_{j-1}^i}{(\Delta\delta_{(j)}^{+2}+\gamma\Delta\delta_{(j)}^2)/2} \quad ....(٣.٢٠)$$

After rearranging equation (٣.٢٠) the following equation is obtained,

$$\theta_j^{i+1}=\frac{1}{1+2\Delta\beta}\left[-\theta_j^{i-1}+\theta_j^i(2+2\Delta\beta-\lambda(\gamma+1))+\lambda(\theta_{j+1}^i+\gamma\theta_{j-1}^i)\right] \quad ....(٣.٢١)$$

Where:

$$\gamma=\frac{\Delta\delta_{(j+1)}}{\Delta\delta_{(j)}} \quad ....(٣.٢٢)$$

And

$$\lambda=\frac{\Delta\beta^2}{\Delta\delta_{(j)}^{+2}+\gamma\Delta\delta_{(j)}^2} \quad ....(٣.٢٣)$$

From the initial condition, equation (٣.٩) is written:

$$\frac{\theta_j^{i+1}-\theta_j^{i-1}}{2\Delta\beta}=0\Rightarrow\theta_j^{i-1}=\theta_j^{i+1} \quad ....(٣.٢٤)$$

For starting substitute Equation (٣.٢٤) in equation (٣.٢١) to get,

$$\theta_j^{i+1}=\frac{1}{2+2\Delta\beta}\left[\theta_j^i(2+2\Delta\beta-\lambda(1+\gamma))+\lambda(\theta_{j+1}^i+\gamma\theta_{j-1}^i)\right] \quad ....(٣.٢٥)$$

From the boundary condition at $\delta=0$ equation (٣.١٠) will be,

$$\theta_{\circ}^{i+1} = \theta_{1}^{i+1} + 2q_{\circ}^{*(i+1)}\Delta\delta_{(1)} + \frac{\Delta\delta_{(1)}}{\Delta t}(q_{\circ}^{*(i+1)} - q_{\circ}^{*(i)})$$ ....(٣.٢٦)

From the boundary condition at $\delta = 1$ equation (٣.١١) is written,

$$\frac{\theta_{j+1}^{i} - \theta_{j-1}^{i}}{2\Delta\delta_{(j)}} = 0 \Rightarrow \theta_{j+1}^{i} = \theta_{j-1}^{i}$$ ....(٣.٢٧)

Substituting equation (٣.٢٧) in equation (٣.٢١) to get,

....(٣.٢٨)

$$\theta_{N}^{i+1} = \frac{1}{1+2\Delta\beta}\left[-\theta_{N}^{i-1}(1-\Delta\beta) + \theta_{N}^{i}(2+2\Delta\beta-\lambda(\gamma+1)) + \lambda(\theta_{N-1}^{i} + \gamma\theta_{N-1}^{i})\right]$$

And substituting equation (٣.٢٧) in equation (٣.٢٥) to get,

$$\theta_{N}^{i+1} = \frac{1}{2+2\Delta\beta}\left[\theta_{N}^{i}(2+2\Delta\beta-\lambda(1+\gamma)) + \lambda(\theta_{N-1}^{i} + \gamma\theta_{N-1}^{i})\right]$$ ....(٣.٢٩)

٣.٩.٢.٢ Non-Dim

The temperature distribution for parabolic heat conduction is found by solving equation (٣.١٣) as:

$$\frac{\theta_{j}^{i+1} - \theta_{j}^{i}}{\Delta\beta} = \frac{1}{2}\frac{\theta_{j+1}^{i} - (\gamma+1)\theta_{j}^{i} + \gamma\theta_{j-1}^{i}}{(\Delta\delta_{(j)}^{+2} + \gamma\Delta\delta_{(j)}^{2})}$$ ....(٣.٣٠)

Rearranging equation (٣.٣٠) gives,

$$\theta_{j}^{i+1} = \theta_{j}^{i}(1-(\frac{\lambda_1}{2})(\gamma+1)) + (\frac{\lambda_1}{2})(\theta_{j+1}^{i} + \gamma\theta_{j-1}^{i})$$ ....(٣.٣١)

Where:

$$\lambda_1 = \frac{\Delta\beta}{(\Delta\delta_{(j)}^{+2} + \gamma\Delta\delta_{(j)}^{2})/2}$$ ....(٣.٣٢)

From the boundary condition at $\delta = 0$ equation (٣.١٠) will be,

$$\theta_{\circ}^{i+1} = \theta_{1}^{i+1} + 2q_{\circ}^{*(i+1)}\Delta\delta_{(1)}$$ ....(٣.٣٣)

After substitute the boundary condition $\delta = 1$ equation (٣.١١), equation (٣.٣١) is written :

$$\theta_N^{i+1} = \theta_N^i (1 - (\frac{\lambda_1}{2})(\gamma + 1)) + (\frac{\lambda_1}{2})(\theta_{N-1}^i + \gamma \theta_{N-1}^i) \qquad \text{....(٣.٣٤)}$$

٣.٩.٢.٣ Non-dimensional Hea

The heat flux distribution is found by solving equation (٣.١٤) by the finite difference method.

$$\frac{q_j^{*(i+1)} - q_j^{*(i)}}{\Delta\beta} + 2q_j^{*(i)} = -\left[\frac{\theta_{j+1}^i + (\gamma^2 - 1)\theta_j^i - \gamma^2\theta_{j-1}^i}{(\Delta\delta_{(j)}^+ + \gamma^2 \Delta\delta_{(j)})}\right] \qquad \text{....(٣.٣٥)}$$

After rearranging equation (٣.٣٥) we get,

$$q_j^{*(i+1)} = q_j^{*(i)} (1 - 2\Delta t) - \lambda_2 \left[\theta_{j+1}^i + (\gamma^2 - 1)\theta_j^i - \gamma^2 \theta_{j-1}^i\right] \qquad \text{....(٣.٣٦)}$$

Where:

$$\lambda_2 = \frac{\Delta\beta}{(\Delta\delta_{(j)}^+ + \gamma^2 \Delta\delta_{(j)})} \qquad \text{....(٣.٣٧)}$$

The boundary condition at $\delta = 0$ and $\delta = 1$ represented by (٣.١٠) and (٣.١١) respectively.

٣.٩.٢.٤ Non-Dimensior

Vazquez and Jou [٣٣],٢٠٠٣, simplify equation (٣.٤) in the form below [see appendix A]. to find the change of entropy directly:

$$\Delta s^* = 0.5\left[\frac{q}{\theta}\right]^2 \qquad \text{....(٣.٣٨)}$$

Then solved it directly by using the explicit finite difference as:

$$(\Delta s^{*})_{j}^{i} = 0.5\left[\frac{q_{j}^{*(i)}}{\theta_{j}^{i}}\right]^{2} \quad ....(٣.٣٩)$$

٣.٩.٣ Case

Aluminum is used to solve the hyperbolic heat conduction, heat flux, entropy change.

٣.٩.٣.١

The temperature distribution through an Aluminum slab is found by solving equation (٣.١) by the finite difference method,

$$\tau\left[\frac{T_{j}^{i+1}+T_{j}^{i-1}-2T_{j}^{i}}{\Delta t^{2}}\right]+\frac{T_{j}^{i+1}-T_{j}^{i}}{\Delta t}=\alpha\,\frac{T_{j+1}^{i}+T_{j-1}^{i}-2T_{j}^{i}}{\Delta x_{(j)}^{2}} \quad ....(٣.٤٠)$$

After rearranging (٣.٤٠) we get,

$$T_{j}^{i+1}=\frac{1}{1+\Delta t/\tau}\left[-T_{j}^{i-1}+T_{j}^{i}(2+\frac{\Delta t}{\tau}-2\lambda_{3})+\lambda_{3}(T_{j+1}^{i}+T_{j-1}^{i})\right] \quad ....(٣.٤١)$$

Where:

$$\lambda_{3}=\frac{\alpha\Delta t^{2}}{\tau\Delta x_{(j)}^{2}} \quad ....(٣.٤٢)$$

after substituting the initial condition (٣.٥) we get,

$$T_{j}^{i+1}=\frac{1}{2+\Delta t/\tau}\left[T_{j}^{i}(2+\frac{\Delta t}{\tau}-2\lambda_{3})+\lambda_{3}(T_{j+1}^{i}+T_{j-1}^{i})\right] \quad ....(٣.٤٣)$$

From the boundary condition (٣.٦) we get,

$$T_{\circ}^{i+1}=\left[T_{1}^{i+1}+\frac{\Delta x_{(1)}\tau}{k\Delta t}(q_{0}^{i+1}-q_{0}^{i})+\frac{\Delta x_{(1)}}{k\tau}q_{0}^{i}\right] \quad ....(٣.٤٤)$$

Substitute the boundary condition $x = L$ (٣.٧) in (٣.٤١) to get,

$$T_{N}^{i+1}=\frac{1}{1+\Delta t/\tau}\left[-T_{N}^{i-1}+T_{N}^{i}(2+\frac{\Delta t}{\tau}-2\lambda_{3})+\lambda_{3}(T_{N-1}^{i}+T_{N-1}^{i})\right] \quad ....(٣.٤٥)$$

And substitute (٣.٧) in (٣.٤٣) to get,

$$T_N^{i+1} = \frac{1}{2+\Delta t/\tau}\left[T_N^i\left(2+\frac{\Delta t}{\tau}-2\lambda_3\right)+\lambda_3\left(T_{N-1}^i+T_{N-1}^i\right)\right] \quad \text{....(٣.٤٦)}$$

٣.٩.٣.٢

The heat flux distribution is found by solving equation (٣.٢) using the finite difference method,

$$\tau\left[\frac{q_j^{i+1}-q_j^i}{\Delta t}\right]+q_j^i = -k\frac{T_{j+1}^i-T_j^i}{\Delta x_{(j)}} \quad \text{....(٣.٤٧)}$$

After rearranging equation (٣.٤٧) we get,

$$q_j^{i+1} = q_j^i\left(1-\frac{\Delta t}{\tau}\right)-\frac{\Delta tk}{\tau\Delta x_{(j)}}\left(T_{j+1}^i-T_j^i\right) \quad \text{....(٣.٤٨)}$$

At $x=0$ the boundary condition is equation (٣.٦) and at $x=L$ is (٣.٧)

٣.٩.٣.٣

Vazquez and Jou [٣٣],٢٠٠٣, simplify equation (٣.٤) in the form below [see Appendix A]. to find the change of entropy directly:

$$\Delta s = \frac{\tau q^2}{2\rho k T_{ne}^2} \quad \text{....(٣.٤٩)}$$

And solved equation (٣.٤٩) directly by using explicit finite difference method,

$$\Delta s_{(j)} = \frac{\tau q_j^{(i)2}}{2\rho k T_{(j)ne}^{(i)2}} \quad \text{....(٣.٥٠)}$$

٣.١٠ Computational Procedure and Computer Program

In this chapter the computer program is developed to calculate the non-equilibrium (hyperbolic) temperature distribution, the entropy change, heat flux, and equilibrium

(parabolic) temperature distribution for finite medium.

٣.١١ Computer Program

A computer program was written in Quick Basic language to perform the numerical solution formulated previously. The program is called "**NEHCFMSCHF"** (Non-Equilibrium Heat Conduction in Finite Medium Subjected by Constant Heat Flux).

٣.١١.١.٢ Program of Non-Dimensional Solution

The program consists of five steps. The first is for uniform mesh for the slab. The second deals with non-dimensional hyperbolic temperature distribution for the slab. The third deals with the non-dimensional heat flux distribution. The fourth is for non-dimensional entropy change. The fifth is for the non-dimensional parabolic temperature distribution. The flowchart of the program is shown in figure(٣-٤).

The inputs of “**NEHCFMSCHF”** for non-dimensional solutions are:

- ❖ The non-dimensional distance (δ).
- ❖ The non-dimensional time (β).
- ❖ The non-dimensional heat flux.
- ❖ The total number of spatial nodes (N).
- ❖ Number of time steps (M).

The outputs of “**NEHCFMSCHF”** for non-dimensional solution are:

- ❖ The slab grid generation.
- ❖ Non-dimensional hyperbolic temperature distribution.

- Non-dimensional heat flux distribution .
- Non-dimensional entropy change .

٣.١١.٢.٢ A Program for the Case Study

The program consists of five steps. The first is for uniform mesh for the slab. The second deals with hyperbolic temperature distribution for the slab. The third deals with the heat flux distribution. The fourth is for entropy change. The fifth is for the parabolic temperature distribution. The flowchart of the program is shown figure(٣-٦).

The inputs of Aluminum solution:

- Slab length (L).
- Max time (t).
- The heat flux (q).
- The total number of spatial nodes (N).
- Number of time steps (M).

The outputs of Aluminum solution:

- The slab grid generation.
- Hyperbolic temperature distribution.
- Heat flux distribution .
- Entropy change.

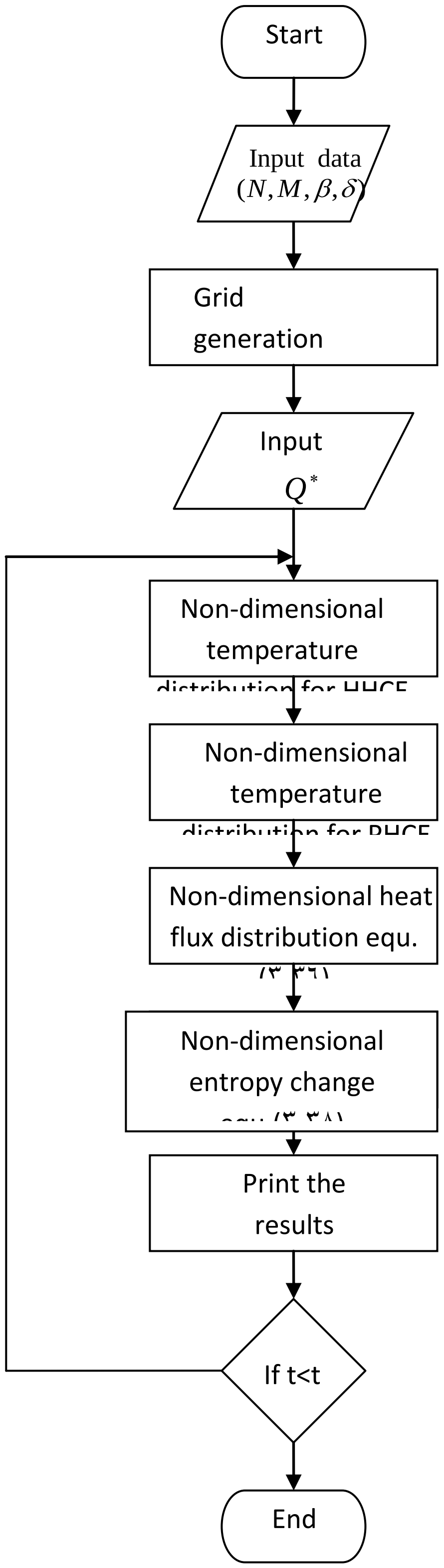


Figure (٣-٥): Flowchart

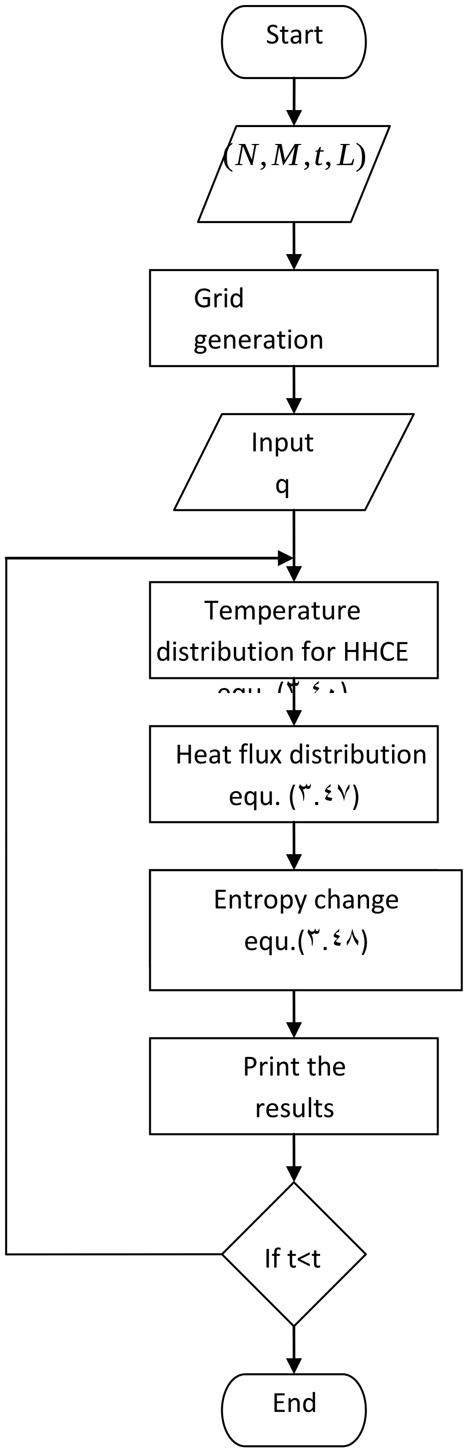


Figure (٣-٦): Flowchart

CHAPTER FOUR 4

RESULTS AND DISCUSSION

٤.١ Non-Dimensional

Non-dimensional solution contains non-dimensional temperature distribution, non-dimensional heat flux distribution, non-dimensional entropy change distribution. the parameters are known as $(\delta = 1, \beta = 2.5)$ and $(Q^* = 0.5, 1.0, 10)$.

٤.١.١ Non-Dimensional Temperature Distribution

Figure (٤-١) shows a general relation of undisturbed and thermal wave regions. At any instant of time an undisturbed and thermal wave regions exit. Any point above the line $(\delta = \beta)$

has a large distance from the wall $(\delta = 0)$ and small time (i.e., $\delta > \beta$), this region is called Undistributed region because at the distance further from the wall than β, the temperature change at the wall has not be felt.

On the other hand, any point below the line $(\delta = \beta)$ has a small distance from the wall and a large time (i.e., $\delta < \beta$) this region called Thermal wave region [٢٦], where the effect of temperature change is felt. This indicates that the line ($\delta = \beta$) represents the separation limit between the undisturbed and wave regions.

Figures from (٤-٢) to (٤-١٣) show the Fourier(diffusion) and non-Fourier(wave) temperature distributions in a finite medium subjected to a constant heat flux (٠.٠٥,١.٠,١٠) against the non-dimensional distance for various values of non-dimensional time. The figures from (٤-٢)

to (٤-٥) for non-dimensional $Q^* = 0.5$, figures from (٤-٦) to (٤-٩) for $Q^* = 1.0$, and figures from (٤-١٠) to (٤-١٣) for $Q^* = 10$ show hyperbolic non-Fourier wave fronts in contrast to the continues Fourier temperature profiles. The largest differences between Fourier and non-Fourier temperature distribution occur at β=٠.١ and ٠.٢(at the start of process). Also, the figures predict the propagation of thermal waves traveling through the medium at a finite speeds of heat propagation. The heat flux is the same for both models (Fourier and non-Fourier) at each time, the difference between the two results is due to the fact that the non-Fourier model requires a finite build –up time for start of heat conduction, while the Fourier model predicts instantaneous propagation through the medium. The discontinuity predicted by the

non-Fourier heat conduction is due to this finite speed. These figures show that for various small duration time (at the start of process) the non-Fourier model predicts higher temperature distributions than the Fourier model. All figures show thermal wave propagates from the left hand side (the source of heat) towards the right hand side (the insolated end) that the temperature distribution of non-Fourier heat conduction has different behaviors. Figure (٤-٤) for $Q^* = 0.5$, figure from (٤-٨) for $Q^* = 1.0$, and figure from (٤-١٢) for $Q^* = 10$ when the wave arrived at the second end (non-dimensional distance $\delta = 1$) at non-dimensional times β is greater than ١.٠ and smaller than ١.٤ the thermal wave is bounces back to the medium which causes an increase in the temperature distribution profile. The reason of the thermal wave bounces back is the isolated end of the medium. It is shown that at non-dimensional time equals to ١.١ the non-Fourier profile approaches the Fourier profile of the non-dimensional temperature distribution. This is due to the forward and backward wave traveling through the system, which are shown in figure(٤-٥) for $Q^* = 0.5$, figure (٤-٩) for $Q^* = 1.0$, and figure (٤-١٣) for $Q^* = 10$ when the non-dimensional time is greater than ١.٤. These figures show that the amplitude of thermal wave is gradually damped as the time increased. It is noticed that at longer times the intensity of the thermal wave decays and approaches the diffusion behavior.

On the other hand, the process where the thermal wave bounces back is not evident in the Fourier model, which displays a monotonic decay to its steady state value. It only predicts

heat flow from $\delta=0$ to $\delta=1$ for all values of non-dimensional time. This is due to instantaneous energy propagation. Fig.(٤-١٤) shows the non-dimensional temperature distribution with a non-dimensional distance at the same non-dimensional time ($\beta=0.2$)for different values of non-dimensional heat flux (٠.٠٥,١.٠,١٠). The figure shows that when the non-dimensional heat flux increases the wave propagate increases due to the increase of the wave speed.

٤.١.٢ Non-Dimensional Heat Flux Distribution

Figures from (٤-١٥) to (٤-٢٦) show the variation of non-dimensional heat flux with non-dimensional distance for various value of a non-dimensional time in a slab subjected to a constant heat flux (٠.٠٥,١.٠,١٠). At $\delta=0$, its noticed that the heat flux is the greater for all values of non-

dimensional time because the input of heat flux in the left hand side. In all figures the wave nature of non-dimensional heat flux is noticed due to the finite speed of heat propagation. The figures explain that along the medium $\delta > 0$, the non-dimensional heat flux is increased with increasing of the non-dimensional time due to the accumulative effect of the heat flux with increasing values of non-dimensional time. When the non-dimensional times are greater than ١.٠ the heat flux wave reached to the right-hand side and bounces back to the medium, which causes an increase in non-dimensional heat flux. This is shown in figure (٤-١٧) for $Q^* = 0.5$, figure(٤-٢١) for $Q^* = 1.0$, and figure (٤-٢٥) for $Q^* = 10$. Figure(٤-١٨) for $Q^* = 0.5$, figure (٤-٢٢) for $Q^* = 1.0$, and figure (٤-٢٦) for $Q^* = 10$ shows that the non-dimensional heat flux profiles increased as the

non-dimensional time increased and reached to the steady state where the maximum values at the left side and the minimum values at the right side. Figure(٤-٢٧) shows the non-dimensional heat flux distribution with the non-dimensional distance at the same non-dimensional time ($\beta = 0.2$)for different values of non-dimensional heat flux (٠.٠٥,١.٠,١٠). The figure shows that when the non-dimensional heat flux increases the non-dimensional heat flux distribution increasing which causes convert the problem from non-Fourier heat conduction to an other problem.

٤.١.٣ Non-Dimensional Entropy Change Distribution

Figures from (٤-٢٨) to (٤-٣٩) show the variation of non-dimensional entropy change with non-dimensional distance for various values of the

non-dimensional time in a finite medium subjected to a different values of constant heat flux(٠.٠٥,١.٠,١٠). All figures show that at $\delta = 0$ the non-dimensional entropy change starts with a maximum value and then decreases with increasing of non-dimensional time, because the values of non-dimensional heat flux constant during all values of non-dimensional time and the non-dimensional temperature increased with increasing of non-dimensional. Also the figures explain that along the medium at $\delta > 0$ the non-dimensional entropy increases with increasing of non-dimensional time, because the local non-dimensional heat flux increases with increasing of non-dimensional time due to the accumulation effect of the heat flux and the increases of non-dimensional temperature with increasing of non-dimensional time. Also, a

wave front is noticed in figures during the values of the non-dimensional time less than ١.٠. when the non-dimensional time greater than ١.٠, the reflection effect takes place which increases the non-dimensional entropy change due to the increase of non-dimensional heat flux and non-dimensional temperature as shown in figure (٤-٣٠) for $Q^* = 0.5$, figure (٤-٣٤) for $Q^* = 1.0$, and figure (٤-٣٨) for $Q^* = 10$. With increasing values of non-dimensional time the effect of reflection is decreased which causes a decreasing in non-dimensional entropy change. Figure (٤-٤٠) shows the non-dimensional entropy change distribution with non-dimensional distance at the same non-dimensional time ($\beta = 0.1$)for different values of non-dimensional heat flux (٠.٥,١.٠,١٠). The figure shows that the entropy change distribution increases, this due to the increases of non-dimensional heat flux and increases the non-dimensional temperature distribution.

٤.٢ Case Study

The temperature distribution, heat flux distribution, and entropy change for an Aluminum media are presented and discussed in this section. And the physical properties of Aluminum are:

$(\tau = 10^{-12}, L = 10^{-5} m, \alpha = 8.418 \times 10^{-5}\, m^2/\sec, k = 204\, w/m.C^\circ, \rho = 2707\, kg/m^3, c_p = 0.896\, kJ/kg.C^\circ)$,

$(Q = 10^8 w, 10^9 w)$ and $(t_{\max} = 30\, pico\,second, pico = 10^{-12}\, second)$.

٤.٢.١ Temperature Distribution

Figures from (٤-٤١) to (٤-٤٨) show the temperature distribution of an Aluminum finite medium subjected to a various values constant heat flux ($100Mw, 1000Mw$) plotted against the distance for various values of time. Figure (٤-٤١)

and figure (٤-٤٢) for $Q = 100Mw$, figure (٤-٤٥) and figure (٤-٤٦) for $Q = 1000Mw$ predict the propagation of thermal waves traveling through the medium at finite speeds of heat propagation. The discontinuity predicted by the non-Fourier heat conduction is due to this finite speed. Figure (٤-٤٣) for $Q = 100Mw$, and figure (٤-٤٧) for $Q = 1000Mw$ show that the thermal wave is bounces back to the medium which causes an increase in the temperature distribution profile which takes place when time is greater than $(3*10^{-12})$. The reason of the thermal wave bounces back to the medium is the isolated end of the Aluminum medium. In figure(٤-٤٤) Aluminum reached to the steady state, and at this heat flux $(Q = 100Mw)$ the Aluminum is heating only but, in figure(٤-٤٨) the Aluminum reached to the steady state at heat flux $(Q = 1000Mw)$ the Aluminum under the time =٢٢

picoseconds is heating above this time the problem convert to phase change . When increasing the heat flux greater than $1000Mw$ the problem becomes from one-dimensional non-Fourier heat conduction to one-dimensional non-Fourier phase change.

٤.٣.٢ Heat Flux Distribution

Figures from (٤-٤٩) to (٤-٥٦) show the variation of heat flux of Aluminum with distance for various values of time in slab subjected to a constant heat fluxes $(100Mw, 1000Mw)$. At $x = 0$, its noticed that the heat flux is the greater for all values of time (the input heat flux at the left end). In all figures the wave nature of heat flux is noticed due to the finite speed of heat propagation. The figures explain that along the medium $x > 0$, the heat flux is increased with increasing of time due to the accumulation

effect of the heat flux . When the values of time are greater than $(3*10^{12}\ \sec ond)$ **the heat flux wave bounces back from the right-hand side to the medium which causes an increase in heat flux and this is shown in figure (٤-٥١) for** $Q = 100Mw$**, and figure (٤-٥٥) for** $Q = 1000Mw$**.**

٤.٣.٣ Entropy Change Distribution

Figures from (٤-٥٧) **to** (٤-٦٢) **show the variation of entropy change of Aluminum with distance for various values of time in a finite medium subjected to a constant heat fluxes** $(100Mw, 1000Mw)$**. The figures show that at** $x = 0$ **the entropy change starts with a maximum value and then decreases with increasing of time, because the values of heat flux constant during all values of time and the temperature increased with increasing of time. The figures explain that along the medium at** $x > 0$ **the entropy change**

increases with increasing of time, because the heat flux increases with increasing of time due to the accumulation effect of the heat flux and the increases of temperature with increasing of time which is shown in figure (٤-٥٧) for $Q = 100 Mw/m^2$, and figure (٤-٦٠) for $Q = 1000 Mw/m^2$ and a wave front is noticed in the figures which are shown in figures (٤-٥٧) and (٤-٦٠). The reflection effect of entropy change is introduced (time=$3*10^{-12}$ sec) in to account which increases the entropy change due to the increase of heat flux as shown in figure(٤-٥٨) for $Q = 100 Mw$, and figure (٤-٦١) for $Q = 1000 w$. With increasing values of time the effect of reflection is decreased which causes a decreasing in entropy change until equilibrium which is shown in figure (٤-٥٩) for $Q = 100 Mw$, and (٤-٦٢) for $Q = 1000 Mw$.

CHAPTER FIVE 5

CONCLUSIONS AND SUGGESTIONS

٥.١ Conclusions

Based on the results obtained from the computer program, which is presented in Chapter Four, and the discussions of the results in Chapter Five, the following conclusions can be drawn:

١- It is found that for short times the non-Fourier temperatures are higher than the corresponding Fourier values. When time increases which means that the relaxation time approaches to zero, the non-Fourier heat conduction converge to Fourier heat conduction.

٢- It is found that the non-Fourier effects are important during the initial stages of the heat transfer process or when the imposed thermal conditions are characterized by short time scales that are less than the magnitude of relaxation time, non-Fourier heat conduction equation is more suitable for dimensions from nano to micro scales and times from Pico to femto second.

٣- It is shown that the equilibrium entropy change in the framework of classical irreversible thermodynamics is not compatible with the non-Fourier heat conduction.

٤- The non-equilibrium heat conduction shows wave type characteristic and sharp discontinuous in the temperature distribution, heat flux and entropy change at the initial stage of the process .

٥- It is found that when the surface heat flux is greater than ١٠Mw, the classical Fourier heat conduction fails and the non-Fourier heat conduction is suitable for this problem.

٥.٢ Suggestions for Further

The following recommendations are suggested for further work:

١- Studying the non-Fourier Heat Conduction Equation with temperature dependent thermal properties (thermal conductivity and specific heat).

٢- Studying the non-Fourier Heat conduction equation with moving heat source.

٣- Using ANSYS to find temperature and heat flux distribution for non-Fourier heat conduction equation.

٤- Studying non-Fourier heat conduction equation in composite media.

٥- Studying Ballistic-Diffusive heat conduction equation in gas flow in microstructure with constant heat flux.

٦- Studying non-Fourier phase change in solids.

٧- Studying two dimensional non-Fourier heat conduction equation.

٨- Studying the extended irreversible thermodynamics of non-Fickian mass transfer or non-Newtonian fluid mechanics.

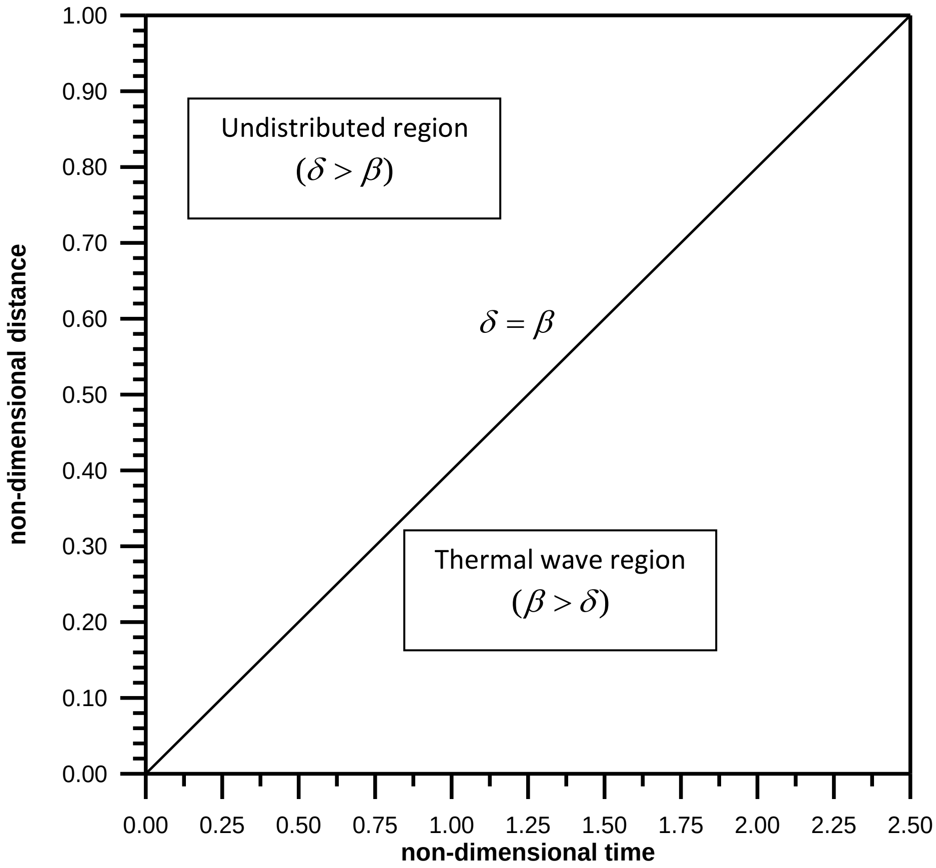


Fig (٤-١): A Relation Between δ and β.

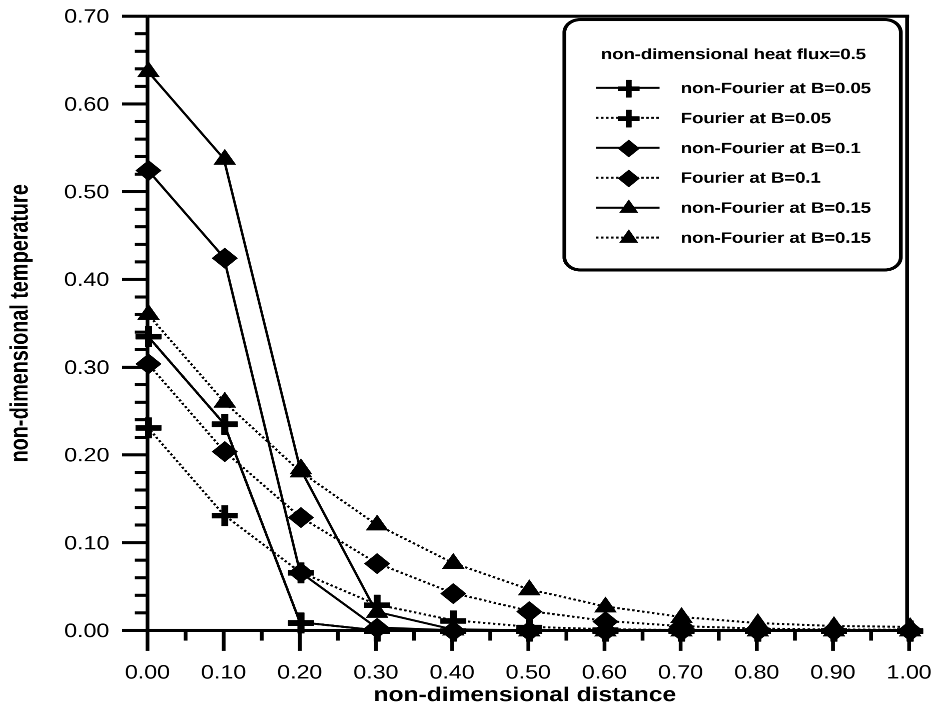


Fig (٤-٢): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.

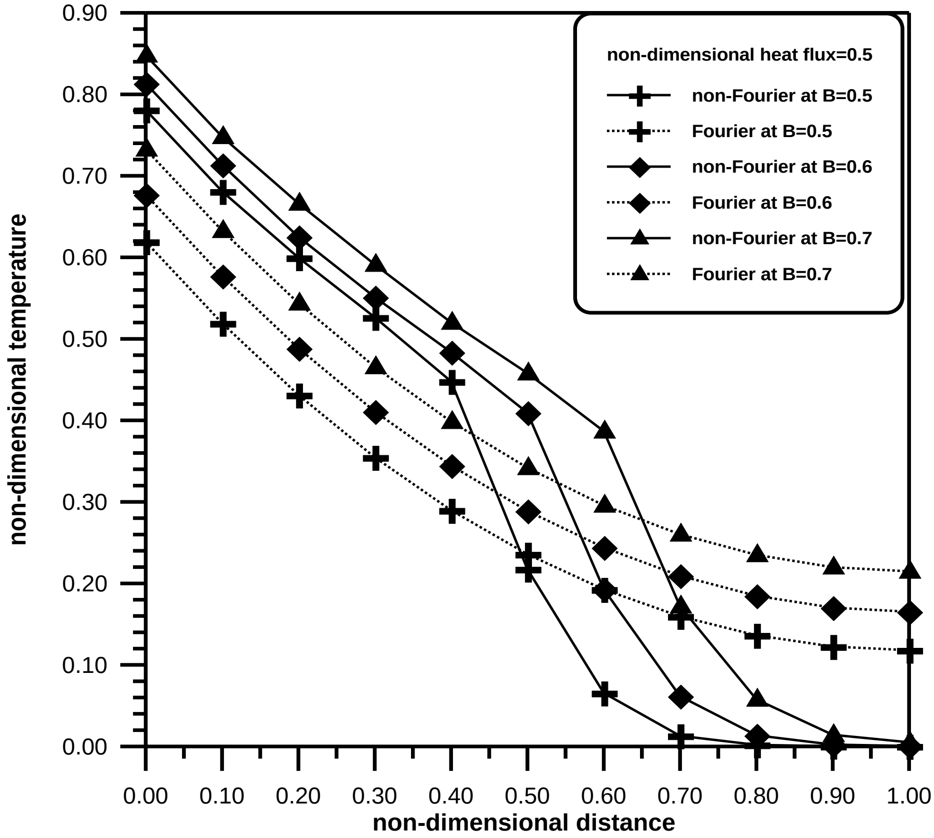


Fig (٤-٣): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.

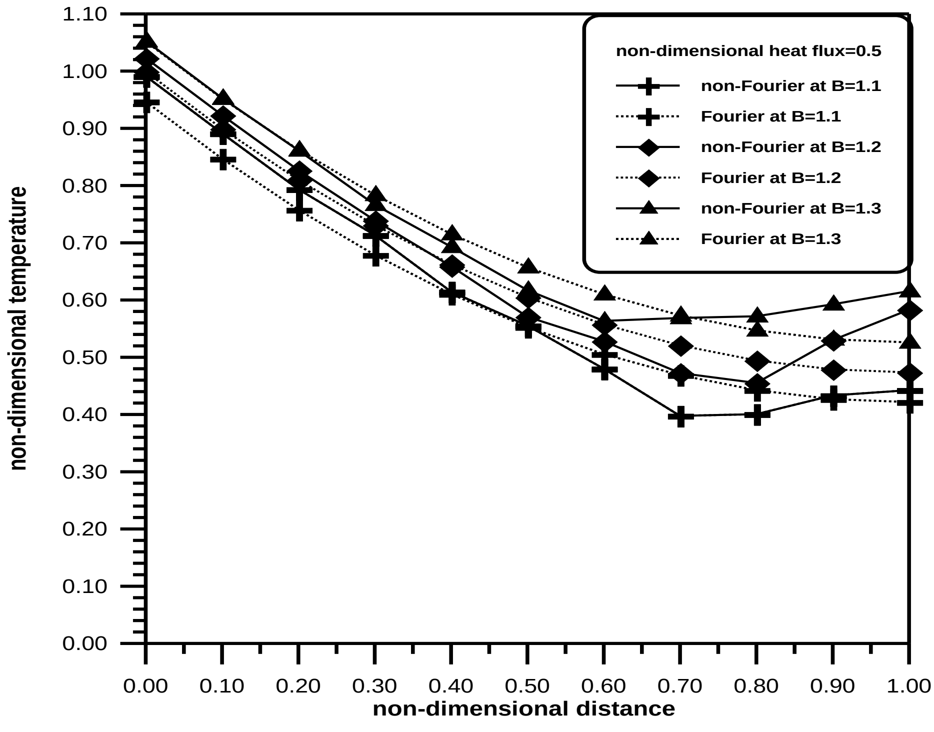
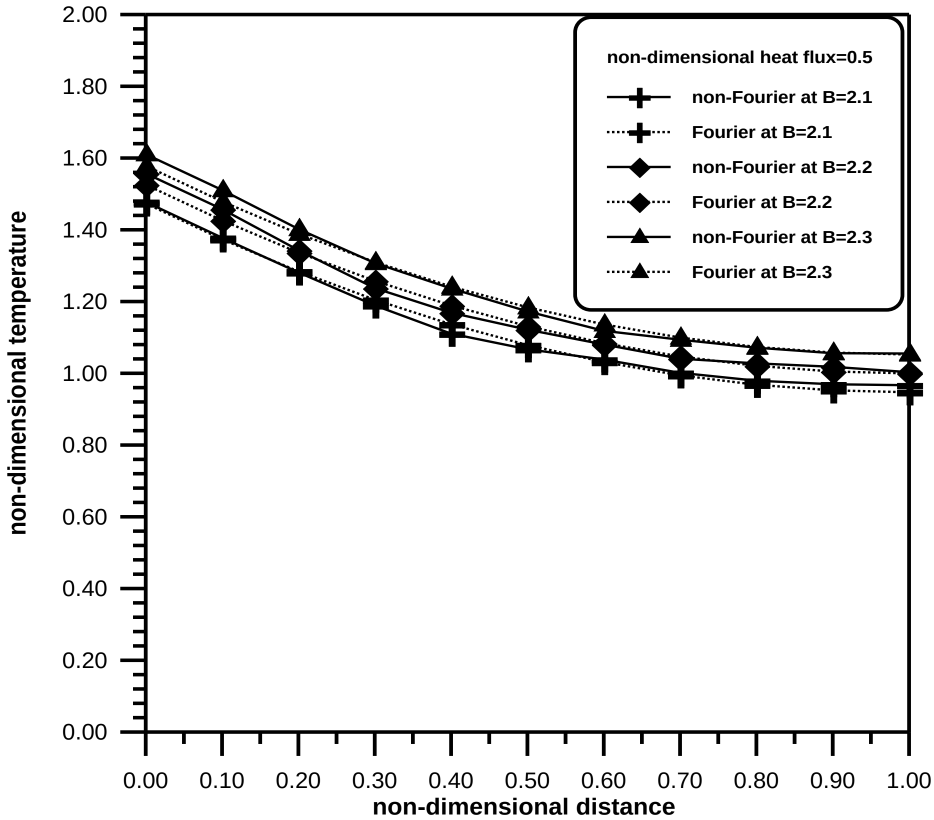
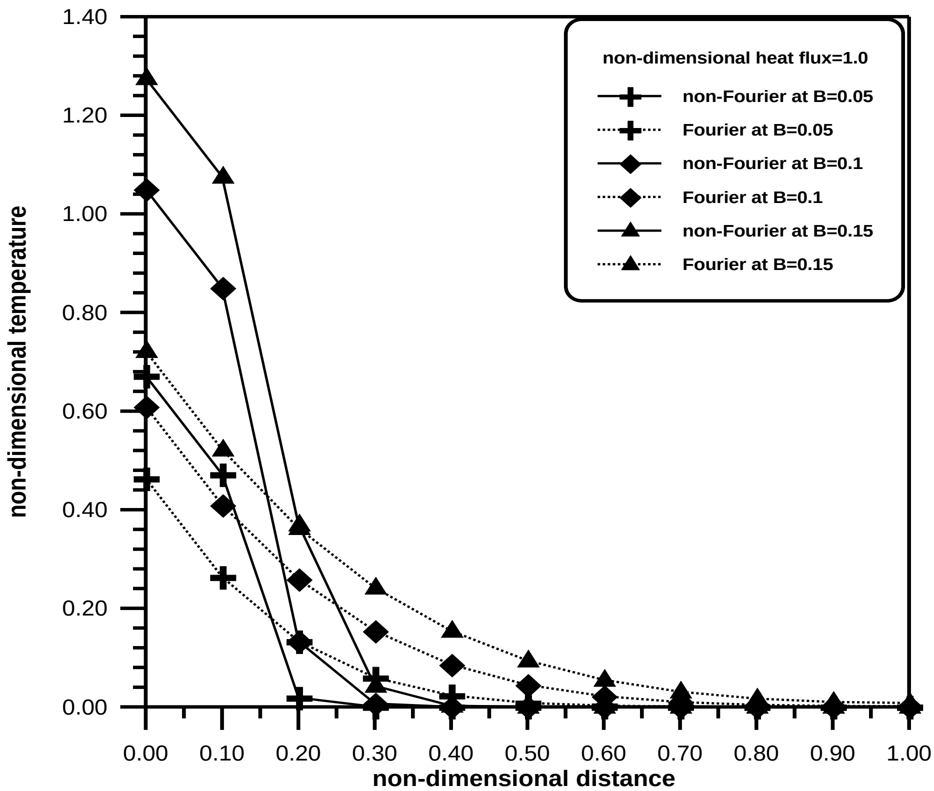


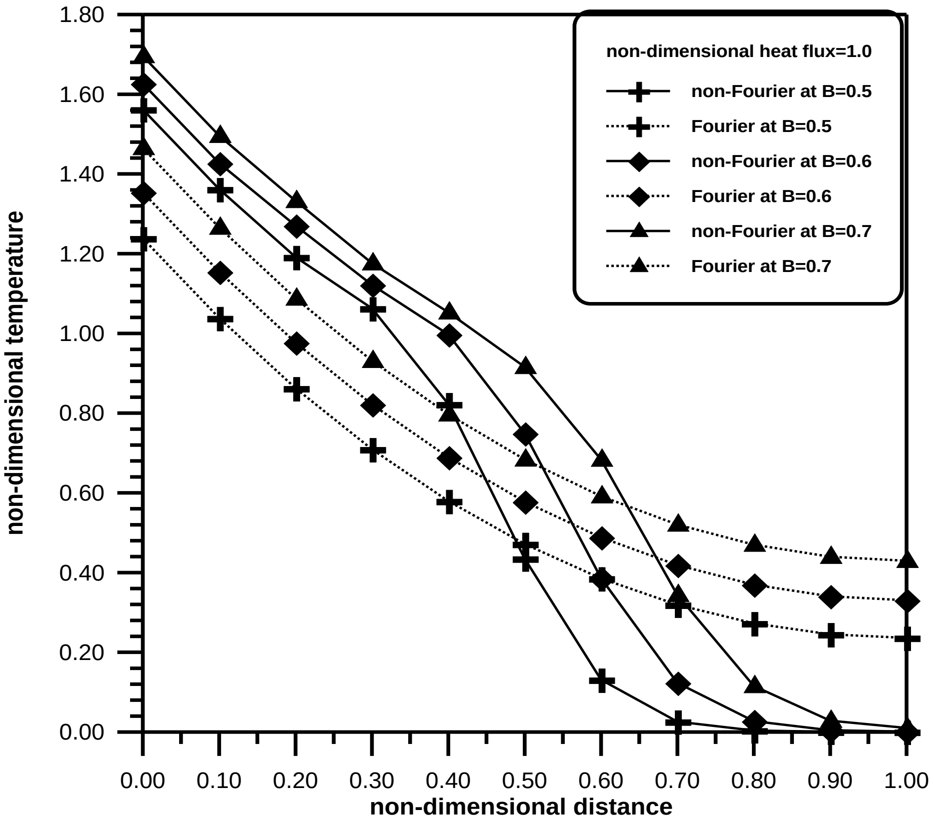
Fig (٤-٤): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



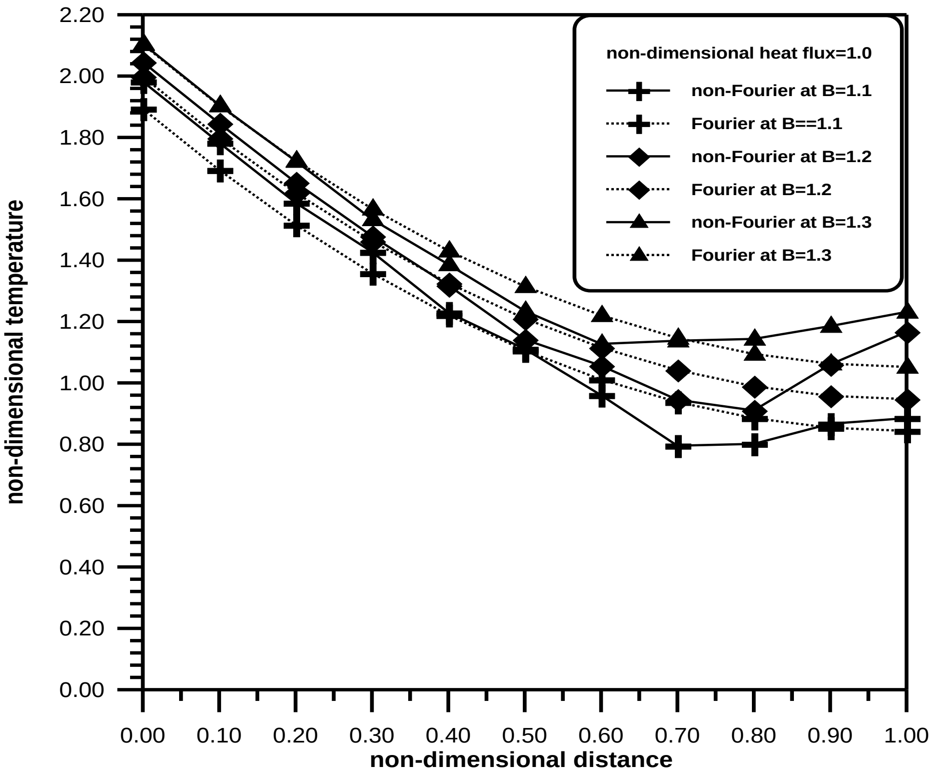
(٤-٥): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



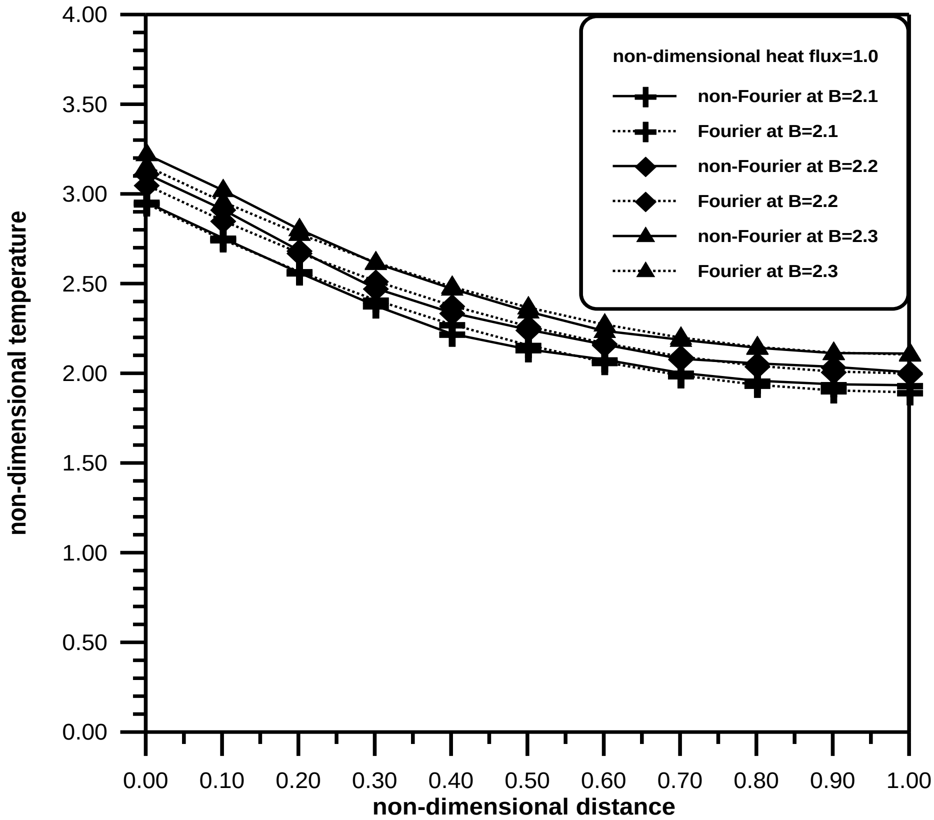
(٤-٦): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



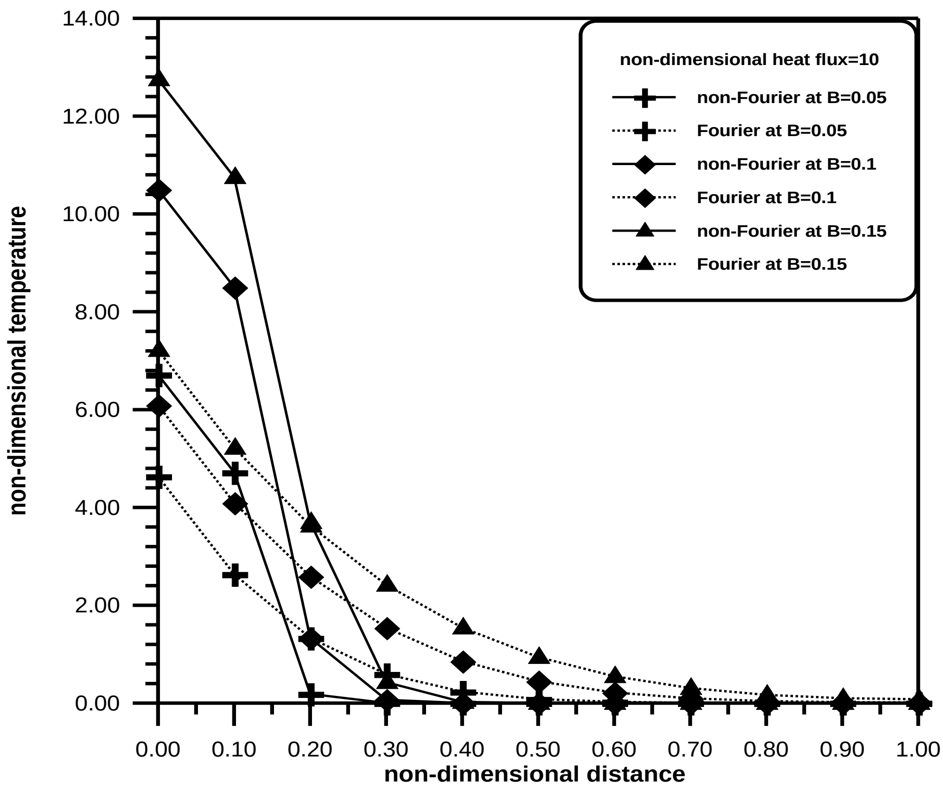
(٤-٧): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux .



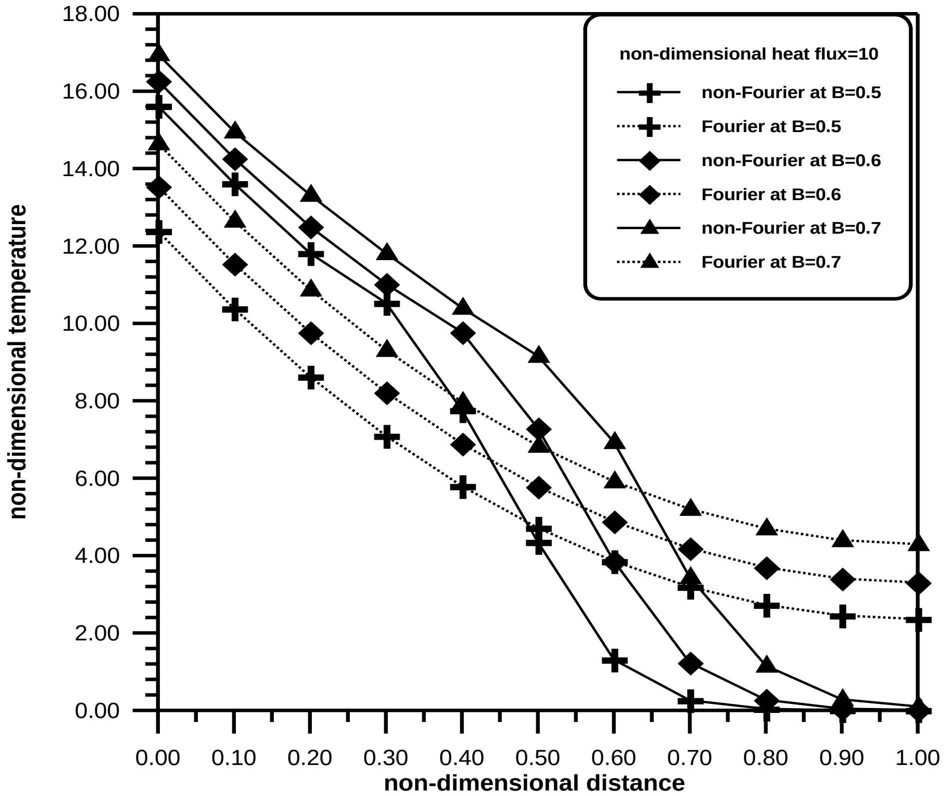
Fig(٤-٨): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



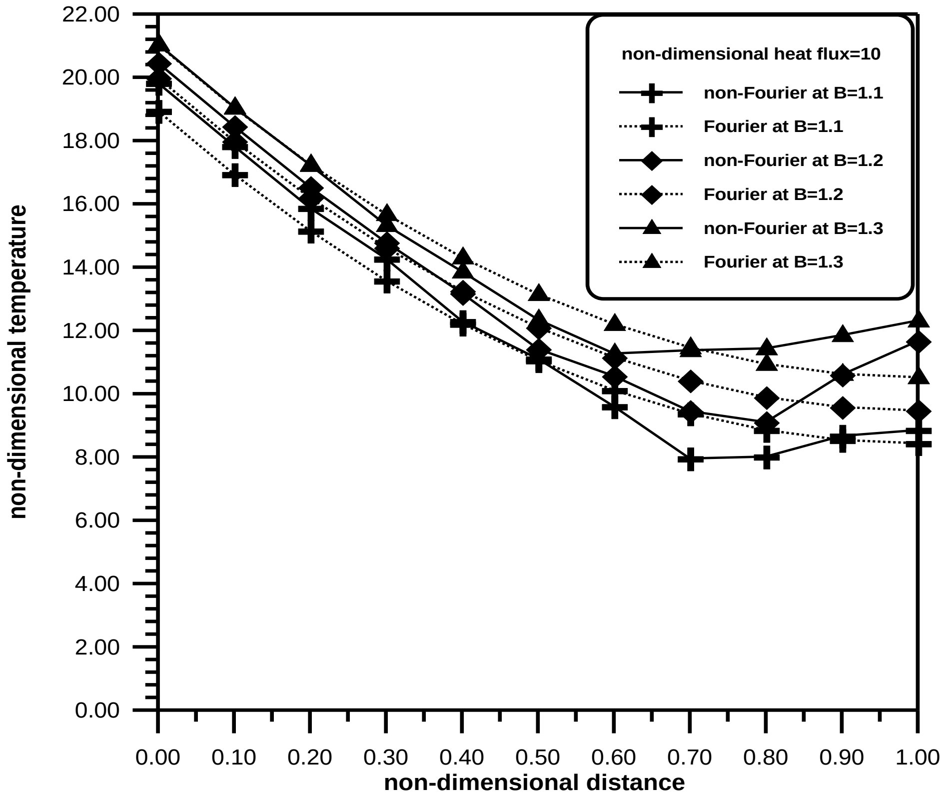
Fig(٤-٩): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



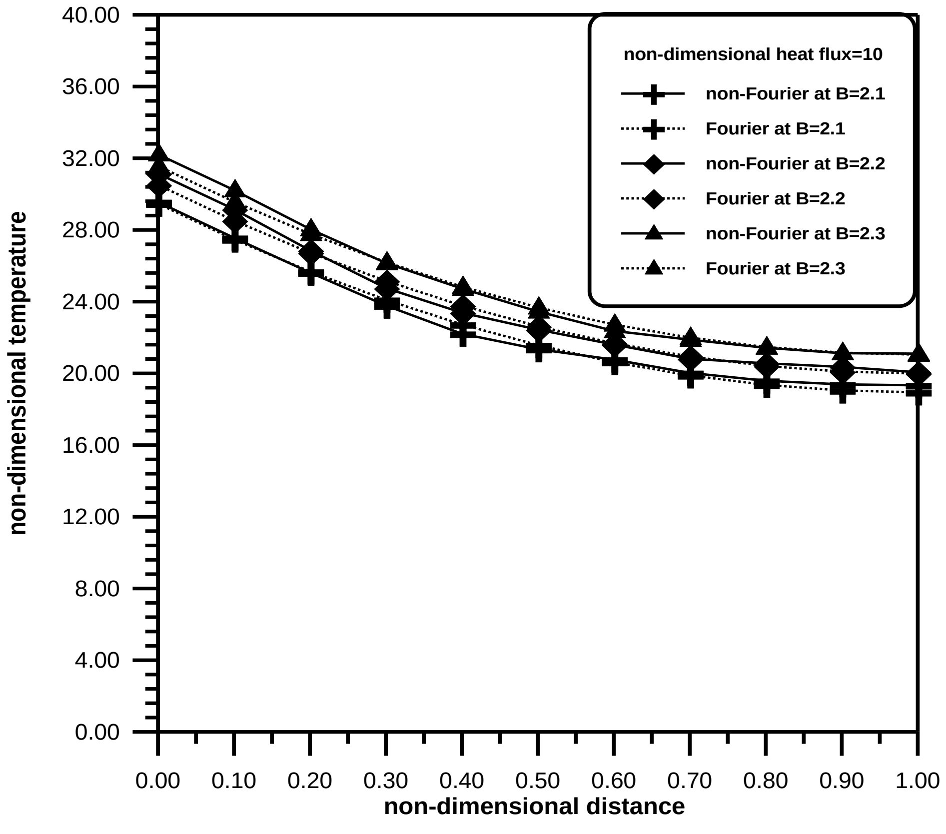
Fig(٤-١٠): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



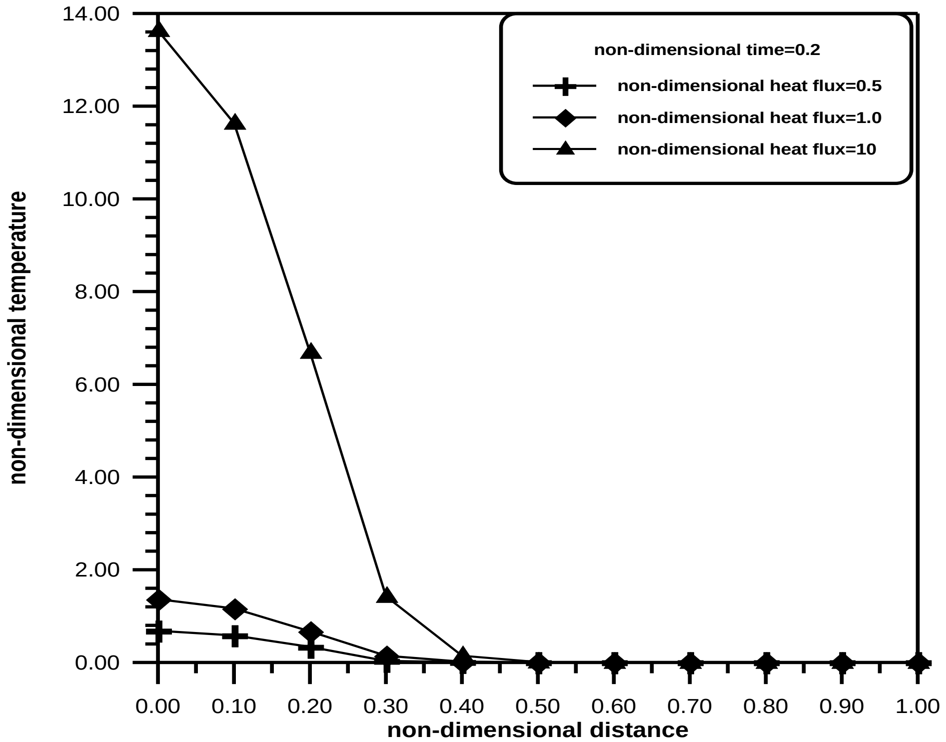
Fig(٤-١١): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



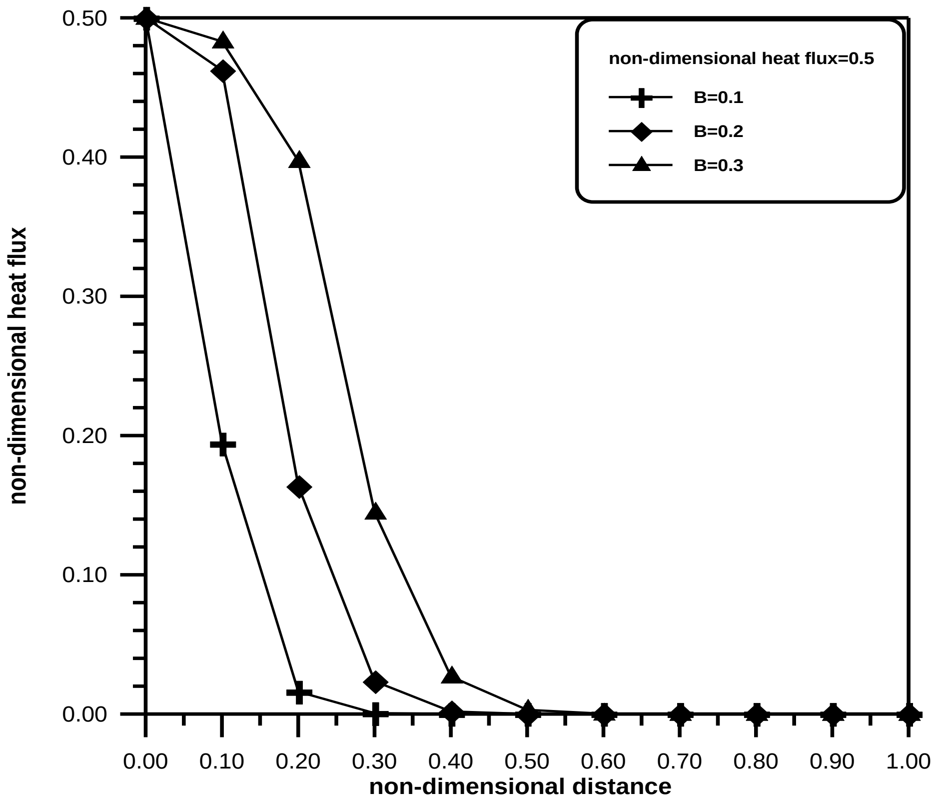
Fig(٤-١٢): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



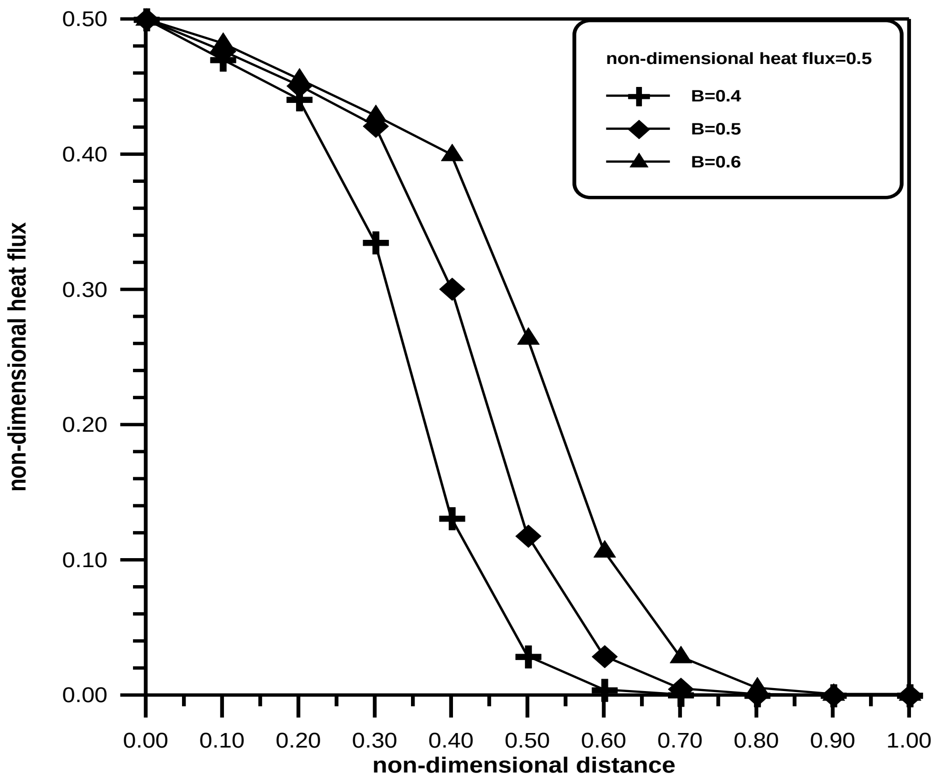
Fig(٤-١٣): Temperature Distribution in a Finite Medium Subjected by Constant Heat Flux.



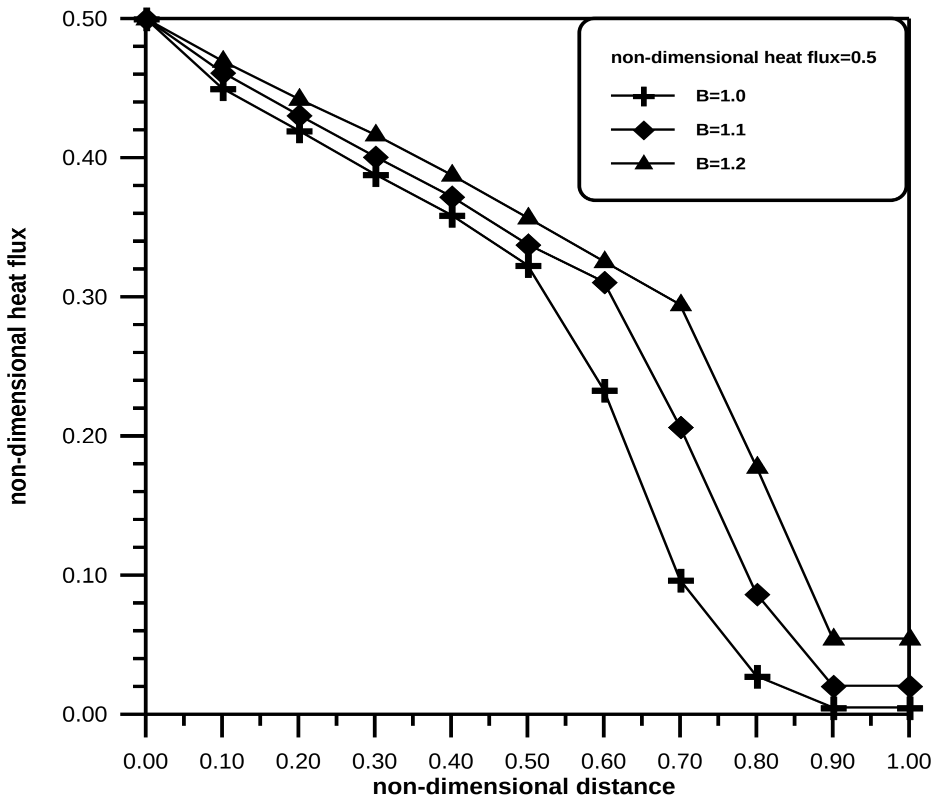
Fig(٤-١٤): Temperature Distribution in a Finite Medium Subjected to a Various Values of Constant Heat Flux at $\beta = 0.2$.



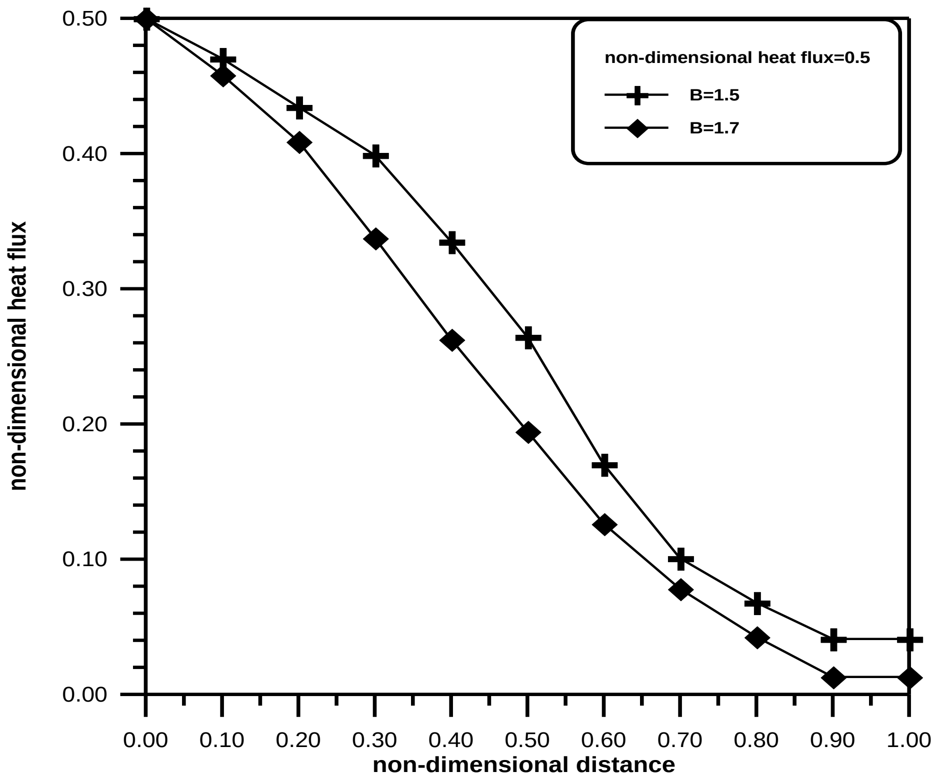
Fig(٤-١٥): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Constant Heat Flux.



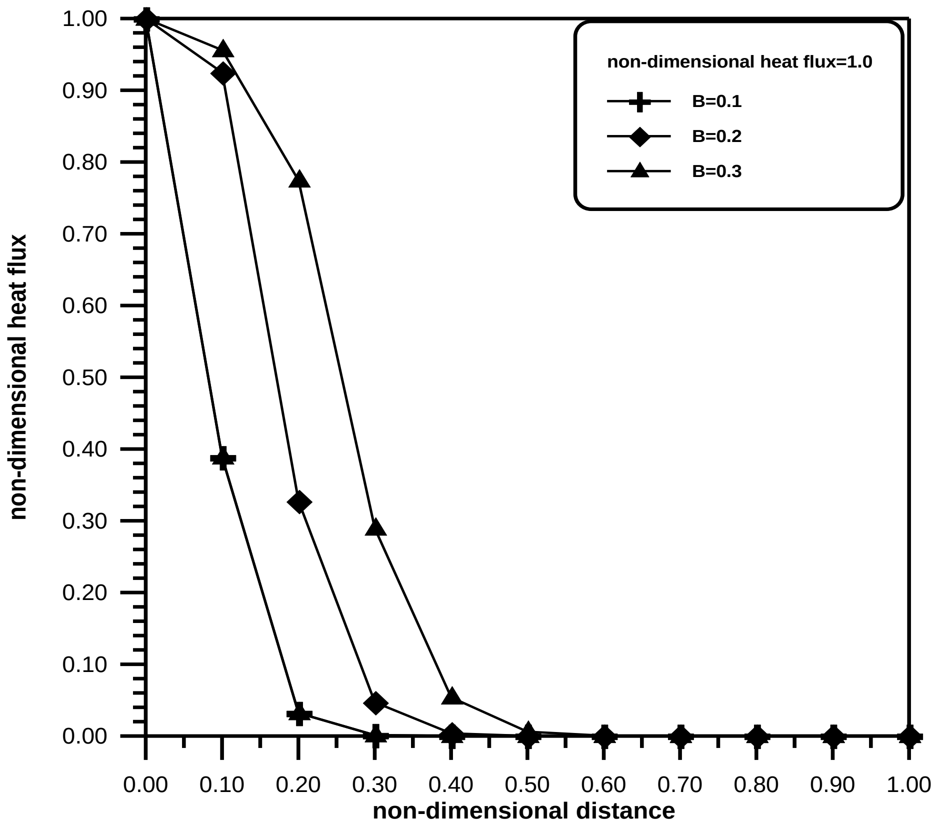
Fig(٤-١٦): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Constant Heat Flux.



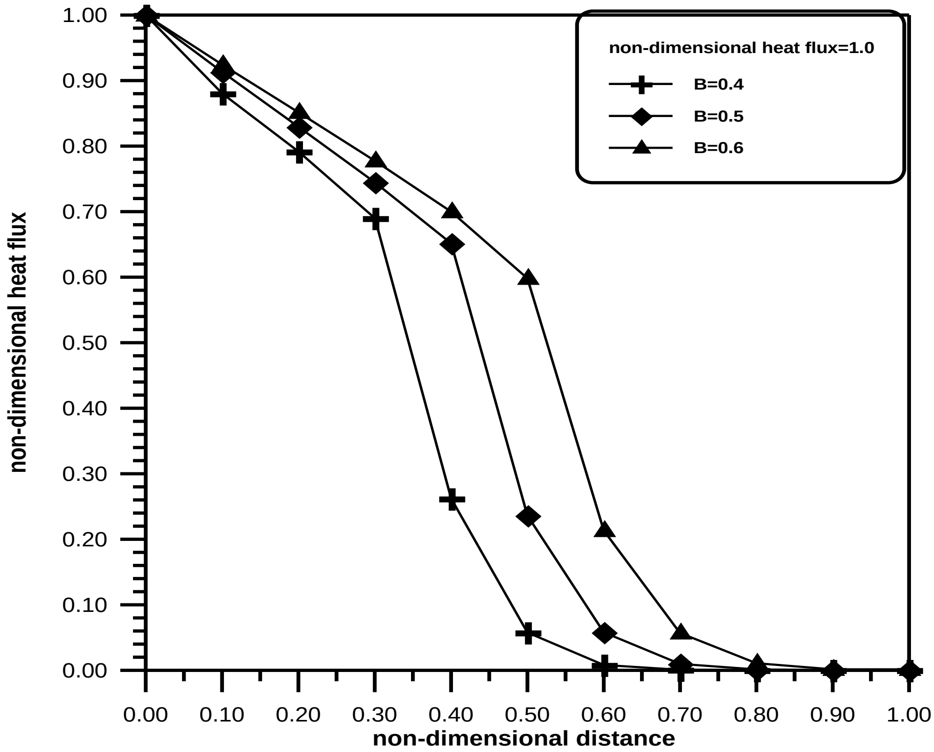
Fig(٤-١٧): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Constant Heat Flux.



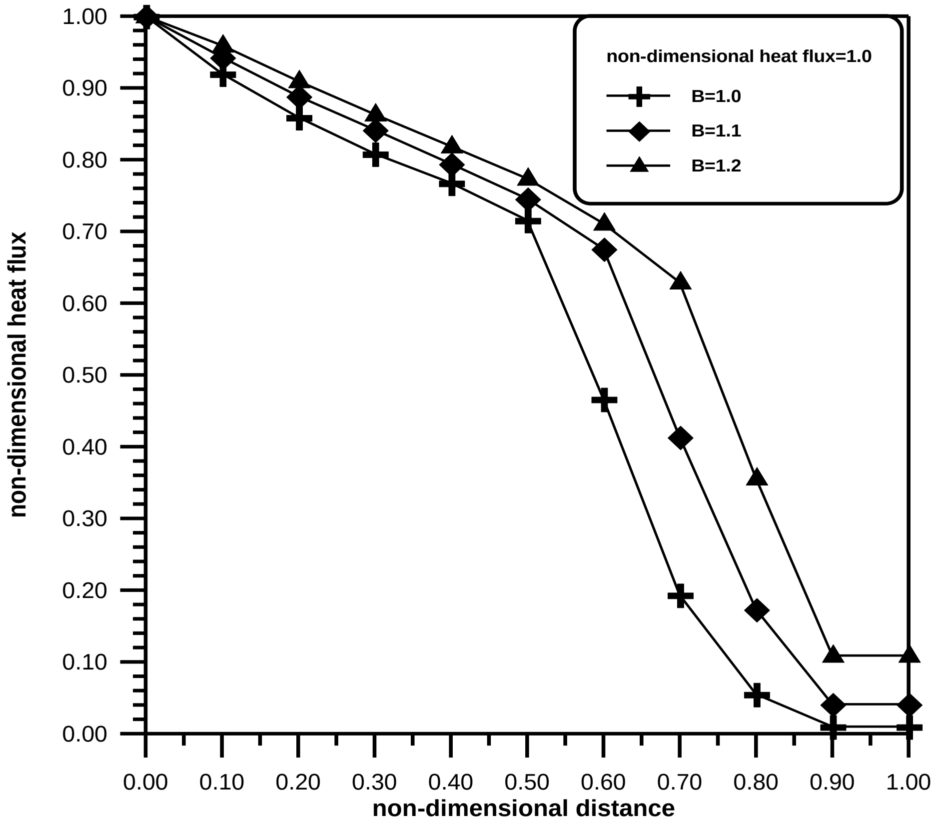
Fig(٤-١٨): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Constant Heat Flux.



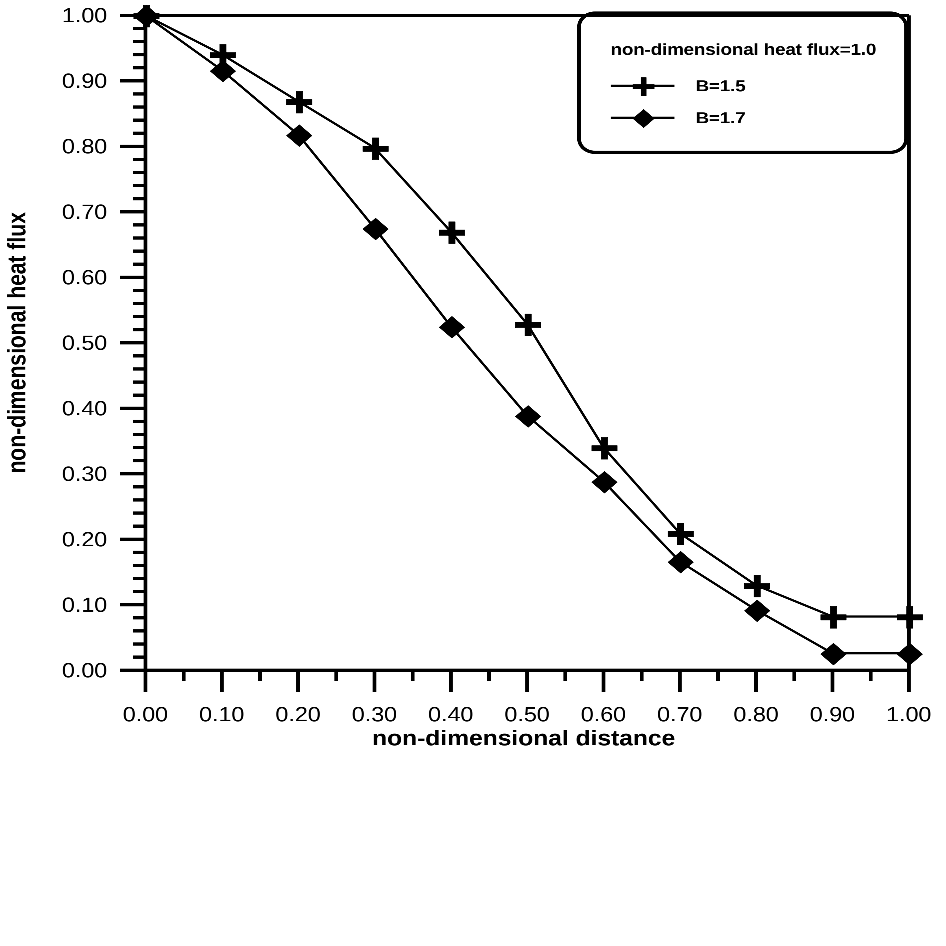
Fig(٤-١٩): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Constant Heat Flux.



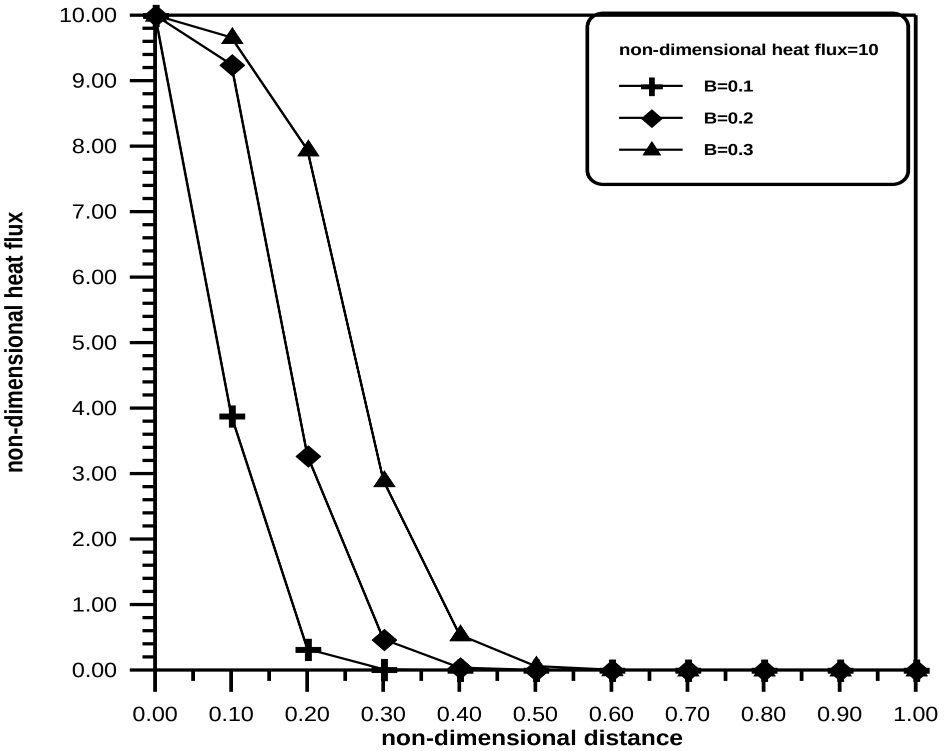
Fig(٤-٢٠): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Constant Heat Flux.



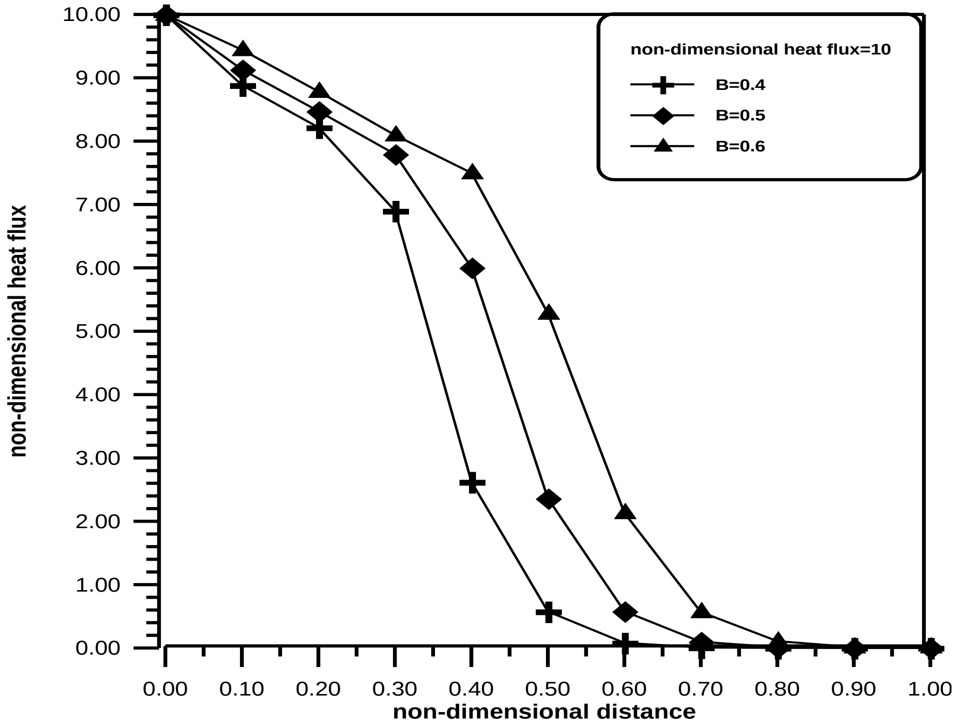
Fig(٤-٢١): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Constant Heat Flux.



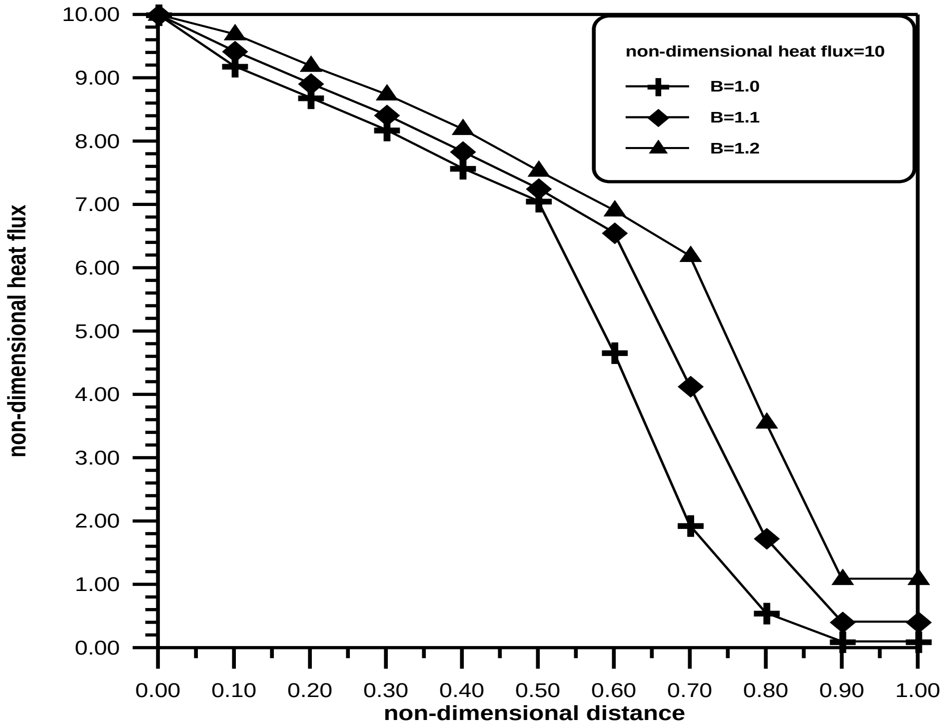
Fig(٤-٢٢): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Constant Heat Flux.



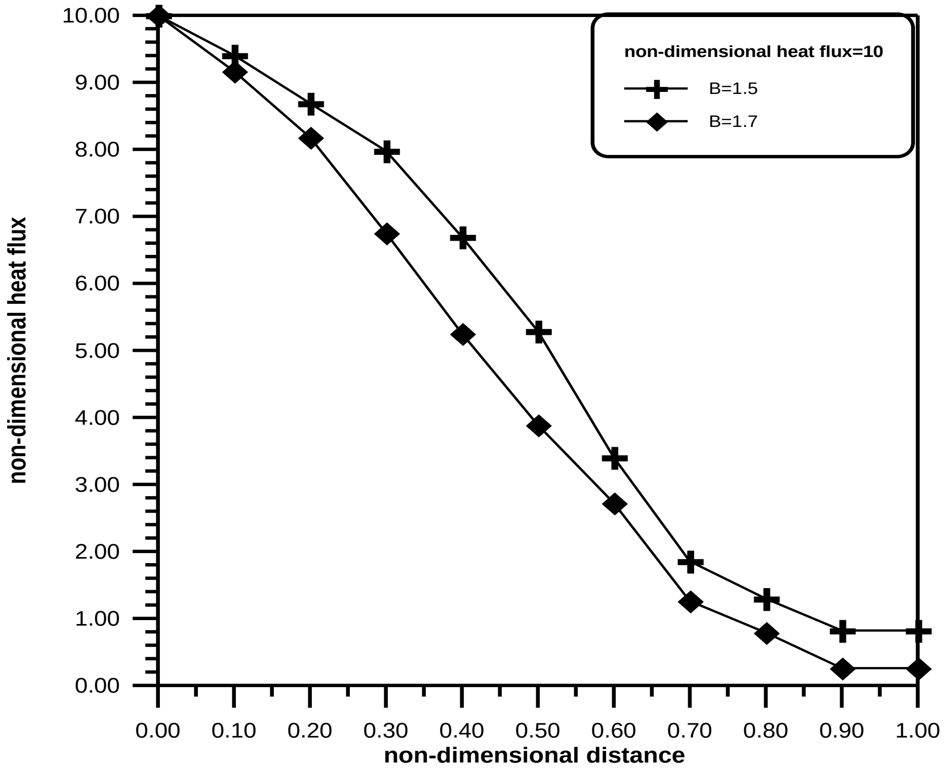
Fig(٤-٢٣): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



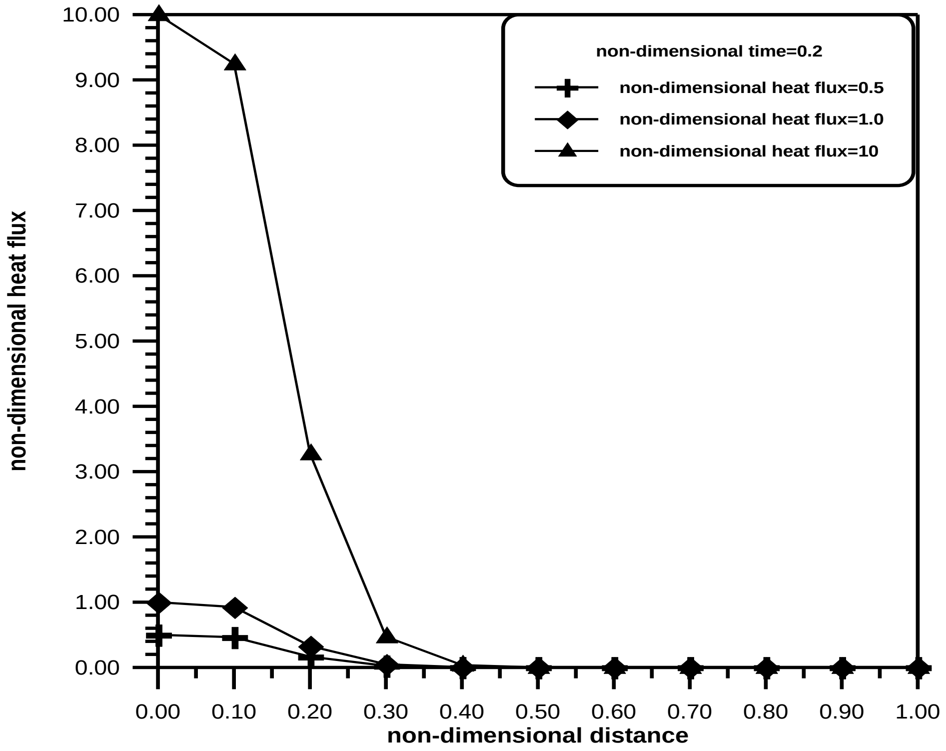
Fig(٤-٢٤): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



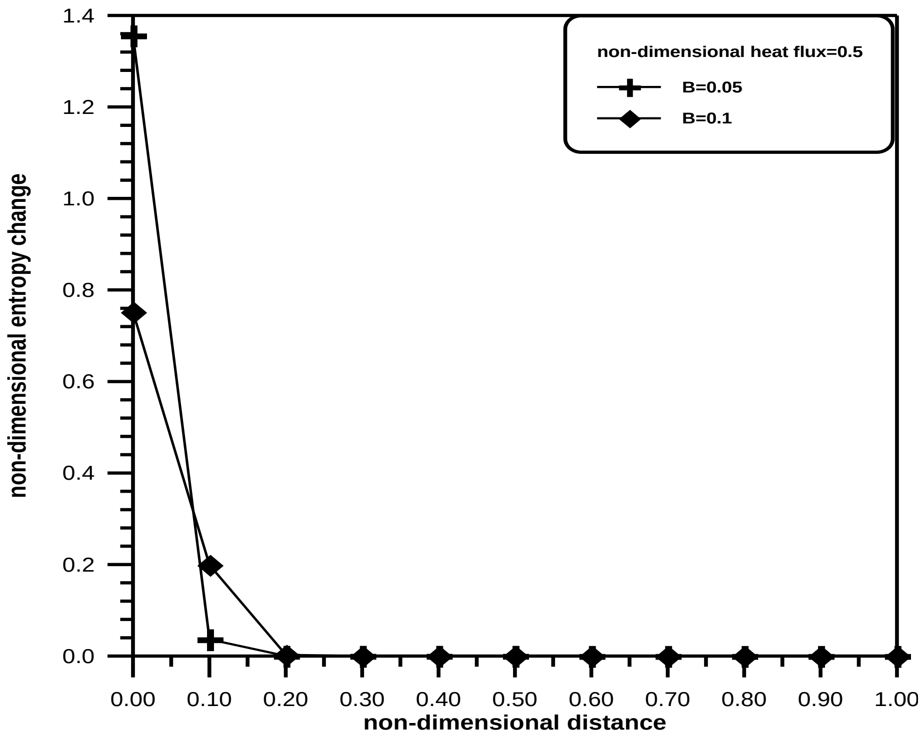
Fig(٤-٢٥): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



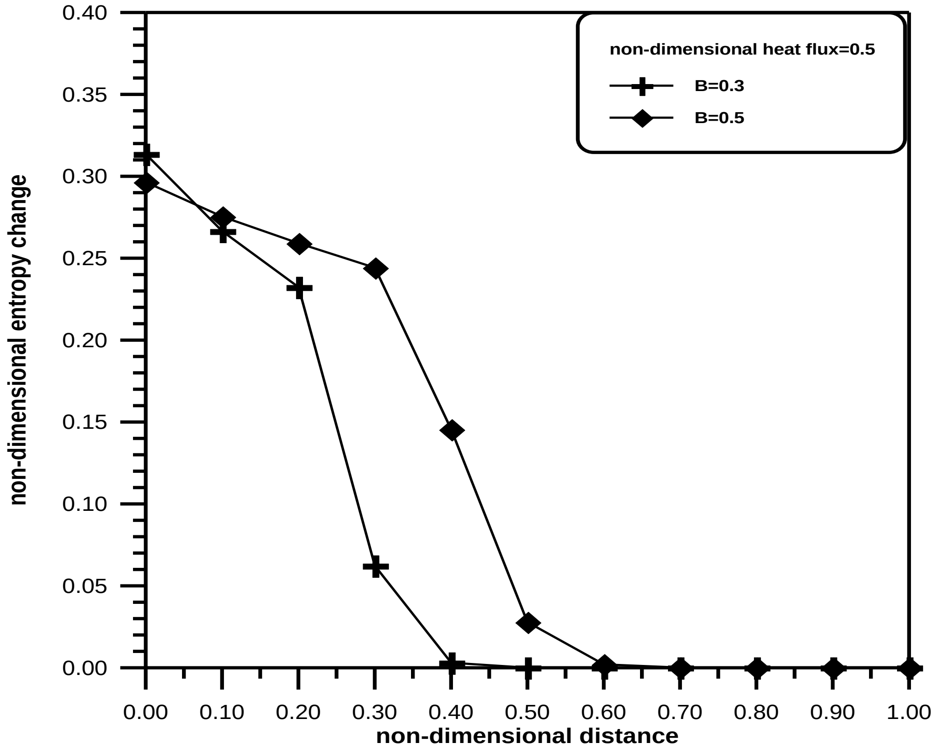
Fig(٤-٢٦): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



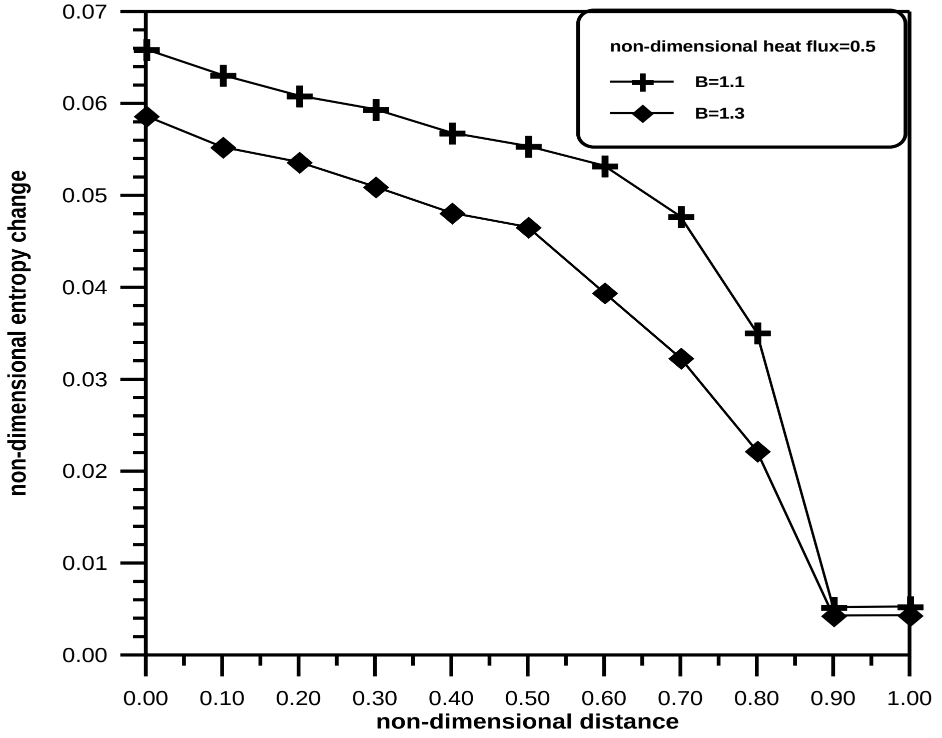
Fig(٤-٢٧): The Variation of Non-Dimensional Heat Flux with Non-Dimensional Distance in a Finite Medium Subjected to a Various Values of Constant Heat Flux at $\beta = 0.2$ **.**



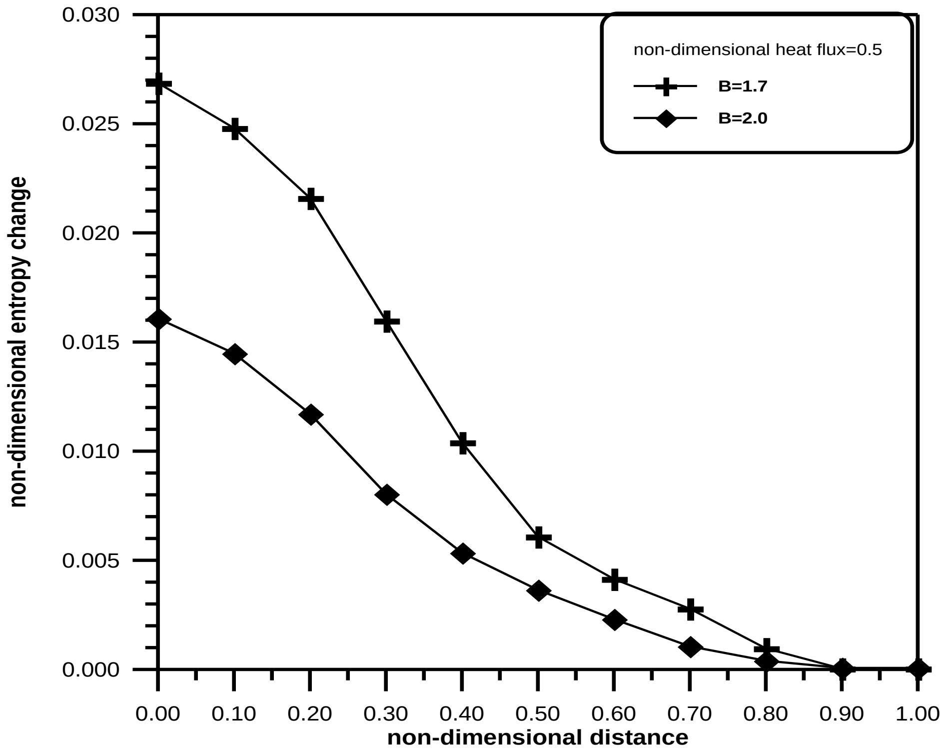
Fig(٤-٢٨): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



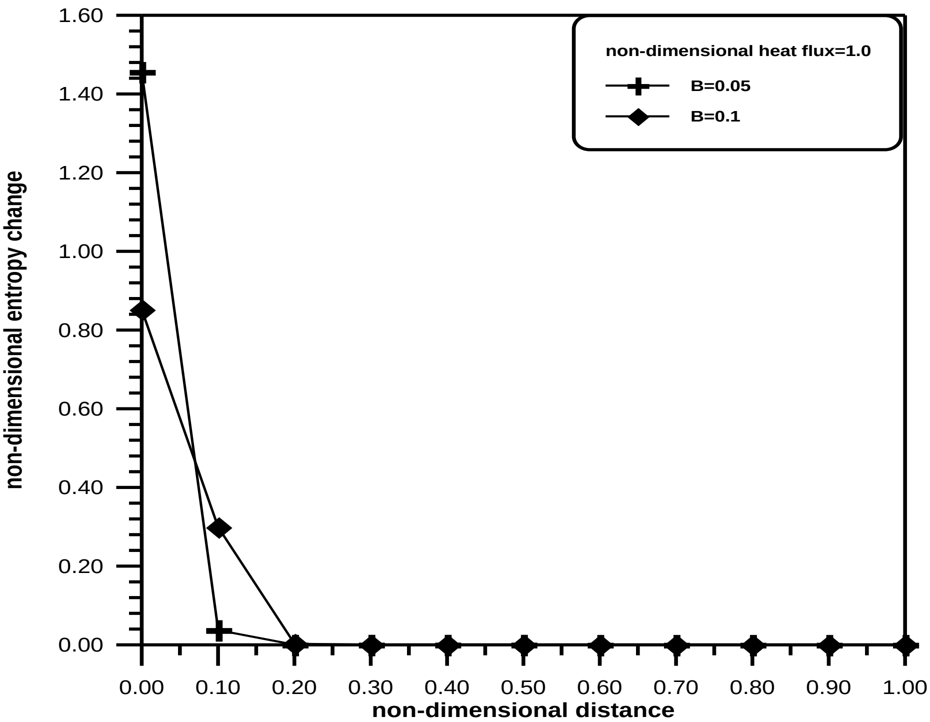
Fig(٤-٢٩): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



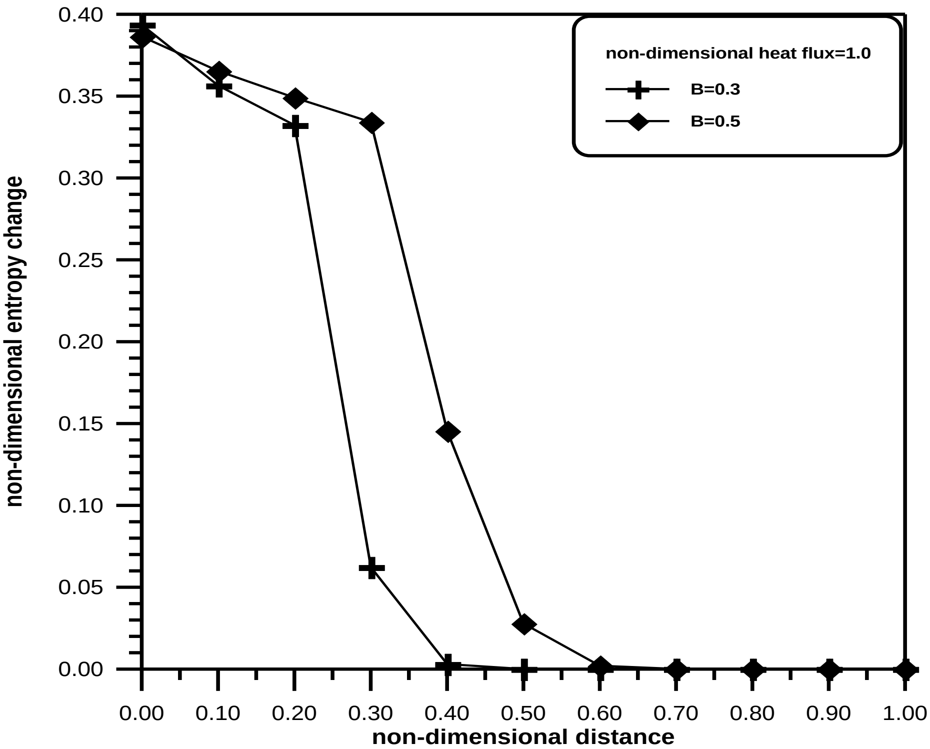
Fig(٤-٣٠): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



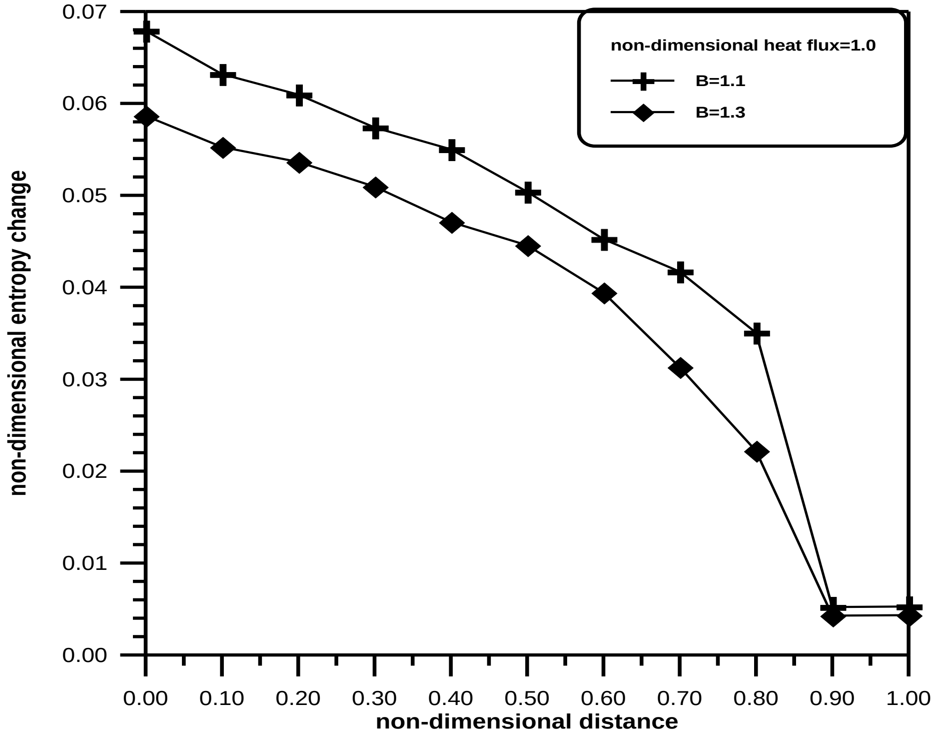
Fig(٤-٣١): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



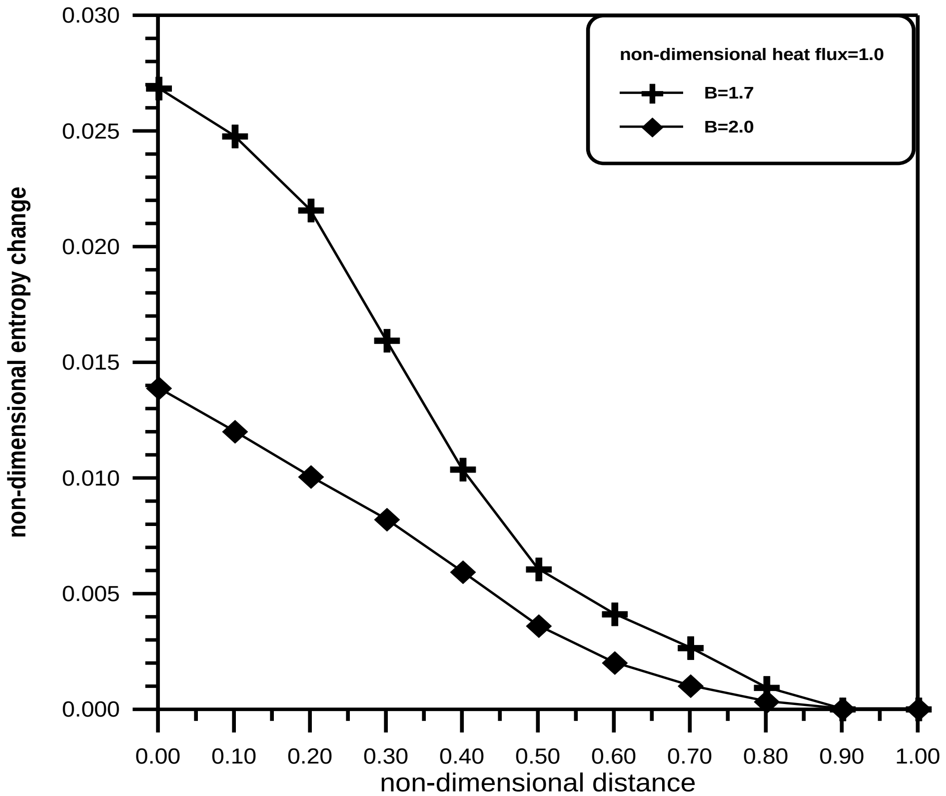
Fig(٤-٣٢): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



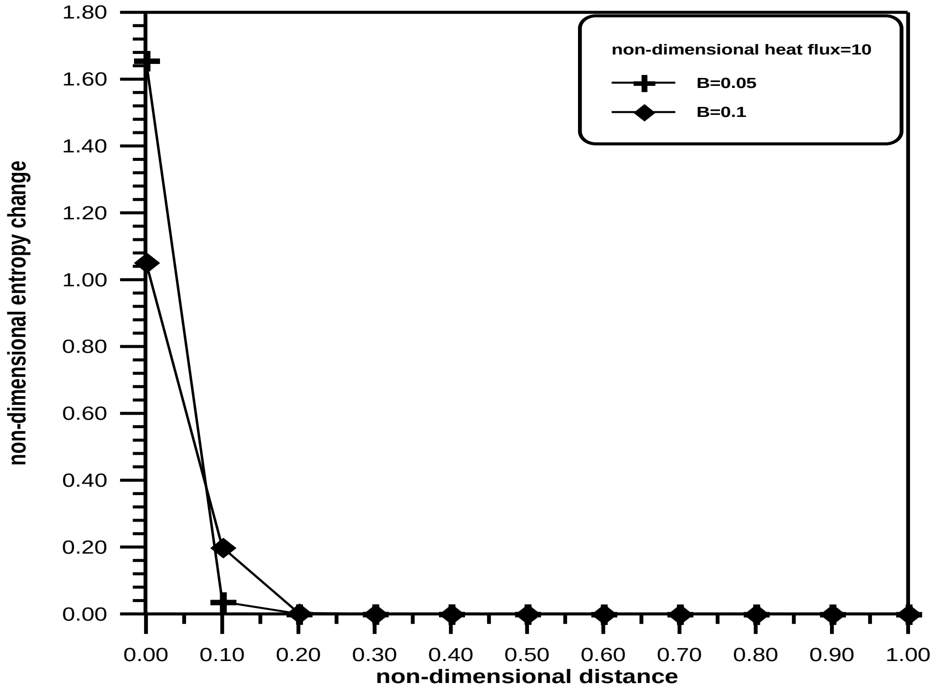
Fig(٤-٣٣): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



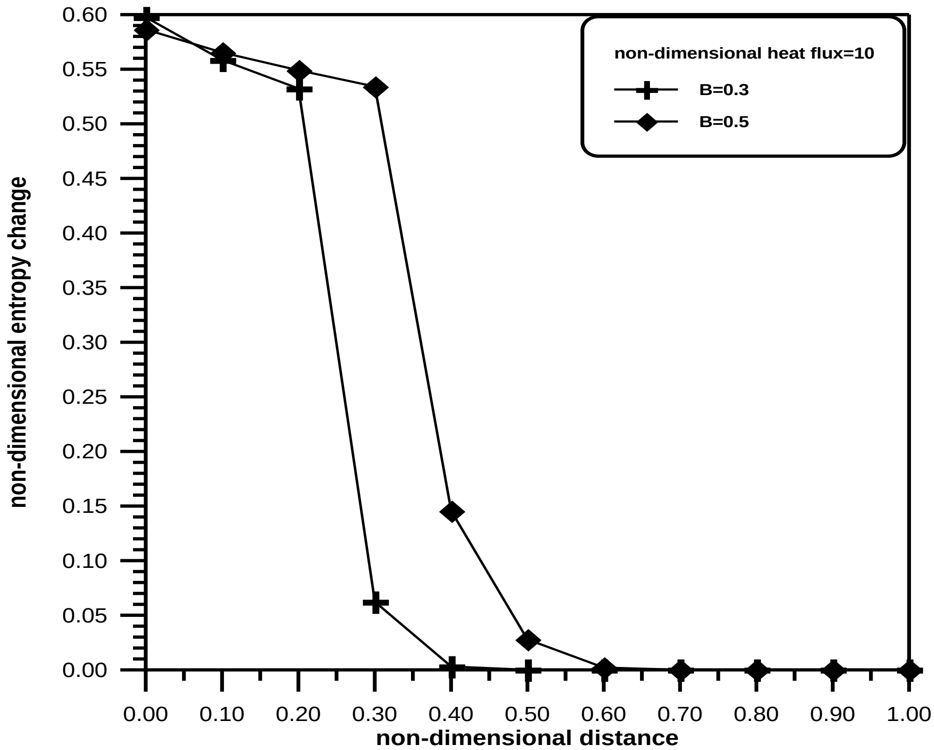
Fig(٤-٣٤): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



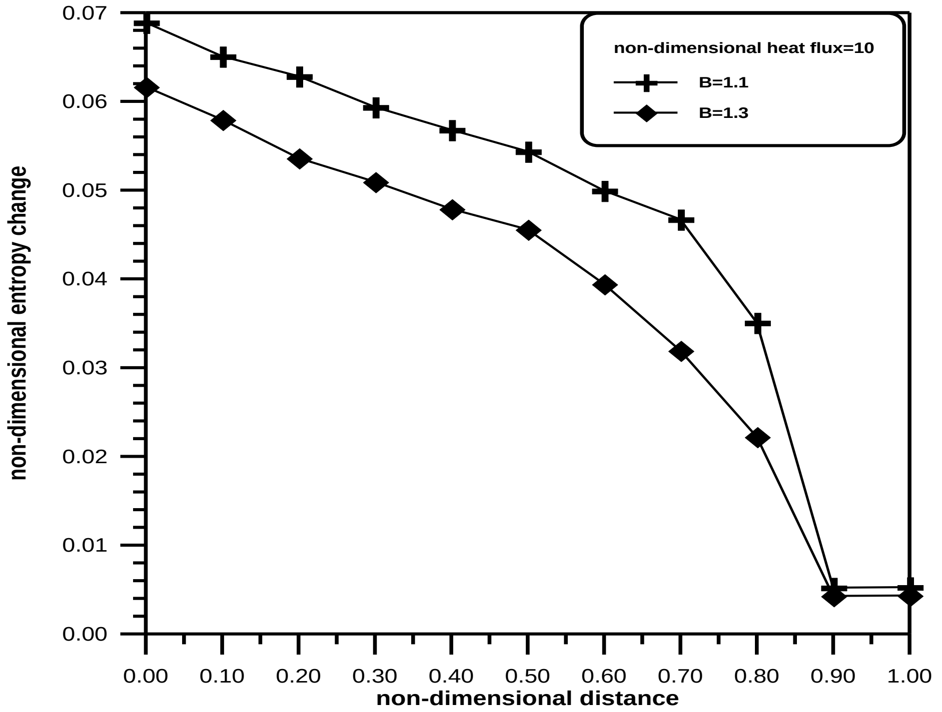
Fig(٤-٣٥): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



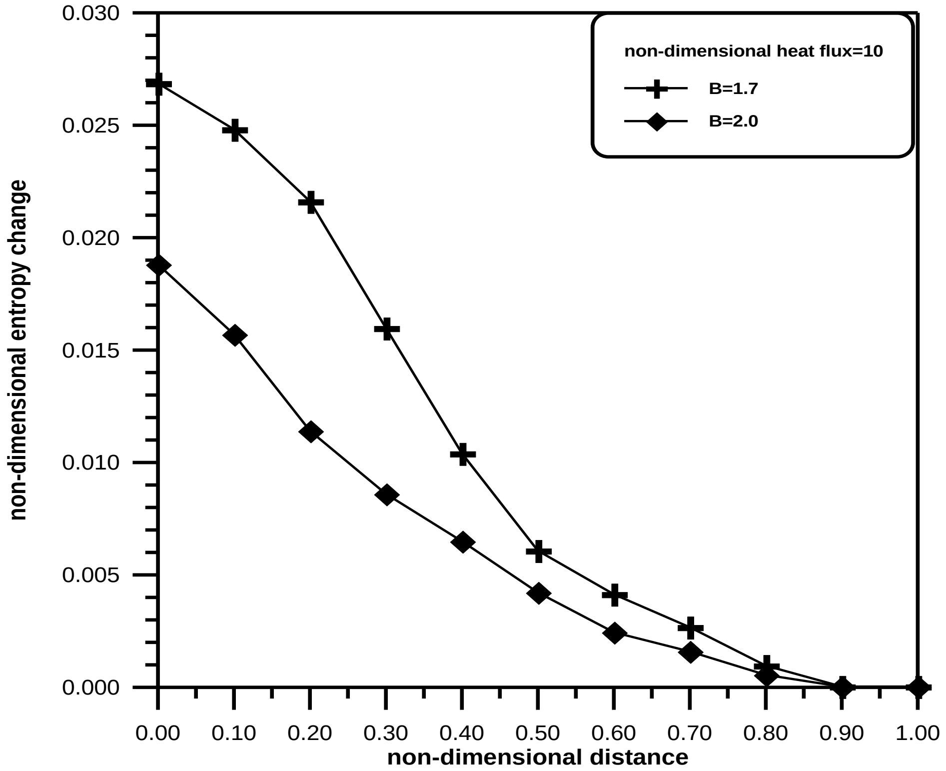
Fig(٤-٣٦): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



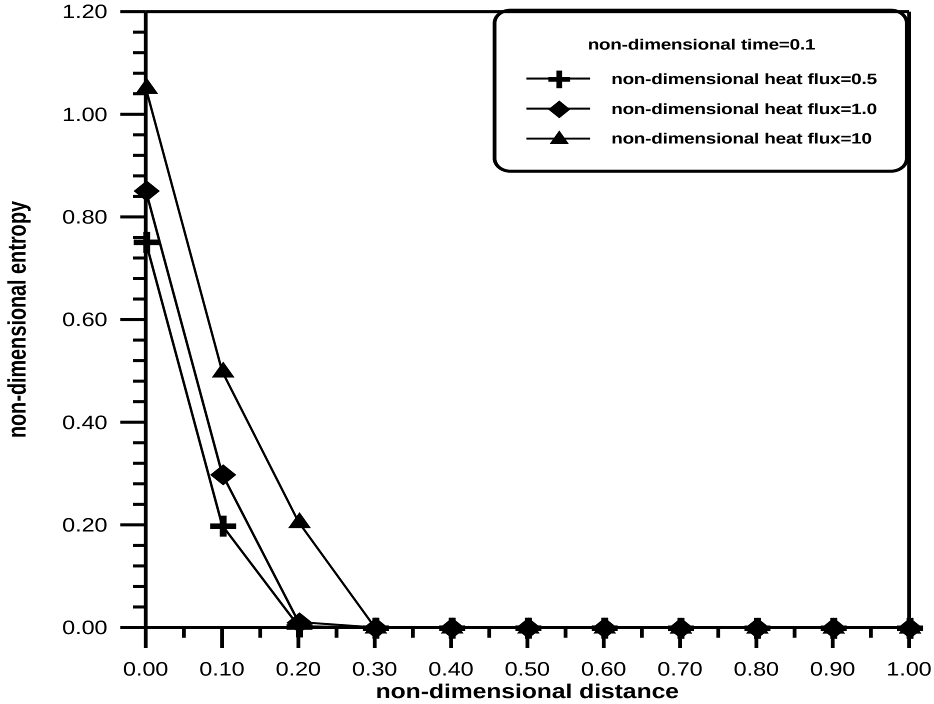
Fig(٤-٣٧): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



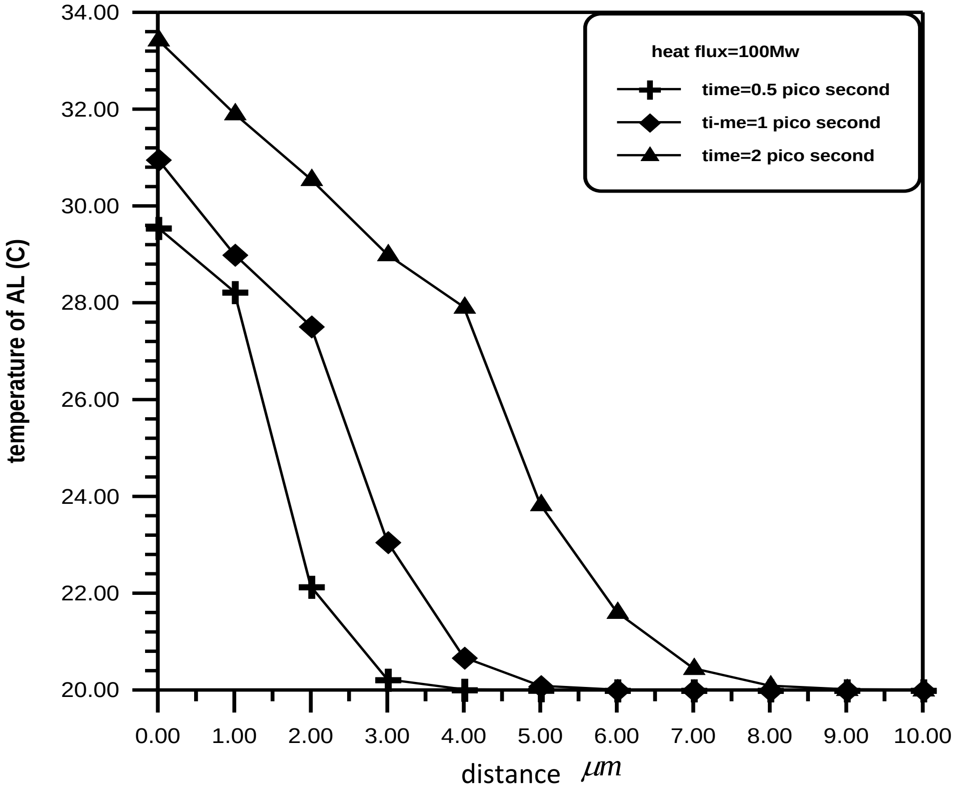
Fig(٤-٣٨): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



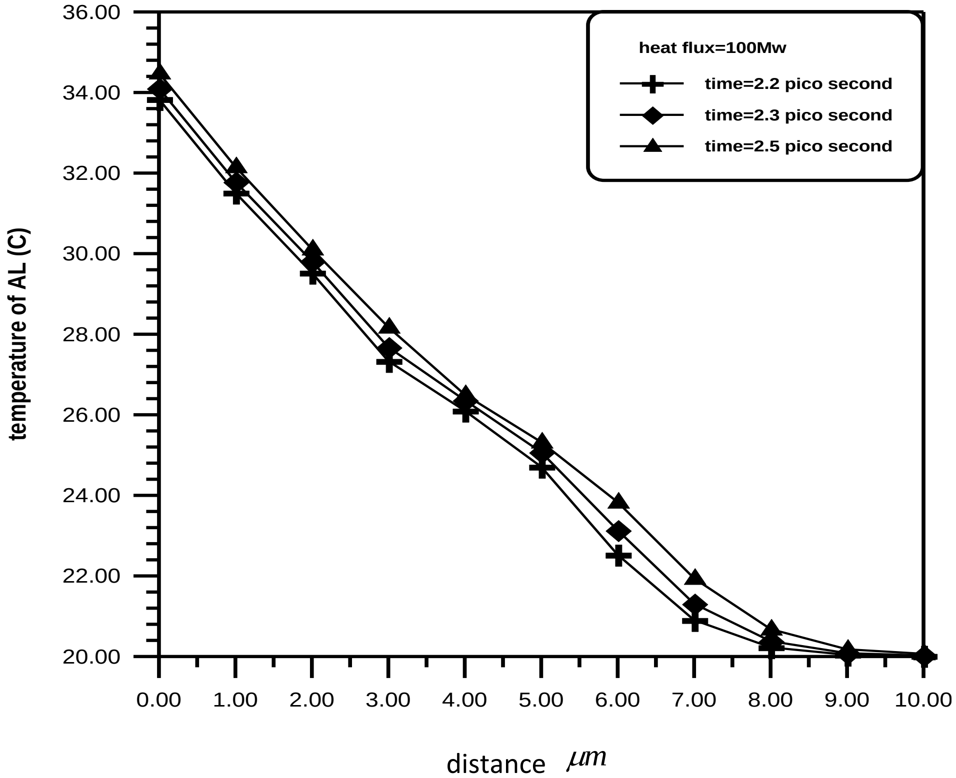
Fig(٤-٣٩): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Constant Heat Flux.



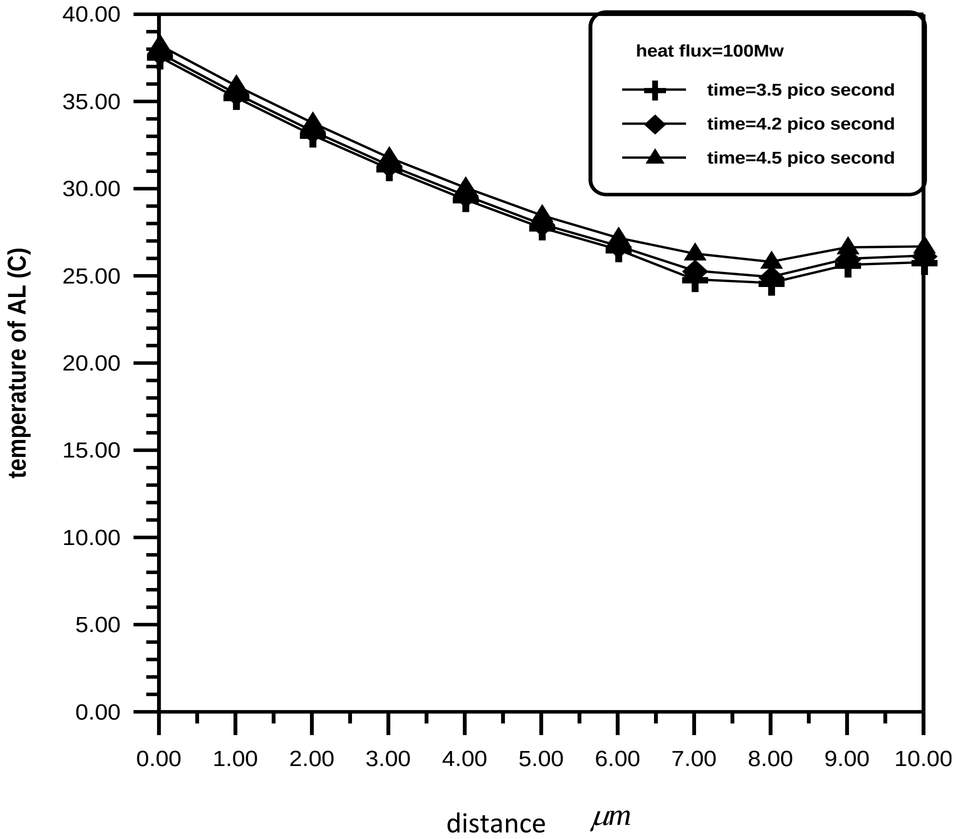
Fig(٤-٤٠): The Variation of Non-Dimensional Entropy with Non-Dimensional Distance in a Finite Medium Subjected a Various Values of A Constant Heat Flux at $\beta = 0.1$.



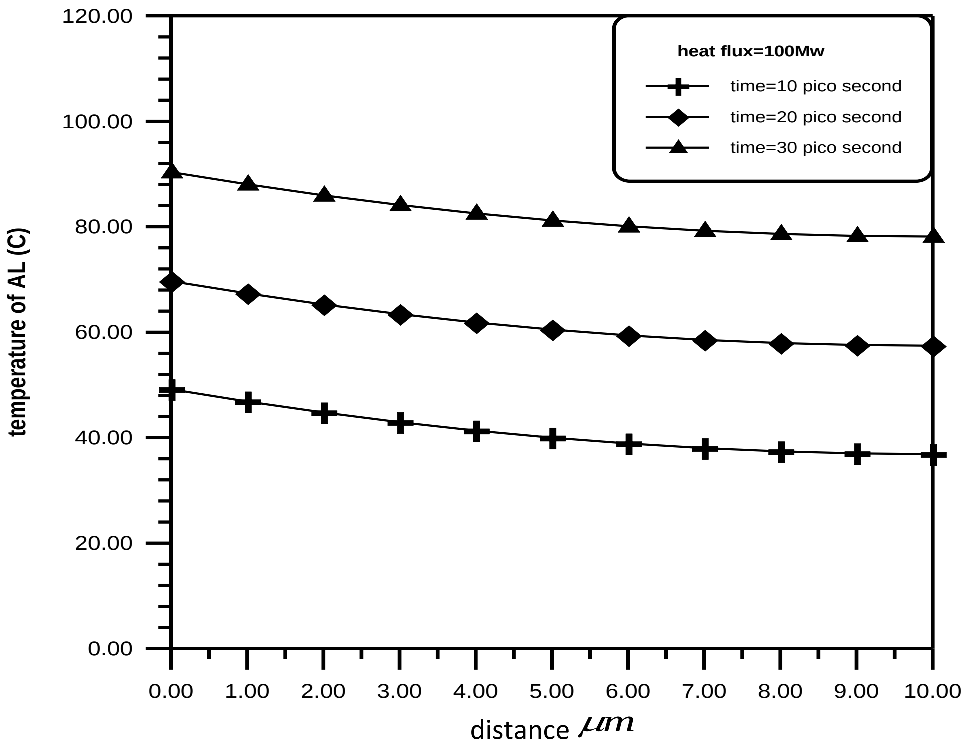
Fig(٤-٤١): Temperature Distribution of Aluminum in a Finite Medium Subjected by Constant Heat flux.



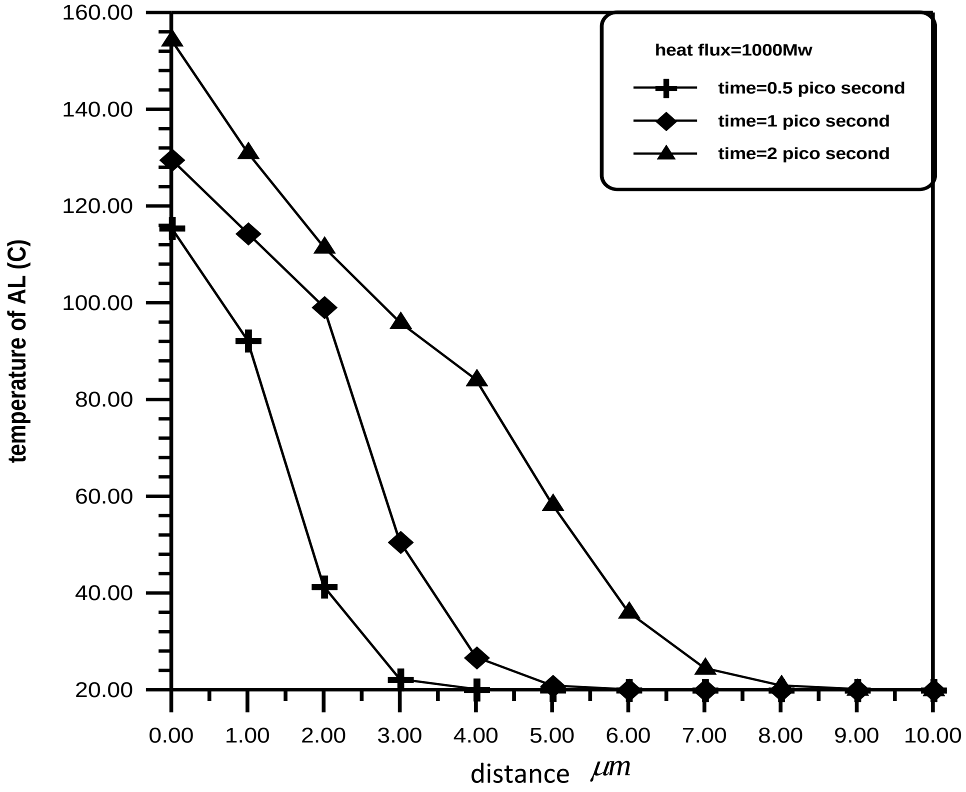
Fig(٤-٤٢): Temperature Distribution of Aluminum in a Finite Medium Subjected by Constant Heat flux.



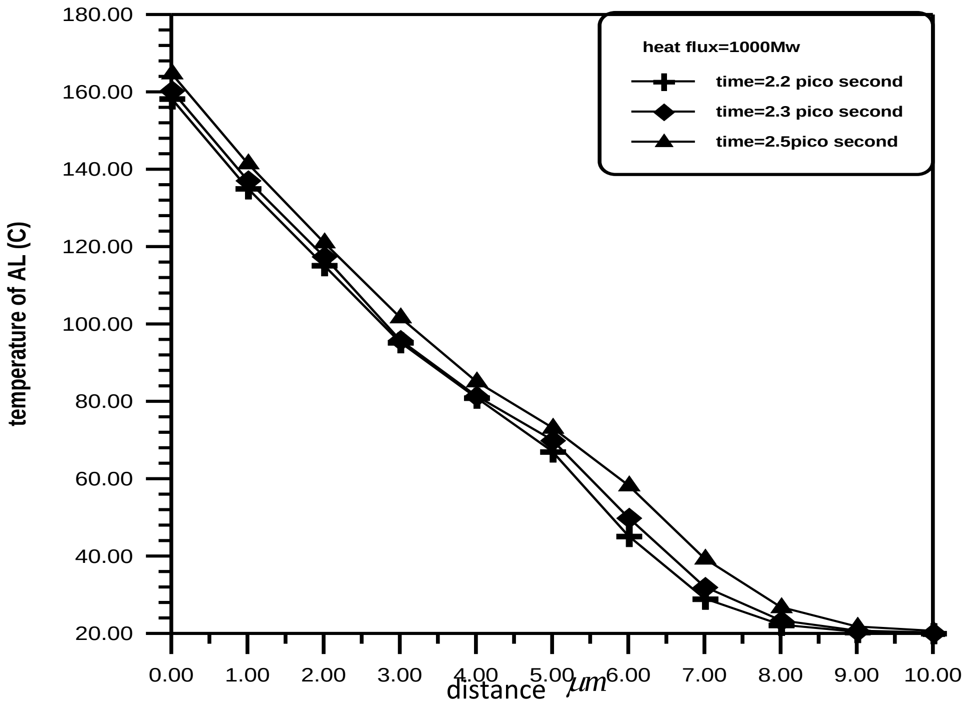
Fig(٤-٤٣): Temperature Distribution of Aluminum in a Finite Medium Subjected by Constant Heat flux.



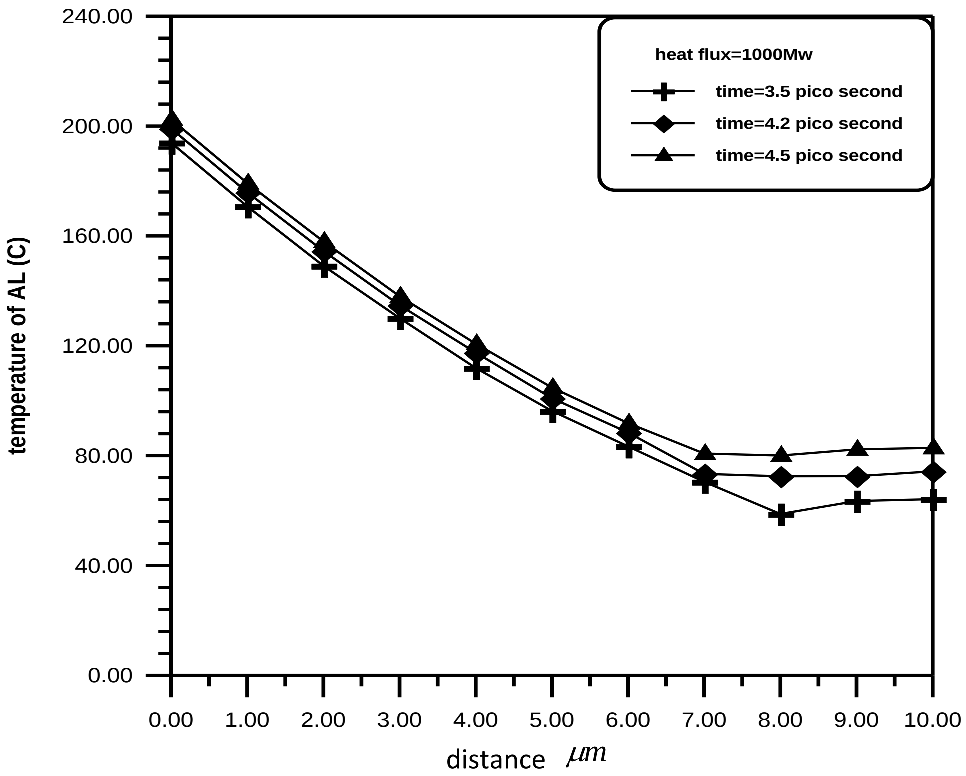
Fig(٤-٤٤): Temperature Distribution of Aluminum in a Finite Medium Subjected by Constant Heat flux.



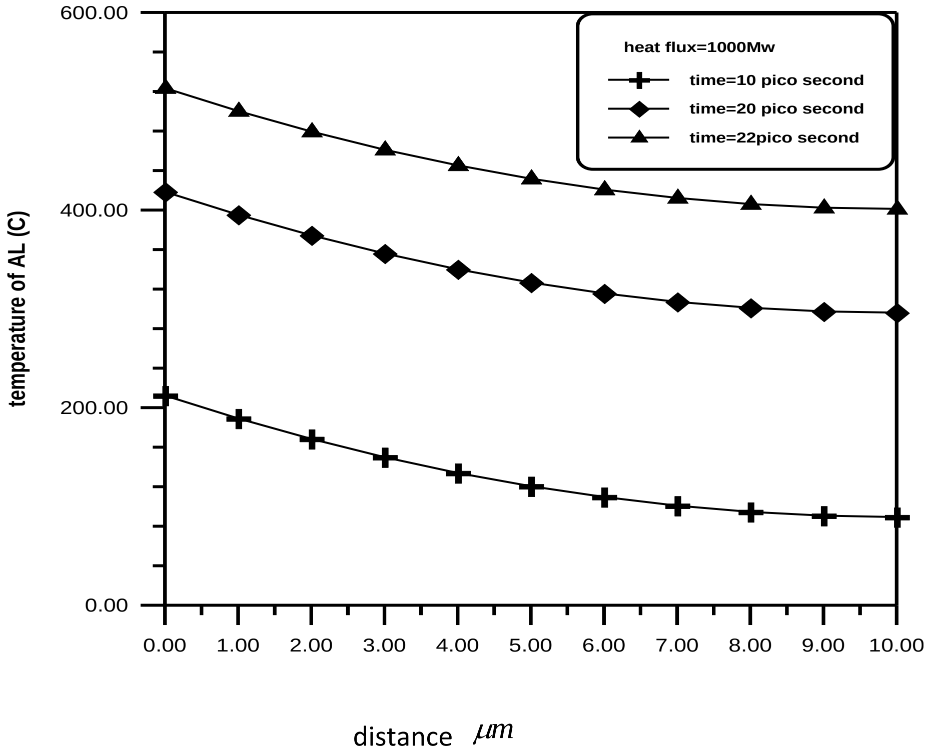
Fig(٤-٤٥): Temperature Distribution of Aluminum in a Finite Medium Subjected by Constant Heat flux.



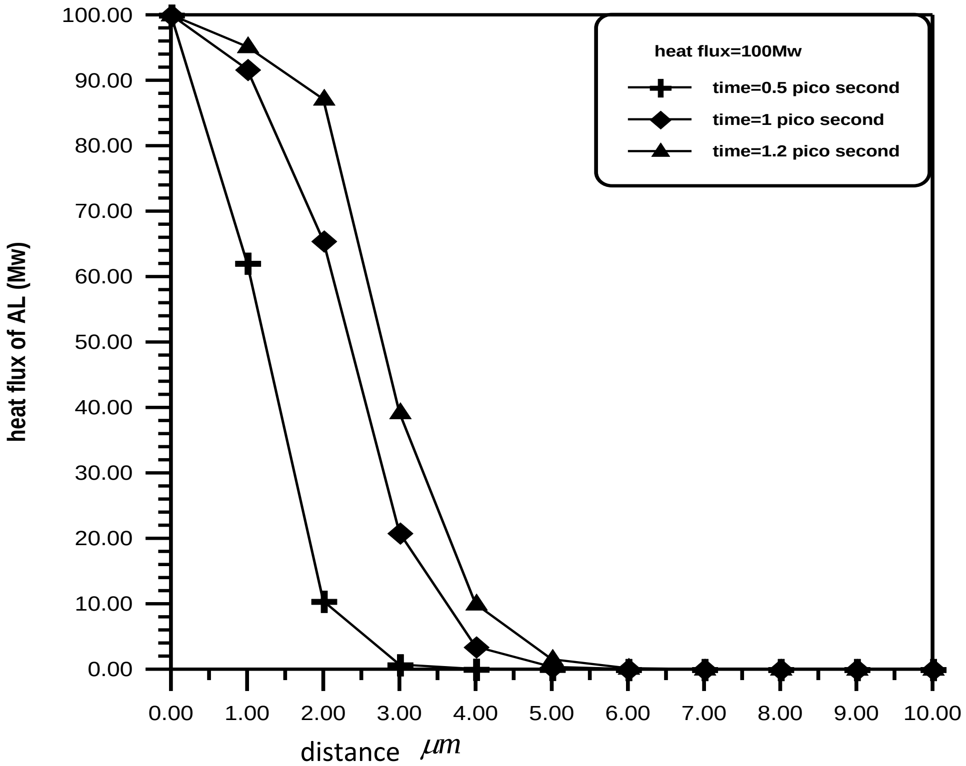
Fig(٤-٤٦): Temperature Distribution of Aluminum in a Finite Medium Subjected by Constant Heat flux.



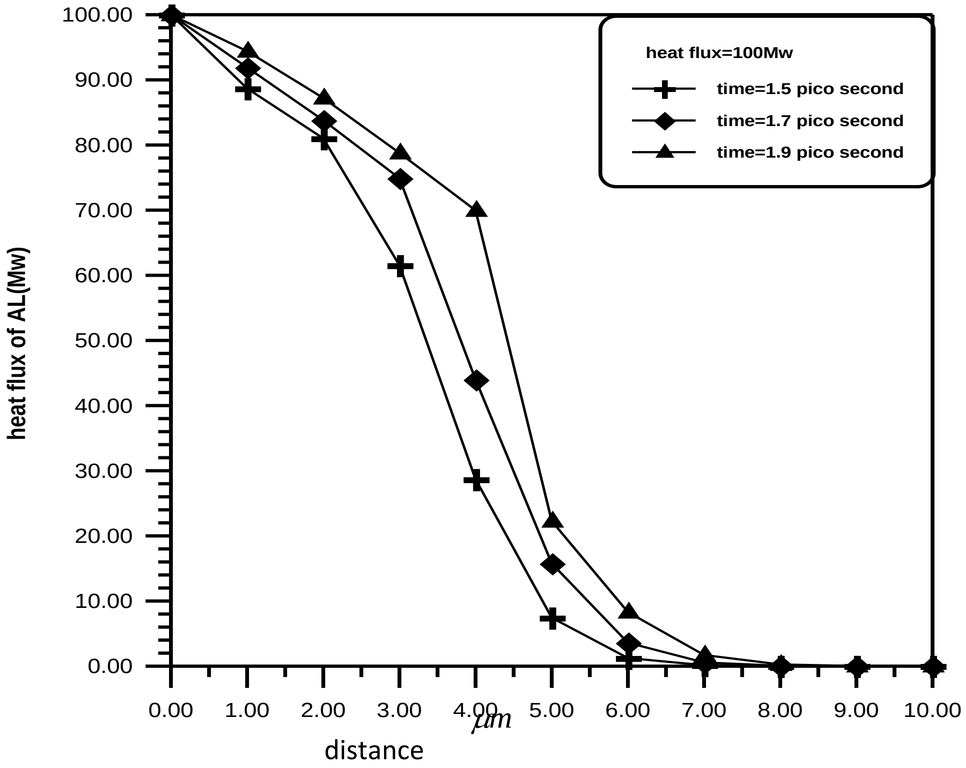
Fig(٤-٤٧): Temperature Distribution of Aluminum in a Finite Medium Subjected by Constant Heat flux.



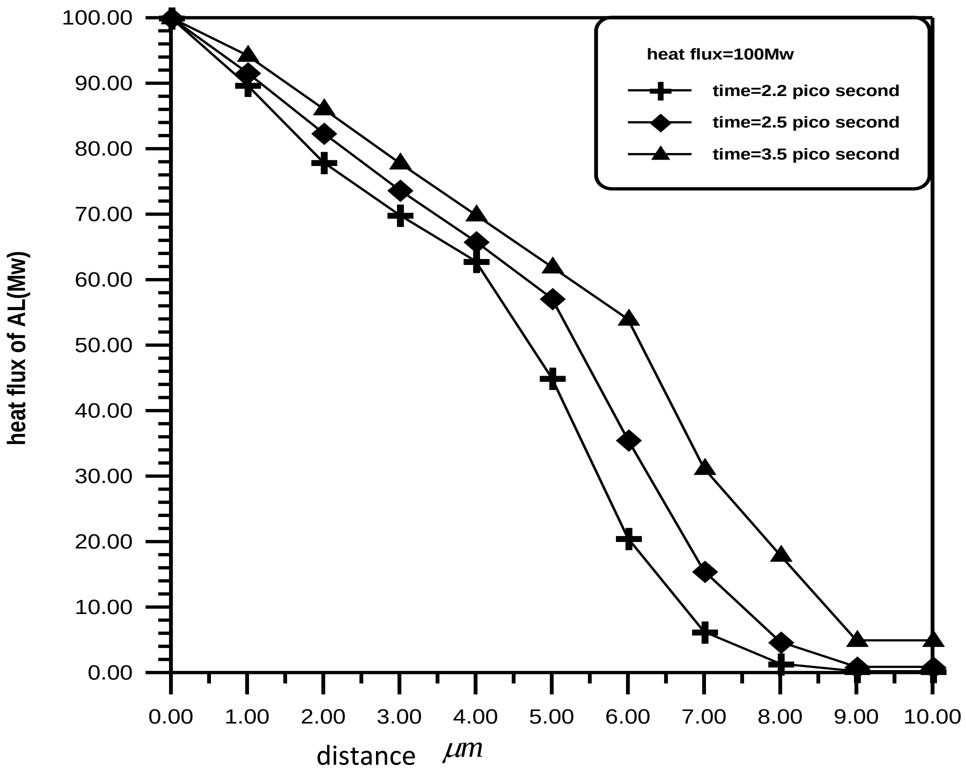
Fig(٤-٤٨): Temperature Distribution of Aluminum in a Finite Medium Subjected by Constant Heat flux.



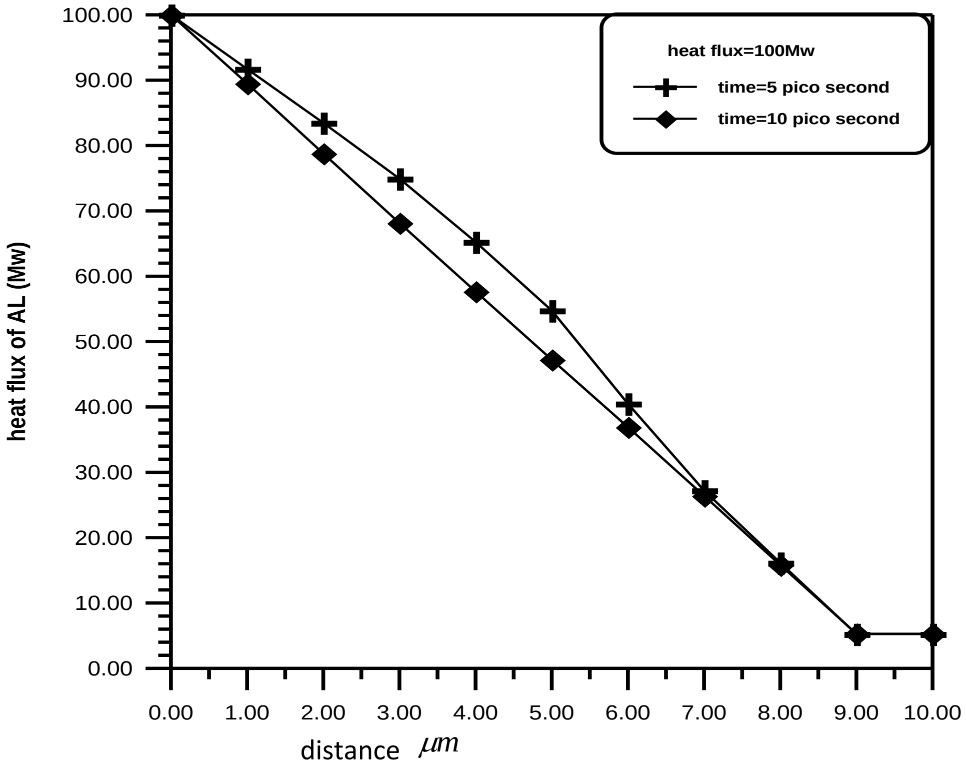
Fig(٤-٤٩): The Variation of Hear Flux of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



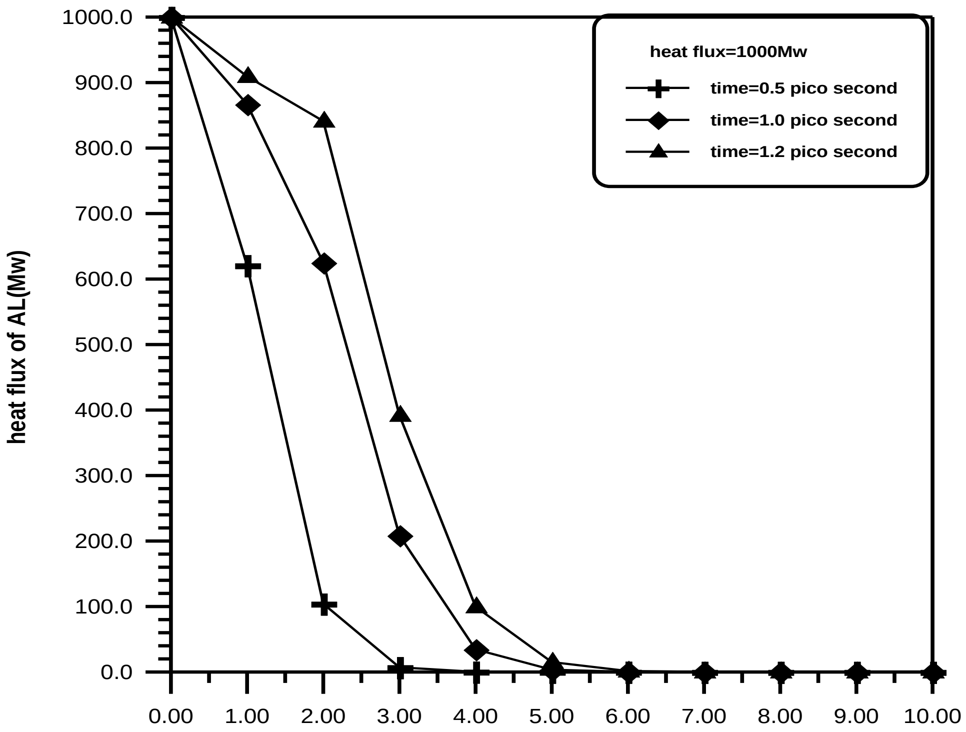
Fig(٤-٥٠): The Variation of Hear Flux of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



Fig(٤-٥١): The Variation of Hear Flux of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.

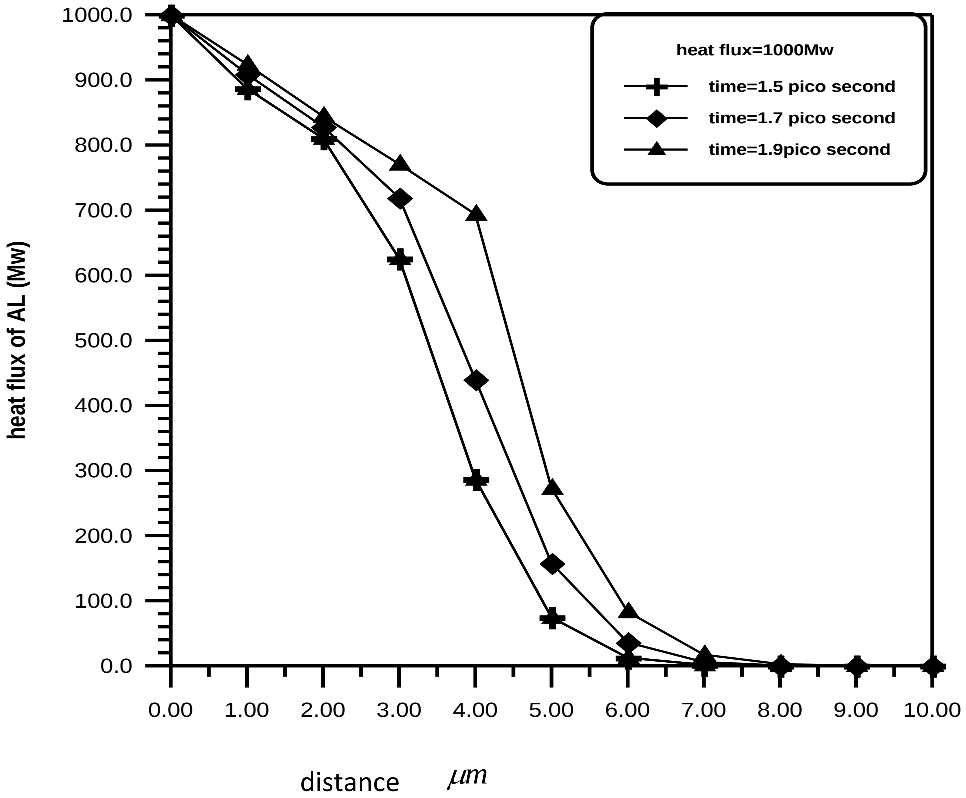


Fig(٤-٥٢): The Variation of Hear Flux of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.

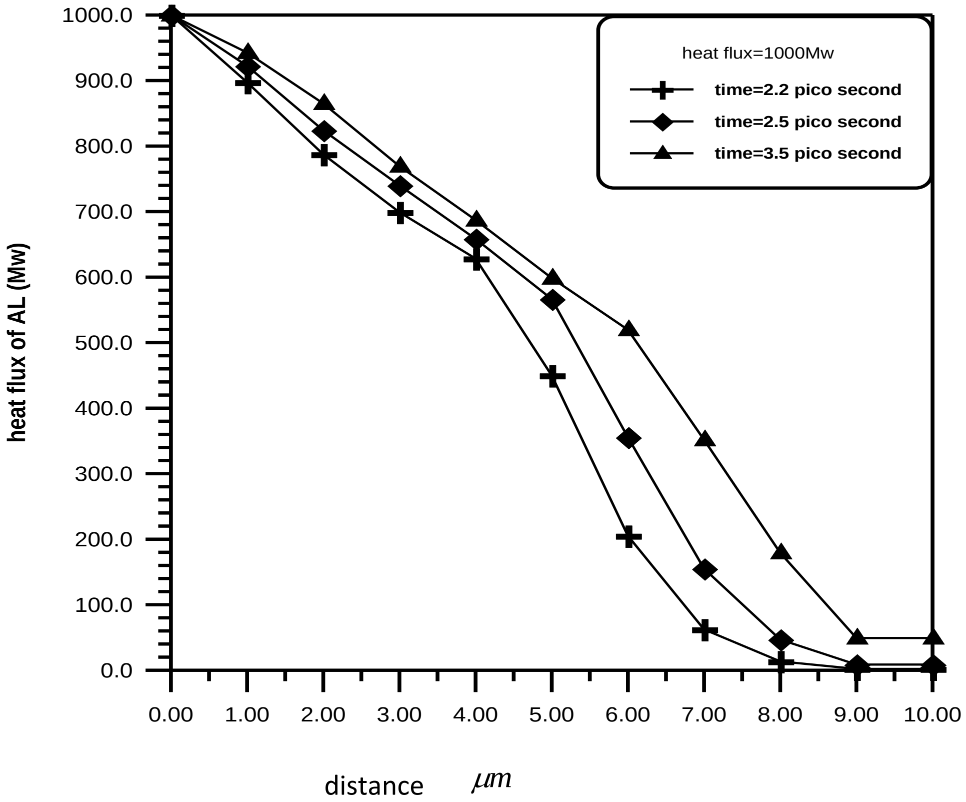


distance μm

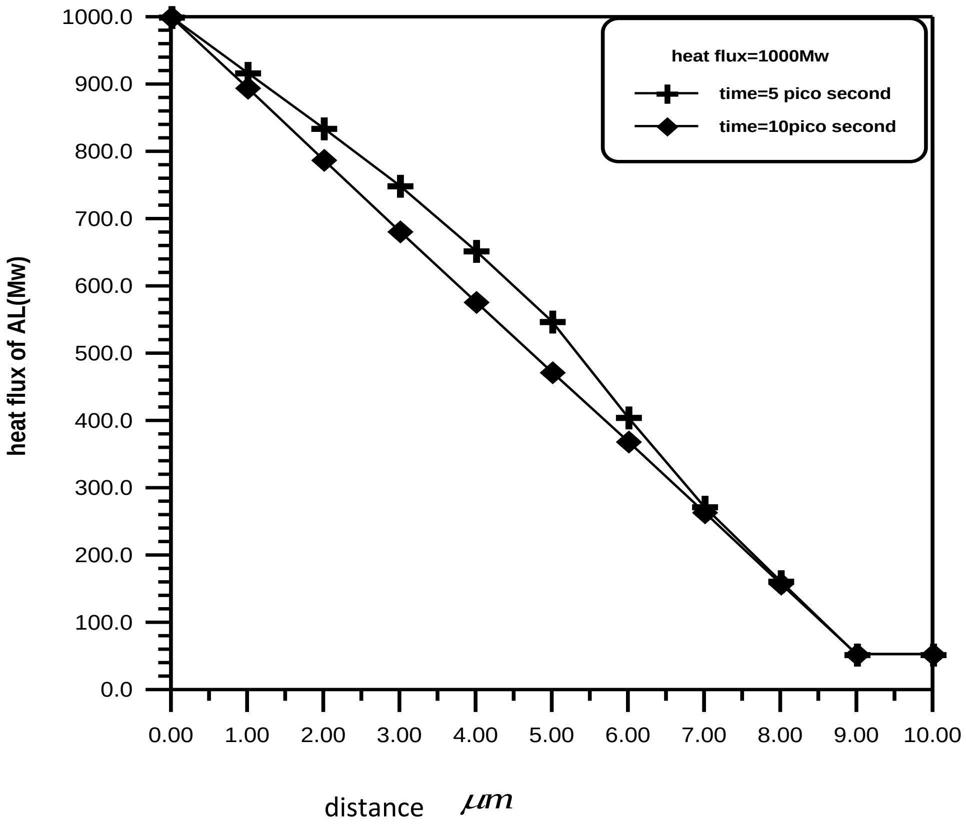
Fig(٤-٥٣): **The Variation of Hear Flux of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.**



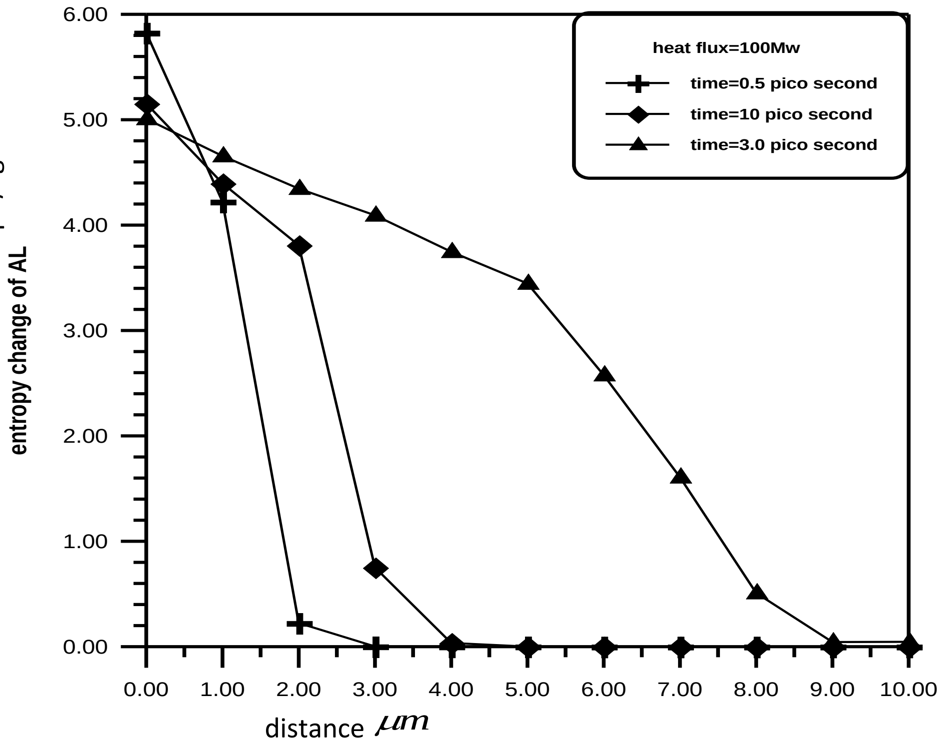
Fig(٤-٥٤): **The Variation of Hear Flux of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.**



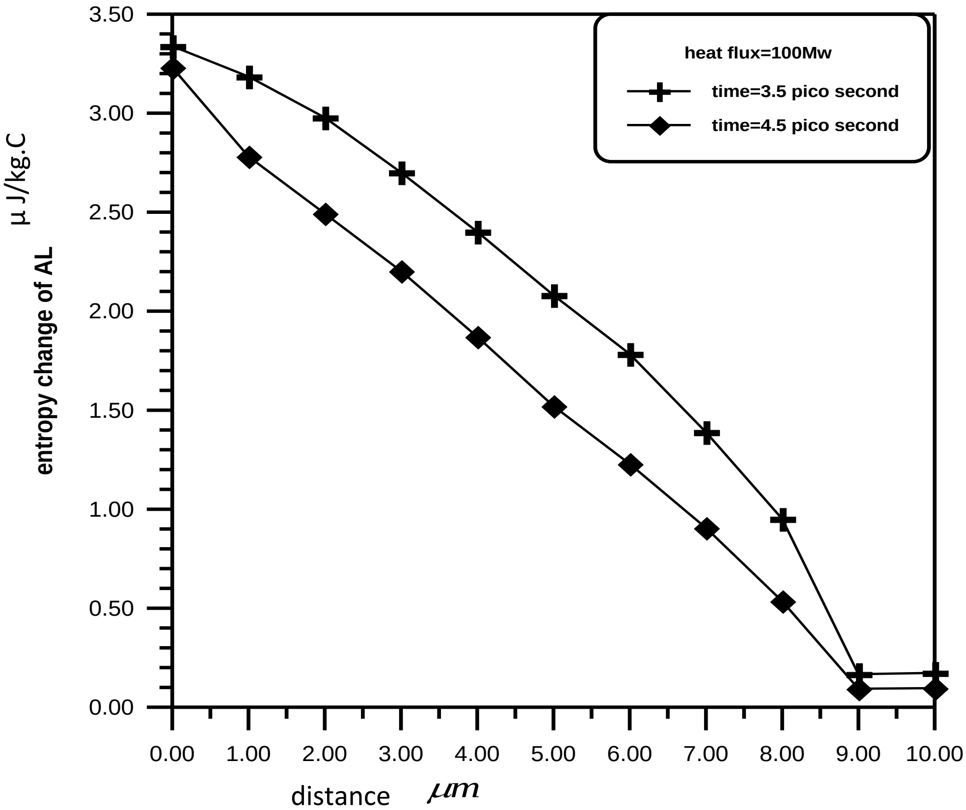
Fig(٤-٥٥): The Variation of Hear Flux of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



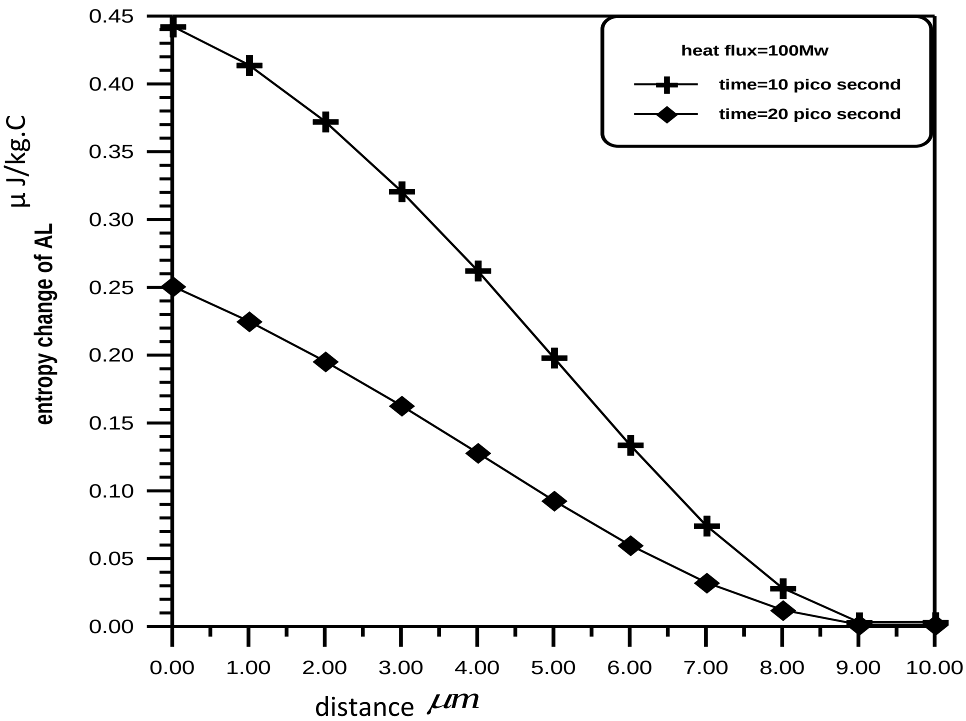
Fig(٤-٥٦): The Variation of Hear Flux of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



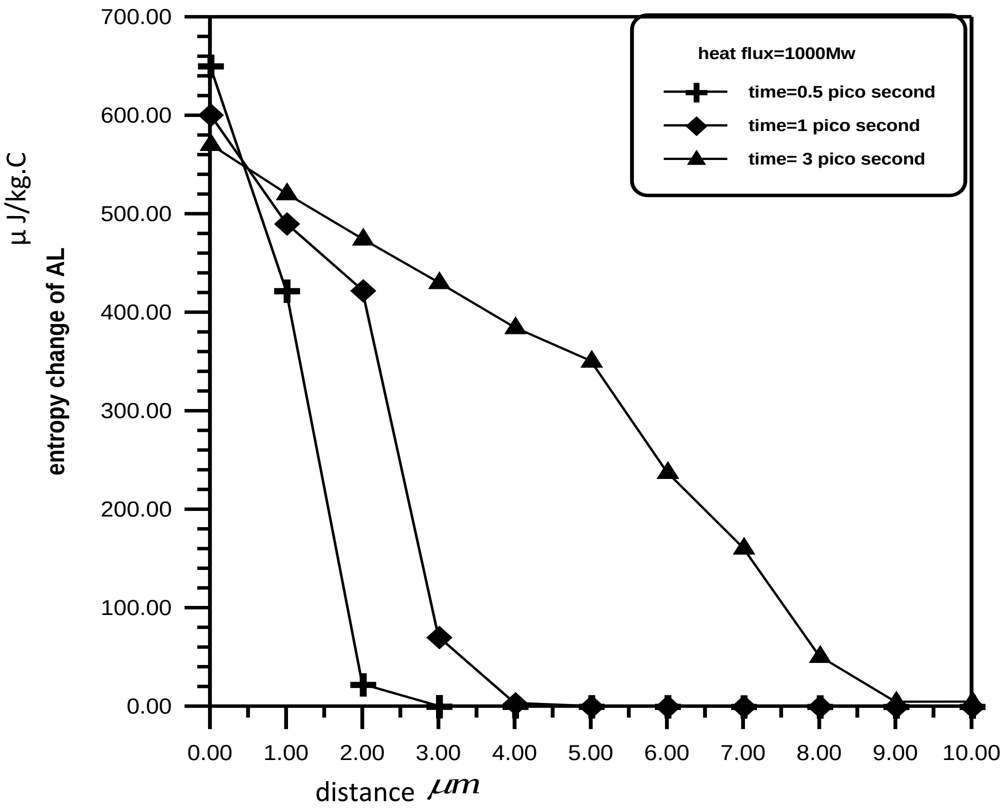
Fig(٤-٥٧): The Variation of Entropy Change of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



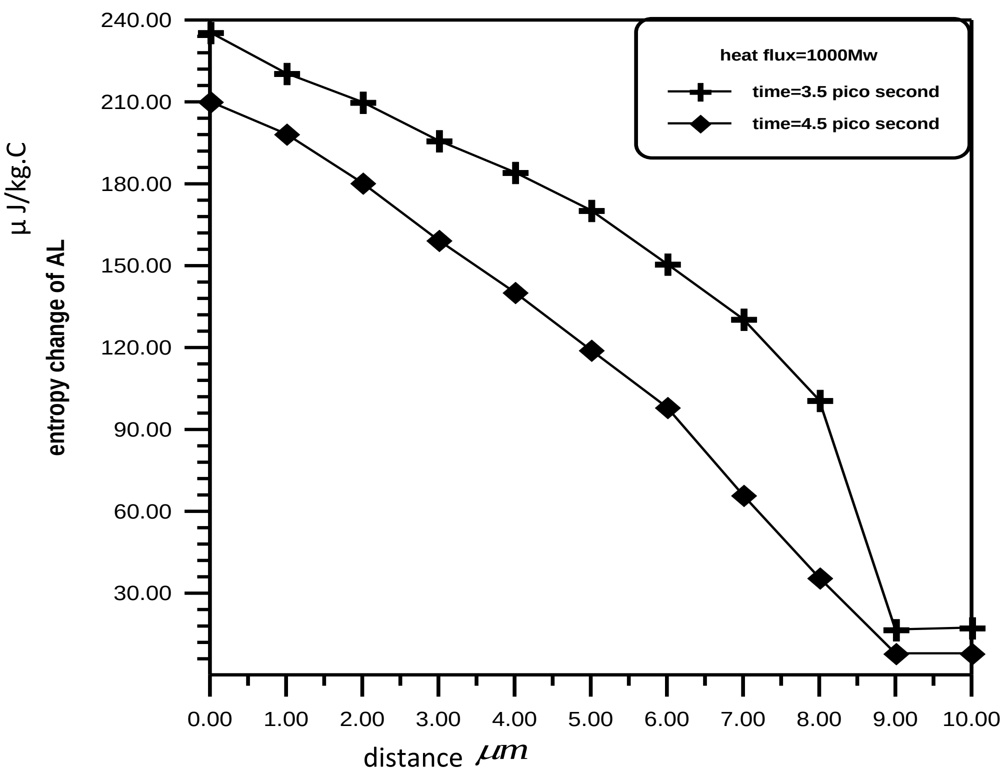
Fig(٤-٥٨): The Variation of Entropy Change of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



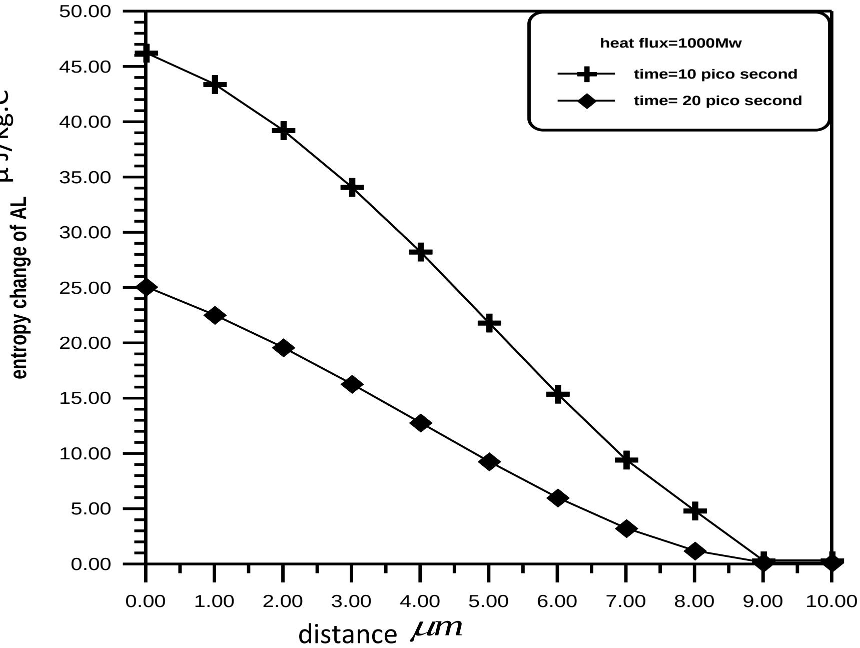
Fig(٤-٥٩): The Variation of Entropy Change of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



Fig(٤-٦٠): The Variation of Entropy Change of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



Fig(٤-٦١): The Variation of Entropy Change of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.



Fig(٤-٦٢): The Variation of Entropy Change of Aluminum with Distance in a Finite Medium Subjected to a Constant Heat Flux.

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