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BY

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B.Sc., ٢٠٠١

الإهداء

إلى:-

نعمة الله السابغة ونقمته الدامغة

الرحمة الموصولة والآية المخزونة

الطلعة الرشيدة و الغرة

الحميدة

ربيع الأنام و نضرة الأيام

الشهاب الثاقب و الأمام الغائب

العلم المنصوب و العلم المصبوب

علم الهدى و كهف الورى

نور الله الذي لا يطفأ

الأوسع كهفاً و

الأكثر علماً

إلى خازن علم الله . . . نور آل محمد

القائم المنتظر (عجل الله فرجه و سهل مخرجه)

أهدت سليمان يوم العرض نملته رجل الجراد التي قد كان في فيها

فترنمت بفصيح القول و اعتذرت إن الهدايا على مقدار مهديها

لينا

بسم الله الرحمن الرحيم

وَيقولون لولا أنزل عليهِ آيةٌ من ربِّه فقُلْ
إنّما الغيبُ لله فانتظروا إني مَعكُمْ من
المُنتظِرينْ

اللهم صلّ على محمد وآل محمد

سورة يونس -(٢٠)

الخلاصة

في هذا المشروع البحثي تم إجراء على تنبؤات الانحناء الزحفي والشد الزحفي عند درجة حرارة الغرفة الثابتة ولفترة زمنية تتراوح بين (١-$١٠^{٩}$) ثانية لقياس خواص الزحف اللزجة-المرنة للبوليمر وحده وللمادة البوليمرية المقواة بألياف قصيرة.

يمثل العمل الحالي دراسة نظرية ماكروميكانيكية تركز على سلوك الزحف الخطي اللزج-المرن لكل من البولي بروبيلين والبولي بروبيلين المقوى بألياف زجاج قصيرة نوع E. حالات التحميل هي أحمال خارجية متمركزة أما التشوهات فهي محددة بانفعالات صغيرة.

في هذا البحث يستخدم الموديل ذو العناصر الثلاثة كموديل رياضي لزج مرن لتمثيل السلوك اللزج المرن الخطي للبولي بروبيلين ولا يصل هذا الموديل مطلقاً إلى مرحلة التشوه الدائمي.

أثناء هذا التحليل تم تطبيق طريقة العناصر المحددة كطريقة عددية للحصول على الخواص الميكانيكية. وقد استخدمت طريقة العناصر المحددة كليركن لاشتقاق المعادلة الأساسية، وقد تم تصميم برنامجين بلغة البيسك السريع للحصول على الخواص الميكانيكية المتمثلة بـ (الانفعال، معامل المرونة، المطاوعة) من خلال تنبؤات الانحناء والشد الزحفيين.

المواد المستخدمة للدراسة الحالية هي:

-بولي بروبيلين غير مقوى متساوي الخواص في جميع لاتجاهات.

-بولي بروبيلين مقوى بألياف زجاج قصيرة موجهة من نوع E.

-بولي بروبيلين مقوى بألياف زجاج قصيرة عشوائية من نوع E.

لقد تم تطبيق مبدأ التكافؤ بين درجة الحرارة والزمن ويمكن ملاحظته من خلال الخواص الميكانيكية لكل من البوليمر والمادة المركبة ذات الأرضية البوليمرية.

معادلة هالبن-تساي أظهرت تحسن في معامل المرونة وتناقص في الانفعال والمطاوعة للمادة المركبة ذات الأرضية البوليمرية المقواة بألياف قصيرة من كلا النوعين اتجاهية وعشوائية من خلال تنبؤات الانحناء والشد الزحفيين.

من ناحية أخرى أظهرت قاعدة المخاليط المعدلة زيادة في قيم الانفعال والمطاوعة مع ثبات في قيم معامل المرونة طبقاً لمنحنيات الشد الزحفي بينما نفس الطريقة أثبتت التأثير القليل للتقوية على خواص المادة المركبة ذات الأرضية البوليمرية من تنبؤات الانحناء الزحفي.

ABSTRACT

In this research project, flexural creep and tensile creep estimations were carried out at constant room temperature and at a time range between (١-١٠^{٩}) seconds to measure the viscoelastic creep properties of polymer and polymeric material reinforced with short fibers.

The present work is a theoretical macromechanics study, which concentrates on linear viscoelastic creep behavior of polypropylene and short **E-glass** fiber reinforced **polypropylene**. The loading cases are externally applied concentrated loads and the deformations have been limited with small strains.

In this investigation, a three-parameter model is used as a mathematical viscoelastic model to represent the linear viscoelastic behavior of polypropylene and will not yet reach to permanent deformation.

Through this analysis, the **finite element method** as a numerical method is applied to obtain the mechanical properties. **Galerkin finite element method** has been adopted for the derivation of governing equation. Two finite element computer programs in **Quick Basic** language have been designed to obtain the mechanical properties that represented by strain, modulus, and compliance from flexural creep and tensile creep estimations.

The material systems that have been chosen for this study are:

-Isotropic unreinforced polypropylene.

-Polypropylene reinforced with aligned short E-glass fibers.

-Polypropylene reinforced with random short E-glass fibers.

The principle of time-temperature equivalence has been applied and observed from the mechanical properties of polymer and PMC.

Halpin-Tsai equation has been shown an improvement the modulus and decrease in the strain and compliance values for PMC reinforced with short fibers in both aligned and random reinforcement through flexural creep and tensile creep estimations.

On the other hand, the modified rule of mixtures has been shown increasing in strain and compliance values and constancy in modulus according to tensile creep estimation while the same method has been proved the slight effect of reinforcement on PMC properties due to flexural creep estimation.

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Finally, I would like to thank everyone help me in anyway for obtaining the present work.

Lina F. Al-Obaidi

CERTIFICATION

We certify that this thesis entitled **"Determination The Creep Properties of Short Fiber Polymer Matrix Composite "** was prepared by **"Lina Fadhil Al-Obaidi"** under our supervision at Babylon University, College of Engineering in partial fulfillment of the requirements for the Degree of Master of Science in Materials Engineering.

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NOMENCLATURE

Symbol	***Description***	***Unit***
A	Cross sectional area of tensile specimen	$m^{٢}$

A^e	Area of ٢-D rectangular element	$m^{٢}$
a	Aspect ratio	-
$a_{١},a_{٢}$	Dimensions of rectangle ٢-D element	m
[B]	Strain-displacement matrix	-
b	Width in tensile and depth in flexural	m
[D]	Plane-stress matrix	GPa
d	Diameter of fiber	m
E	Modulus of elasticity	GPa
e	Number of element	-
{F}	Global force vector	N
$\{f^{(e)}\}$	Element force vector	N
G	Shear modulus	GPa
G_c	Global coordinate	-
G(t)	Relaxation modulus	GPa
h	Thickness in tensile specimen and width in flexural specimen	m
h^e	Thickness of ٢-D rectangle element	m
I	Moment of inertia	$m^{٤}$
J	Creep compliance	$Gpa^{-١}$

k	Bulk modulus	GPa
$\{k^{(e)}\}$	Element stiffness matrix	N.m
[K]	Time dependent global stiffness matrix	N.m
L	Length of specimen	m
L_c	Critical fiber length	m
L_e	Length of linear ١-D element	m
L_f	Fiber length	m
Loc	Local coordinate	-
M	Bending moment	N.m
nn	Node number	-
N	Shape function	-
NDOF	Total number of degree of freedom	-

NET	Total number of elements	-
NNT	Total number of nodes	-
NNX	Number of nodes in x-direction	-
NNY	Number of nodes in y-direction	-
P	External applied load	N
$\{q^{(e)}\}$	Element displacement vector	m

{Q}	Global displacement vector	m
R	Residual term	-
t	Time	Second
t_r	Retardation time	Second
T	Temperature	˚C
V	Volume fraction	-
W	Weighting function	-
y	Distance from the neutral axis to the strained fibers	m

Greek symbols

Symbol	*Description*	*Unit*
ε	Normal strain	-
σ	Normal stress	MPa
σ_{mean}	Mean fiber stress	MPa
ξ,η	Natural coordinates	-
μ	Viscosity of dashpot	Poise
δ	Deflection	m
ν	Poisson’s ratio	-

Subscript

Symbol	Description
b	Bending
C	Composite
c	Critical
el	Elastic
f	Fiber
g	Glass transition
L	Longitudinal
m	Matrix
mean	Mean fiber stress
p	Polymer
r	Random
T	transverse

Superscript

Symbol	Description
e	Element

T	Transpose of matrix

Abbreviations

Symbol	*Description*
Eq.	Equation
Fig.	Figure
FEM	Finite element method
PMC	Polymer matrix composite
PP	polypropylene

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CHAPTER ONE

Introduction

١.١ General

Polymers are not new and some of them have been in existence for over a century but rapid development of polymers has continued to the present day in order to meet the demands of industry. The interest in polymers results from their properties: excellent electrical and thermal insulation properties; they are light, resistant to corrosion, and cheap. Polymers are generally more resistant to chemicals than the metals. As well as they have lower strength, lower moduli, and lower temperature use limits than metals or ceramics do. The reinforcing of polymers to obtain a composite material has been enhanced the mechanical properties of them.

The phenomena of viscoelasticity in materials is studied since [١٣]:

- Viscoelastic solids are used to absorb vibration.
- Viscoelastic effects such as molecular mobility in polymers are linked to physical processes as diffusion, motion of defects such as dislocations, vacancies and other defects, so that viscoelastic measurements are used as a probe into the physics of these processes.
- Viscoelasticity is also used in the design of materials and devices for a variety of purposes including reduction of mechanical shock, and instrument mounts.
- Viscoelasticity is relevant in the phenomena such as ball rebound, sag, and droop.

The subject of composite materials has received considerable attention in recent years because of the unique properties and characteristics displayed by this group of engineering materials. This is especially true in structural applications requiring high strength and lightweight. These materials have been highly successful in areas such as aerospace, transportation, military, medicine, and agriculture [١].

A composite material usually has characteristics that are not depicted by any of its components alone. Generally, the continuous phase is referred to as the matrix, while the distributed phase is called the reinforcement [٢,٣]. The reinforcement is usually harder and stronger than the matrix. The only exception to this rule is the class of materials known as rubber-modified polymers consisting of a rigid polymer matrix, filled with rubber particles [٣].

The properties exhibited by a composite material are strongly influenced by the properties of the individual constituents, shape and size, the bond between the reinforcement and its matrix and any interaction between these constituents when they are combined in composite form [١].

١.٢ General View about Polymer

The term polymer is used to indicate to a compound consists of many repeated structural units. The structure of all polymers consists of long chains; hundreds of molecules in length. These chains usually consist of carbon atoms linked together by covalent bonds [٤,٥].

Polymers of a high average molecular weight, i.e. relatively long chains, are the most useful to the engineer and they are used in industry and referred to as high polymers [٥]. The degree of polymerization of the polymer chain is the number of sub-units, i.e. mers, used to form the chain. The degree of polymerization is not being constant. Thus properties are affected by chain length [٤].

The type of polymerizing reaction that takes place depends upon the particular polymer being produced [٥]. There are two forms of polymerization.

-Addition polymerization: when the reaction takes place without any by-product, polymerizing the monomers can form the polymer under the influence of temperature, pressure, and catalyst. Polypropylene and polyethylene are familiar additional polymers [٥,٦,٧].

-Condensation polymerization: when a by-product is rejected or condensed out during the reaction. The by-product is nonpolymerizable molecule as water or other simple molecule such as HCL. Nylon is a familiar polymer [٥,٧]. condensation

The possible forms of molecular chains of polymer are linear chains, branched chains, and cross-linked chains as shown in Fig. (١.١).

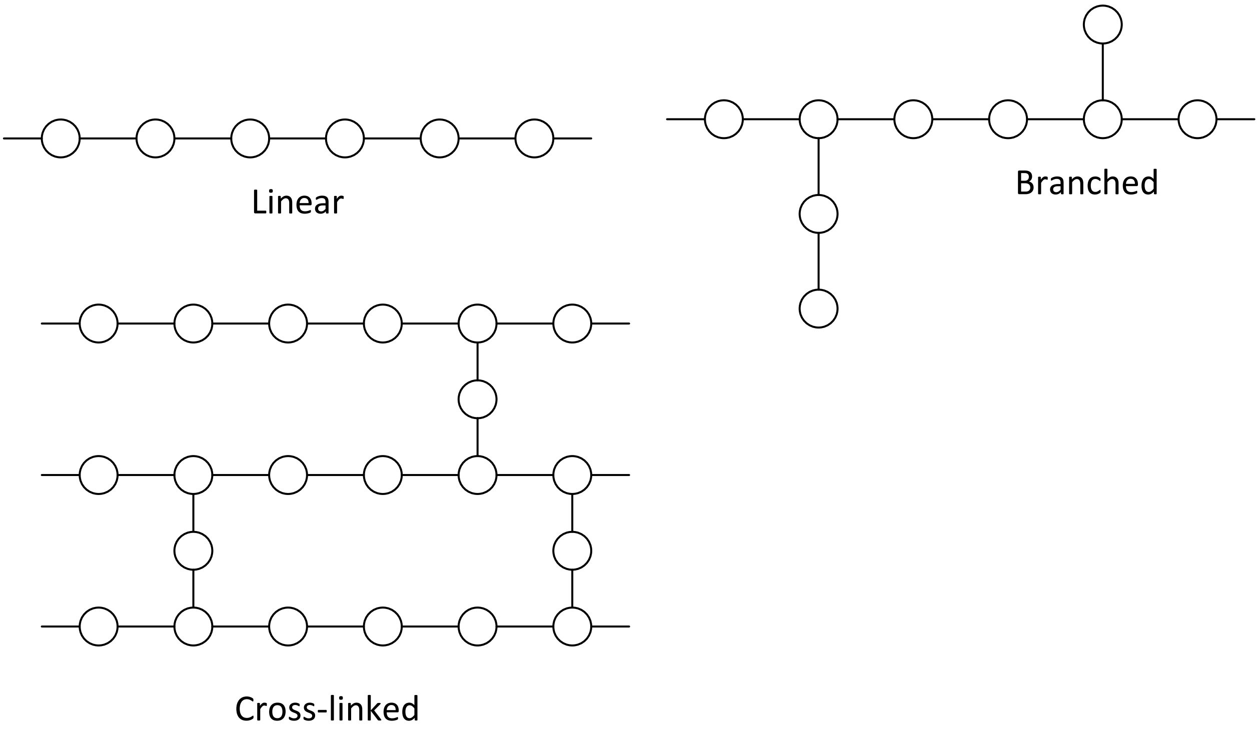


Fig.(١.١):Forms of polymeric chains. Ref [٤]

The term homopolymer is used to describe polymers that are made up of just one monomer. For instance polypropylene is made up of only the

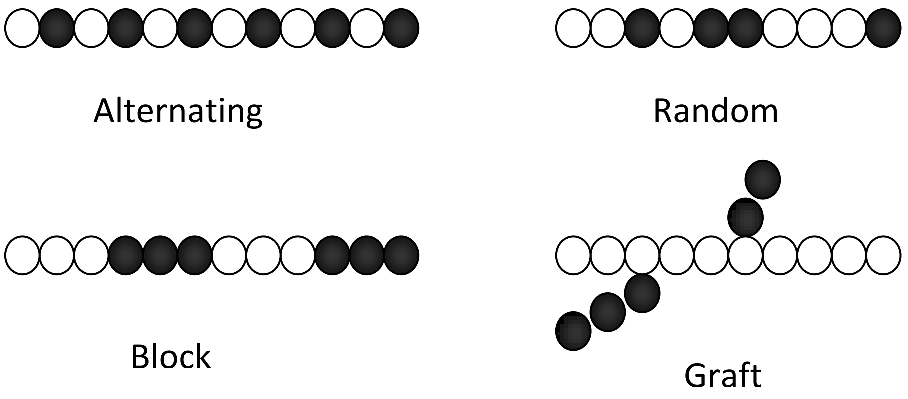


Fig. (١.٢): Structures of copolymers made up of two monomers. Ref. [٤] monomer propylene. The other type of polymer is copolymer, which can be produced by combining two or more monomers in a single polymer chain [٤]. Some types of copolymers can be seen in Fig. (١.٢).

١.٢.١ Structure of Polymers and its Effect on Mechanical Properties

Polymers can be classified depending on the arrangement of the polymer chains into: **amorphous** polymers**, semi crystalline** polymers**, and crystalline** polymers.

Amorphous polymers: they have completely random arrangement. Polymers with side branches or bulky molecules show fewer tendencies to crystallize since the branches or bulky side molecules prevent the molecular chains from becoming closely packed [٤,٥]. When an **amorphous** polymer is heated, it shows no definite melting temperature but progressively becomes less rigid. This is because the arrangement of the chains in the solid is disorderly, just like a liquid, and so there is no structural change occurring at melting [٤]. The strength of such polymers is relatively low. The greater spacing between adjacent molecular chains will increase the toughness of such structures and so display a measure of flexibility [٥].

Semi crystalline polymers: crystalline synthetic polymers are invariably partly **crystalline** and partly **amorphous** [٨]. For a particular polymer the degree of crystallinity depends on the particular type of polymerizing process used and the rate of cooling following polymerization or processing. Hence, if the polymer is cooled under non-equilibrium conditions, the development of large **crystalline** regions is restricted. The degree of crystallinity is usually in the range (٥٠-٩٠٪)[٥].

Depending on their degree of crystallinity, such polymers can show a range of densities and melting points [٢]. So the strength and stiffness of **semi crystalline** polymer will be affected by the degree of crystallinity. A **semicrystalline** polymer will be relatively strong with a reasonable measure of toughness [٥].

Crystalline polymers: The **crystalline** structure is one in which there are orderly arrangements of particle [٤]. The term **crystalline** polymer always implies partially **crystalline** [٨]. Complete crystallization is not obtained [٤]. The reason for this difficulty is the long chain structure of polymers [٢]. The typical properties of a highly **crystalline** polymer are that the presence of crystallinity affects the mechanical behavior of polymer [١] and the strength of this material is high and this polymer will exhibit little flexibility and toughness.

١.٢.٢ Classification of Polymers

Polymers can be classified in terms of the type of linkage both along and between the molecular chains as:

-Thermoplastics (thermosoftening): The Thermoplastic resins offer the possibility of being heated until soft and flowable, then they are simply molded into the desired shape and allowed to cool, so thermoplastic material may be readily molded or extruded because of the absence of cross-links, for example polypropylene and polyethylene [٤, ٥,١]. The mechanical properties of thermoplastics are sensitive to temperature and to sunlight, and exposure to temperature may cause thermal degradation, i.e. breakdown of some bonds [٧].

-Thermosetting plastics (thermo-hardening): such as melamine and epoxy resins; with a thermoset, heat causes the material to char and decompose with no softening. The atoms in a thermoset form a three-dimensional structure of chains with frequent cross-links between chains. The bonds linking the chains are strong and not easily broken. Thus the chains cannot slide over one another. As a consequence, thermosetting polymers are stronger and stiffer than thermoplastics and generally can be used at higher temperature [٤]. It should be noted that the degree of cross-linking varies from one polymer to another [٥]. In general, thermosets have high thermal stability, high dimensional stability, high stiffness, and good resistance to creep, low densities and high electrical and thermal insulating property [٤].

-Elastomers: Elastomers represent a particular group of polymers which consist of very long molecular chains which are folded, coiled and entangled, as well as being subject to a small degree of cross-linking. The difference between thermosets and elastomers is that with thermosets, there are frequent cross-linking bonds between chains while with elastomers there are only occasional bonds. Elastomers are polymers that can show very large, reversible strains when subject to stress [٤].

١.٣ Classification of Polymeric Composite Materials

Composites can be classified based on the type of reinforcement into three categories:

a. Fiber reinforced composites

The fibrous composites have become the most important class of composite materials, as they are capable of achieving high strength. Fibers because of their small cross sectional dimensions are not directly usable in engineering applications. They are, therefore, embedded in matrix materials to form fibrous composite [٣]. The main functions of the fibers in a composite are to carry most of the load applied to the composite and provide stiffness. For this reason, fiber materials have high tensile strength and a high elastic modulus. Aligning all fibers in the same direction gives the directionality to the properties of the composite [٤]. Three types of reinforcing fiber dominate the reinforced plastic scene. These are glass fibers, carbon fibers, and oriented polymeric fibers (Kevlar fibers) [٨].

The function of the matrix in a composite material is usually multifold. Matrices are designed to protect the reinforcing phase from structural damage due to handling, corrosive attack, and environmental degradation. The matrix phase also serves to bind the fibers together, transmit applied stresses to the reinforcing constituents, and stabilize them against buckling in situations where compressive stresses are axially applied to long cylindrical fibers [١,٣]. The matrix also keeps the fibers apart to hinder crack propagation [٤]. Furthermore, the matrix may be selected for its physical properties, such as density, thermal and electrical conductivity (or electrical resistivity), thermal expansivity, melting or softening temperature, and translucency or opacity [١].

Polymeric matrices: include a number of thermosetting and thermoplastic resins, The thermosetting resins commonly used as matrix materials include epoxies, polyesters, phenolics, and melamines. The thermoplastics commonly used as matrix materials for fiber-reinforced plastics include polycarbonates, polyamides (nylon ٦٦), polystyrene, polyethylene, and polypropylene. In general, the polymeric materials display low tensile strength compared with metals and ceramics; the bulk of fiber-reinforced composite materials consists of plastics reinforced with continuous fibers [١].

b. particulate composites

Particulate composites consist of particles of one or more materials suspended in a matrix of another material. Glass spheres are widely used with polymers to give a composite, which is stronger and stiffer than the polymer alone. Incorporating tiny rubber particles in the polymer matrix increases the toughness of some polymers. Polystyrene is toughened in this way by polybutadiene particles to give a product referred to as high impact polystyrene (HIPS)[٤].

c. Laminated composites

Laminated composites consist of layers of at least two different materials that are bonded together. Lamination is used to combine the best aspects of the constituent layers in order to achieve a more useful material [٩]. Laminated fiber reinforced polymer matrix composites are finding increased use in the broad range of industrial applications, particularly in the automotive and aerospace engineering. These materials are attractive for a number of reasons, especially their mechanical properties [١٠].

١.٤ Literature Review

In ٢٠٠٠, Resen, A; *et al* [٢٤] modified the finite element solution for elasticity problems in two-dimensions to linear and nonlinear viscoelastic behavior of solid glassy polymers through changing material properties in each time step. They have been based on creep behavior (constant stress) of two viscoelastic models (three-parameter model and Schapery model). Material under study is polymethyl methacrylate (**PMMA**). They studied three practical cases: uniaxial tensile stress applied on plate with central hole and plate with central crack, and combined loading "tension-shear" stress.

The results that have been obtained from plate with central hole and plate with central crack showed that strain distribution is time dependent, and in the case of plate with central crack, it is clear that the plastic zone propagates as time passes. In the case of combined loading, it is clear that shear component gave non linear strain **(ε > ٠.٥%)** while tensile strain is always lower than shear strain for both cases, tensile strain lies in the linear viscoelastic range.

In ٢٠٠١, Gates; *et al.* [٣٣] studied creep properties of unidirectional carbon fiber reinforced thermoplastic polyimide composite designated as (**IM٧/KB**) by using three accelerated test methods at elevated temperatures sub-Tg temperature. These tests are: -

Flexural creep tests using the DMA (dynamic mechanical analyzer), conventional tensile creep test, and conventional compressive creep test.

They performed tests under similar stresses and identical thermal history condition. They showed that flexural loading mode used in the DMA specimens introduces difficulties of isolating tensile properties from compressive properties of the material.

By comparison of the results from three test programs, they indicated that the DMA flexure creep results were not identical to the tension and compression creep results. The DMA flexural creep test closely resembled the tension and compression creep behavior of polymer matrix composites, and the slight difference between flexure, tension, and compression creep may reflect a real change in material behavior under these modes of loading.

In ٢٠٠٣, Abdel-Magid; *et al.* [٣٤] investigated the flexural creep behavior of two composite systems E-glass/epoxy and E-glass/polyurethane composites. The composite systems were reinforced with continuous fibers and tested along the fiber direction. They performed two sets of flexural creep tests for the materials, one at room temperature and the other at ٥٠ °C. On each set, samples were loaded at various stress levels ranging from ٢٠٪ to ٩٠٪ of the ultimate flexural strength of the material with ٥٪ stress increments.

They have been indicated that the polymer matrix composites exhibit creep behavior when subjected to three-point flexural loading even when the loading is applied along the fiber direction.

١.٥ The Objective of the Present Work

The present work is aimed to investigate numerically the time dependent mechanical properties of semi-crystalline thermoplastic polypropylene and short E-glass reinforced polypropylene in the forms of aligned and random reinforecement. The study is limited by linear viscoelastic behavior with small strains (≤ ٠.٠٠٥).

This work is a theoretical macromechanics study, but the approaches used to obtain stiffness and other mechanical properties of short fiber composite are micromechanics.

A three-parameter model is applied as a theoretical viscoelastic model to predict with the time dependent modulus of polypropylene polymer. A tensile creep with dumbbell specimen and flexural creep with simply supported beam have been taken into consideration.

Two computer finite element programs have been designed to analyze flexural creep behavior and tensile creep behavior of polymer and its composite.

Tensile and flexural creep estimations have been carried out at constant temperature, which is room temperature.

The time dependent mechanical properties understudy (strain, modulus, and compliance) have been determined for each node in flexural estimation and for each element in tensile estimation, so the properties are functions of both time and position.

CHAPTER TWO

The Mechanical Behavior of Polymer and Polymer Matrix Composite

٢.١ General

This chapter is primarily concerned with the mechanical properties of polymeric materials and polymer matrix composites. Polymers are obviously different from metals and other crystalline solids in their mechanical behavior. A polymer undergoes a complex combination of elastic deformation and slow irrecoverable viscous flow, which is generally called viscoelastic behavior.

The addition of fibers to polymeric matrices enhances polymer strength, stiffness, dimensional stability ,and elevated temperature resistance at the expense of ductility [١١]. Furthermore, the combination of high-strength/high-stiffness fibers in polymer matrices offers an outstanding example of the composite principle, that taking the best properties of both materials. The result is a strong, tough, stiff material, which is depending on the matrix and reinforcement type [١٢].

This chapter deals with time-domain transient properties: creep and stress relaxation for polymeric materials and polymer matrix composite.

٢.٢ The Nature of Viscoelasticity

Viscoelasticity is a general property of polymeric solids; the solid is elastic in that it recovers, but is viscous in that it creeps. The major difference between elastic solids, such as metal alloys, and polymers can be stated briefly. Polymers, because of the great length of the molecular chain, cohere as solids even when discrete sections of the chain (or side branches) are undergoing Brownian motion and moving by diffusional jump processes from place to place. In the liquid the whole polymer chain undergoes vigorous Brownian motion; the molecules move as a whole by 'snake-like' motions. In the glass it is clear that although the chain is essentially immobile, limited Brownian motion is possible before the onset of the liquid-like Brownian motion at the glass transition region [٨].

Polymers are viscoelastic at all temperatures, they are never simple elastic solids so that in considering the strains induced in service it is always required to take into account not only the stress, but the time for which it is applied. The viscoelastic properties are also highly temperature-dependent so that the maximum temperature must be clearly specified, and taken into consideration [٨]. Viscoelastic materials have a relationship between stress and strain, which depends on time or frequency [١٢].

٢.٢.١ Glass Transition Temperature (T_g)

If polymers are heated, there is a temperature at which they change from being a stiff, brittle, and a glass-like material to a rubbery material. This temperature is called the glass transition temperature T_g**.**

Below this temperature, segments within the molecular chains are unable to move and the molecular motion is diminished, the volume of

unoccupied space is reduced. The material is stiff with a high elastic modulus and generally rather brittle.

Above this temperature, there is sufficient thermal energy for some motion of segments of the chains to occur and imparts toughness to the polymer. In these temperatures, segments of the long polymer molecules, even though entwined and entangled, exhibit vibrational, rotational, and translational motion that results in a cotinuous rearrangement of the molecule and the creates of the unoccupied space between molecules [٤, ١].

The greater the degree of linking, the higher the glass transition temperature. Thus linear chain molecules tend to have lower glass transition temperatures than molecules with bulky side groups or branches and these, in turn, have lower values than cross-linked polymers [٤].

For rubbery materials $T_g < T_{room}$

For glassy materials $T_g > T_{room}$

The glass transition temperature (T_g**) is as important to polymers as the melting (or freezing) temperature is. For example** T_g **of polystyrene is at approximately ١٠٠ °C; therefore it is glassy and brittle at room temperature. In contrast, a rubber whose** T_g **is at –٧٣ °Cis flexible even in the most severe winter temperatures [١٤].**

The glass transition temperature is sometimes loosely referred to as the softening point. The simplest way of observing the softening point is by means of a sharp, cone-shaped penetrometer. The ratio of the glass transition temperature to the melting point is observed empirically to be ٠.٦ [٨].

Polymers can be grouped into various classes according to their behavior relative to their transition temperatures. Elastomers **must be used**

well above the glass transition temperature (T_g) **in order to retain the .molecular mobility necessary to achieve elasticity**

Amorphous polymers **such as polymethyl methacrylate are utilized for their structural rigidity and consequently must be used at temperatures below** T_g**, while** Crystalline polymers **are used at temperatures well below Tm since changes in crystal structure can occur as Tm is approached. Typically the glass temperature is not important, since it represents only a minor change in the behavior of these polymers, where the higher the degree of crystallinity, the smaller the effect of the glass transition on the mechanical properties of the material[١].**

٢.٣ Viscoelasticity from Mechanical Point of View

The classical theory of elasticity deals with the properties of elastic solids for which, in accordance with Hook's law, stress is always directly proportional to strain in small deformation but independent of the strain rate [١٥].

The elastic solid has a definite shape and is deformed by external new equilibrium shape. On removal of these external forces it forces into a reverts exactly to its original form. The solid stores all the energy that it obtains from the work done by the external forces during deformation. body to its original shape when This energy is then available to restore the these forces are removed [٦].

The classical theory of hydrodynamics deals with properties of viscous liquid, for which, in accordance with Newton's law, the stress is always directly proportional to the strain rate but independent of strain,

itself [١٥]. A viscous liquid, on the other hand, has no definite shape and flows irreversibly under the action of external forces [٦].

If both strain and rate of strain are infinitesimal, system may exhibit behavior, which combines liquid like and solid like characteristics; Materials whose behavior exhibits such characteristics are called viscoelastic [١٥]. Polymers are usually described as viscoelastic materials.

It is difficult to classify polymers as particular types of materials such as a glassy solid or a viscous liquid, since their mechanical properties are so dependent on the conditions of testing, e.g. the rate of application of load, temperature, amount of strain.

A polymer can show all the features of a glassy brittle solid or an elastic rubber or a viscous liquid depending on the temperature and time scale of measurement. At low temperatures, or high frequencies of measurement, a polymer may be glass-like with young's modulus of ($١٠^{٩}$—$١٠^{١٠}$) N/m^{٢} and will break or flow at strains greater than (٠.٥٪).

At high temperatures or low frequencies, the same polymer may be rubber-like with a modulus of ($١٠^{٦}$—$١٠^{٧}$) N/m^{٢}, withstanding large extensions (approximately ١٠٠٪) without permanent deformation. At still higher temperatures, permanent deformation occurs under load, and the polymer behaves like a highly viscous liquid.

In an intermediate temperature or time range, commonly called the glass transition range, the polymer is neither glassy nor rubber-like and it shows an intermediate modulus [٦].

٢.٤ Viscoelastic Behavior

٢.٤.١ Linear Viscoelastic Behavior

A simple definition of linear viscoelastic behavior is that the ratio of stress to strain is a function of time only and not a function of the magnitudes of stress and strain [١٦]. In linear viscoelasticity, effects are simply additive as in classical elasticity, the difference being that in linear viscoelasticity matters at which instant an effect is created [٦].

Stress-strain curves for all viscoelastic solids (time dependent materials) are linear for sufficiently small deformations and strains [١٧]. To ensure that the specimen cross section does not change appreciably.

Generally polymers exhibit the properties of linear viscoelastic behavior at low stresses where strain below (٠.٢-٠.٥)% [١٧]. The end of the region of linear viscoelasticity corresponds to ε = ٠.٠٠٥ [١٨], so above this limit the material exhibits nonlinear viscoelastic behavior. Linear constitutive model is also valid to represent viscoelastic behavior [١٥].

٢.٤.١.١ Transient Properties: Creep and Stress Relaxation

If a polymeric material is subjected to a constant stress, the strain will not be constant but will increase slowly and continuously with time. On .the stress The effect is due to a molecular rearrangement induced by the release of the stress, the molecules slowly recover their former spatial arrangement and the strain simultaneously returns to zero. This effect is termed creep and is a manifestation of a general property of polymeric

solids known as viscoelasticity [٨]. Creep is one of the simplest experimental modalities for characterizing viscoelastic behavior [١٢].

For metals except that the very soft metals like lead, creep effects are negligible at ordinary temperatures. For polymers, creep is often quite significant at ordinary temperatures and even more noticeable at higher temperatures [٤]. All plastics creep to a certain extent. The degree of creep depends on several factors, such as the type of plastic, temperature, and stress level.

If the applied load is released before creep rupture occurs, an immediate elastic recovery will happen, equal to the elastic deformation, followed by a period of slow recovery as shown in Fig. (٢.١), where a constant load is applied at t_0 and removed at t١. The material in most cases does not recover to the original shape and a permanent deformation remains [١٩]. On removing the load from a polymer, the material can recover most, or even all, of the strain through giving it sufficient time. This is different from metals where the strain produced by creep is not recoverable. The time taken to recover depends on the initial strain and the time for which the material was creeping under the load [٤].

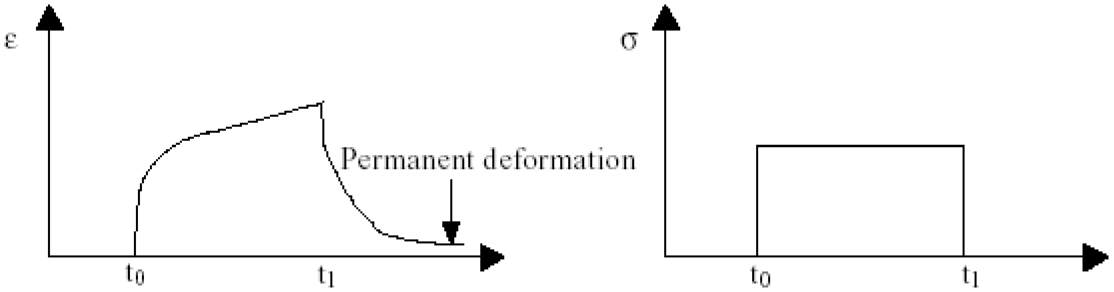


Fig. (٢.١): Creep curve with recovery. Ref. [١٩]

The general form of stress-strain-time relationship can be thought of a ٣-D surface as in Fig. (٢.٢) [١٩]. The ٣-D figure can be transformed into three additional ways by which creep data can be presented for polymers [٤], these ways are:

١. Creep curve: strain-time curve at constant stress.

٢. Isochronous curve: strain-stress curve at constant time.

٣. Isometric (stress relaxation) curve: stress-time curve at constant strain.

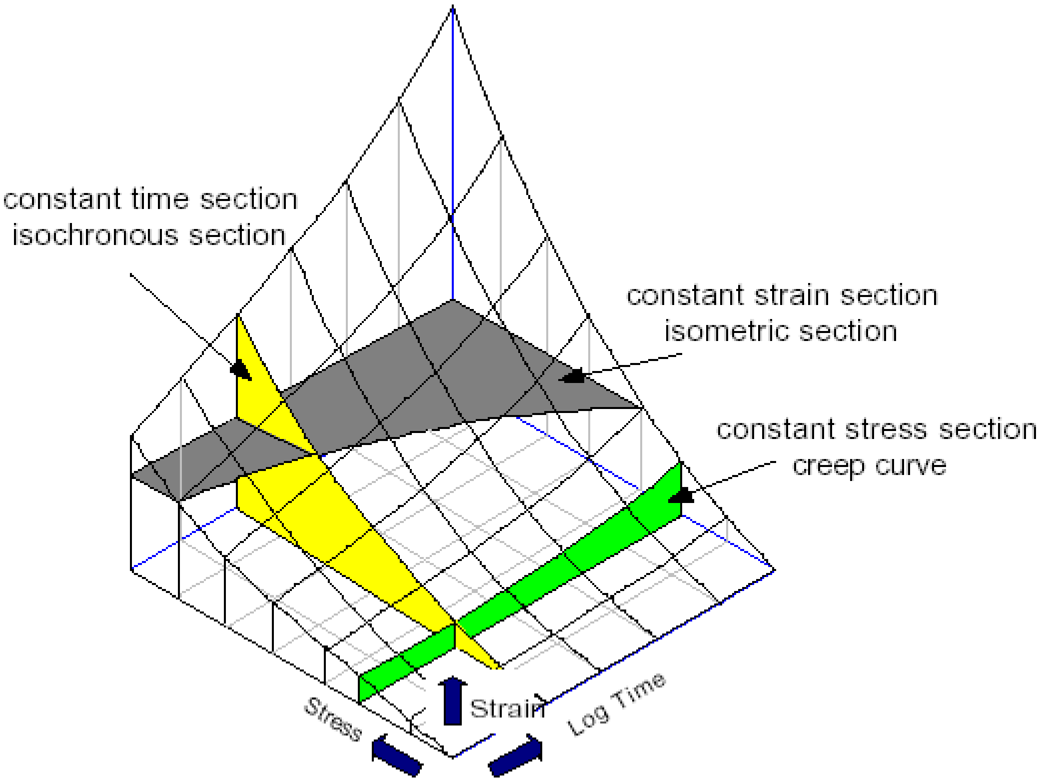


Fig. (٢.٢): ٣-D plot of material behavior. Ref. [٣٨]

The counterpart of creep is stress relaxation, which is defined as a gradual decrease in stress with time under a constant deformation or strain as shown in Fig. (٢.٣). This behavior of a polymer is studied by applying a constant deformation to the specimen and measuring the time dependent stress required for maintaining that strain [١٩].

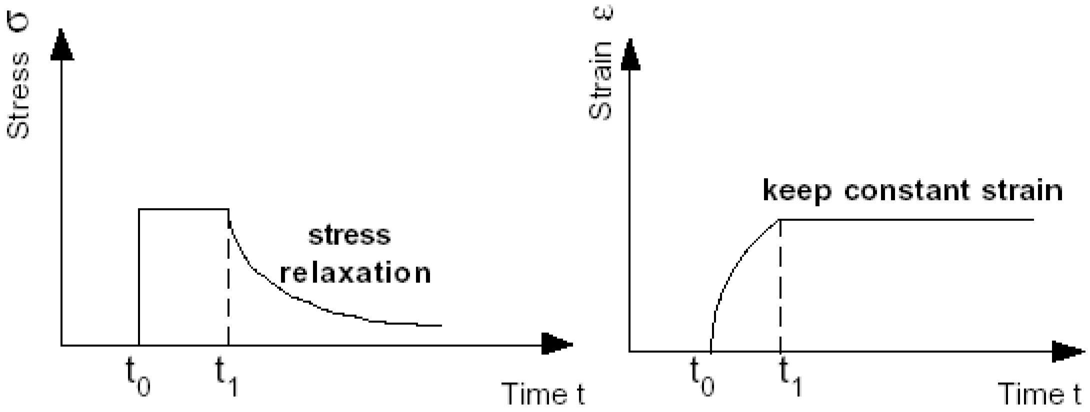


Fig. (٢.٣): Stress relaxation of plastics. Ref. [٣٨]

Relaxation in polymers is of great practical significance when the polymers are used in applications involving seals and gaskets [٢]. The stress relaxation test is more difficult to perform than a creep test and has limited practical applications [١٩]. The rates of relaxation and creep depend on the particular material [٢٠].

٢.٤.١.٢ Creep Compliance and Stress Relaxation Modulus

The creep compliance can be defined as the time dependent strain $\boldsymbol{\varepsilon(t)}$ divided by the constant stress $\boldsymbol{\sigma_o}$ [١٣]. The creep compliance is denoted by J (t) where:

J (t) for a tensile/compressive creep experiment, and **J$_S$ (t)** for a torsion creep experiment [١٣]. In this work, creep compliance is [٦]:

$$J(t)=\frac{\varepsilon(t)}{\sigma_o} \qquad (٢\text{-}١)$$

If a constant stress $\sigma_{١}$ is applied to a viscoelastic specimen, the time dependent strain is recorded as $\varepsilon_{١}(t)$ (see Fig. ٢.٤ a). After some period of time, the load is removed. Suppose the specimen is allowed to recover and a larger stress $\sigma_{٢}$ is applied. The time dependence of the strain $\varepsilon_{٢}(t)$ is shown in Fig. (٢.٤ b). If at a particular time $t_{١}$ and $t_{٢}$ after loading, $\varepsilon_{١}$ and $\varepsilon_{٢}$ are linear with the magnitude of corresponding stresses, $\sigma_{١}$ and $\sigma_{٢}$, the strain-stress relationship that is called isochronous curve can be given by Fig. (٢.٤c) [١٩].

The data for isochronous curves are taken from creep tests at different stresses [١٩]. From creep curve, a vertical line is drawn at some particular time and the stresses needed for the different strains read from the graph. Then a graph of strain against stress can be produced for that time [٤].

Linearity of a material subjected to a series of creep tests at different
is a stresses can be displayed visually by plotting isochronal curve; it straight line if the material is linearly viscoelastic [١٣]. There are always deviations from linearity; a transition from linear to nonlinear viscoelasticity is shown in Fig. (٢.٥) [١٩].

In the linear viscoelastic ragion, the compliance J (t) is independent of stress. The value of the stress used to determine J (t) is of no significance; the compliance that can be observed at time t is exactly the same whether the stress used in the creep experiment is σ١, σ٢ or any arbitrary stress σ

$$J(t_1)=\frac{\varepsilon_1(t_1)}{\sigma_1(t_1)}=\frac{\varepsilon_2(t_1)}{\sigma_2(t_1)}\text{and}J(t_2)=\frac{\varepsilon_1(t_2)}{\sigma_1(t_2)}=\frac{\varepsilon_2(t_2)}{\sigma_2(t_2)}$$

[٨].

Thus the simple loading experiment defines creep compliance J (t) only as the function of time [٦]. If the creep compliance curves for several experiments performed at different stress levels coincide, the viscoelastic material is linear [٢١].

should involve substituting of the creep modulus The design process for stiffness (or Young’s modulus). The creep modulus (Young's modulus) is the apparent stiffness determined by dividing the constant stress over the

$$E(t)=\frac{\sigma_o}{\varepsilon(t)} \qquad (٢\text{-}٢)$$

total deformation to the time defined as [٢٢]:

Fig. (٢.٦) below shows the variation of compliance with time at constant temperature over a very wide time scale for an idealized amorphous polymer. This diagram shows that for short time experiments the observed compliance is $١٠^{-٩}$ m^{٢}/N for a glassy solid that is time independent. At very long times the observed compliance is $١٠^{-٥}$ m^{٢}/N for a rubbery solid, and it is again time independent. At intermediate times the compliance lies between these values and is time dependent; this is the general situation of viscoelastic behavior [٦].

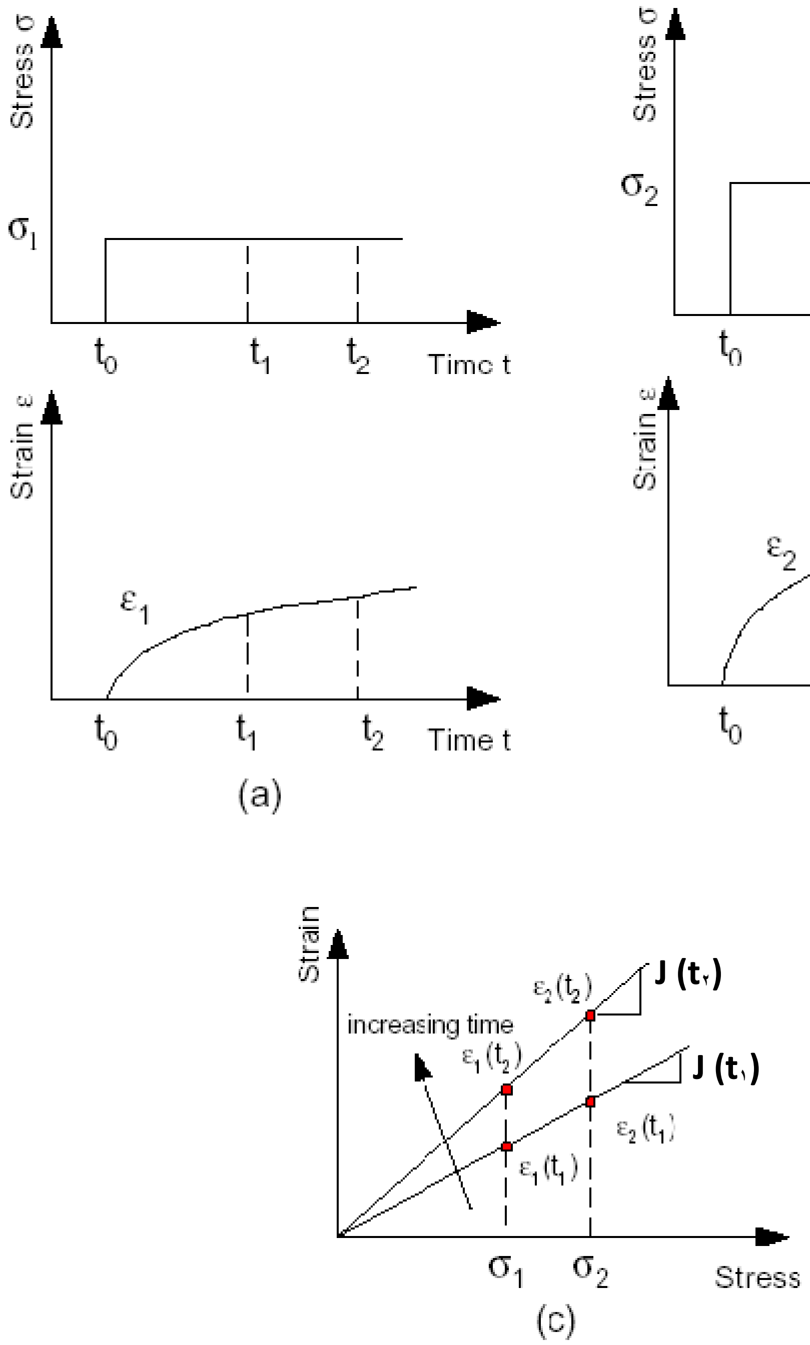


Fig. (٢.٤) Linear viscoelastic creep: (a) constant stress $\sigma_{١}$ leads to time dependent strain $\varepsilon_{١}(t)$ (b) higher stress $\sigma_{٢}$ leads to time dependent strain $\varepsilon_{٢}(t)$ (c) from (a) and (b) the strain at t and t are linear with the stress (isochronous curve). Ref. [١٩]

The stress relaxation modulus is the time dependent stress divided by the constant strain [١٣]. G(t) relaxation modulus in tension/compression, and G_S (t) relaxation modulus in shear.

The assumption of linear behavior enables to define the stress relaxation

$$G(t) = \sigma(t)/\varepsilon \qquad (٢\text{-}٣)$$

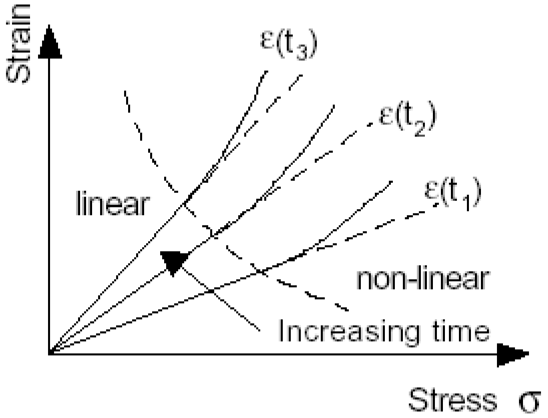


Fig. (٢.٥) Linear-nonlinear transition of strain-stress relationship with respect to different time levels (isochronous curve). Ref. [١٩]

modulus **G (t)** as a decreasing function of time [١٥].

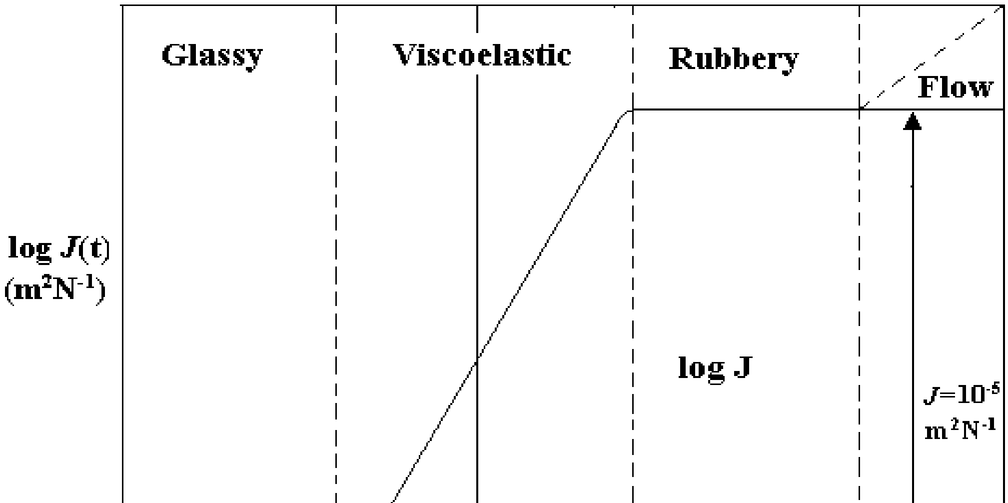
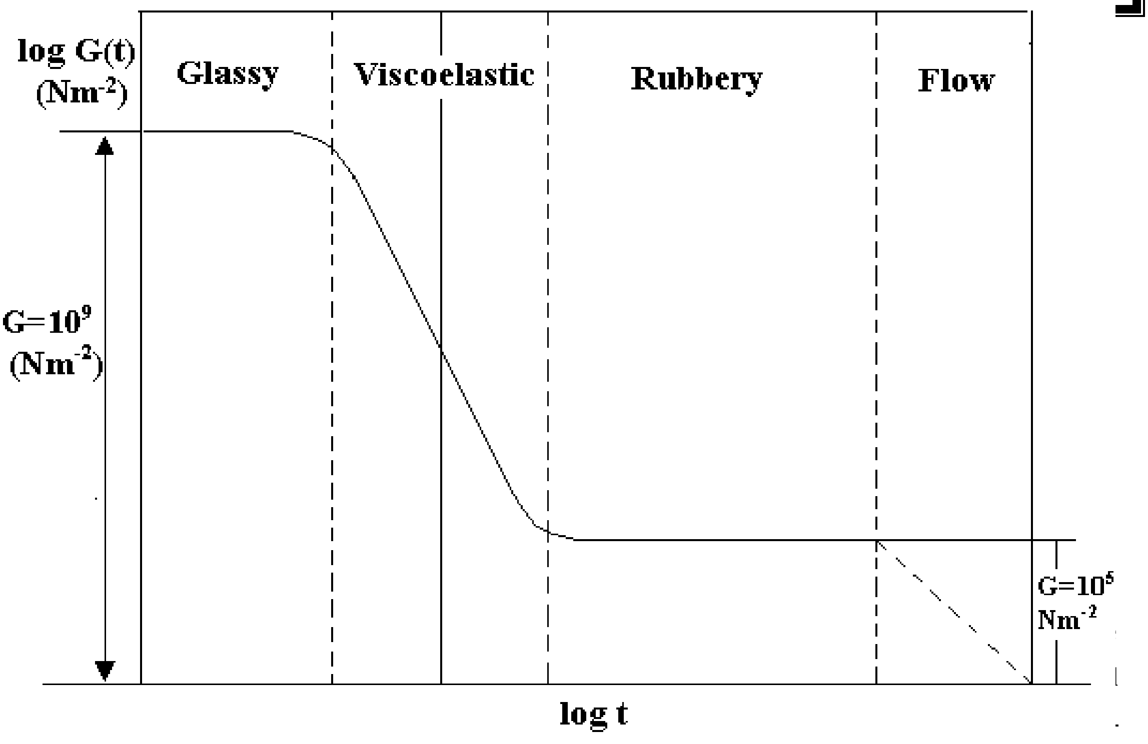


Fig. (٢.٧) is a schematic graph of the stress relaxation modulus as a function of time. The same regions of behavior (glassy, viscoelastic, rubbery, and flow) can be identified. For very short times, the relaxation modulus approaches a maximum limiting value where the material exhibits glassy behavior associated with negligible molecule segmental motions. At longer times, the material experiences a transition to leathery behavior associated with the onset of short-range molecule segmental motions. At still longer times, complete molecule movements are experienced in the rubbery region associated with a further drop in the relaxation modulus. Beyond this point, liquid flow occurs [١١].



is no v

moduli

Fig. (٢.٧): The stress relaxation modulus G (t) as a function of time. Ref

temperature equivalence is applicable to all linear viscoelastic behaviors in polymer [ ].

If the data from several relaxation experiments done at different constant strains give the same relaxation modulus **G (t)**, the material is linearly viscoelastic [٢١]. The stress relaxation modulus at time t is another manifestation of linear viscoelasticity [٨].

The stress data obtained from a stress relaxation test can be used to calculate the stress relaxation modulus by dividing the stress at a particular time by the applied strain [١٩]. The material is linear when **J (t)** is independent of stress, **G (t)** is independent of strain, and otherwise it is nonlinear [١٣]. Relaxation and creep modulus can be plotted against log time to reveal their strong time dependence [١١].

٢.٤.١.٣ Mechanical Models

The linear viscoelastic response of polymeric solids described for many years by a number of mechanical models that have been provided a useful picture of time dependent deformation processes [١١]. It is possible to analyze the deformation of viscoelastic materials with appropriate combinations of elastic springs, which obey Hook's law and viscous dashpots that obey Newton's law of viscosity [٢٣,٦]; the dashpot is imagined as a piston moving in a cylinder of viscous fluid [١٥,١١].

spring and dashpot in the Maxwell model consists of Maxwell model: **series as shown in Fig. (٢.٨).**

For creep, stress is constant (**$\sigma = \sigma_o$**), strain will change with time; so **$d\sigma/dt$ =** ٠[٦]. The application of constant stress causes an instantaneous stretching of the spring, the dashpot extends linearly with time so long as the stress is maintained [١٦]. The final equation of Maxwell model as reported in Ref. [٦] is

$$\varepsilon = \sigma_o t / \mu$$ **(٢-٤)**

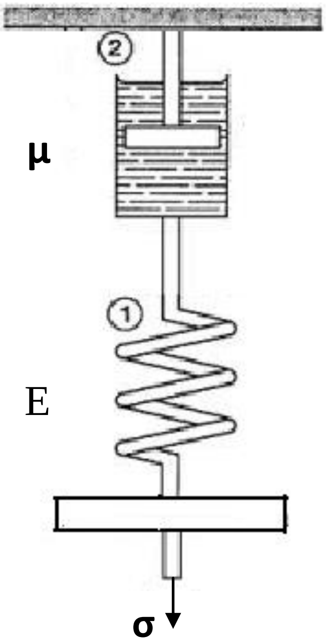


Fig. (٢.٨): Maxwell model. Ref. [١٩]

Eq. **(٢-٤)** corresponds to the response of only viscous element, which is not very realistic form of behavior because in this equation strain increases linearly with time (viscous behavior) [١٥]. This is not generally true for viscoelastic materials where the creep behavior is more complex [٦].

Voigt-Kelvin model: Kelvin or Voigt model consists of a spring and dashpot in parallel [٦] as shown in Fig. (٢.٩). When the stress is applied suddenly in the creep test, the chief resistance is offered by the viscous dashpot, which is initially very resistant.

The initial strain of the system is zero, after a period of time; Kelvin model will display a time dependent deformation behavior [١٦].

$$\varepsilon(t) = \frac{\sigma_o}{E}(1 - e^{-t/t_r}) \qquad (٢-٥)$$

The final equation of Kelvin model is:

Where

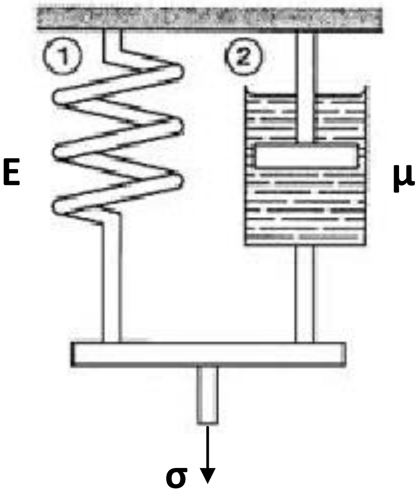


Fig. (٢.٩): Kelvin model. Ref. [١٩]

$t_r = \mu / E$ = **characteristic time constant called the retardation time.**

Maxwell and Kelvin models show certain similarities to actual material response [١٦] because the two models couldn't satisfy the general behavior of viscoelastic solid (creep and stress relaxation) completely if each of them is used alone [١٥] and they do not provide close agreement with real behavior [١٦].

٣-parameters model: The more convenient famous model to represent the viscoelastic behavior is called ٣-parameter model. It is equally valid for creep, stress relaxation, and dynamic response. ٣-parameter model is a modification of Kelvin model [٢٤], which consists of Kelvin model connected in series with a spring as shown in Fig. (٢.١٠)[١٥].

The response of this model contains both initial elastic and decreasing strain rate which represents creep response fully [٢٤].

For this model, total strain = strain due to spring + strain due to Kelvin element [٤].

$$\varepsilon(t)=\frac{\sigma_o}{E_1}+\frac{\sigma_o}{E_2}(1-e^{-t/t_r}) \qquad (٢\text{-}٦)$$

$$J(t)=\frac{\varepsilon(t)}{\sigma_o}=\frac{1}{E_1}+\frac{1}{E_2}(1-e^{-t/t_r}) \qquad (٢\text{-}٧)$$

$$E(t)=\frac{1}{J(t)}=\frac{E_1E_2}{E_1+E_2-E_1e^{-t/t_r}} \qquad (٢\text{-}٨)$$

Where $t_r = \mu_{٢}/E_{٢}$

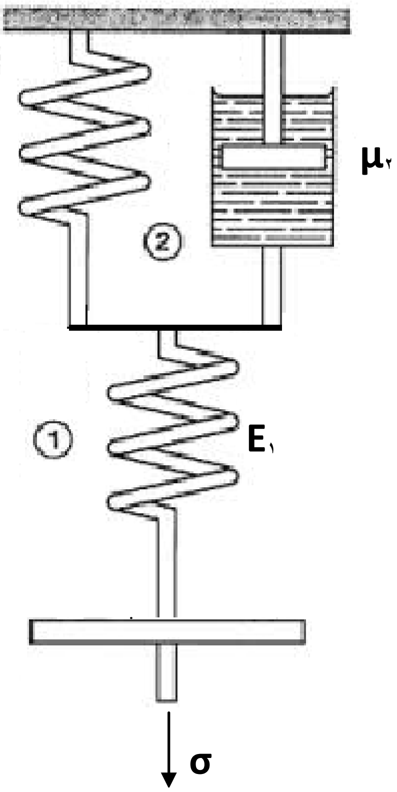


Fig.(٢.١٠) ٣ – parameter model. Ref. [١٩]

In this work, ٣-parameter model will be used as a mechanical model to predict the linear viscoelastic behavior of PP polymer because:

١. It was used in finite element formulation of viscoelastic behavior [٢٤].

٢. It will not yet reach to permanent deformation.

The time dependent modulus of three-parameter model has three constants (**$E_{١}$, $E_{٢}$, $\mu_{٢}$**). These constants have been determined according to experimental curve of polypropylene at constant stress **٨.٣** Mpa and ٢٠°C by the curve fitting procedure.

This experimental curve has been taken from Rollason, E. C. [٧]. The values of the constants of three parameter model for polypropylene polymer are as follows: **$E_{١}$=١.٧**GPa, **$E_{٢}$=٠.٢٣** GPa, and **$\mu_{٢}$=٢.٩ x** $١٠^{١٦}$ N.sec / $m^{٢}$ (poise).

٤-parameters model: Among the available creep models **[٢٥],** considerable improvement can be achieved [١٦] due to the insertion of the Kelvin body between the spring and dashspot of the Maxwell body to construct the ٤-element burger body that can be seen in Fig. (٢.١١) [٢٥].

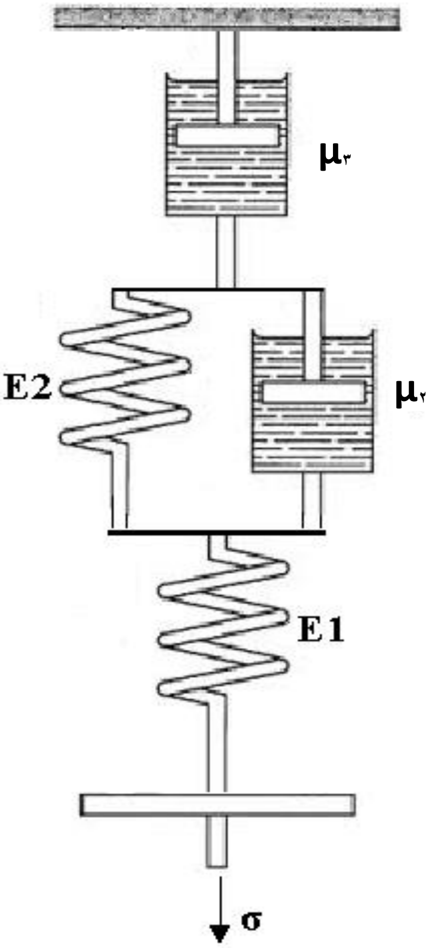


Fig. (٢.١١): ٤-elements model. Ref. [٣٨]

(ε) of such a combination is equal to the In this model, the total strain sums of the strains in each element of the series:

$\varepsilon = \varepsilon_١ + \varepsilon_٢ + \varepsilon_٣$

$$\varepsilon(t) = \frac{\sigma_o}{E_1} + \frac{1}{E_2}\left(1 - e^{-t/t_r}\right) + \frac{\sigma_o t}{\mu_3} \qquad (٢\text{-}٩)$$

Where $t_r = \mu_٢ / E_٢$

Fig. (٢.١٢) represents the general creep behavior of polymers and the regions can be defined as following:

(١) Instantaneous extension of elastic spring.

(٢) Time dependent deformation of Kelvin model.

(٣) Recovery which includes:

(a) Recovery of elastic spring.

(b) Time dependent recovery (slow recovery) of Kelvin model.

(c) Permanent deformation.

Stage **(c)** is not found in ٣-parameter model and it is found in ٤-element model.

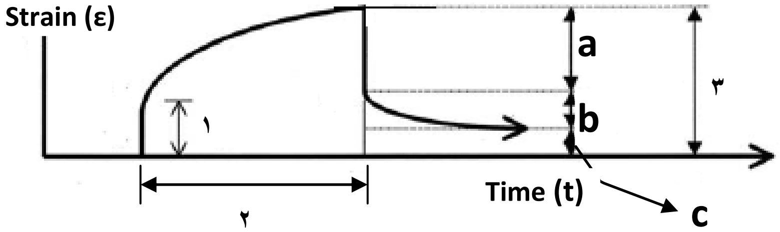


Fig. (٢.١٢): General creep behavior of polymers. Ref. [٢٥]

٢.٤.٢ Non Linear Viscoelastic Behavior

When the stress-strain ratio of a material varies with time and stress, the viscoelastic response is nonlinear [١١]. It is quite usual to observe linear viscoelastic behavior at short times at given stress levels, but the behavior can be markedly nonlinear for long times at the same stress levels [٦]. The transition from linear to nonlinear viscoelastic behavior can be noticed in Fig. (٢.٦).

٢.٥ Factors Affecting the Mechanical Behavior of Composite Material

The factors stated below might be important in determining the properties of the composites but it is seldom to be accounted for all these factors in the development of theoretical descriptions of composite [٣]. The most significant factors in determining mechanical properties are [١٢]:

١. Type of fiber: **When high modulus fibers are used in large concentrations, the principal purpose of a matrix is not to be load-carrying constituent but essentially to bind the fibers together and protect them. The failure mode of such composites is also generally controlled by the fiber [٣]. For example, high-strength, high-modulus graphitic carbon fibers will, of course, produce stronger, stiffer composites than those produced from fiberglass [١٢].**

٢. Fiber volume fraction: **In composites with simple geometric shapes and uniform fiber cross section, the fraction of the total composite volume occupied by fibers (reinforcement phase) is often known or determinable. Thus the volume fraction occupied by matrix material (V_m) is simply the total volume fraction minus the fiber volume fraction which represented by**

(V_m = ١ –V_f) **[١].**

Concentration of reinforcement is usually measured in terms of volume or weight fraction while concentration distribution of the reinforcement refers to their spatial relations to each other [٣]. The composite behaves similarly to the matrix at low fiber volume fractions, while at high fiber volume fractions, the composite behaves more like the fiber phase. The tensile properties of fiber reinforced composites generally begin to decrease when V_f **exceeds about ٠.٨٠. This degradation is due to poor bonding between fibers and matrix, fiber to fiber contact, and voids in the composite, which result from the high percentage of reinforcing phase [١].**

٣. Orientation of fibers: **The alignment of fibers in fiber reinforced composites is also a very important factor. Misalignment of fibers occurs during fabrication simply because short fibers and slender filaments are difficult to align [١]. Orientation of the reinforcement affects the isotropy of the system [٣] where uniaxially aligned fiber composites will have maximum properties along the alignment axis, it will be highly anisotropic and will have poor properties in the transverse direction [١٢].**

٤. Aspect ratio: **A critical aspect ratio is critical fiber length per fiber** diameter (L_c/d), it can be defined as a function of interfacial bond strength and fiber characteristics [٢٦] according to the following relation [٢]:

$$\frac{L_c}{d}=\frac{\sigma_f}{2\tau_i}$$
(٢-١٠)

Where τ_i is shear stress acting along the fiber matrix interface at the fiber end.

To achieve a fiber stress of σ_f at the center of the fiber, the critical aspect ratio (L_c/d) must increase with increasing fiber strength and decreasing shear stress [٢].

The reinforcement is the most effective if it is continuous or at least has a high aspect ratio [١٤]. In practice, most short fiber reinforced material contains fibers whose aspect ratios are of the order of ١٠-١٠٠ where it is shown that an aspect ratio of ١٠٠ is essentially infinite [٢٧].

Continuous Fiber Reinforced Composites ٢.٦

The unidirectional composites have higher strength and modulus in the direction of the fiber axis and are generally very weak in the transverse direction where the longitudinal properties of unidirectional composites are controlled by fiber properties, whereas the transverse properties are matrix-dominated [٣].

The two outstanding features of oriented fibrous composites are their high strength: weight ratio and controlled anisotropy [٣]. Mechanical properties exhibit nonlinearly at higher filler levels of fibers in fiber-reinforced composite [٢٦]. In anisotropic body, the properties are functions of orientation [٩].

When the length of a fiber is much greater than the length over which the transfer of stress takes place, the end effects can be neglected and the fiber may be considered to be infinite in length or continuous. The stress on a continuous fiber can, therefore, be assumed constant over its entire length [٣].

There are two micromechanical approaches to study the macromechanical behavior of composite materials [٩]:

A. The rule of mixtures:

The rule of mixtures indicates that the contributions of the fibers and the matrix to the average composite properties are proportional to their volume fractions [٣]; it is mechanics of material approach [٩]. The stress and elastic modulus of composite are:

$\sigma_{CL} = \sigma_f V_f + \sigma_m V_m$ (٢-١١)

$E_{CL} = E_f V_f + E_m V_m$ (٢-١٢)

Eq. (٢-١١) has been neglected the viscoelasticity of the matrix and has been treated the fibers and matrix as being linearly elastic [٨].

The rule of mixtures has several properties [٣,٨,٢٦,١]:

(١) The prediction of above Eqs. (٢-١١), (٢-١٢) is quite accurate when the applied load is tensile and agrees very well with experimental results, or in other words, The rule of mixtures accurately predicts the stress-strain behavior of a unidirectional composite subjected to longitudinal load [٣].

(٢) The above two Eqs. are used when both the constituents deform elastically [٣]; the viscoelasticity in the case of polymer matrix has been neglected [٨].

(٣) Usually, the rule of mixtures predicts a linear relationship between strength and fiber volume fraction [٢٦].

(٤) The rule of mixtures applies [٢٦] to certain properties of the composite material [١] and it is applied best to nondirectional properties of composite such as specific volume, specific heat, refractive index, and dielectric constant [٢٦].

B. Halpin-Tsai equations:

Halpin-Tsai equations are quite adequate to satisfy the practical requirements for the predictions of transverse composite modulus E_T**, particularly as the variations in composite materials manufacturing processes always cause a variation in the composite moduli [٣], therefore, one cannot hope to precisely predict composite moduli [٣]. Note that the expressions for** E_{CL} **and** ν_{CLT} **are the generally accepted rule of mixture results [٩]. Halpin-Tsai equation for transverse composite modulus can be written as [٣]:**

$$\frac{E_T}{E_m}=\frac{1+\zeta\lambda V_f}{1-\zeta V_f}$$

(٢-١٣)

Where

$$\lambda=\frac{(E_f/E_m)-1}{(E_f/E_m)+\zeta}$$

(٢-١٤)

The predictions of Eq. (٢-١٣) agree very well with some of the more exact solutions. ζ **is a measure of reinforcement and depends on the fiber geometry, packing geometry, and loading conditions [٣]. The values of** ζ **are obtained by comparing Eqs. (٢-١٣) and (٢-١٤) with exact elasticity solutions through curve-fitting techniques. Halpin and Tsai have suggested that a**

value of (ζ = ٢) may be used for fibers with circular or square cross section [٢].

Halpin-Tsai equations have several properties [٢٨,٩]:

a. **Halpin-Tsai equations provide a good approximation due to the fact that they are semianalytical-empirical function.**
b. **These equations are simple, so they can readily be used in the design process.**
c. **The prediction of these equations is apparently quite accurate if the fiber volume fraction** V_f **does not approach one.**
d. **The only difficulty in using Halpin-Tsai equations seems to be in the determination of a suitable value for** ζ.

٢.٧ Discontinuous Fiber Reinforced Composites

The short fiber composite may also be referred to as discontinuous fiber reinforced composite. In short fiber composites, the end effects cannot be neglected and the composite properties are the function of fiber length [٣]. The fiber ends are weak points in the composite (sites of high stress concentration in the matrix) [٨]; the effect of the stress concentration is to lower the composite strength [٣].

In order to achieve the maximum level of tensile strength, the fiber length must be at least equal to a critical length L_c **[٤]. Fiber length is important because the composite having fibers whose length is greater than the critical length will be strong and stiff, and failure occurs due to fiber breakage while composite having fiber length less than the critical will be less stronger and stiffer but tough [٢٦].**

Discontinuous fibers are used only when manufacturing economics dictates the use of a process where the fibers must be in this form, for example injection-molding [٨]. The orientation of short or discontinuous fibers cannot be easily controlled in a composite material. Actually it is extremely difficult to prepare a specimen with uniaxial orientation of short fibers. However, the properties of discontinuous fiber reinforced composites are lowered to a greater degree due to the difficulty in controlling the fiber alignment.

In molded short fiber composites fabricated by compression or injection molding, the fiber orientation throughout the molding varies greatly according to the flow within the mold. Thus the molded part properties will vary from section to section according to the local fiber orientation which can be quite different.

In most cases the fibers are assumed to be randomly oriented in the composite. Randomly oriented short fiber composites are of particular significance because they behave as an isotropic or a quasi-isotropic material, that is, they have the same properties in all directions[٣].

However, in other cases short fibers are distributed and aligned by viscous flow of the surrounding molten polymer due to the molding

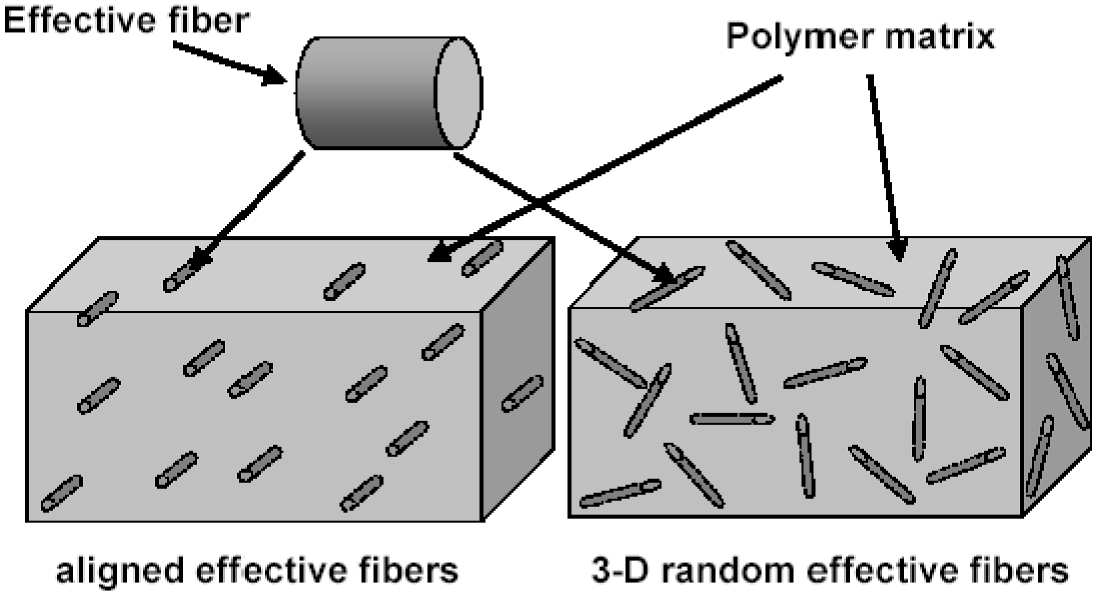


Fig. (٢.١٣): Orientation of effective fibers in short fiber composite. Ref. [٣٠] operation [٨], so considerable orientation can occur in the flow direction and hence induce some anisotropy [٣]. Even at $L_f >> L_c$, the stress that has been transmitted to unoriented fibers is much lower than those transmitted to oriented fibers [٢٩]. Fig. (٢.١٣) shows oriented and random short fiber composite.

Chopped fiber composite as compared with continuous fiber composite has low stiffness and strength, and it does not obey the rule of mixtures [٣١,٣٢]. Some corrections in the value of V_f will be needed to account for the fiber shortness. The extent of correction depends on the length of fibers over which the load gets transferred from the matrix. This adjustment or correction becomes negligible when the fiber length is much greater than a critical length as will be observed [٣].

There are two micromechanical approaches to predict the macromechanical behavior of short fiber composite:

A. Modified rule of mixtures

A modified rule of mixtures can be used, incorporating a length efficiency parameter $\lambda_{١}$ to take account of the fibers not being continuous [٤]. Suppose the fibers are parallel, and a line is imagined drawn across the composite at right angles, as shown in Fig. (٢.١٤), the line intersects fibers at longitudinal positions chosen at random. It follows that the stress

$$\sigma_{CL} = \sigma_{mean} V_f + \sigma_m V_m$$ (٢-١٥)

carried by the composite is no longer given by Eq. (٢-١١) but instead by [٨]

Where σ is the mean fiber stress intercepted:

$$\sigma_{mean} = \frac{1}{L_f} \int_{-L_f/2}^{L_f/2} \sigma_f dx \qquad (٢\text{-}١٦)$$

The strength of fiber σ_f as a function of distance x along the fiber has been obtained from shear lag theory. Shear lag theory assumes perfect bonding between fibers and matrix. The prediction of fiber strength can be stated as follows [٨]:

$$\sigma_f = E_f \varepsilon_f \left\{ 1 - \left[\frac{\cosh\left(na \frac{2x}{L_f} \right)}{\cosh(na)} \right] \right\} \qquad (٢\text{-}١٧)$$

Where

$$n = \sqrt{\left[\frac{2G_m}{E_f \ln(2R/d)} \right]} \qquad (٢\text{-}١٨)$$

The substitution of Eq. (٢-١٧) into Eq. (٢-١٦) and carrying out the integration yields:

$$\sigma_{mean} = E_f \varepsilon_{CL} \left\{ 1 - \left[\frac{\tanh(na)}{na} \right] \right\} \qquad (٢\text{-}١٩)$$

Where n **represents a dimensionless group of constants,** ε_{CL} **is the axial tensile strain in the composite as a whole,** ٢R **is the distance from the fiber to its nearest neighbour measured from its center [٨].**

In order to evaluate n, **assume that the fibers are arranged in a square array, the fiber volume fraction is given by:**

$$V_f = \frac{\pi d^2}{4} \Big/ (2R)^2 \qquad (٢\text{-}٢٠)$$

Then

$$\therefore \frac{2R}{d} = \sqrt{\frac{\pi}{4V_f}} \qquad (٢\text{-}٢١)$$

For most efficient stress transfer to the fibers, na **should be as high as possible. This confirms the desirability of a high aspect ratio** a **but also shows that** n **should be high, the ratio (**G_m/E_f**) should therefore be as high as possible (**G_m **refers to shear in matrix while** E_f **refers to tensile in fiber). Typical values encountered in practice are** a = **٥٠**, n = **٠.٢٤, and hence** na = **١٢ (this satisfies in the case of ٣٠ %** V_f **of glass fibers in a nylon matrix) [٨].**

Thus, Eq. (٢-١٥) will become

$$\sigma_{CL} = E_f \varepsilon_{CL} \left\{ 1 - \left[\frac{\tanh(na)}{na} \right] \right\} V_f + \sigma_m V_m \qquad (٢\text{-}٢٢)$$

Consequently, the axial longitudinal tensile modulus of composite according to modified rule of mixtures (E_{CL}**) is obtained by dividing** σ_{CL} **by** ε_{CL} **that is equal to:**

$$E_{CL} = \lambda_{\text{١}} V_f E_f + V_m E_m \qquad (٢\text{-}٢٣)$$

Where λ١ is a factor corrects the modulus for the shortness of the fibers and is given by:

$$\lambda_1 = 1 - \left[\frac{\tanh(na)}{na} \right] \quad (٢\text{-}٢٤)$$

When na **becomes very large, λ١ approaches ١; as expected since this limit is the case of continuous fibers, but when** na **falls below about ١٠, λ١ is reduced significantly below ١ [٨]. As the length increases, the parameter λ١ approaches ١. The tensile modulus of short fiber composite with the fibers aligned in a longitudinal direction is less than that of the comparable aligned continuous fiber composite [٤].**

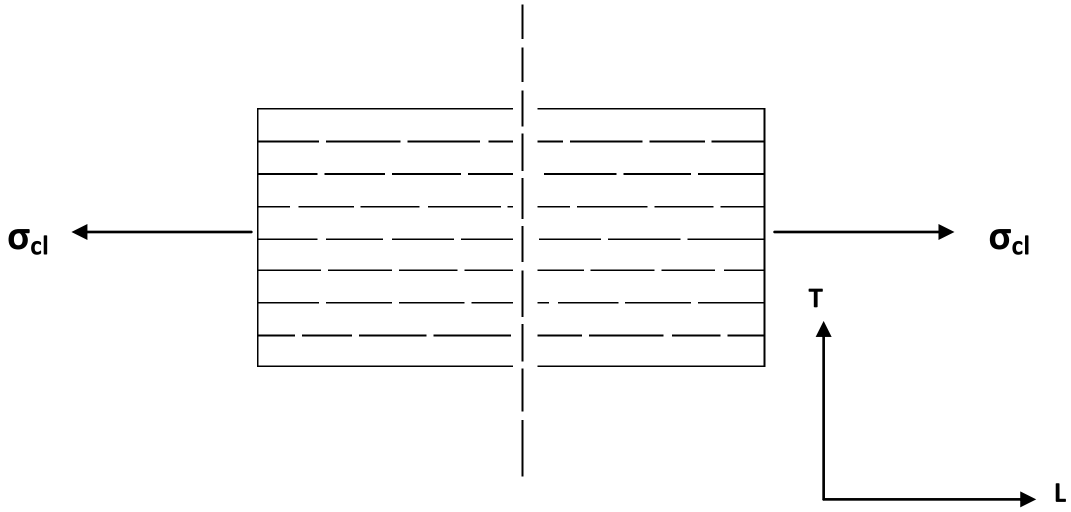


Fig. (٢.١٤): Model of aligned short fiber composite

B. Halpin-Tsai equation

Halpin-Tsai equations have been used to predict the strength and modulus of short fiber composite [٣]. The results are available in the form of curves for specific values of system variables such as fiber aspect ratio (L_f/d**), fiber volume fraction (**V_f**), and properties of the constituents. Whenever a change in any of the system variables takes place, a new set of results has to be obtained [٣].**

Halpin-Tsai equations for predicting the longitudinal and transverse moduli of aligned short fiber composites can be written as [٣]:

$$\frac{E_L}{E_m} = \frac{1+\frac{2L_f}{d}\lambda_L V_f}{1-\lambda_L V_f} \qquad (٢\text{-}٢٥)$$

And

$$\frac{E_T}{E_m} = \frac{1+2\lambda_T V_f}{1-\lambda_T V_f} \qquad (٢\text{-}٢٦)$$

Where

$$\lambda_L = \frac{(E_f/E_m)-1}{(E_f/E_m)+2(L_f/d)} \qquad (٢\text{-}٢٧)$$

And

$$\lambda_T = \frac{(E_f/E_m)-1}{(E_f/E_m)+2} \qquad (٢\text{-}٢٨)$$

It may be pointed out that Eqs. (٢-١٣), (٢-٢٥), and (٢-٢٦) are only particular cases of a general equation. The form of the general equation coincides with that of Eq. (٢-١٣), in which ζ **is a measure of reinforcement. Eqs. (٢-٢٥), (٢-٢٦) can be obtained from Eq. (٢-١٤) by substituting (**$\zeta = ٢L_f/d$**) for the case of longitudinal modulus and (**$\zeta = ٢$**) for the transverse modulus. The transverse modulus of an aligned short fiber composite that predicted due to Halpin-Tsai equation is not influenced by the fiber aspect ratio (**L_f/d**) and its value is the same as that for the transverse modulus of a continuous fiber composite [٣].**

The problem of predicting properties of randomly oriented short fiber composites is more complex. The following empirical equation can be used to predict the modulus of composites containing fibers that are randomly oriented in a plane [٣]

$$E_{Cr} = \frac{3}{8}E_L + \frac{5}{8}E_T \qquad (٢\text{-}٢٩)$$

Where E_L **and** E_T **are respectively the longitudinal and transverse moduli of an aligned short fiber composite having the same fiber aspect ratio and fiber volume fraction as the composite under consideration. Moduli** E_L **and** E_T **can either be calculated experimentally or calculated using Eqs. (٢-٢٥), (٢-٢٦) [٣]. The tensile modulus of composites with non-aligned short length fibers is less than that given on the alignment direction for the same short length fibers when aligned [٤].**

CHAPTER THREE

The Theoretical Prediction of Mechanical Behavior of Linear Viscoelastic Polymer and PMC

٣.١ General

Polymers become an integral part of everyone’s life style with a wide range of applications. The structure of solid polymers below glass transition temperature “T_g” is non-equilibrium inhomogeneous, so that stress and strain analysis of these materials is complicated [٢٤]. Most deformational behavior of thermoplastic under mechanical loads depends to a great extent on time. This influence of time makes the dimensioning of plastic components considerably more complicated than with other materials [١٧]. The aim is the production of strong fiber composite materials seeks to approach the theoretical strengths for ideal materials [٣٥].

In the present work, tensile and flexural creep tests have been performed on polypropylene, aligned and random short E-glass fiber reinforced PP composite in the linear viscoelastic region at room temperature. It is limited to a step function input of the applied load at t = ٠, which then remains constant while the strain, is the function of time. Constitutive equations are derived for the time-dependent response from a thermoplastic polymer and PMC at isothermal loading with small strain.
The present work has been focussing on creep as a basic principle where both polymer and polymer matrix composite have been subjected to tensile creep and flexural creep estimations at ٢٠°C, the time scale used ranging between (١-١٠^{٩}) second.

under Study ٣.٢ Materials

The material systems chosen for this work were homopolymer polypropylene and short E-glass fiber
The properties of fiber reinforced plastics depend in a complex manner on reinforced polypropylene composite.
history [١١]. Both polymer and **PMC** specimens have been fabricated by injection molding in this processing
work.

٣.٢.١ Polypropylene (PP)

by many companies. There are three Polypropylene was first commercialized in ١٩٥٩ and produced forms of symmetry (or stereo-regularity) of vinyl polymers such as polypropylene, polyethylene, and polyvinylchloride. For polypropylene the placement of the **CH٣** groups are seen to take one of three patterns shown in Fig. (٣.١).

There is no change in chemical composition, merely in the placement of **CH٣** groups. The crystals are form, sometimes formed from the syndiotactic form, and never formed from isotactic always formed from the the atactic form [٨]. Syndiotactic is the best for forming crystalline regions, atactic is amorphous and rubbery, and isotactic is crystalline, hard, and stiff material [١٦,٧]. The proportions of these types in a commercial plastic control the properties [٧]. Commercial polypropylene is generally predominantly isotactic with small amounts of the other forms present [٤]. PP is used mainly in its crystalline form; this being a linear polymer with side groups regularly arranged along the chain.

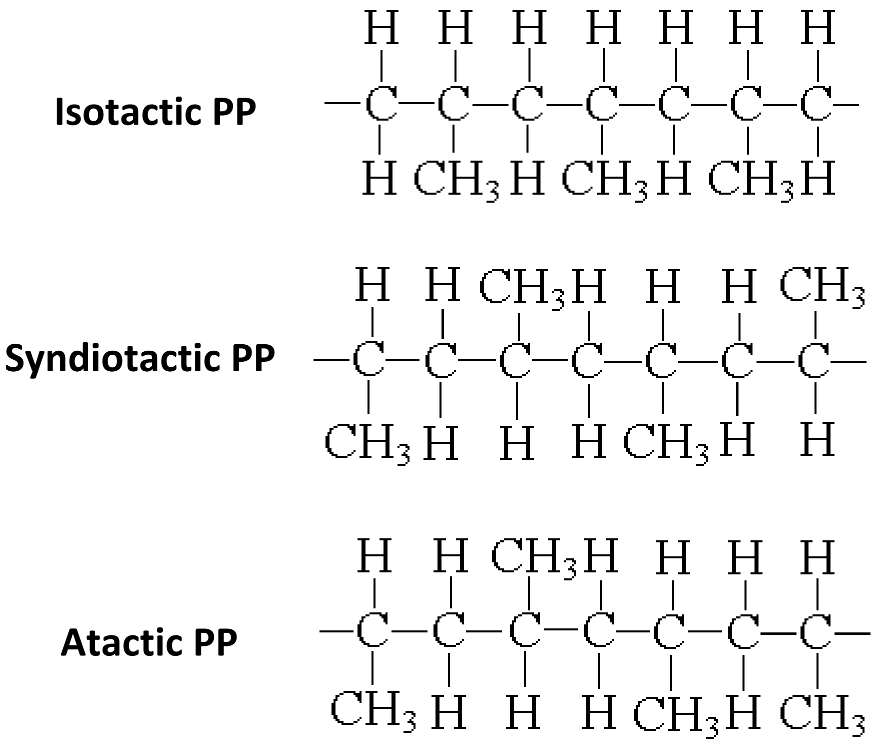


Fig. (٣.١): Forms of polypropylene. Ref. [٤]

Polypropylene has good fatigue resistance and electrical insulation properties [٤]. PP is resistant to cracking (stress corrosion cracking) [٣٦] and to most acids and alkalis even at higher environmental stress The temperatures [١]; it can be used in contact detergents but is attacked by strong oxidizing reagents [٧]. aliphatic chain structure of PP makes it relatively susceptible to oxidation, especially in sunlight, and adequate stabilization by antioxidants is therefore important [٨]. PP is a low-density, hard, stiff material, with good wear resistance, low water absorption, and relatively low cost [١٢].

PP has glass transition that is below room temperature so it is rubbery material; it tends not only to creep under applied stresses, but also to warp in response to molded-in stresses [٨]. Melting temperature of PP ($\mathbf{T_m}$ = ١٧٦°C) [٣٧]. PP has more stiffness than HDPE because of its higher melting point [٦]. Polypropylenes are stiffer, mold well, have high yield stresses and melting points than PE, polypropylenes are more prone to fracture than PE, especially at low temperatures. This is a consequence of the low value of Tg of PP [٨].

Unreinforced polypropylene is a ductile and semicrystalline polymer [٨]. Polypropylene can be spun into filaments, converted into weaves; injection molded [١٢], extruded, blow molded, and its sheet can be thermoformed [٤]. The experimental results emphasize that PP exhibits non-linear viscoelasticity for any deformation of practical importance [٣٨]. In order to improve its performances, PP is frequently added with fillers and/or reinforcements [٣٩].

Because of its characteristics of low density, good processability and environmental resistance, isotactic polypropylene (PP) can be considered as one of the best thermoplastic material that candidates for many industrial applications, like components for the automotive market and cables. Polypropylene is used in air ducting, parcel shelves, air cleaners, many other car applications, and in many domestic appliances (washing
are a good example) [٨]. It is also used in crates, containers, fans, cabinets for radios and TV machine tanks
sets, chair shells [٤], hinges, toys, fibers, pipes, sheets, wire covering [١١], oriented films for packaging [١٨], and valves [٧]. Synthetic fibers of PP are used in clothing, carpets, ropes, and tyre reinforcement [٨]. PP yarns are used as water filters.

٣.٢.٢ Reinforcement: E-Glass Fiber

Glass fibers are manufactured by extruding molten glass at high linear velocity through a large number
a ‘bushing’. The resulting filaments have a round (١٠٠-١٠٠٠) of holes in a platinum plate known as

cross section and then wound up on a mandrel at any even faster rate as they cool and solidify. This has the effect of drawing them down to a much smaller diameter than when they left the bushing. There must be no crystallinity and to ensure the final amorphous microstructure prior to applying final coatings or sizing, the fibers have been cooled rapidly during drawing in order to ensure this, because crystals act as stress raisers in glass, and would greatly weaken the fibers [٨,١٢].

Glass fibers are widely used as fiber reinforcement for plastic. They are of brittle nature, being eminently covalently bonded materials [٤٠]. Glass fibers are characterized by their low cost, clear to white color, good mechanical and electrical properties, high chemical resistance, excellent dimensional stability with operational service to ٥٥٠ °C, and relatively low stiffness [١٢,٤١].

There are several characteristics of glass fibers that make them ideal reinforcement as high tensile strength to weight ratio: they are perfect elastic materials (typical glass fibers have a maximum elongation of ٥ % at break); they are incombustible and also have a low thermal expansion coefficient; they do not absorb moisture and do not swell, stretch, or undergo chemical change through moisture contact and corrosion resistant [٢٦].

Three further characteristics of glass fiber are advantageous in certain applications:

- Resistance to high temperatures, the softening point is about ٨٥٠ C°.
- Transparency to visible light, a composite therefore takes a colour of its matrix.
- Isotropy, for example thermal expansion is identical in axial and radial directions [٨].

due to a loss of A disadvantage of glass fibers is that they are very susceptible to handling damage
This can be caused by rubbing the (١٠-٣٠)% of strength during handling and weaving operations [٤٠].

.surface or by the action of moisture, which gradually dissolves certain of the oxides present at the fiber surface

As a result, the strength of the fibers in use is always lower than that of a freshly drawn fiber by a factor of ٢ or more [٨].

available glass fibers is E-glass [١٢]. Its main constituents are The common type of commercially

SiO٢ (٥٤٪), CaO (١٧.٥٪), Al٢O٣ (١٤.٠٪), B٢O٣ (٨.٠٪), and MgO (٤.٥٪) [٨].

Composite ٣.٢.٣ Short E-Glass Fiber Reinforced PP

Composites, based on thermoplastic resins, are now becoming popular due to their processing advantages. The short fiber composite has become a very common construction material in the engineering of today. Thus, great efforts are made to understand the behavior of these materials, not only on the macro level, but also on the micro level [٤٢].

The mechanical properties of thermoplastics are often inadequate for specific applications [٤٣]; so reinforcing them with glass fibers gives much stronger and stiffer material than the pure polymer [٤]. Glass fiber reinforced plastics suffer from the low elastic modulus [٧] and the high service sensitivity in response to the relative humidity and cracking [٣٦]. Short glass fibers increase the stiffness of PP, increase costs [٨], enhance fracture resistance [٣٩], and greatly reduce ductility. The stiffening effect of the fibers enables PP to be used at temperatures approaching from its melting point [٨].

Short glass fibers embedded within PP aid to limit the propensity of this material to deform under creep conditions, and then contribute also to prevent creep failure in load-bearing applications [٣٩]. The principle that applies equally well to semi-crystalline plastics is: without the reinforcement, creep is a major problem in the temperature range between $\mathbf{T_g}$ and $\mathbf{T_m}$. Glass fiber reinforced polypropylene is a brittle material [٨]. Glass-filled polypropylene is widely used for its enhanced mechanical properties. It is used for food and chemical containers, domestic appliances, furniture, car parts, toys, tubing, and cable sheath[١٢].

٣.٣ Concepts and General Assumptions of the Present Study

There are certain relations between the elastic constants must be satisfied. In this work, the shear modulus of polymer $\mathbf{G_p}$ is defined in terms of the constant elastic modulus $\mathbf{E_p}$, and constant elastic Poisson's ratio $\mathbf{\nu_p}$, and the constant bulk modulus of polymer $\mathbf{k_p}$ as [٩]

$$\mathbf{k_p = \frac{E_p}{3(1-2\nu_p)}} \qquad (٣\text{-}٢)$$

$$G_p = \frac{E_p}{2(1+\nu_p)} \qquad (٣\text{-}١)$$

Poisson’s ratio for isotropic materials is approximately equal to ٠.٣ [٤٧]. For composite

materials, Poisson’s ratio have very significant effects and do not have limited boundaries [٤٤].

Poisson’s ratio is not a material constant but can depend upon time. Poisson's In viscoelastic materials,
ratio of viscoelastic material can increase or decrease with time. The time dependence of Poisson's ratio in

linearly viscoelastic materials that can be determined as [٢٠]

$$\nu(t) = \frac{1}{2} - \frac{E(t)}{6k(t)} \qquad (٣\text{-}٣)$$

It can be assumed that **k (t)** can be approximated as constant k, so

$$\nu(t) = \frac{1}{2} - \frac{E(t)}{6k} \qquad (٣\text{-}٤)$$

ν (t) is an increasing function of time for Where **E (t)** is a monotonically decreasing function,
polymeric materials. Poisson's ratio of the composite structure decreases with time in contrast to the situation in

homogeneous polymer [٢٠]. Glass fibers and polypropylene in the present work are isotropic materials.

Composite materials are heterogeneous, wherein their properties and structure change from point to
point. Because of the inherent heterogeneous nature of composite materials, they are conveniently studied from

two points of view [٩]:

a. Micromechanics: In all the micromechanics results, the predicting of the mechanical properties of the composite material is based on the mechanical properties of its constituent materials.

b. Macromechanics: is the study of composite material behavior wherein the material is presumed homogeneous and the effects of the constituent materials are detected only as averaged apparent properties of the composite.

of some of the microstructure work is that macroscopic homogeneity One of the important conclusions

may not exist for composites[١٠]; therefore, the present work assumes macromechanics behavior while

microstructural considerations will be considered. There are two basic approaches to the micromechanics of composite materials used in this work:

that represented by modified rule of. ١ .Mechanics of materials approach

. mixtures

Elasticity approach which is represented by Halpin-Tsai equation [٩]. ٢.

Polymer and PMC specimens are anisotropic due to injection molding and reinforcing respectively.

The general assumptions of the present work can be stated as follows:

١. The anisotropic behavior in polymer and PMC has not been taken into account because of the lack of theoretical data and the needing of experimental results.

٢. The injection molded PP specimens have been considered to be isotropic.

٣. The injection molded short E-glass fiber/PP composite specimens have been considered to be isotropic in the case of random fiber orientation while oriented fibers will produce directional properties.

٤. The recovery behavior of viscoelastic material has not been considered.

٥. The short fibers have been assumed to be of uniform length, diameter, and properties.

٦. Stress concentrations at fiber ends have been ignored.

٧. The present work has been concerned with the linear viscoelastic behavior in polymer and PMC and does not reach to failure.

٨. The bonding between fiber and matrix is perfect.

٩. The modulus of elasticity **(E_p)** and Poisson's ratio **(ν_p)** for initially elastic matrix are independent of time or at least weak functions of time.

٣.٤ The Mechanical Estimations

Standard estimations for the stiffness of plastics are based on either tensile or flexural measurement [٨]. Creep estimations of polymers are more useful than tensile estimations from design point of view because creep estimation has many varying factors such as stress, strain, time, and temperature [٣٦].

The long-term exposure of polymeric composite materials to extreme use environments, such as pressure, temperature, moisture, and load cycles, results in changes in the original properties of the material. These changes in material properties are translated to structural changes that can have a potentially catastrophic effect on load bearing composite structures. Therefore the study and understanding of the long term exposure on the time dependent viscoelastic properties of polymeric composite structures is important for their proper design, construction, and safe operation [٣٣].

The upper limit of time of a creep estimation is limited by the experimenter’s patience. The simplest method of load application is to use a dead weight. This is appropriate for creep estimations of moderate to long duration [١٣].

٣.٤.١ Flexural Creep Estimation of Polymer and PMC

In this work a three-point flexural creep estimation for a simply supported beams was conducted due to the ASTM D٧٩٠ on specimens of PP and short E-glass fiber reinforced PP composite respectively as shown in Fig. (٣.٢).

All the assumptions of simple bending theory (elementary beam theory) are consistent with the flexure specimen used in this work. In this work, the beam cross section is uniform and symmetrical about the plane of loading. The (x-y) plane is the plane of symmetry that represents the plane of loading. The load is applied in y-direction while the beam is stressed and strained in x-direction.

The case is general bending in this work; where the bending moment varies along the length of the beam. A simplifying assumption will be used that the shear forces that are coming with the varying bending moment do not contribute significantly to the overall deformation [٤١]. If a loaded beam is viscoelastic, the deflections will change with time [٢١].

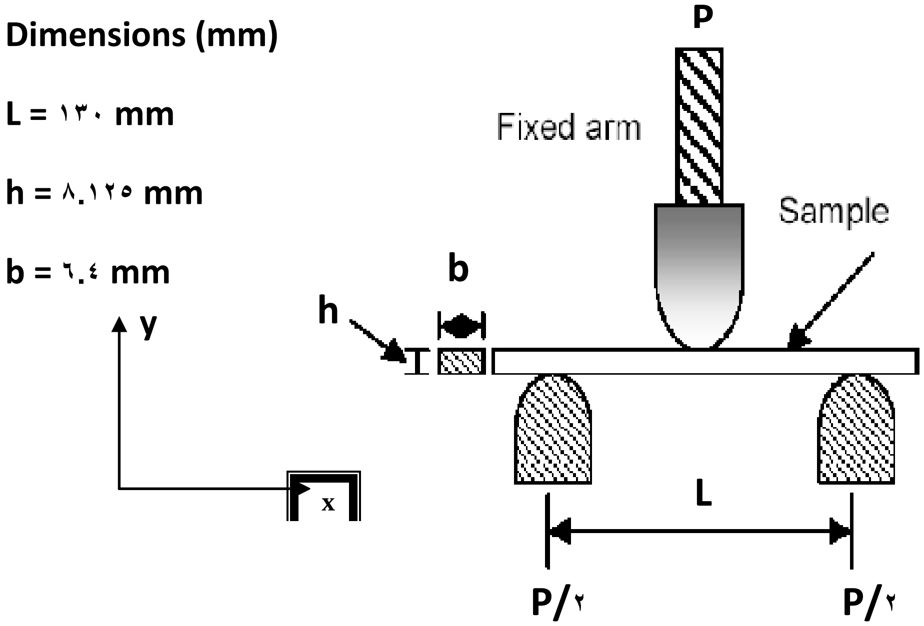


Fig. (٣.٢): Three points flexural test procedure. Ref. [٢٦]

In order to determine the time dependent deflection of a linear viscoelastic beam under sustained loading, one simply multiplies the elastic deflection δ_{el} **(x)** by the constant bending modulus E_b and the creep compliance **J (t)** for the material of the beam. Therefore, the deflection for linearly viscoelastic beams can be formalized as [٢١]

$$\delta(x,t) = \delta_{el}(x)E_bJ(t) \qquad (٣-٥)$$

At the middle region of the beam, maximum deflection and bending moment have been occurred. From elementary beam theory, the following relations can be applied to small deflections of beam [٤٥]:

$$\sigma = -\frac{My}{I} \qquad (٣-٦)$$

$$\varepsilon = \frac{\sigma}{E}$$

$$\varepsilon = \frac{My}{EI} \qquad (٣-٨)$$

The minus sign indicates that there is shortening above the neutral axis and lengthening below. Eq. (٣-٦) shows that for a positive bending moment **M** and positive y, the normal stresses σ_x are compressive; whereas for negative **y**, the stresses are tensile. The procedure of flexural creep estimation of polymer in this work is as follows [٢١]:

$$\varepsilon_p(x,t) = -\frac{M(x)y}{I}J_p(t) \qquad (٣-٩)$$

The creep compliance of viscoelastic beam due to Eq. (٣-٥) is

$$J(t) = \frac{\delta(x,t)}{\delta_{el}(x)E_b} \qquad (٣-١٠)$$

Then

$$\varepsilon_p(x,t) = -\frac{M(x)y}{I}\cdot\frac{\delta(x,t)}{\delta_{el}(x)E_b} \qquad (٣-١١)$$

Where **I** is moment of inertia and can be defined as $\mathbf{I = bh^{٣}} / ١٢$, $\mathbf{E_b}$ bending modulus which is constant material property and independent of load.

The positive bending moment in terms of x can be stated as follows:

$$M(x) = \frac{P}{2}x \tag{٣-١٢}$$

And leads to obtain concave upward. The distance y = -h/٢ for bottom fibers subjected to tensile strain.

$\boldsymbol{\delta_{el}}$ **(x)** is the deflection of a linearly elastic beam with the same loading and boundary conditions as those of the original viscoelastic beam. The elastic deflection in terms of x is [٤٦]:

Where

$$\delta_{el}(x) = \frac{Px}{12E_bI}\left(x^2 - \frac{3}{4}L^2\right) \tag{٣-١٣}$$

$$0 \le x \le \frac{L}{2}$$

δ (x, t) is the viscoelastic deflection of the beam that can be determined from the finite element analysis Finally as will be seen in chapter four.

The time dependent flexural creep modulus of polymer and the time dependent flexural creep compliance of polymer can be determined as [٢٤,٦]:

ε (x, t) at position x and time t must be checked due to the limits of linear viscoelastic Each strain value

$$E_p(t) = \frac{1}{J_p(t)} \tag{٣-١٤}$$

$$J_p(t) = \frac{\varepsilon_p(t)}{\sigma} \tag{٣-١٥}$$

behavior where the strain must be less than ٠.٥٪; therefore, linear viscoelastic behavior is consistent with small deflection.

٣.٤.٢ Tensile Creep Estimation of Polymer and PMC

The tensile estimations were conducted according to ASTM D ٦٣٨ with a specimen shown in Fig.

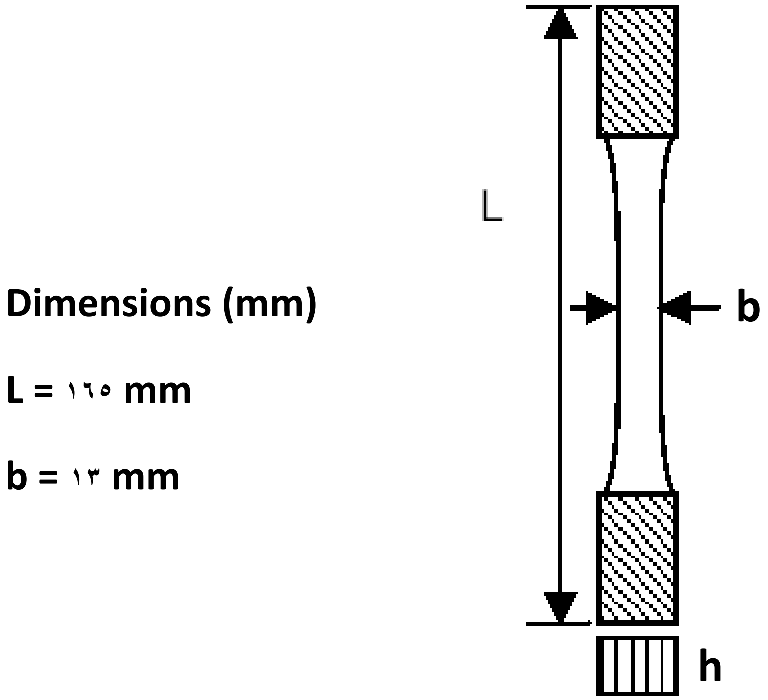


Fig. (٣.٣): Tensile test specimen. Ref. [٢٦]

(٣ . ٣). The standard method for measuring the modulus and yield stress of polymers is a tensile test, in which are attached to the ends of a bar, and pulled apart at constant speed. grips In this type of experiment, some nonlinearity is to be expected in the stress-strain curve, even within the region of linear viscoelastic behavior, because the measurements are made over a range of times [٨]. When the composite is loaded axially, the axial displacements in the fiber and in the matrix are locally different due to the different elastic moduli of the components [٢].

The loading case is point load in tension, and it is uniaxial in plane. The tensile test has the advantages that the stress is uniform in the gauge length and that the corresponding strains can be measured directly [٨]. Linear viscoelastic behavior is consistent with small strain. In tensile creep test for polymer, stress is constant and can be determined as:

Where A is the area of specimen in the stress free state, so **A = b x h** mm^{٢}. The time dependent strain

$$\sigma = \frac{P}{A} \qquad (٣\text{-}١٦)$$

ε_p (t) can be determined by finite element method; and by using Eqs. (٣-١٤) & (٣-١٥) the of polymer time dependent tensile creep modulus and compliance can be determined. In general, the time dependent creep modulus **E (t)** is a decreasing function of time while the time dependent creep compliance **J (t)** is an increasing function [٢٠], and this coincides with the two loading cases flexural creep and tensile creep of polymer and **PMC** [٢].

٣.٥ Orientations of Short Fiber PMC in This Work

This work is interested in the viscoelastic behavior of short fiber PMC. With the appropriate volume the mechanical behavior, such as fraction accompanied by control of fiber orientation and fiber dimensions, strength, toughness, and stiffness, can be optimized, The amount of change depends on both the form the fibers take and the amount. Two types of orientation will be studied in this theoretical work.

Aligned Short Fiber PMC ٣.٥.١

In this work, for tensile creep estimation of aligned short fiber composite, the specimens were loaded parallel to the fibers to determine the viscoelastic creep properties in the fiber dominated longitudinal direction. For flexural creep estimation of aligned short fiber composite, the loading direction is **y** as shown in Fig. (٣.٢) while the specimen were stressed and strained in the fiber dominated longitudinal direction.

The viscoelastic characteristics of composite materials are usually due to a viscoelastic matrix material such as epoxy resin [٩]. PMC material understudy deforms wherein the fiber elastic while the resin matrix is viscoelastic. Hence, the effect of time has been admitted to the modulus of polymer **E_p (t)** according to the two approaches that can be applied to predict the mechanical response of linear viscoelastic behavior for aligned short fiber PMC:

A. Modified Rule of Mixtures

By assuming perfect alignment, the time dependent longitudinal Young's modulus of unidirectional short fiber reinforced composites **E_{CL} (t)** can be obtained as:

$$E_{CL}(t)=\lambda_1(t)V_fE_f+V_mE_p(t) \qquad (٣\text{-}١٧)$$

E_p (t) is the time dependent creep modulus of PP polymer obtained from finite element computer program which will be explained throughout chapter four. λ١ is the factor which corrects the modulus of the shortness of the fibers as can be seen in Eq. (٢-٢٣) with little modification represented by time dependence according to the presence of time dependent shear modulus **G_p (t)** as follows; **V_f** and **V_m** are volume fractions of fiber and matrix respectively; **E_f** modulus of elasticity of E-glass fiber which is isotropic material wherein **E_{Lf}** = **E_{Tf}** = **E_f**. The calculations will be carried out in the following manner

$$\lambda_1(t)=\left\{1-\left[\frac{\tanh(n(t)a)}{n(t)a}\right]\right\} \qquad (٣-١٨)$$

The fiber aspect ratio is **a** and equal to **l/d**

$$n(t)=\sqrt{\frac{2G_p(t)}{E_f Ln(2R/d)}} \qquad (٣-١٩)$$

From Eq. (٣-١٩) above, **n** is dimensionless group of constants (Eq. (٢-١٧)); in this work **n (t)** is time dependent due to the time dependent shear modulus of polymer that can be determined for isotropic material according to:

$$G_p(t)=\frac{E_p(t)}{2(1+\nu_p(t))} \qquad (٣-٢٠)$$

E_p (t) is the same value obtained from finite element method, while **ν_p (t)** for viscoelastic materials mentioned above through Eq. (٣-٤) is specialized to [٢٠]:

$$\nu_p(t)=0.5-\frac{E_p(t)}{6k_p} \qquad (٣-٢١)$$

The bulk modulus of polymer **k_p** is assumed to be constant as mentioned previously due to Eq. (٣-٤). For isotropic material, **k_p** is equal to [٩]:

$$k_p=\frac{E_p}{3(1-2\nu_p)} \qquad (٣-٢)$$

Where **E_p** and **ν_p** are modulus of elasticity and Poisson's ratio for initially elastic matrix. The time dependent longitudinal strains for PMC at the same value of applied stress on polymer and the time dependent creep compliance of composite respectively are as follows:

$$\varepsilon_{CL}(t)=\frac{\sigma}{E_{CL}(t)} \qquad (٣-٢٢)$$

$$J_{CL}(t)=\frac{1}{E_{CL}(t)} \qquad (٣-٢٣)$$

B. Halpin-Tsai Equations

The time dependent longitudinal modulus of aligned short fiber PMC is:

$$E_{CL}(t) = E_p(t)\frac{1+\frac{2L_f}{d}\lambda_L(t)V_f}{1-\lambda_L(t)V_f} \qquad (٣-٢٤)$$

Where

$$\lambda_L(t) = \frac{(E_f/E_p(t))-1}{(E_f/E_p(t))+\frac{2L_f}{d}} \qquad (٣-٢٥)$$

The time dependent longitudinal creep strain and longitudinal creep compliance can be determined due to Eqs. (٣-٢٢) & (٣-٢٣) respectively.

The time dependent transverse modulus of aligned short fiber PMC can be determined as follows:

$$E_{CT}(t) = E_p(t)\frac{1+2\lambda_T(t)V_f}{1-\lambda_T(t)V_f} \qquad (٣-٢٦)$$

Where

$$\lambda_T(t) = \frac{(E_f/E_p(t))-1}{(E_f/E_p(t))+2} \qquad (٣-٢٧)$$

In the same manner, the time dependent transverse creep strain and transverse creep compliance can be determined due to:

$$\varepsilon_{CT}(t) = \frac{\sigma}{E_{CT}(t)} \qquad (٣-٢٨)$$

$$J_{CT}(t) = \frac{1}{E_{CT}(t)} \qquad (٣-٢٩)$$

٣.٥.٢ Random Short Fiber Composite

Applying one approach can do the prediction of mechanical response of linear viscoelastic behavior of random short fiber PMC; this approach is Halpin-Tsai equation. Composite materials made up of randomly oriented short fibers are isotropic materials [٣]. The time dependent modulus of random short fiber PMC can be determined as follows:

Similarly, the time dependent creep strain and creep compliance of random short fiber PMC will be calculated as:

$$E_{Cr}(t) = \frac{3}{8}E_{CL}(t) + \frac{5}{8}E_{CT}(t) \qquad (٣\text{-}٣٠)$$

$$\varepsilon_{Cr}(t) = \frac{\sigma}{E_{Cr}(t)} \qquad (٣\text{-}٣١)$$

$$J_{Cr}(t) = \frac{1}{E_{Cr}(t)} \qquad (٣\text{-}٣٢)$$

CHAPTER FOUR

Finite Element Approach and Computer Programs

٤.١ General

The finite element method not only accommodates complex geometry and boundary conditions, but it also has proven successful in representing various types of complicated material properties that are difficult to incorporate into other numerical methods. For example, formulations in solid mechanics have been devised for anisotropic, nonlinear, time-dependent, or temperature-dependent material behavior. The systematic generality of the finite element procedure makes it a powerful and versatile tool for a wide range of problems [٤٧].

The FEM is applicable to time dependent and time independent problems [٤٨] and to both linear and nonlinear problems [٤٩]. The idea of the FEM is to solve the continuous problem approximately [٤٥]. The basic differential equation for the deflection of beam will be developed in this chapter and solution of this equation will be illustrated in detail.

The tensile test in this work represents solid mechanics problem while the flexural test represents structural analysis problem [٤٨]. The assumption of a perfect bond means that the finite element analysis takes no account of the adhesion properties of the interface [٢٢].

The reliability of finite element prediction is critically dependent on the accuracy of the constitutive model representing the viscoelastic behavior [٢٤], which is represented in this work, by ٣-parameter model. This work is applying the numerical FEM to measure the viscoelastic creep properties of polymers and polymeric composite materials; the finite element equations have been obtained by using Galerkin formulation.

Galerkin weighted– residual method performs the best out of all the approximate solution techniques while the other global approximate solution methods resulted in even larger errors; Galerkin method requires that the shape functions themselves be used as the weighting function as in Eq. (٤-١) [٤٩]

$$W_i(x)=N_i(x) \qquad (٤-١)$$

٤.٢ General Procedure of Finite Element Method for Flexural Problem

٤.٢.١ Finite Element Equation for Flexural Estimation

In this work, the detailed study of flexural test is concentrated on Eq. (٤-٢) below. The one-dimensional linear differential equation for deflection of a simply supported beam subjected to concentrated load is [٤٨]:

$$EI\frac{d^2\phi}{dx^2}-\frac{Px}{2}=0 \qquad (٤\text{-}٢)$$

This equation is fundamental in elastic beam deflections and it is called

$$0\le x\le\frac{L}{2}$$

moment-curvature relation [٤١], it is applicable in the range:

The finite element equations are obtained using Galerkin's formulation [٤٨]. The evaluation of the weighted residual integral generates a system of linear

$$-\int_0^L W(x)(EI\frac{d^2\phi}{dx^2}-\frac{Px}{2})dx=0 \qquad (٤\text{-}٣)$$

equations:

A negative one has multiplied the integral so that the results can be written in a more convenient form. **EI** in Eq. (٤-٢) is a bending stiffness or flexural rigidity term composed of the elastic modulus of the beam material **E** and the moment of inertia **I** [٤٨].

After defining the weighting function, the next step is to evaluate the residual integral from Eq. (٤-٣). Using the sequence of nodes r, s, **and m that can be seen from Fig. (٤.١) below, Eq. (٤-٣) will become**

$$R_s=R_s^{(e)}+R_s^{(e+1)}=-\int_{X_r}^{X_s}\left[N_s(EI\frac{d^2\phi}{dx^2}-\frac{Px}{2})\right]^{(e)}dx-\int_{X_s}^{X_m}\left[N_s(EI\frac{d^2\phi}{dx^2}-\frac{Px}{2})\right]^{(e+1)}dx=0$$

(٤-٤)

Because **Ws** = ٠ for **x** < **X**$_r$ and **x > X**$_m$, the integral splits into two parts because **Ws (x)** is defined by two separate equations within the interval

$$X_r \le x \le X_m$$

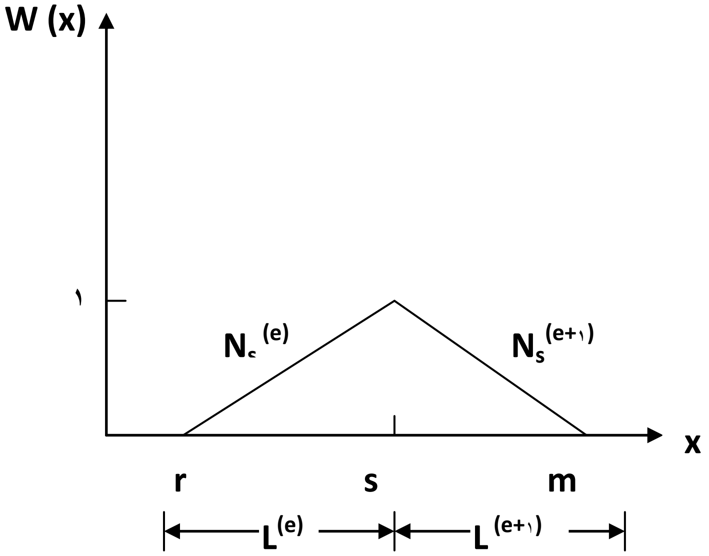


Fig. (٤.١): The weighting function for an interior node. Ref. [٤٨]

The terms **R**$_s$$^{(e)}$ and **R**$_s$$^{(e+١)}$ represent the contributions of element (e) and (e +١) to the residual equation for node **s**. The integral of second derivative term is not defined. This difficulty can be circumvented, however, by changing the second derivative term into a new term. Consider the first integral in Eq. (٤-

$$\frac{d}{dx}(N_s\frac{d\phi}{dx}) = N_s\frac{d^2\phi}{dx^2} + \frac{dN_s}{dx}\frac{d\phi}{dx} \qquad (٤\text{-}٥)$$

٤) and note that

$N_s\frac{d^2\phi}{dx^2}$ Substituting for integral gives **(٤-٦)**

dx ` $\overset{s}{X}$ dx' dx dx

A similar set of operations applied to the first term of the second integral in Eq. (٤-٤) produces

$$R_s^{(e+1)} = -\int_{X_s}^{X_m} \frac{d}{dx}(EI.N_s \frac{d\phi}{dx})^{(e+1)} dx + \int_{X_s}^{X_m} (EI \frac{dN_s}{dx}\frac{d\phi}{dx})^{(e+1)} dx + \int_{X_s}^{X_m} N_s \frac{P}{2} x dx \quad (٤-٨)$$

The first terms in each of Eqs. (٤-٧) & (٤-٨) can be simplified because they represent the interelement terms, which are related to the presence of derivative boundary conditions. The complete residual equation is

$$R_s = \int_{X_r}^{X_s} EI \frac{dN_s}{dx}\frac{d\phi}{dx})^{(e)} dx + \int_{X_r}^{X_s} (N_s \frac{P}{2} x dx)^{(e)} + \int_{X_s}^{X_m} (EI \frac{dN_s}{dx}\frac{d\phi}{dx})^{(e+1)} dx + \int_{X_s}^{X_m} (N_s \frac{P}{2} x dx)^{(e+1)} \quad (٤-٩)$$

Evaluation of the integrals in Eq. (٤-٩) yields the residual equation for an interior node. The equations for the first and last nodes can be obtained from these operations [٤٨].

$$\phi^{(e)} = N_r^{(e)}\Phi_r + N_s^{(e)}\Phi_s \quad (٤-١٠)$$

$$\phi^{(e)} = \left(\frac{X_s - x}{L_e}\right)\Phi_r + \left(\frac{x - X_r}{L_e}\right)\Phi_s \quad (٤-١١)$$

Starting with element (e)

And

Substitution of the appropriate terms and evaluating the first integral in Eq. (٤-

$$\frac{dN_s^{(e)}}{dx} = \frac{1}{L_e} \quad (٤-١٢)$$

$$\frac{d\phi^{(e)}}{dx} = \frac{1}{L_e}(-\Phi_r + \Phi_s) \quad (٤-١٣)$$

$$\int_{X_r}^{X_s} (EI \frac{dN_s}{dx}\frac{d\phi}{dx})^{(e)} dx = \left(\frac{EI}{L_e}\right)^{(e)} (-\Phi_r + \Phi_s) \quad (٤-١٤)$$

٩) yields

The second integral in Eq. (٤-٩) can be evaluated as

$$\int_{X_r}^{X_s} N_s \frac{P}{2} x dx = \int_{X_r}^{X_s} \frac{x - X_r}{L_e} x dx \tag{٤-١٥}$$

Then

$$= \frac{P}{2L_e} \int_{X_r}^{X_s} x^2 dx - X_r x dx \tag{٤-١٦}$$

Eq. (٤-١٦) after a series of simplification and ordering will become:

$$= \frac{P}{12L_e} \left[2X_s^3 - 2X_r X_s^2 + X_r^3 - X_r X_s^2 \right] \tag{٤-١٧}$$

Then

$$= \frac{P}{12L_e} \left[2X_s^2 (X_s - X_r) + X_r^3 - X_r X_s^2 \right] \tag{٤-١٨}$$

So

$\mathbf{L_e = X_s - X_r}$

And

$$= \frac{P}{12L_e} \left[2X_s^2 . L_e + X_r^3 - X_r X_s^2 \right] \tag{٤-١٩}$$

The final formula of the second term of Eq. (٤-٩) is

$$\int_{X_r}^{X_s} N_s \frac{P}{2} x dx = \frac{P}{12L_e}\left[X_s^2(2L_e - X_r) + X_r^3\right]$$ (٤-٢٠)

The residual equation of element (e) in terms of **r**, **s** is:

$$R_s^{(e)} = -\left(\frac{EI}{L_e}\right)\Phi_r + \left(\frac{EI}{L_e}\right)\Phi_s + \frac{P}{12L_e}\left[X_s^2(2L_e - X_r) + X_r^3\right]$$ (٤-٢١)

To evaluate the third term of Eq. (٤-٩)

$$\phi^{(e+1)} = N_s^{(e+1)}\phi_s + N_m^{(e+1)}\phi_m$$ (٤-٢٢)

$$\phi^{(e+1)} = \left(\frac{X_m - x}{L_e}\right)\Phi_s + \left(\frac{x - X_s}{L_e}\right)\Phi_m$$ (٤-٢٣)

Thus

$$\frac{dN_s^{(e+1)}}{\text{And}} = -\frac{1}{L_e}$$ (٤-٢٤)

$$\frac{d\phi^{(e+1)}}{dx} = \frac{1}{L_e}(-\Phi_s + \Phi_m)$$ (٤-٢٥)

The third integral in Eq. (٤-٩) yields:

$$\int_{X_s}^{X_m}\left(EI\frac{dN_s}{dx}\frac{d\phi}{dx}\right)^{(e+1)} dx = \left(\frac{EI}{L_e}\right)^{(e+1)}(\Phi_s - \Phi_m)$$ (٤-٢٦)

Finally, the evaluation of the fourth integral of Eq. (٤-٩) is as follows:

$$\int_{X_s}^{X_m} N_s \frac{P}{2} xdx = \int_{X_s}^{X_m} \frac{X_m - x}{L_e} \frac{P}{2} xdx$$ (٤-٢٧)

Eq. (٤-٢٧) after a series of modification and ordering will become:

$$= \frac{P}{12L_e}\left[2X_s^2(X_s - X_m) + X_m^3 - X_m X_s^2\right]$$ (٤-٢٨)

$$L_e = X_m - X_s$$

$$\int_{X_s}^{X_m} N_s \frac{P}{2} xdx = \frac{P}{12L_e}\left[-X_s^2(2L_e + X_m) + X_m^3\right]$$ (٤-٢٩)

The residual equation of element (e +١) in terms of **s, m**

$$R_s^{(e+1)} = \left(\frac{EI}{L_e}\right)\Phi_s - \left(\frac{EI}{L_e}\right)\Phi_m + \frac{P}{12L_e}\left[-X_s^2(2L_e + X_m) + X_m^3\right]$$ (٤-٣٠)

The nodal residual equation of node s in terms of r. m after evaluation is

$$R_s = -\left(\frac{EI}{L_e}\right)^{(e)}\Phi_r + \left[\left(\frac{EI}{L_e}\right)^{(e)} + \left(\frac{EI}{L_e}\right)^{(e+1)}\right]\Phi_s - \left(\frac{EI}{L_e}\right)^{(e+1)}\Phi_m$$ (٤-٣١)

$$+ \frac{P}{12L_e}(-X_s^2(2L_e + X_m) + X_m^3 + X_s^2(2L_e - X_r) + X_r^3)$$

٤.٢.٢ Finite Element Modeling for Flexural Specimen

Solving of beam deflection problems, in addition to the differential equations, requires boundary conditions prescribing. There are several types of homogeneous boundary conditions. For roller or pinned support at the end, neither deflection δ nor moment M can exist [٢١]. The boundary conditions must also not change with time. For a simply supported beam, boundary conditions at supports are δ = ٠, M=٠.

For a simply supported beam understudy, the boundary conditions are:

a. At **x = ٠** ,**δ_{el}** = ٠, **δ(x, t)** = ٠, **M** = ٠.
b. At **x = L/٢** ,**δel (x)** = maximum, **δ(x, t)** = maximum, **M** = maximum.
c. At **x =L** ,**δel (x)** = ٠, **δ(x, t)** = ٠, **M** = ٠.

For flexural creep estimation, the simply supported beam is descretized by using one-dimensional linear elements. Each element has two nodes and there is one degree of freedom for each node, which represents transverse deflection, the total number of nodes is equal to the total number of equations to be solved for the assemblage. Because of symmetry about y-axis, only half of beam length was analyzed and studied. Its length is ٦٥ mm. The finite element mesh for simply supported beam is shown in Fig. (٤.٢).

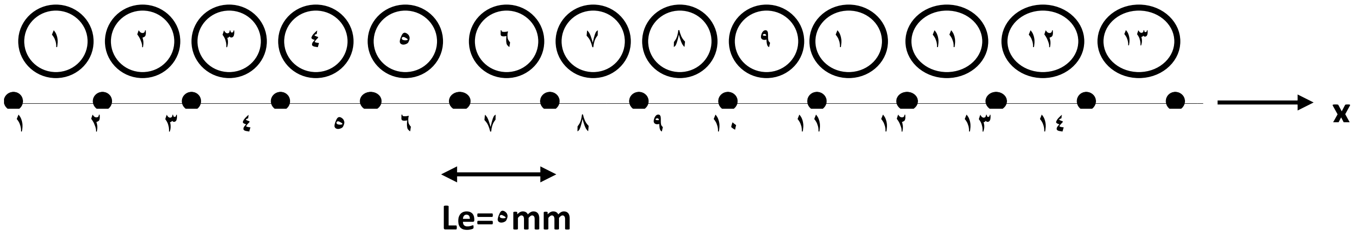


Fig. (٤.٢): Finite element mesh for flexural specimen.

٤.٣ General Procedure of Finite Element Method for Tensile Estimation

٤.٣.١ Finite Element Equation for Tensile Estimation

The Galerkin approach according to the principle of virtual work will be considered. This procedure was mentioned with several derivations through Ref. [٤٥]. For two-dimensional tensile estimation, the following relations will be applied whereas element stiffness matrix is:

$$k^e = A^e h^e B^T DB \qquad (٤\text{-}٣٢)$$

$$A^e = a_{١} . a_{٢} \qquad (٤\text{-}٣٣)$$

A^e is area of element and can be determined as follows:

Whereas **$a_{١}$** and **$a_{٢}$** are length and width of rectangular element respectively and can be seen clearly in Fig. (٤.٣) where **$a_{١}$**=٥mm & **$a_{٢}$**=٣.٢٥mm, **h^e** is the thickness of element. [**B**] is strain-displacement matrix for all elements as a constant equation; [**B**]^T is the transpose of matrix [**B**]; [**D**] is the plane-stress matrix.

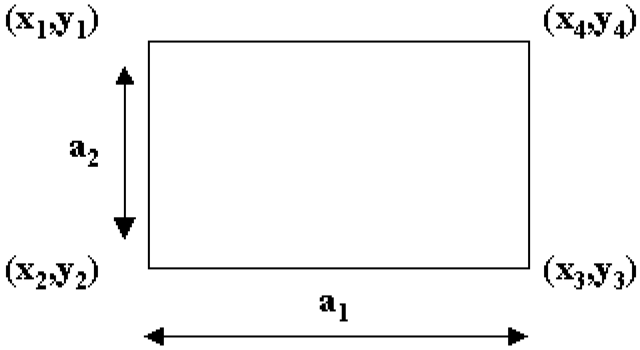


Fig.(4.3): Cartesian coordinates of rectangle element

The finite element solution for elasticity problems is modified through changing material properties in each time step. The key is to replace the elastic constitutive law by another viscoelastic, i.e. Young’s modulus will become time dependent quantity and its value will change with time according to the three-parameter model used. Plane stress matrix **[D]** for linear viscoelastic behavior is as follows:

$$[D(t)] = \frac{E_p(t)}{1-\nu_p^2(t)} \begin{bmatrix} 1 & \nu_p(t) & 0 \\ \nu_p(t) & 1 & 0 \\ 0 & 0 & \dfrac{1-2\nu_p(t)}{2} \end{bmatrix} \qquad (٤\text{-}٣٤)$$

In the current algorithm, at each time step a new stiffness matrix has been calculated according to the change in material properties with time represented in [**D]** **(t)** matrix.

The reduction of a ٣-D problem to a ٢-D problem occurs in two ways: plane stress and plane strain. Plane stress exists when the elastic body is very thin and there are no loads applied in the coordinate direction parallel to the thickness [٤٨]. The assembly rules for the global stiffness matrix and global load vector are [٤٧]:

$$[K] = \sum_{e=1}^{NET} [k^e] \qquad (٤\text{-}٣٥)$$

$$\{F\} = \sum_{e=1}^{NET} \{f^e\} \qquad (٤\text{-}٣٦)$$

Where **NET** is the total number of elements. For assembly procedure, the element connectivity is useful in adding element stiffness values in **[k^e]** into corresponding global locations in global stiffness matrix **[K]** [٤٧]. The global force **{F}** consists from the contribution of body forces, traction forces, and point loads. In this study, there are no body forces and traction forces and hence, the externally applied load **P** is added directly to corresponding location of global force vector **{F}**. The nodal point displacements are unknown. The displacement can be used then to calculate strains, and other quantities of interest.

The process of assembling the elements is unaffected by the linearity or nonlinearly of the material behavior or by the specific character of the problem [٤٧]. The assembly process is direct stiffness method. Field variables are displacements in tensile test and deflections in flexural test which represent the principle unknowns of the problem. It can be extracted the **q** vector from the global **Q** vector, an operation performed frequently in a finite element program. The general Finite element equation is:

$$[K]\{Q\} = \{F\} \qquad (٤\text{-}٣٧)$$

٤.٣.٢ Shape Function and Its Derivatives

In this work, the two dimensional finite element formulations of shape function will be used. For two dimensional element used in this work, the element shape functions are written in terms of natural coordinates (ξ,η) to get a continuity between elements and because they are more convenient for analytical and numerical integration [٤٨].

The shape function information is general information, which can be used in solving solid mechanics problems as well as differential equations [٤٨]. The derivatives of shape functions with respect to global coordinate system are included in the integral together with Jacobean of coordinate transformation between the normalized (or natural) coordinate and global coordinate systems. One usually needs a conversion of the derivative of shape function since the shape functions are defined in terms of the normalized or natural coordinate [٥٠].

In order to evaluating the strains, partial derivatives of **u** and **v** are to be taken with respect to **x** and **y**, the strain-displacement relations [٤٥] are:

$$\varepsilon = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} \qquad (٤\text{-}٣٨)$$

The shape functions in terms of the natural coordinates ζ and η are [٤٨,٥٠]:

$$N_1 = \frac{1}{4}(1-\xi)(1+\eta)$$

$$N_2 = \frac{1}{4}(1-\xi)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1-\eta) \qquad (٤\text{-}٣٩)$$

$$N_4 = \frac{1}{4}(1+\xi)(1+\eta)$$

The shape function can be simplified

$$N_1 = \frac{1}{4}(1-\xi+\eta-\xi\eta)$$

$$N_2 = \frac{1}{4}(1-\xi-\eta+\xi\eta)$$

$$N_3 = \frac{1}{4}(1+\xi-\eta-\xi\eta) \qquad (٤\text{-}٤٠)$$

$$N_4 = \frac{1}{4}(1+\xi+\eta+\xi\eta)$$

The displacements inside the element are now written using the shape functions and the nodal values of the unknown displacement field [٤٥]

$$u = N_1q_1 + N_2q_3 + N_3q_5 + N_4q_7$$

Or $N_1q_2 + N_2q_4 + N_3q_6 + N_4q_8$ (٤-٤١)

$$u = Nq$$ (٤-٤٢)

For the linear rectangular element, the coordinates x, y can also be represented in terms of nodal coordinates using the same shape functions.

$$x = \sum_{i=1}^{4} N_i(\xi,\eta)x_i$$

$$y = \sum_{i=1}^{4} N_i(\xi,\eta)y_i$$ (٤-٤٣)

This is isoparametric representation [٤٥]. The general relation is [٥٠]:

From Eq. (٤-٤٠) and Eq. (٤-٤٣):

And

$$y = \frac{1}{4}(y_1 + y_2 + y_3 + y_4) + \frac{\xi}{4}(-y_1 - y_2 + y_3 + y_4) + \frac{\eta}{4}(y_1 - y_2 - y_3 + y_4)$$

$$+ \frac{\xi\eta}{4}(-y_1 + y_2 - y_3 + y_4)$$ (٤-٤٥)

$$x = \frac{1}{4}(x_1 + x_2 + x_3 + x_4) + \frac{\xi}{4}(-x_1 - x_2 + x_3 + x_4) + \frac{\eta}{4}(x_1 - x_2 - x_3 + x_4)$$

$$+ \frac{\eta\xi}{4}(-x_1 + x_2 - x_3 + x_4$$ (٤-٤٤)

Eqs. (٤-٤٤) & (٤-٤٥) express x and y as functions of natural coordinates

$$\frac{\partial x}{\partial \xi} = \frac{1}{4}(-x_1 - x_2 + x_3 + x_4) + \frac{\eta}{4}(-x_1 + x_2 - x_3 + x_4)$$

$$\frac{\partial x}{\partial \eta} = \frac{1}{4}(x_1 - x_2 - x_3 + x_4) + \frac{\xi}{4}(-x_1 + x_2 - x_3 + x_4)$$ (٤-٤٦)

$$\frac{\partial y}{\partial \xi} = \frac{1}{4}(-y_1 - y_2 + y_3 + y_4) + \frac{\eta}{4}(-y_1 + y_2 - y_3 + y_4)$$

$$\frac{\partial y}{\partial \eta} = \frac{1}{4}(y_1 - y_2 - y_3 + y_4) + \frac{\xi}{4}(-y_1 + y_2 - y_3 + y_4)$$

(ξ,η). By using the notation **x_{ij} = x_i - x_j** and **y_{ij} = y_i - y_j** whereas in the present work, the corresponding values are shown in Fig. (٤.٣) which are represented by **$a_١$= $x_٤$-$x_١$=$x_٣$-$x_٢$** while **$a_٢$ = $y_٤$-$y_٣$ = $y_١$-$y_٢$**. These relations can be used to simplify the following derivatives:

By ordering Eq. (٤-٤٦) and with simplification:

$$\frac{\partial x}{\partial \xi}=\frac{1}{4}(a_1+a_1)+\frac{\eta}{4}(a_1-a_1)=\frac{a_1}{2}=2.5$$

$$\frac{\partial x}{\partial \eta}=\frac{1}{4}(a_1-a_1)+\frac{\xi}{4}(-a_1+a_1)=0$$

$$\frac{\partial y}{\partial \xi}=\frac{1}{4}(a_2-a_2)+\frac{\eta}{4}(-a_2+a_2)=0 \qquad (٤-٤٧)$$

$$\frac{\partial y}{\partial \eta}=\frac{1}{4}(a_2+a_2)+\frac{\xi}{4}(-a_2+a_2)=\frac{a_2}{2}=1.625$$

The partial derivatives of shape functions that are defined in terms of normalized or natural coordinates [٥٠] are:

$$\frac{\partial N_i}{\partial \xi}=\frac{\partial N_i}{\partial x}\frac{\partial x}{\partial \xi}+\frac{\partial N_i}{\partial y}\frac{\partial y}{\partial \xi}$$
$$\frac{\partial N_i}{\partial \eta}=\frac{\partial N_i}{\partial x}\frac{\partial x}{\partial \eta}+\frac{\partial N_i}{\partial y}\frac{\partial y}{\partial \eta} \qquad (٤-٤٨)$$

$$\begin{Bmatrix}\dfrac{\partial N_i}{\partial \xi}\\[2mm] \dfrac{\partial N_i}{\partial \eta}\end{Bmatrix}=\begin{bmatrix}\dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi}\\[2mm] \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta}\end{bmatrix}\begin{Bmatrix}\dfrac{\partial N_i}{\partial x}\\[2mm] \dfrac{\partial N_i}{\partial y}\end{Bmatrix} \qquad (٤-٤٩)$$

And in matrix notation

The Jacobean matrix of the transformation is [٤٥]:

$$J=\begin{bmatrix}\dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi}\\[2mm] \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta}\end{bmatrix} \qquad (٤-٥٠)$$

By substituting the derivatives of x and y will be:

$$\mathbf{J}=\begin{bmatrix}2.5 & 0\\ 0 & 1.625\end{bmatrix}$$ (٤-٥١)

$$\begin{Bmatrix}\dfrac{\partial N_i}{\partial x}\\ \dfrac{\partial N_i}{\partial y}\end{Bmatrix}=\mathbf{J}^{-1}\begin{Bmatrix}\dfrac{\partial N_i}{\partial \xi}\\ \dfrac{\partial N_i}{\partial \eta}\end{Bmatrix}$$ (٤-٥٢)

Also from Eq. (٤-٤٩)

Where **J**$^{-١}$ is the inverse of the Jacobean **J** that is [٤٥]:

$$\mathbf{J}^{-1}=\frac{\mathbf{adjJ}}{\mathbf{det\,J}}$$ (٤-٥٣)

Where

$$\mathbf{det\,J=(2.5)(1.625)=4.0625}$$ (٤-٥٤)

So

$$\mathbf{J}^{-1}=\frac{1}{4.0625}\begin{bmatrix}1.625 & 0\\ 0 & 2.5\end{bmatrix}$$ (٤-٥٥)

Therefore, Eq. (٤-٥٢) will become:

$$\begin{Bmatrix} \dfrac{\partial N_i}{\partial x} \\ \dfrac{\partial N_i}{\partial y} \end{Bmatrix} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.615 \end{bmatrix} \begin{Bmatrix} \dfrac{\partial N_i}{\partial \xi} \\ \dfrac{\partial N_i}{\partial \eta} \end{Bmatrix}$$

(٤-٥٦)

The ordering of nodes can be shown in Fig. (٤.٥); the respective derivative quantities due to Eqs. (٤-٤٠) are:

$$\frac{\partial N_1}{\partial \xi} = \frac{1}{4}(-1-\eta) = -0.5$$

$$\frac{\partial N_1}{\partial \eta} = \frac{1}{4}(1-\xi) = 0.5$$

(٤-٥٧a)

At node (١): η = ١, ξ = -١

$$\frac{\partial N_2}{\partial \xi} = \frac{1}{4}(-1+\eta) = -0.5$$

$$\frac{\partial N_2}{\partial \eta} = \frac{1}{4}(-1+\xi) = -0.5$$

(٤-٥٧b)

At node (٢): η = -١, ξ = -١

At node (٣) :η = -١, ξ = ١

$$\frac{\partial N_3}{\partial \xi}=\frac{1}{4}(1-\eta)=0.5$$
$$\frac{\partial N_3}{\partial \eta}=\frac{1}{4}(-1-\xi)=-0.5$$

(٤-٥٧c)

$$\frac{\partial N_4}{\partial \xi}=\frac{1}{4}(1+\eta)=0.5$$
$$\frac{\partial N_4}{\partial \eta}=\frac{1}{4}(1+\xi)=0.5$$

(٤-٥٧d)

At node (٤): **η** = ١, **ξ** = ١

According to Eqs. (٤-٤٢), (٤-٥٦), and (٤-٥٧); the strain displacement matrix **[B]** will be derived as follows whereas:

Strain in x-direction is:

$$\varepsilon_x=\frac{\partial u}{\partial x}=\frac{\partial N}{\partial x}q$$

(٤-٥٨)

$$\varepsilon_x=\begin{bmatrix}\frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x}\end{bmatrix}\begin{Bmatrix}q_1\\ q_3\\ q_5\\ q_7\end{Bmatrix}$$

(٤-٥٩)

And in the same manner, strain in y-direction

$$\varepsilon_y=\frac{\partial v}{\partial y}=\frac{\partial N}{\partial y}q$$

(٤-٦٠)

$$\varepsilon_y=\begin{bmatrix}\frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial y}\end{bmatrix}\begin{Bmatrix}q_2\\ q_4\\ q_6\\ q_8\end{Bmatrix}$$

(٤-٦١)

And the shear

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial N}{\partial y} q + \frac{\partial N}{\partial x} q \qquad (٤\text{-}٦٢)$$

$$\gamma_{xy} = \begin{bmatrix} \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{Bmatrix} \qquad (٤\text{-}٦٣)$$

The matrix **[B]** is a (٣ x ٨) strain-displacement matrix relating the three strains to the eight nodal displacements of the element and is given by:

$$\varepsilon = Bq \qquad (٤\text{-}٦٤)$$

It must be noted that the matrix [B] is constant for all elements.

٤.٣.٣ Finite Element Modeling for Tensile Specimen

For tensile specimen, the two-dimensional linear rectangle element is used. Specifically, rectangle element has four nodes at the corners and each node has two degrees of freedom or (two field variables) represent the vertical and horizontal displacements. So, each element has eight degrees of freedom. According to the tensile specimen that is clearly specified through Fig. (٤.٤). The analysis due to FEM has been limited by the middle region that represented by gauge dimensions (٥٠ mm by ١٣ mm) where the elongation and failure usually occurred. Because of symmetry, half of tensile specimen was discretized where length is ٢٥ mm and width is ١٣ mm. Each element has the following dimensions: $\mathbf{a}_{١}$ = ٥mm, $\mathbf{a}_{٢}$ = ٣.٢٥mm. The nodal numbering is counter clockwise as can also be shown in Fig. (٤.٥).

The displacement components of local node j are taken as $\mathbf{q}_{٢j-١}$ in the x-direction and $\mathbf{q}_{٢j}$ in the y-direction respectively. The global displacement vector

$$\mathbf{Q} = \left[\mathbf{Q}_1, \mathbf{Q}_2, \ldots\ldots, \mathbf{Q}_{\mathbf{NDOF}}\right] \qquad (٤-٦٥)$$

denoted as:

Where NDOF is the total number of degrees of freedom.

Most standard finite element codes use the convention of going around the element in a counterclockwise direction to avoid calculating a negative area [٤٥].

The present study uses the natural coordinates (ξ,η) in the finite element analysis of rectangular element as shown in Fig. (٤.٥). These coordinates are

$$-1 \le \xi \le 1 \text{and} -1 \le \eta \le 1$$

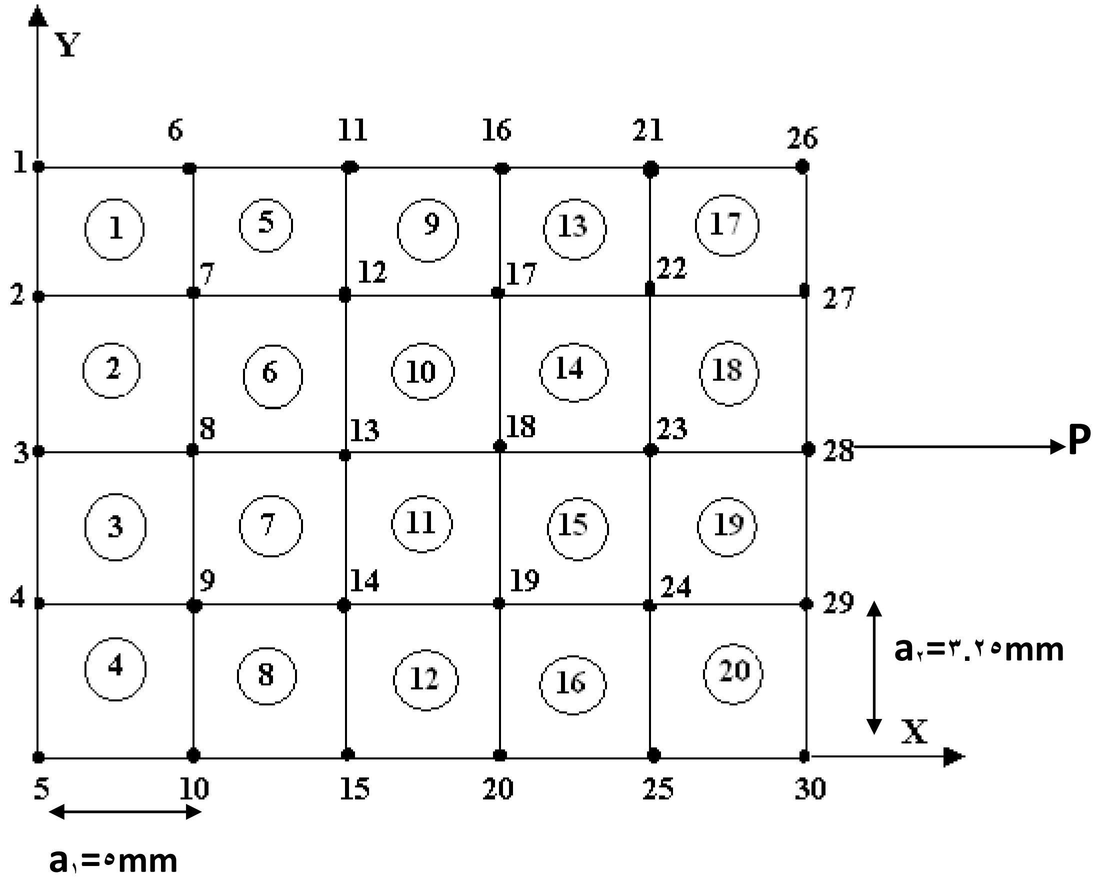


Fig. (٤.٤): Finite element mesh for tensile specimen

located at the center of the element, where:

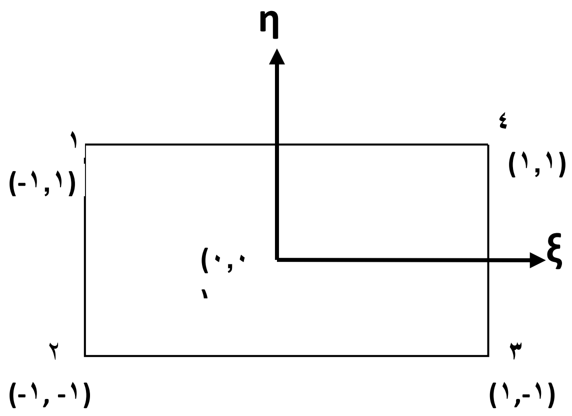


Fig. (٤.٥): A natural coordinate system for the rectangle element. Ref. [٤٨]

٤.٤ Computer Programming

This work concentrates on the theoretical determinations of the mechanical properties of linear viscoelastic polymer and linear viscoelastic polymer reinforced with short fibers. These properties have been investigated numerically by using finite element method and computer programs have been written in quick basic language.

In this study, there are two main computer programs, which are specially designed and each program has several subroutines. The main parts of the computer programs control the reading of the input data, call different subroutines to perform the calculation and print the output. The details of these programs and their subroutines will be discussed in the next section.

٤.٤.١ "QSFEFPC" Program

The "**QSFEFPC**" program is a quasi-static finite element computer program has been designed to solve flexural beam problem of polymer and its composite reinforced with short fibers through the linear viscoelastic region. The procedure of this program can be seen as a flowchart in Fig. (٤.٦).

The function of each subroutine in the computer program "**QSFEFPC**" will be described as follows:

١.**"GENF":** this subroutine is used for generating of local coordinate, global coordinate, element connectivity, and nodal numbering system for the half of flexural specimen, which is simply supported beam. Fig. (٤.٧) denotes the flowchart of this subroutine.

٢.**"STRESS":** this subroutine is used to calculate the flexural stress of a simply supported beam subjected to concentrated load, which varies with position x. The procedure of this subroutine can be shown through Fig. (٤.٨).

٣.**"STRAIN":** the purpose of this subroutine is to determine flexural strain based on the deflection of elastic beam which varies with position x and on the time dependent nodal deflection values that have been obtained from finite element method. At each time step a new set of results will be obtained. For this subroutine, Fig. (٤.٩) can be shown.

٤.**"COMPR":** this subroutine uses the modified rule of mixtures to obtain the time dependent longitudinal modulus of aligned short fiber composite. Fig. (٤.١٠) shows the schematic flowchart of this subroutine.

٥.**"COMPHT":** the function of this subroutine is to compute the time dependent behavior of the longitudinal, transverse, and random modulus of aligned and random short fiber composite respectively due to Halpin-Tsai equation. The procedure of the subroutine can be shown as a flowchart in Fig. (٤.١١).

٤.٤.٢ "QSFETPC" Program

The "**QSFETPC**" program is a quasi-static finite element computer program has been designed to solve tensile problem of polymer and its composite reinforced with short fibers. The procedure of this program is shown through flowchart in Fig. (٤.١٢).

The role of each subroutine in the computer program "**QSFETPC**" will be described as follows:

١. **"GENT١":** the purpose of this subroutine is to generate global and local coordinate, total number of nodes in **x** and **y** direction. Fig. (٤.١٣) refers to the flowchart of this subroutine.

٢. **"GENT٢":** this subroutine generates the descritization of tensile specimen (finite element mesh) and nodal numbering. This subroutine is called at each time step. Fig. (٤.١٤) shows the flowchart of this subroutine.

٣. "**COMPR"** and **"COMPHT"** are mentioned previously as flowcharts in Figs. (٤.١٠) and (٤.١١) respectively.

In order to **RUN** the **"QSFEFPC"** and **"QSFETPC"** programs, the following data must be specified:

١. Externally applied load.
٢. Volume fractions of fiber and matrix.
٣. Aspect ratio and diameter of fiber.
٤. Elastic Poisson’s ratio and elastic modulus of polymer.
٥. Elastic modulus of fiber.

٦. Constants of three-parameter model.

٧. Total number of degrees of freedom.

٨. Dimensions of specimens.

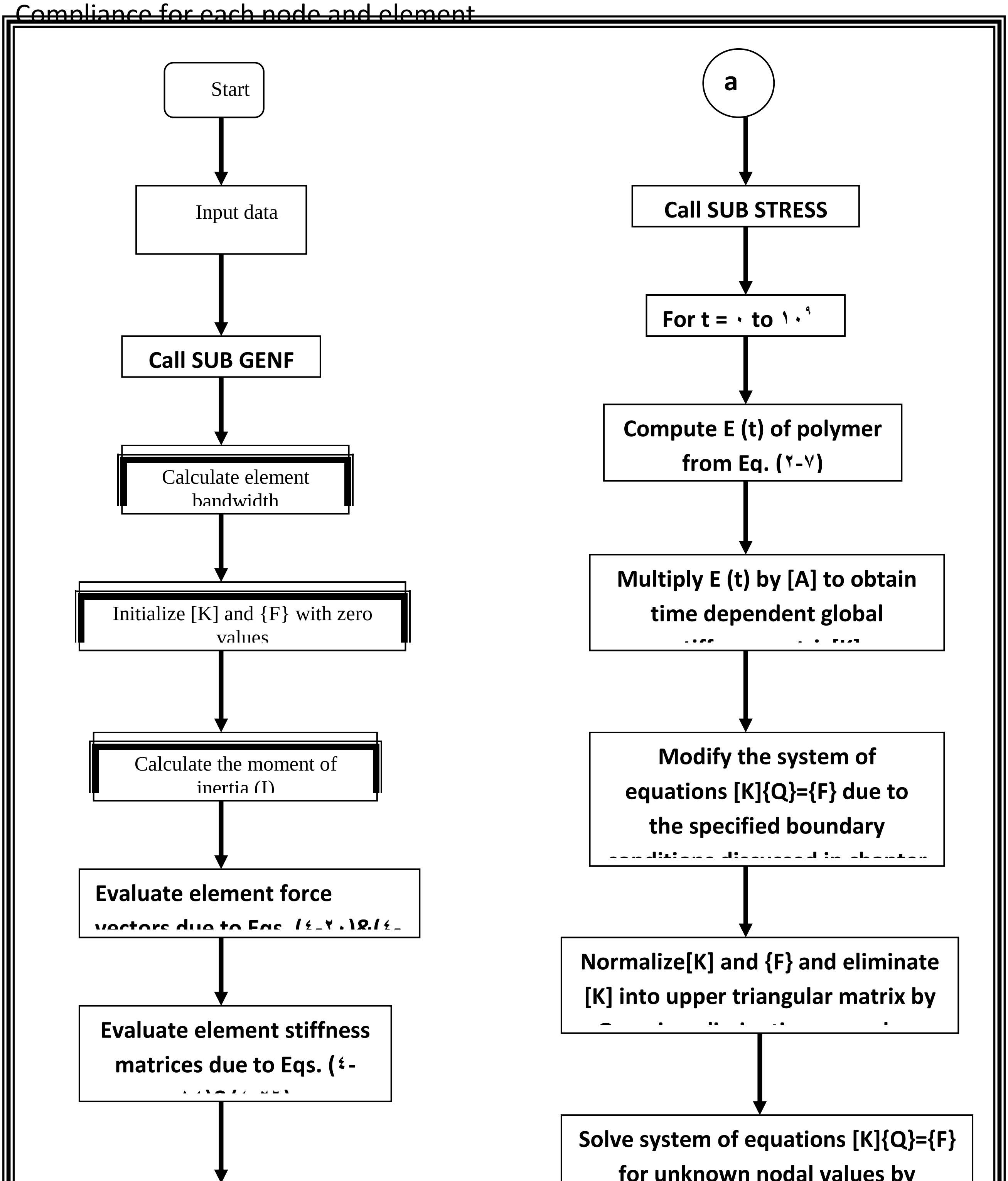
٩. Total number of elements.

The outputs of the " **QSFEFPC**" and "**QSFETPC**" at each time step are:

a- Strain for each node and element.

b- Modulus for each node and element.

c- Compliance for each node and element.



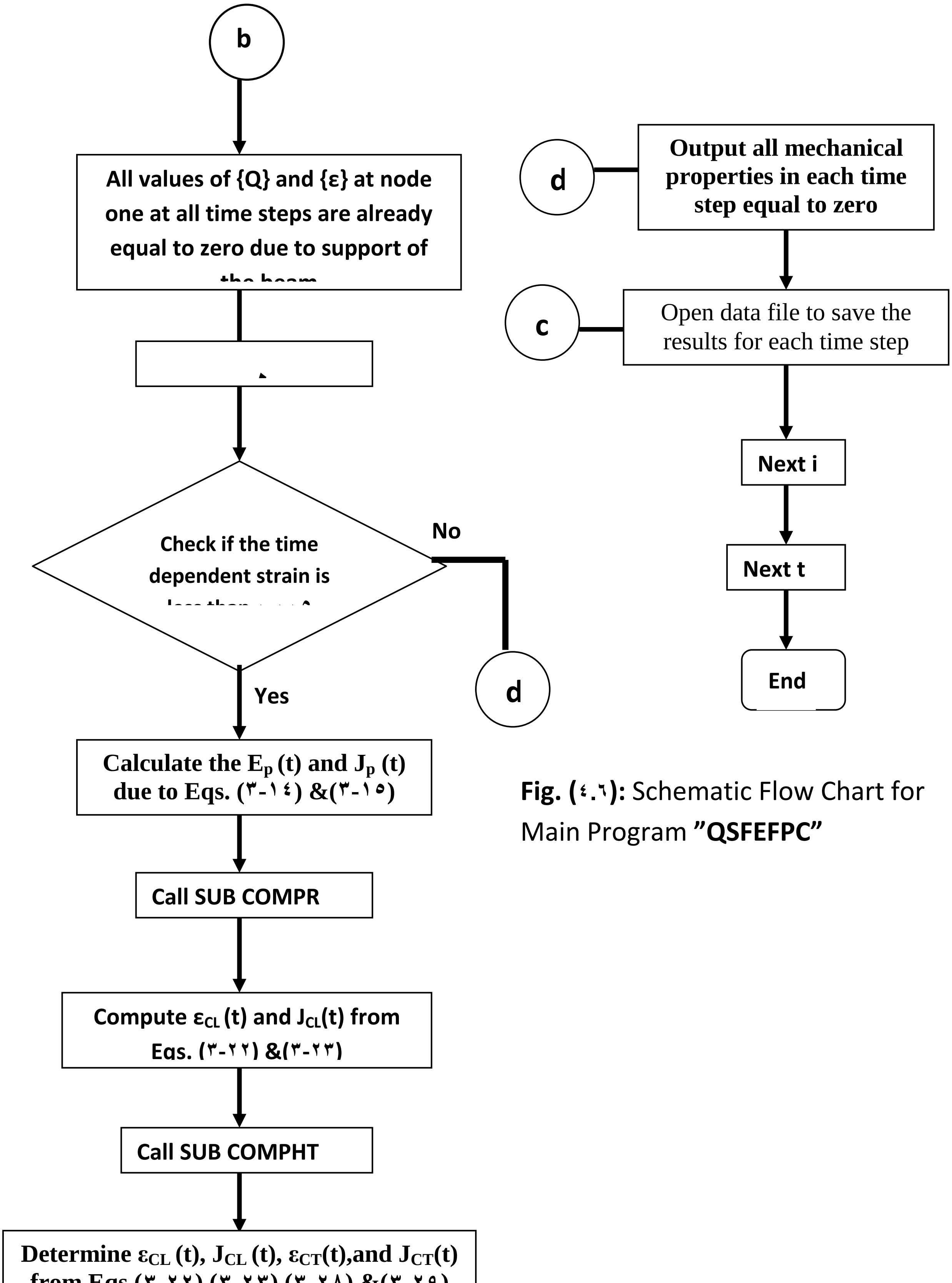
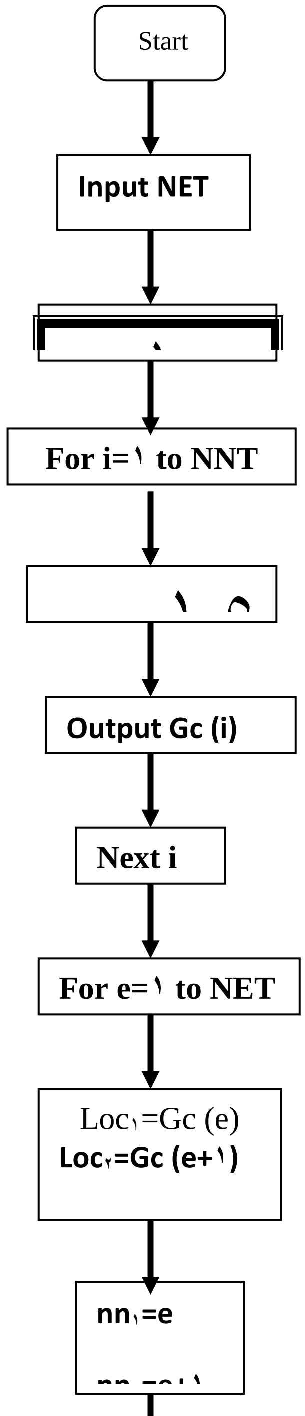
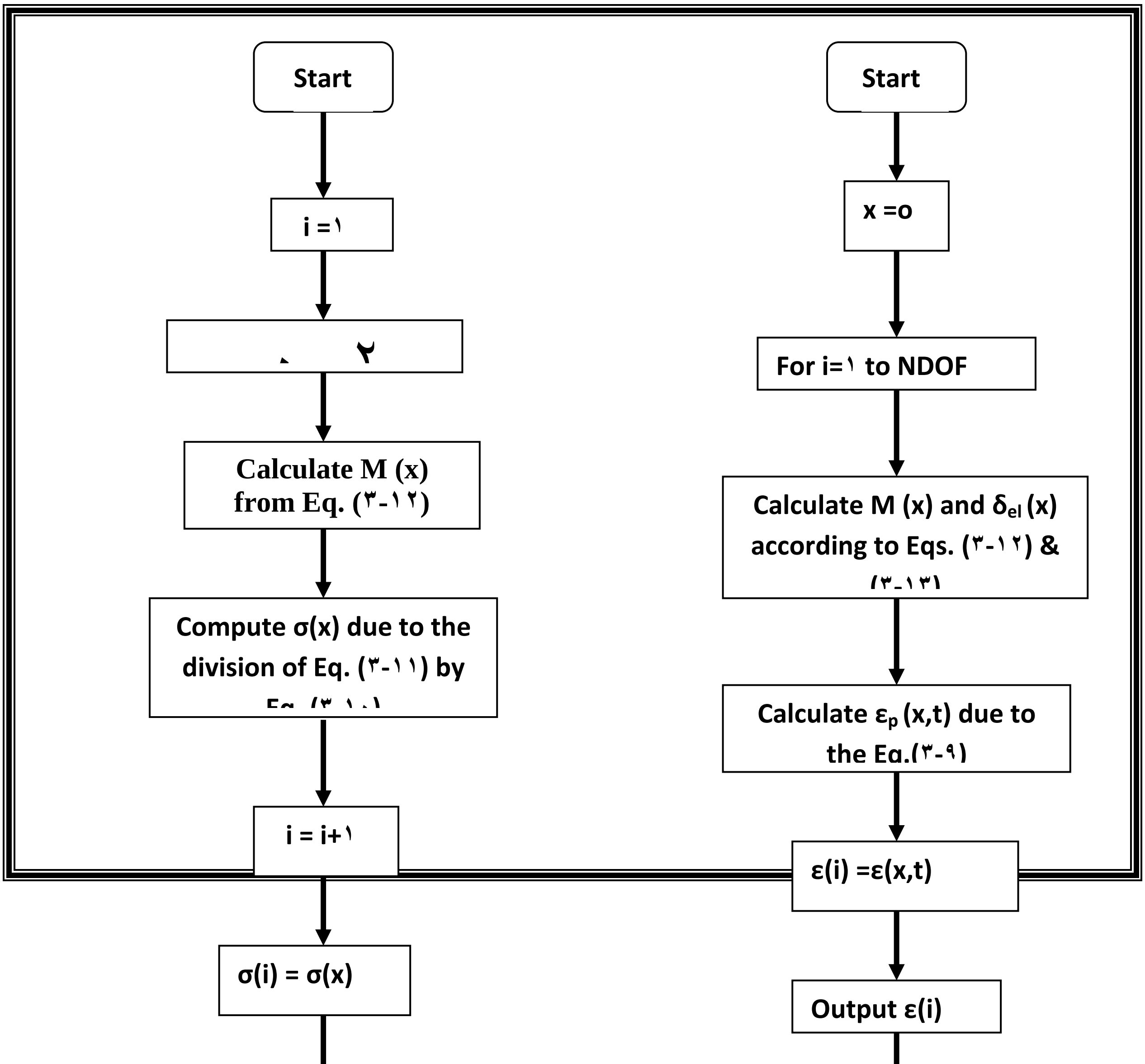
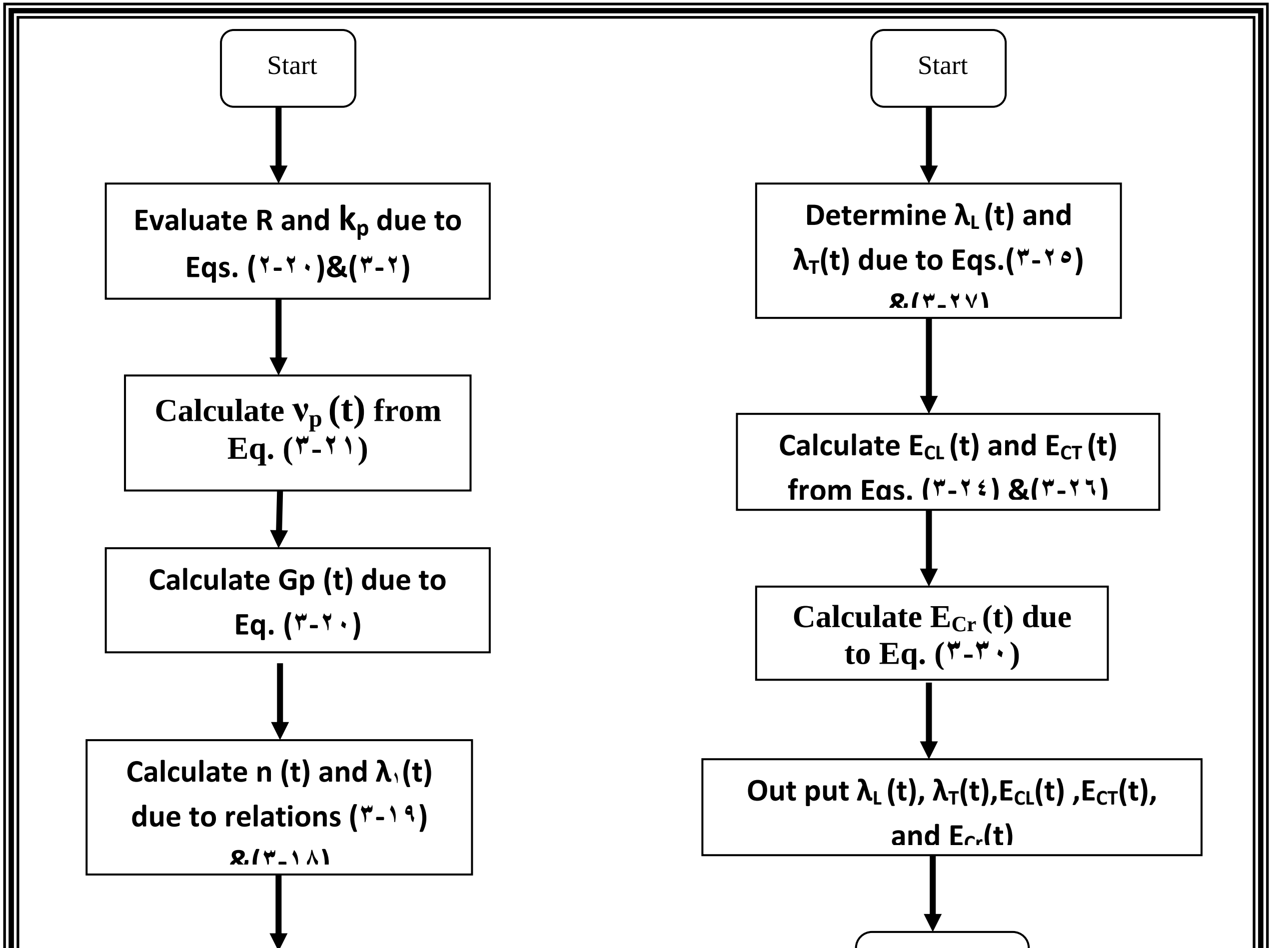
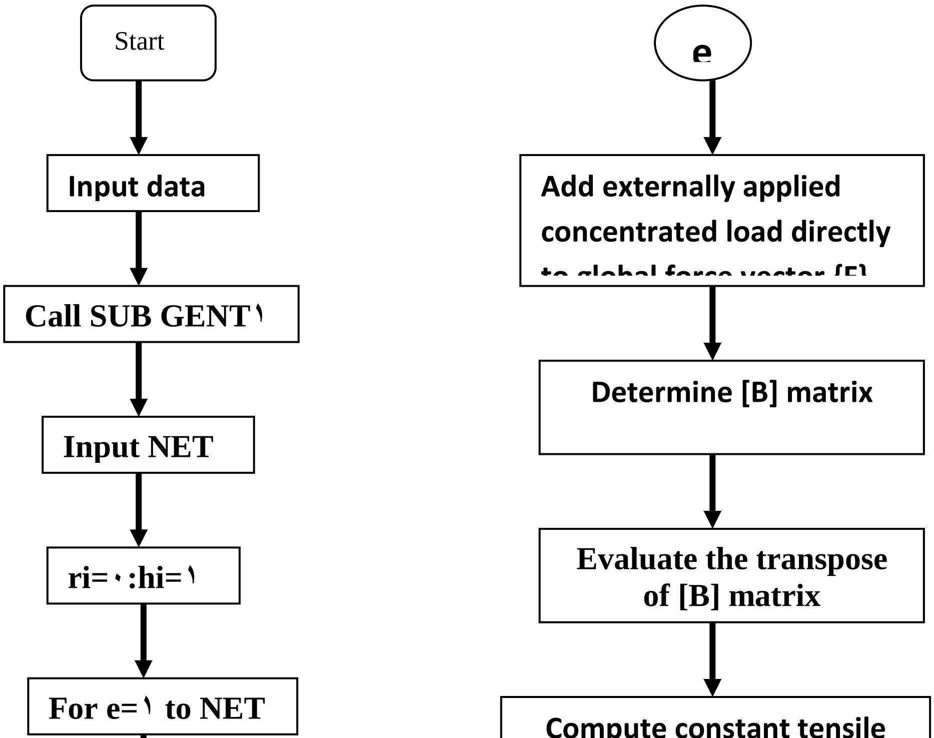


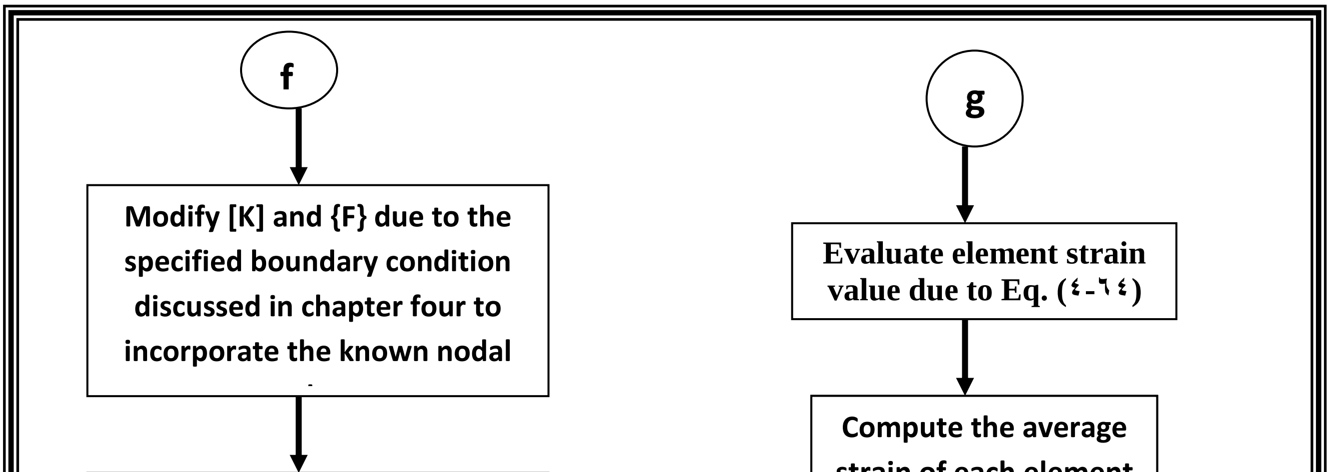
Fig. (٤.٦): Schematic Flow Chart for Main Program **"QSFEFPC"**


Start
Input NET
For i=١ to NNT
Output Gc (i)
Next i
For e=١ to NET
Loc١=Gc (e)
Loc٢=Gc (e+١)
nn١=e


Start
i =١
٢
Calculate M (x) from Eq. (٣-١٢)
Compute σ(x) due to the division of Eq. (٣-١١) by
i = i+١
σ(i) = σ(x)
Start
x =o
For i=١ to NDOF
Calculate M (x) and δel (x) according to Eqs. (٣-١٢) &
Calculate εp (x,t) due to the Eq.(٣-٩)
ε(i) =ε(x,t)
Output ε(i)


Start
Evaluate R and k_p due to Eqs. (٢-٢٠)&(٣-٢)
Calculate ν_p (t) from Eq. (٣-٢١)
Calculate Gp (t) due to Eq. (٣-٢٠)
Calculate n (t) and λ_1(t) due to relations (٣-١٩) &(٣-١٨)
Start
Determine λ_L (t) and λ_T(t) due to Eqs.(٣-٢٥) &(٣-٢٧)
Calculate E_{CL} (t) and E_{CT} (t) from Eqs. (٣-٢٤) &(٣-٢٦)
Calculate E_{Cr} (t) due to Eq. (٣-٣٠)
Out put λ_L (t), λ_T(t), E_{CL}(t), E_{CT}(t), and E_{Cr}(t)


Start
Input data
Call SUB GENT١
Input NET
ri=٠:hi=١
For e=١ to NET
e
Add externally applied concentrated load directly to global force vector [F]
Determine [B] matrix
Evaluate the transpose of [B] matrix
Compute constant tensile


f
Modify [K] and {F} due to the specified boundary condition discussed in chapter four to incorporate the known nodal
g
Evaluate element strain value due to Eq. (٤-٦٤)
Compute the average strain of each element

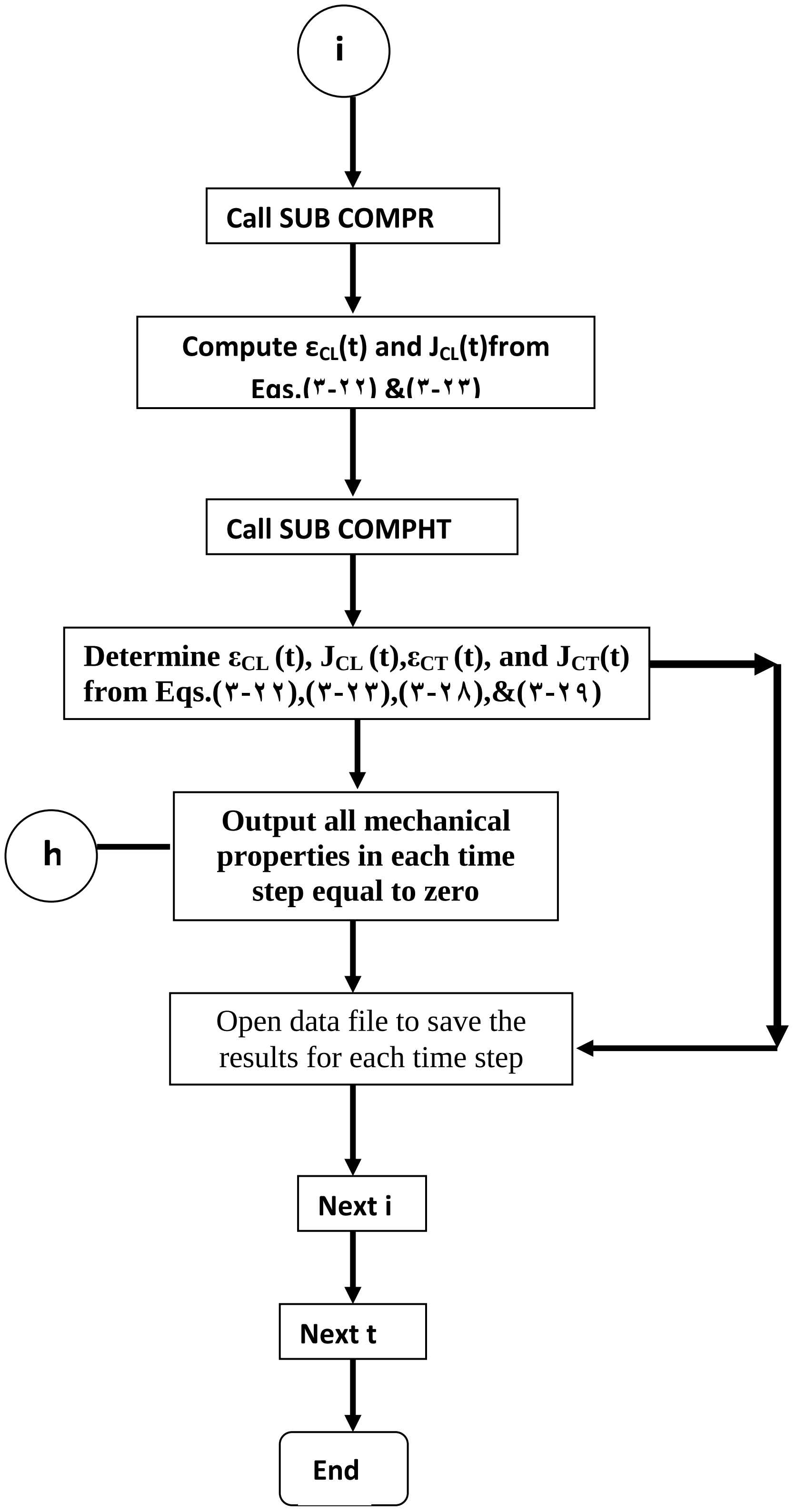
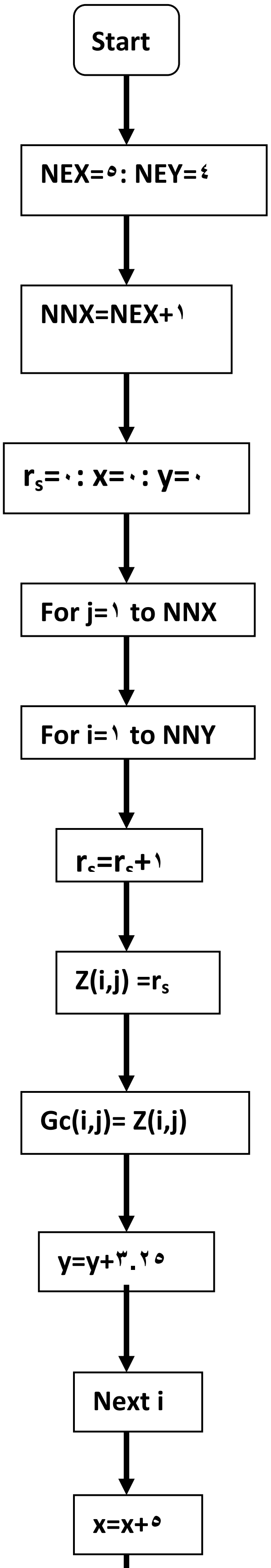


Fig. (٤.١٢): Schematic Flow Chart for Main Program **“QSFETPC”**



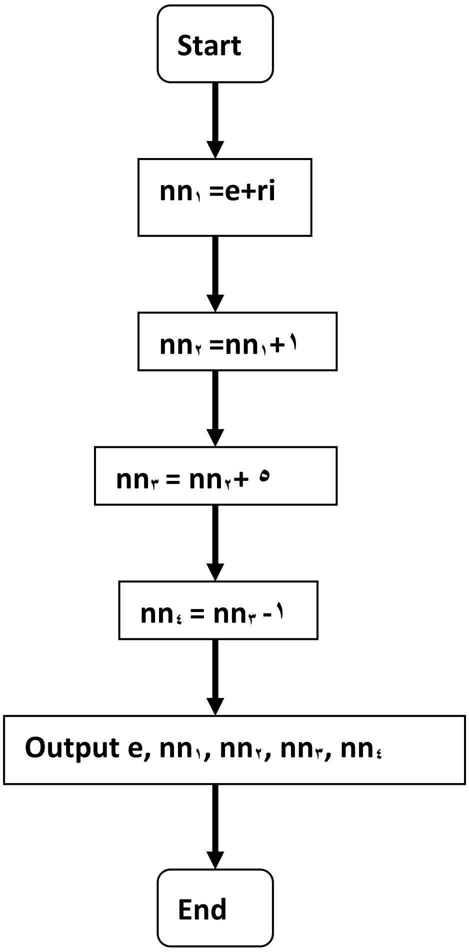


Fig. (٤.١٤): GENT٢ SUB

CHAPTER FIVE

Results and Discussion

٥.١ General

This chapter presents and discusses the results that have been obtained by using the special designed two computer programs. Firstly: "**QSFEFPC**" represents the quasi-static finite element program of flexural estimation on polymer and its composite reinforced with short fiber. Secondly: "**QSFETPC**" represents quasi-static finite element program for tensile estimation on polymer and its composite.

Materials that have been chosen as a case study are polypropylene polymer, polypropylene reinforced with aligned short E-glass fibers, and polypropylene reinforced with random short E-glass fibers.

The modified rule of mixtures as a mechanics of materials approach has been applied on aligned composite material while Halpin-Tsai equation as an elasticity approach has been applied on aligned and random composite materials.

The principle of time-temperature equivalence has been applied on the mechanical properties (strain, modulus, and compliance) for both polymer and PMC.

The following data have been considered in the present analysis:

- Volume fraction, **V_f** = ٠.٢.
- Aspect ratio, **a =** ٤٠ [٥١].
- Fiber diameter, **d =** ١٤ μm [٣٩].
- Elastic modulus of E-glass fiber, **E_f** = ٧٠ GPa [٥٢].
- Tensile modulus of polypropylene, **E_p** = ١.٢ GPa [٥٣].
- Flexural modulus of polypropylene, **E_b** = ١.٤ GPa [٥٤].
- Poisson's ratio of polypropylene, **ν_p** = ٠.٣ [٢٨].
- Externally applied load in tensile test, **P =** ٥٠ KN.
- Concentrated load applied for flexural test, **P =** ٢٥٠ N.

The values of externally applied loads have been chosen where there are excess over the linear viscoelastic limits to show the estimations that have been carried out in the present work.

٥.٢ Flexural Creep Estimation

In this test, a concentrated load of ٢٥٠ N, an aspect ratio of ٤٠, a volume fraction of ٠.٢, and a period of time between (١-$١٠^{٩}$) seconds have been considered. In general the results show the typical curves for polymer and **PMC** of strain and compliance as increasing functions of time as can be seen from Figs. (٢.٢) & (٢.٦) respectively and the curves of modulus as decreasing functions of time as can be seen from Fig. (٢.٧).

٥.٢.١ Flexural Creep of Polypropylene

- Flexural Creep Strain

Fig. (٥.١) shows the time dependent flexural strain of polypropylene polymer specimen from node ٢ to node ١٤ according to Eq. (٣-١١). The closer nodes to the applied load have larger strains whereas strain increases with time starting from node ٢ to node ١٤. Node ١٤ is the location of applied load and maximum deflection in the beam. Maximum strain of node ١٣ is ٤.٥x١٠$^{-٣}$.

- Flexural Creep Modulus

Fig. (٥.٢) refers to the effect of time on flexural modulus of polypropylene polymer due to Eq. (٣-١٤). Flexural modulus have been decreased with time beginning from node ٢ to node ١٤. Minimum modulus for node ١٣ is ٢٥ GPa.

The flexural modulus of polypropylene is low due to the basic characteristic of polymers that represented with low stiffness.

- Flexural Creep Compliance

Fig. (٥.٣) represents the time dependent flexural compliance of polypropylene that is obtained from Eq. (٣-١٥). The flexural compliance values along the specimen are very close to each other. The maximum compliance is observed at node ١٣ and equal to **٤.٢x١٠**$^{-٢}$ GPa$^{-١}$.

The increase in strain and compliance curves and the decrease in modulus curves against time have been corresponded to the typical curves whereas the principal of time-temperature equivalence is applicable for polymer and its composites.

٥.٢.٢ Longitudinal Flexural Creep of Aligned Short E-Glass/PP Composite (Halpin-Tsai Equation).

- Flexural Creep Strain

Fig. (٥.٤) is denoting the time dependence of longitudinal flexural strain for aligned short E-glass fiber reinforced polypropylene due to Halpin-Tsai equation (Eq. (٣-٢٢)). Strain will increase with the closing to the mid-point of the specimen where the concentrated load has been applied.

- Flexural Creep Modulus

Fig. (٥.٥) denotes the effect of time on longitudinal flexural modulus of aligned short E-glass fiber reinforcing polypropylene according to Halpin-Tsai equation (Eq. (٣-٢٤)). For all nodes (٢-١٤), flexural modulus curves are decreasing with time.

- Flexural Creep Compliance

Fig. (٥.٦) shows the effect of time on longitudinal flexural compliance for short E-glass fibers reinforcing polypropylene according to Halpin-Tsai equation (Eq. (٣-٢٣)).

Maximum values of longitudinal flexural properties have been obtained at node ١٣. Maximum strain is equal to **٠.٢١x١٠**$^{-٣}$, minimum modulus is **٥٠٠** GPa, and maximum compliance is equal to **٠.١٩٨x١٠**$^{-٢}$.

These values have been shown an improvement in PMC properties and this has been reflected by the clarity of reinforcement effect. This improvement in PMC properties proves the accuracy and success of Halpin-Tsai equation.

٥.٢.٣ Transverse Flexural Creep of Aligned Short E-Glass/PP Composite (Halpin-Tsai Equation).

- Flexural Creep Strain

Fig. (٥.٧) refers to the time dependent flexural strain of aligned short E-glass reinforcing polypropylene in transverse direction according to Halpin-Tsai equation (Eq. (٣-٢٨)).

- Flexural Creep Modulus

Fig. (٥.٨) represents the time dependence of transverse flexural modulus of aligned short E-glass fibers reinforcing polypropylene that obtained from Halpin-Tsai equation through Eq. (٣-٢٦).

- Flexural Creep Compliance

Fig. (٥.٩) shows the effect of time on transverse flexural compliance of aligned composite material that consists of short E-glass fiber in polypropylene matrix. The method used is Halpin-Tsai equation that has been represented by Eq. (٣-٢٩).

Maximum transverse mechanical properties at node ١٣ are: $٢.٥x١٠^{-٣}$ for strain, ٥٠ GPa for modulus, and $٢.٤x١٠^{-٢}$ $GPa^{-١}$ for compliance. Polypropylene polymer results are: $٤.٥x١٠^{-٣}$ for strain, ٢٥ GPa for modulus, and $٤.٢x١٠^{-٢}$ $GPa^{-١}$ for compliance.

The comparison between these properties shows that there is a good improvement in lateral stiffness values and decrease in strain and compliance values.

In general, the lateral mechanical properties are very close to the results of PP polymer alone because the lateral properties of aligned composite are matrix dominated.

٥.٢.٤ Longitudinal Flexural Creep of Aligned Short E-Glass/PP Composite (Modified Rule of Mixtures)

- Flexural Creep Strain

Fig. (٥.١٠) gives the time dependent longitudinal flexural strain of aligned short E-glass fiber reinforcing polypropylene by applying modified rule of mixtures (Eq. (٣-٢٢)).

- Flexural Creep Modulus

Fig. (٥.١١) shows the effect of time on longitudinal flexural modulus of aligned short E-glass fiber reinforced polypropylene by using the modified rule of mixtures (Eq. (٣-١٧)).

- Flexural Creep Compliance

Fig. (٥.١٢) refers to the effect of time on longitudinal flexural compliance of aligned composite material consists of short E-glass fiber reinforced polypropylene polymer due to Eq. (٣-٢٣).

Maximum longitudinal properties of (strain, modulus, and compliance) respectively are ٣.٢٥x١٠$^{-٣}$, ٣١GPa, ٣.١x١٠$^{-٢}$ GPa$^{-١}$. PP polymer properties (strain, modulus, compliance) are ٤.٥x١٠$^{-٣}$, ٢٥**GPa**, ٤.٢x١٠$^{-٢}$ GPa$^{-١}$.

It is clear that the deformation of aligned composite is less than the deformation of polymer but very near to it. On the other hand, there is little improvement in stiffness and decrease in strain and compliance.

The rule of mixtures gives accurate results of composite reinforced with aligned continuous fibers. According to the present study, the modified rule of mixtures is less accurate in predicting of the mechanical properties of composite reinforced with aligned short fibers.

This behavior has been attributed to the slight effect of reinforcement where the aligned composite properties are very near to the polymer properties.

٥.٢.٥ Flexural Creep of Random Short E-Glass/PP Composite (Halpin-Tsai Equation)

- Flexural Creep Strain

Fig. (٥.١٣) represents the flexural creep strain versus time for random short E-glass reinforced polypropylene by applying Halpin-Tsai equation (Eq. (٣-٣١)).

- Flexural Creep Modulus

Fig. (٥.١٤) shows the effect of time on flexural creep modulus of random short E-glass reinforced polypropylene obtained from Halpin-Tsai equation (Eq. (٣-٣٠)).

- Flexural Creep Compliance

Fig. (٥.١٥) shows the time dependent flexural creep compliance of random short E-glass reinforced polypropylene by the using of Halpin-Tsai equation (Eq. (٣-٣٢)).

The random short E-Glass/PP composite properties are: ٠.٤٩x١٠$^{-٣}$ for strain, ٢٥ GPa for modulus, and ٠.٤٧x١٠$^{-٢}$ GPa$^{-١}$ for compliance. The properties of PP polymer have been stated as ٤.٥x١٠$^{-٣}$ for strain, ٢٥ GPa for modulus, and ٤.٢x١٠$^{-٢}$ for compliance.

By comparing these results it can be concluded that the effect of reinforcement in random case is small with respect to polymer alone. The stiffness values are the same without improvement while strain and compliance values are lesser than PP polymer. Random composite material is isotropic and its mechanical properties are equal in all directions.

Maximum values of strain and compliance and minimum values of modulus have been obtained at **t =** $١٠^{٩}$ seconds. The values of all the mechanical properties understudy have been found out of the linear viscoelastic limits at node ١٤.

There is a checking has been made through the computer programs. This checking states that if there is excess over the linear viscoelastic limits (**ε >** ٠.٥٪), the mechanical property will decrease to zero due to the present study that is limited with linear viscoelastic behavior.

There is constancy in flexural curves for (strain, modulus, and compliance) during the periods ($١-١٠^{٦}$, $١-١٠^{٥}$, $١-١٠^{٦}$) seconds. These regions have been represented the regions of glassy behavior of PP polymer and PP reinforced with short E-glass fibers in its two forms aligned and random.

٥.٣ Tensile Creep Estimation

In this test, an external applied point load of ٥٠ KN, an aspect ratio of ٤٠, and a volume fraction of ٠.٢ are considered. The period of time is between ($١-١٠^{٩}$) seconds. The typical curves of polymer and **PMC** for strain and compliance are increasing with time as in Figs. (٢.٢) & (٢.٦) while the typical curves of modulus are decreasing functions of time as can be seen from Fig. (٢.٧). So, the present results are identical to the typical curves.

٥.٣.١ Tensile Creep of Polypropylene

- Tensile Creep Strain

Fig. (٥.١٦) denotes the effect of time on tensile strain of polypropylene for half of tensile specimen which descretized into ٢٠ elements using Eq. (٤-٦٤).

At t > ١٠^{٦} seconds, the tensile strain increases in a positive direction for some elements with a maximum value of **٣.٨x١٠**$^{-٣}$ at node number ١٦. Tensile strain increases in a negative direction for the other elements with a maximum value of –**١.٩٥x١٠**$^{-٣}$ at node number ١٨.

- Tensile Creep Modulus

Fig. (٥.١٧) concentrates on the time dependent tensile modulus of polypropylene that obtained from Eq. (٣-١٤).

The tensile moduli of a number of elements have been decreased in the positive part and a minimum value is equal to ٥٠٠ GPa at elements (١,٣,٦,٨). Other number of elements has tensile modulus curves in the negative part with a minimum value of –١٠٠٠ GPa at element (٤,١٨).

Because of the connection between strain and modulus, negative values of modulus have been denoted the lateral contraction which relates with the longitudinal extension.

For element (١٠), the tensile modulus is a decreasing function in the negative part. At **t =** ١٠^{٩} seconds, the greatest modulus of element ١٠ for unreinforced polypropylene is equal to ٧٠٠٠٠ GPa. The reason is that element ١٠ has been located in the middle region of tensile specimen; therefore, its deformation will be less than the deformation of elements that have been located around it, so element ١٠ is undergoing small deformation. The dependence of modulus calculations on strain is the reason behind the high values of stiffness of this element in all cases understudy.

- Tensile Creep Compliance

Fig. (٥.١٨) gives the effect of time on tensile compliance of polypropylene polymer according to Eq. (٣-١٥).

Some of the tensile compliances have been increased in the positive part while the other increased in the negative part. The maximum positive compliance of element ١٦ equals to $٠.٣٢x١٠^{-٤}$ GPa$^{-١}$ and the maximum negative compliance of element ١٨ is equal to $-٠.١٦x١٠^{-٤}$ GPa$^{-١}$.

The negative compliances have been related with negative moduli that are attributed to the happening of lateral contraction coincident with the longitudinal extension according to tensile creep estimation.

The principle of time-temperature equivalence has been applied through the present work. The strain and compliance curves have been increased against time while the modulus curves have been decreased versus time according to the typical case.

٥.٣.٢ Longitudinal Tensile Creep of Aligned Short E-Glass/PP Composite (Halpin-Tsai Equation).

- Tensile Creep Strain

Fig. (٥.١٩) denotes the time dependent longitudinal tensile strain of aligned short E-glass fiber reinforcing polypropylene matrix by applying Halpin-Tsai equation according to Eq. (٣-٢٢).

The tensile strain curves of several elements have been located in the positive part with a largest value of $٠.١٧٥x١٠^{-٣}$ at element ١٦. The tensile strain curves of the other elements have been located in the negative part with a smallest value of $-٠.٠٩x١٠^{-٣}$ at element ١٨.

It must be noted that the strains of aligned composite in longitudinal direction are less than the polymer alone whereas the reinforcement with fibers has been restricted the mobility and consequently the deformation of polymer.

- Tensile Creep Modulus

Fig. (٥.٢٠) determines the time dependent longitudinal tensile modulus of aligned composite consists of short E-glass fiber reinforced polypropylene by using Halpin-Tsai equation according to Eq. (٣-٢٤). The smallest positive modulus of elements (١,٣) is equal to ١٠٠٠٠ GPa while the smallest negative modulus of elements (٤,١٨) is –٢٠٠٠٠ GPa.

The tensile modulus of element ١٠, which is located in the middle region of tensile specimen, is the largest modulus in longitudinal direction, which is equal to ١٤٨٠٠٠٠ GPa and its value has been attributed to the very small strain corresponded to the same element.

- Tensile Creep Compliance

Fig. (٥.٢١) represents the effect of time on longitudinal tensile compliance of aligned short E-glass fiber reinforced polypropylene. The method used is Halpin-Tsai equation by applying Eq. (٣-٢٣).

Maximum positive compliance of ٠.٠١٥**x**$١٠^{-٢}$ GPa$^{-١}$ has been obtained at element ١٦ and maximum negative compliance of – ٠.٠٠٧٦**x**$١٠^{-٢}$ GPa$^{-١}$ has been obtained at element ١٨.

The previous results have been proven an improvement in stiffness and decrease in strain and compliance for PMC as compared with the polymer results.

So, Halpin-Tsai equation is accurate method in predicting of the longitudinal properties of composite material reinforced with short fibers whereas the reinforcement is effective in the resulting properties.

The elements (١٦,١٩) have the greatest positive values for both strains and compliances because of their location that is very near to the applied load. The moduli of these elements are constants and very near to zero.

The elements (١٣,١٨) have the greatest negative values of strains and compliances because of their location that is very near to the applied load.

٥.٣.٣ Transverse Tensile Creep of Aligned Short E-Glass/PP Composite (Halpin-Tsai Equation).

- Tensile Creep Strain

Fig. (٥.٢٢) shows the time dependent transverse tensile strain of aligned composite using Eq. (٣-٢٨). The constituents are aligned short E-glass fiber reinforced polypropylene matrix.

Maximum positive transverse strain at element ١٦ is equal to **٢.٢٥x١٠**$^{-٣}$ while maximum negative transverse strain at element ١٨ is equal to **–١.١٥x١٠**$^{-٣}$. For comparison, PP polymer results have been represented by maximum positive transverse strain at element ١٦ that is equal to **٣.٨x١٠**$^{-٣}$ and maximum negative transverse strain at element ١٨ is equal to **–١.٩٥x١٠**$^{-٣}$.

- Tensile Creep Modulus

Fig. (٥.٢٣) represents the effect of limited period of time on the transverse tensile modulus of elements (١-٢٠) according to (Eq. (٣-٢٦)).

The tensile modulus curves in the positive part have a minimum value is equal to ١٠٠٠ GPa at elements (١,٣,٦,٨). On the other hand, the tensile modulus curves in the negative part have a minimum value is equal to –١٠٠٠ GPa at element ١٨.

The stiffness has inverse relation with strain. So, element ١٠ gives the highest tensile stiffness that is equal to ١٢٥٠٠٠ GPa.

For PP polymer, minimum positive transverse modulus is equal to ٥٠٠ GPa at the elements (١,٣,٦,٨) while minimum negative transverse modulus is equal to –١٠٠٠ GPa at element ١٨.

On the other hand, PP polymer has a tensile modulus of element ١٠ that is equal to ٧٠٠٠٠ GPa.

- Tensile Creep Compliance

Fig. (٥.٢٤) concentrates on the time dependent transverse tensile compliance by applying Halpin-Tsai equation (Eq. (٣-٢٩)).

The tensile compliances of several elements have been located in the positive part and maximum compliance of element ١٦ equals to **٠.١٨x١٠**$^{-٢}$ GPa$^{-١}$ while the tensile compliances of other elements have been located in the negative part and maximum compliance of element ١٨ is equal to **– ٠.٠٩٥x١٠**$^{-٢}$ GPa$^{-١}$.

For PP polymer, maximum positive compliance is equal to **٠.٣٢x١٠**$^{-٢}$ at element ١٦ while maximum negative compliance is equal to **– ٠.١٦x١٠**$^{-٢}$.

From Halpin-Tsai equation, it is clear that there is slight improvement in the mechanical properties of PMC in transverse direction. The transverse properties of PMC are very near to the properties of unreinforced polymer. The deformation of polymer in transverse direction with respect to the aligned fibers dominates the overall deformation of composite.

٥.٣.٤ Longitudinal Tensile Creep of Aligned Short E-Glass/PP Composite (Modified Rule of Mixtures)

- Tensile Creep Strain

Fig. (٥.٢٥) refers to the effect of time on longitudinal tensile strain of aligned short E-glass fiber reinforcing polypropylene matrix throughout Eq. (٣-٢٢).

The maximum positive strain has been obtained from element ١٦ that is equal to **٤.٥x١٠**$^{-٣}$ and maximum negative strain is equal to –**٢.٥x١٠**$^{-٣}$ at element ١٨.

For PP polymer, maximum positive tensile strain at element ١٦ is **٣.٨x١٠**$^{-٣}$ while maximum negative tensile strain at element ١٨ is equal to –**١.٩٥x١٠**$^{-٣}$.

- Tensile Creep Modulus

Fig. (٥.٢٦) concentrates on studying the longitudinal tensile modulus of aligned composite against time due to Eq. (٣-١٧).

The smallest positive modulus is ٥٠٠ GPa and has been located at elements (١,٣,٦,٨) while the smallest negative modulus has been obtained at elements (٤,٥,١٨) and is equal to –١٠٠٠ GPa. The greatest modulus that can be obtained from modified rule of mixtures is equal to ٥٧٠٠٠ GPa at element ١٠.

For PP polymer, minimum positive modulus is equal to ٥٠٠ GPa at the elements (١,٣,٦,٨) while minimum negative modulus is equal to –١٠٠٠ GPa at elements (٤,١٨). On the other hand, maximum modulus at element ١٠ is equal to ٧٠٠٠٠ GPa.

- Tensile Creep Compliance

Fig. (٥.٢٧) represents the effect of time on longitudinal tensile compliance curves of aligned short E-glass fiber reinforced polypropylene by applying modified rule of mixtures due to Eq. (٣-٢٣). The largest compliance is equal to **٠.٣٨x١٠**$^{-٢}$ GPa$^{-١}$ for element ١٦ and the greatest compliance is equal to –**٠.٢x١٠**$^{-٣}$ GPa$^{-١}$ for element ١٨. For unreinforced PP polymer, maximum positive compliance has been obtained at element ١٦ is equal to **٠.٣٢x١٠**$^{-٢}$ GPa$^{-١}$ while maximum negative compliance has been obtained at element ١٨ is equal to **–٠.١٦x١٠**$^{-٢}$ GPa$^{-١}$.

The comparison between the mechanical properties of polymer and PMC according to modified rule of mixtures shows a decrease in stiffness values that must be increased and constancy in compliance values while the strain values of PMC is greater than the unreinforced polymer.

This behavior is wrong and means that there is no effect of reinforcement and consequently the modified rule of mixtures is inaccurate in predicting of the longitudinal properties of aligned short fiber composite.

The inaccuracy of modified rule of mixtures to predict the mechanical properties that have been obtained from ٢-D tensile test is very clear as comparing with the mechanical properties that have been obtained from ١-D flexural test.

٥.٣.٥ Tensile Creep of Random Short E-Glass/PP Composite (Halpin-Tsai Equation)

- Tensile Creep Strain

Fig. (٥.٢٨) denotes the effect of time on tensile strain curves of all elements for a specimen of a random short E-glass reinforced polypropylene. The method used is Halpin-Tsai equation through Eq. (٣-٣١).

The greatest positive strain is equal to **٠.٥٥x١٠**$^{-٣}$ at element ١٦ while the maximum negative strain is equal to **– ٠.١٧٥x١٠**$^{-٣}$ has been obtained at element ١٨.

The properties of PP polymer have been stated as maximum positive strain at element ١٦ is equal to **٣.٨x١٠**$^{-٣}$ while maximum negative strain at element ١٨ is equal to **–١.٩٥x١٠**$^{-٣}$.

- Tensile Creep Modulus

Fig. (٥.٢٩) shows the effect of time on tensile modulus of random short E-glass reinforced polypropylene for all elements according to Halpin-Tsai equation (Eq. (٣-٣٠)).

From tensile test, the smallest positive modulus of elements (١,٣,٦,٨) is equal to ٥٠٠٠ GPa while the smallest negative modulus of elements (٤,١٨) is equal to -١٠٠٠٠ GPa. The greatest modulus of random short fiber composite equals to ٦٢٥٠٠٠ GPa can be obtained at element ١٠.

PP polymer results are the smallest positive modulus is equal to ٥٠٠ GPa at elements (١,٣,٦,٨) while the smallest negative modulus is equal to –١٠٠٠ GPa at element ١٨. On the other hand, maximum positive modulus for polymer has been obtained at element ١٠ and equal to ٧٠٠٠٠ GPa.

- Tensile Creep Compliance

Fig. (٥.٣٠) shows the time dependent tensile compliance of random short E-glass fibers reinforced polypropylene for all elements determined according to Halpin-Tsai equation (Eq. (٣-٣٢)).

The maximum positive compliance is **٠.٠٣٥x**$١٠^{-٢}$ GPa$^{-١}$ for element ١٦ while the maximum negative compliance equals to **–٠.٠١٧٥x**$١٠^{-٢}$ GPa$^{-١}$ at element ١٨

For PP polymer, maximum positive compliance has been obtained at element ١٦ is equal to **٠.٣٢x**$١٠^{-٢}$ GPa$^{-١}$ while the negative compliance is equal to **– ٠.١٦x**$١٠^{-٢}$ GPa$^{-١}$ at element ١٨.

The above results have been shown good effectiveness of the reinforcing fibers in random case where the material is isotropic and the mechanical properties are equal in all directions.

It can be noted that an improvement in stiffness values of random short fiber PMC and decrease in deformation that has been reflected through strain and compliance values of PMC where the presence of reinforcement will restrict the deformation of polymer.

The sided elements (٨,١٢,١٦,٢٠) and (٥,٩,١٣,١٧) have horizontal displacements in x direction while vertical displacements are equal to zero; their deformation is high against low stiffness.

The other sided elements (١,٢,٣,٤) have vertical displacements in y direction while horizontal displacements are equal to zero; these elements have high deformation versus low modulus.

The elements in the middle region of tensile specimen around element ١٠ have displacements in x and y directions. They have amount of deformation less than the sided elements and greater than element ١٠ at the expense of stiffness where element ١٠ has the greatest modulus in tensile specimen. These elements are (٦,٧,١١,١٤,١٥).

The maximum and minimum values of the parameters understudy have been obtained at **t=**١٠^{٩} seconds. In element ١٩ for all materials understudy, the mechanical properties (strain, modulus, and compliance) have been found out of the linear viscoelastic limits at a period of time between (١٠^{٨}-١٠^{٩}) seconds.

The tensile strain and compliance curves are constants during the time periods (١-١٠^{٦}) and (١٠-١٠^{٦}) seconds while the tensile modulus curves are time independent through the periods (١٠-١٠^{٥}) and (١٠-١٠^{٦}) seconds. These regions have been represented the regions of glassy behavior of PP polymer and PP reinforced with short E-glass fibers in its two forms aligned and random.

The results that have been obtained from tensile test for all the mechanical properties (strain, modulus, and compliance) are with positive and negative signs. Positive values of several elements are denoting to the happening of extension in the applied load direction while negative values of other elements are referring to contraction in the lateral direction with respect to the direction of applied load.

The present strain results that have been obtained from flexural creep and tensile creep estimations for polypropylene polymer are coincident with the strain results that have been obtained by **Rollason, E. C.** [٧], **Resen, A. S.; *et al.*** [٣٨], and **Resen, A. S.; *et al.*** [٢٤]. This reflects the real and accurate results of strain as well as modulus and compliance results that are depending on the strains; this behavior can be observed from Fig. (٥.٣١).

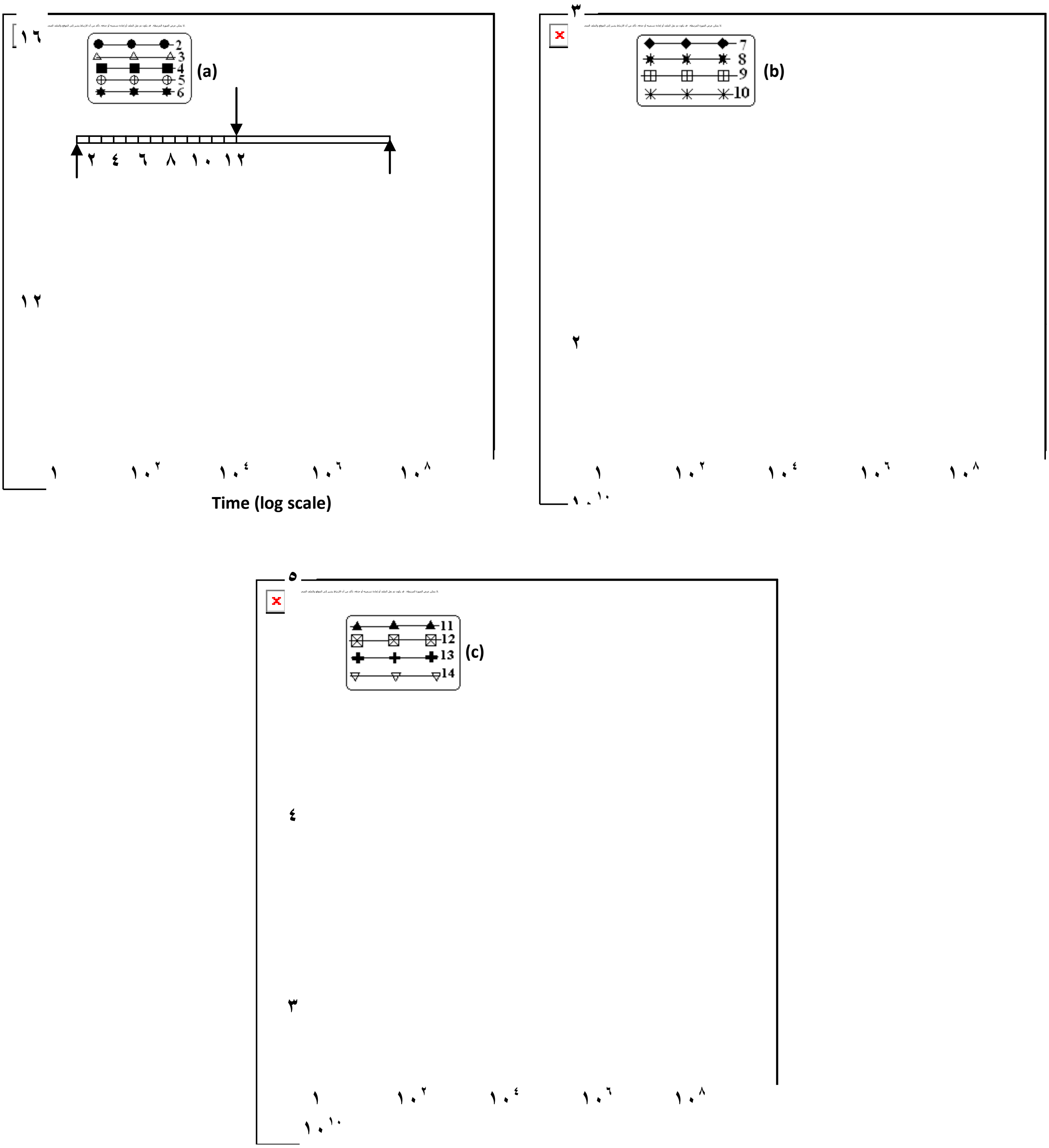


Fig. (٥.١): Time dependent flexural strain for polypropylene polymer. (a) nodes (٢-٦); (b) nodes (٧-١٠);(c) nodes (١١-١٤)

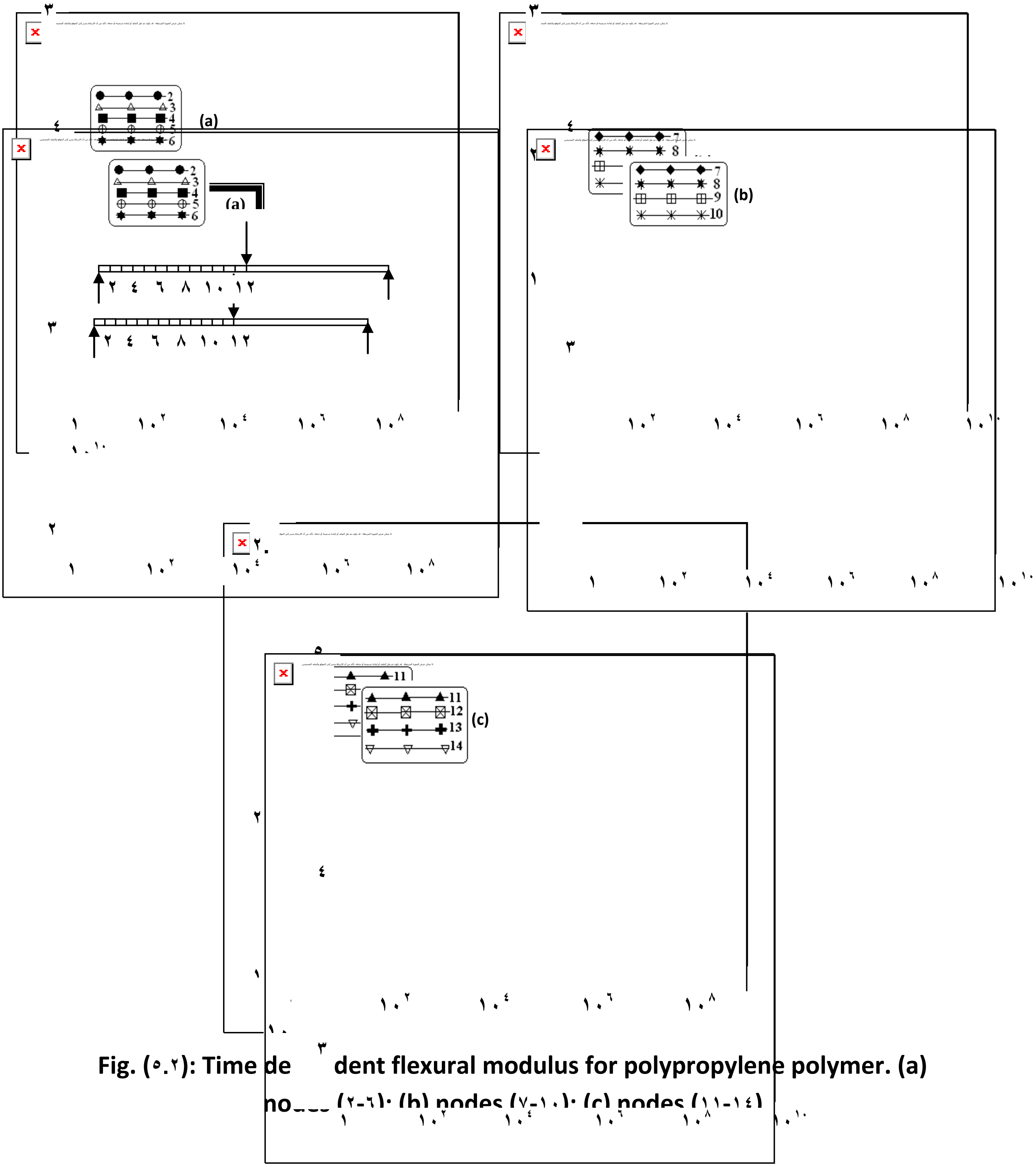


Fig. (٥.٢): Time de dent flexural modulus for polypropylene polymer. (a) no (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

Fig. (٥.٣): Time dependent flexural compliance for polypropylene polymer. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

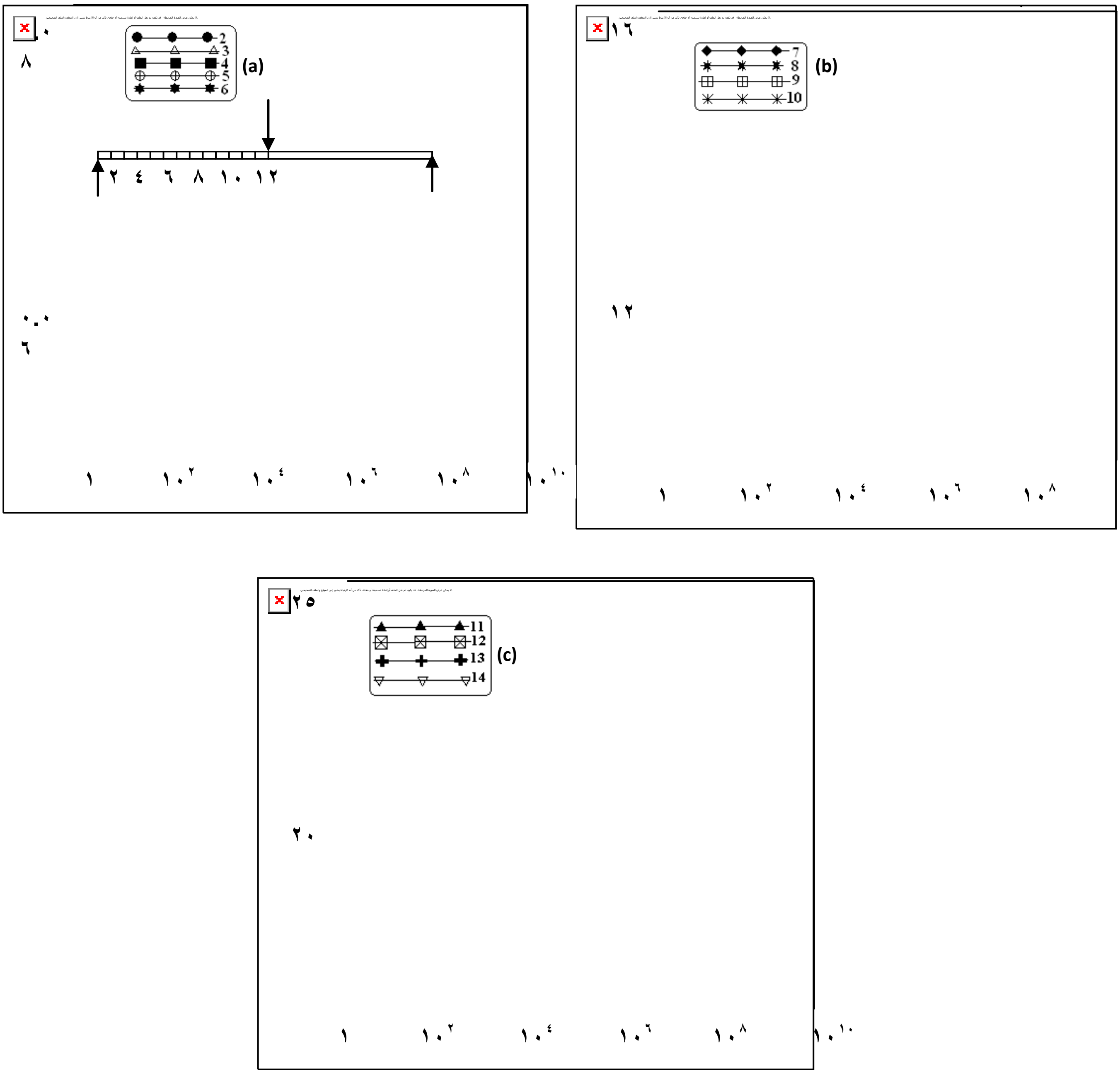


Fig. (٥.٤): Time dependent longitudinal flexural strain for aligned short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

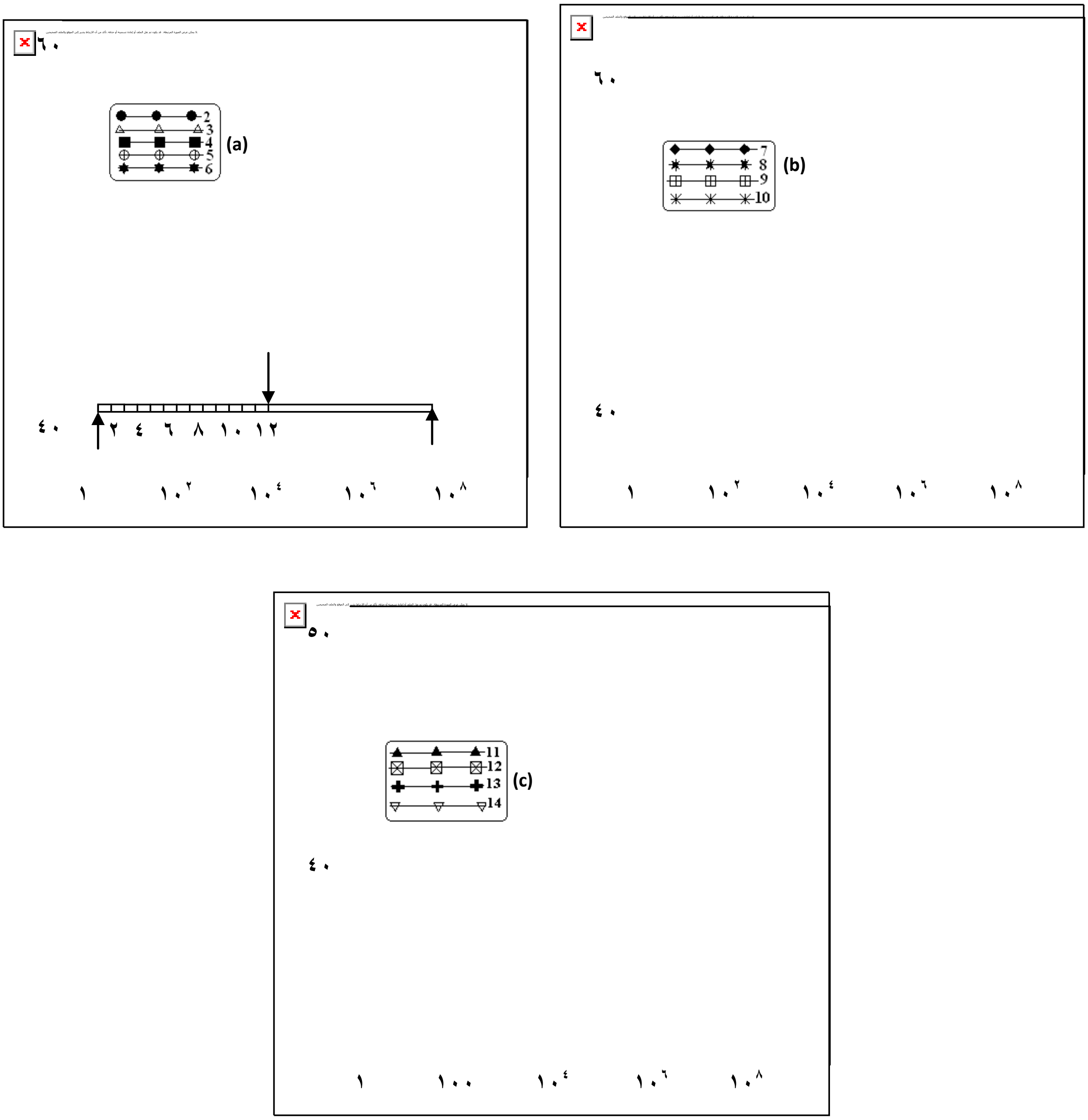


Fig. (٥.٥): Time dependent longitudinal flexural modulus for aligned short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes

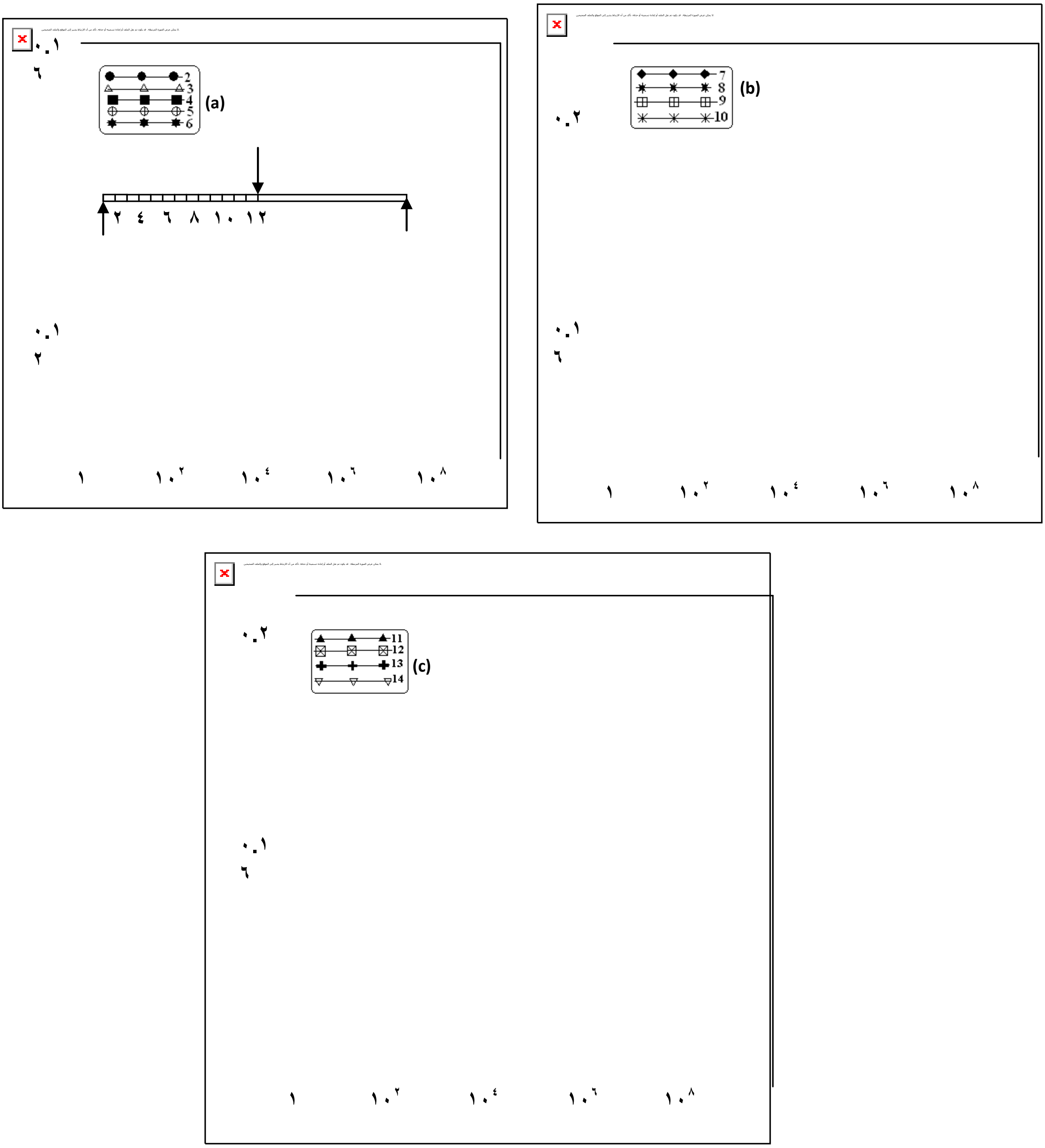


Fig. (٥.٦): Time dependent longitudinal flexural compliance for aligned short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

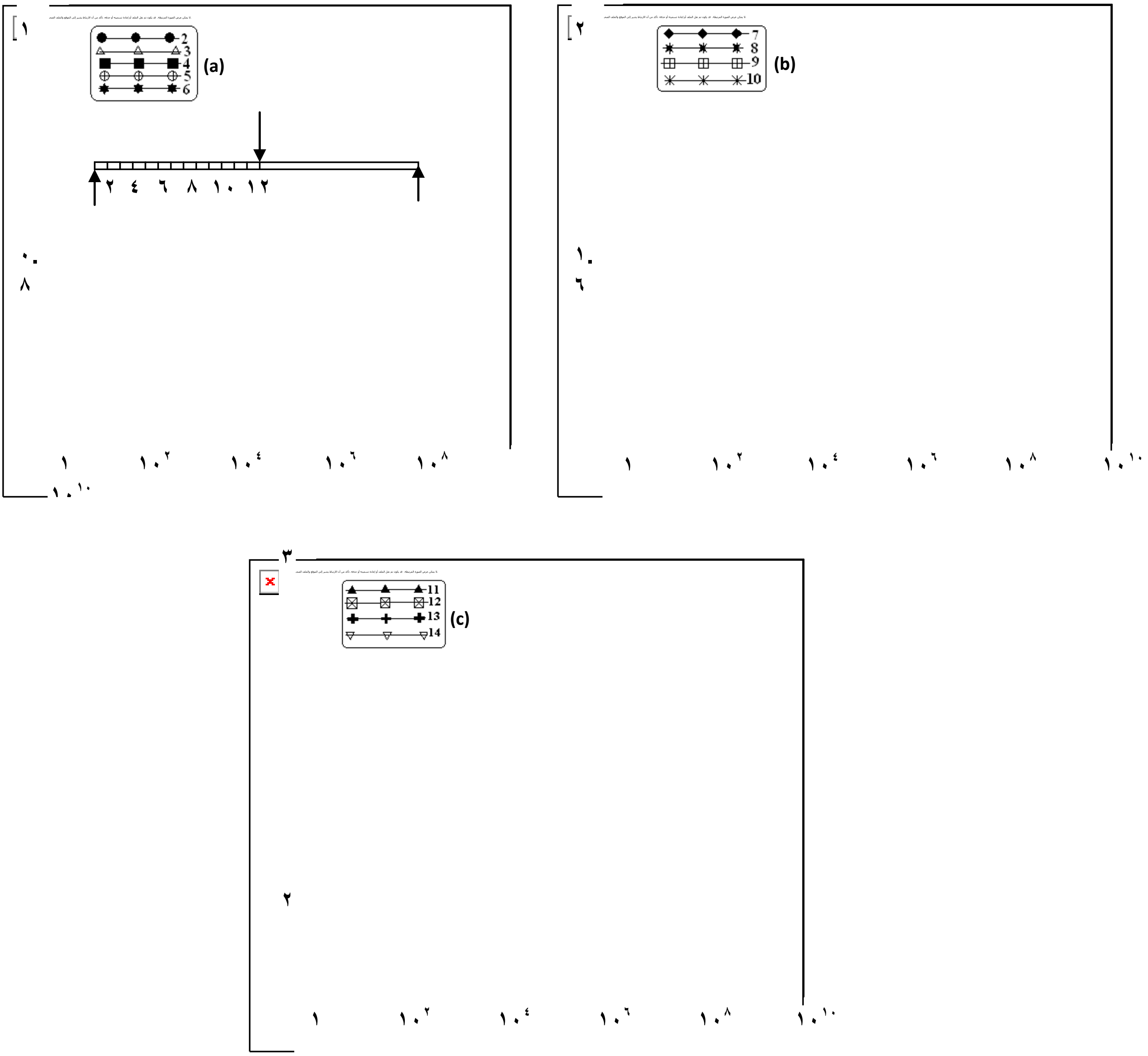


Fig. (٥.٧): Time dependent transverse flexural strain for aligned short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

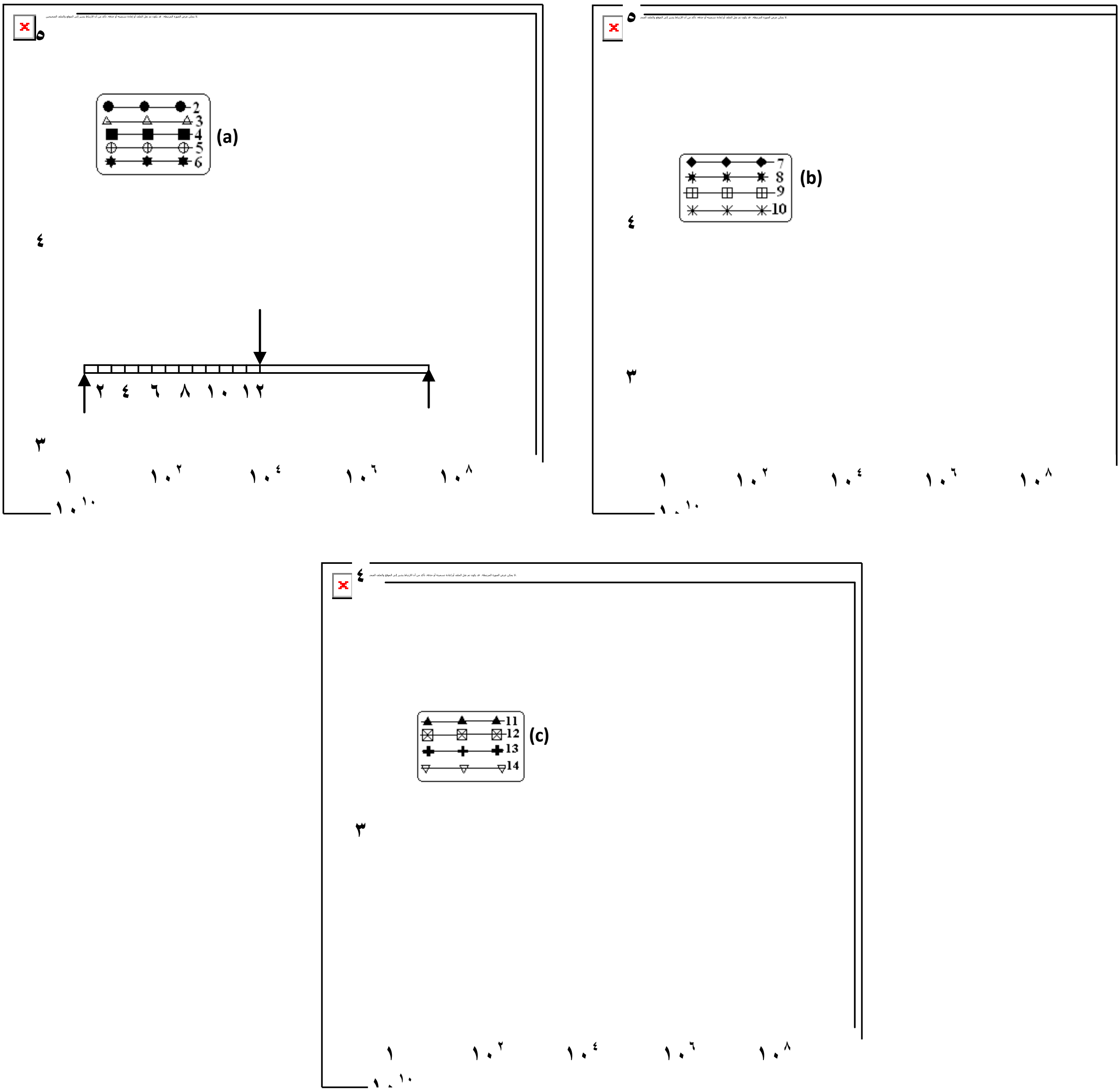


Fig. (٥.٨): Time dependent transverse flexural modulus for aligned short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

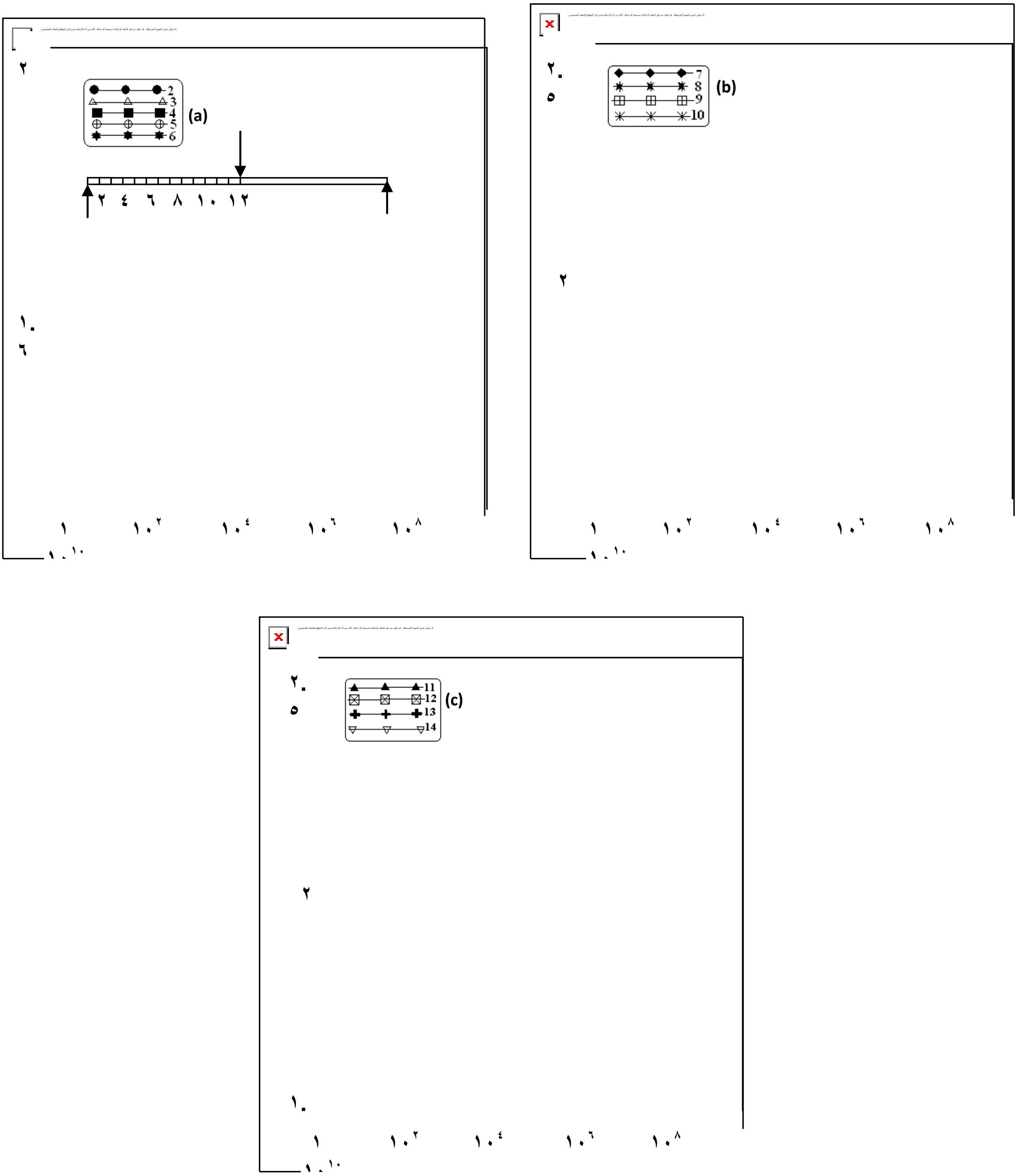


Fig. (٥.٩): Time dependent transverse flexural compliance for aligned short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes

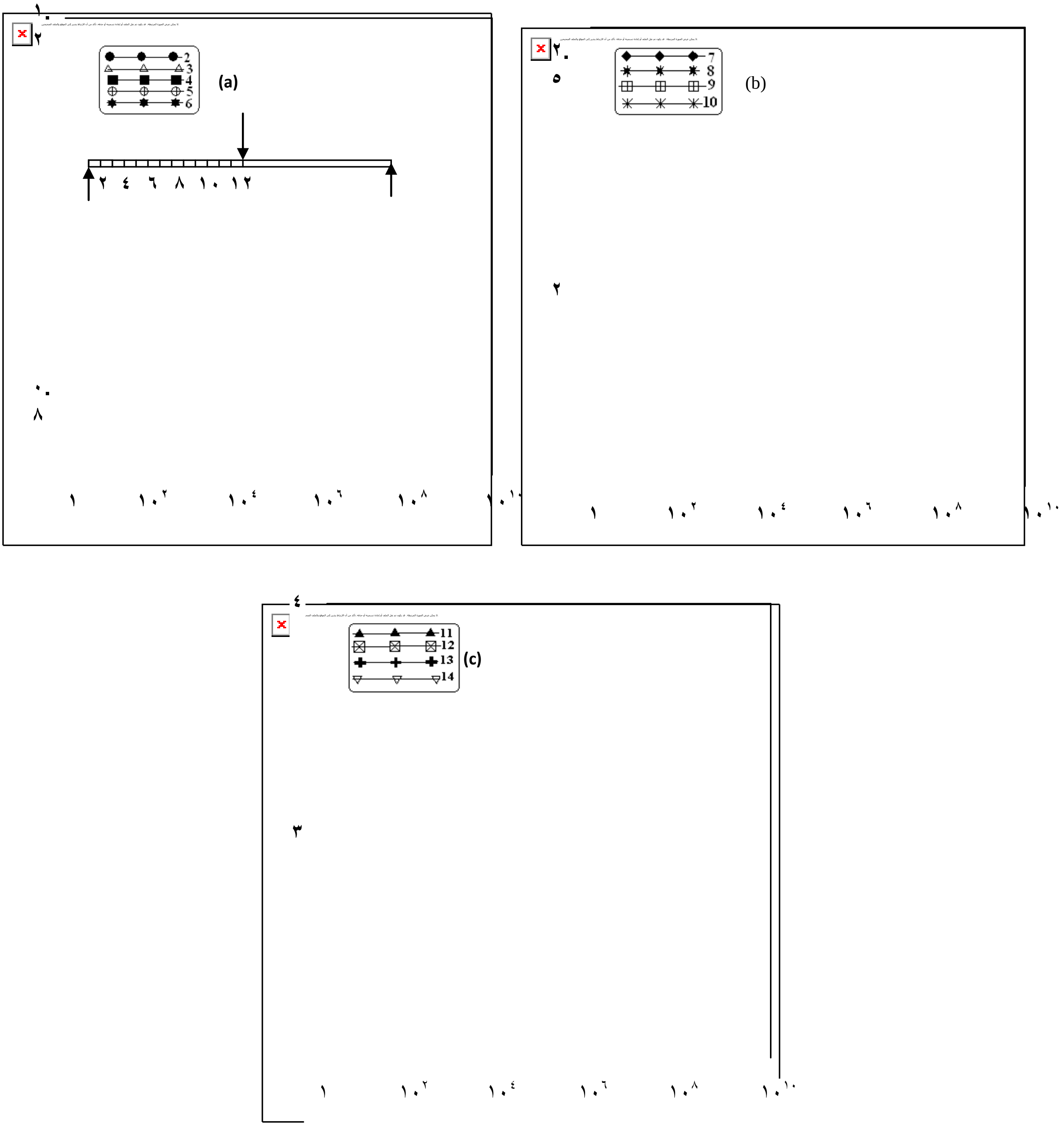


Fig. (٥.١٠): Time dependent longitudinal flexural strain for aligned short E-glass/PP composite {modified rule of mixtures}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes

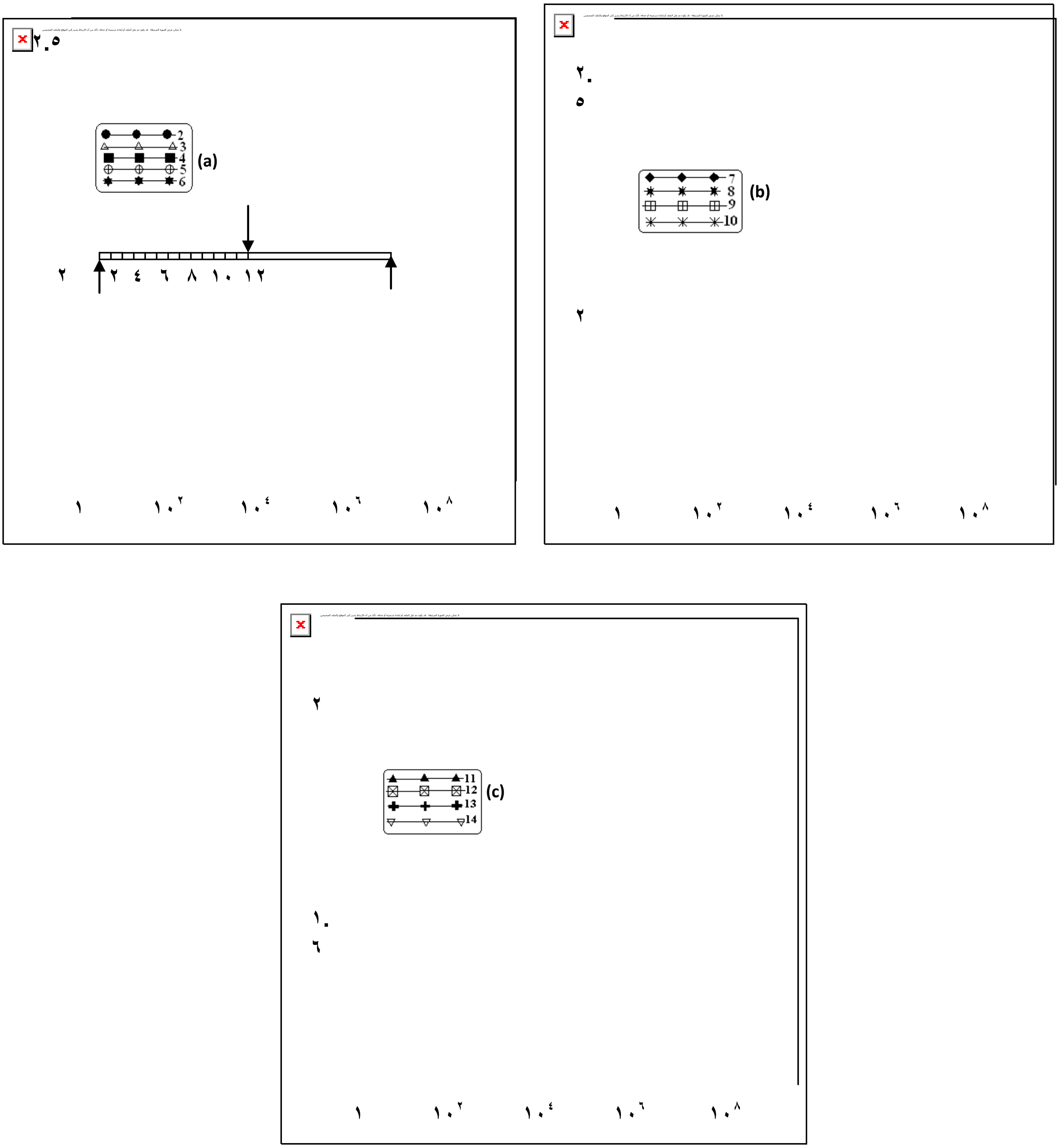


Fig. (٥.١١): Time dependent longitudinal flexural modulus for aligned short E-glass/PP composite {modified rule of mixtures}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

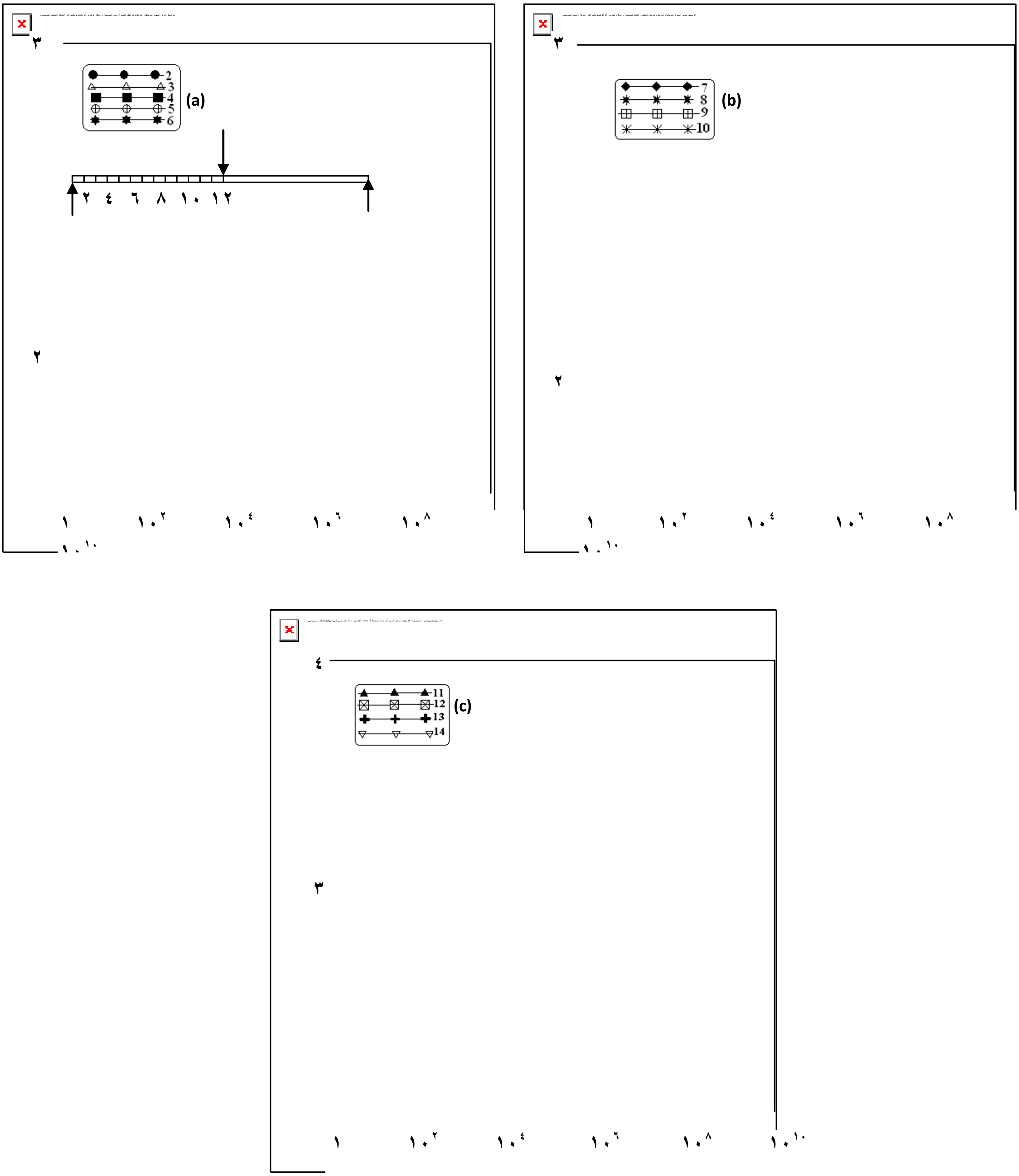


Fig. (٥.١٢): Time dependent longitudinal flexural compliance for aligned short E-glass/PP composite {modified rule of mixtures}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

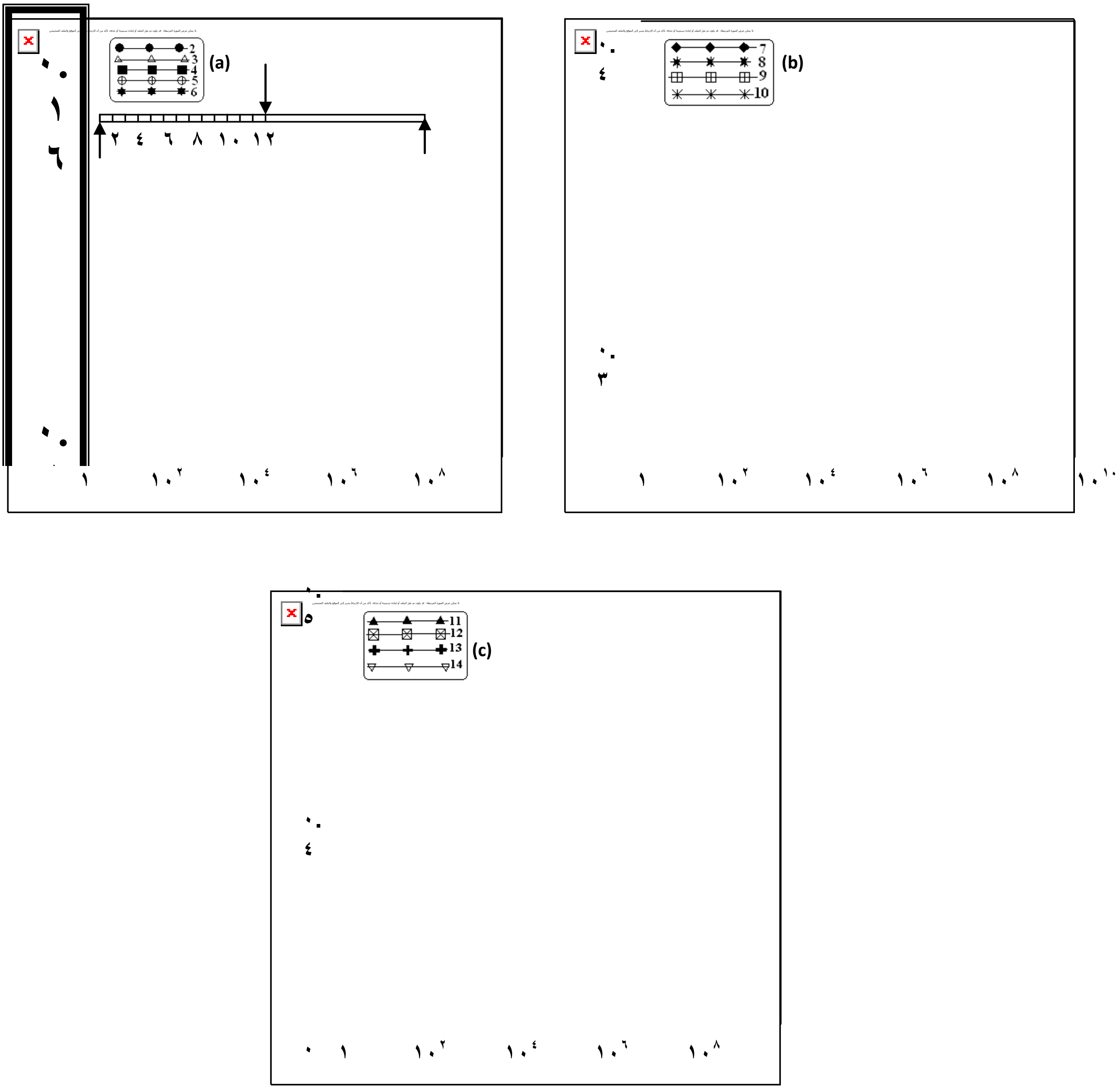


Fig. (٥.١٣): Time dependent random flexural strain for short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

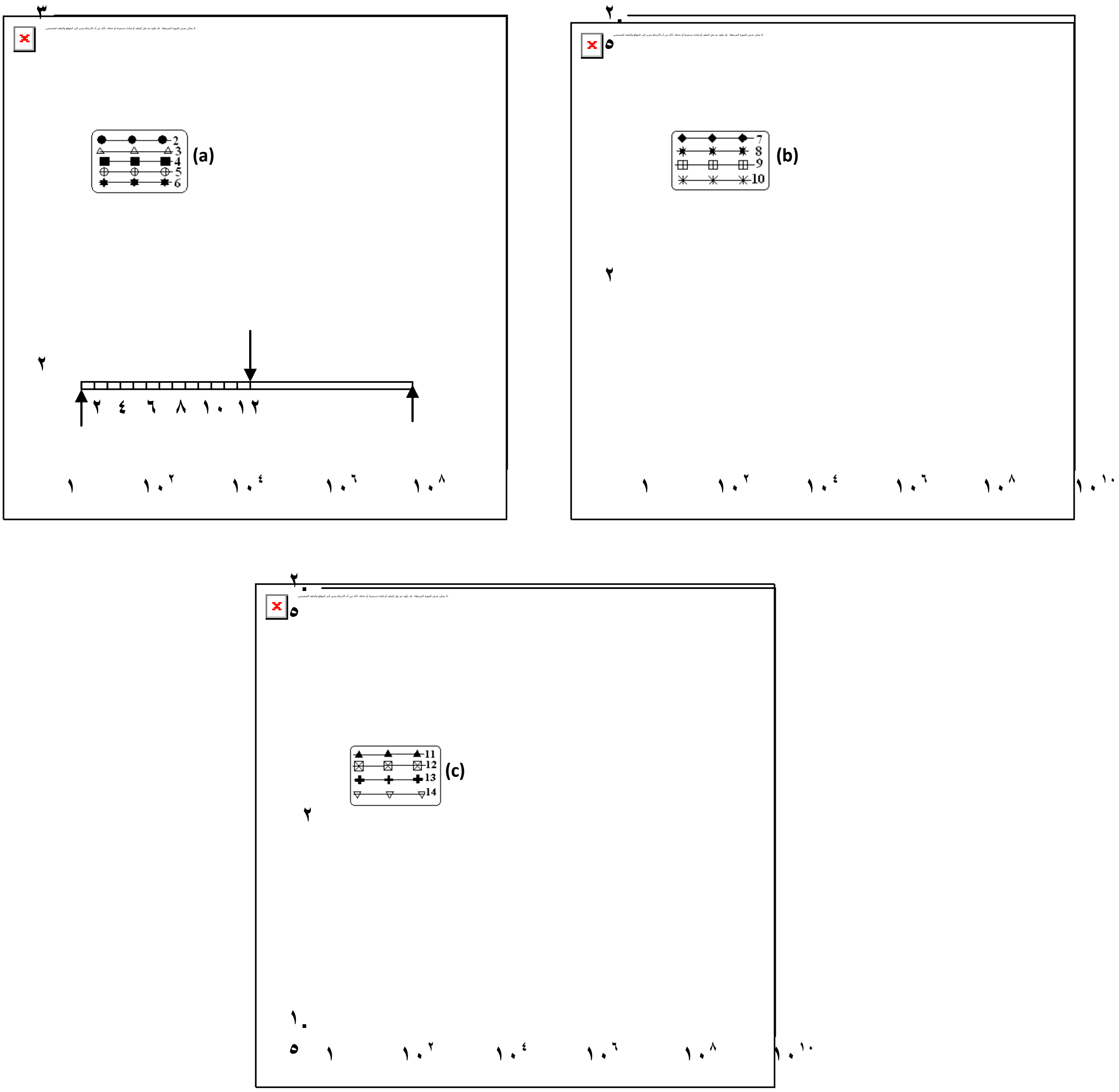


Fig. (٥.١٤): Time dependent flexural modulus for random short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)

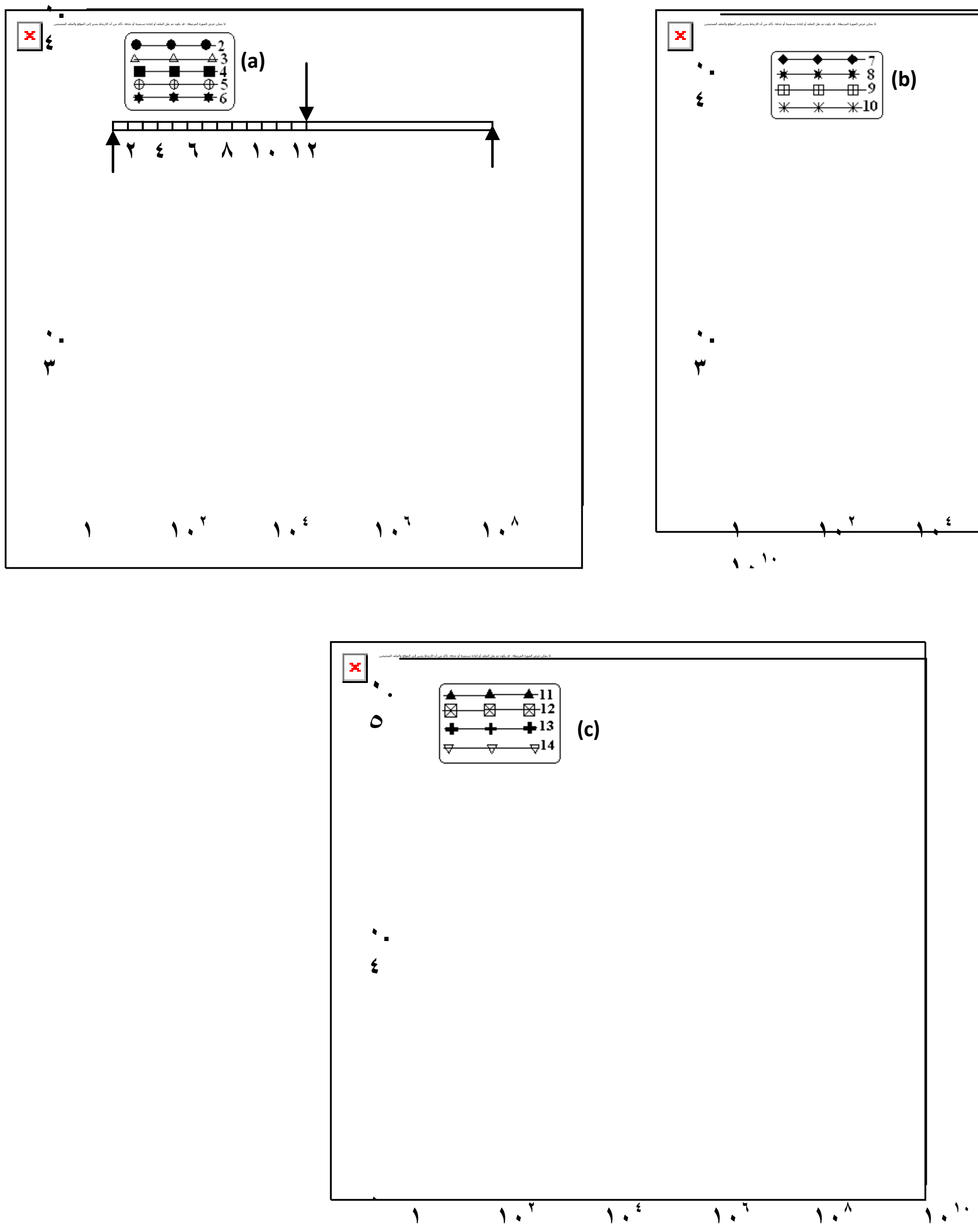
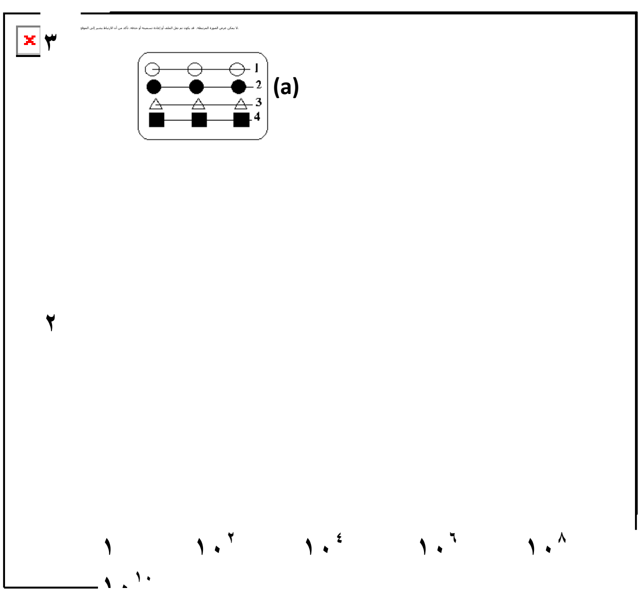


Fig. (٥.١٥): Time dependent flexural compliance for random short E-glass/PP composite {Halpin-Tsai equation}. (a) nodes (٢-٦); (b) nodes (٧-١٠); (c) nodes (١١-١٤)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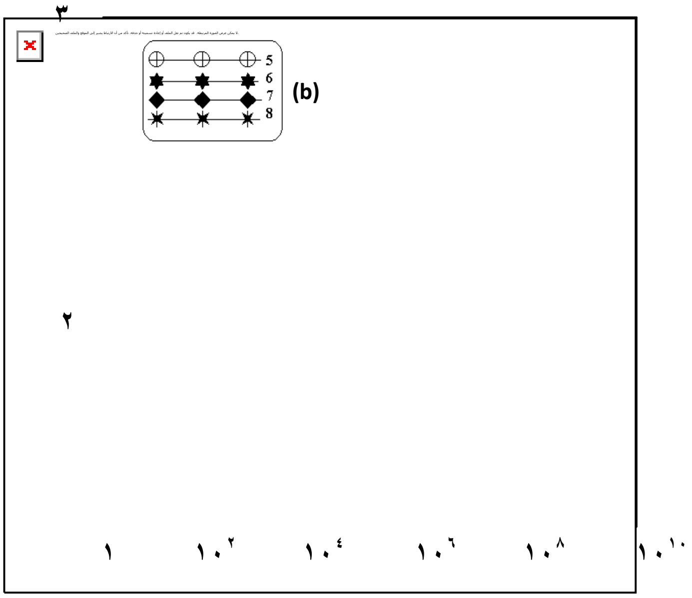
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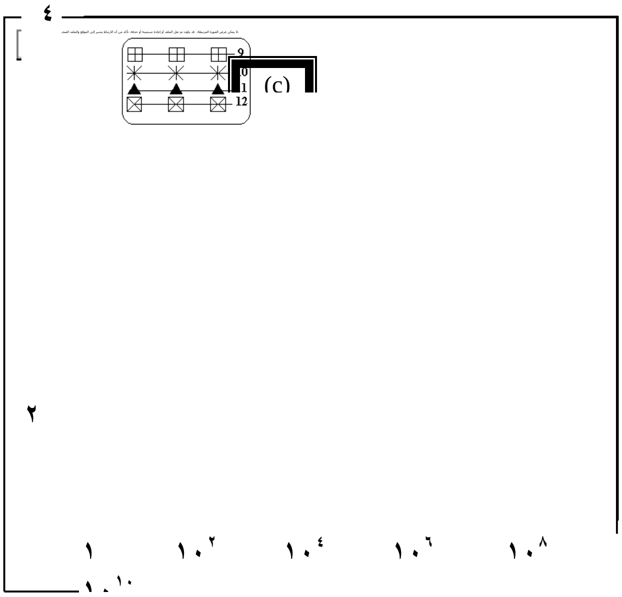
(b)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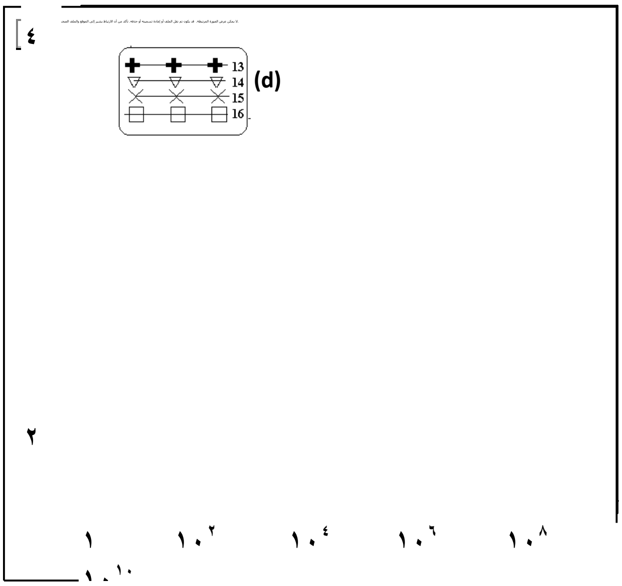
(c)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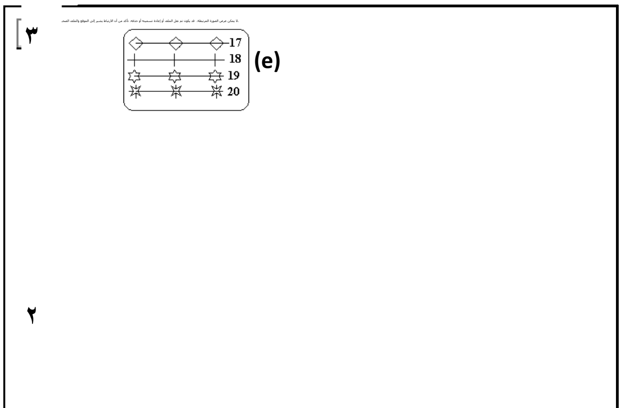
(d)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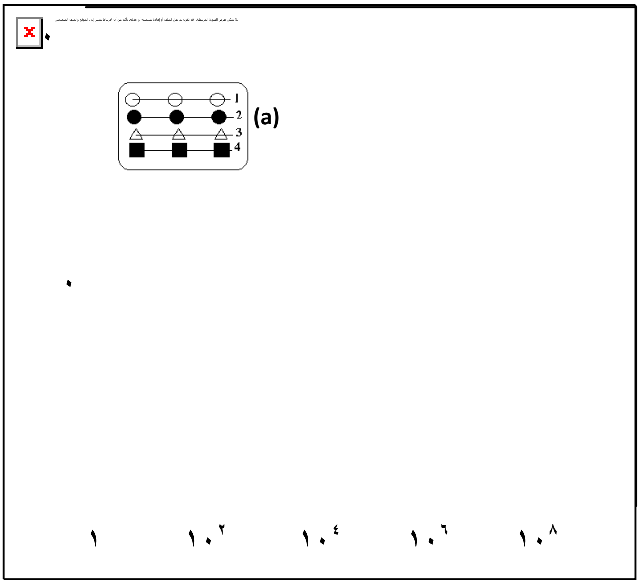
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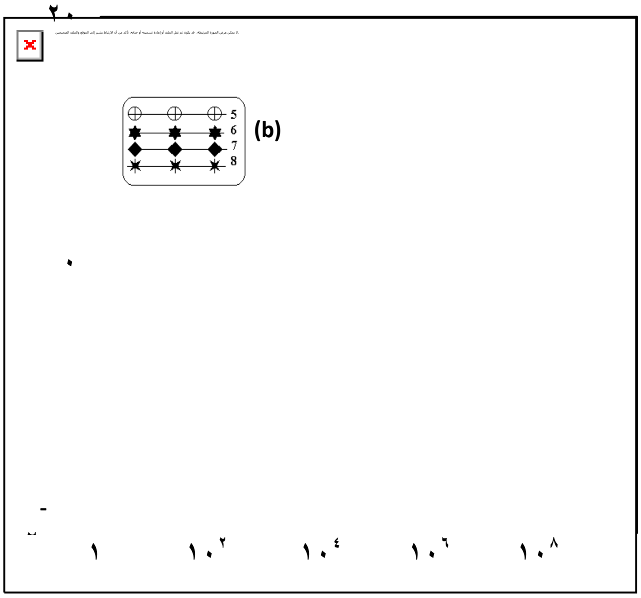
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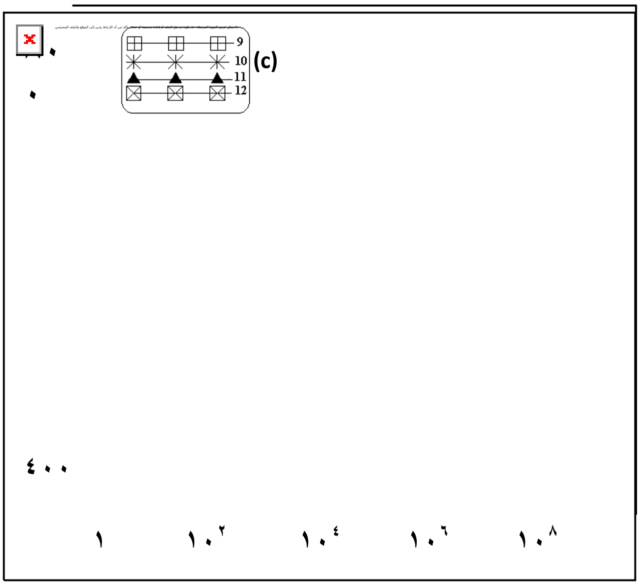
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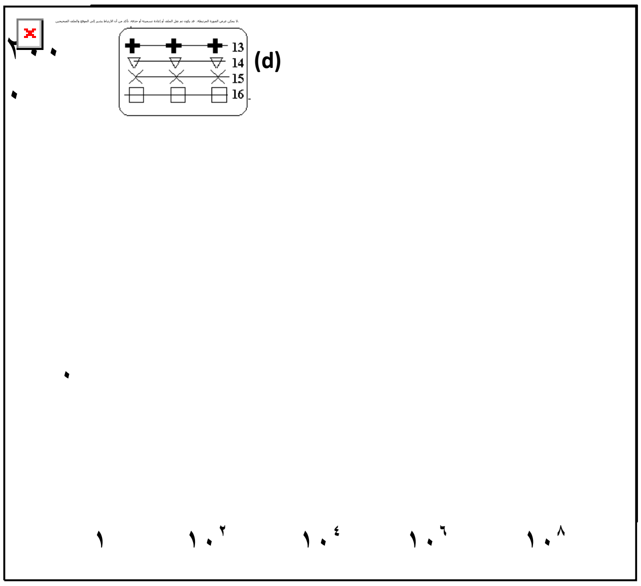
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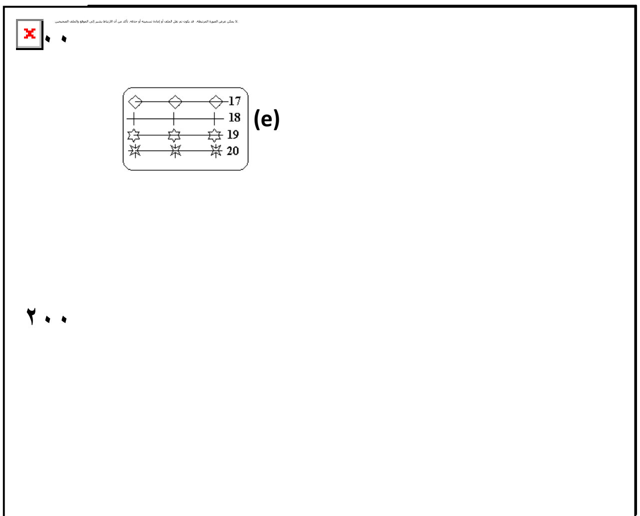
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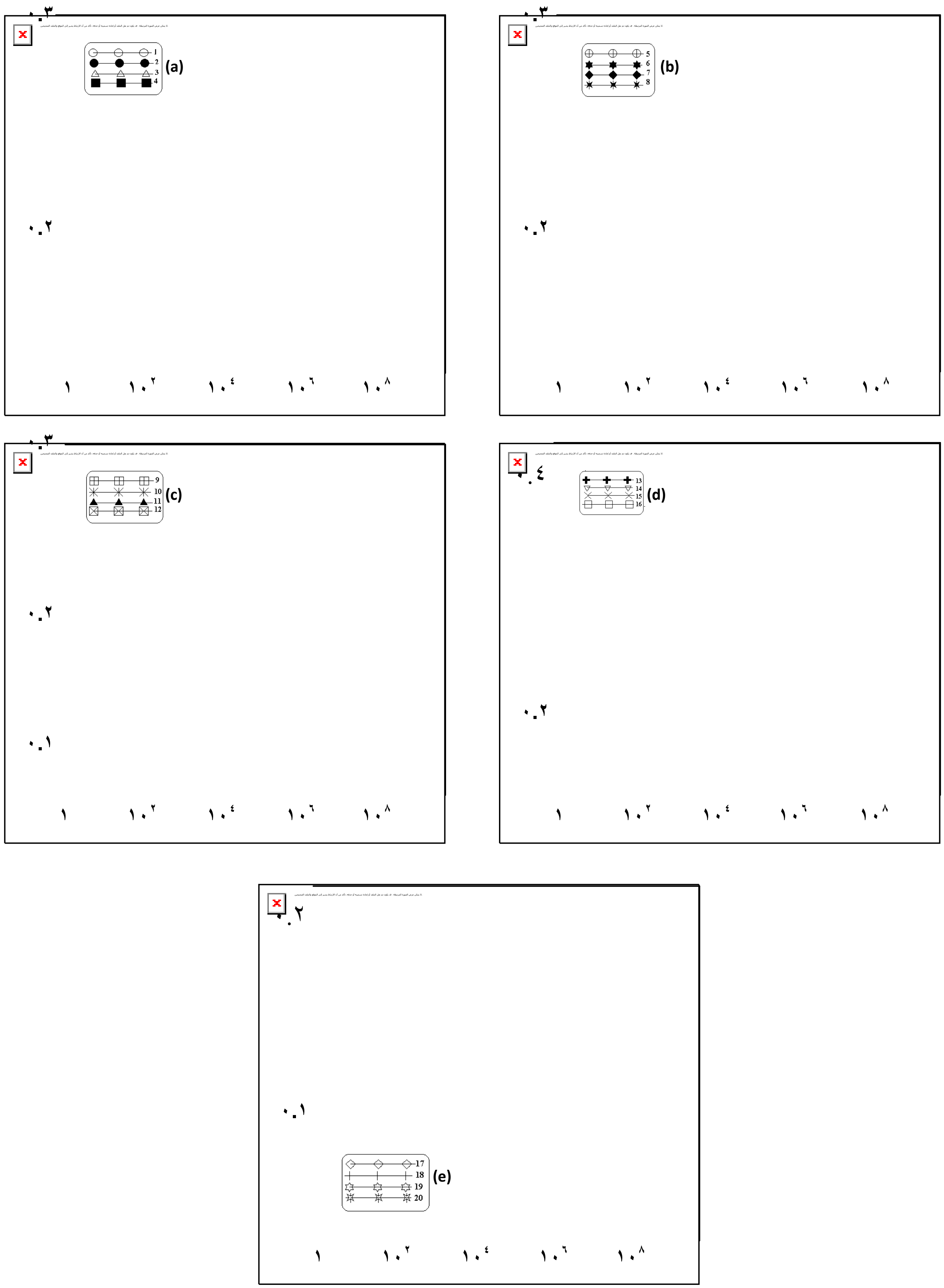


Fig. (٥.١٨): Time dependent tensile compliance for polypropylene polymer. (a) elements (١-٤); (b) elements (٥-٨); (c) elements (٩-١٢);(d) elements (١٣-١٦); (e)

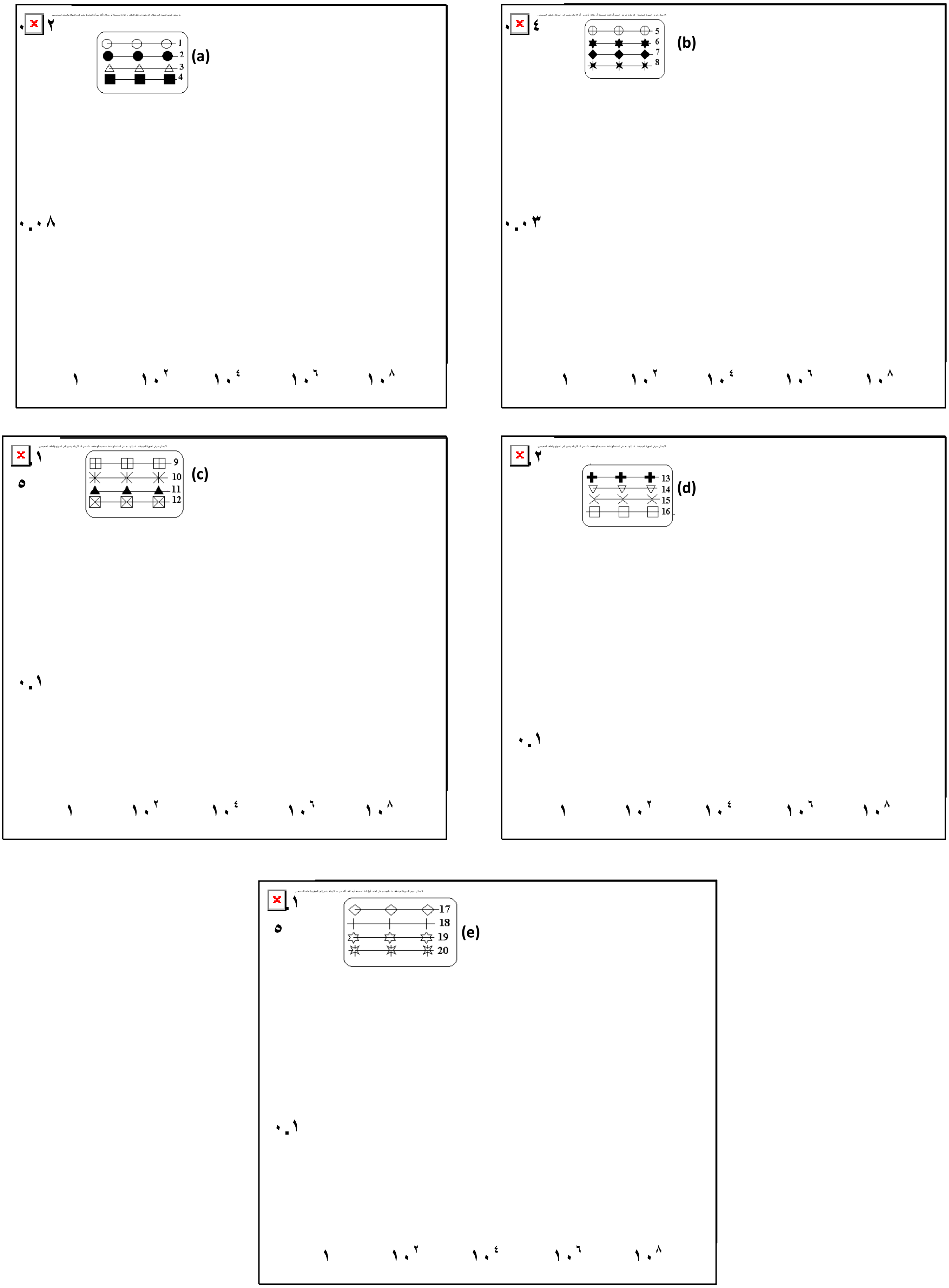
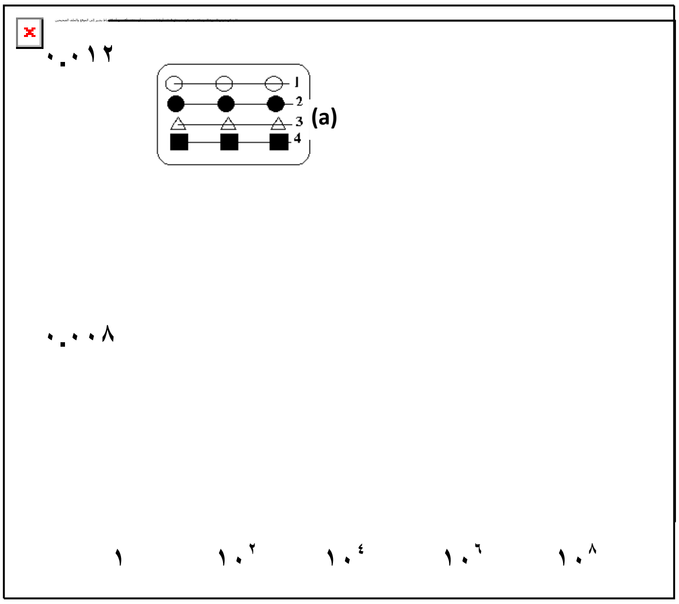


Fig. (٥.١٩): Time dependent longitudinal tensile strain for aligned short E-glass/PP composite {Halpin-Tsai equation}. (a) elements (١-٤); (b)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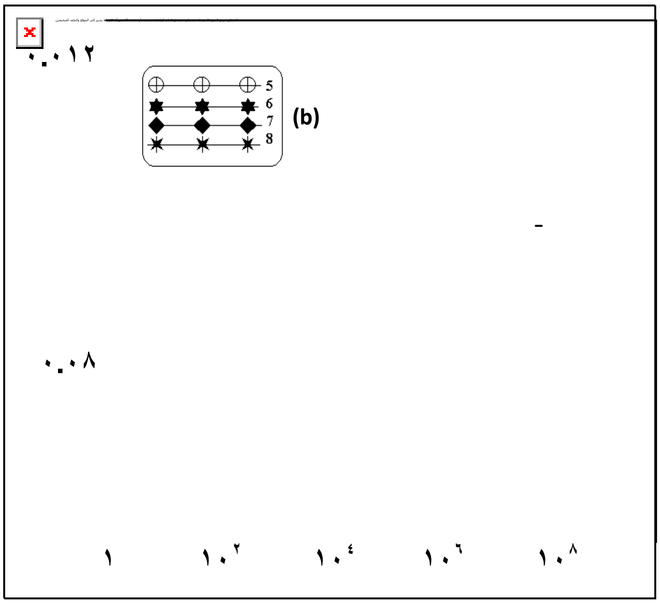
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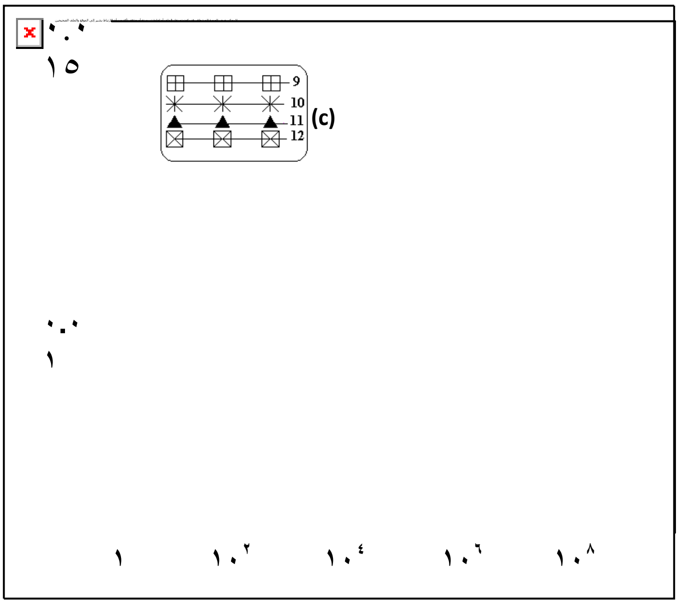
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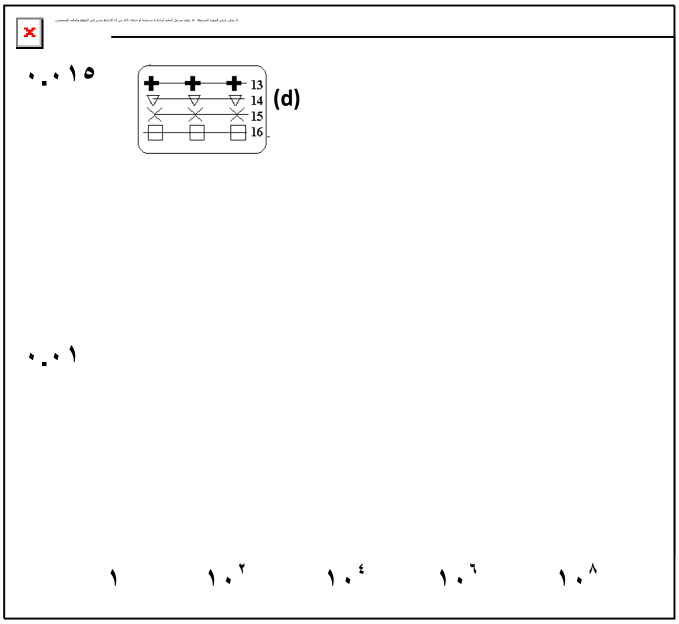
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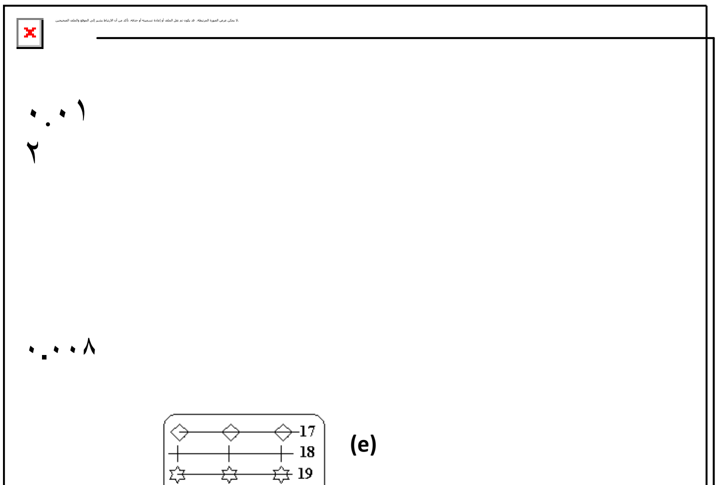
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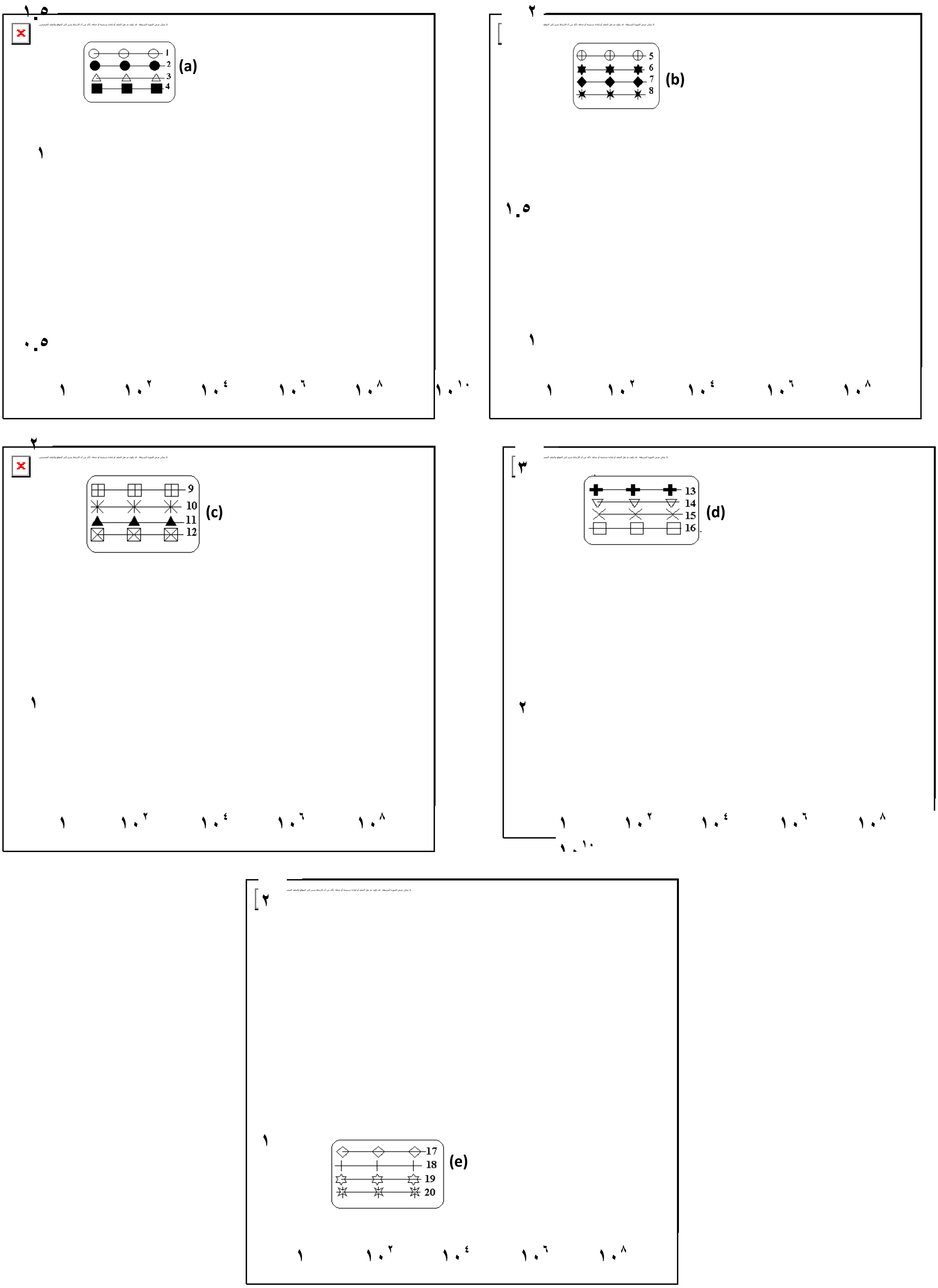
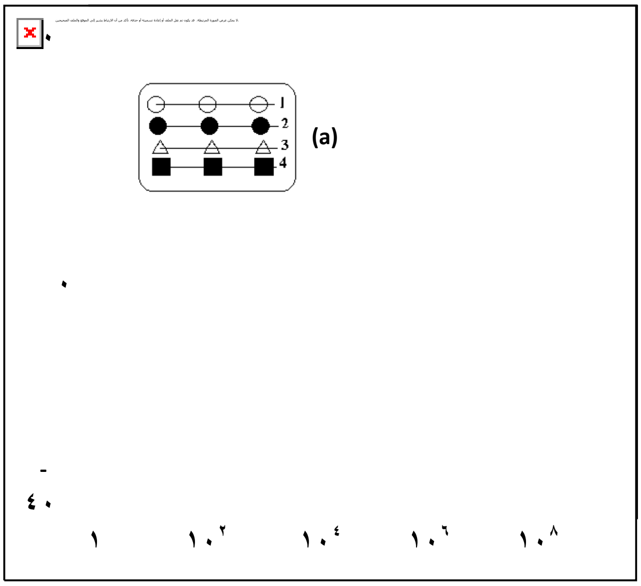


Fig. (٥.٢٢): Time dependent transverse tensile strain for aligned short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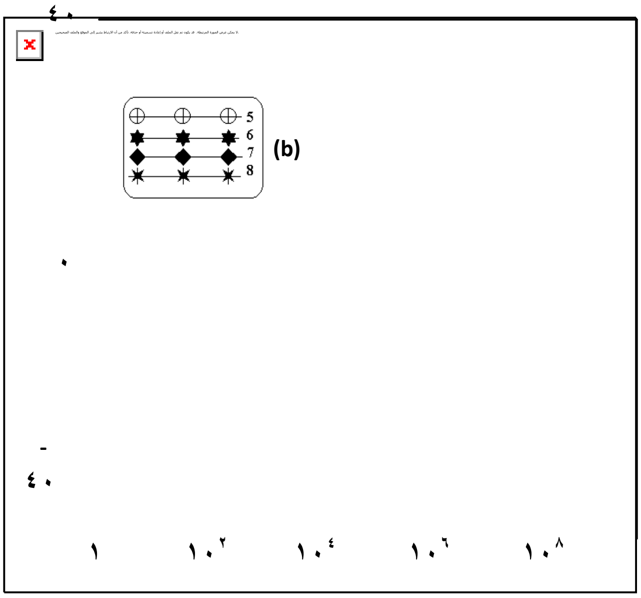
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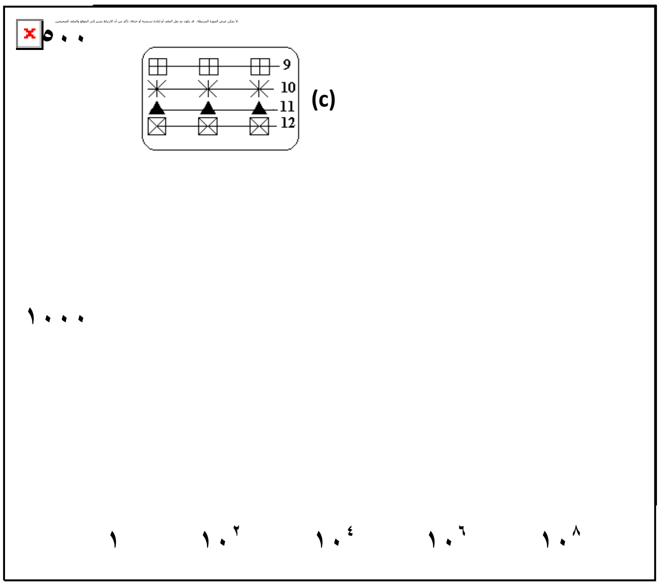
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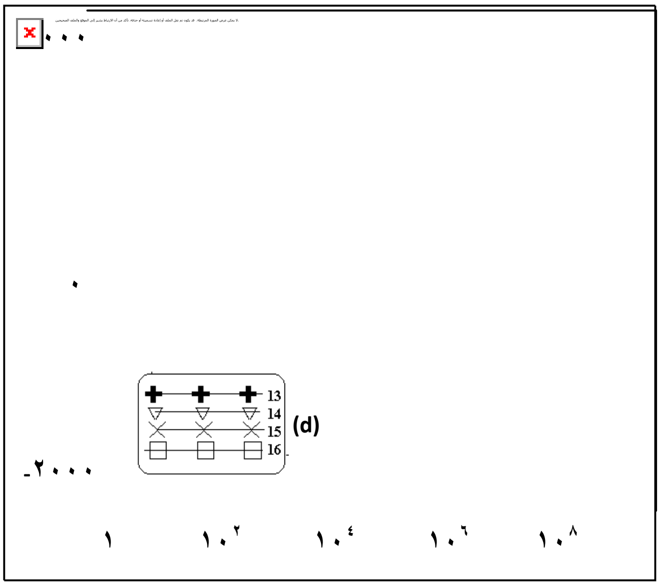
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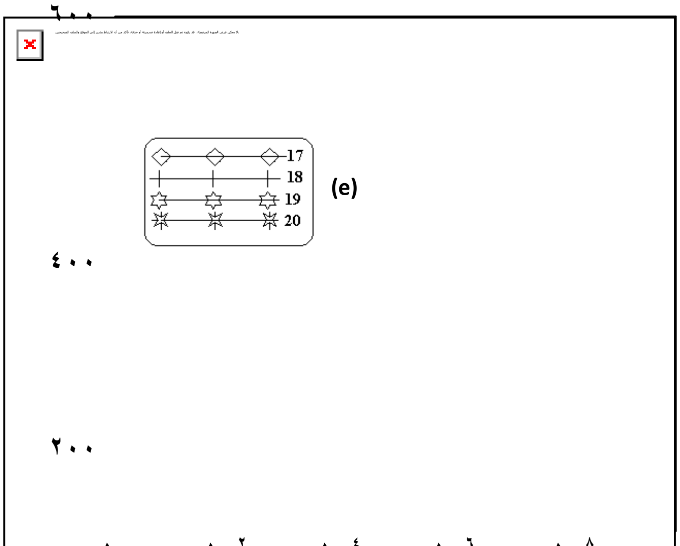
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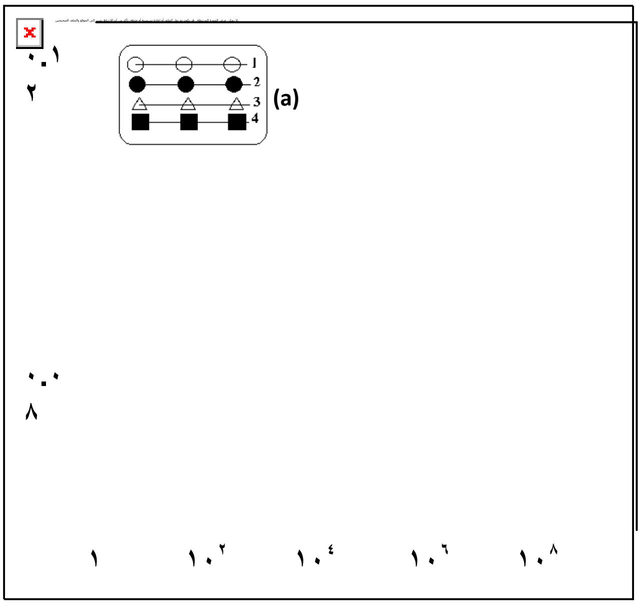
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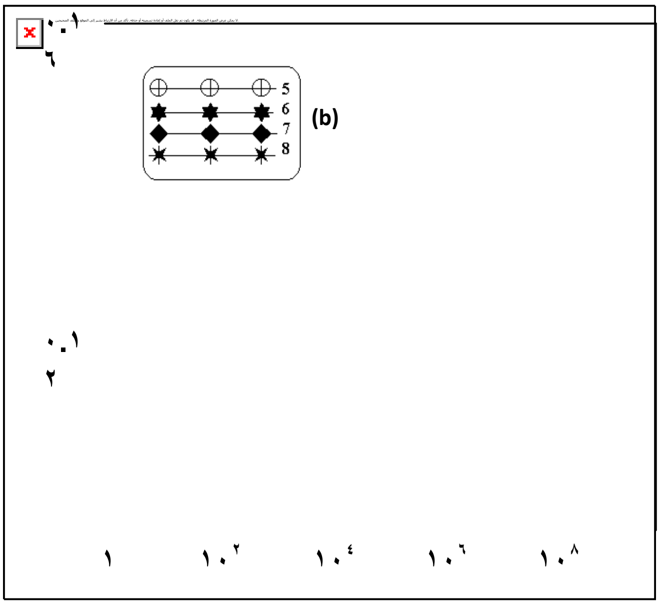
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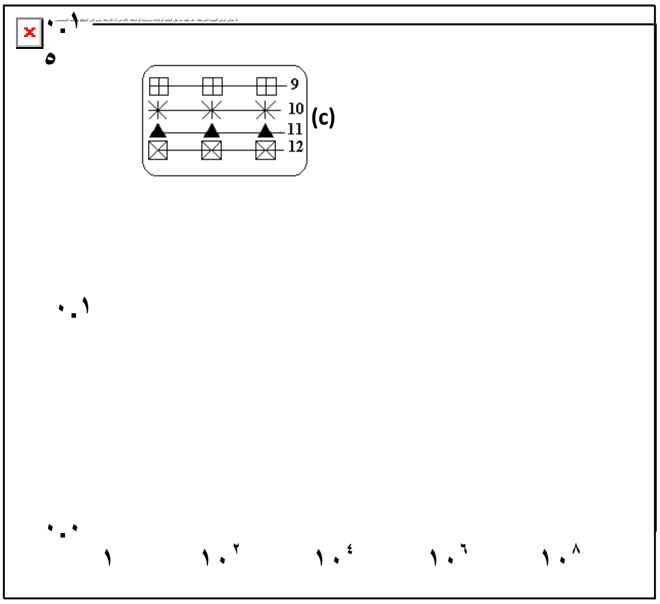
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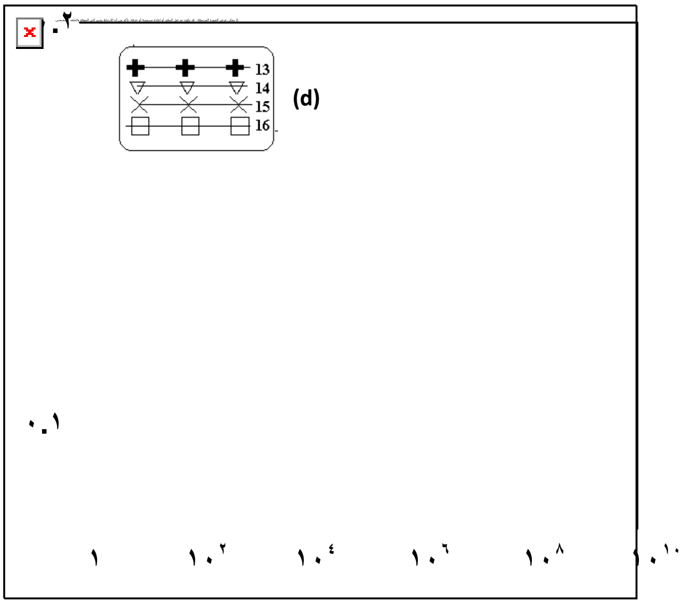
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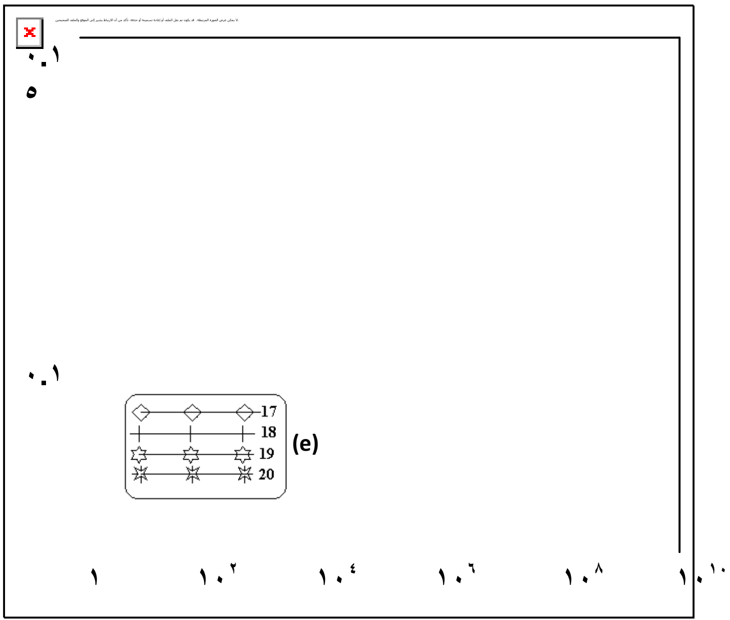
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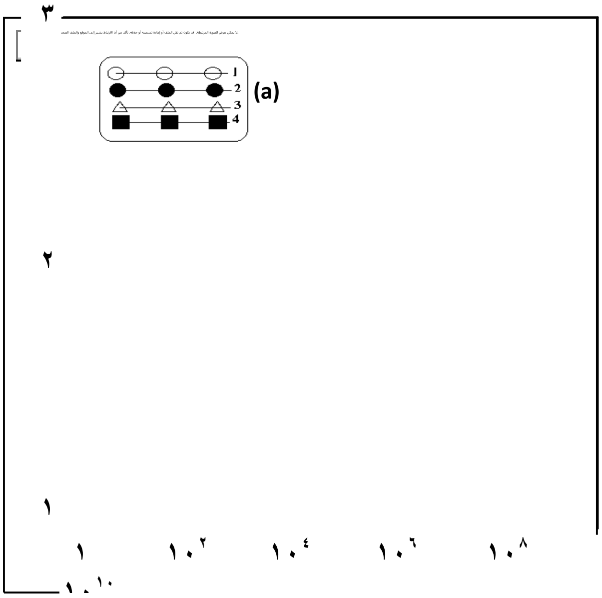
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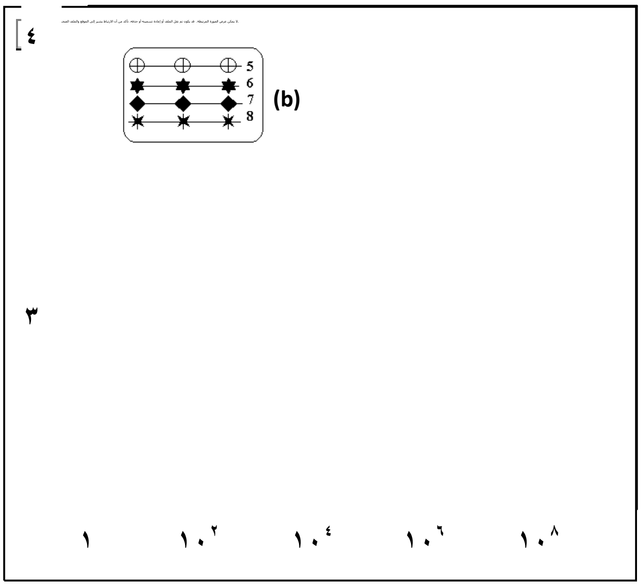
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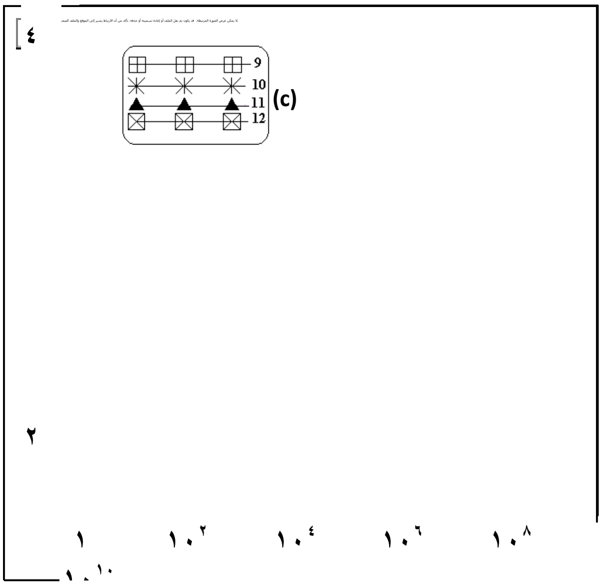
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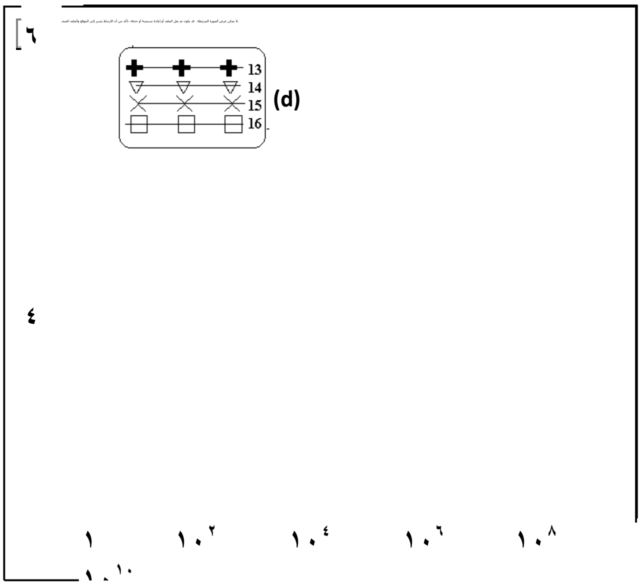
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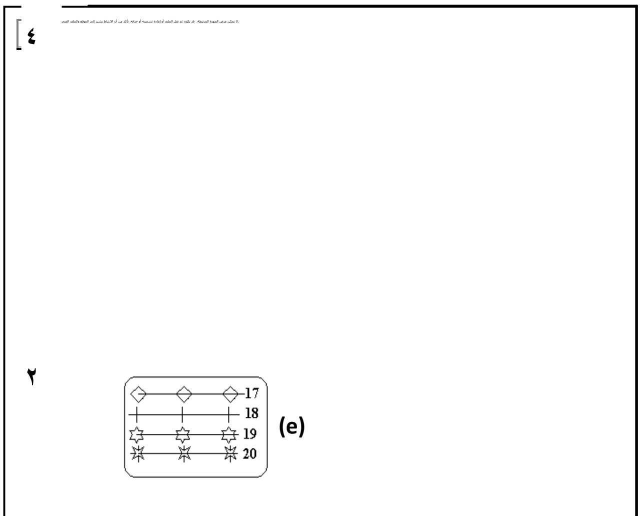
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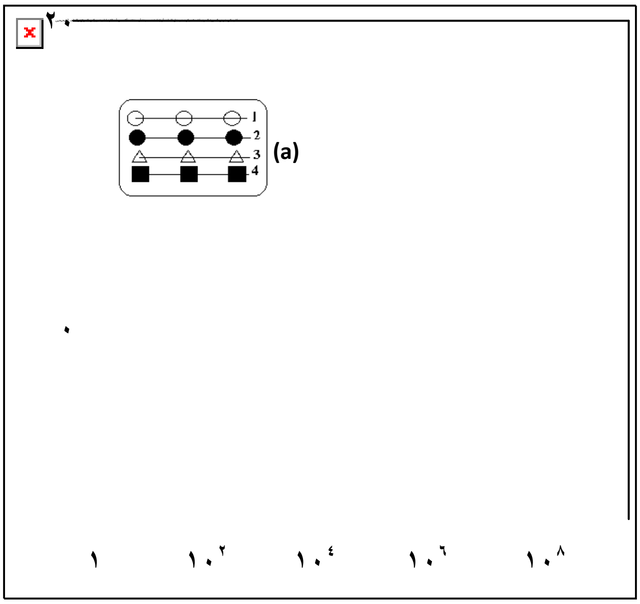
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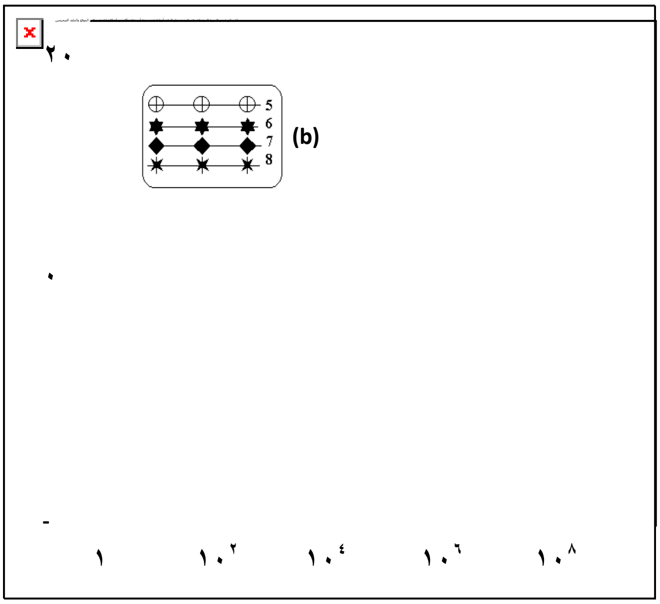
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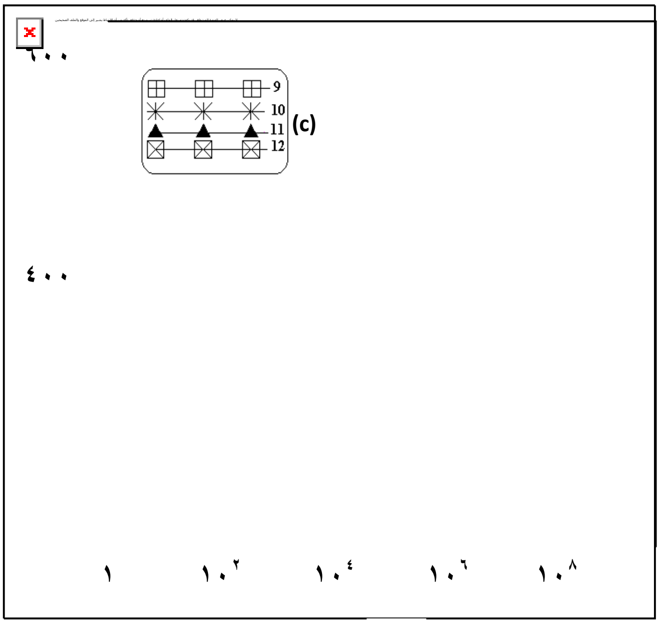
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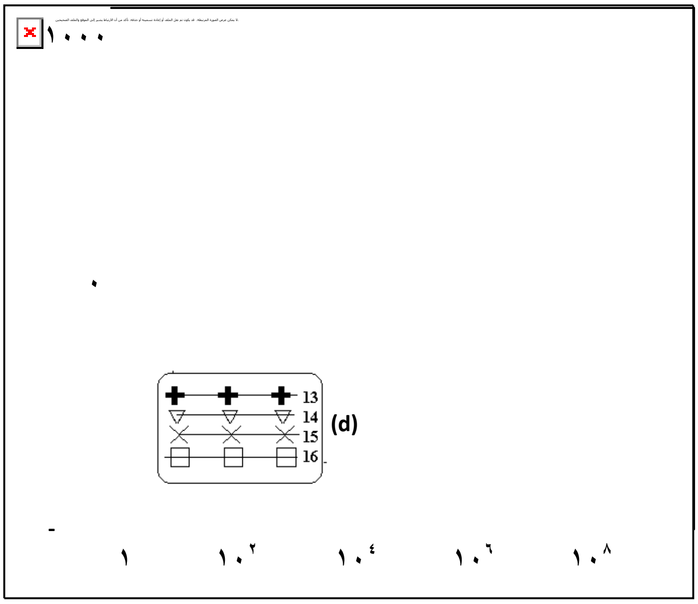
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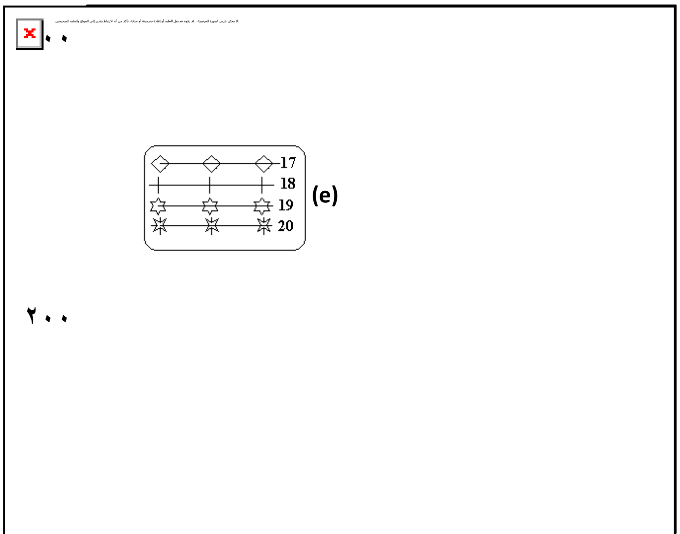
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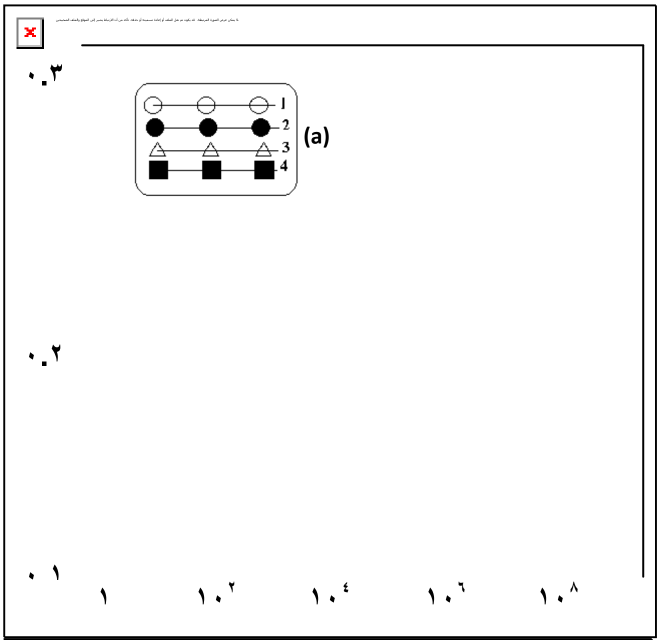

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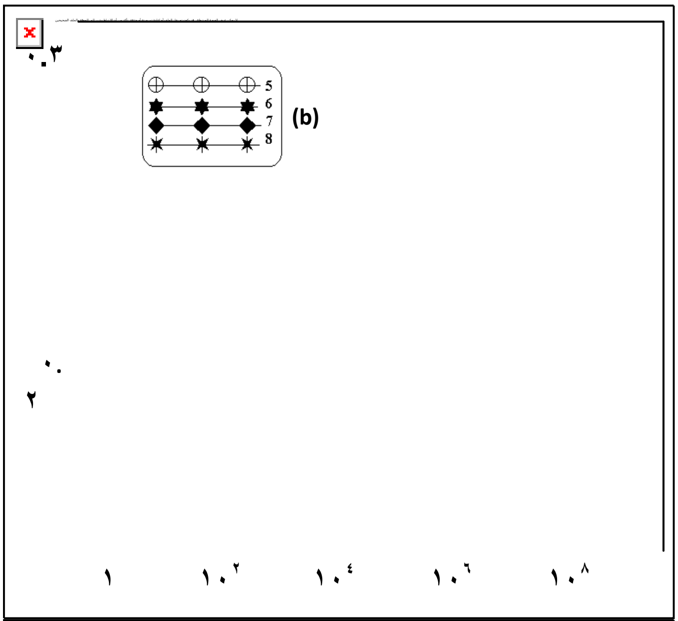

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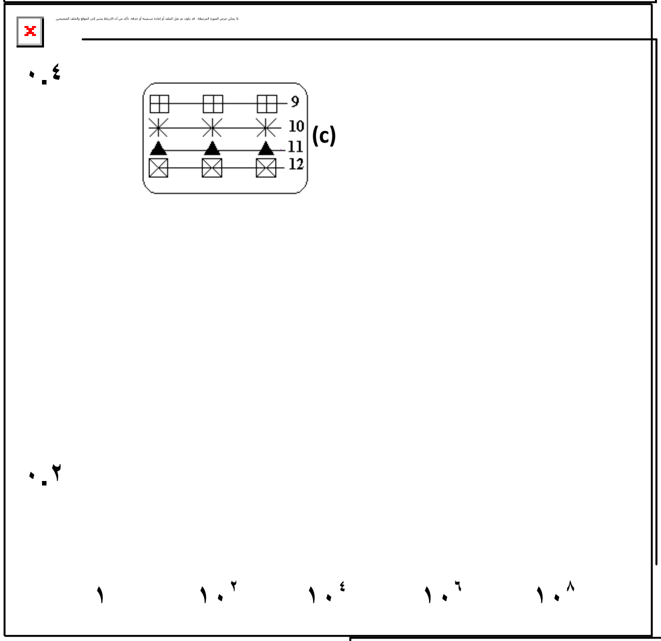

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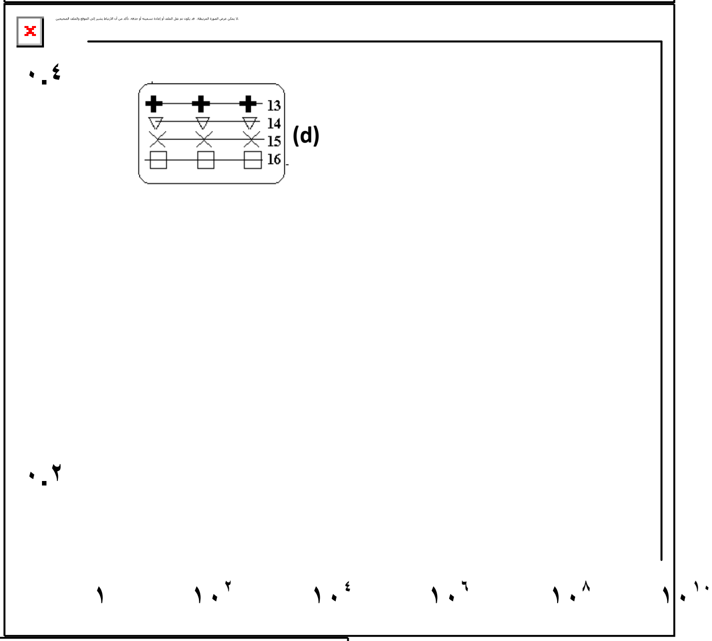

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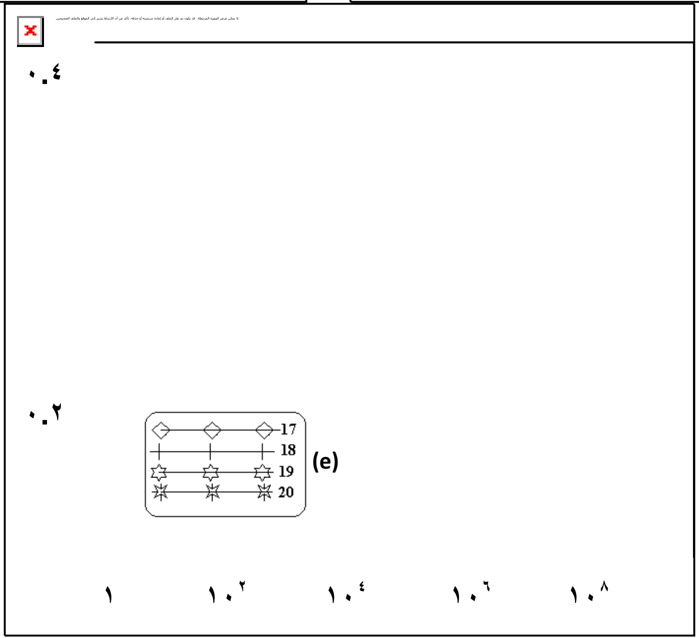

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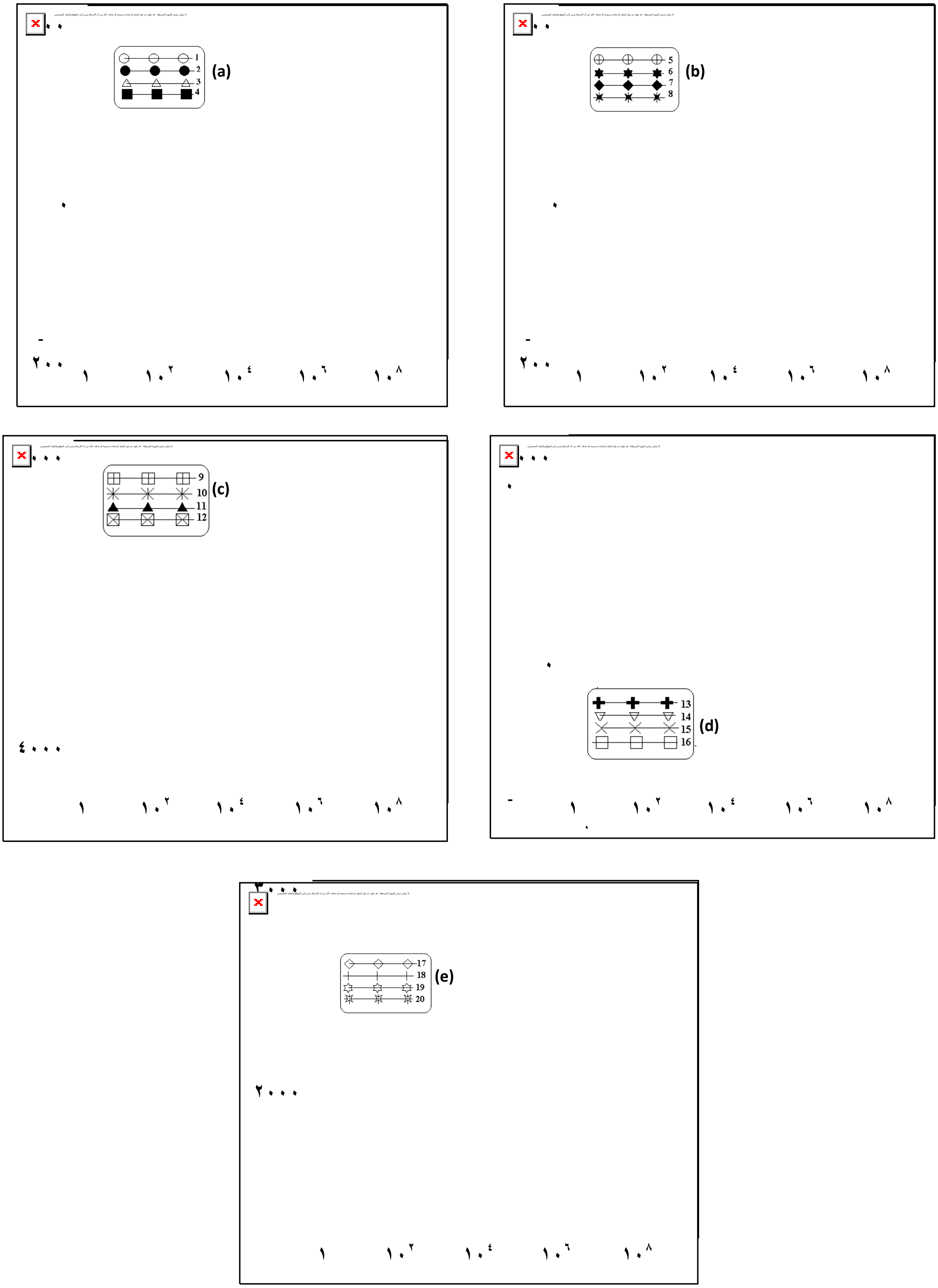

Fig. (٥.٢٩): Time dependent tensile modulus for random short E-glass / PP composite {Halpin-Tsai equation}. (a) elements (١-٤); (b) elements (٥-٨); (c) elements (٩-١٢); (d) elements (١٣-١٦); (e) elements (١٧-٢٠)

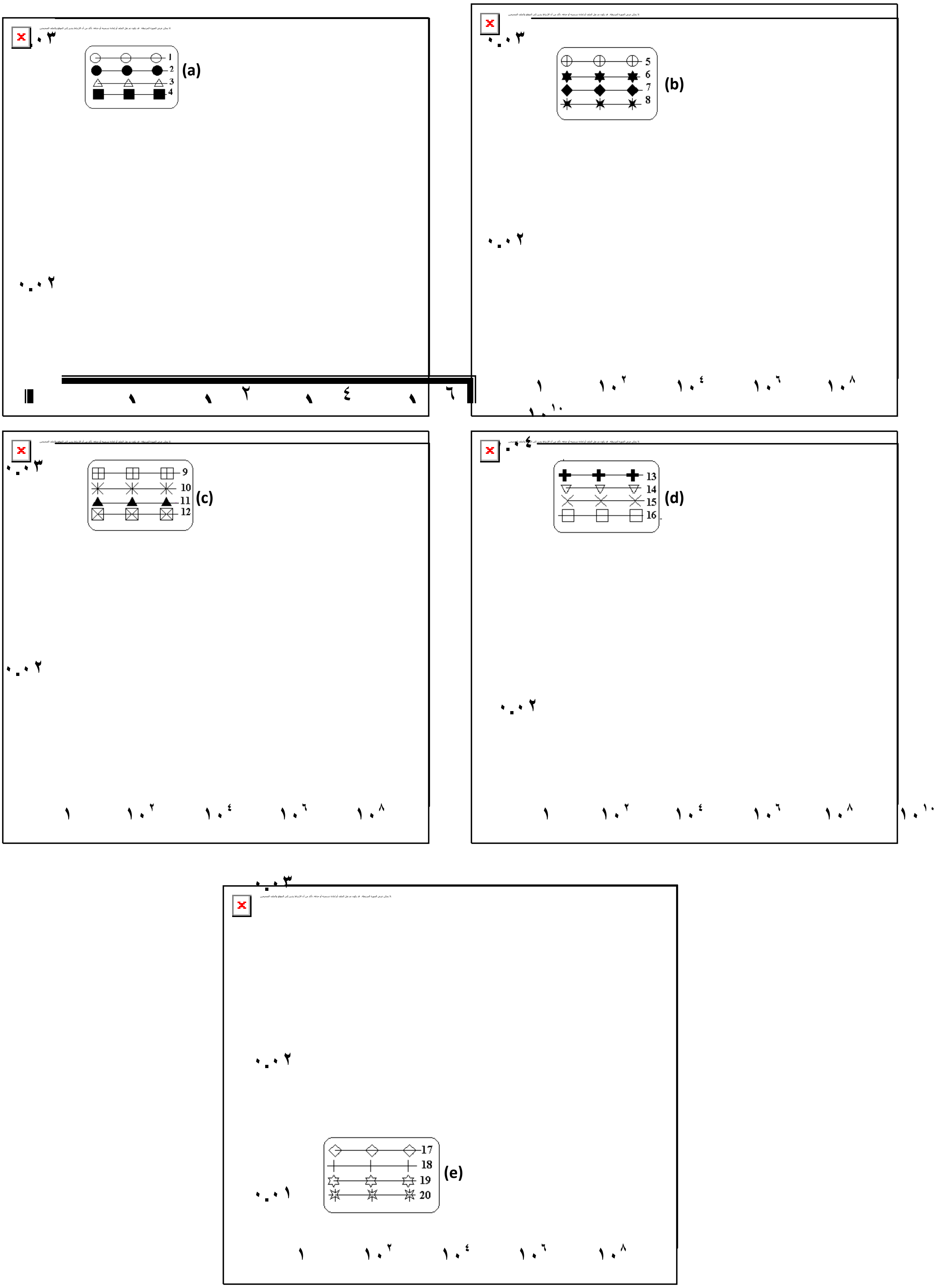

Fig. (٥.٣٠): Time dependent tensile compliance for random short E-glass / PP composite {Halpin-Tsai equation}. (a) elements (١-٤); (b) elements (٥-٨); (c) elements (٩-١٢); (d) elements (١٣-١٦); (e) elements

CHAPTER SIX

Fig. (٥.٣١): Tensile creep of homopolymer PP at ٢٠ºC. Ref [٧]

Conclusions and Suggestions

٦.١ Conclusions

According to the mechanical properties of unreinforced polypropylene and short E-glass/PP composite in both aligned and random reinforcement, the following conclusions can be stated:

١. The region of glassy behavior of polypropylene polymer alone and polypropylene reinforced with short E-glass fibers in the forms of aligned and random reinforcement is limited by the time period (١-١٠٦) seconds with slight differences through this range.

٢. It can be observed that using a wider range of time either in minutes or hours may give three regions in the curves of mechanical properties (strain, modulus, and compliance) versus time, these regions are glassy, viscoelastic, and rubbery.

٣. All the mechanical properties that have been obtained from tensile and flexural creep estimations are identical to the time dependent typical behavior of polymer and **PMC**, where strains and compliances are increasing functions with time and the modulus curves are decreasing with time. This behavior reflects the success of the theoretical analysis that has been applied with its assumptions in the present work.

٤. Polypropylene polymer results are accurate and reasonable whereas the values of strain and compliance are the greatest and the values of modulus are the smallest; the low stiffness of polymers is the main objective of reinforcing them with fibers.

٥. The main objective of reinforcement is satisfied through this work, whereas the deformation of reinforced polymer is lowered as a result to fiber presence because the fibers bear the major portion of the load while the polymer matrix bears a minor portion of the load due to its low modulus.

٦. The reinforcement of polypropylene polymer by short E-glass fiber in its two forms, aligned and random, gives low values of strain and compliance and improves modulus according to Halpin-Tsai equation.

٧. Element ١٩ from tensile specimen and node ١٤ from flexural specimen are excess over the limits of linear viscoelastic behavior because these locations are the nearest to the externally applied loads, and hence the regions of fast deformation, weakness, and consequently failure.

٨. Halpin-Tsai equation is more accurate than modified rule of mixtures in the theoretical prediction of the mechanical properties of the understudy composite material reinforced with short fibers.

٩. The longitudinal properties of aligned short fiber composite that were calculated from modified rule of mixtures are inaccurate and the effect of reinforcement is weak where the deformations of aligned composite are very close to the results of unreinforced polymer and the moduli are lesser.

٦.٢ Suggestions for Future Work

١. Experimental work should be investigated and compare it with the present theoretical study.

٢. Studying the creep behavior in tensile and flexural tests as a function of temperature.

٣. Extending the time period into minutes and hours to obtain the full dependence on time that represented by (glassy, viscoelastic, and rubbery) behavior.

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