جمهورية العراق

وزارة التعليم العالي والبحث العلمي

جامعة بابل

كلية الهندسة

دراسة سلوك المساند عكسية الدوران ذات الحلقة العائمة

رسالة

مقدمة إلى كلية الهندسة في جامعة بابل وهي جزء من متطلبات نيل درجة ماجستير علوم في الهندسة الميكانيكية

(مكانيك تطبيقي)

أعدت من قبل

مصطفى باقر هنين الخفاجي

بكالوريوس علوم في الهندسة الميكانيكية

١٤٢٨ ه   ٢٠٠٧ م

Republic of Iraq

Ministry of Higher Education

and Scientific Research

University of Babylon

College of Engineering

AN INVESTIGATION INTO THE BEHAVIOR OF COUNTER ROTATING FLOATING RING JOURNAL BEARING

A Thesis

Submitted to the College of Engineering of

University of Babylon in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering

(Applied Mechanics)

By

Mustafa Baqir Hunain

Al – Khafagi

(B. Sc)

٢٠٠٧ *A.M* ١٤٢٨ *A.H*

بسم الله الرحمن الرحيم

يرفع الله الذين آمنوا مِنكم والذين أُتوا العِلمَ دَرَجاتٍ والله بما تعملونَ خبيرٌ

صدق الله العلي العظيم
سورة المجادلة , الآية

الخلاصة

المساند عكسية الدوران ذات الحلقة العائمة هي نوع خاص من المساند و التي يدور فيها المحور و الركيزة باتجاهين متعاكسين و بسرعة نسبية. تبقى الحلقة العائمة تدور بسرعة محتثة بين المحور و الركيزة.

تم دراسة مختلف المتغيرات المؤثرة على خصائص أداء هذا المسند (نسبة سرعة الحلقة العائمة إلى سرعة المحور, نسبة الخلوص, نسبة أنصاف الأقطار, و كذلك تأثير السرعة العكسية للركيزة) بثبوت و تغير درجة الحرارة, حيث لم تستأثر هذه العوامل المؤثرة على أداء المسند بالاهتمام الكافي خلال الأعمال السابقة. و على ما يبدو أيضا بأنه لا توجد دراسة تأخذ بنظر الاعتبار تأثير تغير لزوجة الزيت على أداء المسند بتغير درجة الحرارة.

تم تطوير و استخدام التحليل الحراري المبسط للمسند المركزي لدراسة تأثير درجة حرارة طبقة الزيت على المتغيرات المختلفة التي تؤثر على أداء المحمل.

تم حل المعادلات الحاكمة و التي تتمثل بمعادلة رينولدز و التي تم تحويرها لتتضمن تأثير طبقتي الزيت (المحور و الحلقة, و الحلقة و الركيزة) مع معادلة الطاقة المبسطة باستخدام شروط حدية مناسبة و معادلات الاتزان لكلا الطبقتين. تم تضمين تأثير درجة الحرارة عن طريق اختيار معادلة مناسبة تحدد علاقة لزوجة الزيت بدرجة الحرارة.

تضمنت المعالجة الرياضية حلا عدديا للمعادلات الحاكمة للمسالة باستخدام طريقة الفرو قات المحددة. و لهذا الغرض تم كتابة برنامجين بلغة فورتران (٩٠) لحل المعادلات الحاكمة أعلاه لكلا الحالتين (بثبوت و تغير درجة الحرارة). إن دراسة سلوك هذا المسند اظهر تأثير واضح لدرجة الحرارة لا يمكن إهماله على أداء المسند قيد الدراسة.

تشير النتائج إلى أن سرعة الحلقة و معامل الاحتكاك يزدادان مع زيادة نسبة سرعة الركيزة إلى المحور,تقليل نسبة الخلوص و نسبة أنصاف الأقطار.

أظهرت النتائج المستحصلة من هذه الدراسة بأن لدرجة الحرارة تأثير لا يستهان به و لا يمكن إهماله على أداء المساند عكسية الدوران ذات الحلقة العائمة. إن سرعة الحلقة تقل مع القيم

العالية لدرجات الحرارة الناتجة بسبب تأثير قص الزيت بالنسبة, للمسند العادي ذو الحلقة العائمة, ثم تزداد سرعة الحلقة عندما تزداد سرعة الدوران العكسي للركيزة. معامل الاحتكاك سوف يقل في كلا الحالتين السابقتين.

ABSTRACT

A counter rotating floating ring journal bearing is a special type of journal bearing in which both the journal and the bearing are counter rotates about its centers in a proportional speed. A floating ring was maintained to rotate with an induced speed between the journal and the bearing.

The effect of different parameters on the performance of such bearing, namely ring to journal speed ratio, bearing to journal speed ratio, clearance ratio, and radii ratio gets a little attention in the previous works. Also, it seems that there is no work, up to our knowledge, dealing with the effect of variable viscosity due to the variation of oil film temperature.

Hence, a simplified thermal analysis of a plain journal bearing was extended and used to study the effect of oil film temperature on different bearing performance parameters.

The classical Reynolds equation which was modified to include the effect of the two oil films (journal – ring and ring – bearing oil films) with a simplified energy equation are solved with an appropriate steady state equilibrium conditions. The effect of the oil film temperature has been included by using a suitable equation of state. The computational treatment consists of a numerical solution for the governing equations of the problem, using a finite difference technique.

Two computer programs have been prepared for thermal and isothermal cases, which were written in (FORETRAN ٩٠) language, to solve the governing equations of the problem.

The results indicate that the ring speed and the coefficient of friction increases with increasing the bearing to journal speed ratio, decreasing the clearance and radii ratios.

II

This investigation reveals that the oil film temperatures have a pronounced effect which cannot be neglected on the performance of counter rotating floating ring journal bearing. For conventional floating ring journal bearing (i. e when ω_b = ٠.٠) the ring speed decreases with higher values of temperature resulting due to the oil shearing effect then the ring speed increases as the counter rotating speed of the bearing increases. In both the above cases the coefficient of friction shown to be decreases.

ACKNOWLEDGMENT

(In The Name of Allah, The Gracious, The Merciful)

All thanks are due to ***Allah*** *for mercy, blessing, and assistance in preparing this work.*

I would like to thank my supervisors ***Dr. Basim A. Abass*** *and* ***Dr. Adil A. Alwan*** *for their guidance, advices and continuous encouragement throughout this research. Their patience and encouragement in preparation of this thesis and the extra time given a busy schedule are greatly appreciated.*

The researcher is also indebted with thanks to ***Dr. Karima. E. Amori****, the lecturer at the Mechanical Engineering Department, University of Baghdad,* ***Mr. Nzar Q. Mohammed****, the lecturer at the Mechanical Engineering Department, University of Baghdad and* ***Miss. Lekaa' H. Abd – Al Shaheed****, the lecturer at the Mechanical Engineering Department, University of Babylon, for their help and great support.*

My thanks are due to ***Mr. Ali Abd Al Ameer****, the lecturer at the Mechanical Engineering Department, University of Kufa, for his assistance and great support.*

Grateful acknowledgments are due to my family and my friends for their help, patience and encouragement during the period of my study.

Mustafa

٢٠٠٧

Appendix A

Derivation of Energy Equation

Many form of a full energy equation can be found in the literatures, and such that given in Ref. **[٢٥]**:-

$$J\left\{\left[\left(\frac{\rho u h}{2}-\frac{h^3}{12\upsilon}\frac{\partial p}{\partial x}\right)\frac{\partial(C_v T)}{\partial x}-\frac{h^3}{12\upsilon}\frac{\partial p}{\partial z}\frac{\partial(C_v T)}{\partial z}\right]-\left[\frac{\partial}{\partial x}\left(hk\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z}\left(hk\frac{\partial T}{\partial z}\right)+\frac{\partial}{\partial y}\left(hk\frac{\partial T}{\partial y}\right)\right]\right\}=$$

$$\rho\frac{\upsilon u^2}{h}+\frac{h^3}{12\rho\upsilon}\left[\left(\frac{\partial p}{\partial x}\right)^2+\left(\frac{\partial p}{\partial z}\right)^2\right] \qquad \text{(A – ١)}$$

Neglecting the heat conduction through the oil film, the above equation can be rewritten as:-

$$6JupC_v h\left[\left(1-\frac{h^2}{6\mu u}\frac{\partial p}{\partial x}\right)\frac{\partial T}{\partial x}-\left(\frac{h^2}{6\mu u}\frac{\partial p}{\partial z}\right)\frac{\partial T}{\partial z}\right]+J\left[K_b(T-T_b)+K_R(T-T_R)\right]=$$

$$\frac{12\mu u^2}{h}\left\{1+\frac{h^4}{12\mu^2 u^2}\left[\left(\frac{\partial p}{\partial x}\right)^2+\left(\frac{\partial p}{\partial z}\right)^2\right]\right\} \qquad \text{(A- ٢)}$$

In the above equation, properties such as C_v and K are taken constant; radiation has neglected altogether.

If the second group of terms with square brackets is eliminated, the equation reduced to its adiabatic version, in which all the heat generation stored in and carried away by the lubricant. Hence, the adiabatic energy equation corresponding to this case can be given by Ref. **[٢٥]**:-

$$[1-\gamma_\theta]\frac{\partial T}{\partial \theta}-\gamma_\zeta=\frac{E}{\beta}\frac{\overline{\mu}}{\overline{h}^2}\left[1+3(\gamma_\theta)^2+3(\gamma_\zeta)^2\right] \qquad (A-٣)$$

Where;

$$\gamma_\theta=\frac{\overline{h}^2}{\overline{\mu}}\left(\frac{\partial\overline{p}}{\partial\theta}\right) \qquad \gamma_\zeta=\frac{\overline{h}^2}{\overline{\mu}}\left(\frac{\partial\overline{p}}{\partial\zeta}\right)$$

Appendix B

Gauss – Seidel Method

The equation used to calculate the pressure field in the lubricant film is equation (٣.٣٧), which can be written as follows:-

$$p_{i,j}^n=\frac{A1_i\,p_{i+1,j}^{n-1}+A2_i\,p_{i-1,j}^{n}+A3_i\,p_{i,j+1}^{n-1}+A4_i\,p_{i,j-1}^{n}-A5_i}{A6_i} \qquad (B-١)$$

Where, n is the iteration number. This equation is presented in a form for ascending order of (i), and (j).

One method usually used to accelerate the rate of convergence of the iterative process, is to consider the residual, which is the right – hand side of equation (B – ١) less ($p_{i,j}^{n-1}$), multiplied by a relaxation factor (R_f), and add to the current value of ($p_{i,j}^{n-1}$), to yield the following, **[١١, ١٢]**;

$$p_{i,j}^{n} = (1-R_f)p_{i,j}^{n-1} + R_f\left[\frac{A1_i\, p_{i+1,j}^{n-1} + A2_i\, p_{i-1,j}^{n} + A3_i\, p_{i,j+1}^{n-1} + A4_i\, p_{i,j-1}^{n} - A5_i}{A6_i}\right] \qquad (B-٢)$$

Where, ٠ < R_f < ٢

For, ٠ < R_f < ١, the scheme is termed successive under – relaxation, for ١ < R_f < ٢, it is termed successive over – relaxation, and for R_f =١, it is reduced to the ordinary Gauss – Seidel method. The successive over relaxation scheme is the most widely used iterative method for solving the Reynolds' equation for incompressible fluid **[٢٦]**.

To terminate the iterative process, a convergence criterion is chosen, so as to give the following, **[١١, ١٢]**;

Error < ١٠$^{-٥}$ (B– ٣)

Where, the error can be evaluated as, **[١١, ١٢, ٢٩]**;

$$Error = \frac{\sum\sum\left|p_{i,j}^{n} - p_{i,j}^{n-1}\right|}{\sum\sum\left|p_{i,j}^{n}\right|} \qquad (B-٤)$$

NOMENCLATURE

The following symbols are generally used throughout the text:-

Symbol	Description	Units
c_1	Journal – Ring Mean Radial Clearance	m
c_2	Ring – Bearing Mean Radial Clearance	m
Cp	Heat Capacity of Lubricant at Constant Pressure	J/Kg. ^{o}C
C_v	Heat Capacity of Lubricant at Constant Volume	J/Kg. ^{o}C
$(D)_{kk}$	Diameter, $(D)_{kk} = (2*R)_{kk}$	m
e_1	Journal Eccentricity, $(e_1= c_1* \varepsilon_1)$	m
e_2	Ring Eccentricity, $(e_2= c_2* \varepsilon_2)$	m
F_j	Frictional Force at the Journal Surface	N
f	Coefficient of Friction	-
$(h)_{kk}$	Oil – Film Thickness	m
$\bar{h}$	Dimensionless Oil – Film Thickness,($\bar{h}=h/c$)	-
J	Mechanical Equivalent of Heat	-
K	Heat Transfer Coefficient	W/m^2.^{o}C
k	Heat Conductivity	W/m.^{o}C
L	Length of Bearing	m
N_1	Number of Nodes Around the Bearing Circumference	-
N_2	Number of Nodes Along the Bearing Length	-
N_c	Number of Circumferential Nodes to the Cavitations Start	-
N_b	Bearing Rotational Speed	r.p.m

N_r	Ring Rotational Speed	r.p.m
N_j	Journal Rotational Speed	r.p.m
O_b	Bearing Center	-
O_r	Ring Center	-
O_j	Journal Center	-
$(p)_{kk}$	Oil – Film Pressure	$N/m^{٢}$
R_j	Journal Radius	m
$R_{١}$	Ring Inner Radius	m
$R_{٢}$	Ring Outer Radius	m
R_f	Over – Relaxation factor	-
S	Reference Sommerfeld Number,($S=\left(\frac{\mu_o \omega_j L R_1}{\pi W_1}\right)\left(\frac{R_1}{c_1}\right)^2$)	-
$T_{r١}$	Frictional Torque at the Inner Surface of Ring	N.m
$T_{r٢}$	Frictional Torque at the Outer Surface of Ring	N.m
$(T)_{kk}$	Temperature of the Oil Film	^oC
$(T_{max})_{kk}$	Maximum Temperature of the Oil Film	^oC
T_o	Inlet lubricant Temperature	^oC
$(U)_{kk}$	Linear Surface Velocity	m/s
$(W)_{kk}$	Load Carrying Capacity of the Bearing	N
$(W_r)_{kk}$	Load Component in Radial Direction	N
$(W_t)_{kk}$	Load Component in Tangential Direction	N
$(x_c)_{kk}$	x – Coordinates to the Position of Cavitation	m
x, y, z	Cartesian Coordinates, (x = R*θ)	m

Greek Symbols

Symbol	Description	Units
$(\alpha)_{kk}$	Ring to Journal Speed Ratio, ($\alpha = \omega_r/\omega_j$)	-
γ	Bearing to Journal speed Ratio, ($\gamma = \omega_b/\omega_j$)	-
γ_θ	Pressure Gradient Term	-
γ_ξ	Pressure Gradient Term	-
β	Viscosity – Temperature Constant	١/ ^{o}C
$\Delta\theta$	Increment in θ– Direction	rad
Δx	Increment in x – Direction, ($\Delta x = R*\Delta\theta$)	m
Δz	Increment in z – Direction	m
$\varepsilon_{\text{١}}$	Journal –Ring Eccentricity Ratio	-
$\varepsilon_{\text{٢}}$	Ring –Bearing Eccentricity Ratio	-
$(\mu)_{kk}$	Absolute Viscosity of Lubricant	pa . s
μ_o	Absolute Viscosity of Inlet Lubricant	pa . s
$\bar{\mu}$	Dimensionless Absolute Viscosity,($\bar{\mu} = \mu/\mu_o$)	-
υ	Kinematics Viscosity of Lubricant ($\upsilon = \mu/\rho$)	$m^{\text{٢}}/s$
θ	Angular Coordinate in Bearing	degree
$(\theta_c)_{kk}$	Angular Coordinate to the Position of Cavitation	degree
φ	Pressure Extent after θ=١٨٠°	degree
ρ	Density of Lubricant	$kg/m^{\text{٣}}$

$(\tau)_{kk}$	Shear Stress of the Oil Film	$N/m^{٢}$
$(\psi)_{kk}$	Attitude Angle	degree
ω_j	Journal Rotational Speed	rad/s
ω_r	Floating Ring Rotational Speed	rad/s
ω_b	Bearing Rotational Speed	rad/s

Subscripts

b	Referring to Bearing
c	Referring to the Position of Cavitation
j	Referring to Journal
i , j	Grid Number in Circumferential and Axial Direction, Respectively
kk	=١ Referred for Journal – Ring Oil – Film =٢ Referred for Ring – bearing Oil – Film
R	Referring to Runner
r	Referring to Floating Ring
α	Angular Position of End of Pressure Curve Beyond Minimum Film Thickness

<u>*LIST OF CONTENTS*</u>

EXAMINING COMMITTEES CERTIFICATE

*We certify that this thesis entitled **"An Investigation into the Behavior of Counter Rotating Floating Ring Journal Bearing "** and as examining committee, examined the student " **Mustafa Baqir Hunain Al _ Khafagi"** in its contents and that in our opinion it meets the standard of thesis for the degree of Master of Science in Mechanical Engineering.*

Signature: *Signature:*

Signature:

Name: ***Lecturer***

Dr. Basim A. Abass Al-Maksosi

(Supervisor)

Date: / /٢٠٠٧

Signature:

Name: ***Asst. Prof.***

Dr. Adil A. Alwan Al- Moosawy

(Supervisor)

Date: / /٢٠٠٧

Signature:

Name: ***Lecturer***

Dr. Hatem H. Obeid Al-Taie

(Member)

Date: / /٢٠٠٧

Signature:

Name: ***Asst. Prof.***

Dr. Alaa' M. Hussan Al-Jesany

(Member)

Date: / /٢٠٠٧

Signature:

Name: ***Asst. Prof.***

Dr. Ahmed A. Al – Rajihy

(Chairman)

Date: / /٢٠٠٧

Approval of the Mechanical Engineering Department

Head of the Mechanical Engineering

Signature:

Name: ***Asst. Prof. Dr. Adil A. Alwan Al- Moosawy***

Date: / /٢٠٠٧

Approval of the College Engineering

Dean of the College of Engineering

Signature:

Name: ***Prof. Dr. Abdul alwahid K. Rajih Al- Bakri***

Date: / /٢٠٠٧

CERTIFICATION

*We certify that this thesis entitled "**An Investigation into the Behavior of Counter Rotating Floating Ring Journal Bearing** " which is prepared by "**Mustafa Baqir Hunain Al - Khafagi**" has been carried out completely under our supervision at the University Of Babylon, Department of Mechanical Engineering, as a partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering.*

Signature: *Signature:*

Lecturer *Asst. Prof.*

Dr. Basim A. Abass *Dr. Adil A. Alwan*

Al-Maksosi *Al-Moosawy*

(Supervisor) *(Supervisor)*

Date / /٢٠٠٧ *Date / /٢٠٠٧*

Chapter One

Introduction

١.١ *Introduction*

Circular journal bearing with the journal and bearing counter rotating at the same speeds has no load capacity and can not serve as a bearing. An example of counter rotating journal bearing system can be found in the inter shaft bearing of the front drive turbo – shaft engine. **Vance** in ١٩٨٨ refers that, it is sometimes desirable from an aerodynamic design standpoint to have the gas generator shaft and power turbine shaft rotating in opposite directions, but this may be another problem for the inter – shaft bearing, especially a fluid – film bearing. A fluid film bearing with journal and bearing counter rotating at equal speeds has zero load capacity. Hence, it is concluded that fluid film bearings can not be used as a counter rotating inter – shaft bearings. The solutions of such problem are to insert a ring and kept free to move in lubricant between the journal and bearing.

A floating ring journal bearing is a special type of hydrodynamic lubricated journal bearing in which a ring is maintained floating in lubricating fluid between the shaft journal and a rigid housing. With the presence of two films, floating ring journal bearings have been considered as less friction loss, better stability characteristics and high damping characteristics than those of conventional journal bearings. Hence, it was found that floating ring journal bearings are attractive for high speed turbomachinery applications. Counter rotating floating ring journal bearing seems to have the advantages mentioned above.

Since the experimental work done by **Pinkus** in ١٩٦٢, it seems that there is a little work dealing with the performance of such bearings. Although, **Yeon – Min et al** in ٢٠٠١ investigated the operating characteristics of counter rotating floating ring journal bearing, the effect of different parameters affecting the performance of such bearing have not been taken into consideration.

Moreover, the performance of the counter rotating floating ring journal bearing is analyzed with an isothermal finite bearing theory i.e. neglecting the effect of oil temperature rise due to the friction heating on the performance of the bearing.

The purpose of the present work is to establish the effect of different parameters, namely clearance ratio (c_1/c_2), radii ratio (R_2/R_1), and bearing to journal speed ratio (γ) on the performance of such bearings. Also, the effect of the temperature induced due to the friction heating was studied during this work. A steady state solution has been obtained theoretically with a finite length bearing theory. The classical forms of Reynolds' equation with a simplified, energy equation are solved simultaneously to satisfy the equilibrium conditions, using the finite difference technique.

For this purpose, two computer programs are written in Fortran ٩٠ to solve the governing equations of the problem. The first was used to solve the isothermal case. The results obtained in this case are checked by comparing it with that obtained by **Yeon – Min et al** in ٢٠٠١. A good agreement between these results was obtained. The second was used to the thermal case, in which the variation of oil viscosity with the film temperature has been considered. An iterative scheme (Gauss – Seidel method) with successive over relaxation has been adopted to solve the problem numerically.

١.١ *Main objectives of the present work*

١. Study the effect of different parameter, namely clearance ratio, radii ratio, and bearing to journal speed ratio on the performance of counter rotating floating ring journal bearing.

٢. Study the effect of viscosity variation due to the friction heating on the performance of such bearing.

٣. Preparing two computer programs used to evaluate the effect of the above parameters on the performance of the bearing for two cases (isothermal and thermal).

Chapter Two

Literature Survey

The review of the literature related to the present work can be classified into the following categories:-

٢. ١. *Lubrication of Floating Ring Journal Bearings*

Tanaka and Hori, (١٩٧٢), [١], studied the oil-whip suppressing effect. They investigated the stability characteristics of floating ring journal bearing theoretically and experimentally. They derive a theory based on the assumptions of infinitely short bearing and film rupture in negative pressure region. The experimental work carried out to study the suppressing effect of the floating ring journal bearing. They concluded that the stability of the floating ring bearing was improved by using a stiffer shaft and larger radii and clearance ratio. The bearing has better stability than the conventional cylindrical bearing; this is attributed to the high damping effect of the outer film. It was found that the theoretical results are in good agreement with the experimental results.

Rohde and Ezzat, (١٩٨٠), [٢], studied the frictional performance of the dynamically loaded floating ring journal bearing and investigated their feasibility for automotive applications. The results of their analysis showed that the frictional power losses decreased with increasing the inner and outer clearances (c_1), and (c_2), and there existed a clearance beyond which there was a minor reduction in power losses. The inner and outer minimum film thickness in the floating ring design could be greater or less than those experienced in the ordinary journal bearing depending on the operating condition and on the radial clearances. The drop of the ring speed increases with increasing inner clearance (c_1), since under this condition the inner film frictional torque available to drive the ring was decreased. The floating ring concept resulted in an average friction reduction of (٣٥%) in the main bearing and (٢٥%) in the connecting rod bearing, as compared to conventional journal bearings. Finally, the results indicate that the adoption of the floating ring design concept had the potential for reducing power loss in automotive engine bearings.

Mokhtar, (١٩٨١), [٣], investigated theoretically the performance characteristics and design data for infinitely long floating ring journal bearings. He treated this bearing analytically with lubricant side leakage effects neglected. Results obtained indicated that the ring dimensions are a dominant factor in deciding the final bearing behavior and that the oil film thickness between the ring and the bearing housing is much thinner than that between the journal and the ring. The increased ring-bearing clearance (c_2) relative to journal-ring clearance (c_1) helped in generating higher ring speeds. Higher ring induced speeds were found, for the same journal eccentricity, with thinner ring thickness. The ring eccentricity ratio (ε_2) was shown to have higher values than the journal eccentricity ratio (ε_1), which may reach unity, while the journal eccentricity ratio was well below. This was attributed to the induced ring speed being necessarily less than the journal speed. This type of bearing should work at journal eccentricities well below (٠.٥) if ring-bearing metallic contact has to be avoided. The hydrodynamic action in the journal –ring interfacial oil film was enhanced with both the journal and ring rotating while at the ring-bearing interfacial oil film poorer hydrodynamic action would result. The floating ring bearing showed less frictional power less than the ordinary bearing, but that might be achieved at the expense of the load carrying capacity.

Li and Rohde, (١٩٨١), [٤], analyzed the steady – state and dynamic characteristics of floating ring journal bearing. The stability characteristics of the bearing, based on linear theory, were given. The transient problem, in which the equation of motion for the bearing system integrated in real time, was studied. They employed the finite bearing theory rather than that short bearing assumption in their study which concluded that the effect of clearance ratio ($c_٢/c_١$) on journal eccentricity ($\varepsilon_١$) was quite small. The ring speed decreased with decreasing the values of Sommerfeld number, this decrease was considerably more rapid with increasing the loads for larger values of clearance ratio ($c_٢/c_١$). Since under those conditions, the outer film eccentricity ratio ($\varepsilon_٢$) was greater than the inner film eccentricity ratio ($\varepsilon_١$). For small values of clearance ratio ($c_٢/c_١$) the outer film became considerably stiffer, hence it could operate at smaller eccentricity ratio than the inner film. At greater values of clearance ratio ($c_٢/c_١$) the outer film could support less load at a given eccentricity ratio.

Higher ring speeds and smaller journal eccentricity ratio ($\varepsilon_{١}$) associated with larger values of ($c_{٢}/c_{١}$) resulted in smaller coefficients of friction.

Athre K. et al, (١٩٨٢), [٥], presents a comparative study for stability characteristics of a single floating bush bearing using the short bearing and infinitely bearing approximation. They concluded that the short bearing approximation showed a higher range of stable operation, which means that making the bearing more slender, (i.e., decreasing L/D ratio), improved the stability characteristics of the bearing. Generally the infinite length bearing approximation gave a lower limit for the stability rang of operation than the short bearing approximation. Stiffer shaft and larger clearance ratio ($c_{٢}/c_{١}$) means more stable rotor-bearing system.

Li, (١٩٨٢), [٦], discuss some information about the design of the floating ring journal bearing supporting a high speed flexible rotor. The analysis was used to calculate the transient response of a rotor supported by two floating ring bearings. It has been shown during this analysis that the power consumption decreased with increasing clearance and that it was more sensitive to change in the inner clearance ($c_{١}$) than in the outer clearance ($c_{٢}$). The ring rotated faster when the inner clearance ($c_{١}$) was reduced. The power consumption in floating ring journal bearing was seen to be lower than that in conventional journal bearing since the shear force at the journal surface became lower due to the rotation of the floating ring.

Wilkock, (١٩٨٣), [٧], compared the single film journal bearing and the floating ring journal bearing from laminar to turbulent conditions. The general case of the floating ring journal bearing under the requirement that capacity number and the load should be the same as for the single film bearing had been examined. The load carrying efficiency of the bearing was also investigated. The analysis showed that the floating ring design made possible significant power saving over the full rang from laminar to turbulent conditions.

Hussain A. J., (١٩٨٤), [٨], carried out an experimental and theoretical investigation into the behavior of a statically loaded multiple floating ring journal bearing with lubricant side leakage neglected. The main conclusions were that the multiple ring journal bearing had better load carrying capacity as the number of rings increased than that obtained in single ring journal bearing and it was possible form of journal bearing, suitable mainly for a high speed and a low load application. The performance characteristics of the bearing became unaffected as the number of rings increased above three. It was found that two and three ring journal bearing had about (٤٤%) and (٥٩.٢%) less energy dissipation respectively than that obtained from the single ring journal bearings having the same (L/D), clearance, and radii ratios. However the above percentages decreased with increasing clearance, and radii ratio. The limiting values of the journal eccentricity ratio of the single and two floating ring journal bearing were below (٠.٥) and (٠.٣) respectively.

Abass B. A., (١٩٨٩), [٩], made a theoretical and experimental investigations into the behavior of the floating ring journal bearing lubricated with bi – phase (liquid – solid) lubricant using an infinitely short bearing approximation. Solid additives used in this work were in the form of wood and graphite particles of various particle size and concentration. They concluded that the ring speed increased linearly with the journal speed. A considerable increase in load carrying capacity and sharp reduction in attitude angle with slight increase in friction coefficient had been achieved in comparison with the bearings lubricated with pure oil.

Dong and Zhao, (١٩٩٠), [١٠], analyzed the performance of floating ring journal bearing with non-stationary load. In this work the experiment was carried out in which the main bearings of a S١٩٥ four stroke diesel engine was replaced by floating ring bearings successfully. It had been invariably insisted that floating ring bearings were only suitable to a relatively high speed and light load rotating machinery. Their observation was dealing with static properties of the bearing and it was possible to use in engines, where economic benefit can be gained by reducing power loss. Floating ring bearings could also be in a good working order, even when subjected to low speeds and heavy, non-stationary loads.

Nacy S. M., (١٩٩٣), [١١], made a theoretical and experimental studies for the floating ring conical journal bearing. The problem of this bearing was solved by assuming the conical floating and rotating ring is in equilibrium conditions. The main conclusions were that the floating ring conical journal bearing has less frictional losses and load carrying capacity when compared with the equivalent conical journal bearing. Floating ring conical journal bearing posses superior stability characteristics as compared with an equivalent conical journal bearing. The stability characteristics are dominated by the two lubricant films, namely journal – ring and ring – bearing lubricant films. The decreasing in the clearance ratio ($c_{٢}/c_{١}$) tends to enhance the stability characteristics, reduce the load carrying capacity, and decrease the frictional force.

Mohammed N. Q., (٢٠٠٠), [١٢], presented a study on the hydrodynamic plain cylindrical floating ring journal bearing**s** operating in the super laminar flow regime. In this work a theoretical analysis was carried out to obtain the conventional bearing parameters under the steady state operation. The oil film forces on this floating ring were obtained by solving Reynolds equation for both inner and outer oil films, by using the finite difference method. They showed that this bearing has a small attitude angle than that of the equivalent journal bearing when it works in super laminar regime, which might be indicated an improvement in the stability of the floating ring journal bearing. The load carrying capacity was less than that of an equivalent journal bearing. The floating ring journal bearing exhibits less frictional losses than that in an equivalent journal bearing, especially when the two bearing operating in the super laminar flow regime. Decreasing the clearance ratio ($c_{١}/c_{٢}$) tends to enhance the stability characteristics, reduce the load carrying capacity, decrease the frictional force and increase the side leakage flow rate for both laminar and superlaminar operation. Decreasing the radii ratio ($R_{٢}/R_{١}$) tends to reduce stability characteristics, increase the load carrying capacity, decrease the frictional force and increase the side leakage flow rate for both laminar and superlaminar operations.

Abd – Al Shaheed L. H., (٢٠٠٥), [١٣], analyzed the steady state characteristics of the porous floating ring journal bearing working under hydrodynamic lubrication conditions. The computational treatment consists of numerical solutions for the governing equation of the problem under different operating conditions namely, permeability, supply pressure, and the geometry of the ring and bearing. The theoretical results obtained, showed that the Sommerfeld number increases with increasing the values of the radii ratio while, decreases with increasing the supply pressure and clearance ratio. The load carrying capacity increases with decreasing radii ratio. The journal – ring eccentricity ratio becomes higher than the ring – bearing eccentricity ratio at higher permeability, clearance ratio and lower radii ratio. The ring speed increases with decreasing the clearance and radii ratios. The coefficient of friction decreases with increasing the values of the supply pressure, the clearance ratio, and decreases with decreasing the values of the radii ratio. The attitude angle of both films increases with increasing values of the permeability parameters, the supply pressure and the clearance ratio, and increases with decreasing the radii ratio.

٢. ٢. *Thermal Effect in Journal Bearings*

Pinkus and Bupara, (١٩٧٩), [١٤], Provides a solution for finite journal bearings with two axial groove offered a method of including variable viscosity in bearing analysis by the use of a simple energy equation uncoupled from the Reynolds equation . The relevant adiabatic solutions is made independent of the specifications of the lubricant used and its initial conditions. The effect of variable viscosity on the performance of misaligned bearings is also examined. The results showed that the effect of a decreasing viscosity was to lower the load capacity of the bearings. The friction factor would be lower than that in isothermal bearings. The side flow rate showed always increase with decreasing oil viscosity. The effect of misalignment was to increase the maximum temperature.

Mitsui et al, (١٩٨٦), [١٥], studied experimentally the effect of various factors on the bearing temperature for the journal bearing and found that the significant parameters are journal speed, lubricant viscosity and clearance ratio. The results were compared with the theoretical analysis by the presented authors and good agreement was obtained over wide rang of operating conditions. It had been shown that the temperature distributions remain nearly constant within the bearing width. The maximum temperature increases remarkably with the increase of the journal speed, and the decrease of eccentricity ratio.

Prabhu and Rajalingham, (١٩٨٧), [١٦], investigated the influence of temperature on journal bearing characteristics, through the strong dependence of viscosity on temperature. They were rendered difficult by the complicated nature of the thermal boundary conditions, the flow in the cavitated region, and by the computational requirements for the simultaneous solution of the Reynolds equation and the energy equation. It was found the judicious use of the experimentally observed fact that neglecting the variation of temperature in the axial direction drastically simplifies the investigation without impairing its accuracy. The main conclusions of this work showed that for a given load and speed, the dependence of viscosity on temperature increases the eccentricity ratio, peak pressure and flow rate, whereas it decreases the attitude angle, friction coefficient and temperature especially at high speeds.

Vijayaraghavan and Brewe,(١٩٩٨), [١٧], investigated the effect of rate of viscosity variation due to the change in temperature and pressure on the performance of an axial grooved journal bearing. A total thermohydrodynamic model with heat transfer within the fluid film and heat dissipation to the ambient through the journal and bush is utilized. They showed that as the film and bearing temperature are concerned, the maximum occurs near the minimum film thickness. The effect of viscosity – pressure at eccentricity ratio less than (٠.٧) is negligible, about (٢٠ %) change in the rate of viscosity variation produces a (٥–١٠ %) variation in load capacity, (٦ – ١٠ %) in peak pressures at the same eccentricity ratio. It also increases the side leakage, power loss, and the temperature for the same load.

Albert E. Yousif et al, (٢٠٠٠), [١٨], adopted a theromhydrodynamic theory to determine the film temperature distribution of (liquid – solid) biphase lubricants including (MoS_2) and carbon black additives. The theoretical results were in a good agreement with the experimental findings up to a certain solid particle additive concentration. The results also indicate that bearing temperature field was significantly affected by the presence of solid particles in oil. Moreover, the effects of solid particles diameter, their percentage concentration by weight, journal speed, and bearing load on film temperatures were examined. The results obtained showed that at maximum film temperature has a maximum limit at a certain value of journal speed, bearing load, and solid particle concentration. The maximum film temperature increases with increasing solid particle concentration by weight. The eccentricity ratio decreases with increasing maximum film temperature. The frictional torque increases with increasing the solid particle concentration.

Hussain A. J. et al, (٢٠٠٢), [١٩], presented a theoretical analysis to obtain the performance parameters of the conventional bearing and studied the effect of Iraqi standard oil viscosity grade on the steady state and dynamic performance of the circular journal bearings under the superlaminar regime. The theromhydrodynamic lubrication and the finite length bearing theories have been adopted. The mixing temperature of the circulating oil with the incoming oil temperature in the groove region, and the cavitation effects were taken into account. The main conclusion of this study showed that the steady state performance was strongly affected by the values of thermal parameter and global Reynolds number. The increase of oil viscosity increases the sommerfeld number and the bush temperature, while it decreases the non – dimensional pressure, friction coefficient and attitude angle. In the superlaminar flow regime the friction coefficient and attitude angle are greater than that in laminar regime while the sommerfeld numbers are less. In the dynamic operation it is found that the stability region increases with increasing oil viscosity grade.

Andres and Kerth, (٢٠٠٤), [٢٠], analyzed a thermal flow for prediction of the steady state and transient response of an automotive turbocharger supported on floating ring journal bearings. They put forward a physical model and fast computation programs in order to improve the design and performance of turbochargers. The model includes the effects of lubricant heating and bearing clearance changes due to the bearing power consumption. The floating ring bearing analysis renders the floating ring speeds and bearing force coefficients for use in a rotordynamic model predicting the system stability. They showed that the predicted ring speed ratio decreases as the shaft speed increases, mainly owing to thermal effects on the operating clearances and film viscosities. The test results showed more dramatic reductions in ring speeds and the prediction for exit lubricant temperature, power losses and floating ring speeds agreed well with measurements obtained in an automotive turbocharger test rig, therefore, the floating ring bearings offer lower power consumption and cooler operating conditions than plain journal bearing or semi – floating ring bearings.

Abd _ AL Ameer A., (٢٠٠٦), [٢١], represented a theoretical analysis to obtain the conventional bearing performance parameters. He studied the effect of lubricant viscosity variation on the performance of the finite width plain journal bearing with Newtonian and non – Newtonian lubricant in the steady state operation conditions using adiabatic solutions. The main conclusions of this work are, for Newtonian lubricant, the decrease of the lubricant viscosity due to the temperature effect decreases the attitude angle and shear force while increases the side leakage flow rate. The variation of viscosity was more significant for higher values of eccentricity ratio. For the non – Newtonian lubricant, the adiabatic solutions showed that the load capacity, shear force and side flow rate were greatly varied compared with that given by the isoviscous solutions.

٢.٣ *Lubrication of Counter Rotating Journal Bearings*

Cheong and Kim, (٢٠٠١), [٢٢], analyzed the steady state performance of the counter – rotating floating ring journal bearings with the isothermal finite bearing theory, and investigate the effect of counter – rotating speed of the bearing on the performance of bearing. It was shown that counter – rotating floating ring journal bearings have considerable load capacity at the same counter – rotating speeds, while conventional circular journal bearings with one fluid film can not. They investigated the relationship between the frictional torques exerted on the ring due to the inner and outer films and rotational speed of the ring, and identified the stability of the equilibrium state and clarified the operating characteristics of the counter – rotating floating ring journal bearing according to the method of acceleration and deceleration of the rotational speeds of the journal and bearing. It was theoretically confirmed that floating ring journal bearing could be used in counter – rotating journal – bearing system and becomes good substitutes for rolling bearings in counter – rotating systems.

٢.٤ *Concluding Remark*

It seems from the above review that there is a little work related to the performance of counter rotating floating ring journal bearing working under different conditions. Also, there is nothing (to our knowledge) about the thermal performance of counter rotating floating ring journal bearing. The present work represents an attempt to study the performance of the counter rotating floating ring journal bearing for L/D= ١.٠ under different working conditions, namely clearance ratio ($c_{١}/c_{٢}$= ٠.٥, ١, ١.٥), radii ratio, ($R_{٢}/R_{١}$= ١.٢٥, ١.٥), and bearing to journal speed ratio, (γ = ٠.٠, -٠.٥, -١.٠). Hence, the purpose of this work is to analyze the steady state performance of counter rotating floating ring journal bearing with thermal and the isothermal finite bearing theory.

Chapter Three

Theoretical Analysis

٣. ١. *Introductions*

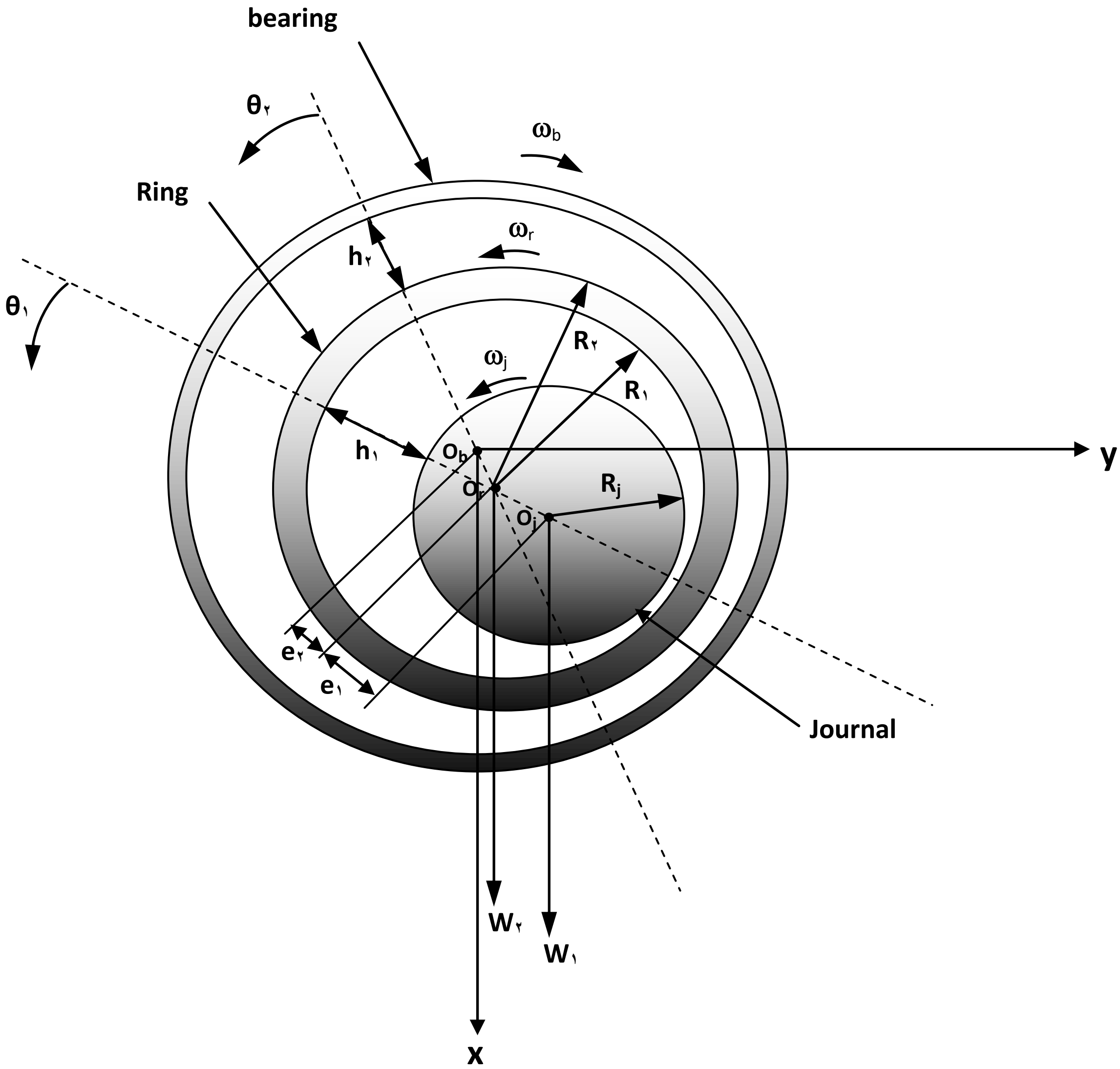
Theoretical analysis presented in this chapter form an attempt to build up a mathematical model to investigate the behavior of the counter rotating floating ring journal bearing under different working conditions. Isothermal case and thermal case have been implemented.

The main governing equations with appropriate assumptions and boundary conditions are used to investigate the performance of hydrodynamically lubricated counter rotating floating ring journal bearing are described in the following articles.

٣. ٢. *Geometry of the Bearing*

The geometry of the counter rotating floating ring journal bearing with the suitable coordinate system used during this work are shown schematically in Fig. (٣.١).

The journal rotates with a constant angular velocity (ω_j) about its center (O_j), while the bearing rotates with a constant angular velocity (ω_b) about its center (O_b). The ring rotates about its center (O_r) with an induced angular velocity (ω_r). Rotational direction of the journal is set to be positive.



o_b Bearing Center

o_r Ring Center

Figure (٣.١) Counter – rotating floating ring journal bearing.

٣. ٣. *Coordinates System*

Two dimensional cylindrical coordinates (θ, z) were adopted to describe the behavior of the oil films, where :-

θ : coordinate in circumferential direction.

z : coordinate in axial direction.

٣. ٤. *Assumptions*

The following assumptions are made to derive the governing equations used to solve the problem:-

١. All the assumptions used in the derivation of Reynolds equation are valid.

٢. No slip at the boundaries (always valid, since the velocity of the oil layer adjacent to the boundary is the same of the boundary).

٣. Lubricant behaves as a Newtonian fluid.

٤. Surface roughness effect is neglected.

٥. Heat transfer through the solid boundaries is neglected.

٦. Shaft is rigid and there is no misalignment.

٧. All the heat generated by the viscous shearing is converted out of bearing by oil.

٨. Axial temperature gradient is ignored.

٣. ٥. *Governing Equations*

The governing equations adopted in this work can be presented as follows:-

٣. ٥. ١. *Reynolds Equation*

The fluid film forces are obtained by solving the basic lubricant equation for the Pressure distribution the so called Reynolds equation which can be written for viscous flow as follows, **[١٠, ٢٢]** :-

$$\left[\frac{1}{R}\frac{\partial}{\partial\theta}\left(\frac{h^3}{\mu R}\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)\right]_{kk}=6(\omega_{kk}+\omega_{kk+1})\left[\frac{\partial h}{\partial\theta}\right]_{kk} \qquad (٣.١)$$

Where;

kk =١, $\omega_{kk} = \omega_j$ and $\omega_{kk+1} = \omega_r$ refer to the journal – ring oil film.

kk =٢, $\omega_{kk} = \omega_r$ and $\omega_{kk+1} = \omega_b$ refer to the ring – bearing oil film.

٣. ٥. ٢. *Oil Film Thickness*

The oil film profile can be approximated by the following equation, **[٢٧]**:-

$$h_{kk} = [c(1+\varepsilon\cos\theta)]_{kk} \qquad (٣.٢)$$

٣.٥.٣. *Energy Equation*

The assumption that the lubricant viscosity remains constant through out the hydrodynamic film is a crude approximation which allowed the derivation and algebraic solution of the Reynolds equation. In practice, the bearing temperature is raised by frictional heat and the lubricant viscosity varies accordingly. Hence, the thermal effects play a major role in bearing operation and can not be ignored. Most hydrodynamic bearings operate under a condition between isothermal and adiabatic heat transfer. Pinkus **[٢٥]**, shows that , according to the experimental results, (٩٩٪) of the heat generated due to the shearing effect within the oil film of a journal bearing is converted away by the oil, while the heat lost by conduction through the bearing surface is not more than (١٪).

The present work provides an analysis to the counter rotating floating ring journal bearing including the effect of variable viscosity on the performance of the bearing

The energy equation used by Pinkus **[٢٥]** is adopted through this analysis to study the effect of oil film temperature on the performance of the counter rotating floating ring journal bearing is:-

$$[1-\gamma_\theta]\frac{\partial T}{\partial\theta} - \gamma_\zeta = \frac{E}{\beta}\frac{\bar{\mu}}{\bar{h}^2}\left[1+3(\gamma_\theta)^2+3(\gamma_\zeta)^2\right] \qquad (٣.٣)$$

Where;

$$\gamma_\theta = \frac{\bar{h}^2}{\bar{\mu}}\left(\frac{\partial \bar{p}}{\partial \theta}\right) \qquad (٣.٤)$$

$$\gamma_\zeta = \frac{\bar{h}^2}{\bar{\mu}}\left(\frac{\partial \bar{p}}{\partial \zeta}\right) \qquad (٣.٥)$$

A brief derivation for the above equation can be found in Appendix A.

McCallion, et al, **[٢٨]**, shows that neglecting the pressure gradient terms, (γ_θ) and (γ_ζ), in equation (٣.٣) does not introduce any significant error in bearing performance over the full range of possible eccentricity ratios.

Hence evaluating the oil film temperature can be made regardless to the pressure gradient effects, (γ_θ) and (γ_ζ).

The temperature within the oil film at any angular position can be evaluated as **[١٤, ٣٤]** :-

$$T_{kk} = \left[T_o + \frac{1}{\beta}\ln\left(1 + EI_\theta\right)\right]_{kk} \qquad (٣.٦)$$

Where;

E_{kk} : is a constant given by

$$E_{kk} = 2(\omega_{kk+1} + \omega_{kk})\left[\left(\frac{R}{c}\right)^2\left(\frac{\beta\mu_o}{\rho Cp}\right)\right]_{kk} \qquad (٣.٧)$$

I_θ: Is the integral of the angle between the inlet oil hole and the section in which it is required to calculate the temperature, it can be calculated as, **[٢٥]**;

$$I_{\theta(kk)} = \left[\int \frac{1}{\bar{h}^2} d\theta\right]_{kk} = \left[\frac{1}{(1-\varepsilon^2)}\left(-\frac{\varepsilon \sin\theta}{1+\varepsilon\cos\theta} + \frac{1}{(1-\varepsilon^2)^{\frac{1}{2}}}\cos^{-1}\left(\frac{\varepsilon + \cos\theta}{1+\varepsilon\cos\theta}\right)\right)\right]_{kk} \qquad (٣.٨)$$

٣.٥.٤. *Viscosity – Temperature Relationship*

It is well known that oil viscosity varies more markedly with temperature than with pressure within the low pressure range of (٠-٠.١)Gpas [٣٢, ١٤], therefore, the following viscosity-temperature relationship has been adopted through this work as used by **[٣١, ٣٦]** :-

$$\mu_{kk} = \mu_o e^{-\beta(T_{kk} - T_o)} \qquad (٣.٩)$$

٣.٥.٥. *Boundary conditions*

Pressure distribution through the inner and outer oil films are obtained by solving the Reynolds equation subjected to the following boundary conditions, **[١٢, ٢٢]** :-

$(p)_{kk} = 0$ at $(\theta)_{kk} = 0$ (at position of maximum film thickness). (٣.١٠)

$(p)_{kk} = \left(\frac{\partial p}{\partial \theta}\right)_{kk} = 0$ at $(\theta)_{kk} = (\theta_c)_{kk}$ (at film rapture boundaries). (٣.١١)

$(p)_{kk} = 0$ at $z = 0$ and $z = L$ (at axial ends). (٣.١٢)

Where:

$(\theta_c)_{kk} = 180 + (\varphi)_{kk}$ (٣.١٣)

$(T)_{kk} = T_o$ at $(\theta)_{kk} = 0,2\pi$ (at position of maximum film thickness). (٣.١٤)

٣. ٦. *Bearing parameters*

The solution of the Reynolds equation with the appropriate boundary conditions yields the bearing pressure distribution. Integrate the pressure distribution in radial and tangential directions give the load components in these directions, **[١١, ٣٣]**;

$$(W_r)_{kk} = \left[\int_0^L \int_0^{\theta_c} Rp\cos\theta d\theta dz\right]_{kk} \quad (٣.١٥)$$

$$(W_t)_{kk} = \left[\int_0^L \int_0^{\theta_c} Rp\sin\theta d\theta dz\right]_{kk} \quad (٣.١٦)$$

The total load carried by the bearing can be evaluated as:-

$$(W)_{kk} = \sqrt{(W_r)_{kk}^2 + (W_t)_{kk}^2} \qquad (٣.١٧)$$

The attitude angle for both oil films (inner and outer oil films) can be evaluated as:-

$$(\psi)_{kk} = \tan^{-1}\left(\mp \frac{W_t}{W_r}\right)_{kk} \qquad (٣.١٨)$$

The positive sign would be used for outer oil film (ring – bearing oil film) at γ = - ٠.٥, -١.٠, because of the counter rotating of the bearing.

The reference sommerfeld number can be expressed from the following expression**, [٢٢, ٢٧]**:-

$$S = \left(\frac{\mu_o \omega_j L R_1}{\pi W_1}\right)\left(\frac{R_1}{c_1}\right)^2 \qquad (٣.١٩)$$

The shear stress, for laminar flow and Newtonian lubricant, at the bearing – ring surfaces and the ring – journal surfaces can be written as, **[٣٣]**:-

$$(\tau)_{kk} = \mu \frac{\partial u}{\partial y}\Big|_{h_{kk},0} \qquad (٣.٢٠)$$

Where;

u: is the lubricant velocity field across the inner and outer oil films, which can be evaluated as, **[١٢, ٣٣]**:-

$$(u)_{kk} = \frac{1}{2\mu_{kk}}\left(\frac{\partial p}{\partial x}\right)_{kk}\left(y^2 - yh_{kk}\right) + \left(1 - \frac{y}{h}\right)_{kk}(u)_{kk+1} + \frac{y}{h_{kk}}(u)_{kk} \qquad (٣.٢١)$$

$$\left(\frac{\partial u}{\partial y}\right)_{kk} = \frac{1}{2\mu_{kk}}\left(\frac{\partial p}{\partial x}\right)_{kk}\left(2y - h_{kk}\right) + \frac{1}{h_{kk}}\left(u_{kk} - u_{kk+1}\right) \qquad (٣.٢٢)$$

Substituting equation (٣.٢٢) into equation (٣.٢٠) to get;

$$(\tau)_{kk} = \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)_{kk}\left(2y - h_{kk}\right) + \left(\frac{\mu}{h}\right)_{kk}\left(u_{kk} - u_{kk+1}\right) \qquad (٣.٢٣)$$

For the counter rotating floating ring journal bearing, shear stress induced at the journal surface can be expressed as, **[١٢, ٣٣]**:-

$$\tau_j\Big|_{y=h_1} = \frac{h_1}{2}\left(\frac{\partial p}{\partial x}\right)_1 + \left(\frac{\mu}{h}\right)_1\left(u_j - u_r\right) \qquad (٣.٢٤)$$

The friction force at the journal surface is obtained by integrating the shear stress equation defined above over the journal surface area as follows, **[١٢]** ;

$$F_j = \int_0^L \tau_j \, dx dz \qquad (٣.٢٥)$$

$$F_j = \int_0^L \int_0^{x_{c1}} \left(\frac{h_1}{2}\left(\frac{\partial p}{\partial x}\right)_1 + \left(\frac{\mu}{h}\right)_1\left(u_j - u_r\right)\right) dx_1 dz \qquad (٣.٢٦)$$

Hence, the friction coefficient can be defined as:-

$$f = \frac{F_j}{W_j} \qquad (٣.٢٧)$$

The shear stress induced at the inner ring surface can be written as:-

$$\tau_1\Big|_{y=0} = -\frac{h_1}{2}\left(\frac{\partial p}{\partial x}\right)_1 + \left(\frac{\mu}{h}\right)_1 \left(u_j - u_r\right) \qquad (٣.٢٨)$$

The frictional torque acting on the inner ring can be evaluated as, **[٢٠]**:-

$$T_{r1} = R_1 \int_0^L \int_0^{x_{c1}} \left(-\frac{h_1}{2}\left(\frac{\partial p}{\partial x}\right)_1 + \left(\frac{\mu}{h}\right)_1 \left(u_j - u_r\right) \right) dx_1 dz \qquad (٣.٢٩)$$

The shear stress induced at the outer ring surface can be written as:-

$$\tau_2\Big|_{y=h_2} = \frac{h_2}{2}\left(\frac{\partial p}{\partial x}\right)_2 + \left(\frac{\mu}{h}\right)_2 \left(u_r - u_b\right) \qquad (٣.٣٠)$$

Then the frictional torque acting on the outer ring can be evaluated as, **[٢٠]**:-

$$T_{r2} = R_2 \int_0^L \int_0^{x_{c2}} \left(\frac{h_2}{2}\left(\frac{\partial p}{\partial x}\right)_2 + \left(\frac{\mu}{h}\right)_2 \left(u_r - u_b\right) \right) dx_2 dz \qquad (٣.٣١)$$

٣.٧. Steady State Performance

To calculate the steady state performance of the bearing, it is necessary to find out the steady state equilibrium positions of the journal and ring centers. With floating ring fully immersed in the lubricating oil filling the bearing, as shown in Fig. (٣.١), the hydrodynamic action would eventually forces the ring to rotate at a speed governed by bearing assembly and bearing parameters. To calculate the load carrying capacity and the frictional torque on the journal, it is necessary to find those positions of the journal and the ring centers with respect to the bearing center at which the ring is in steady state equilibrium, that is the eccentricity ratios, attitude angles for inner and outer films, and the ratio of ring to journal speeds (α) must be established. This is done by performing the force balance and the torque balance at the ring. In steady state operating regimes, the frictional torques acted on the inner and outer ring surfaces must be equal and opposite, thus, **[٢٢]**;

$$T_{r١} = T_{r٢} \qquad (٣.٣٢)$$

Also, the hydrodynamic film forces at both oil films must be support the applied load. The developed hydrodynamic pressure at journal-ring and ring-sleeve films should be capable of supporting the applied load and keeping the ring floating without metal to metal contact, i.e., the magnitude and the direction of inner and outer films loads must be equal, thus, **[٢٢]**;

$$W_{١} = W_{٢} \qquad (٣.٣٣)$$

Equations (٣.٣٢) and (٣.٣٣) represent two equilibrium equations with three unknowns, namely journal – ring eccentricity ratio ($\varepsilon_{١}$), ring-bearing eccentricity ratio ($\varepsilon_٢$), and the ring speed. Hence, equations (٣.٣٢) and (٣.٣٣) may solve iteratively to evaluate these unknowns. The values of the journal – ring eccentricities ratio are assumed in order to determine the other unknowns namely the ring-bearing eccentricity ratio and the ring speed iteratively.

٣.٨. *Numerical solution*

The governing equations mentioned before are solved numerically by using the finite difference method.

This section is dealing with the construction of the numerical solution of the governing equations, the computation algorithm and the structure of the computer program which prepared to solve the problem as follows :-

٣.٨.١. *Grid Generation*

Solution of the partial differential equations can be greatly simplified by a well-constructed grid. The oil film pressures distribution (P_{kk}) can be obtained by solving equation (٣.١) for the inner and outer oil films.

This equation is discretized by a mesh size of ($N_١$ =٥٠) in circumferential direction, and ($N_٢$ =٢٠) along the length of the bearing. The finite difference grid can be shown schematically in Fig. (٣.٢).

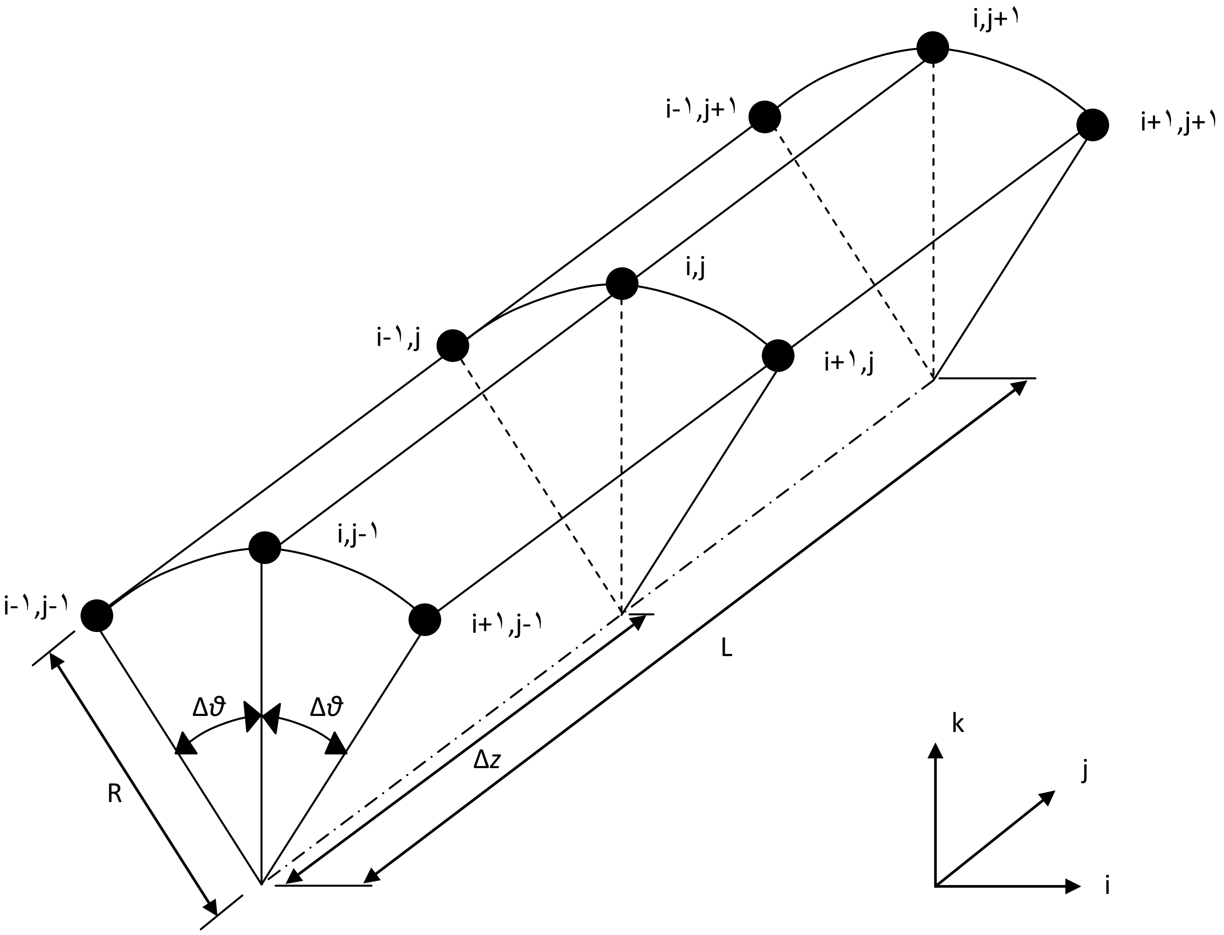


Figure (٣.٢) Finite difference grid of the oil film.

٣.٨. ٢. *Disecritization of the Governing Equations*

The Reynolds equation (٣.١) can be rewritten as follows:-

$$\left[\frac{1}{R}\frac{\partial}{\partial\theta}\left(\frac{h^3}{\mu R}\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)\right]_{kk}=6\left(\omega_{kk}+\omega_{kk+1}\right)\left[\frac{\partial h}{\partial\theta}\right]_{kk}$$

The finite difference analog to Reynolds equation for two cases can be written as, **[١١, ١٢]**:-

$$\left[\frac{1}{R^2}\frac{1}{\Delta\theta^2}\left[\frac{h^3_{i+0.5,j}}{\mu_{i+0.5,j}}p_{i+1,j}+\frac{h^3_{i-0.5,j}}{\mu_{i-0.5,j}}p_{i-1,j}\right]+\frac{1}{\Delta z^2}\left[\frac{h^3_{i,j+0.5}}{\mu_{i,j+0.5}}p_{i,j+1}+\frac{h^3_{i,j-0.5}}{\mu_{i,j-0.5}}p_{i,j-1}\right]\right.$$

$$\left.-\left[\frac{1}{R^2}\frac{1}{\Delta\theta^2}\left[\frac{h^3_{i+0.5,j}}{\mu_{i+0.5,j}}+\frac{h^3_{i-0.5,j}}{\mu_{i-0.5,j}}\right]+\frac{1}{\Delta z^2}\left[\frac{h^3_{i,j+0.5}}{\mu_{i,j+0.5}}+\frac{h^3_{i,j-0.5}}{\mu_{i,j-0.5}}\right]\right]p_{i,j}\right]_{kk}=$$

$$6\left(\omega_{kk}+\omega_{kk+1}\right)\frac{\left\lfloor h_{i+0.5,j}-h_{i-0.5,j}\right\rfloor_{kk}}{\Delta\theta} \qquad (٣.٣٤)$$

Equation (٣.٣٤) can be simplified as:-

$$\left(p_{i,j}\right)_{kk}=\left(\frac{A1_i\,p_{i+1,j}+A2_i\,p_{i-1,j}+A3_i\,p_{i,j+1}+A4_i\,p_{i,j-1}-A5_i}{A6_i}\right)_{kk} \qquad (٣.٣٥)$$

Where;

$$\left(A1_i\right)_{kk}=\left(\frac{h^3{}_{i+0.5}}{R^2\mu_{i+0.5}\Delta\theta^2}\right)_{kk} \qquad (٣.٣٦)$$

$$\left(A2_i\right)_{kk}=\left(\frac{h^3{}_{i-0.5}}{R^2\mu_{i-0.5}\Delta\theta^2}\right)_{kk} \qquad (٣.٣٧)$$

$$\left(A3_i\right)_{kk}=\left(\frac{h^3{}_i}{\mu_i\Delta z^2}\right)_{kk} \qquad (٣.٣٨)$$

$$\left(A4_i\right)_{kk}=\left(\frac{h^3{}_i}{\mu_i\Delta z^2}\right)_{kk} \qquad (٣.٣٩)$$

$$\left(A5_i\right)_{kk}=6\left(\omega_{kk}+\omega_{kk+1}\right)\left(\frac{h_{i+0.5}-h_{i-0.5}}{\Delta\theta}\right)_{kk} \qquad (٣.٤٠)$$

$$\left(A6_i\right)_{kk}=\left(A1_i+A2_i+A3_i+A4_i\right)_{kk} \qquad (٣.٤١)$$

The film thickness can be represented by:-

$$\left(h_i\right)_{kk}=\left(c\left(1+\varepsilon\cos\theta_i\right)\right)_{kk} \qquad (٣.٤٢)$$

$$\left(h_{i+0.5}\right)_{kk}=\left(c\left(1+\varepsilon\cos\theta_{i+0.5}\right)\right)_{kk} \qquad (٣.٤٣)$$

$$\left(h_{i-0.5}\right)_{kk}=\left(c\left(1+\varepsilon\cos\theta_{i-0.5}\right)\right)_{kk} \qquad (٣.٤٤)$$

And the oil viscosity can be represented using equation (٣.٩):-

$$\left(\mu_i\right)_{kk} = \mu_o e^{-\beta\left(T_i - T_o\right)_{kk}} \qquad (٣.٤٥)$$

$$\left(\mu_{i+0.5}\right)_{kk} = \mu_o e^{-\beta\left(T_{i+0.5} - T_o\right)_{kk}} \qquad (٣.٤٦)$$

$$\left(\mu_{i-0.5}\right)_{kk} = \mu_o e^{-\beta\left(T_{i-0.5} - T_o\right)_{kk}} \qquad (٣.٤٧)$$

Also the temperature distribution can be written from equation (٣.٦):-

$$\left(T_i\right)_{kk} = \left(T_o + \frac{1}{\beta}\ln\left(1 + EI_i\right)\right)_{kk} \qquad (٣.٤٨)$$

$$\left(T_{i+0.5}\right)_{kk} = \left(T_o + \frac{1}{\beta}\ln\left(1 + EI_{i+0.5}\right)\right)_{kk} \qquad (٣.٤٩)$$

$$\left(T_{i-0.5}\right)_{kk} = \left(T_o + \frac{1}{\beta}\ln\left(1 + EI_{i-0.5}\right)\right)_{kk} \qquad (٣.٥٠)$$

Where;

$$E_{kk} = 2\left(\omega_{kk+1} + \omega_{kk}\right)\left[\left(\frac{R}{c}\right)^2\left(\frac{\beta\mu_o}{\rho Cp}\right)\right]_{kk} \qquad (٣.٥١)$$

$$\left(I_i\right)_{kk} = \left[\frac{1}{\left(1-\varepsilon^2\right)}\left(-\frac{\varepsilon\sin\theta_i}{1+\varepsilon\cos\theta_i} + \frac{1}{\left(1-\varepsilon^2\right)^{\frac{1}{2}}}\cos^{-1}\left(\frac{\varepsilon + \cos\theta_i}{1+\varepsilon\cos\theta_i}\right)\right)\right]_{kk} \qquad (٣.٥٢)$$

$$\left(I_{i+0.5}\right)_{kk} = \left[\frac{1}{\left(1-\varepsilon^2\right)}\left(-\frac{\varepsilon\sin\theta_{i+0.5}}{1+\varepsilon\cos\theta_{i+0.5}} + \frac{1}{\left(1-\varepsilon^2\right)^{\frac{1}{2}}}\cos^{-1}\left(\frac{\varepsilon + \cos\theta_{i+0.5}}{1+\varepsilon\cos\theta_{i+0.5}}\right)\right)\right]_{kk} \qquad (٣.٥٣)$$

$$\left(I_{i-0.5}\right)_{kk} = \left[\frac{1}{\left(1-\varepsilon^2\right)}\left(-\frac{\varepsilon\sin\theta_{i-0.5}}{1+\varepsilon\cos\theta_{i-0.5}} + \frac{1}{\left(1-\varepsilon^2\right)^{\frac{1}{2}}}\cos^{-1}\left(\frac{\varepsilon + \cos\theta_{i-0.5}}{1+\varepsilon\cos\theta_{i-0.5}}\right)\right)\right]_{kk} \qquad (٣.٥٤)$$

And the angles can be represented by:-

$$(\theta_i)_{kk} = (i-1)\Delta\theta \qquad (٣.٥٥)$$

$$(\theta_{i+0.5})_{kk} = (i-0.5)\Delta\theta \qquad (٣.٥٦)$$

$$(\theta_{i-0.5})_{kk} = (i-1.5)\Delta\theta \qquad (٣.٥٧)$$

Equation (٣.٣٥) was solved iteratively for the pressure distribution ($P_{i,j}$)$_{kk}$ using the Gauss – Siedel iterative method with the successive over relaxation scheme which accelerates the rate of convergence of the iterative process. It is the most widely used iterative method for solving the Reynolds equation for incompressible fluid, **[٢٦].** The procedure of the solution has been shown in appendix B.

The load carrying capacity can be calculated from the pressure distribution over the bearing surface. Equations (٣.١٥), and (٣.١٦) can be expressed in the following numerical form;

$$(W_r)_{kk} = \left[\sum_{j=1}^{N_2-1}\sum_{i=1}^{N_1-1}\left[\frac{p_{i+1,j+1}+p_{i,j+1}+p_{i+1,j}+p_{i,j}}{4}\right]\cos\theta_{i-0.5}\Delta x\Delta z\right]_{kk} \qquad (٣.٥٨)$$

$$(W_t)_{kk} = \left[\sum_{j=1}^{N_2-1}\sum_{i=1}^{N_1-1}\left[\frac{p_{i+1,j+1}+p_{i,j+1}+p_{i+1,j}+p_{i,j}}{4}\right]\sin\theta_{i-0.5}\Delta x\Delta z\right]_{kk} \qquad (٣.٥٩)$$

Where;

$$(\theta_i)_{kk} = (i - 0.5)\Delta\theta \qquad (٣.٦٠)$$

Hence;

$$(W)_{kk} = \sqrt{(W_r)_{kk}^2 + (W_t)_{kk}^2} \qquad (٣.٦١)$$

Then consequently, the attitude angle is:-

$$(\psi)_{kk} = \tan^{-1}\left(\mp \frac{W_t}{W_r}\right)_{kk} \qquad (٣.٦٢)$$

The numerical representation of the journal friction, given by equation (٣.٢٦) can be written as:-

$$F_j = \sum_{j=1}^{N_2}\sum_{i=1}^{N_c}\left[\left(\frac{h_i}{2}\right)_1\left(\frac{p_{i+1,j} - p_{i,j}}{\Delta x}\right)_1 + \left(\frac{\mu_i}{h_i}\right)_1\left(u_j - u_r\right)\right]\Delta x_1 \Delta z \qquad (٣.٦٣)$$

The numerical representation of the frictional torques acting at the inner and outer surfaces of the ring, given by equations (٣.٢٩), and (٣.٣١), can be expressed numerically as:-

$$T_{r_1} = R_1\left[\sum_{j=1}^{N_2}\sum_{i=1}^{N_c}\left[\left(\frac{-h_i}{2}\right)_1\left(\frac{p_{i+1,j}-p_{i,j}}{\Delta x}\right)_1+\left(\frac{\mu_i}{h_i}\right)_1\left(u_j-u_r\right)\right]\Delta x_1\Delta z\right] \qquad (٣.٦٤)$$

$$T_{r_2} = R_2\left[\sum_{j=1}^{N_2}\sum_{i=1}^{N_c}\left[\left(\frac{h_i}{2}\right)_2\left(\frac{p_{i+1,j}-p_{i,j}}{\Delta x}\right)_2+\left(\frac{\mu_i}{h_i}\right)_2\left(u_r-u_b\right)\right]\Delta x_2\Delta z\right] \qquad (٣.٦٥)$$

٣. ٩. *Computation Algorithm*

The numerical calculation described in the previous articles can be transformed into a computational algorithm, as follows;

٣. ٩. ١. *Isothermal Case:-*

Choose material, geometry, dimensions, and then,

١. Perform the grid generation for the bearing oil films in (θ, z) directions.

٢. Choose a value for the journal – ring eccentricity ratio ($\varepsilon_١$).

٣. Assume initial value for the ring – bearing eccentricity ratio ($\varepsilon_٢$).

٤. Assume initial value for the ring speed.

٥. Calculate the pressure distribution through the inner oil film by solving Reynolds equations (٣.٣٥), using Gauss – Seidel iterative method with over relaxation factor, then test the convergence using the convergence criterion (B – ٣)

٦. Calculate the load carrying capacity of the journal – ring oil film using equation (٣.٦١).

٧. Calculate the frictional force at the journal surface of the oil film using equation (٣.٦٣).

٨. Calculate the frictional torque at the inner surface of the journal – ring oil film using equation (٣.٦٤).

٩. Repeat the above procedure from step (٥), with the ring – bearing oil film and calculate the load carrying capacity of the ring – bearing oil film using equation (٣.٦١), then evaluate the frictional torque at the outer surface of the ring – bearing oil film using equation (٣.٦٥).

١٠. If $|W_{١}-W_{٢}|<١٠^{-٣}$ and $|T_{r١}-T_{r٢}|<١٠^{-٣}$ then move to the next step, otherwise modify the value of ring speed.

١١. If ($N_r<N_j$) then return to step ٥, otherwise modify the value of the ring – bearing eccentricity ratio ($\varepsilon_{٢}$) and return to step ٤.

١٢. Calculate the value of reference Sommerfeld number, attitude angle and coefficient of friction.

١٣. If ($\varepsilon_{٢}>٠.٩٥$) then move to the next step, otherwise modify the value of ring – bearing eccentricity ratio ($\varepsilon_{٢}$) and then return to step ٤.

١٤. Print the output results.

٣. ٩. ٢. *Thermal Case:-*

Choose material, geometry, dimensions, inlet temperature of the bearing and then,

١. Perform the grid generation for the bearing oil films in (θ, z) directions.

٢. Choose a value for the journal – ring eccentricity ratio ($\varepsilon_{١}$).

٣. Assume initial value for the ring – bearing eccentricity ratio ($\varepsilon_{٢}$).

٤. Assume initial value for the ring speed.

٥. Calculate the temperature distribution through the journal – ring oil film using equations (٣.٤٨), (٣.٤٩), and (٣.٥٠).

٦. Calculate the viscosity distribution through the journal – ring oil film using equations (٣.٤٥), (٣.٤٦), and (٣.٤٧).

٧. Calculate the pressure distribution through the inner oil film by solving Reynolds equations (٣.٣٥), using Gauss – Seidel iterative method with over relaxation factor, then test the convergence using the convergence criterion (B– ٣).

٨. Calculate the load carrying capacity of the journal –ring oil film using equation (٣.٦١).

٩. Calculate the frictional force at the journal surface of the oil film using equation (٣.٦٣).

١٠. Calculate the frictional torque at the inner surface of the journal – ring oil film using equation (٣.٦٤).

١١. Repeat the above procedure from step (٥), with the ring – bearing oil film and calculate the load carrying capacity of the ring bearing oil film using equation (٣.٦١), then evaluate the frictional torque at the outer surface of the ring – bearing oil film using equation (٣.٦٥).

١٢. If $|W_{١}-W_{٢}|<١٠^{-٣}$ and $|T_{r١}-T_{r٢}|<١٠^{-٣}$ then move to the next step, otherwise modify the value of ring speed.

١٣. If ($N_r<N_j$) then return to step ٥, otherwise modify the value of the ring – bearing eccentricity ratio ($\varepsilon_{٢}$) and return to step ٤.

١٤. Calculate the value of reference Sommerfeld number, attitude angle and coefficient of friction.

١٥. If ($\varepsilon_{٢}>٠.٩٥$) then move to the next step, otherwise modify the value of ring – bearing eccentricity ratio ($\varepsilon_{٢}$) and then return to step ٤.

١٦. Print the output results.

٣.١٠. *Computer Program*

The computer programs were implemented to predicate theoretical results by solving the governing equation of the problem. The programs are written in a suitable programming language (FORTRAN-٩٠).

١. Prog.١: A computer program utilized to predict the steady state characteristics of the counter rotating floating ring journal bearing in isothermal case. The flowchart for this computer program can be shown in Fig. (٣.٣).

٢. Prog.٢: A computer program utilized to predict the steady state characteristics of the counter rotating floating ring journal bearing in thermal case. The flowchart for this computer program can be shown in Fig. (٣.٤).

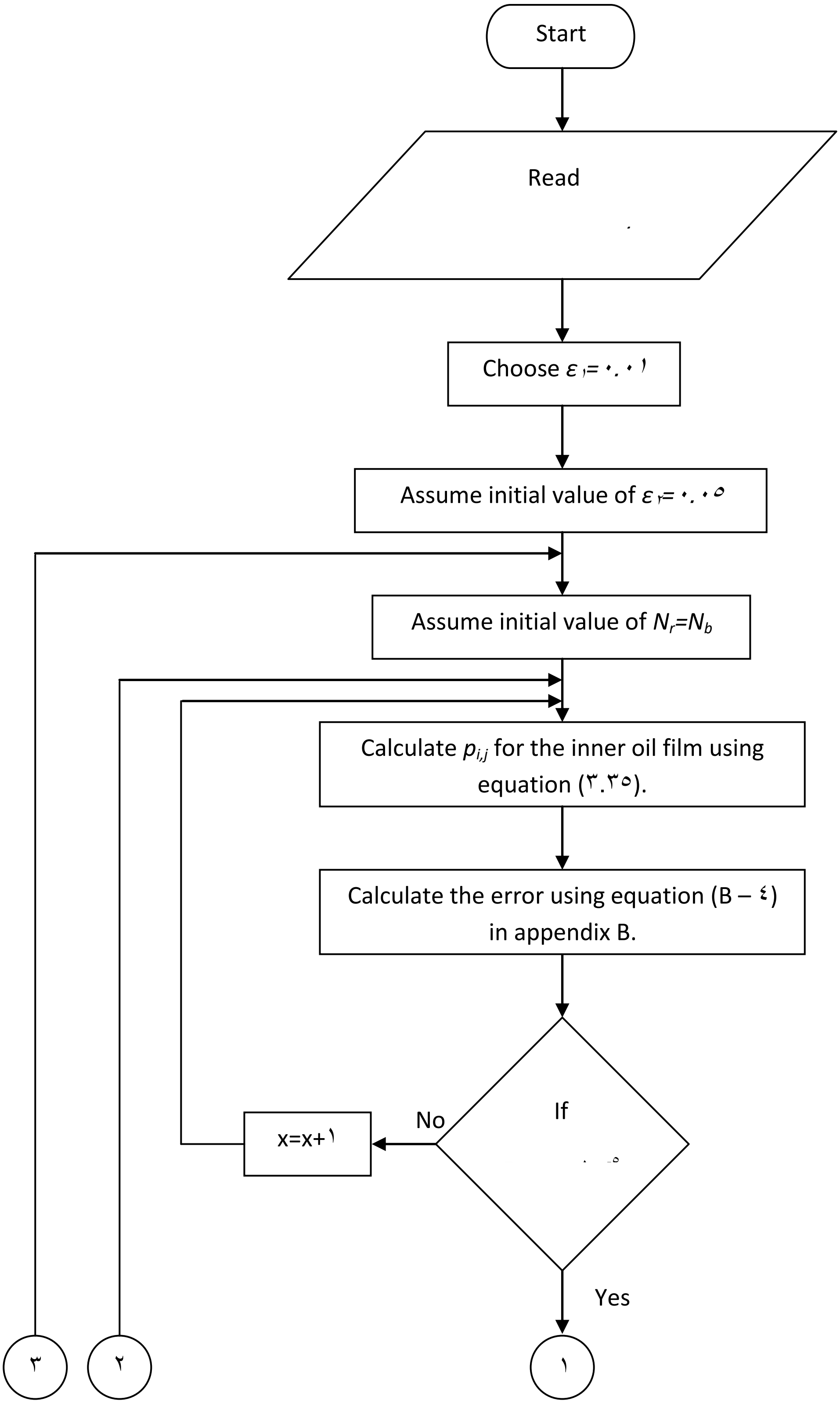


Figure (٣. ٣) Flowchart for Prog. ١ continued.

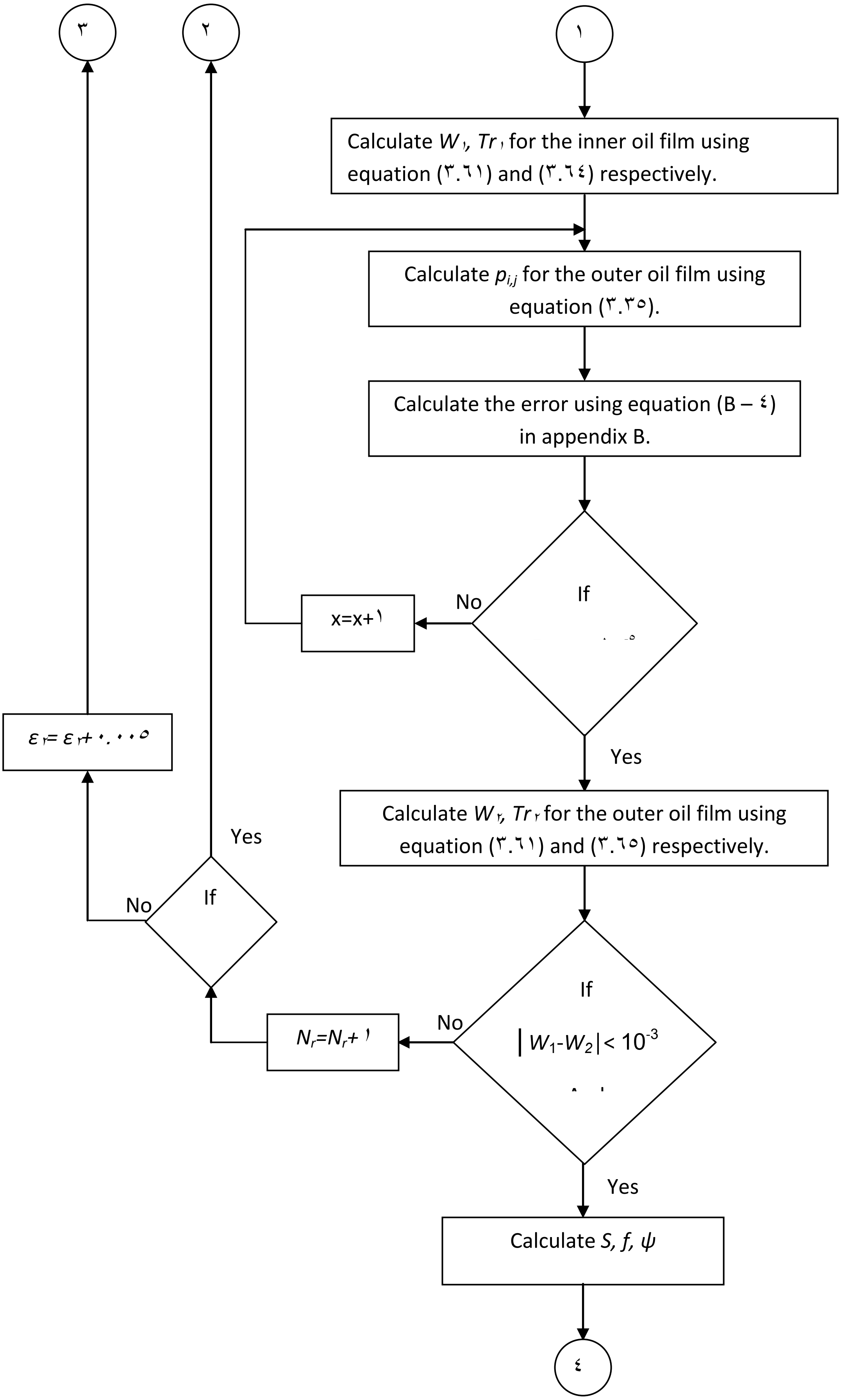


Figure (٣. ٣) continued.

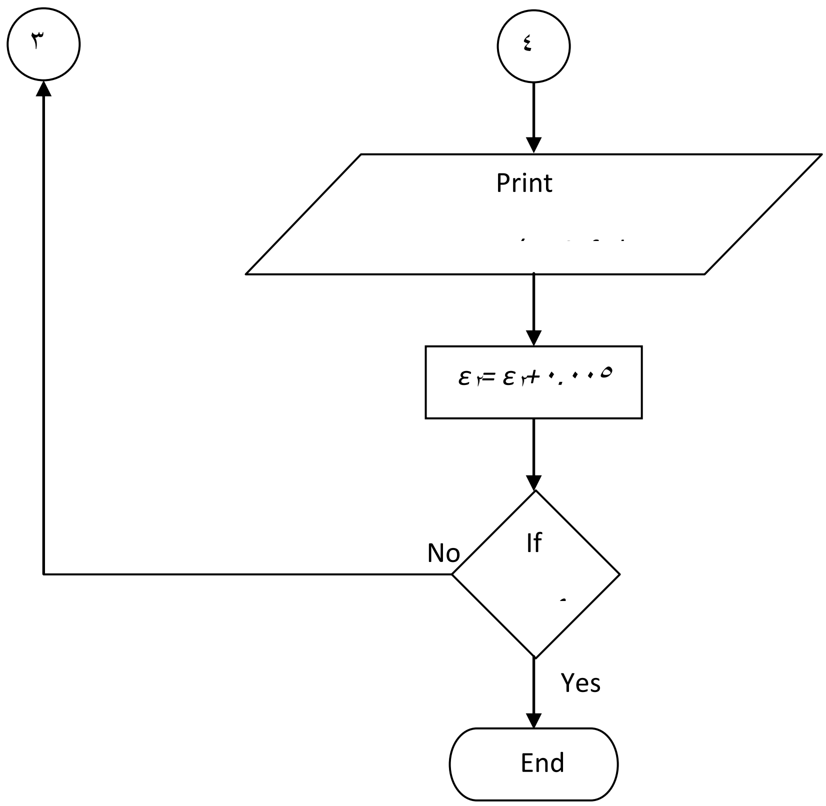


Figure (٣. ٣).

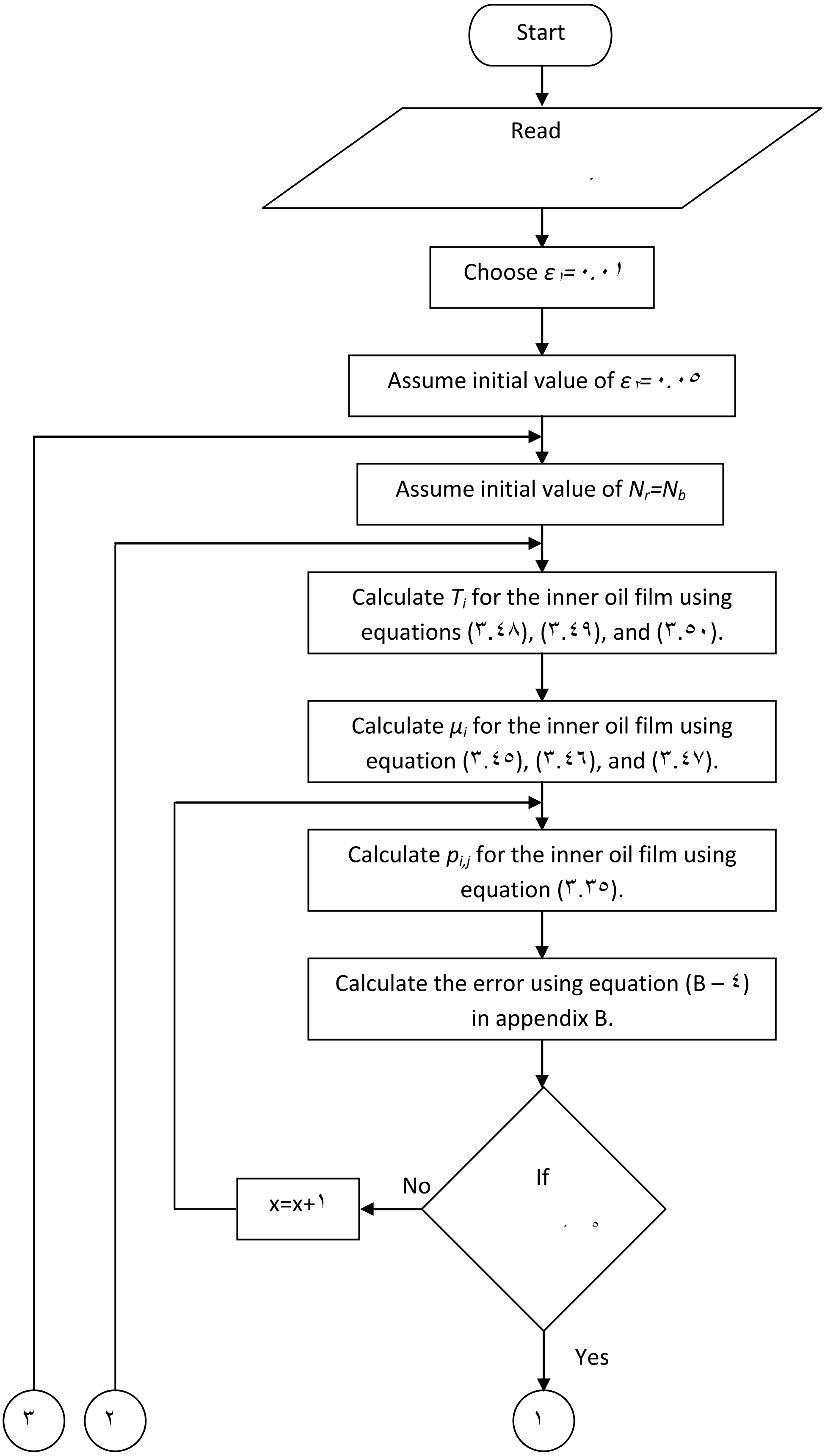


Figure (٣. ٤) Flowchart for Prog. ٢ continued.

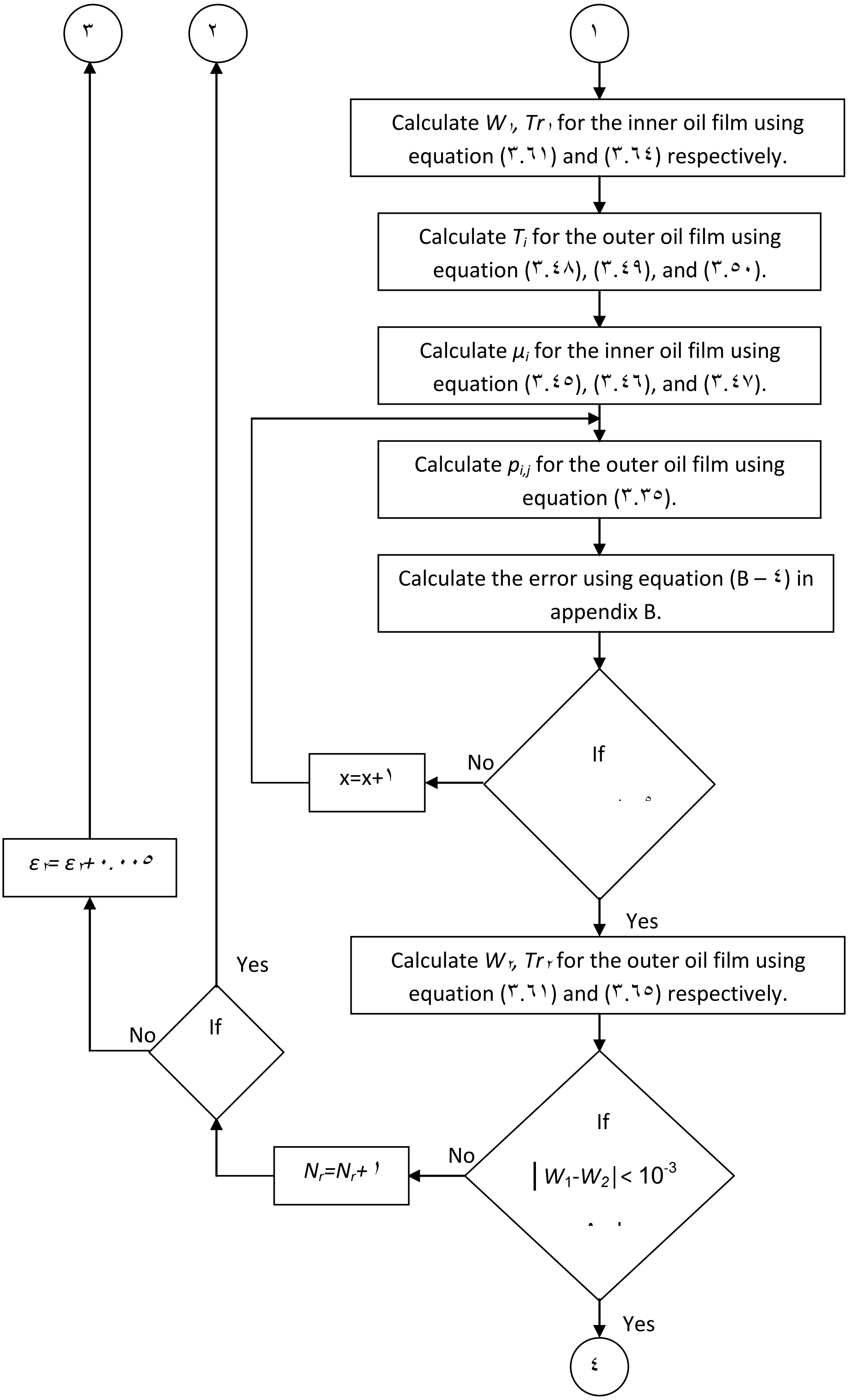


Figure (٣. ٤) continued.

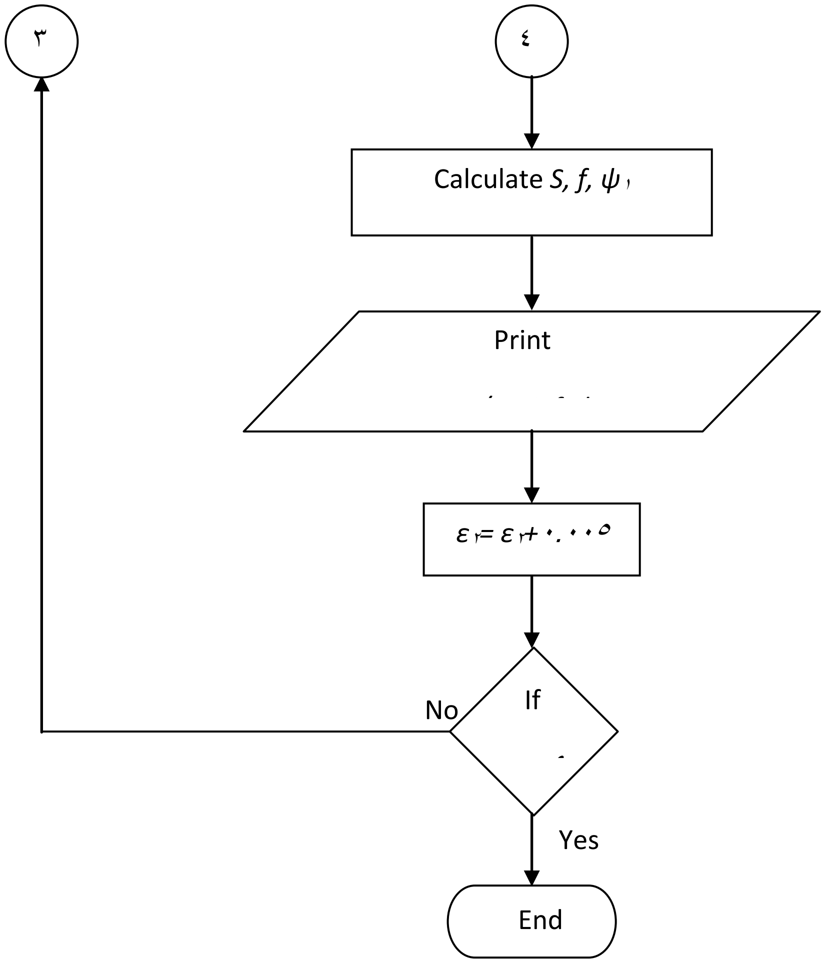


Figure (٣. ٤).