التحليل اللاخطي للركائز تحت تأثير الاحمال الستاتيكيه و الديناميكيه

اطروحه

مقدمه الى كلية الهندسه

في جامعة بابل كجزء من متطلبات نيل

شهادة الماجستير في علوم الهندسه المدنيه

من قبل

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كانون الثاني٢٠٠٦

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NONLINEAR STATIC AND FREE VIBRATION ANALYSIS OF DRIVEN PILES

thesis

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Ahmed Noor Mohammed

LIST OF CONTENT

List of Symbols

A	**Cross sectional area**
[C]	**Damping matrix**
DO.F.	**Degree of freedom**
E	**Modulus of elasticity**
f	**Cyclic natural frequency.**
I	**Moment of Inertia**
K_s	**Subgrade reaction**
K_f	**Frictional subgrade reaction**
[Ki]	**Incremental stiffness matrix**
$[k_{٠}]$	**conventional linear stiffness matrix**
$[K_p]$	**Initial stress matrix**

K١] and [K٢] **Nonlinear stiffness matrices**

[K_T] **Tangent stiffness matrix**

L **Length**

M٠ **Mass of pile material per unit length**

[M] **Mass matrix**

[M_a], [M_r], [M_t] **Cosistent mass matrix for Axial,Rotational and translation inertia respectively.**

P **Axial Force**

V **vertical displacement**

U **The strain energy stored in the pile-soil system.**

T **Natural period(١/f)**

ω **Natural frequency**

Θ **rotation Angle**

Δ **Deformation**

ρ $\sqrt{\frac{K_f}{EA}}$

Note :Any other symbol may be explained where it appears in the text.

الخـلاصـــة

في هذه الدراسة ,تم تناول التحليل النظري اللاخطي للركائز المسوقه داخل التربه و المتاثره بانواع مختلفه من الاحمال باستخدام طريقة العناصر المحدده(Finite Element Method), الدراسة تتضمن جزئين ,تحليل استاتيكي و تحليل ديناميكي .

في التحليل الأستاتيكي ,علاقة القوة-التشوة اوجدت اعتمادا على تقنية الحمل المتزايد و المسيطر عليه بأستخدام تكرارات نيوتن- رافسون (Newton-Raphson Technique) لتحقيق التوازن في نهاية كل مرحله من مراحل التحميل.

في هذه الدراسة اشتقت مصفوفة الجسائه للركيزه تحت تاثير العزم و القوه الجانبيه اضافة الى القوة المحوريه.

كما تضمن الاشتقاق التأثير الهندسي اللاخطي.مثلت التربه بنوعين من النوابض جانبيه و اخرى احتكاكيه.

في تحليل الاهتزاز الحر تم استخدام طريقة التكرار المعكوس (Inverse Iteration) لحساب التردد الطبيع Natural) للركائز.و قد تم التعبير عن الخواص الديناميكيه باستخدام مصفوفتي الكتله المتجمعه و الكتله المتوافقه (Lumped and consistent) للتعبير عن خواص الكتله.

عمل نموذج اختباري للفحص لغرض ايجاد التردد الطبيعي و يجاد مدى مطابقته للتحليل النظري .

في هذه الدراسه تم تطور برنامج حاسوب بلغة(Quick Basic ٤.٥) لاستيعاب حالات عدة من التحميل و طرق اسناد مختلفة,و لقد تم تقيم أداء البرنامج و مدى الدقه من خلال تحليل مجموعه من الامثله التي تتضمن بيانات لمسائل ركائز, كنتيجه للاطروحه تم التوصل الى عدت استنتاجات, واحده من هذه الاستنتاجات ان الجسائه اللجانبيه للتربه تؤثر حمل الفشل للركيزه فهي تزيده بمقدار (٪٧.٥).

ABSTRACT

In the present study, a theoretical non-linear analysis of driven piles subjected to different types of static loading is presented. Finite element method is used. The study includes two parts, static analysis and free vibration analysis.

In the static analysis, the load-deflection relations are obtained by using an incremental-iterative technique based on Newton-Raphson (N-R) methods for iteration process. Exact stiffness matrix for prismatic pile elements under moment as well as axial force and lateral force is derived in the present study. Two types of springs represent the soil effect frictional and lateral. Geometric nonlinearity is also included in the derived stiffness matrix. In free vibration analysis, the inverse iteration method was used. System mass matrix has been represented by using both of consistent and lumped models. Test is carried out in the present study in order to find the natural frequency experimentally.

In the present study, a computer program of (Al-Bidairi) is modified in Quick Basic ٤.٥ language to deal with the considered problem. This program is used to analyze several cases of piles those where tested theoretically and experimentally by others. Consequently, it is that average difference in results is (٦.١٪) for static analysis and (١.٥٪) for free vibration analysis.

As a result of this investigation, several conclusions are obtained. One of these conclusions, that the lateral soil stiffness may increase the failure load of the pile under vertical load to about (٧.٥٪) for sand.

CHAPTER ONE

INTRODUCTION

١-١ General

Soil-structure interaction is an important topic in the development of a performance based design procedure. With the rapid advances of computing technology, finite element analysis is assuming as a more important method in engineering practice. The advantage of the finite element analysis lies in its ability to accommodate complex soil stratigraphy and its potential for solving three-dimensional soil-structure interaction problems. However, to be successfully it is used in practical design; the soil model should be simple and can be easily calibrated by conventional field or lab testing. On the other hand, the model should be able to realistically capture the most important aspects of soil-structure nonlinearities [١].

Piles can be defined as slender foundation i.e. having relatively small cross sectional dimensions with respect to their length .The most important function of the piles is to transmit loads to a deep soil layer which has the required bearing capacity. There are several cases when pile represents the most economic a structural solution from which [٢]:

a- A soil layers having a reliable bearing capacity can be found at a great depth only.

b- The layers immediately beneath the structure can be washed out, scours may occur.

c- For construction where the superstructure transmits great concentrated loads to the foundation.

d- For structures transmitting unusually high vertical and /or horizontal loads.

e- For structures which are very sensitive to unequal settlements.

f- For offshore constructions.

g- For sites with a high ground water table.

١-٢ Non linear behavior of structures

The behavior of a deep foundation depends on a set of complex factors such as the nonlinear constitutive behavior of soils including the effect of pore water pressure, soil pile-superstructure interaction including slip and separation at the pile-soil interface, characteristics of the loading, superstructure compliance …etc. When the amplitude of loading is large, most of these factors control the behavior. Accuracy and reliability of the predicted behavior depend not only on the analysis method employed, but also on the accuracy with which the model parameters (or soil properties) are determined.

However the non linear behavior of a body or a structure occurs in two different forms. The first is ***material or physical nonlinearity*** which results from the nonlinear stress-strain relations and in which only small displacements and small strains are considered .The second is the ***geometric nonlinearity*** which results from the nonlinear strain –displacement relations and when the large displacements are considered .In the present study, only the geometrical nonlinearity will be considered.

In comparison with a linear problem, that has a constant stiffness matrix, the stiffness matrix of the geometrically nonlinear problem contains terms that are nonlinear functions of the deformations of the structure, and this matrix is called the ***tangent stiffness matrix*** [٣].

١-٣ Dynamic analysis of piles

All real physical structures, when subjected to loads or displacements, dynamically. The additional inertia forces, ***from Newton's second law***, behave the mass times the acceleration. If the loads or displacements are are equal to slowly then the inertia forces can be neglected and a static load applied very justified. Hence, static analysis is a simple extension of dynamic analysis can be analysis.

In addition, all real structures potentially have an infinite number of displacements. Therefore, the most critical phase of a structural analysis is to model, with a finite number of massless members and a create a computer displacements that will simulate the behavior of finite number of node (joint) structural system, which can be accurately the real structure. The mass of a for linear elastic structures the estimated, is lumped at the nodes. Also, experimental data, can be stiffness properties of the members, with the aid of the dynamic approximated with a high degree of confidence. However, conditions loading, energy dissipation properties and boundary (foundation) cases of for many structures are difficult to estimate. This is always true for the seismic input or wind loads.

The dynamic analysis of linear elements with distributed mass, such as a pile, rod, shaft, beam or beam-column have been extensively investigated over the past decades.

The dynamic loads on piles may act due to construction operations (pile driving),operation of reciprocating and rotary machines and hammer ,earthquakes, loading due to wave action of water, bomb blasts, fast moving traffic, or loading due to wind. The nature of each of these loads is quite different from the nature of the loads in the other cases [٢].

The present study deals with the free vibration analysis to determine the fundamental natural frequency.

١-٤ Finite Element Analysis of piles.

The finite element method is a numerical procedure for obtaining solutions to a large number of the problems encountered in engineering analysis .It has two primary subdivisions. The first utilizes discrete elements to obtain the joint displacements and a member of forces of a structural framework. The second uses the continuum elements to obtain approximate solutions to heat transfer, fluid mechanics and solid mechanics problems.

The finite element method combines several mathematical concepts and may include a system of linear or non linear equations .The number of equations is usually very large, may be from ٢ to ٢٠,٠٠٠ or more and requires the computational power of the digital computer .The method has a little practical value if a computer is not a available[٤].

١-٥ Aims and Scope:

In heartland and the south of Iraq, the soil layers consist the sedimentation layers with low bearing capacity in addition to the high water table. Piles represent the most economic structural solution for this problem for large structures such as multistory buildings, bridges and water tanks...etc.

The main aims of the present work are as follows:

١-Derive an exact stiffness matrix and mass matrix for prismatic pile element subjected to moment, horizontal load and vertical load, including the geometric nonlinearity.

٢- Fabricate a model to test the natural frequency experimentally.

٣-Present a comprehensive study about the geometrically nonlinear finite element analysis of pile under different kinds of static and free vibration analysis.

٤-Study the effect of soil properties on the structural behavior.

The thesis includes seven chapters: the present chapter is an introductory one. Chapter two is concerned with literature review. Chapter three includes formulation and derivation of stiffness and mass matrices related to the finite element method. Chapter four contains computational techniques and numerical solutions. Chapter five introduces the experimental test to find the natural frequency.

In chapter six, several application, and verifications of the adopted method of analysis are presented.

Finally, in chapter seven the conclusions and the recommendations are written.

CHAPTER TWO

LITERATURE REVIEW

This review covers two kinds of literatures; the first one includes the development and the methods in studying the analysis of pile and the second shows the studies about beam –column on elastic foundation.

٢-١ Pile Analysis Researches:

Rausche et.al. (١٩٩٢) [٦] investigated the dynamic soil resistance on pile and presented a pure analysis program for the prediction of pile stresses and blow counts of a pile driven by an impact hammer. The GRLWEAP software was shown to produce good simulations of hammer and pile behavior .For accurate predictions, a good knowledge of both the static and dynamic soil resistance behavior must also exist.

The commonly used wave equation program (GRLWEAP is based on the earlier introduced WEAP program [Goble, Rausche ١٩٧٦]) offers several options for soil damping calculation.

The paper investigates various damping models and compares results. It compares GRLWEAP calculated force –velocity histories and evaluates the sensitivity of the bearing graph results relative to the various damping models.

Paikowsky et.al (١٩٩٤) [٧] worked out a dynamic analysis of plugged pipe pile in clay. A new approach was investigated in witch the spatial stress transformation within the soil plug was modeled using an axi-symmetric wave propagation formulation .A two-dimensional finite difference solution was developed for that formulation .This numerical solution was implemented in a computer program called PWAR(plug wave analysis program), a case study was then used to examine the applicability of the solution and to determine if the static capacity of the pile could be predicted more accurately. The PWAP analysis was performed on a pipe pile driven in empire clay using dynamic records taken from a well documented case study.

Rădulescu et.al (١٩٩٦) [٨] studied pile foundation design conditions for two fixed coupled offshore structures on the Romanian Black Sea continental platform. This paper described the design conditions for a special offshore platform operating on the Romanian Black Sea continental plateau. This offshore platform consists of two fixed jackets connected at the operating level. One of these jackets has been built few years ago, the design of its four - pile foundation representing, for the time being, a current work.

In order to fully use the existing facilities and to better exploit the oil deposit it has been decided to joint a new jacket to the first one.

The results of the checking of the piles belonging to the existing jacket under the new load conditions and of the sizing of the piles of the added jacket are presented.

Daniel et.al (١٩٩٨) [١٠] performed soil-pile-superstructure interaction in liquefying sand and soft clay this dissertation describes the results of a study on the dynamic response of pile foundations in liquefying sand and soft clay during strong shaking. The research consisted of: (١) a series of dynamic centrifuge tests of pile supported structures; (٢) a critical study of modeling techniques and limitations; (٣) back-calculation of p-y behavior; and (٤) comparison of pseudo-static analyses to the dynamic centrifuge model tests.

These dynamic model tests were among the first performed using the new shaking table on the ٩ m radius centrifuge at UC Davis. The results of the modeling study presented here will benefit other current and future projects utilizing the large centrifuge.

Lee et.al (١٩٩٩) [١١] performed numerical analysis of a piled foundation in idealised granular material. The first part of this research is aimed at modeling the basic case of a pile in a granular material. In this study, the pile was assumed to be an end bearing pile, not a friction pile. The slip elements were, therefore, applied around the pile in the numerical analysis using Crisp. The numerical analysis was carried out in order to compare the displacement patterns and failure mechanisms in terms of strain fields with the laboratory ٢D pile load test using a photogrammetry technique. The new Mohr-Coulomb soil model based on linear elastic perfectly plastic model was used to introduce the dilation effect into this idealized granular material. In connection with the new Mohr-Coulomb soil model, all parameters were assumed. Stiffness and dilation angle were then changed to approach the real experimental behavior of this idealized material.

Jamaa et.al (٢٠٠٠) [١٢] introduced a method for three dimensional interaction analysis of pile –soil system in time domain. They presented a new numerical approach to analyze the pile-soil interaction system taking into account the effects of the behavior of the soil at infinity and at the same time accounting for the non-linearity exists in the soil adjacent to the pile. Hence, it is a time domain analysis. It is an accurate and efficient approach to analyze the pile –soil system tacking into account the presence of near-pile nonlinearities and in the same time considering the effects of soil behavior at infinity.

Rausche et.al (٢٠٠٠) [١٣] presented combining pile design and dynamic installation analysis. This procedure requires soil resistance vs. depth input which thus far had to be pre calculated in a separate analysis either manually or using another computer program. The program has been expanded to include this pre-analysis .This paper presented the method itself, its correlation with static tests and another similar method. It also briefly discussed limitations and special consideration that make this method somewhat different from other static pile analysis formulas. In this program the analysis deals with pile element without considering the geometric non-linearity. However, for meaningful results, this so-called drivability analysis requires a much more detailed soil parameter input than the original approach.

Yang et.al (٢٠٠٠) [١٦] Presented the numerical analysis of pile behavior under lateral loads in layered elastic-plastic soils. That paper represented results from a finite element study on the behavior of a single pile in elastic-plastic soils. The pile behavior in uniform sand and clay soils as well as cases with sand layer in clay deposit and clay layer in sand deposit were analyzed and cross compared to investigate layering effects. . Finite element results were used to generate p - y curves and then compared with those obtained from methods commonly used in practice. Based on the results presented, it is concluded that three dimensional finite element analysis using very simple elastic-plastic soil models can predict the pile head defection with a very good accuracy.

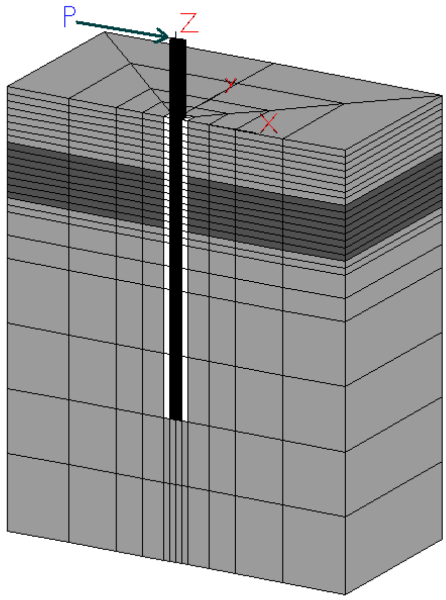


Figure ٢-٢: Mesh of Single Pile Model, Side View and Top Eight Layers of Finite Elements as Yang and Jeremic Represent Their Model

Khodair et.al (٢٠٠١) [١٤] Introduced the analysis of pile soil interaction and presented a ٣D finite element (FE) model of the Scotch Road integral abutment bridge located in Trenton, New Jersey. The bridge has been instrumented during construction, and experimental data that are used to verify the FE model. Herein, the model has been used to study the substructure, which consists of an integral abutment supported by a single row of piles. The behavior of piles has been studied extensively using both laboratory tests and theoretical studies.

Rausche (٢٠٠٢) [١٥] introduced modeling of the pile driving. This paper summarizes current analytical method and explains how wave equation analysis can be used in a manner comparable to the analysis of impact driven piles. Hammer and soil modeling details are discussed with a few examples demonstrate capabilities and limitation of methods and for the wave equation approach sensitivity of results to important soil resistance parameter. The wave equation approach offers a rational means of analysis over a wide range of hammer frequencies and a simple approach to representing soil resistance forces.

Alwash (٢٠٠٢) [٢] introduced the structural behavior of pile during driving. In this research, the analysis of the pile driving problem is performed by using finite element method. Two models for idealized the pile –soil system were adopted. The first one is the lumped spring model. In this model the pile is represent by a series of truss element while the soil is idealized by frictional springs lumped at the nodes and bearing spring connected to the last node. The other model is the consistent spring model. In this model the frictional soil springs are distributed uniformly along the elements of the pile that embedded into soil to form pile element. Newmark method is used to show the time dependent dynamic problem.

Jeong et.al (٢٠٠٢) [١٧] studied simplified ٣D analysis of laterally loaded pile groups and presented a simple and concise algorithm to analyze laterally loaded three-dimensional pile groups which proposed by considering both pile-soil-pile and pile-cap interactions. Beam-column method, one of the most practical approaches, was used for numerical modeling of the pile-soil-pile system. The nonlinear characteristics of the pile soil- pile interaction are modeled by discrete nonlinear load transfer curves, which account for the group interaction effects (p-multiplier concept). In addition to the pile-soil-pile interaction, an analytical method is developed to consider the group effect that comes from the pile-cap interaction including geometric configuration of the piles in a group and connectivity conditions between piles and the cap.

Zhang et.al (٢٠٠٢) [١٨] presented a Nonlinear Dynamic single pile-soil interaction analysis, This paper presented a method to take into account the effects of Nonlinearities due to both strain-induced and pore water pressure induced softening which are important aspects in dynamic pile-soil interaction. While the model of beam on Winkler foundation is widely used for its analysis, the above nonlinearities have not yet been satisfactorily represented.

In which soil response is expressed in terms of multiple steady state modal shape functions. By using adequate number of these steady-state shape functions, the time-domain response can be captured.

Choi et.al (٢٠٠٢) [١٩] performed the finite element analysis of pile-porous media Interaction presented the elasto-plastic bounding surface constitutive model with damage evolution as a property of cohesive soil is proposed and described as incremental relations between the changes of effective stress and strain of the soil. Finite element spatial discritization, time integration, implementation of the model and the iterative procedures for solving the nonlinear incremental formulation are described. From the finite element code developed as u-p-U form, based on the porous media theory and bounding surface plasticity, the pile-soil interactive behavior is simulated during pile driving, consolidation, and subsequent loading. The numerically simulated results are compared with high quality field and experimental data.

P´ersio et.al (٢٠٠٣) [٢٠] performed dynamic axial response of single piles embedded in transversely isotropic media. In this research, the response of piles with circular section embedded in transversely isotropic half spaces to time harmonic axial loads is analyzed. The pile is modeled by the finite element technique as a series of bar elements. The soil is modeled by an indirect formulation of the Boundary Element Method. The BEM formulation uses influence functions which are displacements and stresses due to loads distributed along circular and cylindrical surfaces inside a transversely isotropic elastic half space.

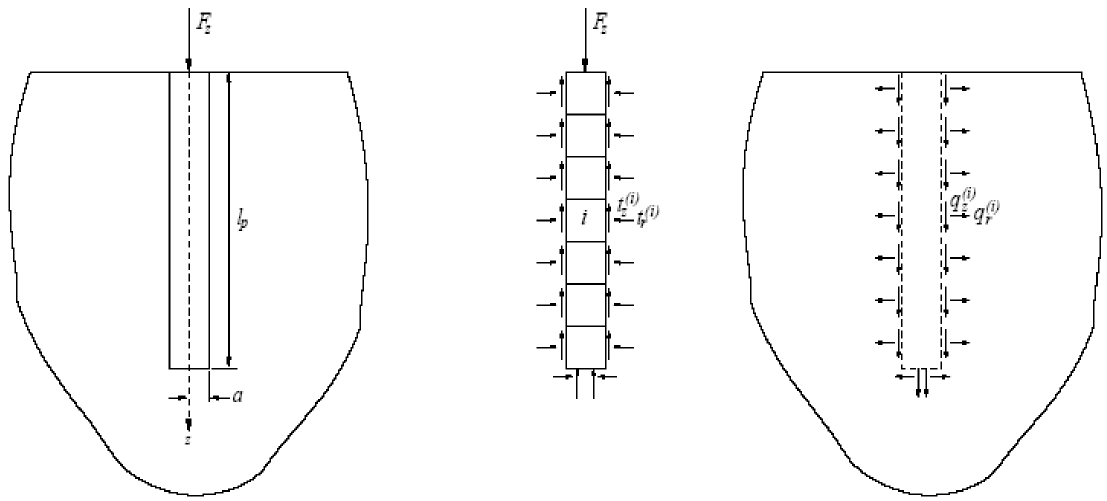


Figure ٢-٣ :P´ersio and Barros١ Model

This approach has two main advantages: only the soil-pile interface needs discretization and the load sources can be applied directly along the true pile surface. The coupling between the two models is set at the middle point of the bar elements, rather than at the end points. This leads to a much more stable numeric solution and to a smoother load transfer profile. The results for the isotropic case are compared with numerical values obtained by other researchers. The influence of the soil anisotropy is addressed.

Rausche et.al (٢٠٠٣) [٢٢] studied the dynamic determination of the pile capacity with a presentation for determining the axial static pile capacity from dynamic measurements of force and acceleration made under the impact (if a large hammer). The basic equation for calculation of the force resisting pile penetration is derived. The limitations of the basic resistance equation are discussed. It is possible to prove that the Case Pile Wave Analysis Program (CAPWAP) resistance force distribution is unique. Using the assumption that the resistance to penetration can be divided into static and dynamic parts, an expression is developed for calculating the dynamic resistance to penetration. The resulting method requires the selection of a “damping” constant which is shown empirically, to relate to soil size distribution .A correlation of the case method capacity and the capacity observed in static load test is given for ٦٩ statically tested piles that were also tested dynamically.

Maharaj et.al (٢٠٠٣) [٢٣] introduced the effect of raft size and pile length on load-settlement behavior of ax symmetric piled raft foundation. The nonlinear finite element analyses have been done to see the effect of raft size and pile length on the load settlement behavior of ax symmetric piled raft foundation.

Analyses have been done by NLAXIFEM-Nonlinear ax symmetric finite element software. The piles in the piled raft foundation have been represented as an equivalent annulus of same volume. The raft, pile and soil have been discretized into four nodded isoperimetric finite elements. The soil has been modeled as Von Mises elastoplastic medium. The load carrying capacity of raft foundation is found to increase with an increase in the size of the raft.

Wang et.al (٢٠٠٤) [١] discussed numerical analysis of piles in elasto-plastic soils under axial loading. A finite element model was developed to simulate nonlinear response of pile /drilled piers under axial loading. The nonlinear stress-strain behaviors of soils are modeled via Drucker-Prager and von Mises type plasticity. A parametric study is carried out to address the influence of various factors, such as soil friction approximation, effect of interface element and shear strength profile …etc. on the prediction of pile behavior in elasto - plastic soils and key issues in the simulation are critically reviewed. Soil-structure interaction is an important topic in the development of a performance based design procedure.

Miyamoto et.al (٢٠٠٤) [٢٥] studied Seismic design of a structure supported on pile foundation considering dynamic soil- structure interaction In this paper the pile foundation responses were clarified by experimental studies using ground motions induced by large-scale mining blasts and nonlinear analyses of soil-pile foundation-superstructure system.

Nonlinear responses of the soil-pile structure system were obtained for various levels of liquefaction in the test pit.

The vibration test method employed in this research was found to be very useful and effective for investigating the dynamic behavior of large model structures under severe ground motions.

Wang et.al (٢٠٠٥) [٢٦] introduce the nonlinear analysis of drilled piers under dynamic and static axial loading, This paper presented the results of numerical simulation of a series of static and dynamic tests on drilled piers. It implemented a nonlinear soil model based on multi-axial cyclic bounding surface plasticity within a general finite element.

The model requires a minimal number of parameters that can be easily obtained through conventional site investigations. The results of the simulations show that the model can reasonably capture modulus reduction and hysteretic damping, the nonlinear response of the soil, and the model is suitable for fully nonlinear analysis of soil-pile system under multi-directional shaking.

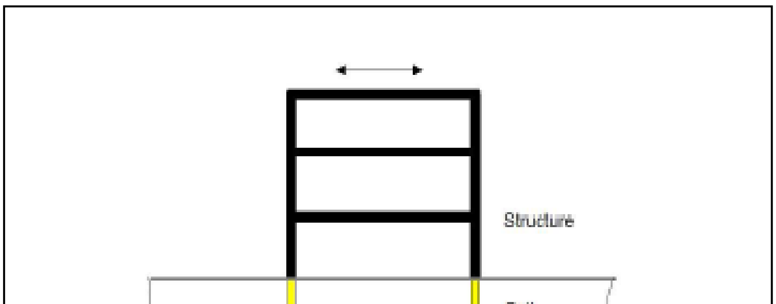


Figure ۲-٤ Foundation soil structure system used by Wang and Sitar.

Ramachandran (۲۰۰٥) [۲۷] discussed analysis of pile foundations under seismic loading. This research studies the interaction between piles in pile groups accounting for soil nonlinearity and pile-soil gapping effects. A ۳D finite element analysis is used in this research.

٢-٢ Beam-Column Analysis Researches:

Sirosh et.al (١٩٨٩) [٥] introduced a general finite element for beam –columns on elastic foundation and presented the analysis of beam on elastic foundation under large axial compressive or tensile force. The reduction or increase in bending stiffness of beam in case of large compressive or tensile force are presented, which is sometime referred to as the beam –column or the P-Delta effect, and it may be treated by a linear analysis if the magnitude of the axial force is assumed to remain constant during bending. This assumption is adopted here to derive the stiffness matrix and equivalent nodal loads of individual members.

A general finite element is derived for beam or beam -column with or without a continuous Winkler type elastic foundation. The need to discretize members to shorter elements for convergence towards an exact solution is eliminated by employing in the derivation of the element exact shape functions obtained from the equation of the elastic line.

Aristizabal et.al (١٩٩٧) [٩] studied classical stability of beam columns with semi rigid connections on elastic foundation, this study explains The stability analysis of beam-columns under compressive end loads with end sides ways uninhibited, partially inhibited, and totally inhibited, including the effects of semi rigid connections and a uniformly distributed lateral elastic foundation (Winkler's type) along its entire span is presented in a classical manner.

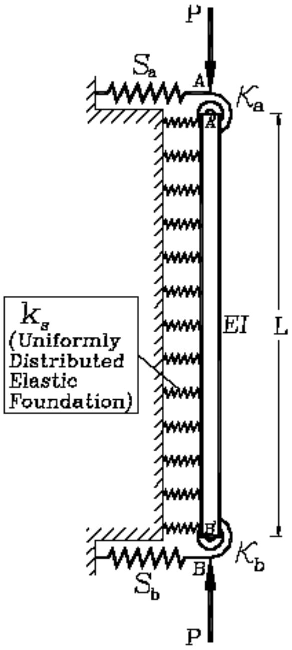


Figure ٢-١: Model of Aristizabal and Ochoa

The stability analysis consists of an eigen-value solution to a ٤×٤ matrix for a column with end side sway uninhibited or partially inhibited at both ends, and to a ٣×٣ matrix for column with end side sway inhibited at one or both ends, respectively. The effects of end lateral bracing at one or both ends of the beam-column on its buckling load are presented and analyzed.

Al Khafaji (٢٠٠٣) [٢١] introduced a Large displacement and a post –bulking analysis of plane steel structures with non-prismatic members resting on elastic foundation. This thesis presents analysis adopts the beam-column approach and models the structure's elements as a beam-column elements. The formulation of the beam-column elements are based on the Eulerian approach, taking into account the influence of axial force on the bedding stiffness. Also, changes in the member chord length due to axial deformation and flexural bowing are taken in to account.

Brandenberg et.al (٢٠٠٤) [٢٤] studied Open Sees Beam on Nonlinear Winkler Foundation Modeling of Pile Groups in Liquefied and Laterally Spreading Ground in Centrifuge Tests. This paper presented the beam on nonlinear Winkler foundation (BNWF) analyses of pile groups in liquefied and laterally spreading ground were performed by using the Open Source for Earthquake Engineering Simulation (Open Sees). Piles were modeled using elastic beam-column elements.

٢-٣ THE SUMMARY

The nonlinear finite element analysis of piles driven into elastic soil and subjected to vertical as well as lateral loads, including the geometric nonlinearity is not considered in the available literature review and will be presented in the present study. Also, free vibration analysis of such piles is considered in this research.

CHAPTER THREE

THEORY AND DERIVATION

٣-١ INTRODUCTION

In this chapter, a theoretical basis for the analysis of piles subjected to static and dynamic loads is included. The pile is assumed to carry vertical load as well as transverse loads. At this chapter, Winkler model is adopted to present the analytical model of the elastic foundation and in the finite element approach .The chapter deals also with the derivation of tangent stiffness matrix .The nonlinear stiffness matrix of the considered pile element is composed of the beam-column on elastic foundation element derived in the present study and that of the pile element under axial force derived by Alwash. The derivation of stiffness matrix of the beam –column on elastic foundation is presented in this study. Also the derivation of the mass matrix for the beam –column on elastic foundation is also presented.

٣-٢ Winkler Model

The sugared reaction model of the soil behavior was originally proposed by Winkler (١٨٦٧); characterizes the soil as a series of unconnected linearly elastic springs. The deflection at any point (or any spring) occurs only where loading exists at that point. The obvious disadvantage of this soil model is the lack of continuity. Real soil at least to some extent is continuous. A further disadvantage is that the spring modulus of the model (modulus of subgarde reaction) is dependent on the size of foundation. In spite of these drawbacks, the subgarde reaction approach has been widely employed in foundation practice because it provides a relatively simple means of analysis and enables factors, such as non-linearity, variation of soil stiffness with depth and layering of the soil profile to be taken into account. Soil configuration can be presented by using two types of subgarde reaction modulus along the foundation length, the normal and the tangential.

The normal sugared reaction modulus is defined as the load required to acting normally on a unit area of the elastic foundation to produce a unit normal displacement. The tangential subgarde reaction modulus is defined as the load required to acting tangentially on a unit area of elastic foundation to produce a unit horizontal displacement.

٣-٣ Static analysis

The stiffness matrix for beam on elastic foundation including the geometric nonlinearity is presented in this chapter. Then, the stiffness matrix of axial pile is combined with beam-column on elastic foundation matrix in order to find the final pile stiffness matrix under any type of loading.

٣-٣-١ finite element approach

In this approach, the tangential incremental stiffness matrix in Lagrangian coordinates system has been derived by using the principle of stationary potential energy which leads finally to the expression [Chajes and Churchill ١٩٨٧]:

$$[K_i]\{\Delta X\} = \{\Delta P\} \qquad (٣-١)$$

Where [Ki] is the incremental stiffness matrix that relates the incremental displacements {ΔX} to the corresponding incremental element forces {ΔP}.

The incremental stiffness matrix in equation (٣-١) consists of the sum of four distinct matrices [٣٠], that is:

$$(٣-٢)\ [K_i] = [K_0] + [Kp] + \frac{AE}{2}[K_1] + \frac{AE}{3}[K_2]$$

in which

$[k_{.}]$ = conventional linear stiffness matrix.

$[K_p]$ = initial stress matrix, which is linear function of the axial force presented at beginning of the incremental steps.

$[K_1]$ and $[K_2]$ = nonlinear stiffness matrices and quadratic functions of the incremental element displacements respectively.

In some instances the nonlinear effects due to the change in element force and displacement occurring during the incremental step are very small compared with the nonlinear effects of the forces and deformations that exist at the beginning of the step. When this is the case, $[K_1]$ and $[K_2]$ can be neglected and the stiffness matrix takes the form:

$$[K_i] = [K_0] + [K_p] \qquad (٣-٣)$$

Since this stiffness matrix, $[k_i]$, depends only on the internal forces and deformations existing at the beginning of the load step, it is commonly referred to as the tangent stiffness matrix, $[K_T]$.

٣-٣-٢ FORMULATION OF STIFFNESS MATRIX:

According to castigliano's first theorem, for an element having strain energy U, the force F exists at node i where the nodal degree of freedom e presents is:

$$(٣-٤)\ F_i = \frac{\partial U}{\partial e_i}$$

Castigliano's second theorem indicated that the stiffness coefficient K_{ij} of such element is (j is the other node on element):

$$(٣-٥)\ K_{ij} = \frac{\partial F_i}{\partial e_j} = \frac{\partial^2 U}{\partial e_i \partial e_j}$$

The strain energy for flexural element is:

$$(٣-٦)\ U = \int_0^l \frac{EI}{2} \left(\frac{d^2 y}{dx^2}\right)^2 dx$$

if EI is constant then equation(٣-٦) becomes:

$$(٣\text{-}٧)\ U=\frac{EI}{2}\int_0^l\left(\frac{d^2y}{dx^2}\right)^2 dx$$

By differentiation with respect to e_i:

$$(٣\text{-}٨)\ \frac{\partial U}{\partial e_i}=EI\int y''\frac{\partial y''}{\partial e_i}dx$$

Then, differentiation with respect to e_j:

$$(٣\text{-}٩)\ \frac{\partial^2 U}{\partial e_i\partial e_j}=EI\int\left(y''\frac{\partial^2 y''}{\partial e_i\partial e_j}+\frac{\partial y''}{\partial e_j}\frac{\partial y''}{\partial e_i}\right)dx$$

Now:

$y=a_١ f(x)_١+a_٢ f(x)_٢ +a_٣ f(x)_٣ +a_٣f(x)_٤$ (٣-١٠)

$$(٣\text{-}١١)\ y=\begin{bmatrix}a_1 & a_2 & a_3 & a_4\end{bmatrix}\begin{Bmatrix}f(x)_1\\ f(x)_2\\ f(x)_3\\ f(x)_4\end{Bmatrix}$$

$$(٣\text{-}١٢)\ y=[a]\{Z(x)\}$$

Applying the boundary conditions that:

at x=٠ then $y=y_i$, $y`=\theta_i$

at x = l then $y = y_j$, $y` = \theta_j$

(٣-١٣) $\{e\} = [R]\{a\}$

(٣-١٤) $\{a\} = [R]^{-1}\{e\}$

Let $[R]^{-1} = [G]$ (٣-١٥)

(٣-١٦) $\therefore\ y = [Z(x)][G]\{e\}$

(٣-١٧) $y'' = [Z''(x)][G]\{e\}$

is linear function of (e) then ″Since y

(3-18) $\frac{\partial^2 y''}{\partial e_i \partial e_j} = 0$

Substitute from equation (3-18) in equation (3-8) then

(3-19) $K_{ij} = \frac{\partial^2 U}{\partial e_i \partial e_j} = EI\int_0^l \frac{\partial y''}{\partial e_i} \cdot \frac{\partial y''}{\partial e_j} dx$

(3- $[K] = EI\int_0^l \{\frac{\partial y''}{\partial e}\}_{4x1} \left\lfloor \frac{\partial y''}{\partial e} \right\rfloor_{1x4} dx$

20)

Then, in a detailed matrix form:

(3-21)
$$[K] = EI\int_0^l \begin{Bmatrix} \frac{\partial y''}{\partial e_1} \\ \frac{\partial y''}{\partial e_2} \\ \frac{\partial y''}{\partial e_3} \\ \frac{\partial y''}{\partial e_4} \end{Bmatrix} \left\lfloor \frac{\partial y''}{\partial e_1} \quad \frac{\partial y''}{\partial e_2} \quad \frac{\partial y''}{\partial e_3} \quad \frac{\partial y''}{\partial e_4} \right\rfloor dx$$

to find $\frac{\partial y''}{\partial e_i}$

(3-22) $$y'' = \left\lfloor Z''(x) \right\rfloor [G]\{e\}$$

(3-23) $$y'' = \lfloor e \rfloor [G]^T \{Z''(x)\}$$

(٣-٢٤) $$\begin{Bmatrix} \frac{\partial y''}{\partial e} \end{Bmatrix} = [I][G]^T \{Z''\}$$

(3-25) $$\begin{Bmatrix} \frac{\partial y''}{\partial e} \end{Bmatrix}_{4x1} = [G]^T_{4x4} \{Z''\}_{4x1}$$

(٣-٢٦) $$\left[\frac{\partial y''}{\partial e} \right]_{1x4} = \left[Z'' \right]_{1x4} [G]_{4x4}$$

Sub Eq (3-26) and Eq (3-25) in Eq (3-21):

(3-27)
$$[K] = EI\int_0^l [G]^t . \{Z''(x)\} \left\lfloor Z''(x) \right\rfloor . [G] dx$$

[G] is constant

$$(3-\therefore [K] = EI[G]^t \int_0^l \{Z''(x)\} \lfloor Z''(x) \rfloor .dx.[G]$$

28)

٣-٣-٣ Derivation of stiffness matrix for the Beam - column element on Elastic Foundation.

A beam column of length L, cross section area A and modulus of elasticity E and x is the distance from beam head, is laying on soil having a variable side resistance spring constant K_s. If a static axial load P, lateral load ΓP (where n is ≥٠) and moment M are applied at first end.

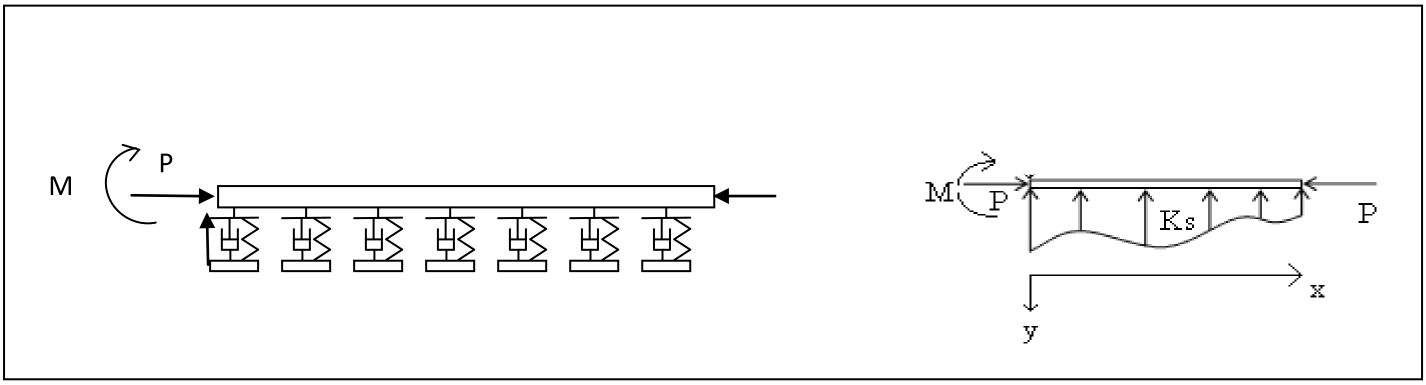


Fig (3-1) beam on elastic foundation with applied loads

The differential equation of the beam-column on the elastic foundation based on the principle of minimum potential energy (**Aristizabal and Ochoa (١٩٩٧)**) is:

$$(٣\text{-}٢٩)\ EI\frac{d^4y}{d^4x} + P\frac{d^2y}{d^2x} + k_s y = 0$$

By solve the Eq.(٣-٢٩) :

$$(٣\text{-}٣٠)\ EI\ m^4 + P\ m^2 + k_s = 0$$

$$(٣\text{-}٣١)\quad m_{1,2,3,4} = \pm\sqrt{\frac{P}{2EI} \pm \sqrt{\left(\frac{P}{2EI}\right)^2 - \frac{k_s}{EI}}}$$

$$y = A e^{m_1 x} + B e^{m_2 x} + C e^{m_3 x} + D e^{m_4 x} \quad (٣\text{-}٣٢)$$

Where A, B, C and D are constants determined from the boundary conditions.

The solution of the differential equation (٣-٣٢) as **Aristizabal and Ochoa** mentioned at their paper:

For $0 \le P \le \sqrt{k_s EI}$:

$$(٣\text{-}٣٣)\quad y = \left(A e^{\alpha x} + B e^{-\alpha x}\right)\cos\beta x + \left(C e^{\alpha x} + D e^{-\alpha x}\right)\sin\beta x$$

And for $P \ge \sqrt{k_s EI}$:

$$(٣\text{-}٣٤)\quad y = A\cos\gamma x + B\cos\varphi x + C\sin\gamma x + D\sin\varphi x$$

Where

$$(٣\text{-}٣٥)\quad \alpha = \sqrt{\frac{\sqrt{k_s}}{4EI} - \frac{P}{4EI}}$$

$$\beta = \sqrt{\sqrt{\frac{k_s}{4EI}} + \frac{P}{4EI}} \quad (٣\text{-}٣٦)$$

$$(٣\text{-}٣٧)\quad \gamma = \sqrt{\frac{P}{2EI} - \sqrt{\left(\frac{P}{2EI}\right)^2 - \frac{k_s}{EI}}}$$

$$(٣\text{-}٣٨)\quad \varphi = \sqrt{\frac{P}{2EI} + \sqrt{\left(\frac{P}{2EI}\right)^2 - \frac{k_s}{EI}}}$$

This solution depends on three assumptions:

١-The pile material is homogenous, therefore the modulus of elasticity is constant.

٢-The stiffness of elastic foundation is constant (linear) along the length of Beam-column element.

٣-The axial load is constant.

٣-٣-٤ Tangent matrix for $0 \le P \le \sqrt{k_s EI}$:

$$y(x) = \begin{bmatrix} e^{\alpha x}\cos\beta x & e^{-\alpha x}\cos\beta x & e^{\alpha x}\sin\beta x & e^{-\alpha x}\sin\beta x \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = [z(x)]\{a\}$$

(٣-٣٩)

To find θ derive equation (٣-٣٤) with respect to x:

$$\begin{aligned} y' = {} & A\left(\alpha e^{\alpha x}\cos\beta x \quad \beta e^{\alpha x}\sin\beta x\right) + B\left(-\alpha e^{-\alpha x}\cos\beta x + \beta e^{-\alpha x}\sin\beta x\right) \\ & + C\left(\alpha e^{\alpha x}\sin\beta x + \beta e^{\alpha x}\cos\beta x\right) + D\left(\beta e^{-\alpha x}\cos\beta x \quad \alpha e^{-\alpha x}\sin\beta x\right) \end{aligned}$$

(٣-٤٠)

Apply boundary conditions:

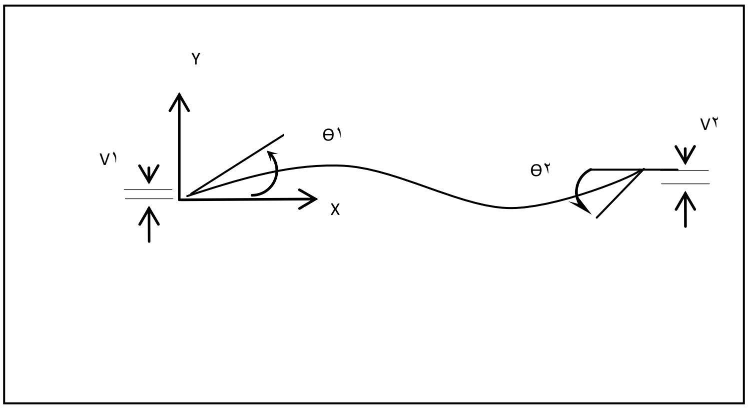


Fig (3-2) beam-column Deformations

At x=٠, $v = v_1$, $\theta = \theta_1$

At x=l , $v = v_2$, $\theta = \theta_2$

Substitute into equations (٣-٣٤) and (٣-٤٠)

(٣-٤١) $v_1 = A + B$

$\theta_1 = A\alpha \quad B\alpha + C\beta + D\beta$ (٣-٤٢)

(٣- $v_2 = \left(A\, e^{\alpha l} + B\, e^{\quad \alpha l}\right)\cos\beta l + \left(C\, e^{\alpha l} + D\, e^{\quad \alpha l}\right)\sin\beta l$

٤٣)

$$\theta_2 = A\left(\alpha\, e^{\alpha l}\cos\beta l \quad \beta\, e^{\alpha l}\sin\beta l\right) + B\left(\quad \alpha\, e^{\quad \alpha l}\cos\beta l \quad \beta\, e^{\quad \alpha l}\sin\beta l\right)$$
$$+ C\left(\alpha\, e^{\alpha l}\sin\beta l + \beta\, e^{\alpha l}\cos\beta l\right) + D\left(\beta\, e^{\quad \alpha l}\cos\beta l \quad \alpha\, e^{\quad \alpha l}\sin\beta l\right)$$

(٣-٤٤)

in matrices form:

$$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha & -\alpha & \beta & \beta \\ e^{\alpha l}\cos\beta l & e^{-\alpha l}\cos\beta l & e^{\alpha l}\sin\beta l & e^{-\alpha l}\sin\beta l \\ \alpha\, e^{\alpha l}\cos\beta l - & -\alpha\, e^{-\alpha l}\cos\beta l - & \alpha\, e^{\alpha l}\sin\beta l + & \beta\, e^{-\alpha l}\cos\beta l - \\ \beta\, e^{\alpha l}\sin\beta l & \beta\, e^{-\alpha l}\sin\beta l & \beta\, e^{\alpha l}\cos\beta l & \alpha\, e^{-\alpha l}\sin\beta l \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} \quad (٣\text{-} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix}$$

٤٥)

$$[R]=\begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha & -\alpha & \beta & \beta \\ e^{\alpha l}\cos\beta l & e^{-\alpha l}\cos\beta l & e^{\alpha l}\sin\beta l & e^{-\alpha l}\sin\beta l \\ \alpha e^{\alpha l}\cos\beta l- & -\alpha e^{-\alpha l}\cos\beta l- & \alpha e^{\alpha l}\sin\beta l+ & \beta e^{-\alpha l}\cos\beta l- \\ \beta e^{\alpha l}\sin\beta l & \beta e^{-\alpha l}\sin\beta l & \beta e^{\alpha l}\cos\beta l & \alpha e^{-\alpha l}\sin\beta l \end{bmatrix}$$

(٣-٤٦)

$$[G]=\begin{bmatrix} G11 & G12 & G13 & G14 \\ G21 & G22 & G23 & G24 \\ G31 & G32 & G33 & G34 \\ G41 & G42 & G43 & G44 \end{bmatrix}=[R]^{-1}$$

(٣-٤٧)

where

$$G11=\frac{2\alpha^2\sin^2\beta l-\beta^2e^{-2\alpha l}+2\alpha\beta\sin\beta\cos\beta l+\beta^2}{3\alpha^2\sin^2\beta l-\beta^2e^{2\alpha l}-\beta^2e^{-2\alpha l}+2\beta^2}$$

(٣-٤٨)

$$G21=\frac{-2\beta\sin\beta l\cos\beta l+2\alpha\sin^2\beta l}{3\alpha^2\sin^2\beta l-\beta^2e^{2\alpha l}-\beta^2e^{-2\alpha l}+2\beta^2}$$

(٣-٤٩)

$$G31=\frac{\beta^2\cos\beta l(e^{-\alpha l}-e^{\alpha l})-\alpha\beta\sin\beta l(e^{-\alpha l}+e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2e^{2\alpha l}-\beta^2e^{-2\alpha l}+2\beta^2}$$

(٣-٥٠)

$$G41=\frac{-\beta\sin\beta l(e^{-\alpha l}-e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2e^{2\alpha l}-\beta^2e^{-2\alpha l}+2\beta^2}$$

(٣-٥١)

$$G12=\frac{-2\alpha\beta\sin\beta l\cos\beta l+2\alpha^{2}\sin^{2}\beta l+\beta^{2}-\beta^{2}e^{2\alpha l}}{3\alpha^{2}\sin^{2}\beta l-\beta^{2}e^{2\alpha l}-\beta^{2}e^{-2\alpha l}+2\beta^{2}}$$

(٣-٥٢)

$$G22=\frac{-2\alpha\sin^{2}\beta l}{3\alpha^{2}\sin^{2}\beta l-\beta^{2}e^{2\alpha l}-\beta^{2}e^{-2\alpha l}+2\beta^{2}}$$

(٣-٥٣)

$$G32=\frac{-\beta^{2}\cos\beta l(e^{-\alpha l}-e^{\alpha l})+\alpha\beta\sin\beta l(e^{-\alpha l}+e^{\alpha l})}{3\alpha^{2}\sin^{2}\beta l-\beta^{2}e^{2\alpha l}-\beta^{2}e^{-2\alpha l}+2\beta^{2}}$$

(٣-٥٤)

$$G42=\frac{\beta\sin\beta l(e^{-\alpha l}-e^{\alpha l})}{3\alpha^{2}\sin^{2}\beta l-\beta^{2}e^{2\alpha l}-\beta^{2}e^{-2\alpha l}+2\beta^{2}}$$

(٣-٥٥)

$$G13=\frac{\alpha\beta e^{-2\alpha l}-2\alpha^{2}\sin\beta l\cos\beta l+\alpha\beta(\sin^{2}\beta l-\cos^{2}\beta l)}{3\alpha^{2}\sin^{2}\beta l-\beta^{2}e^{2\alpha l}-\beta^{2}e^{-2\alpha l}+2\beta^{2}}$$

(٣-٥٦)

$$G23=\frac{-2\alpha\sin\beta l\cos\beta l-\beta e^{-2\alpha l}+\beta}{3\alpha^{2}\sin^{2}\beta l-\beta^{2}e^{2\alpha l}-\beta^{2}e^{-2\alpha l}+2\beta^{2}}$$

(٣-٥٧)

$$G33=\frac{2\alpha^{2}e^{-\alpha l}\sin\beta l+\alpha\beta\cos\beta l(e^{\alpha l}-e^{-\alpha l})+\beta^{2}\sin\beta l(e^{-\alpha l}-e^{\alpha l})}{3\alpha^{2}\sin^{2}\beta l-\beta^{2}e^{2\alpha l}-\beta^{2}e^{-2\alpha l}+2\beta^{2}}$$

(٣-٥٨)

$$G43 = \frac{-\beta\cos\beta l(e^{\alpha l} - e^{-\alpha l}) + 2\alpha\sin\beta l e^{-\alpha l}}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$

(٣-٥٩)

$$G14 = \frac{-2\alpha^2\sin\beta l\cos\beta l - \alpha\beta e^{2\alpha l} + \alpha\beta(\cos^2\beta l + \sin^2\beta l)}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$

(٣-٦٠)

$$G24 = \frac{2\alpha\sin\beta l\cos\beta l + \beta - \beta e^{2\alpha l}}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$

(٣-٦١)

$$G34 = \frac{2\alpha^2 e^{\alpha l}\sin\beta l - \beta^2\sin\beta l(e^{-\alpha l} - e^{\alpha l}) - \alpha\beta\cos\beta l(e^{-\alpha l} - e^{\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$

(٣-٦٢)

$$G44 = \frac{-2\alpha e^{\alpha l}\sin\beta l + \beta\cos\beta l(e^{\alpha l} - e^{-\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$

(٣-٦٣)

To get [Z"] differentiate equation (٣-٤٠) with respect to x:

$$\begin{aligned} y'' = {} & A\left[\left(-\alpha\beta e^{\alpha x}\sin\beta x + \alpha^2 e^{\alpha x}\cos\beta x\right) - \left(\beta^2 e^{\alpha x}\cos\beta x + \alpha\beta e^{\alpha x}\sin\beta x\right)\right] \\ & + B\left[\left(\alpha\beta e^{-\alpha x}\sin\beta x + \alpha^2 e^{-\alpha x}\cos\beta x\right) - \left(\beta^2 e^{-\alpha x}\cos\beta x - \alpha\beta e^{-\alpha x}\sin\beta x\right)\right] \\ & + C\left[\left(\alpha\beta e^{\alpha x}\cos\beta x + \alpha^2 e^{\alpha x}\sin\beta x\right) + \left(-\beta^2 e^{\alpha x}\sin\beta x + \alpha\beta e^{\alpha x}\cos\beta x\right)\right] \\ & + D\left[\left(-\beta^2 e^{-\alpha x}\sin\beta x - \alpha\beta e^{-\alpha x}\cos\beta x\right) - \left(\beta\alpha e^{-\alpha x}\cos\beta x - \alpha^2 e^{-\alpha x}\sin\beta x\right)\right] \end{aligned}$$

(٣-٦٤)

or $y'' = A(a_1) + B(a_2) + C(a_3) + D(a_4)$ (٣-٦٥)

Then find

$$\int_0^l \{z''(x)\} \cdot [z''(x)]\,dx = \int_0^l \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} \cdot [a_1 \quad a_2 \quad a_3 \quad a_4]\,dx \qquad (٣\text{-}٦٦)$$

$$\int_0^l \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} \cdot [a_1 \quad a_2 \quad a_3 \quad a_4]\,dx = \int_0^l \begin{bmatrix} a_1a_1 & a_1a_2 & a_1a_3 & a_1a_4 \\ a_2a_1 & a_2a_2 & a_2a_3 & a_2a_4 \\ a_3a_1 & a_3a_2 & a_3a_3 & a_3a_4 \\ a_4a_1 & a_4a_2 & a_4a_3 & a_4a_4 \end{bmatrix} dx \qquad (٣\text{-}٦٧)$$

$$\int_0^l \begin{bmatrix} a_1a_1 & a_1a_2 & a_1a_3 & a_1a_4 \\ a_2a_1 & a_2a_2 & a_2a_3 & a_2a_4 \\ a_3a_1 & a_3a_2 & a_3a_3 & a_3a_4 \\ a_4a_1 & a_4a_2 & a_4a_3 & a_4a_4 \end{bmatrix} dx = \begin{bmatrix} z1 & z2 & z3 & z4 \\ z2 & z5 & z6 & z7 \\ z3 & z6 & z8 & z9 \\ z4 & z7 & z9 & z10 \end{bmatrix} \qquad (٣\text{-}٦٨)$$

where

$$z1=\left[\frac{\left(\alpha^2-\beta^2\right)^2}{2}+2\alpha^2\beta^2\right]\cdot\left(\frac{e^{2\alpha l}-1}{2\alpha}\right)+\left[\frac{\left(\alpha^2-\beta^2\right)^2}{2}-2\alpha^2\beta^2\right]\cdot$$

$$\left[\frac{1}{\left(1+\frac{\alpha^2}{\beta^2}\right)}\cdot\left(\frac{\alpha e^{2\alpha l}}{2\beta^2}\cos 2\beta l+\frac{e^{2\alpha l}}{2\beta}\sin 2\beta l-\frac{\alpha}{2\beta^2}\right)\right]-\left[2\alpha\beta\left(\alpha^2-\beta^2\right)\right]\cdot$$ (٣-٦٩)

$$\left[\frac{1}{\left(1+\frac{\alpha^2}{\beta^2}\right)}\cdot\left(\frac{\alpha e^{2\alpha l}}{2\beta^2}\sin 2\beta l-\frac{e^{2\alpha l}}{2\beta}\cos 2\beta l+\frac{1}{2\beta}\right)\right]$$

(٣- $$z2=\left[\frac{\left(\alpha^2-\beta^2\right)^2}{2}-2\alpha^2\beta^2\right]\cdot 1+\left[\frac{\left(\alpha^2-\beta^2\right)^2}{2}+2\alpha^2\beta^2\right]\cdot\frac{\sin 2\beta l}{2\beta}$$

٧٠)

$$z3=\left[\frac{\left(\alpha^2-\beta^2\right)^2}{2}-2\alpha^2\beta^2\right]\cdot\left[\frac{1}{\left(1+\frac{\alpha^2}{\beta^2}\right)}\cdot\left(\frac{\alpha e^{2\alpha l}}{2\beta^2}\sin 2\beta l-\frac{e^{2\alpha l}}{2\beta}\cos 2\beta l+\frac{1}{2\beta}\right)\right]$$

(٣-

$$+\left[2\alpha\beta\left(\alpha^2-\beta^2\right)\right]\left[\frac{1}{\left(1+\frac{\alpha^2}{\beta^2}\right)}\cdot\left(\frac{\alpha e^{2\alpha l}}{2\beta^2}\cos 2\beta l+\frac{e^{2\alpha l}}{2\beta}\sin 2\beta l-\frac{\alpha}{2\beta^2}\right)\right]$$

٧١) $$z4=\left[-2\alpha\beta\left(\alpha^2-\beta^2\right)\right]\cdot 1-\left[\frac{\left(\alpha^2-\beta^2\right)^2}{4}+\alpha^2\beta^2\right]\cdot\left(\cos 2\beta l-1\right)$$

(٣-٧٢)

$$z5=-\left[\frac{\left(\alpha^2-\beta^2\right)^2}{2}+2\alpha^2\beta^2\right]\cdot\left(\frac{e^{-2\alpha\beta}-1}{2\alpha}\right)+\left[\frac{\left(\alpha^2-\beta^2\right)^2}{2}-2\alpha^2\beta^2\right]\cdot$$

$$\left[\frac{1}{\left(1+\frac{\alpha^2}{\beta^2}\right)}\cdot\left(\frac{-\alpha e^{-2\alpha l}}{2\beta^2}\cos 2\beta l+\frac{e^{-2\alpha l}}{2\beta}\sin 2\beta l+\frac{\alpha}{2\beta^2}\right)\right]+\left[2\alpha\beta\left(\alpha^2-\beta^2\right)\right]\cdot \qquad (٣-٧٣)$$

$$\left[\frac{1}{\left(1+\frac{\alpha^2}{\beta^2}\right)}\cdot\left(\frac{-\alpha e^{-2\alpha l}}{2\beta^2}\sin 2\beta l-\frac{e^{-2\alpha l}}{2\beta}\cos 2\beta l+\frac{1}{2\beta}\right)\right]$$

$$z6=\left[2\alpha\beta\left(\alpha^2-\beta^2\right)\right]\cdot l-\left[\frac{\left(\alpha^2-\beta^2\right)^2+4\alpha^2\beta^2}{4\beta}\right]\cdot\left(\cos 2\beta l-1\right) \qquad (٣-٧٤)$$

$$z7=\left[\frac{\left(\alpha^2-\beta^2\right)^2}{2}-2\alpha^2\beta^2\right]\cdot\left[\frac{1}{\left(1+\frac{\alpha^2}{\beta^2}\right)}\cdot\left(\frac{-\alpha e^{-2\alpha l}}{2\beta^2}\sin 2\beta l-\frac{e^{-2\alpha l}}{2\beta}\cos 2\beta l+\frac{1}{2\beta}\right)\right]$$

$$-\left[2\alpha\beta\left(\alpha^2-\beta^2\right)\right]\left[\frac{1}{\left(1+\frac{\alpha^2}{\beta^2}\right)}\cdot\left(\frac{-\alpha e^{-2\alpha l}}{2\beta^2}\cos 2\beta l+\frac{e^{-2\alpha l}}{2\beta}\sin 2\beta l+\frac{\alpha}{2\beta^2}\right)\right]$$

(٣-٧٥)

$$z8=\left[\frac{\left(\alpha^{2}-\beta^{2}\right)^{2}}{2}+2\alpha^{2}\beta^{2}\right]\cdot\left(\frac{e^{2\alpha l}-1}{2\alpha}\right)-\left[\frac{\left(\alpha^{2}-\beta^{2}\right)^{2}}{2}-2\alpha^{2}\beta^{2}\right]\cdot$$

$$\left[\frac{1}{\left(1+\frac{\alpha^{2}}{\beta^{2}}\right)}\cdot\left(\frac{\alpha e^{2\alpha l}}{2\beta^{2}}\cos 2\beta l+\frac{e^{2\alpha l}}{2\beta}\sin 2\beta l-\frac{\alpha}{2\beta^{2}}\right)\right]+\left[2\alpha\beta\left(\alpha^{2}-\beta^{2}\right)\right]\cdot \qquad (٣\text{-}٧٦)$$

$$\left[\frac{1}{\left(1+\frac{\alpha^{2}}{\beta^{2}}\right)}\cdot\left(\frac{\alpha e^{2\alpha l}}{2\beta^{2}}\sin 2\beta l-\frac{e^{2\alpha l}}{2\beta}\cos 2\beta l+\frac{1}{2\beta}\right)\right]$$

$$z9=\left[\frac{\left(\alpha^{2}-\beta^{2}\right)^{2}}{2}-2\alpha^{2}\beta^{2}\right]\cdot l-\left[\frac{\left(\alpha^{2}-\beta^{2}\right)^{2}}{2}+2\alpha^{2}\beta^{2}\right]\cdot\frac{\sin 2\beta l}{2\beta} \qquad (٣\text{-}٧٧)$$

$$z10=-\left[\frac{\left(\alpha^{2}-\beta^{2}\right)^{2}}{2}+2\alpha^{2}\beta^{2}\right]\cdot\left(\frac{e^{-2\alpha\beta}-1}{2\alpha}\right)-\left[\frac{\left(\alpha^{2}-\beta^{2}\right)^{2}}{2}+2\alpha^{2}\beta^{2}\right]\cdot$$

$$\left[\frac{1}{\left(1+\frac{\alpha^{2}}{\beta^{2}}\right)}\cdot\left(\frac{-\alpha e^{-2\alpha l}}{2\beta^{2}}\cos 2\beta l+\frac{e^{-2\alpha l}}{2\beta}\sin 2\beta l+\frac{\alpha}{2\beta^{2}}\right)\right]-\left[2\alpha\beta\left(\alpha^{2}-\beta^{2}\right)\right]\cdot \qquad (٣\text{-}٧٨)$$

$$\left[\frac{1}{\left(1+\frac{\alpha^{2}}{\beta^{2}}\right)}\cdot\left(\frac{-\alpha e^{-2\alpha l}}{2\beta^{2}}\sin 2\beta l-\frac{e^{-2\alpha l}}{2\beta}\cos 2\beta l+\frac{1}{2\beta}\right)\right]$$

Substitute the equation (٣-٦٨) and [G] values at equation (٣-٢٨) to get the stiffness matrix.

٣-٤-٥ Tangent matrix for $P>\sqrt{k_s EI}$:

Differentiate equation (٣-٣٦) with respect to x to have:

=-A γ sinγx –B ϕ sinϕx +C γ cosγx +D ϕ cosϕx (٣-٧٩)′y

Then apply the boundary conditions that:

=Θ١ to be substitute into equations (٣-٣٤) and (٣-٧٩) in order to ′At x=٠ y=v١, y have

V١=A+B (٣-٨٠)

Θ١=C γ +D ϕ (٣-٨١)

=Θ_2′At x=l y=v٢, y

To be Substitute in to at equations (٣-٣٤) and (٣-٧٩) to get:

$V_۲$=Acosγl +B cosϕl + C sinγl +D sin ϕl (۳-۸۲)

$Θ_۲$=Aγsinγl – B ϕ sinϕl +C γ cosγl + D ϕ cosϕl (۳-۸۳)

In a matrix form:

$$\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \gamma & \varphi \\ \cos\gamma l & \cos\varphi l & \sin\gamma l & \sin\varphi l \\ -\gamma\sin\gamma l & -\varphi\sin\varphi l & \gamma\cos\gamma l & \varphi\cos\varphi l \end{vmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix}$$

(۳-۸٤)

$$[R] = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \gamma & \varphi \\ \cos\gamma l & \cos\varphi l & \sin\gamma l & \sin\varphi l \\ -\gamma\sin\gamma l & -\varphi\sin\varphi l & \gamma\cos\gamma l & \varphi\cos\varphi l \end{vmatrix}$$

(۳-∴ ۸٥)

$$[G'] = \begin{bmatrix} G'11 & G'12 & G'13 & G'14 \\ G'21 & G'22 & G'23 & G'24 \\ G'31 & G'32 & G'33 & G'34 \\ G'41 & G'42 & G'43 & G'44 \end{bmatrix}$$

(۳-۸٦)

Where

$$G'11 = \frac{\phi\gamma\cos\varphi l\cos\gamma l - \phi\gamma - \phi^2\sin\phi l\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$

(۳-۸۷)

$$G'21 = \frac{-\phi\sin\gamma l\cos\phi l + \gamma\sin\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$

(۳-۸۸)

$$G'31 = \frac{\gamma\phi\cos\phi l - \varphi\gamma\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$ (٣-٨٩)

$$G'41 = \frac{-\gamma\sin\phi l + \phi\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$ (٣-٩٠)

$$G'12 = \frac{-\phi\gamma + \gamma^2\sin\gamma l\sin\phi l + \gamma\phi\cos\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$ (٣-٩١)

$$G'22 = \frac{\varphi\sin\gamma l\cos\phi l - \gamma\sin\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$ (٣-٩٢)

$$G'32 = \frac{-\gamma\phi\cos\phi l + \phi\gamma\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$ (٣-٩٣)

$$G'42 = \frac{\gamma\sin\phi l - \phi\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$ (٣-٩٤)

$$G'13 = \frac{-\varphi^2\sin\phi l\cos\gamma l + \phi\gamma\sin\gamma l\cos\phi l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$ (٣-٩٥)

$$G'23 = \frac{\gamma\sin\gamma l\sin\phi l + \phi\cos\phi l\cos\gamma l - \phi}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$ (٣-٩٦)

(٣- $G'33 = \dfrac{-\phi\gamma\sin\gamma l + \phi^2 \sin\phi l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$

٩٧)

(٣- $G'43 = \dfrac{-\phi\cos\gamma l + \phi\cos\phi l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$

٩٨)

(٣- $G'14 = \dfrac{\phi\gamma\cos\gamma l\sin\phi l - \gamma^2 \sin\gamma l\cos\phi l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$

٩٩)

(٣- $G'24 = \dfrac{\gamma\cos\gamma l\cos\phi l + \phi\sin\phi l\sin\gamma l - \gamma}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$

١٠٠)

(٣- $G'34 = \dfrac{\gamma^2 \sin\gamma l - \gamma\phi\sin\phi l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$

١٠١)

(٣- $G'44 = \dfrac{\gamma^2 \cos\gamma l - \gamma\cos\phi l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$

١٠٢)

Then, the second derivative of y will be:

$=-A\,\gamma^2\cos\gamma x - B\,\phi^2\cos\phi x - C\gamma^2\sin\gamma x - D\,\phi^2\sin\phi x$ (٣-″y

١٠٣)

(x)} in order to substitute in to equation (3-″(x) ﺭ. {z″Determine the value of ∫ ﻟz

28)

$$[Z''(x)]\{z''(x)\}=\begin{vmatrix} \gamma^4\cos^2\gamma x & \gamma^2\varphi^2\cos\gamma x\sin\varphi x & \gamma^4\cos\gamma x\sin\gamma x & \varphi^2\gamma^2\sin\varphi x\cos\gamma x \\ \varphi^2\gamma^2\cos\varphi x\cos\gamma x & \varphi^4\cos\varphi x & \varphi^2\gamma^2\cos\varphi x\sin\gamma x & \varphi^4\cos\varphi x\sin\varphi x \\ \gamma^4\sin\gamma x\cos\varphi x & \varphi^2\gamma^2\sin\gamma x\cos\varphi x & \gamma^4\sin^2\gamma l & \gamma^2\varphi^2\sin\gamma x\sin\varphi x \\ \varphi^2\gamma^2\sin\varphi x\cos\gamma x & \varphi^4\sin\varphi x\cos\varphi x & \varphi^2\gamma^2\sin\varphi x\sin\gamma x & \varphi^4\sin^2\varphi x \end{vmatrix}$$ (٣-١٠٤)

Thus

$$\int_0^1\{y''(x)\}\cdot[y''(x)]dx = \int_0^1\begin{bmatrix} a_1a_1 & a_1a_2 & a_1a_3 & a_1a_4 \\ a_2a_1 & a_2a_2 & a_2a_3 & a_2a_4 \\ a_3a_1 & a_3a_2 & a_3a_3 & a_3a_4 \\ a_4a_1 & a_4a_2 & a_4a_3 & a_4a_4 \end{bmatrix}dx$$ (٣-١٠٥)

$$\int_0^1\begin{bmatrix} a_1a_1 & a_1a_2 & a_1a_3 & a_1a_4 \\ a_2a_1 & a_2a_2 & a_2a_3 & a_2a_4 \\ a_3a_1 & a_3a_2 & a_3a_3 & a_3a_4 \\ a_4a_1 & a_4a_2 & a_4a_3 & a_4a_4 \end{bmatrix}dx = \begin{bmatrix} X1 & X2 & X3 & X4 \\ X2 & X5 & X6 & X7 \\ X3 & X6 & X8 & X9 \\ X4 & X7 & X9 & X10 \end{bmatrix}$$ (٣-١٠٦)

Thus

Where:

(٣-١٠٧) $$X1=\frac{\gamma^4}{2}\left(l+\frac{1}{2\gamma}\sin 2\gamma l\right)$$

(٣-١٠٨) $$X2==\frac{\gamma^2\varphi^2}{2}\left(\frac{\sin(\gamma-\varphi)l}{\gamma-\varphi}+\frac{\sin(\gamma+\varphi)l}{\gamma+\varphi}\right)$$

(٣-١٠٩) $$X3=\frac{\gamma^3}{2}\sin^2\gamma l$$

(٣-١١٠) $$X4=\frac{\gamma^2\varphi^2}{2}\left(\frac{\cos(\varphi-\gamma)l}{\gamma-\varphi}-\frac{\cos(\varphi+\gamma)l}{\varphi+\gamma}-\frac{1}{\gamma-\varphi}+\frac{1}{\gamma+\varphi}\right)$$

$$(٣\text{-}١١١)\quad X5 = \frac{\varphi^4}{2}\left(1 + \frac{\sin \varphi l}{\varphi}\right)$$

$$(٣\text{-}١١٢)\quad X6 = \frac{\gamma^2\varphi^2}{2}\left(\frac{\cos(\varphi-\gamma)}{\gamma-\varphi} - \frac{\cos(\varphi+\gamma)}{\gamma+\varphi} - \frac{1}{\gamma-\varphi} + \frac{1}{\gamma+\varphi}\right)$$

$$(٣\text{-}١١٣)\quad X7 = \frac{\varphi^3}{2}\sin^2 \varphi l$$

$$(٣\text{-}١١٤)\quad X8 = \frac{\gamma^4}{2}\left(1 - \frac{1}{2\gamma}\sin 2\gamma l\right)$$

$$(٣\text{-}١١٥)\quad X9 = \frac{\gamma^2\varphi^2}{2}\left(\frac{\sin(\varphi-\gamma)l}{\varphi-\gamma} - \frac{\sin(\varphi+\gamma)l}{\varphi+\gamma}\right)$$

$$(٣\text{-}١١٦)\quad X10 = \frac{\varphi^4}{2}(1 - \frac{1}{2\varphi}\sin 2\varphi l)$$

Substitute all the values of $\int_0^1 a_i a_j$ at equation (٣-١٠٦), and substitute from equation (٣-١٠٦) and [G] value into equation (٣-٢٤) to get the stiffness matrix for such cases.

٣-٤-٦ THE STIFFNESS MATRIX OF PILE UNDER AXIAL LOAD:

The stiffness matrix for pile under axial load with frictional spring is (Alwash ٢٠٠٢):

$$[K] = \frac{K_f}{\left(e^{-\rho l} - e^{\rho l}\right)^2}\begin{bmatrix} \frac{e^{2\rho l} - e^{-2\rho l}}{\rho} & 2\frac{e^{-\rho l} - e^{\rho l}}{\rho} \\ 2\frac{e^{-\rho l} - e^{\rho l}}{\rho} & \frac{e^{2\rho l} - e^{-2\rho l}}{\rho} \end{bmatrix} \qquad (٣\text{-}١١٧)$$

٣-٤-٧ THE FINAL STIFFNESS MATRIX:

In this section the super position between the stiffness matrix of the beam on elastic foundation and the stiffness matrix of pile under axial load will be carried out to find the final stiffness matrix for the pile element subjected to lateral as well as vertical load. Six degree of freedom (u١, v١, θ١, u٢, v٢ and θ٢) per node and is constructed as follows:

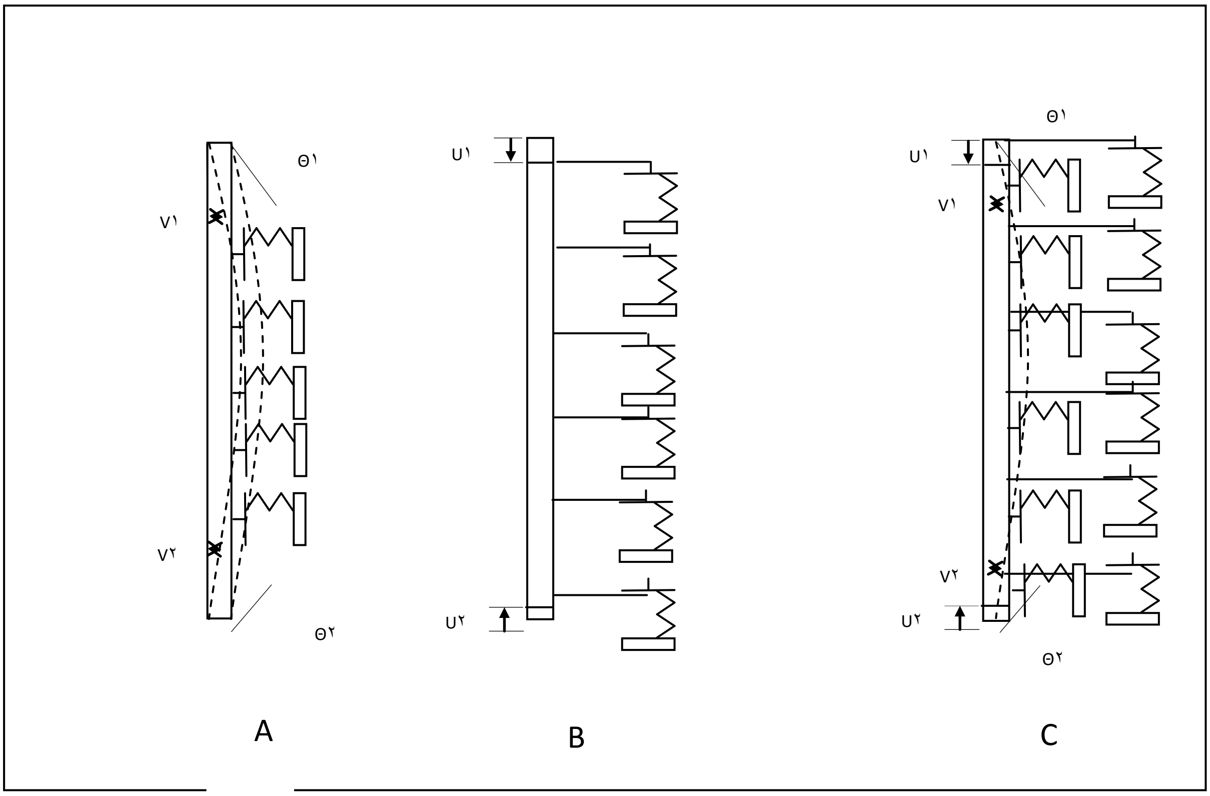


Fig (3-3) a-beam on elastic foundation b-pile under axial load c-pile after super position

If one put the stiffness matrix of equation (٣-١١٧) in the following form:

$$(٣\text{-}١١٨)\ [K]=\begin{bmatrix} O_{1,1} & O_{1,2} \\ O_{2,1} & O_{2,2} \end{bmatrix}$$

And the stiffness matrix of beam-column on elastic foundation as:

$$(٣\text{-}١١٩)\ [K]=\begin{bmatrix} B_{1,1} & B_{1,2} & B_{1,3} \\ B_{2,1} & B_{2,2} & B_{2,3} \\ B_{3,1} & B_{3,2} & B_{3,3} \end{bmatrix}$$

The final stiffness matrix [K_G] will be:

$$(٣\text{-}١٢٠)\ [K_G]=\begin{bmatrix} O_{1,1} & 0 & 0 & O_{1,2} & 0 & 0 \\ 0 & B_{1,1} & B_{1,2} & 0 & B_{1,3} & B_{1,4} \\ 0 & B_{2,1} & B_{2,2} & 0 & B_{2,3} & B_{2,4} \\ O_{2,1} & 0 & 0 & O_{2,2} & 0 & 0 \\ 0 & B_{3,1} & B_{3,2} & 0 & B_{3,3} & B_{3,4} \\ 0 & B_{4,1} & B_{4,2} & 0 & B_{4,3} & B_{4,4} \end{bmatrix}$$

And the equation of displacement will be:

$$(٣\text{-}١٢١)\ \begin{bmatrix} F_x1 \\ F_y1 \\ M1 \\ F_x2 \\ F_y2 \\ M2 \end{bmatrix}=\begin{bmatrix} O_{1,1} & 0 & 0 & O_{1,2} & 0 & 0 \\ 0 & B_{1,1} & B_{1,2} & 0 & B_{1,3} & B_{1,4} \\ 0 & B_{2,1} & B_{2,2} & 0 & B_{2,3} & B_{2,4} \\ O_{2,1} & 0 & 0 & O_{2,2} & 0 & 0 \\ 0 & B_{3,1} & B_{3,2} & 0 & B_{3,3} & B_{3,4} \\ 0 & B_{4,1} & B_{4,2} & 0 & B_{4,3} & B_{4,4} \end{bmatrix}\begin{bmatrix} u1 \\ v1 \\ \theta 1 \\ u2 \\ v2 \\ \theta 2 \end{bmatrix}$$

٣-٥ Dynamic analysis

The dynamic equation of the motion and the derivative of mass matrix for the pile are presented. The equation of the motion for nonlinear system can be formulated by expressing the equilibrium of the effective forces associated with each of its degrees of freedom. In general, four types of force will be involved, the externally applied load $\{F_i(t)\}$ and the internal force resulting from motion which are inertia force $\{f_{Ii}\}$,damping $\{f_{Di}\}$and internal force $\{f_{si}\}$thus the dynamic equilibrium may be expressed as (Clough and penzien ١٩٧٥):

$$\{F_{Ii}\} + \{f_{Di}\} + \{f_{si}\} = \{F_i(t)\} \quad (٣-١٢٢)$$

In which:

$$(٣-١٢٣)\ \{f_I\} = [M]\{\ddot{w}\}$$

$$(٣-١٢٤)\ \{f_D\} = [C]\{\dot{w}\}$$

$$(٣-١٢٥)\ \{f_s\} = [T]\{w\}$$

Substitute eqs. (٣-١٢٣), (٣-١٢٤) and (٣-١٢٥) in to equation (٣-١٢٢) give the complete dynamic equilibrium of system i.e:

$$(٣-١٢٦)\ [M]\{\ddot{w}\} + [C]\{\dot{w}\} + [T]\{w\} = \{F(t)\}$$

where

[M]: mass matrix

[C]: damping matrix

[T]: stiffness matrix

: Nodal acceleration vector of the nodal point of the system $\{\ddot{w}\}$

: Nodal velocity vector of the nodal point of the system $\{\dot{w}\}$

: Nodal displacement vector of the nodal point of the system $\{w\}$

٣-٥-١ FORMULATION OF MASS MATRIX

Mass matrix can be defined in two basic types, lumped and consistent the two methods will be considered in this section.

٣-٥-١-١ LUMPED MASS MATRIX:

In this method it is assumed that the entire mass is concentrated at the nodal points at which the translational displacements are defined. The usual procedure for defining the lumped mass to be located at each node is to assume that the pile is divided into segments, and the nodes serving as connection points. The total mass concentrated at any node, is, the summation of the nodal contributions from all the segments attached to that node (Clough and Penzien ١٩٧٥).

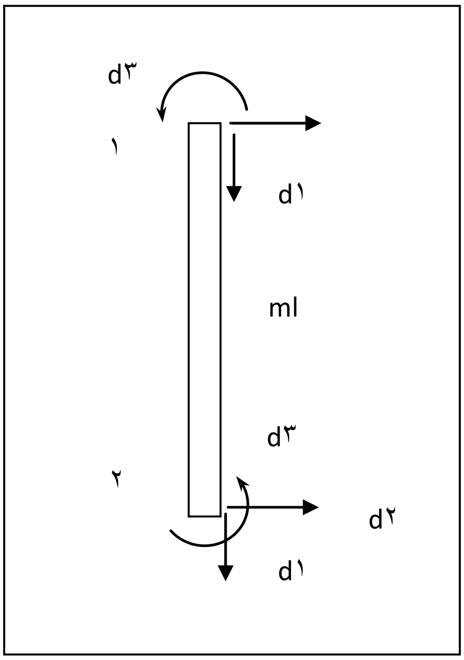


Fig (3-4) lumping of mass at element nodes

The lumped mass matrix for the uniform pile element as shown below can be obtained by placing half of the total element mass as a particle at each node.

Thus the lumped mass matrix for that element can be given as (Cook ET al.١٩٨٩):

$$(٣\text{-}١٢٧)\ [M]=\frac{ml}{2}\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&\frac{L^2}{12}&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&\frac{L^2}{12}\end{bmatrix}$$

The third and sixth diagonal terms in the mass matrix refer to the rotary inertia, which can be neglected in most cases assuming that the particle mass lumps have no rotary inertia.

The zero diagonal terms $M_{i,j}(i\neq j)$ of the lumped matrix vanish because the inertia force at any node (i) due to a unit acceleration at another node(j) is always equal to zero.

٣-٥-١-٢ CONSISTENT MASS MATRIX:

The element consistent mass matrix can be derived by making use of the finite element concept by procedure similar to that of the derivation of element stiffness matrix.

For the pile element the inertia force per unit length $F_I(x)$ along the pile due to acceleration $\ddot{w}(x)$ is:

$$(٣\text{-}١٢٨)\ F_I(x)=m_0\,\ddot{w}(x)$$

Where, m. is the mass per unit length.

٣-٥-١-٢-١ CONSISTENT MASS MATRIX FOR BEAM ON ELASTIC FOUNDATION:

For the beam on the elastic foundation the acceleration $\ddot{w}(x)$ may be expressed by the same function derived for the displacement w(x) same equation (٣-٣٣) for $0 \le P \le \sqrt{k_s EI}$ and equation (٣-٣٩) for $P > \sqrt{k_s EI}$, hence

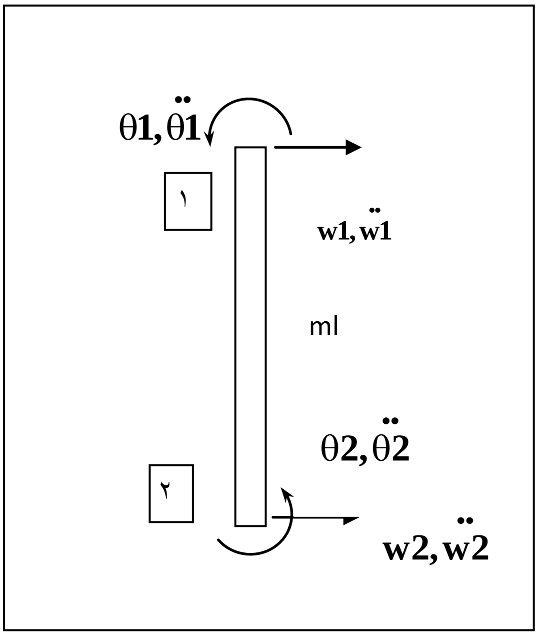


Fig (3-5) Nodal displacement and acceleration for abeam element $0 \le P \le \sqrt{k_s EI}$

$$\ddot{w} = \left(A e^{\alpha x} + B e^{-\alpha x}\right)\cos\beta x + \left(C e^{\alpha x} + D e^{-\alpha x}\right)\sin\beta x \quad (٣-١٢٩)$$

And for $P > \sqrt{k_s EI}$

$$(٣\text{-}١٣٠)\quad \ddot{w} = A\cos\gamma x + B\cos\varphi x + C\sin\gamma x + D\sin\varphi x$$

In order to solve the dynamic equation of motion the inertia force $F_I(x)$ in Eq. (٣-١٢٨) should be transformed to the nodal force.

It is possible to express the acceleration $\ddot{w}(x)$, the acceleration $\ddot{\mathbf{w}}(\mathbf{x})$ may obtain as:

$$(٣\text{-}١٣١)\quad \ddot{w}(x) = [z(x)][G]\{\ddot{e}\}$$

The matrices $[z(x)]\text{and}[G]$ have the same forms (٣-٣٩) and (٣-٤٧) for $0 \le P \le \sqrt{k_s EI}$ and eq (٣-٣٤) and eq (٣-٨٦) for $P > \sqrt{k_s EI}$, and the matrix $\{\ddot{e}\}$ is the nodal accelerations vector.

The equation (٣-١٣١) can be re-written as:

$$(٣\text{-}١٣٢)\quad \ddot{w}(x) = [f_1 f_2 f_3 f_4]\begin{Bmatrix}\ddot{w}\\ \ddot{\theta}\\ \ddot{w}\\ \ddot{\theta}\end{Bmatrix}$$

Where

For $0 \le P \le \sqrt{k_s EI}$

$$f_1 = e^{\alpha x}\cos\beta x\,\frac{2\alpha^2\sin^2\beta l-\beta^2 e^{-2\alpha l}+2\alpha\beta\sin\beta\cos\beta l+\beta^2}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{-\alpha x}\cos\beta x\,\frac{-2\beta\sin\beta l\cos\beta l+2\alpha\sin^2\beta l}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{\alpha x}\sin\beta x\,\frac{\beta^2\cos\beta l(e^{-\alpha l}-e^{\alpha l})-\alpha\beta\sin\beta l(e^{-\alpha l}+e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{-\alpha x}\sin\beta x\,\frac{-\beta\sin\beta l(e^{-\alpha l}-e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$

(٣-١٣٣)

$$f_2 = e^{\alpha x}\cos\beta x\,\frac{-2\alpha\beta\sin\beta l\cos\beta l+2\alpha^2\sin^2\beta l+\beta^2-\beta^2 e^{2\alpha l}}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{-\alpha x}\cos\beta x\,\frac{-2\alpha\sin^2\beta l}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{\alpha x}\sin\beta x\,\frac{-\beta^2\cos\beta l(e^{-\alpha l}-e^{\alpha l})+\alpha\beta\sin\beta l(e^{-\alpha l}+e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{-\alpha x}\sin\beta x\,\frac{\beta\sin\beta l(e^{-\alpha l}-e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$

(٣-١٣٤)

$$f_3 = e^{\alpha x}\cos\beta x\,\frac{\alpha\beta e^{-2\alpha l}-2\alpha^2\sin\beta l\cos\beta l+\alpha\beta(\sin^2\beta l-\cos^2\beta l)}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{-\alpha x}\cos\beta x\,\frac{-2\alpha\sin\beta l\cos\beta l-\beta e^{-2\alpha l}+\beta}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{\alpha x}\sin\beta x\,\frac{2\alpha^2 e^{-\alpha l}\sin\beta l+\alpha\beta\cos\beta l(e^{\alpha l}-e^{-\alpha l})+\beta^2\sin\beta l(e^{-\alpha l}-e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$
$$+e^{-\alpha x}\sin\beta x\,\frac{-\beta\cos\beta l(e^{\alpha l}-e^{-\alpha l})+2\alpha\sin\beta l e^{-\alpha l}}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}$$

(٣-١٣٥)

$$f_4 = e^{\alpha x}\cos\beta x\,\frac{-2\alpha^2\sin\beta l\cos\beta l - \alpha\beta e^{2\alpha l} + \alpha\beta(\cos^2\beta l + \sin^2\beta l)}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$
$$+ e^{-\alpha x}\cos\beta x\,\frac{2\alpha\sin\beta l\cos\beta l + \beta - \beta e^{2\alpha l}}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$
$$+ e^{\alpha x}\sin\beta x\,\frac{2\alpha^2 e^{\alpha l}\sin\beta l - \beta^2\sin\beta l(e^{-\alpha l} - e^{\alpha l}) - \alpha\beta\cos\beta l(e^{-\alpha l} - e^{\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$
$$+ e^{-\alpha x}\sin\beta x\,\frac{-2\alpha e^{\alpha l}\sin\beta l + \beta\cos\beta l(e^{\alpha l} - e^{-\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2}$$

(٣-١٣٦)

For $P > \sqrt{k_s EI}$

$$f_1 = \cos\gamma x\,\frac{\phi\gamma\cos\varphi l\cos\gamma l - \phi\gamma - \phi^2\sin\phi l\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$
$$+ \cos\varphi x\,\frac{-\phi\sin\gamma l\cos\phi l + \gamma\sin\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$
$$+ \sin\gamma x\,\frac{\gamma\phi\cos\phi l - \varphi\gamma\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$
$$+ \sin\varphi x\,\frac{-\gamma\sin\phi l + \phi\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}$$

(٣-١٣٧)

$$
\begin{aligned}
f_2 = {} & \cos\gamma x \frac{-\phi\gamma + \gamma^2 \sin\gamma l \sin\phi l + \gamma\phi\cos\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \cos\varphi x \frac{\varphi\sin\gamma l\cos\phi l - \gamma\sin\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \sin\gamma x \frac{-\gamma\phi\cos\phi l + \phi\gamma\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \sin\varphi x \frac{\gamma\sin\phi l - \phi\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}
\end{aligned}
$$

(٣-١٣٨)

$$
\begin{aligned}
f_3 = {} & \cos\gamma x \frac{-\varphi^2 \sin\phi l\cos\gamma l + \phi\gamma\sin\gamma l\cos\phi l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \cos\varphi x \frac{\gamma\sin\gamma l\sin\phi l + \phi\cos\phi l\cos\gamma l - \phi}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \sin\gamma x \frac{-\phi\gamma\sin\gamma l + \phi^2\sin\phi l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \sin\varphi x \frac{-\phi\cos\gamma l + \phi\cos\phi l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}
\end{aligned}
$$

(٣-١٣٩)

$$
\begin{aligned}
f_4 = {} & \cos\gamma x \frac{\phi\gamma\cos\gamma l\sin\phi l - \gamma^2\sin\gamma l\cos\phi l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \cos\varphi x \frac{\gamma\cos\gamma l\cos\phi l + \phi\sin\phi l\sin\gamma l - \gamma}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \sin\gamma x \frac{\gamma^2\sin\gamma l - \gamma\phi\sin\phi l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\
& + \sin\varphi x \frac{\gamma^2\cos\gamma l - \gamma\cos\phi l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l}
\end{aligned}
$$

(٣-١٤٠)

Now, if the beam element is subjected to unit nodal acceleration, $\ddot{w}(x)_1$ then the equation (٣-١٢٥) becomes:

For $0 \le P \le \sqrt{k_s EI}$

$$F_1(x) = m_0\Big(e^{\alpha x}\cos\beta x\frac{2\alpha^2\sin^2\beta l-\beta^2 e^{-2\alpha l}+2\alpha\beta\sin\beta\cos\beta l+\beta^2}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2} + e^{-\alpha x}\cos\beta x\frac{-2\beta\sin\beta l\cos\beta l+2\alpha\sin^2\beta l}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2} + e^{\alpha x}\sin\beta x\frac{\beta^2\cos\beta l(e^{-\alpha l}-e^{\alpha l})-\alpha\beta\sin\beta l(e^{-\alpha l}+e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2} + e^{-\alpha x}\sin\beta x\frac{-\beta\sin\beta l(e^{-\alpha l}-e^{\alpha l})}{3\alpha^2\sin^2\beta l-\beta^2 e^{2\alpha l}-\beta^2 e^{-2\alpha l}+2\beta^2}\Big)\ddot{w}_1 \quad (٣\text{-}١٤١)$$

For $P > \sqrt{k_s EI}$:

$$F_1(x) = m_0\Big(\cos\gamma x\frac{\phi\gamma\cos\varphi l\cos\gamma l-\phi\gamma-\phi^2\sin\phi l\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2)-2\gamma\varphi+2\phi\gamma\cos\phi l\cos\gamma l} + \cos\varphi x\frac{-\phi\sin\gamma l\cos\phi l+\gamma\sin\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2)-2\gamma\varphi+2\phi\gamma\cos\phi l\cos\gamma l} + \sin\gamma x\frac{\gamma\phi\cos\phi l-\varphi\gamma\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2)-2\gamma\varphi+2\phi\gamma\cos\phi l\cos\gamma l} + \sin\varphi x\frac{-\gamma\sin\phi l+\phi\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2+\gamma^2)-2\gamma\varphi+2\phi\gamma\cos\phi l\cos\gamma l}\Big)\ddot{w}_1 \quad (٣\text{-}١٤٢)$$

To determine the mass matrix coefficients m١١, m٢١, m٣١ and m٤١virtual displacements in w١, θ١, and w٢ and θ٢ direction are applied separately.

For example, to determine the mass matrix Coefficient m٢١, a virtual displacement corresponding to the unit displacement in the θ١ –direction of (δθ١=١) is proceeded to apply the principle of virtual work for an elastic system as shown in fig (٣-٦).

Equating distributed inertial force, $F_I(x)$, gives:

$$Pa\delta\theta_1 = \int_0^l F_I(x)\delta w(x)dx \qquad (٣\text{-}١٤٣)$$

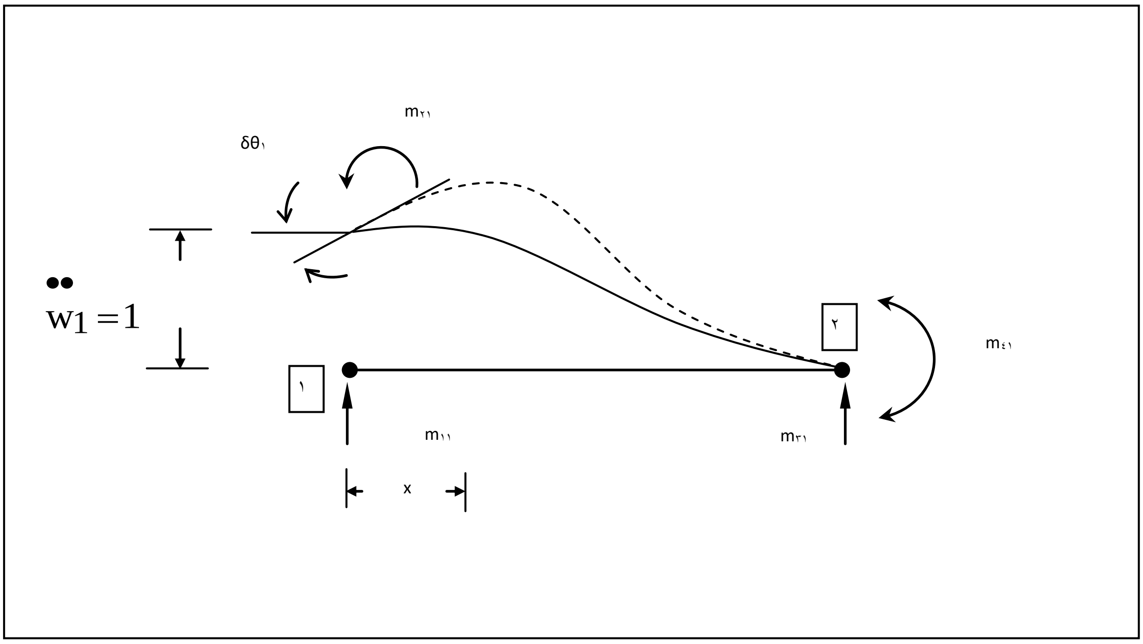


Fig (3-6) beam element supporting inertial load due to acceleration $\ddot{w}_1$, *undergoing virtual displacement* $\delta\theta 1=1$.

In which Pa is the real inertia force existed before applying the virtual displacement δθ١, which can be expressed as:

$$Pa = m_{٢١}\ddot{w}_1 = m_{٢١} \qquad (٣\text{-}١٤٤)$$

And δw(x) is the virtual displacement of a section at distance x.

Using the same displacement field in equation (٣-١٤١) to represent the displacement δw(x), with a unit nodal displacement in other direction gives:

For $0 \le P \le \sqrt{k_s EI}$

$$\begin{aligned}\delta w(x) = (&e^{\alpha x}\cos\beta x \frac{-2\alpha\beta\sin\beta l\cos\beta l + 2\alpha^2\sin^2\beta l + \beta^2 - \beta^2 e^{2\alpha l}}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ &+ e^{-\alpha x}\cos\beta x \frac{-2\alpha\sin^2\beta l}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ &+ e^{\alpha x}\sin\beta x \frac{-\beta^2\cos\beta l(e^{-\alpha l} - e^{\alpha l}) + \alpha\beta\sin\beta l(e^{-\alpha l} + e^{\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ &+ e^{-\alpha x}\sin\beta x \frac{\beta\sin\beta l(e^{-\alpha l} - e^{\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2})\delta\theta_1\end{aligned}$$

(٣-١٤٥)

For $P > \sqrt{k_s EI}$ **:**

$$\begin{aligned}\delta w(x) = (&\cos\gamma x \frac{-\phi\gamma + \gamma^2\sin\gamma l\sin\phi l + \gamma\phi\cos\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ &+ \cos\varphi x \frac{\varphi\sin\gamma l\cos\phi l - \gamma\sin\phi l\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ &+ \sin\gamma x \frac{-\gamma\phi\cos\phi l + \phi\gamma\cos\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ &+ \sin\varphi x \frac{\gamma\sin\phi l - \phi\sin\gamma l}{\sin\gamma l\sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l})\delta\theta_1\end{aligned}$$

(٣-١٤٦)

Substituting Equations (٣- ١٤٤), (٣-١٤١) and (٣-١٤٥) for $0 \le P \le \sqrt{k_s EI}$ and Equation (٣-١٤٦), Equation (٣-١٤٤) and Equation (٣-١٤٢) for $P > \sqrt{k_s EI}$ at Equation (٣-١٤٣), yields:

For $0 \le P \le \sqrt{k_s EI}$

$$m_{21} = \int_0^l m_0 \begin{pmatrix} e^{\alpha x}\cos\beta x \dfrac{2\alpha^2\sin^2\beta l - \beta^2 e^{-2\alpha l} + 2\alpha\beta\sin\beta\cos\beta l + \beta^2}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ + e^{-\alpha x}\cos\beta x \dfrac{-2\beta\sin\beta l\cos\beta l + 2\alpha\sin^2\beta l}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ + e^{\alpha x}\sin\beta x \dfrac{\beta^2\cos\beta l(e^{-\alpha l} - e^{\alpha l}) - \alpha\beta\sin\beta l(e^{-\alpha l} + e^{\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ + e^{-\alpha x}\sin\beta x \dfrac{-\beta\sin\beta l(e^{-\alpha l} - e^{\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \end{pmatrix} * \begin{pmatrix} e^{\alpha x}\cos\beta x \dfrac{-2\alpha\beta\sin\beta l\cos\beta l + 2\alpha^2\sin^2\beta l + \beta^2 - \beta^2 e^{2\alpha l}}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ + e^{-\alpha x}\cos\beta x \dfrac{-2\alpha\sin^2\beta l}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ + e^{\alpha x}\sin\beta x \dfrac{-\beta^2\cos\beta l(e^{-\alpha l} - e^{\alpha l}) + \alpha\beta\sin\beta l(e^{-\alpha l} + e^{\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \\ + e^{-\alpha x}\sin\beta x \dfrac{\beta\sin\beta l(e^{-\alpha l} - e^{\alpha l})}{3\alpha^2\sin^2\beta l - \beta^2 e^{2\alpha l} - \beta^2 e^{-2\alpha l} + 2\beta^2} \end{pmatrix} dx \qquad (٣-١٤٧)$$

For $P > \sqrt{k_s EI}$

$$m_{21} = \int_0^l m_0 \begin{pmatrix} \cos\gamma x \dfrac{\phi\gamma\cos\varphi l\cos\gamma l - \phi\gamma - \phi^2 \sin\phi l \sin\gamma l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ + \cos\varphi x \dfrac{-\phi\sin\gamma l\cos\phi l + \gamma\sin\phi l\cos\gamma l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ + \sin\gamma x \dfrac{\gamma\phi\cos\phi l - \varphi\gamma\cos\gamma l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ + \sin\varphi x \dfrac{-\gamma\sin\phi l + \phi\sin\gamma l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \end{pmatrix} * \begin{pmatrix} \cos\gamma x \dfrac{-\phi\gamma + \gamma^2 \sin\gamma l\sin\phi l + \gamma\phi\cos\phi l\cos\gamma l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ + \cos\varphi x \dfrac{\varphi\sin\gamma l\cos\phi l - \gamma\sin\phi l\cos\gamma l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ + \sin\gamma x \dfrac{-\gamma\phi\cos\phi l + \phi\gamma\cos\gamma l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \\ + \sin\varphi x \dfrac{\gamma\sin\phi l - \phi\sin\gamma l}{\sin\gamma l \sin\phi l(\phi^2 + \gamma^2) - 2\gamma\varphi + 2\phi\gamma\cos\phi l\cos\gamma l} \end{pmatrix} dx \qquad (٣\text{-}١٤٨)$$

Solve the Eqs (٣-١٤٧) and (٣-١٤٨) to get $m_{٢١}$.

In order to put Eqs (٣-١٤٧) and (٣-١٤٨) in a general form for all mass coefficients, the acceleration fields $\ddot{w}_1$, and the displacement ,w(x) should be put in matrix forms as:

$$(٣\text{-}١٤٩)\quad w(x) = [f]\{d\}$$

$$(٣\text{-}١٥٠)\quad \ddot{w}(x) = [f]\{\ddot{d}\}$$

In which $[f]$ is (١x٤) matrix with same values of f_1, f_2, f_3 and f_4 at eq (٣-١٣٣) to the eq (٣-١٤٠)

And $\{d\}$ and $\{\ddot{d}\}$ are the displacement and acceleration vectors with:

$d_1 = v_1$; $d_2 = \theta_1$; $d_3 = v_2$; $d_4 = \theta_2$ (٣-١٥١)

$$\ddot{d}_1 = \ddot{v}_1 ; \ddot{d}_2 = \ddot{\theta}_1 ; \ddot{d}_3 = \ddot{v}_2 ; \ddot{d}_4 = \ddot{\theta}_2 \quad (٣-١٥٢)$$

Applying the same procedure, the consistent mass coefficients may be determined from the following equation:

$$mij = m_0 \int_0^l f_{ij} dx \quad (٣-١٥٣)$$

Using the equation (٣-١٥٣) gives the following relationship between the initial force and acceleration of the nodal coordinates:

$$\{p\} = [M_t]\{\ddot{d}\} \quad (٣-١٥٤)$$

where,

$\{p\}$ =the inertial force vectors at nodal coordinates.

$[M_t]$ =the consistent mass matrix for translation inertia.

In the same way, the consistent mass coefficients for the rotational inertia can be written in condensed forms as:

$$\{p\} = [M_r]\{\ddot{d}\} \quad (٣-١٥٥)$$

And for axial inertia by using same way from can get for axial pile:

(٣- $$\{p\} = \frac{m_0}{\sinh^2(-\omega l)} \begin{bmatrix} MM1 & MM2 \\ MM2 & MM1 \end{bmatrix} \{\ddot{d}\}$$

١٥٦)

where MM١ and MM٢ expressions are:

(٣-١٥٧) $$MM1 = \frac{\sinh(2\omega l)}{4\omega} - \frac{1}{2}$$

(٣- $$MM2 = \frac{\sinh^3(\omega l)}{2\omega} + \left(\frac{1}{2} - \frac{\sinh(2\omega l)}{4\omega}\right)\cosh(\omega l)$$

١٥٨)

Finally combining the mass matrix in eqs (٣-١٥٤), (٣-١٥٥) and (٣-١٥٦) gives the complete consistent mass matrix [M_c] for a beam as [Cook etal.١٩٨٩]

[M_c] = [M_t] + [M_r]+ [M_a] (٣-١٥٩)

CHAPTER FOUR

COMPUTATION TECHNIQUES AND NUMERICAL SOLUTIONS

٤-١ Introduction:

This chapter deals with the configuration and strategy for solving the nonlinear simultaneous equations of the problem. Also, this chapter concerns with the adopted solution technique and the numerical method which is used for the analysis of the present problem.

The first part of this chapter presents the non prismatic presentation for the member. The second part deals with the static analysis and presents the basic strategies for the solution of nonlinear problems. The third part deals with the dynamic analysis.

٤-٢ THE EFFECT OF NON PRISMATIC MEMBER:

In this study the effect of non prismatic member will be presented by using steps method.

This method divides each member to (n) segments, each segment has different cross section area and different moment of inertia .Also, the thickness of each segment is (L/n) .The width of segment i is :

$$h_i = h - i\left(\frac{h - h_0}{n}\right) \quad (٤\text{-}١)$$

Where

i: segment number

n: total number of segment

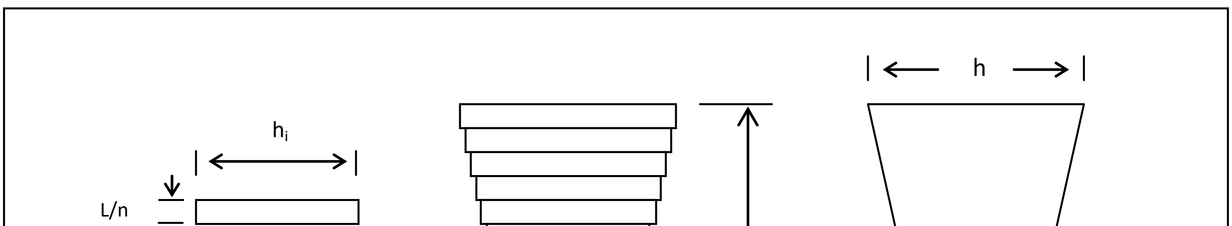


Figure (٤- ١) steps method for non prismatic pile

٤-٣ STATIC ANALYSIS

There are several techniques for solving nonlinear problems; this part presents and reviews two of them:

٤-٣-١ Stepwise procedures (or Incremental procedures)

The basic of the stepwise or incremental procedure is the subdivision of the load in to many small partial loads or increments. Usually these load increments are of equal magnitude, but in general they need not be equal.

During the application of each load increment the equations are assumed to be linear, i.e. tangent stiffness matrix is used. Based on the last configuration of the structure (beginning of load step) .It is assumed to have a fixed value throughout each increment. The solution for each step of loading is obtained as an increment of the displacement (x). These displacement increments are accumulated to give the total displacement at any stage of loading, and incremental process approximates the nonlinear problem as a series of linear problems, that nonlinearity is treated as piecewise linear (Desai and Abel ١٩٧٢).

In writing the equations for the incremental method, the total load may be divided into n increments, so the total effective load, (P_a), is:

$$(P_a)=\sum_{j=1}^{n}(\Delta P_j) \qquad (٤\text{-}٢)$$

Where the symbol Δ is used to indicate a finite increment .Hence, after the application of the i^{th} increment, the load and the displacement vectors are given by:

$$(٤\text{-}٣) \quad (P_i)=\sum_{j=1}^{i}(\Delta P_j)$$

$$(٤\text{-}٤) \quad (X)=\sum_{J=1}^{I}(\Delta X_J)$$

To compute the increments of the displacements, a fixed value of the stiffness which is evaluated at the end of the previous increment, may be used as:

For i=١, ٢…,n (٤-٥) $[T_{i-1}]\{\Delta X_i\} = (\Delta P_i)$

Where:

(٤-٦) $[T_{i-1}] = [T(\{X_{i-1}\},(P_{i-1}))]$

In which:

[$T_{i-١}$] = the tangent stiffness matrix at the end of the i-١ th increment.

= the vectors of incremental applied loads and the corresponding $\{\Delta X_i\},(\Delta P_i)$ change in deformations respectively.

=the internal forces and deformations obtained at the end of the i- $\{X_{i-1}\},(P_{i-1})$ ١ th load step.

The incremental procedure is schematically indicated in fig (٤-٢)

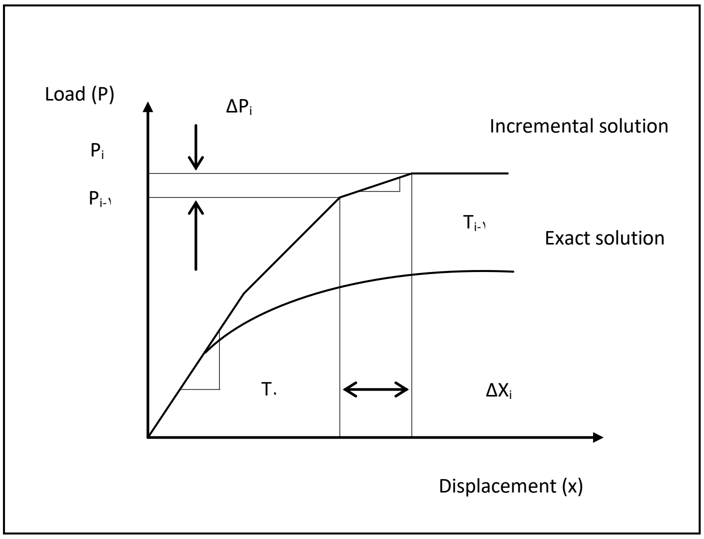


Figure (٤- ٢) basic incremental procedure

The accuracy of the incremental procedure can be improved by assuming small load increments. However, since a new incremental tangent stiffness matrix must be computed for each step, the increased accuracy is purchased at the cost of computational effort [Chajes and Churchill ١٩٨٧].It is more practical to use large increments at the beginning of the analysis and small increments at final steps of the analysis because most structures have a linear part at the beginning of the load deflection curve and a nonlinear behavior at final steps.

٤-٣-٢ ITERATIVE PROCEDURES

The iterative procedure is a sequence of calculation in which the structure is fully loaded in one step to get initial approximate solution which is then improved step by step by using an iteration process until equilibrium is satisfied. After each iteration, the portion of the total loading which is not balanced is calculated and used in the next iteration to compute an additional increment of the displacements. This process is repeated until equilibrium is approximated to some acceptable degree. Essentially, the iterative procedure consists of successive corrections to the solution until equilibrium under the total load (P_a) is satisfied.

٤-٣-٢-١ Incremental Iterative method:

The conventional N-R method is one of the oldest and best known methods used in solving nonlinear problems.

In this method as [Desai and Abel ١٩٧٠] mention the necessary load (P_i) for the i^{th} cycle of iteration is:

(٤-٧) as $\{P_i\} = \{P_a\} - \{P_{e,i-1}\}$

shown in the fig (٤-٣):

$\{P_a\}$=the total load to be applied.

$\{P_{e,i-1}\}$=the equilibrated load after the previous iteration.

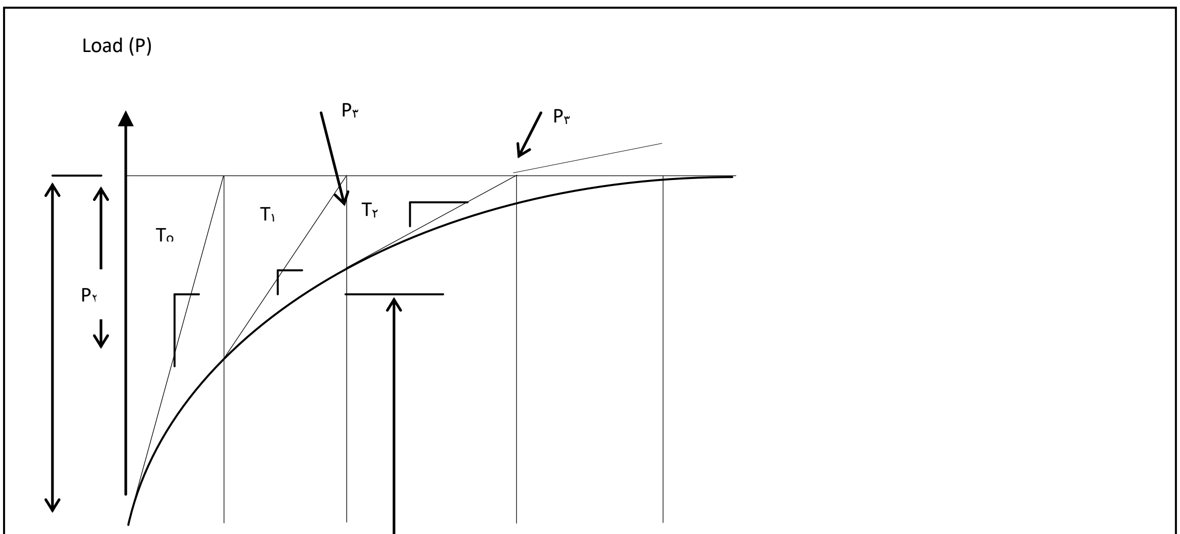
An increment to the displacement is computed during the i^{th} iteration by using the relation.

(٤-٨) $[T_{i-1}]\{\Delta X_i\} = \{P_i\}$

And the total displacement after the i^{th} iteration is computed from:

(٤-٩) $\{X_i\} = \sum_{j=1}^{i} \{\Delta X_j\}$

Finally, $\{P_{e,i-1}\}$ is calculated as the load necessary to maintain the displacements $\{X_i\}$.The procedure is repeated until the increments of the displacements $\{\Delta X_i\}$ or the unbalanced forces, $\{P_i\}$, become sufficiently close to null according to some pre-selected criterion.



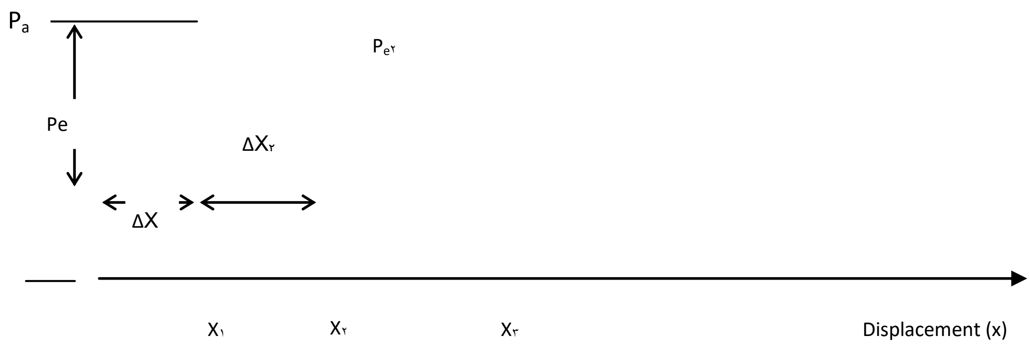


Figure (٤- ٣) conventional N-R method

In the above iterative procedure, the tangent stiffness matrix in Eq. (٤-٨) can be computed from the configuration at the end of the previous iteration which represents the slope of the load-deflection curve at the point ({$X_{i-١}$},{$P_{i-١}$}).At the beginning of the analysis [T٠] represents the tangent stiffness at point ({X٠},{P٠}).

Instead of computing a different stiffness for each iteration a modified N-R technique has been employed, which utilizes only the initial stiffness, [T٠].

The modified procedure (figure ٤-٤) necessitates a greater number of the iterations; however, there is a substantial saving of computation because it is not necessary to invert a new stiffness at each cycle [Cook at al.١٩٨٩]

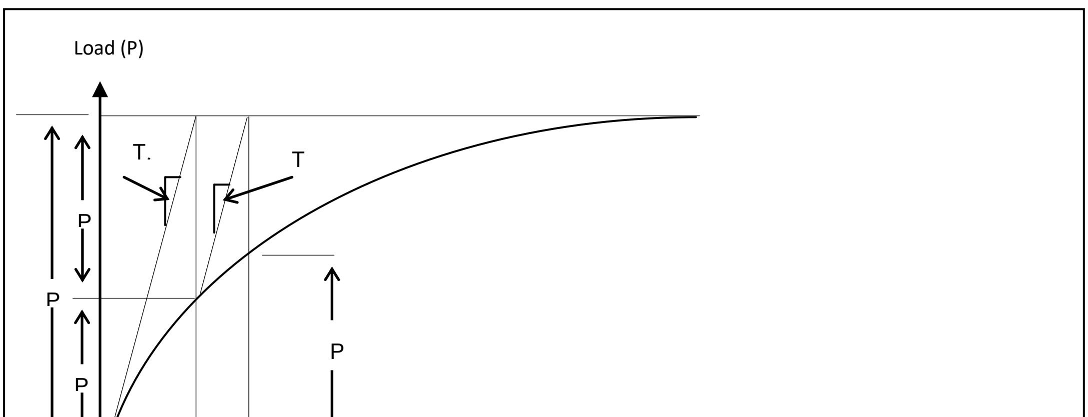


Figure (٤-٤) Modified N-R Method

The above two procedures can be combined together to get the so called combined conventional and modified N-R method in which the stiffness is held constant for several iterations and is updated when the rate of convergence begins to deteriorate (number of iterations exceeds maximum limit)[Cook et al ۱۹۸۹] , Figure(٤-٥) illustrates the combined N-R method.

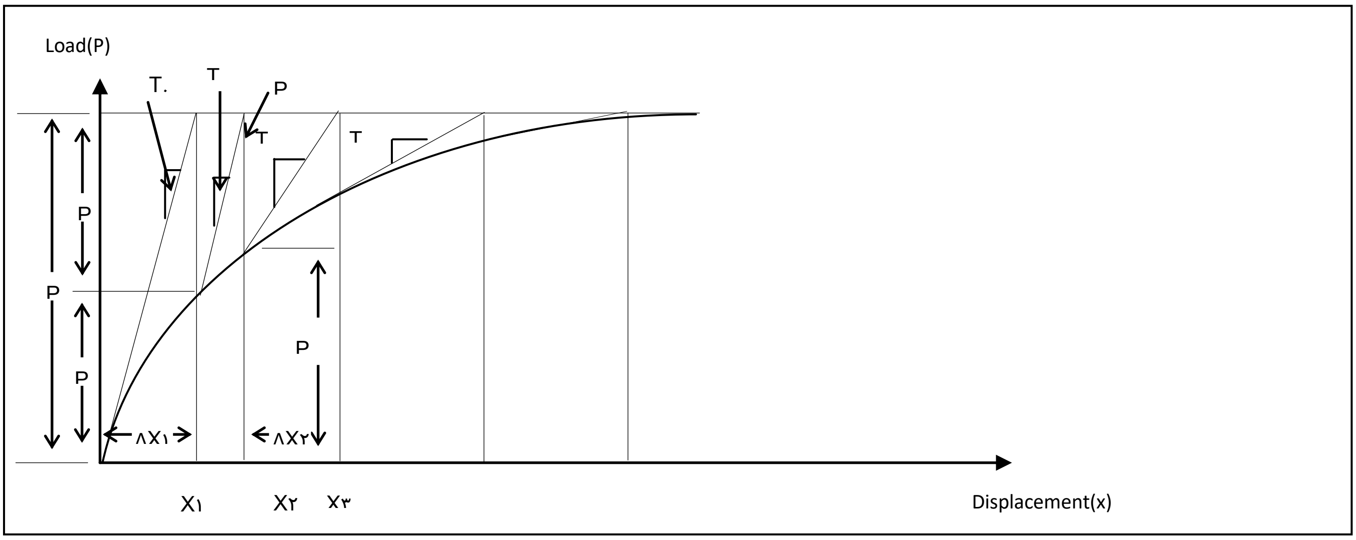


Figure (٤-٥) Combined N-R method

In present study the incremental iterative has been used.

٤-٤ Dynamic Analysis

For nonlinear systems, the only applicable method for solving the second order differential equation of motion is the direct time –integration method, in which the approach is to write the equation of motion, at specific instance of time [Cook et al ١٩٨٩]:

$$(٤\text{-}١٠)\quad [M]_n \left\{\ddot{X}\right\}_n + [C]_n \left\{\dot{X}\right\}_n + [T]_n \{X\}_n = \{F(t)\}_n$$

Where

$\{f\}_n$ = internal forces vector at time nΔt.

$[T]_n$ = tangent stiffness matrix at time nΔt.

$\{X\}_n$ = displacements vector at time nΔt.

$\left\{\dot{X}\right\}_n$ = velocities vector at time nΔt.

= accelerations vector at time nΔt. $\left\{\ddot{X}\right\}_n$

The direct numerical integration is based on two ideas:

١- The dynamic equilibrium equation Eq (٤-١٠) is satisfied at discrete time intervals Δt.

٢- The variation of displacements, velocities and accelerations within each time interval Δt, is assumed .Different forms of these assumed variations give rise to different direct integration schemes, each of which has different accuracy ,stability and cost [Subbaraj and Dokainish ١٩٨٩].

The available direct procedures can be further subdivided into

١-explicit method; have the form [Cook et al ١٩٨٩]

$$(٤\text{-}١١)\ \{X\}_{n+1} = f(\{X\}_n, \{\dot{X}\}_n, \{\ddot{X}\}_n, \{X\}_{n-1}, \ldots)$$

And hence, permit $\{X\}_{n=١}$ to be determined in terms of completely historical information consisting of displacements and time derivatives of displacement at time nΔt and before.

٢-Implicit methods have the form:

$$(٤\text{-}١٢)\quad \{X\}_{n+1} = f(\{\dot{X}\}_{n+1}, \{\ddot{X}\}_{n+1}, \{X\}_n, \ldots)$$

And hence, computation of $\{X\}_{n+١}$ requires knowledge of the time derivatives of $\{X\}_{n+١}$ which are unknown.

Implicit algorithms tend to be numerically stable, permitting large time steps, but the cost per time step is high and storage requirements tend to increase dramatically with the number of D.O.F. of the structure. On the other hand, explicit algorithms tend to be inexpensive per step and require less storage than implicit algorithm, but the numerical stability requires small time steps to be employed [Subbaraj and Dokainish ١٩٨٩].

Generally, implicit algorithms are more effective for structural dynamic problems, in which the response is controlled by relatively a small number of low frequency mode, while explicit algorithms are very efficient for wave propagation problems, in which the contribution of intermediate and high frequency structural modes to the response is important [Subbaraj and Dokainish ١٩٨٩].The preset study deals with undemped free vibration of piles in order to obtain the natural frequencies with the corresponding mode shape vectors.

٤-٤-١ Calculation of natural frequencies.

In the dynamic analysis ,the natural frequency ω of the vibration is important to give an idea about the oscillation of the system with time and to determine the natural period T of the vibration which represents the time for which the vibration repeats itself ,where:

$$(٤\text{-}١٣)\ T = \frac{2\pi}{\omega}$$

To determine the natural frequencies of the structure, a free vibration problem (no external applied loads) for the un-damped case is assumed [Weaver and Johnston ١٩٨٧]:

$$(٤\text{-}١٤)\ [M]\{\ddot{X}\} + [T]\{X\} = 0$$

Assuming harmonic motion that is:

; i=١,٢,٣ ...n (٤-١٥) $\{X_i\} = \{\phi_i\} \sin \omega_i t$

where

n= the number of degrees of freedom of the system.

=the mode shape vector for i^{th} mode of vibration. $\{\phi_i\}$

ω_i =the angular natural frequency of mode i.

Differentiating Eq. (٤-١٦) twice with respect to time yields:

(٤-١٦) $\{\ddot{X}_i\} = -\omega_i^2 \{\phi_i\} \sin \omega_i t$

Then, substituting eqs(٤-١٦)and (٤-١٧) into Eq.(٤-١٥) yields, after canceling the term ($\sin\omega_i t$):

$([T] - \omega_i^2 [M]) \{ \phi_i \} =$ ٠ (٤-١٧)

Eq(٤-١٨) has the form of the algebraic eigenvalue problem ($T\phi = \lambda M \phi$) .From the theory of homogeneous equations, nontrivial solutions exist only if the determinant of the coefficient matrix is equal to zero.[Gere and Weaver ١٩٨٣]thus:

(٤-١٨) $\left| [T] - \omega_i^2 [M] \right| = 0$

Expansion of the determinant yields a polynomial of order n called characteristic equation. The n roots of this polynomial ω_i^2 are the characteristic value or eigenvalue.

The cyclic natural frequencies, f_i is then obtained from:

(٤-١٩) $f_i = \dfrac{\omega_i}{2\pi}$

Substitution of these roots (one at each time) into the homogeneous equation (٤-١٨) produces the characteristic vectors or eigenvectors, { ϕ_i } within arbitrary constants.

A number of solution algorithms have been developed for the solution of eigenvalue problem .One of most important techniques is the inverse iteration method.

٤-٤-١-١ Inverse iteration method

This technique is very effective in calculating the smallest eigenvalue and the corresponding eigenvector which are the most important egien pair in dynamic structural.

The basic steps for solving the eigenvalue problem of the form ($T\phi = \lambda M\phi$) using the inverse iteration method is [Bathe ١٩٨٢]:

١-Asumming a starting iteration vector, {X١} almost with all terms equal to one.

٢- Assuming that:

{Y١}= [M] {X١} (٤-٢٠)

٣-Evaluating for each iteration step k=١, ٢...

(٤-٢١) $\{\bar{X}_{k+1}\} = [T]^{-1}\{Y_k\}$

(٤-٢٢) $\{\bar{Y}_{k+1}\} = [M]\{\bar{X}_{k+1}\}$

(٤-٢٣) $\rho_d\left(\{\bar{X}_{k+1}\}\right) = \dfrac{\{\bar{X}_{k+1}\}^T\{Y_k\}}{(\{\bar{X}_{k+1}\}^T\{\bar{Y}_{k+1}\})}$

$$(٤\text{-}٢٤)\ \{Y_{k+1}\} = \frac{\{\bar{Y}_{k+1}\}}{(\{\bar{X}_{k+1}\}^T\{\bar{Y}_{k+1}\})^{0.5}}$$

Where the superscript T denotes transpose

٤- The value of $\rho_d\left(\{\bar{X}_{k+1}\}\right)$ in equation (٤-٢٤) represents an approximation to the eigenvalue λ١ denoting the current approximation for λ١ by λ_1^{k+1} , i.e.

= $\rho_d\left(\{\bar{X}_{k+1}\}\right)$ the convergence will occur when: λ_1^{k+1}

$$(٤\text{-}٢٥)\quad \frac{\left|\lambda_1^{(k+1)} - \lambda_1^{(k)}\right|}{\lambda_1^{(k+1)}} \le 10^{-s}$$

Where

S: is the number of significant digits of the desired accuracy.

٥- If m is the last iteration cycle, then the smallest eigenvalue λ١ and the corresponding eigenvector will be respectively:

$$(٤\text{-}٢٦)\ \lambda_1 = \rho_d(\{\bar{X}_{m+1}\})$$

$$(٤\text{-}٢٧)\quad \{\phi_1\} = \frac{\{\bar{X}_{m+1}\}}{(\{\bar{X}_{m+1}\}^T\{\bar{Y}_{m+1}\})^{0.5}}$$

CHAPTER FIVE

EXPERIMENTAL WORK

٥-١ The experimental test:

To verify the ability of the program a ٦٠٠ mm length of steel rod with width ٤٠mm and thickness ٦mm as cantilever supported by welded at bottom with steel base as shown in figure (٥-١) has been chosen to be an experimental model of the present study to obtain the natural frequency and compare it with the natural frequency that was obtained from the theoretical study.

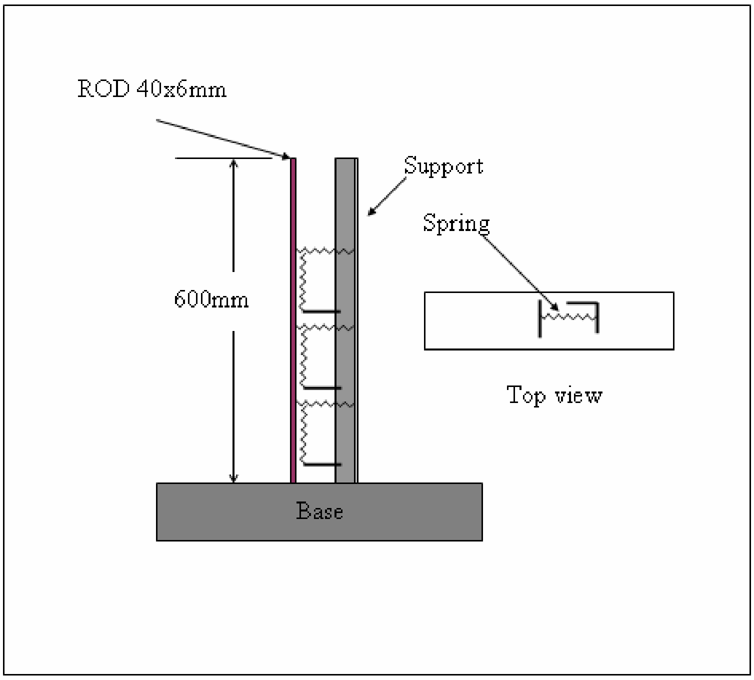


Figure (٥-١): experimental model

There are three cases of tests presented in this study. The first one obtained the natural frequency for the model without the effect of the elastic foundation. The second case symbolized the pile with the elastic foundation by using four elements and the third case indicated the pile with the elastic foundation effect by using three elements.

٥-٢ Soil Representation:

The soil resistance is represented by using springs, as shown in figure(٥-٢) .There are two springs for each node ,vertical one (parallel to the rod) which represents the frictional resistance of soil and horizontal spring (perpendicular to the rod) which represents the elastic foundation effect.

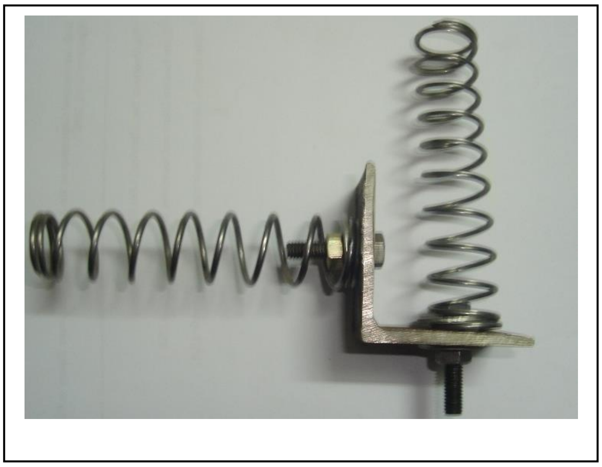


Figure: (٥-٢) spring system that used to represent the elastic foundation

٥-٣ The recording tool:

Because of there is no available instrument to determine the natural frequency; in present study a manual tool had been used for recording the waves of vibration.

The recording tool is a tool product by fabricated two cylinder fixed by plastic pipes to bots with a steel angle as shown in the figure (٥-٣), the steel angle welded vertically to a steel base.

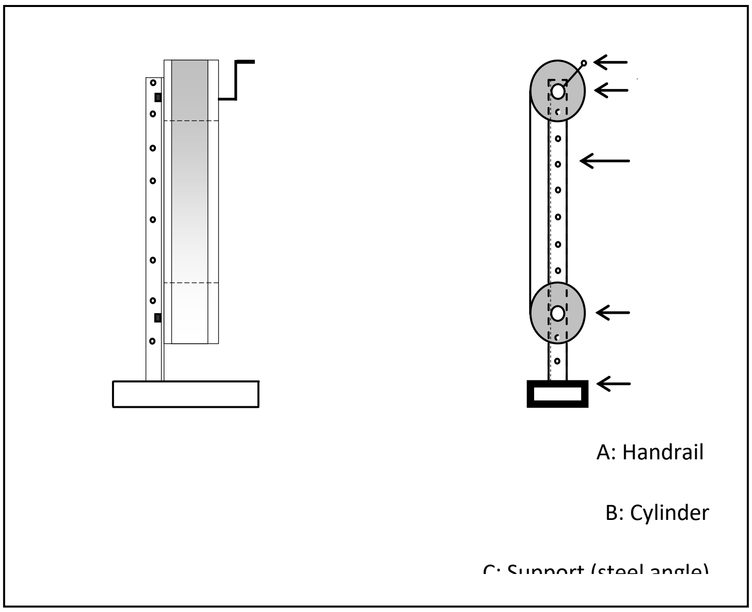


Figure: (5-3) Recording tool

The bottom cylinder is the roll of a recording paper and the recording paper stick by the top cylinder and the top cylinder has a handrail to use it to moving up the recording paper.

٥-٢ The steps of the test:

١- Find the stiffness of the spring by fixing the spring at the top and indicate the location of the bottom then suspend a load at the bottom and locate the displacement happen by this load. Then, increase the load and find out the corresponding displacement of the final load. Obtain the axial stiffness by draw the relationship between load and displacement as figure (٥-٤). From this figure the stiffness of the spring that is used at this study is ٠.٩٨١ N/mm.

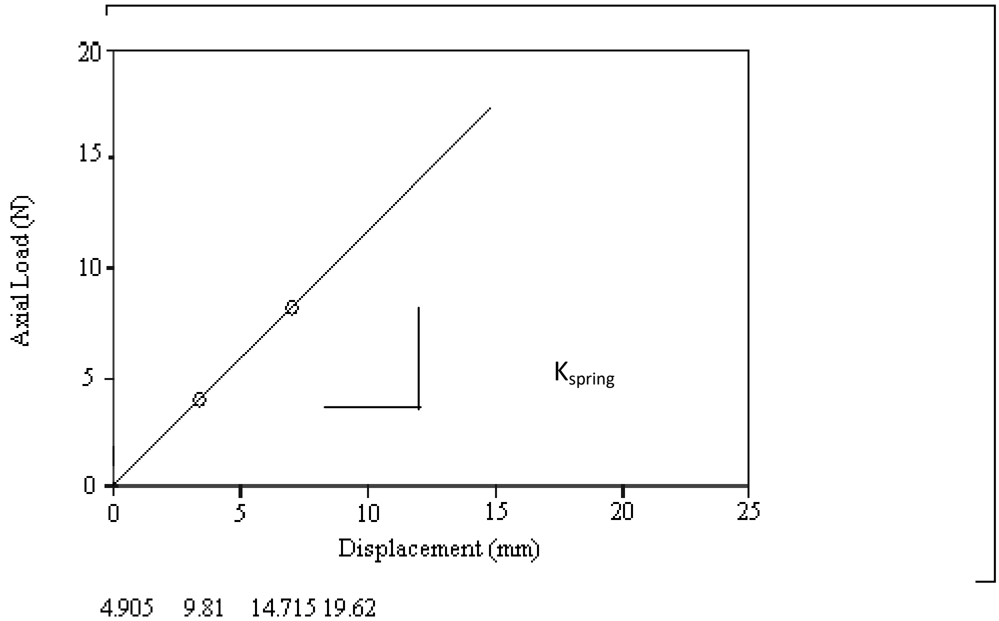


Figure (٥-٤): The relationship between loads and displacement for spring

٢- By putting the recording tool in front of the rod closer to let the pin cored attached the record paper as in fig (٥-٥).

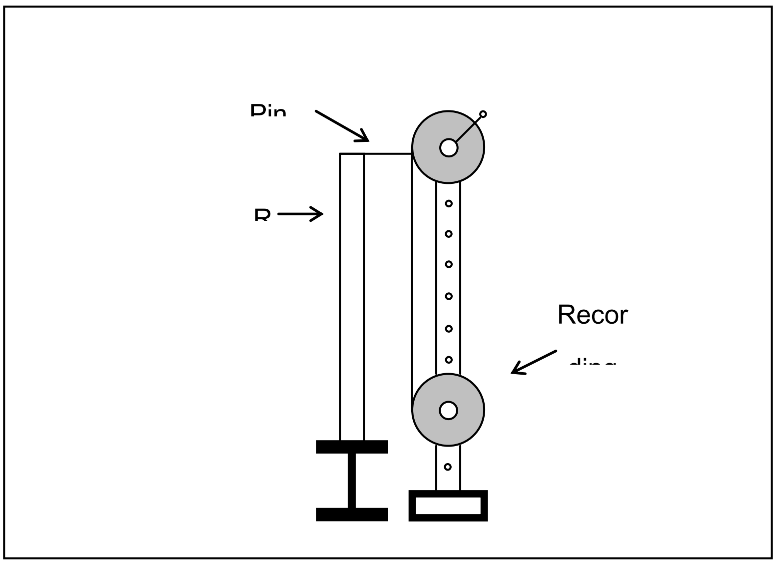


Fig (5-5) Recording tool and the model

٣- Impact the rod laterally without any spring as shown in figure (٥-٦) to find the natural frequency for the model and record the wave that was draws from this movement (calculate the number of cycles) and record the movement time. Figure (٥-٧) shows the recorded movements.

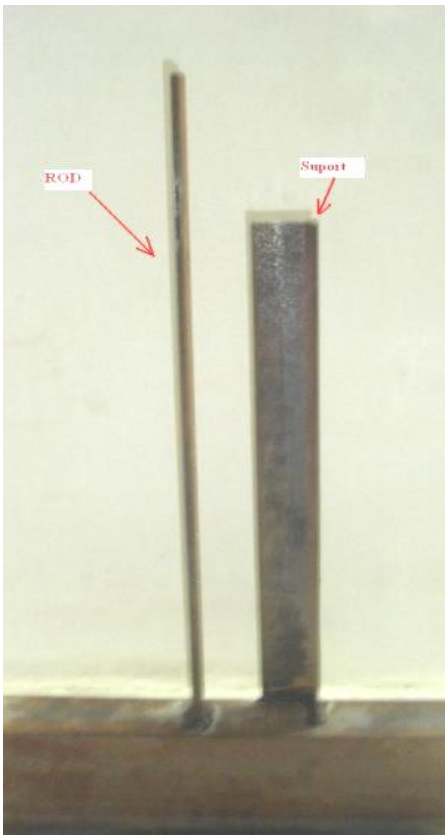


Figure (٥-٦): Model without any springs

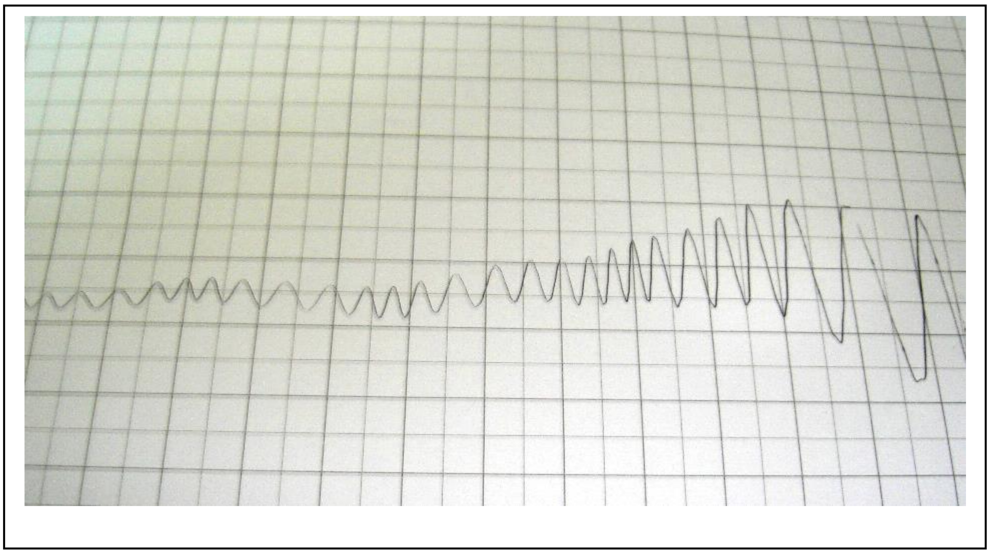


Figure (٥-٧): Recorded movement

٤- Calculate the natural period T by the relation ship:

$$T = \frac{\text{Total time of movement (free vibration)}}{\text{Number of cycle}} \quad (٥\text{-}١)$$

The natural frequency obtained by using the relation ship:

$$\omega = \frac{1}{T} \quad (٥\text{-}٢)$$

٥- Then, Fixing the springs at the three nodes by bolts (the vertical and horizontal springs).The distance between the two nodes is ١٥٠mm c/c as shown in figure (٥-٨). Similar retest in the steps ٢ and ٣ to obtain the natural frequency for the pile (four elements) with the effect of elastic foundation.

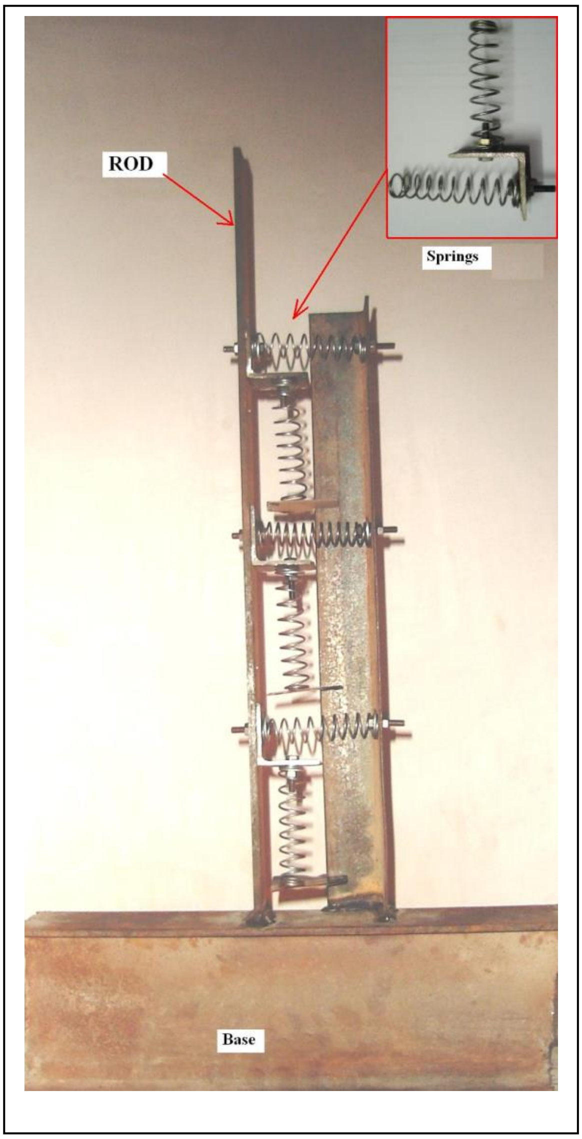


Figure (٥-٨): the model of pile with three systems of springs

٦- By changing the six springs by four (replace the three points of contact to two points .The distance between two bolts is ٢٠٠mm c/c) as shown in figure (٥-٩). To obtain the natural frequency for the pile of three elements a retest is followed the similar as in steps ٢and ٣.

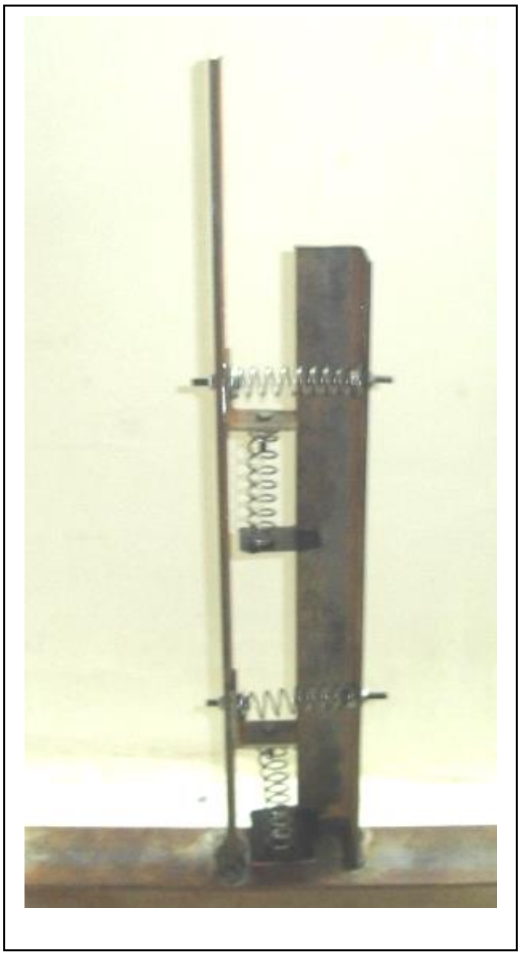


Figure (٥-٩) the model of pile with two systems of spring

Table (٥-١) shows the natural frequencies which was obtained from the test and theoretical analysis for the considered model.

Table (٥-١) the natural frequencies from the test and theoretical analysis for the model.

Case	Experimental Natural frequency Cycle/sec	Theoretical Natural frequency Cycle/sec
Without springs	١٢.٤٦٨٨	١٤.١٥٥٥
Two springs	١١.٦٢٤٢	١٣.٠٢١٢
Three springs	١٠.٥٤٢٨	١٢.١٣٨٧

The maximum difference between the theoretical frequency and experimental frequency one is (١٣.٤%) and this belongs to two sources:

١-The experimental frequency is a combination of the infinite number of frequencies in addition to the fundamental natural frequency that found theoretically and this will reduce this frequency.

٢- The experimental frequency is based on damped free vibration while the theoretical is based on undamped free vibration.

CHAPTER SIX

COMPUTER PROGRAM AND APPLICATIONS

٦-١ Introduction

This chapter presents a description of the computer program of Al Bidairi developed in the present study. The computer program was first introduced by ***Al-Rawanduzi*** (١٩٩٦), and it was designed to deal with large displacement elastic stability analysis of plane frames under static and dynamic loads.

In the present study, the previous version of the computer program has been adopted and developed to deal with the pile element inside different layers of soil .This program is developed to deals with the **non –prismatic** pile under vertical as well as lateral loads and moment. The stiffness matrix as well as the miss matrix those were derived in the present study for non-prismatic pile element under lateral and vertical loads, are involved in the present version of program. Also; the program is modified to find the natural frequencies of piles.

٦-٢ Properties and abilities of the program

The computer program has been developed as a stated previously, to deal with large displacement elastic stability analysis of the piles with different end conditions subjected to vertical and lateral loads. The free vibration problem is also considered in the present study. The properties and abilities of this program may be summarized as:

١-Use either proportional or non-proportional load increments in the static part of analysis.

٢-Use different methods of analysis, those are:

a - Linear analysis.

b- Nonlinear analysis by linear –incremental method.

c- Nonlinear analysis by incremental-iterative methods using either N-R or modified N-R method for iteration process.

٣- Calculate the relationship between the determinant of the tangent stiffness matrix and the load factor, which can be used to determine the approximate elastic critical load by the extrapolation method presented in chapter four.

٤- Use the Eulerian coordinates system in constructing the member tangent stiffness matrix, in which the coordinates are updated after each iteration according to the member last configuration.

٥- Use two different types of mass representation (lumped and consistent).

٦-Solve the eigenvalue problem of free vibration by using the inverse iteration method to determine the fundamental natural frequency.

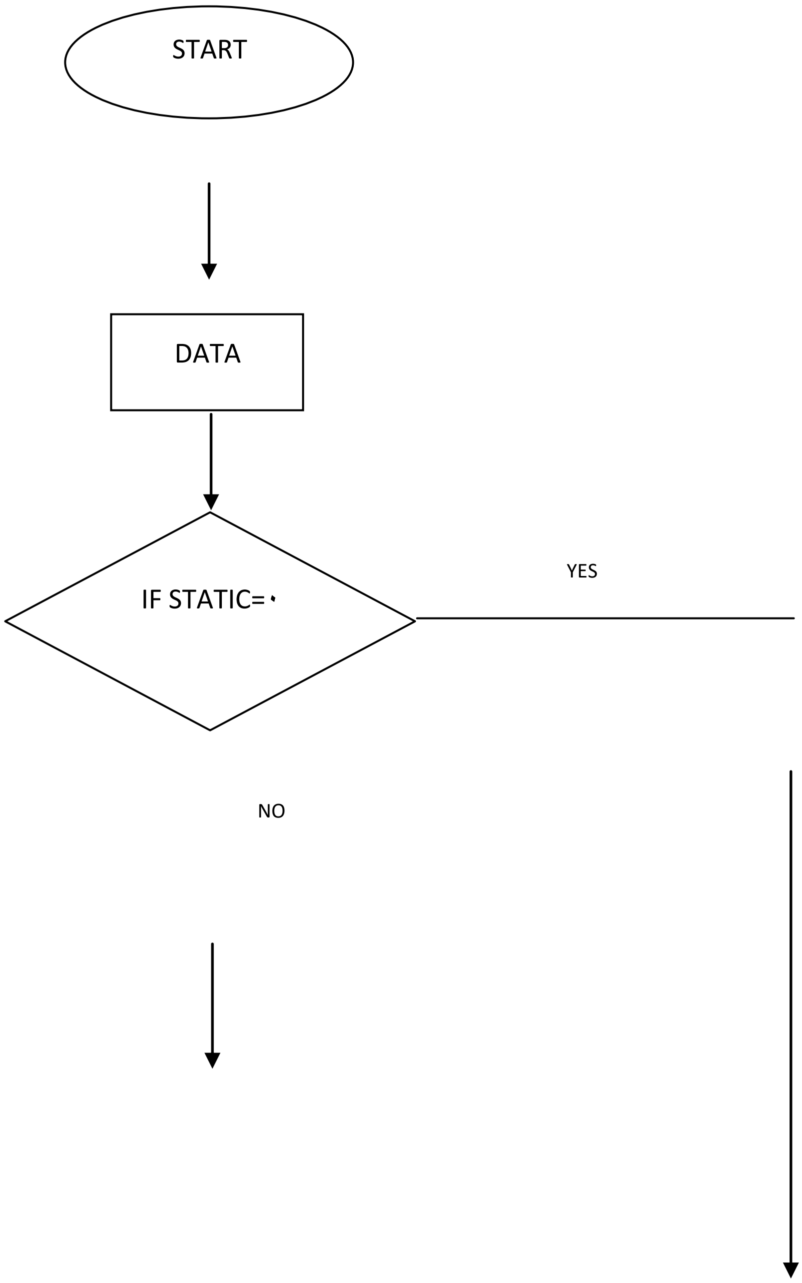
٧-Record the central processing unit time needed for completing the analysis .This make it easy to compare the different methods used in static and dynamic analysis and then to choose the most economical one, that is ,the most time saving one for the same accuracy.

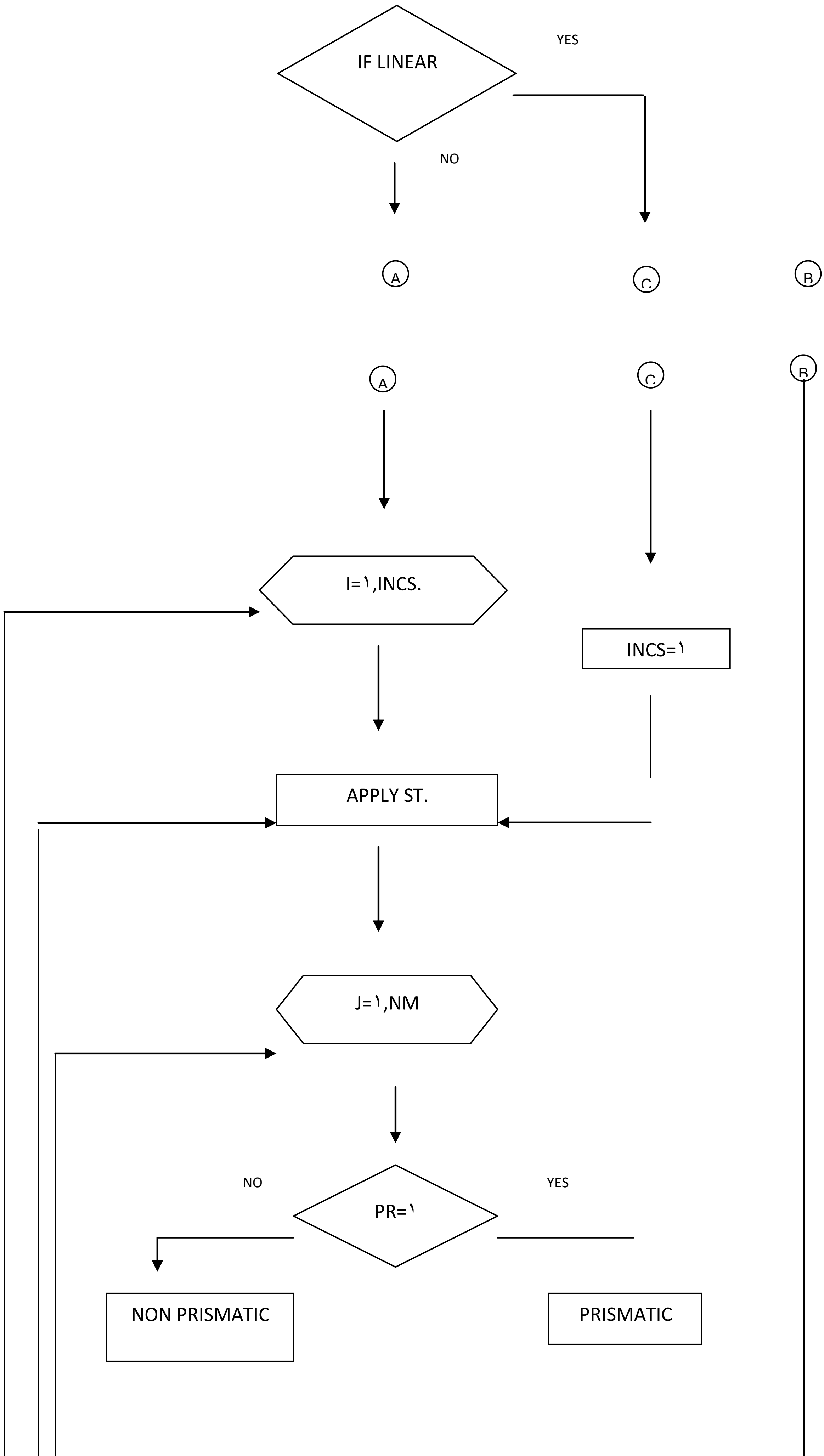
٦-٣ STRUCTURE OF THE PROGRAM

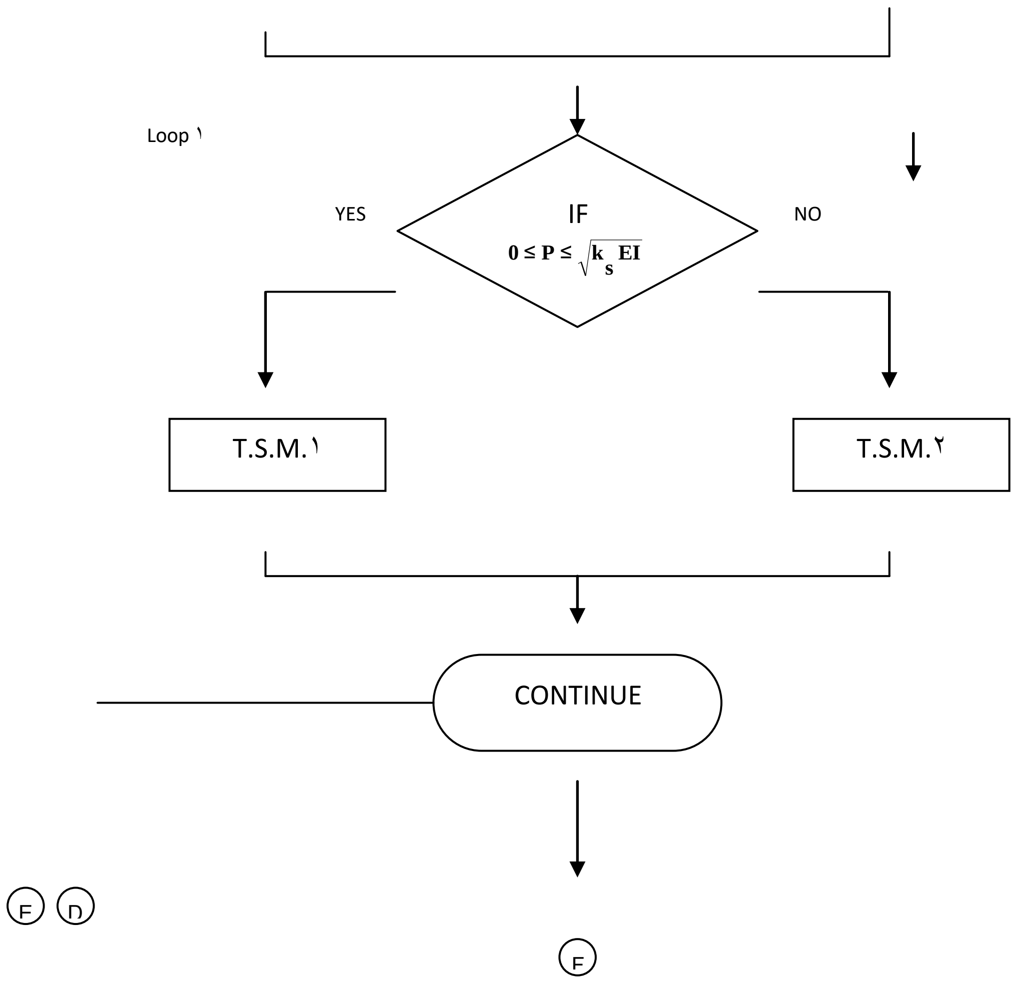
The program is coded in ***Quick Basic language (Version ٤.٥)***.

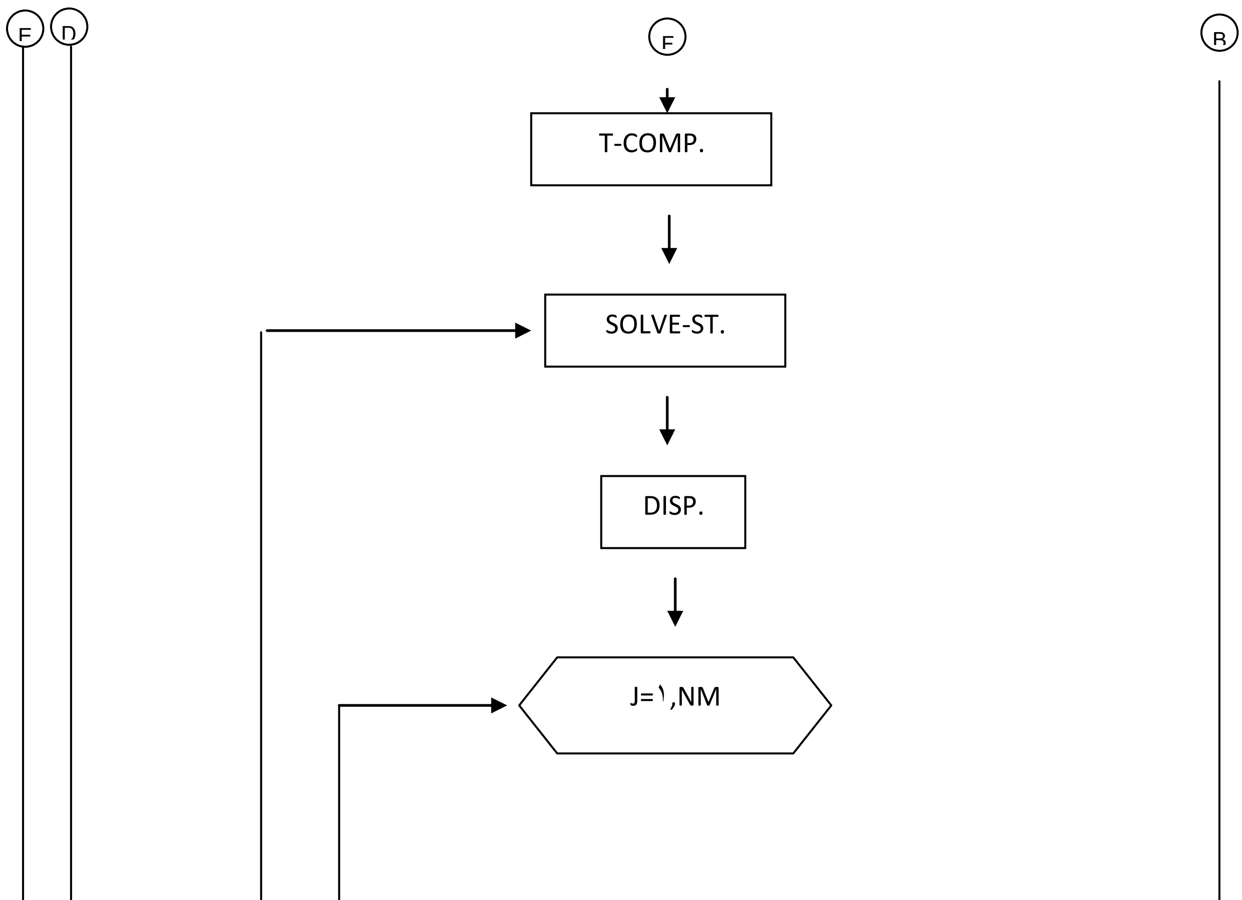
It consists of a main routine and many subroutines. Each one of these subroutines deals with a specified stage of the analysis which may be repeated many times in the main routine.

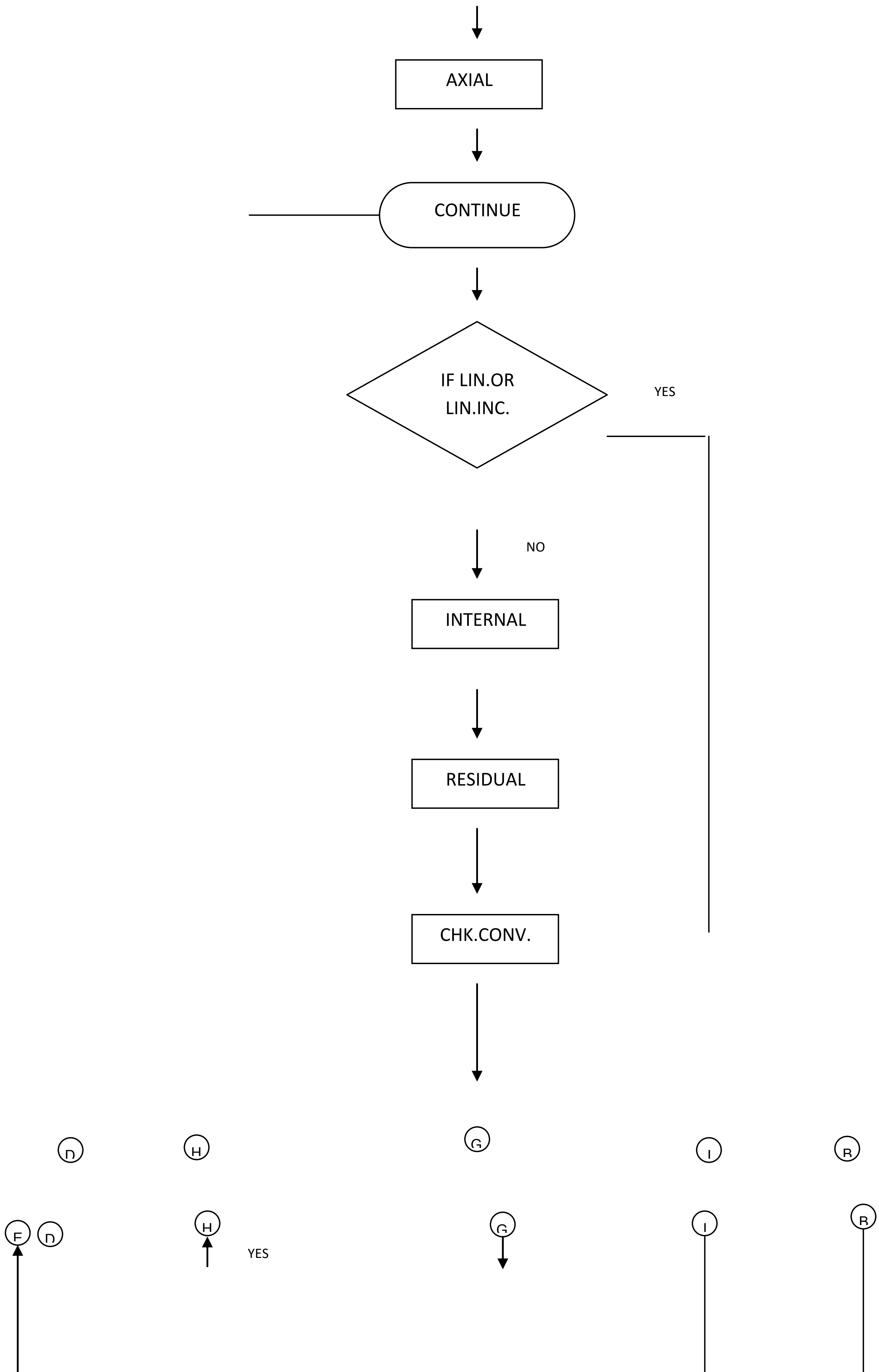
The sequence of the operations in the main routine is presented in the Flow-Chart in figure (٦-١) and a brief description for the main items used in the flow chart is shown in table (٦-١):

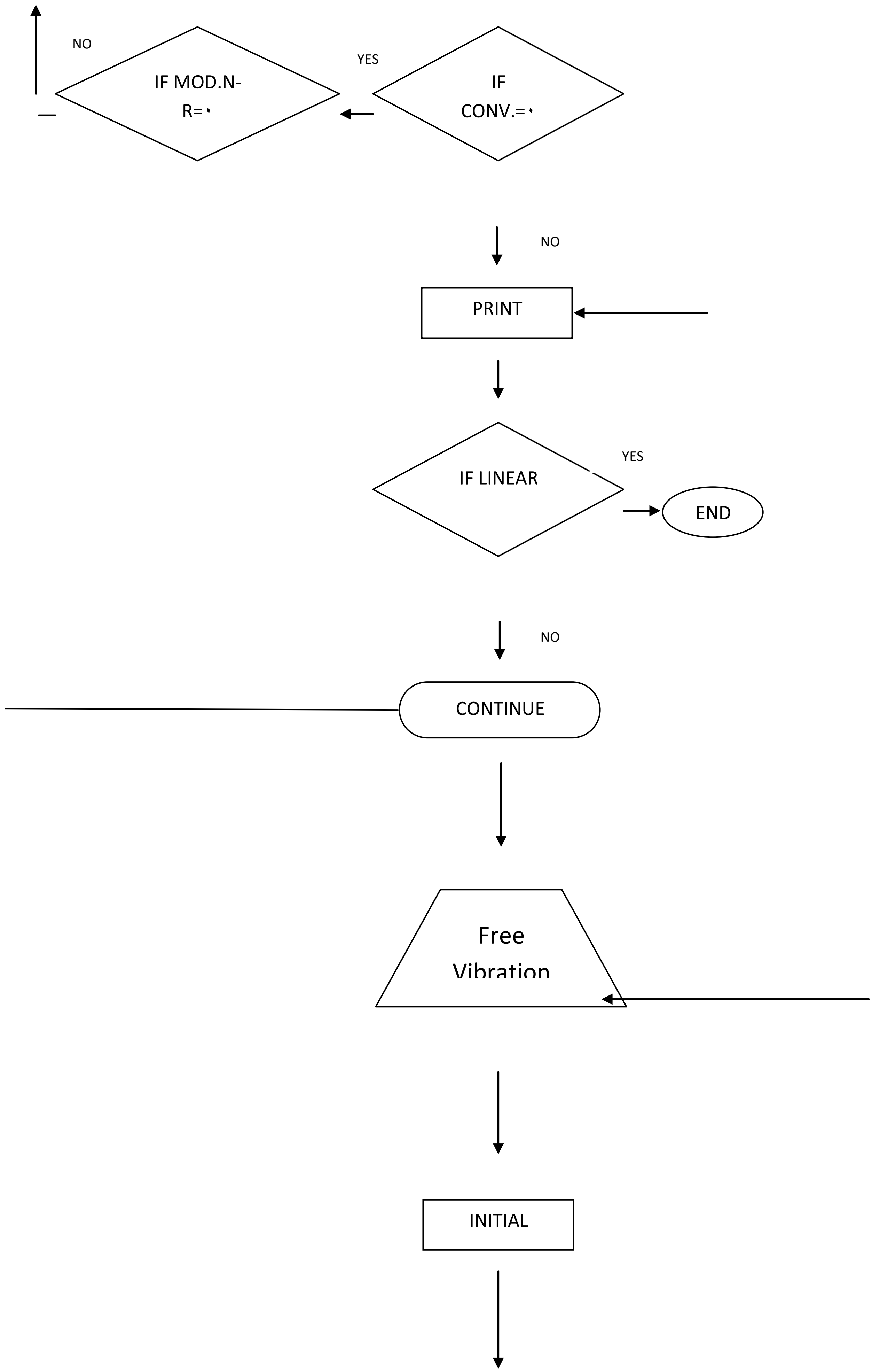


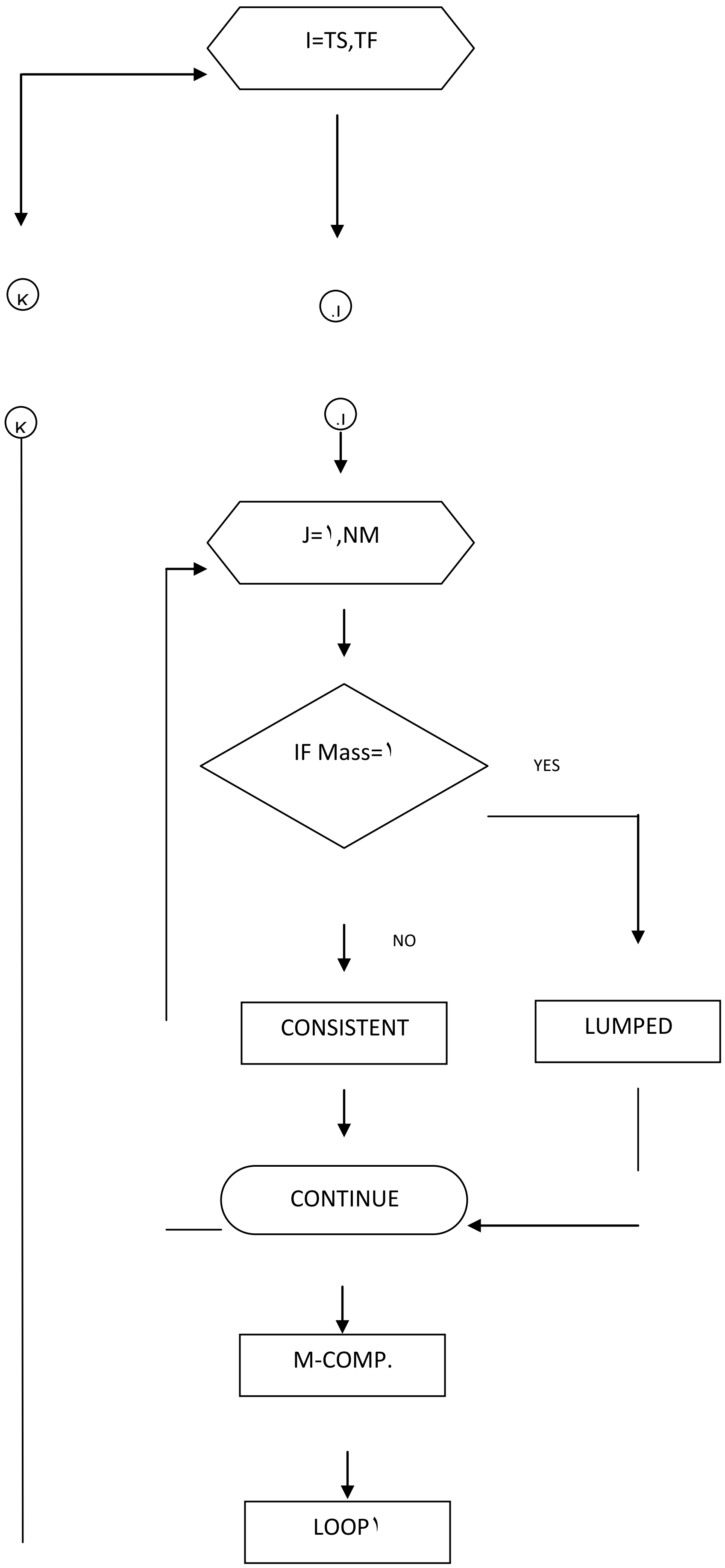

IF LINEAR
YES
NO
A
C
B
A
C
B
I=١,INCS.
INCS=١
APPLY ST.
J=١,NM
NO
YES
PR=١
NON PRISMATIC
PRISMATIC


Loop ١
YES
IF
0 ≤ P ≤ √(k_s EI)
NO
T.S.M.١
T.S.M.٢
CONTINUE
E
D
E


E
D
E
B
T-COMP.
SOLVE-ST.
DISP.
J=١,NM


AXIAL
CONTINUE
IF LIN.OR
LIN.INC.
YES
NO
INTERNAL
RESIDUAL
CHK.CONV.
D
H
G
I
B
E
D
H
YES
G
I
B


NO
IF MOD.N-R=٠
YES
IF CONV.=٠
NO
PRINT
IF LINEAR
YES
END
NO
CONTINUE
Free Vibration
INITIAL


I=TS,TF
K
J
K
J
J=١,NM
IF Mass=١
YES
NO
CONSISTENT
LUMPED
CONTINUE
M-COMP.
LOOP١

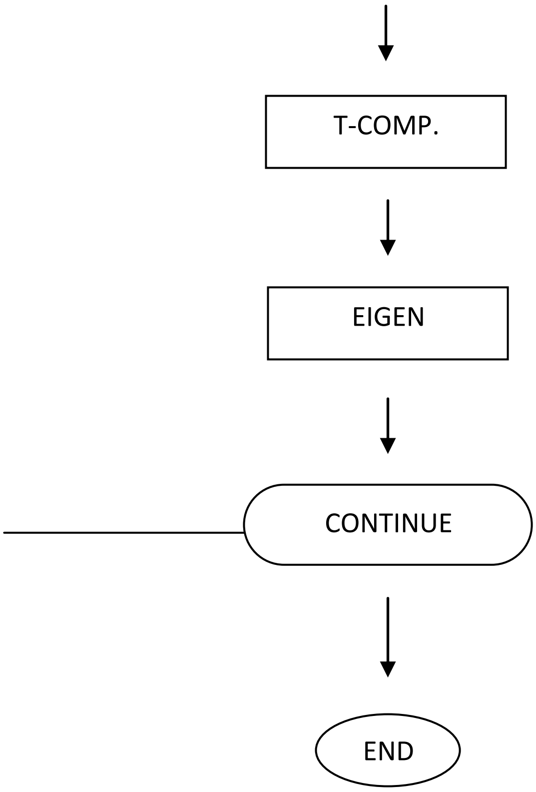


Figure (٦- ١) the flow –Chart of the Computer program

Table (٦- ١) A description for the main items used in the Flow-Chart of the computer program.

No.	**Item**	**Description**
١	Data	Read all data concerning pile geometry and properties, applied loads and method of analysis
٢	If static =٠	If YES: There is free vibration If NO: There is static loading
٣	If linear	If YES: Linear analysis is used. If No: Nonlinear analysis is used.
٤	INCS.	Number of load increments.

٥	Apply-St	Calculate the static applied load vector.
٦	NM	Number of elements.
٧	PR	If YES: use prismatic member in the analysis If No: calculate the properties of each segment and analysis by steps method.
٨	T.S.M.١	Evaluate the tangent stiffness matrix for each element of $0 < \mathbf{P} < \sqrt{\mathbf{k_s\ EI}}$.
٩	T.S.M.٢	Evaluate the tangent stiffness matrix for each element of $\mathbf{P} > \sqrt{\mathbf{k_s\ EI}}$.
١٠	T-Comp.	Evaluate the pile tangent stiffness matrix
١١	Solve-St	Solve the set of equation for the incremental displacement by Gauss elimination. Also find the determinate of the T.S.M.
١٢	Disp.	Evaluate the total displacement vector.
١٣	Axial	Calculate the member axial force by iterative process. Also find the member end moments.
١٤	If Lin.Inc.	If YES: the linear incremental method is used. If NO: the incremental-iterative method is used.
١٥	Internal	Evaluate the internal force vector of the system.
١٦	Residual	Calculate the out-of-balance load vector.
١٧	Chk.Conv.=٠	Check the convergence of the solution using the displacement criterion.
١٨	If Conv.=٠	If YES: The solution does not converge. If No: the solution converges.
١٩	If Mod.N-R	If YES: The modified N-R is used for the iteration process. If NO: The conventional N=R method is used.

٢٠	Print	Print all the desired results.
٢١	Intitial	Set the initial conditions at time for acceleration, velocity and displacement.
٢٢	TS.Tf	Time step and end of the time chosen for the dynamic analysis respectively.
٢٤	If mass=١	If YES: Lumped mass matrix is used. If NO: The consistent mass is used.
٢٥	Lumped	Calculate the member mass lumped matrix.
٢٦	Consistent	Calculate the member consistent mass matrix.
٢٧	M-Comp.	Evaluate the pile mass matrix.
٢٨	Eigen	Solve the eigen value problem and find the natural frequencies of the system.

٦-٤ Numerical Examples

In order to verify the reliability of the computer program, seven examples are used to be solved and the results are compared with those by other researchers. These examples represent different types of loading conditions for the pile and beams on elastic foundation.

The first three examples deals with static analysis .The next two examples deals with the dynamic free vibration analysis and obtain the natural frequencies. The last two are explaining the pile under static loads in case of prismatic and non-prismatic case.

٦-٤-١ Example No. ١

The first example chosen to be solved in the present study represents a square aluminum (E=٧٠ MPa) pile ٢.٦٨٤ m length with width of ٠.٤٢٩ mm embedded in uniform sand media under an axial load media (the specification of the pile is shown in figure (٦-٢)) .This example had been solved by (Yang and Jeremic ٢٠٠٠) using finite element method .In the present study ,pile was analyzed using (٥ elements) and equal load increments each of (٤٠KN).The N-R method has been used for the iteration process with a tolerance error of (١x ١٠$^{-٤}$). Figure (٦-٣) shows the load-displacement curves for point (A), where a good agreement has been shown between the present study and the previous solution obtained by (Yang and Jeremic ٢٠٠٠).

The maximum percentage difference between the present study and That of Yang and Jeremic is (٥.٩٪) due to the analysis of previous study depend on ٣D FEM while the present study used ٢D FEM.

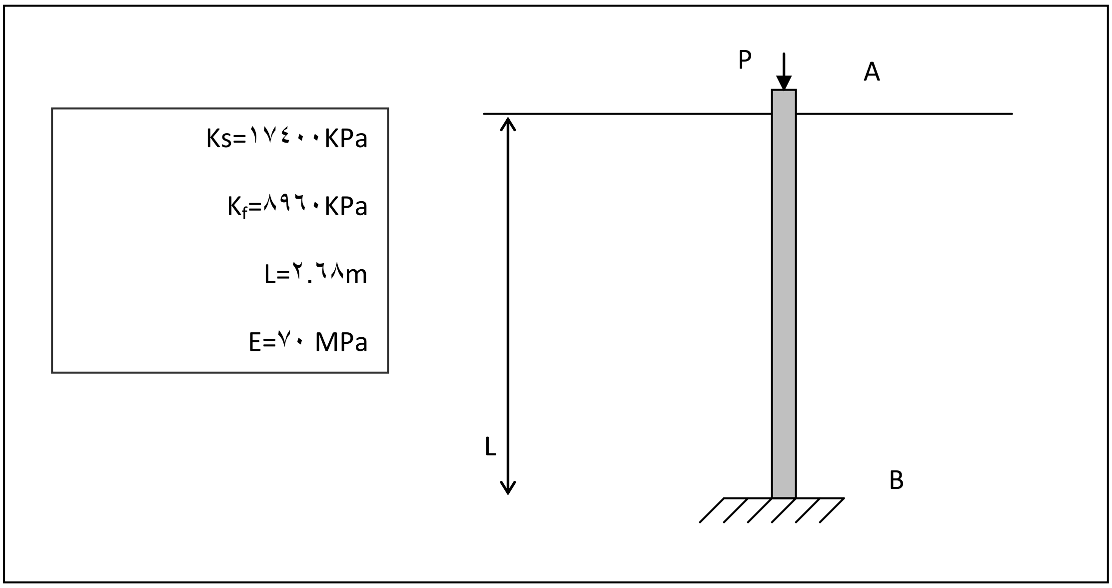


Figure (6-2) the geometry and loading condition for pile in example No.1

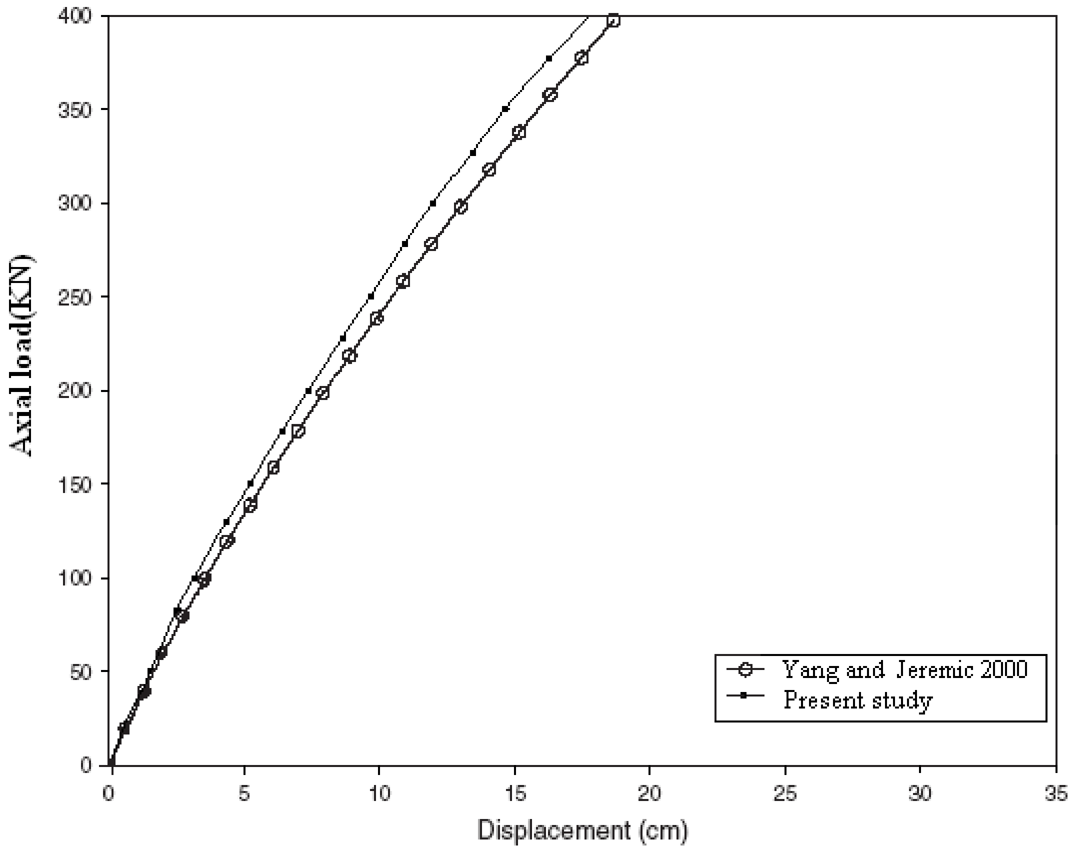


Figure (6-3) the load-displacement Curve for point A of example No.1

٦-٤-٢ Example No.٢

In this example ,the geometry and properties of the lateral loaded aluminum pile are embedded in uniform clay media shown in figure (٦-٤).This problem had been solved by (Yang and Jeremic),by using ٣D finite element method .In the present study ,pile was analyzed using (٥ elements) and equal load increments each of (٢٠KN) . N-R method has been used for the iteration process with a tolerance error of (١x ١٠$^{-٤}$).Figure (٦-٥) shows the load-displacement curves of point A (the top of pile), where a good a agreement has been shown between the present study and previous solution as obtained by (Yang and Jeremic).

The differences in the results occur because of the same reason that was mentioned before.

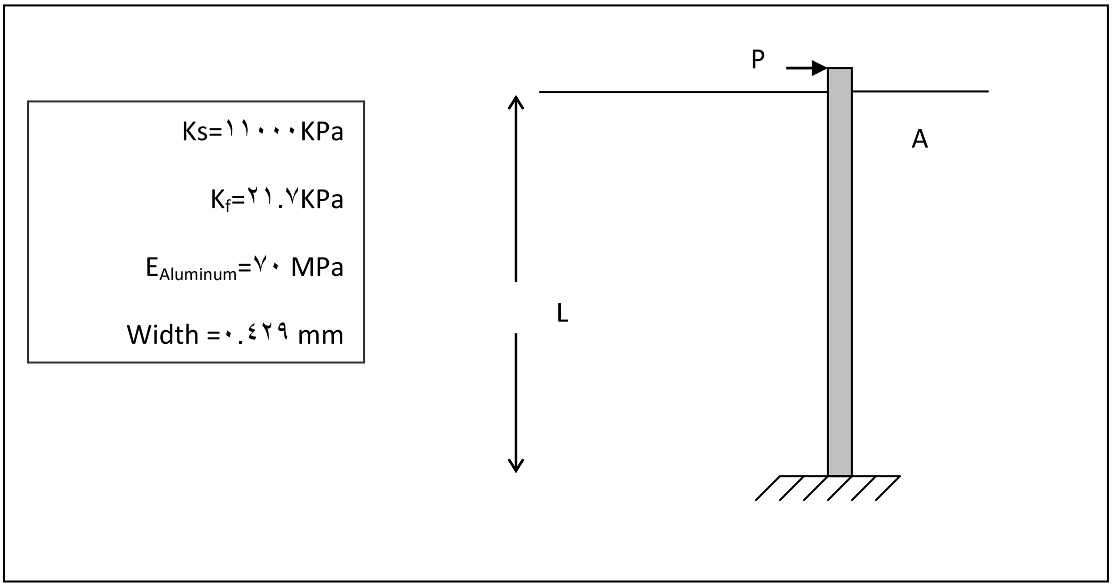


Figure (6-4) the geometry, properties and loading condition for pile in example No.2

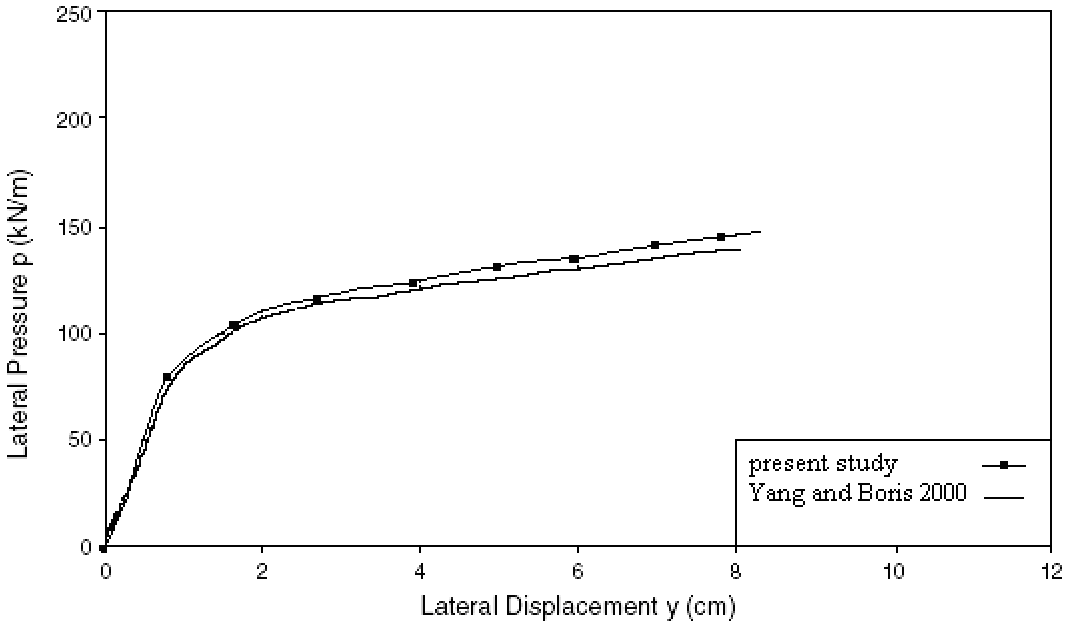


Figure (6-5) the load-displacement curves for point A of example No.2

٦-٤-٣ Example No. ٣

A beam shown in the figure (٦-٦) with it geometry and properties was chosen to be analyzed in the present study to indicate the validity of computer program. The problem had been solved by (Al-Khafaji) by using beam-column approach.

In the present study, this beam on elastic foundation was analyzed using (١٠ elements) and equal load increments each of (٢٥KN).

The results is shown in figure (٦-٧).It can be seen that the present solution have a good agreement with pervious study.

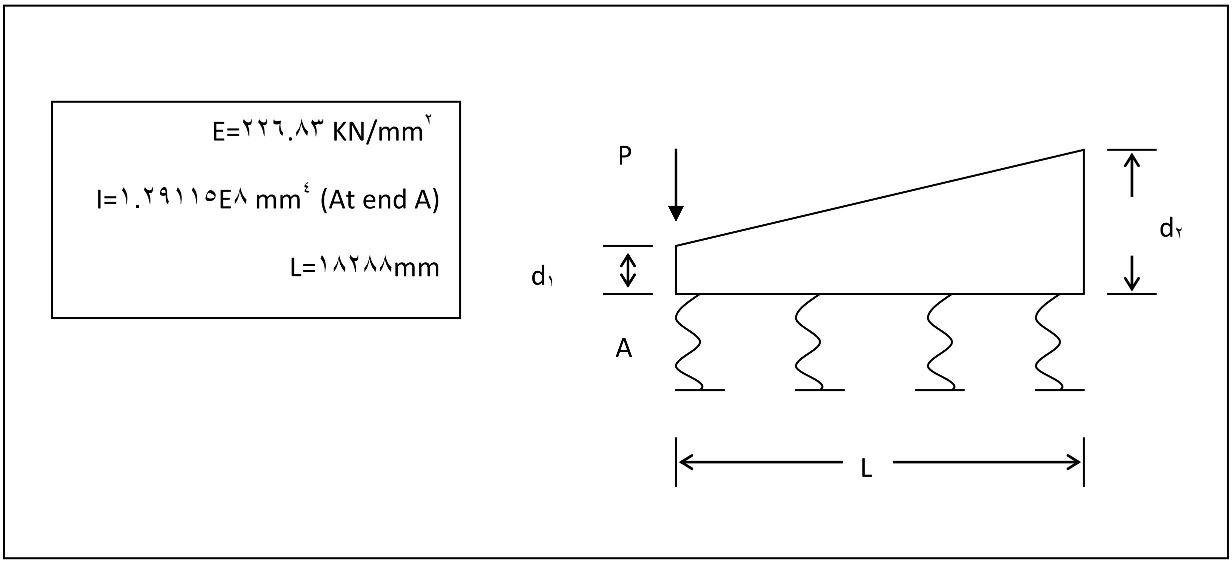


Figure (6-6) the geometry, properties and loading condition for beam on elastic foundation in example No.3

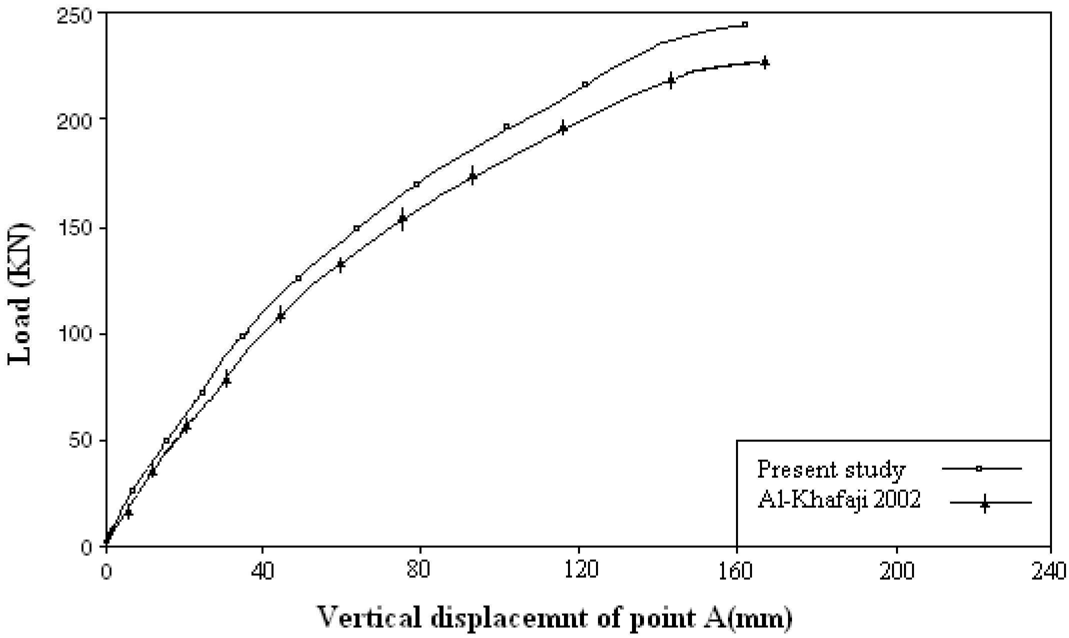


Figure (6-7) the load-displacement curves of example

٦-٤-٤ Example No. ٤

The same pile at example No.١ has been chosen to be acquainted with the effective of the lateral soil stiffness on the failure load.

Figure (٦-٨) is shown the load- displacement curve for the pile with lateral soil stiffness effect and for pile without lateral soil stiffness effect. The displacement shown at figure represents the vertical displacement for point A.

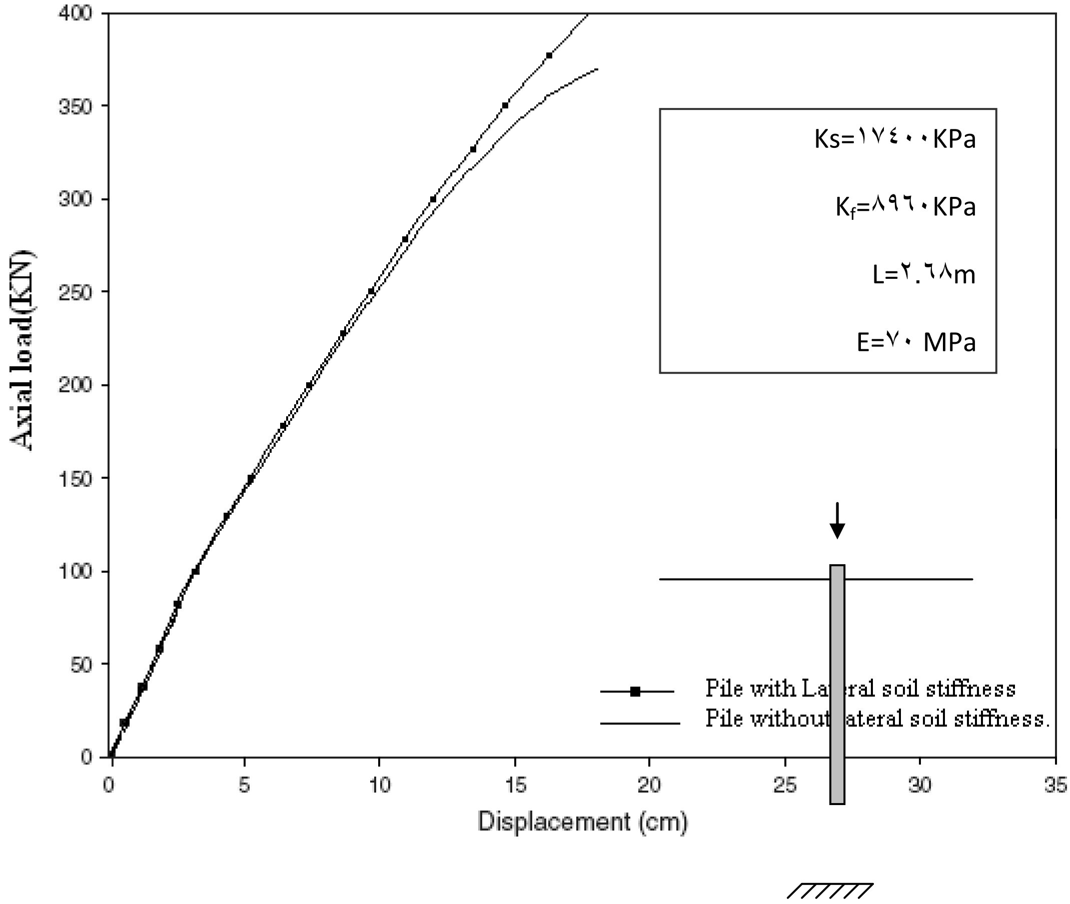


Figure (6-8) the load-displacement curves of example No4.

٦-٤-٥ Example No.٥

A fixed ended beam as shown in figure (٦-٩) was chosen to verify the reliability of the dynamical part of the computer program.

This example has been given by (Al-Bidairi ١٩٩٨) to obtain the natural frequency.

Table (٦-٢) gives the fundamental natural frequency that obtained by (Al Bidairi) and the present study, A good an agreement has been shown between the present study and previous solution obtained by (Al-Bidairi ١٩٩٨).With difference of not more than (٢.٤٪)

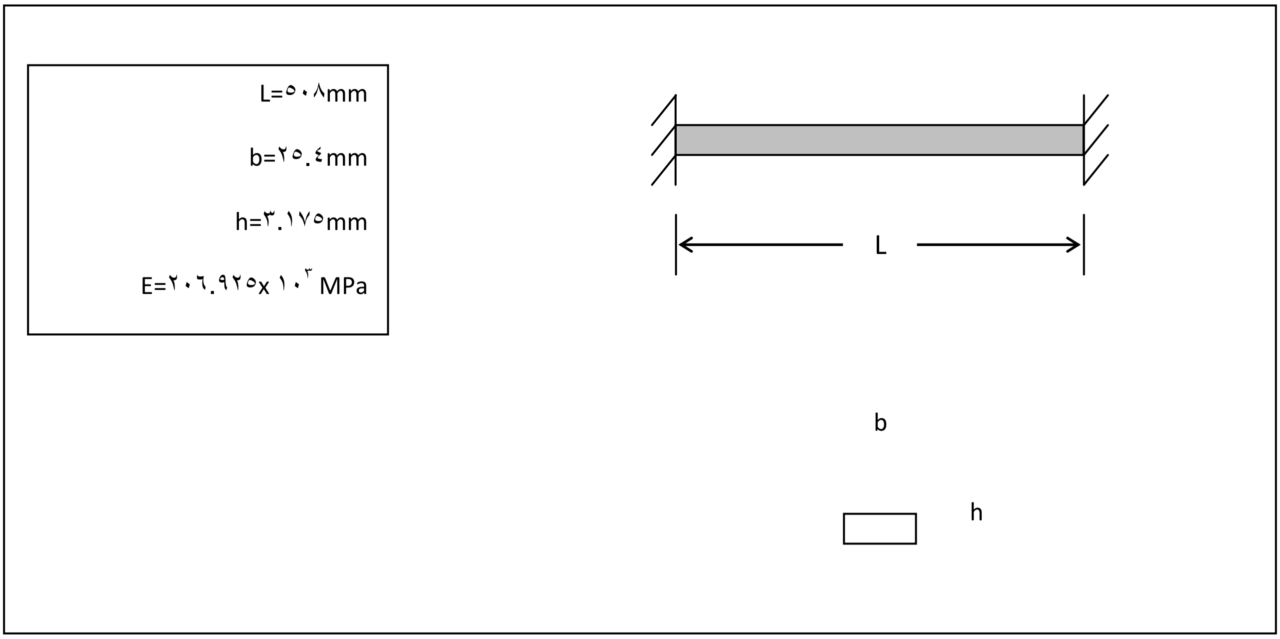


Figure (6-9) the geometry, properties and loading condition for pile in example No.5

The table (٦- ٢) the natural frequency for example No. ٥

Type of Mass Matrix	Al-Bidiari Natural frequency Cycle/sec	Present study Natural frequency Cycle/sec
١-lumped(٢ elements)	٩٦.٧٦٧٧	٩٩.١٠٣٥
٢-Lumped (٤ elements)	١٠٦.٩٢٨	١٠٨.٣٧٧٧
٣-Lumped (٨elements)	١٠٩.٥٩٨٩	١١٠.٣٢٦٣
٤-consistent (٢ elements)	١١٢.٢٧٣٧	١١١.٠٨٢٤

٦-٤-٦ **EXAMPLE NO.٦**

The geometry and properties of the beam in the figure (٦-١٠) show the example that was given by (kolousek١٩٧٣) .In the previous study were used ٥ elements to analyze the problem, while the present study ١٠ elements were used to analyze the problem.

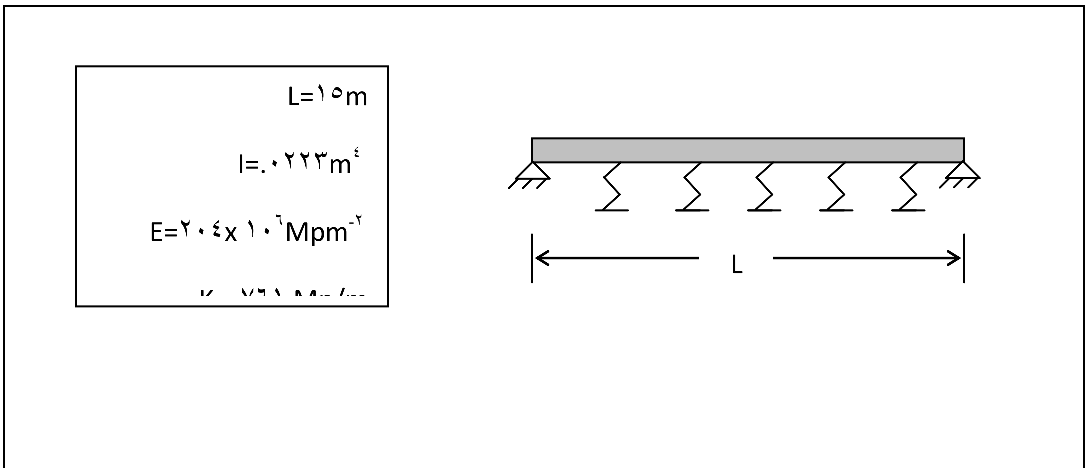


Figure (6-10) the geometry, properties and loading condition for pile in example No.6

The table (٦-٣) shows the natural frequency that was obtained by (Kolousek ١٩٧٣) and the present study, A good agreement has been shown between the present study and the previous solution obtained by (Kolousek ١٩٧٣).

Table (٦- ٣) the natural frequencies for example No. ٦

Reference	Natural frequency Cycle/sec
Kolousek	٢.٧
Present study	٢.٦٦٦

٦-٤-٧ Example No.٧

A non-prismatic steel pile fixed at bottom and free at top under bending moment and the axial load had been chosen to be the seventh example ,the geometry and properties are shown in figure (٦-١١).

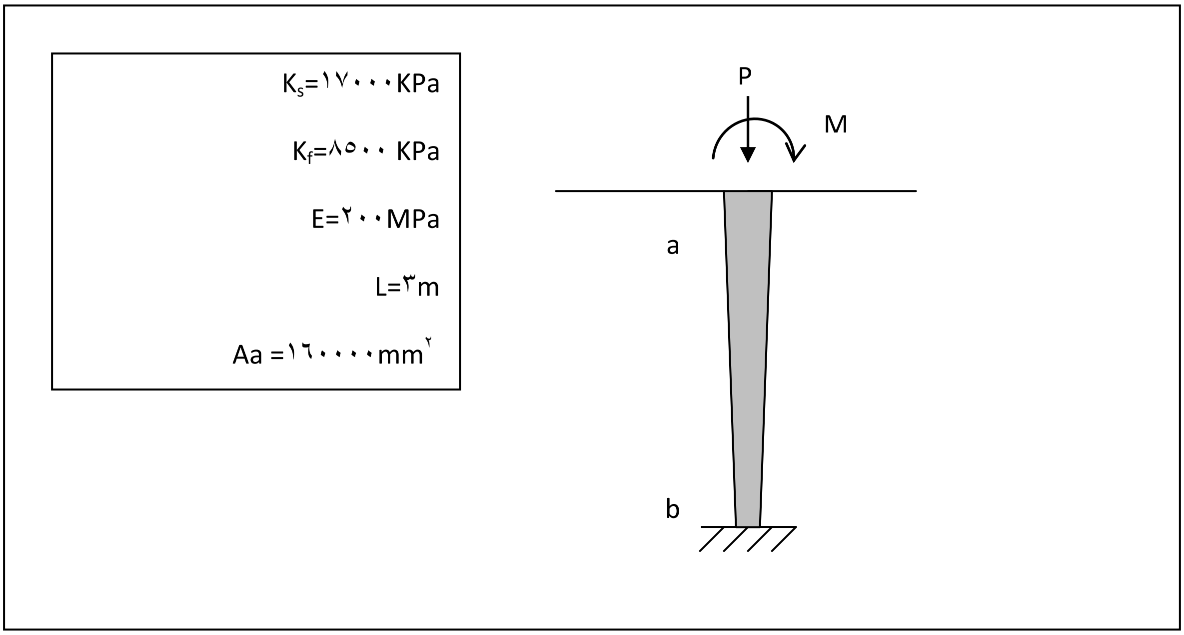


Figure (6-11) the geometry and properties for example No. 7

The Figure (٦-١٢) shows the load-vertical displacement curve. The example had been analyzed by seven elements and increment equal load is (٦٠ KN) for the axial load with fixing the magnitude of the moment.

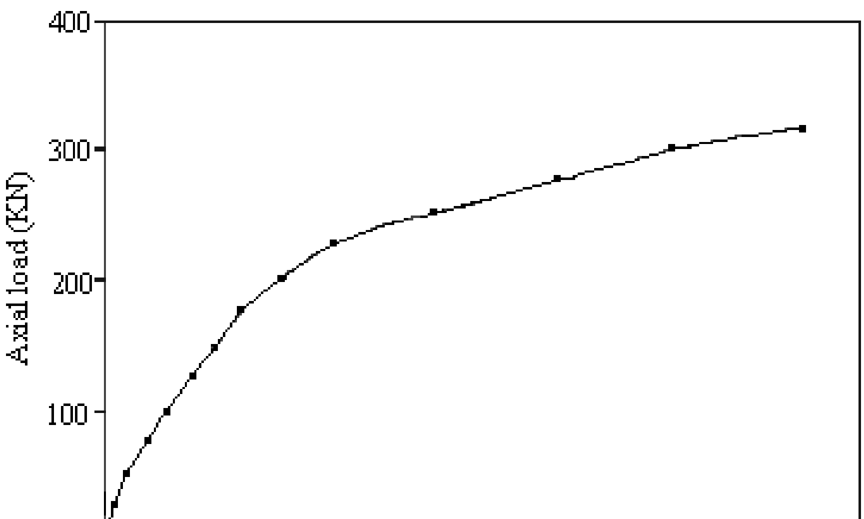


Figure (6-12) the load-vertical displacement curve for exampleNo.7

CHAPTER SEVEN

Conclusions And Recommendations

٧-١ Conclusions

Based on the results obtained in the present study, several conclusions may be drawn. These may be summarized as follows:

١-This study shows that the large displacement elastic behavior of steel pile subjected to static loads and the free vibration behavior can be accurately predicted by using the present finite element modeling.

٢-Using different types of pile mass matrices gives a considerable difference in the obtained solutions. The consistent mass matrix is shown to be more accurate than the lumped mass matrix to obtaining the fundamental natural frequency. Increasing the number of elements with a lumped mass matrix makes the solution approaches to that obtained by the consistent mass matrix. The difference between the results that obtained by the present study by using consistent mass matrix and those the other researchers is about (١.٢ %).

٣-The elastic foundation decreases the natural frequency for the pile.The percentage of drop in the fundamental natural frequency may reach to (١٤.٢٨٪) experimentally.

٤-The lateral soil stiffness may increase the failure loads of the pile to about (٧.٥٪) for clay.

٥-Experimentlly, the natural frequency of the considered model is well matched with the result that obtained theoretically with difference about (١١٪).

٧-٢ RECOMMENDATIONS:

The following recommendation may be considered as an extension for the present study:

١- The tangent stiffness matrix presented in this study may be modified to include more effects such as the variation of the moment of inertia along the pile.

٢- The present study has been concerned with free vibration .A promising field to extending the present work is to study the dynamic behavior of pile under different dynamic types of loads.

٣- The present work may be extended by considering the post-critical behavior of the pile.

٤- Further research may be directed for investigating the elasto-plastic behavior of the piles.

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