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Appendix 1

The all simulation program contains the subroutines CONTROL, EXPT, CHOOSE ,SAMPLING and TERMINAL .

CONTROL is the main procedure and contains the do loop whose iteration gives the replications of the experiment and the code for accumulation of data , computation of statistics and input/ output.

EXPT is the executive program in the event simulation environment . Each call to EXPT yields replication. At the end of an execution of EXPT, the following information is available:

1. The index of the selection population: SEL-POP, SAMP.
2. The number of successes and failures observation in each population: NS(1),..,NS(K) and NF(1),…….,NF(K).

The interaction among the investigator CONTROL and EXPT is as follows : The investigator specifies the input constants by means of values of k , nmax ,p(i),rp(i) and np(i) (i=1,….,k) (where rp and np means rthe prior information r' and n' respectively). Together with a seed for the random number generator and number of replications to be performed ,these are put in a fail for input to CONTROL . For the given input values
CONTROL calls EXPT for as many replications as required and tabulates the observation values.

The number of the replications effects the precision of the estimator from the simulation.

To create the event simulation environment , EXPT invokes the procedures CHOOSE, SAMPLE and TERMENAT each of which corresponds to aspect of the sequential procedure .

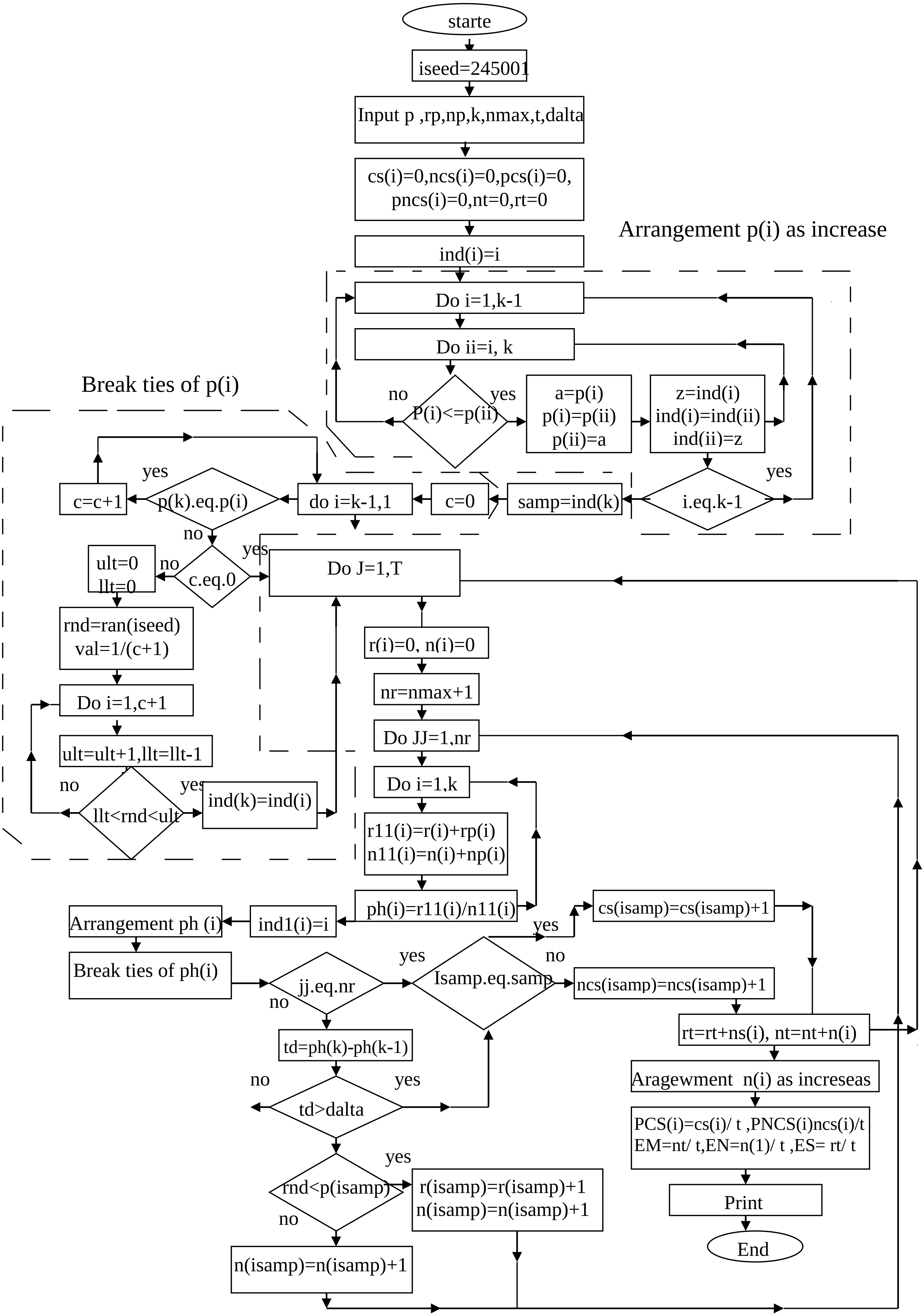
Now, we have two groups of programs. The first group contains the sampling rules A_1,A_2,A_3,A_4,A_5,A_6 and δ_G and the same stopping and terminal rules DS and DT. From this groups selected the scheme δ_f(A4) program which has been expressed by using the first flow chart . The other programs followed the same method with some different in SAMPLE subroutine.

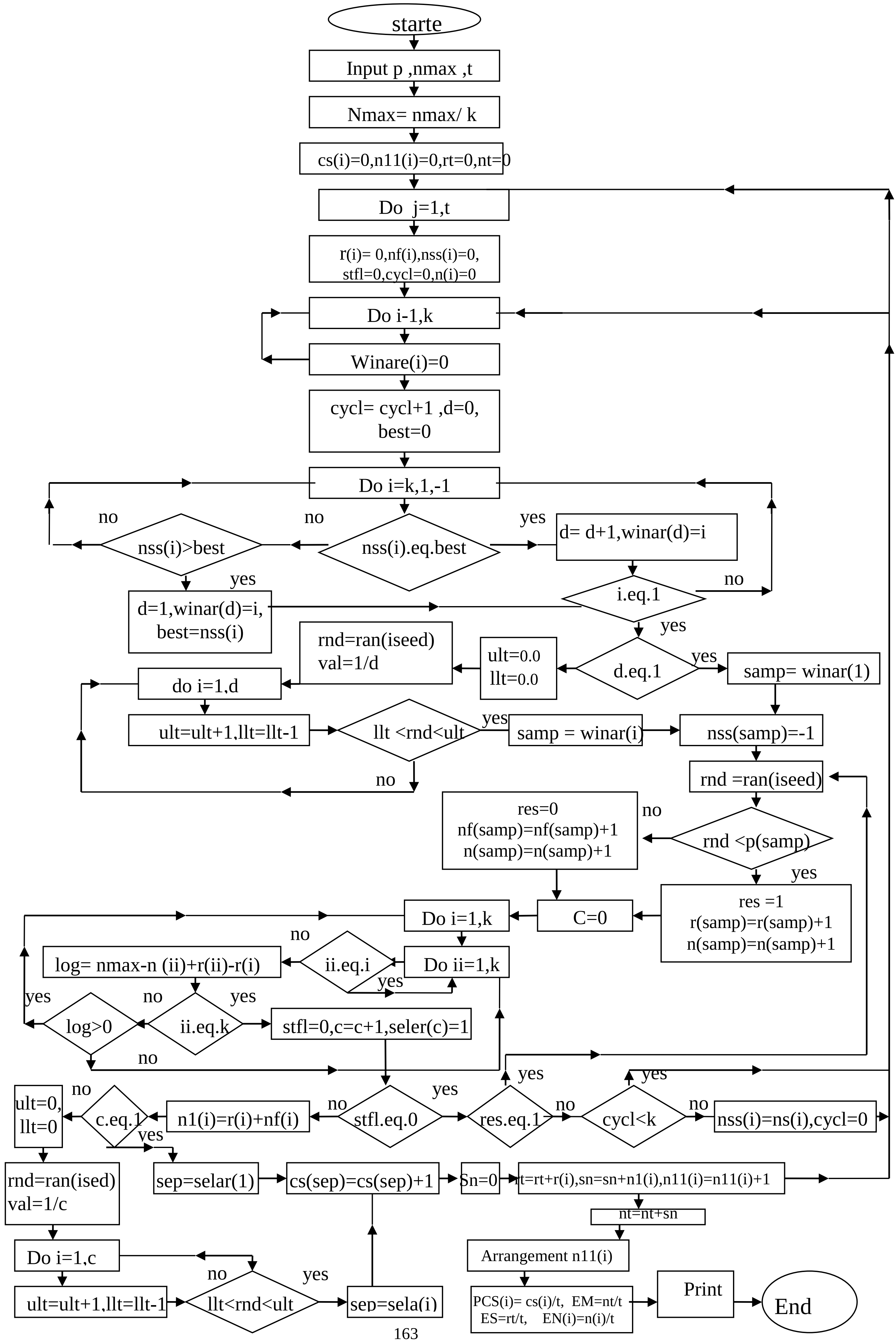
The second group of programs contains the combination of the sampling rules BKR,PWCR,PCWR and GS(VT), the same stopping rules BKS and modification of it and terminal decision rule BKT . The program of the scheme (BKR,BKS,BKT) shows by the second flow chart ,and the others programs followed the same method with some different in CHOOSE and SAMPLE subroutines.

Appendix 2

Listing of the computer code for the simulation of

the schemes $\delta_f(A_4)$ and (BKR,BKS,BKT).


starte
iseed=245001
Input p ,rp,np,k,nmax,t,dalta
cs(i)=0,ncs(i)=0,pcs(i)=0,
pncs(i)=0,nt=0,rt=0
Arrangement p(i) as increase
ind(i)=i
Do i=1,k-1
Do ii=i, k
no
yes
P(i)<=p(ii)
a=p(i)
p(i)=p(ii)
p(ii)=a
z=ind(i)
ind(i)=ind(ii)
ind(ii)=z
Break ties of p(i)
yes
c=c+1
p(k).eq.p(i)
do i=k-1,1
c=0
samp=ind(k)
i.eq.k-1
yes
no
ult=0
llt=0
no
c.eq.0
yes
Do J=1,T
rnd=ran(iseed)
val=1/(c+1)
r(i)=0, n(i)=0
nr=nmax+1
Do i=1,c+1
Do JJ=1,nr
ult=ult+1,llt=llt-1
no
yes
ind(k)=ind(i)
llt<rnd<ult
Do i=1,k
r11(i)=r(i)+rp(i)
n11(i)=n(i)+np(i)
ph(i)=r11(i)/n11(i)
cs(isamp)=cs(isamp)+1
Arrangement ph (i)
ind1(i)=i
yes
yes
no
Break ties of ph(i)
jj.eq.nr
Isamp.eq.samp
ncs(isamp)=ncs(isamp)+1
no
rt=rt+ns(i), nt=nt+n(i)
td=ph(k)-ph(k-1)
no
yes
Aragewment n(i) as increseas
td>dalta
PCS(i)=cs(i)/ t ,PNCS(i)ncs(i)/t
EM=nt/ t,EN=n(1)/ t ,ES= rt/ t
yes
rnd<p(isamp)
r(isamp)=r(isamp)+1
n(isamp)=n(isamp)+1
Print
no
End
n(isamp)=n(isamp)+1


starte
Input p ,nmax ,t
Nmax= nmax/ k
cs(i)=0,n11(i)=0,rt=0,nt=0
Do j=1,t
r(i)= 0,nf(i),nss(i)=0,
stfl=0,cycl=0,n(i)=0
Do i-1,k
Winare(i)=0
cycl= cycl+1 ,d=0,
best=0
Do i=k,1,-1
no
nss(i)>best
no
nss(i).eq.best
yes
d= d+1,winar(d)=i
yes
d=1,winar(d)=i,
best=nss(i)
i.eq.1
no
yes
rnd=ran(iseed)
val=1/d
ult=0.0
llt=0.0
d.eq.1
yes
samp= winar(1)
do i=1,d
ult=ult+1,llt=llt-1
llt <rnd<ult
yes
samp = winar(i)
nss(samp)=-1
no
rnd =ran(iseed)
res=0
nf(samp)=nf(samp)+1
n(samp)=n(samp)+1
no
rnd <p(samp)
yes
res =1
r(samp)=r(samp)+1
n(samp)=n(samp)+1
Do i=1,k
C=0
no
log= nmax-n (ii)+r(ii)-r(i)
ii.eq.i
Do ii=1,k
yes
yes
no
yes
log>0
ii.eq.k
stfl=0,c=c+1,seler(c)=1
no
no
yes
yes
yes
ult=0,
llt=0
c.eq.1
yes
n1(i)=r(i)+nf(i)
no
stfl.eq.0
res.eq.1
no
cycl<k
no
nss(i)=ns(i),cycl=0
rnd=ran(ised)
val=1/c
sep=selar(1)
cs(sep)=cs(sep)+1
Sn=0
rt=rt+r(i),sn=sn+n1(i),n11(i)=n11(i)+1
nt=nt+sn
Do i=1,c
Arrangement n11(i)
no
yes
ult=ult+1,llt=llt-1
llt<rnd<ult
sep=sela(i)
PCS(i)= cs(i)/t, EM=nt/t
ES=rt/t, EN(i)=n(i)/t
Print
End

1.1 Prelude

Multiple decision problems are encountered in many real- life situations. The classical approach to solve these problems is the testing hypothesis, but in many cases, the goal is not only rejecting or accepting the homogeneity hypothesis but to select the best among several alternatives or ordering them according to their performance. For example, we may be interested in choosing the best of several drugs (treatments) or choosing best candidate from several alternatives. The statistical techniques by which these problems can be solved are known as Ranking and Selection procedures. Therefore, Ranking and Selection procedures are statistical techniques for comparing some number k of populations [17, 24], we assume at the outset that the populations are not all the same and can be ordered in some meaningful way (according to their bestness), from worst to best, or from smallest to largest [14,16,37]

The Selection procedures are designed specifically to identify the best single population, or the best subset of – populations, or some subset of populations that includes the best population, or like [16].

Also we can define Ranking and Selection as: selecting the system with the largest or smallest expected performance measure (selection of the best). [42, 43]

This thesis deals with Binomial selection problem where the goal is to select the best Binomial population among k Binomial populations. The approach is Bayesian where some prior information about the parameter of the underlying distribution is involved. Some Monte Carlo simulation results are obtained.

Through this thesis, we shall generally assume the following conditions:

1. There is a prior knowledge regarding the parameters of interest.

2. The procedures are truncated (closed) where there exists an upper bound of number of observations carried out until a decision is taken. Evidently, the procedure will terminate with probability one. The fact that these procedures are closed increases their potential for use in read- life applications.
3. The outcomes of observations are independent and the probabilities for these outcomes remain constant from observation to another observation (stationary). [36]

1.2 The statement of the problem

If there is $\prod_i$(i=1,……,k) are k Binomial populations. The quality of the ith population is characterized by a real- valued parameter p_i, the unknown probability of a success in a single trial from population i, where $0 \le p_i \le 1$ (i=1,….k). The problem is to select the best of these Binomial populations on the basis of sequence of observations. The values of $p_{[i]}$(i= 1,….,k) are assumed to be unknown to us. Moreover, we do not know which population is associated with $p_{[k]}$.The best population is defined to be the one with the highest probability of success that is the population associated with $p_{[k]}$. The ranked success probabilities $p_1, \ldots, p_k$ are denoted by

$$p_{[1]} \le p_{[2]} \le \cdots \le p_{[k]}$$

Observations may be obtained sequentially, singly or in groups of constants size.

Oure goal is to design selection procedures using Bayesian approach that enable us to select the population associated with $p_{[k]}$.

The statistical formulation as stated above is typical of many well-known practical problems in many situations in real life. For examples, Selecting the best production method for certain product among several alternative methods, or selecting the best treatment among several alternatives (treatments).

1.3 Review of literature

In the last three decades, there has been an increasing interest in the development of the selection procedures to solve the problem of selecting the best of k Binomial populations. This interest has stemmed from the potential applicability of these procedures in medical trials and related fields of applications.

Sobel and Hnyett (1957)[44] is considered as a fundamental paper in Binomial selection studies. In this paper they proposed a single sampling procedure in which an equal number of observations n^* are taken from each population and the population having the most successes is selected as the better population with ties broken by randomization. They employed the idea of indifference zone approach which was developed by Bechhofer (1954)[6] to solve the problem of selection in Normal populations. This classical approach requires that the probability of making a correct selection is greater than or equal to some preassigned value, p^*, and the true difference between the largest and next to the largest is a preassigned number Δ^* formally ,

$$P(CS) \geq p^* \qquad \text{....(1.1)}$$

whenever :

$$p_{[k]} - p_{[k-1]} \geq \Delta^* \qquad 0 < \Delta^* < 1, \ \frac{1}{k} < p^* < 1 \ \text{.....................(1.2)}$$

Where CS (for correct selection) denotes the final selection of a population with probability of success $p_{[k]}$.

With the condition above, called p^*, Δ^*–condition, we will be at least 100 p^* percent sure of selecting the better parameter whenever the largest parameter $p_{[k]}$ is at least Δ^* better than the second largest $p_{[k-1]}$.

The P(CS) is minimized when

$$p_{[k]} - p_{[k-1]} = \Delta^*,$$

This is called the least favorable configuration (LFC) of the population parameters $p_1, p_2, \ldots\ldots\ldots p_k$. The value of n^* is then chosen to guarantee (1.1) when the parameter values in the least favorable configuration.

The problem of allocating (assigning) observations among patients in clinical trials has been investigated using other approaches by many authors. Armitage (1975) [3] developed closed sequential procedures. Anscombe (1963)[1], Colton (1963)[13] and Canner (1970)[10] used loss functions to produce a decision theoretic approach using a fully sequential procedure. Pocock (1977)[40] has developed a group sequential design for clinical trials in which the data are analysed at less frequent intervals and which may lead to an early decision, or stopping of a clinical trial, if large treatment differences are observed.

Other workers using the indifference zone approach are Taylor and David (1962)[48] who discussed a multistage procedure for this problem. Paulson (1967)[39] who proposed an open sequential procedure, which permitted the elimination of "non- contending" population and Bechhofer, Kiefer and Sobel (1968)[4] who proposed an open sequential procedure employing a vector at a time sampling rule (VT). The application of the play the winner-sampling rule (PWR) to problem of allocating observations among treatments appeared first in Zelen (1969)[49].

Later a great deal of attention has been paid to the sequential procedures for this problem using different sampling rules such as PWR and VT sampling rules. Kiefer and weiss (1971)[32]. Hoel (1972)[21], Nebenzahl and Sobel (1972)[38], Berry and Sobel (1973)[9], Fushimi(1973), Kiefer and Weiss (1974)[33]and Tamhance (1985)[47] proposed and studied closed sequential procedures for selecting the better of the two Binomial populations. Procedure for selecting the best population of k≥2 Binomial population were considered by Sobel and Weiss (1972)[46]. Hoel and Sobel (1972)[21] and Hoel, Sobel and Weiss (1975)[22].

At the same time, another approach to the Binomial selection problem has been suggested in the classical framework; it is known as the subset selection approach. Here the goal is to select a subset containing the best population with a preassigned probability p^*. This approach is useful for the situation when we have very large number of populations and the procedures require more observations than that available. Therefore it is desirable to select a subset consisting of the best for further extensive investigation. Gupta, Huyett and Sobel (1957)[45], Gupta and Huang (1976)[19] are among those who studied this problem using this approach. Goal and Rubin (1977)[18] gave a general Bayesian decision theoretic approach for selecting a subset containing the population.

The main difference between the subset selection approach and the indifference zone approach is that in the subset selection we have no indifference zone and the least- favorable configuration is simply the worst configuration with all parameters are equal.

Although the literature on Binomial selection problems is large, the literature using Bayesian approach to solve the problems is rather scarce. Important contributions were made By Bland and Bratcher (1968), Bartcher and Bland (1975)[7], who developed Bayesian fixed sample size procedures to solve the problem of ranking Binomial probabilities where more than two populations are compared. In Madin (1986)[36],where

Bayesian sequential schemes are developed for selecting the better of two Binomial populations.

Bechhofer and kulkarni(1981)[6] proposed a very interesting closed sequential procedure avoiding the (p^*,Δ^*-condition) of the indifference zone approach. In a subsequent paper, Bechhofer and Kulkarni (1982)[5] gave exact numerical results for the performance characteristics of the procedures given in Bechhofer and Kulkarni (1981). Bechhofer and Frisardi (1982)[15] investigated these procedures employing Monte Carlo simulation. Law and Kelton (1991)[24] provide an excellent introduction to Ranking and Selection (R&S) with corresponding references to more mathematically intense treatments. Likewise Sanchenz (1997) [12]gives an overview of R&S with several sample scenarios and an extensive list of references. Chick (1997)[12] presents a Bayesian analysis of selecting.The best simulated system. Inoue and Chick (1998)[11] compare Bayesian and frequents approaches for selecting the best system. Chick and Inove (1998) extend Chick′s (1997) work to the study of sampling Costs and value of information arguments to improve the computational efficiency of identifying the best system.

Recently, there are many researches concern with the application of ranking and selection, for instance in the medical field, In 2004 Jennison presented a studying about adaptive sequential designs [28],and in the same year, he presented another paper involves a studying about group sequential monitoring of clinical trials[29]. In 2005 the investigator he did a research about the group sequential selection procedure with elimination and Data- dependent treatment allocation[30], also he taken par with Bruce Turubull to publish a paper cleans with the subject of adaptively in clinical trial designs old and new[31].

Pichitlamken and Nelson (2001) and Boesel et. at. (2003) presented more on the application of Ranking and Selection [43].

1.4 Some Basic ideas and concepts

This section presents some concepts and ideas which are useful in constructing the proposed selection procedures.

1.4.1 Bayesian approach

This approach requires that we specify a prior probability density function $\pi(p_i)$, expressing our beliefs about p_i before we obtain the data. It would be very convenient if p_i is assigned a prior distribution which is a member of a family of distributions closed under Binomial sampling or a member of the conjugate family. The conjugate family in this case is the family of Beta distributions.

Accordingly, let p_i is assigned Beta prior distribution with parameters r_i' and n_i', $B(r_i', n_i')$, the normalized density function is given by

$$\pi(p_i) = \frac{p_i^{r_i'-1}(1-p_i)^{n_i'-r_i'-1}}{B(r_i', n_i'-r_i')} \qquad \text{..........(1.4.1)}$$

$$0 \le p_i \le 1, \quad 1 \le r_i' \le n_i' \quad , \qquad i = 1, \ldots\ldots, k$$

Where $B(r_i', n_i'-r_i')$ is the complete Beta function. It is also assumed that p_i are a prior distribution independent. The parameters r_i' and n_i' not be integer. However it is convenient from this point, we assume that r_i' and n_i' are integers so that we can replace the Gamma functions by the factorial terms in our formulation of the schemes [23].

If our prior distribution beliefs can not be presented in a form of (1.4.1), the analysis which follows will not be applicable and the form of the probability density function will have to be calculated directly from the assessment of prior distributions is not pursed here, we assume that the statistician has already chosen his prior distribution.

In addition to the prior information, we obtain some sampling information from the population $\prod_i$(i=1,……,k). In doing so, we assume that we observe the number of successes r_i, obtained in n_i trials giving probability function.

$$f(r_i / p_i, n_i) = \binom{n_i}{r_i} p_i^{r_i} (1 - p_i)^{n_i - r_i} \quad \text{………………………….(1.4.2)}$$

$$r_i \in [0, \ldots\ldots, n_i]$$

The posterior probability density function is derived from the prior probability function and the assumed sampling model by means of Bayes theorem mentioned earlier.

$$\pi(p / r_i'', n_i'') = \frac{f(r_i'' \backslash p_i, n_i'') \pi(p_i)}{\int_{p_i} f(r_i'' \backslash p_i, n_i) \pi(p_i) dp_i} \quad \text{……………………….(1.4.3)}$$

where $r_i'' = r_i + r_i'$, $n_i'' = n_i + n_i^`$, $i = 1,\ldots\ldots,k$

If the sample size n_i taken from population Π_i is large, then the actual choice of prior parameters (r_i', n_i') has little effect on the posterior density function which can be well approximated by a Beta probability density function with parameters r_i and n_i. In this case it is sufficient to take the uniform prior distribution $\pi(p_i)=1$, to express vague knowledge about the parameters of interest.

As the Beta family is conjugate with the Binomial sampling, it is unnecessary to revise a Beta prior distribution on the basis of a sample from a Bernoulli process using Bayes theorem. Given the prior distribution and the sample results, we need simply note that

$$r_i'' = r_i + r_i' \quad \text{and} \quad n_i'' = n_i + n_i'$$

are the parameters of the posterior Beta density function [23].

1.4.2 Sequential procedure

A sequential procedure has three elements: sampling rule, stopping rule and Terminal decision rule [15,17].

These elements are now defined as follows.

(a). sampling rule.

The sampling rule prescribes which observations are taken from which population. There are different sampling rules such as one at a time or group at a time [15].

(b). Stopping rule.

The stopping rule prescribes when sampling should be terminated. At this stage one population is chosen as the best.

(c). Termination rule.

A decision is made at the time when sampling is terminated .

D_i: p_i is the best, $(i=1,\ldots\ldots,k)$.

1.5 Outline of the thesis

The plan of the thesis can be summarized as follows: Chapter two deals with Bayesian sequential procedure Based on posterior Estimates where Fully sequential schemes δ_f and Group sequential schemes δ_G are considered. In addition, Bechhofe and Kulkarni (BK) and its conjugate are described in this chapter which also involves the modifications of the stopping rules for BK schemes

There are further selection procedures based on sampling rule such as(the Cyclic Play-the -winner- sampling rule(PWCR), the Cyclic play-the-Loser-sampling rule (PLCR) and Play-the-clear -winner –sampling rule (PCWR)). These are presented in chapter two.

Chapter Three contains Monte Carlo simulation studies to investigate the performance of all schemes in terms of some performance measure such as the probability of correct selection and expected sample size.

Discussion of simulations results and some concluding remarks are presented in chapter four.

2.1: Bayesian sequential procedure Based on Posterior Estimates

In this section some Bayesian sequential suboptimal schemes are proposed with decision criteria based on the posterior probabilities of p_i, i=1,….,k, these are prompted by the need for a quick, easy schemes, to select the best of k Binomial populations, which allow for the incorporation of prior information about the populations with the sampling information.

This section contains the description and formulation of the schemes: fully sequential (δ_f) and group sequential (δ_G).

2.1.1:Description of fully sequential and group sequential schemes (δ_f and δ_G)

Suppose that observations are taken sequentially one at a time (δ_f) or group at a time (δ_G), with a maximum sample size of n_i, and are assumed to be independent with probability

$$f(r_i) = \binom{n_i}{r_i} P_i^{r_i} (1 - P_i)^{n_i - r_i} \qquad , i = 1, 2, \ldots, k$$

where p_i is the probability of success for Π_i. Further, assume that p_i is assigned a Beta prior distribution with integer parameters r_i', n_i' (or $B(r_i', n_i')$) with density function proportional to;

$$p_i^{r_i'-1} q_i^{n_i'-r_i'-1} \; , 1 \le r_i' \le n_i' - 1, \quad q_i = 1 - p_i$$

then after r_i successes in n_i trials on $\prod_i$ (i=1, …..,k), the posterior distribution of p_i is Beta with integer parameters r_i'' , n_i'' where $r_i'' = r_i' + r_i$, $n_i'' = n_i' + n_i$ and the posterior expectation of p_i or the predictive probability that the next trial results in a success is $\hat{p}_i = \frac{r_i''}{n_i''}$.[13].

Let $\hat{p}_{[1]} \le \hat{p}_{[2]} \le .... \le \hat{p}_{[k]}$ be the ordered values of $\hat{p}_1, \hat{p}_2, ..., \hat{p}_k$. These Bayesian sequential suboptimal schemes, termed δ-schemes with δ_f refers to the fully sequential scheme and δ_G to the group sequential scheme, are based merely on

$$\delta = \hat{p}_{[k]} - \hat{p}_{[k-1]} \qquad \text{.........(2.1)}$$

and a preassigned constant $\delta_o (0 \le \delta_o \le 1)$ serving as an appropriate distance measure of the differences among the populations.

2.1.2 Formulation of the fully sequential scheme (δ_f)

As we mentioned, in these schemes we sample observations sequentially one at a time. The difference δ is used both to determine the type of observations and the stopping rules and the terminal decision rules. For all schemes D_i is made at termination, where

$D_i : p_i = p_{[k]}$ (i = 1, ………, k),

and for all sequential schemes sampling is stopped once $\delta > \delta_o$.

Let DS and DT denote to the stopping and terminal decision rules for these schemes respectively.

The fully sequential scheme is considered under the following six sampling rules $A_1 – A_6$, when $\delta \le \delta_o$.

A_1 : sample from population [k-1] ($\prod_{[k-1]}$) at next trial if δ>0

sample from population [k] ($\prod_{[k]}$) at next trial if δ<0

sample at random from populations ($\prod_{[k]}, \prod_{[k-1]}$), if δ=0

A_2: A modification of A_1 to break ties, there are unlikely to be ties for non-integer prior parameters.

$\prod_{[k-1]}$ if δ>0 or δ=0 and $\lambda = n_{[k]} - n_{[k-1]} > 0$.

$\prod_{[k]}$ if δ<0 or δ=0 and λ < 0.

At random from ($\prod_{[k-1]}$, $\prod_{[k-1]}$) if δ=0 and λ = 0

A_3 : $\prod_{[k-1]}$ if $\delta>0$ or $\delta=0$ and $\lambda < 0$

$\prod_{[k]}$ if $\delta< 0$ or $\delta=0$ and $\lambda > 0$

at random from it if $\delta=0$ and $\lambda = 0$

A_4 : $\prod_{[k-1]}$ if $\delta<0$

$\prod_{[k]}$ if $\delta>0$

$(\prod_{[k]} , \prod_{[k-1]})$ if $\delta=0$

A_5 : $\prod_{[k-1]}$ if $\delta<0$ or $\delta=0$ and $\lambda > 0$

$\prod_{[k]}$ if $\delta>0$ or $\delta=0$ and $\lambda < 0$

$(\prod_{[k]} , \prod_{[k-1]})$ if $\delta=0$ and $\lambda = 0$

A_6 : $\prod_{[k-1]}$ if $\delta<0$ or $\delta=0$ and $\lambda < 0$

$\prod_{[k]}$ if $\delta>0$ or $\delta=0$ and $\lambda > 0$

$(\prod_{[k-1]} , \prod_{[k]})$ if $\delta=0$ and $\lambda = 0$

2.1.3: Formulation of group sequential Scheme (δ_G)

This scheme is similar to δ_f as it depends on the posterior estimates of the parameters p_i (i=1, ….., k). the observations here are taken sequentially in group of kn observations at each stage, n on each population until a decision is reached or N is reached.

The stopping and terminal decision rules used are DS and DT described in section 2.2. Here there is no choice of sampling among the populations as we continue sampling with all populations if $\delta \leq \delta_o$ is satisfied, otherwise, we stop sampling and proceeds to the terminal decision rule DT. If the sampling has not stopped before N according to this stopping rule then it should be terminated at N.

2.2: Bechhofer and Kulkarni Selection procedure (BK) and (BK*) with modification

This section concerns with considering the selection scheme of Bechhofer and Kulkarni (1982) (BK) and some modifications to their stopping rule. As well as it deals with conjugate of selection scheme of Bechhofer and Kulkarni (BK^*).

2.2.1: Description of (BK) and (BK^*) Schemes

Bechhofer and Kulkarni (BK) (1982)[5] proposed a closed sequential scheme for selecting the best of k ($k \geq 2$) Binomial populations. This scheme consists of three rules: Sampling rule (BKR), a stopping rule (BKS) and terminal decision rule (BKT).

In this procedure each observation corresponds to a stage thus, at stage m, a total of N observations have been taken from all the populations.

The following notation is used to define the Bechhofer and Kulkarni procedur (BKR, BKS, BKT) and it's conjugate procedure (BKR^*, BKS, BKT). Suppose the experiment is at stage m, that is the m^{th} observation has been taken. Let,

$n_{i,m}$ be the number of observations taken from $\prod_i$.

$z_{i,m}$ be the number of successes yielded by $\prod_i$.

$F_{i,m}$ be the number of failures yielded by $\prod_i$.

Note that $n_{i,m} - z_{i,m} = F_{i,m}$

The procedure (BKR, BKS, BKT) is as follows. Sampling rule (BKR) is:

Part 1. Determine

$$F = \left\{ \Pi_i : 1 \leq i \leq k, n_{i,m} < n, F_{i,m} = \min_{1 \leq J \leq k} \left\{ F_{j,m} \right\} \right\}$$

i.e. the set of populations $\prod_i$, with the fewest failures, If only one such population exists, then take the next observation from it and proceed to BKS. Otherwise proceed to part2.

Part 2: Find the set

$$\left\{ \Pi_{i'} : \Pi_{i'} \in F \quad and \quad z_{i,m} = \max_{j \in F} \left\{ z_{j,m} \right\} \right\}$$

i.e. among the populations having the fewest, the $\prod_i$, are those havig the most successes. If there is only one such population, take the next observation from it, otherwise, select observation from it.

Stopping rule BKS:

After taking observation (m+1),consider the inequalities for $1 \le i \le k$

$$z_{i,m+1} \ge n - n_{i,m+1} + z_{j,m+1} \quad for \quad 1 \le j \le k \quad and \quad j \ne i \qquad ...(2.2)$$

If the inequalities hold for any i then stop sampling and proceed to BKT, otherwise, return to part1 of BKR and continue sampling.

Sampling stops when there is at least one population such that no other population can have a greater number of successes in the n available observations.

Terminal rule (BKT):

The terminal decision rule is to select the population i for which the inequalities (2.2) holds. If more than one such population exists, select one at random from among them. Thus, population is selected such that no other population can have a greater number of successes in the n available observations.

We use the following notations for several random variable associated with (BKR, BKS, BKT) when all sampling has terminated:

$N_{[i]}$: is the number of observations taken from the population associated with $p_{[i]}$ $(1\leq i \leq k)$

N : is the total number of observations $N=n_1 + n_2 \ldots\ldots + n_i$

N_s and N_F: are the total numbers of successes and failures observed in the experiment.

Some remarks about this procedure have to be noted. These are.

(i) The inequalities (2.2) with BKR imply $n\leq N \leq kn-1$. To see that N attains the lower bound suppose that the first population sampled from yields n consecutive successes, then BKS calls for stopping, and the population that yielded these successes is selected.

(ii) A consequence of the definition of BKR is that sampling proceeds in cycles. Each cycle, except possibly that last, consists of a sequence of observations taken from each of the k populations. Within a cycle sampling is continued from a given population until it yields a failure or until n observations are taken from that populations. At the start of a cycle, all populations have the same number of failures (this is the same as the number of cycles) so part 2 of BKR applies. Therefore, the first sample is taken from the population having the most successes, randomizing to break ties. Sampling from this population continues until a failure is observed or until n observations have been taken from that population,then part1 of BKR requires that sampling with to one of remaining populations.

The next observation is taken from that population having the largest number of successes among the (k-1) remaining populations (randomizing to break ties). Observations are taken from this population until n observations have been taken from that population. The process is repeated for the (k-2) remaining populations and so on until a failure is observed in all (k) populations or until BKS requires that sampling stop. If sampling is not stopped by BKS a new cycle is begun.

The conjugate procedure of Bechhofer and Kulkarni (BK^*) consists of three rules: conjugate sampling rule BKR^*, stopping rule BKS and the terminal decision rule BKT.

BKR^* is the conjugate sampling rule of BKR. The definition of BKR^* is as for BKR with the roles of successes and failures exchanged.

The sampling rule BKR^* can be described as follows:

Part1. Determine

$$G = \left\{ \Pi_i : 1 \le i \le k, n_{i,m} < n, z_{i,m} = \min_{1 \le j \le k} \left\{ z_{j,m} \right\} \right\}$$

i.e the set of populations having the fewest successes. If only one such population exists, take the next observations from it and proceed to BKS. Otherwise, proceed to part2.

Part2. Determine

$$\left\{ \Pi_{i'} : \Pi_{i'} \in G, F_{i,m} = \max_{j \in G} \left\{ n_{j,m} - z_{j,m} \right\} \right\}$$

where $F_{i,m} = n_{i,m} - z_{i,m}$ *and* $F_{i,m} = n_{j,m} - z_{j,m}$

i.e the set of populations having the most failures among those having the fewest successes. If there is one such population, take the next observation from it, otherwise, select at random a population from this set and take the next observation from it. Sampling with BKR^* may also represented by cycles exchanges the roles of successes and failures in the description of sampling in cycles with BKR.

2.2.2 Some modifications to the stopping rule BKS

Each stopping rule calculates the potential future number of successes on each population in different ways:

1. The stopping rule BKS1

In many practical problems it is not possible to have all the samples of the same size , since in some cases the investigator has no control over the sample sizes. Hence another modification can be made to the stopping rule BKS so that maximum number of observations on each population is not restricted.

The first stopping rule becomes:

$z_i \geq z_j + (N - n_i - n_j)$ where $i, j = 1, \ldots\ldots, k \ \ and \ i \neq j$ …………(2.3)

Stop and take decision, in the event of a tie i.e $r_i = r_j$ after the maximum of N-1 observations, make the decision randomly.

2. The stopping rule BKS_2

A modification to the stopping BKS. Using the posterior estimates of $p_i(i = 1, \ldots., k)$. The stopping rule becomes

$z_i \geq z_j + (N - n_i)\hat{p}_j$ where $i, j = 1, \ldots\ldots, k \ \ and \ i \neq j$ …………...(2.4)

Then stop and proceed to BKT.

3. The stopping rule BKS_3

A modification of the stopping rule BKS_1 using the posterior estimate of $p_i(i = 1, \ldots., k)$. Therefore, the stopping rule becomes:

$z_i \geq z_j + (N - n_i - n_j)\hat{p}_j$ where $i, j = 1, \ldots\ldots, k \ \ and \ i \neq j$ ………(2.5)

stop and proceed to BKT.

2.3: Further Selection Procedure

we discuss some further selection schemes which are combinations of the various sampling, stopping and terminal decision rules already given in the previous section. Let GS(VT) denotes the group sequential sampling rule where vector of observations, one observation from each population, are taken at a time. The investigation also covers three further sampling rules.

2.3.1 The Cyclic Play the winner-sampling rule (PWCR)

The play-the-winner rule (PWR) is a betting strategy used by gamblers for years. This sampling rule was first suggested by Robbins (1956)[41] in a discussion of the two armed-bandit problem (Büringer et al., (1980)) [8]and its application to the problem of allocating observations among treatments appeared in Zelen (1969)[49]. According to this rule the observations are taken sequentially one at a time. At the outset one of the populations is randomly selected and the first observation is taken from this population. If there exists some prior information on the populations then the first observation is taken from the population which has the largest prior mean, or in the event of a tie, at random. In subsequent stages a success generates another trial on the same population and a failure causes a switch to the other population. It is assumed that each observation has instantaneous response. The PWCR orders the k populations at random at the outset and use this ordering in a cyclic manner. After each success, we sample from the same population; after each failure, we switch to the next population in the ordering scheme. After the k^{th} population, we complete the cycle by go back to the first population. The use of PWR and PWCR causes a bias in a favor of sampling the best population.

2.3.2The Cyclic Play the Loser_sampling rule (PLCR)

It is a conjugate of PWCR, The PLCR order orders the k populations at random at the outset and use this ordering in a cyclic manner. After each failure, we sample from the same population; after each success, we switch to the next population in the ordering scheme.

After the k^{th} population, we complete the cycle by go back to the first population. The use of PLCR causes a bias in a favor of sampling from the worse population.

2.3.3 Play the clear – winner-sampling rule (PCWR)

This rule was proposed by Arkles and Srinivasan (1979)[2] and is a modification to PWCR where by observations are taken sequentially either from all or one of the population at each stage. At the first stage one observation is made from each of the populations. In subsequent stages sampling takes place on one population depending on the outcome of the preceding stage. At the i^{th} stage (i=2,3,……) observations either made on all populations if at the $(i\text{-}1)^{th}$ stage given success results on all populations or made on one population Π_i if at the previous stage $(i\text{-}1)^{th}$ given this population Π_i success result.

For some applications, it is of interest to compare the schemes mentioned in the previous chapters under certain criteria. If the schemes are to be used in clinical trials, measure such as the probability of correctly selecting the best treatment at termination or the expected numbers of trials on the poorer treatment are of more use. To calculate these and other measures, some Monte Carlo (MC) simulations were curried out to assess the effectiveness of our procedures.

3.1 Description of the Monte Carlo (MC) studies

In this section, we briefly describe the method of MC simulation as it is applied to our procedures. Monte Carlo studies have been carried out to investigate some of the performance characteristics of the proposed procedures. Computer programs, which simulate the operations of these procedures, were written in Fortran Power Station.

The simulation programs perform a large number of runs t (t=100,000), which are assumed to be independent, in order to obtain MC estimates with high precision. At each run mutually independent Bernouli observations are generated by using the assumed probability model under $p_i(i=1,2,..........,k)$ specified in advance and then the selection procedure is applied. The observed values of several performance measures are accumulated. At the end of all runs, these accumulated values are divided by t to obtain the MC estimates of the performance characteristics of interest.

The library function is used to generate a uniform variate $x(0<x<1)$. Population Π_i with probability of success and failure if $x \geq p_i$ $(i=1,....,k)$. Formally, A Binomial B(n,p) random variable r can be written as $r = \sum_{i=1}^{n} x_i$, where x_i are independent Bernoulli random variables, each taking the values $x_i = 1$ with probability p or $x_i = 0$ with probability (1-p). Thus, to simulate such an r, we need just simulate n independent u(0,1) random variables, $u_1, u_2, \ldots u_n$ and set $x_i = 1$ if $u_i < p$ and $x_i = 0$ if $u_i \geq p$.

The values of $p_i(i = 1,2,\ldots\ldots\ldots,k)$ are fixed, where for each run of 100,000 trials the same $p_i(i = 1,2,\ldots\ldots\ldots,k)$ are used.

With the observed value p_i, the values of x can be considered as the observed values of a random variable, possessing the Bernoulli distribution that should be simulated. According to the suboptimal sampling schemes, the following quantities are required for input.

(i) For the fully sequential scheme δ_f :

N, k, priors information , δ_o

(ii) For the Bechhofer and Kulkarni schemes and other sampling rules:

N, k, prior information .

(iii) Fot the group sequential scheme δ_G :

N, k, priors in formation ,δ_o, group size:

As measures of performance of the proposed procedures we shall use the following quantities.

1.P(CS): probability of correct selection

In a MC experimentation the population that has the greatest probability of success is known to us, so we can check if the procedure gives a correct selection. After t repetitions we estimate the probability of correct selection by the fraction of correct selections in the t replications. It can be computed as follows:

$P(Di/Di)$: The proportion of number of times when the procedure stops and takes decision Di given decision Di is true in t repetitions.

$$P(CS) = \sum_{i=1}^{k} p\left(D_i / D_i\right)$$, where D_i : $p_i = p_{[k]}$, i=1,…..k

2. $E(M)$: Expected sample size, where (M) denoted the actual number of observations taken from the given population. An estimate of *M* is given by

$$E(M) = \sum_{j=1}^{t} M_j / t$$

where M_j denotes the number of observations taken from the k populations in the jth run.

3. $E\left(N_{(1)}\right)$: Expected number of observations on the inferior population.

An estimate of $N_{(1)}$ is given by

$$E\left(N_{(1)}\right) = \sum_{j=1}^{t} N_{(1)j} / t$$

where $N_{(1)j}$ is the number of observations assigned to the inferior population in the jth run. $E\left(N_{(1)}\right)$ is not given for the group sequential schemes since $E\left(N_{(1)}\right) = E(M)/k$.

4. $E(S)$: Expected number of successes. An estimate of *S* is given by:

$$E(S) = \sum_{j=1}^{t} \left(\sum_{i=1}^{k} R_i \right)_j \Bigg/ t$$

where $\sum_{i=1}^{k} R_i$ is the number of the successes gained in the jth run with R_i successes from population $\Pi_i (i = 1, \ldots, k)$.

These simulation studies provide a comprehensive picture of the performance of the schemes mentioned before over a broad range of values of $\underset{\sim}{p} = (p_{[1]}, p_{[2]}, \ldots\ldots, p_{[k]})$ for k= 3, 4, 5 and 9 populations, and for a range of N-values of practical interest. We have limited our study to equally spaced p-values and equal p- values; the values chosen permit meaningful sensitivity studies for one p- vector to another.

In this thesis it is an important to recall that the symbol of greater than (>) refers to the favorites of an schemes.

The contents of this chapter can be summarized as follows:

The performance of the suboptimal selection schemes δ_f, δ_G and $F_{\delta\delta}$ are given is section 4.1.

The performance of the sampling rule BKR with stopping rules BKS and modifications are discussed in section 4.2. the performance of the sampling rules PWCR, PCWR and GS (VT) all these with stopping rules BKS, BKS_1, BKS_2 and BKS_3 are discussed in section 4.3, 4.4 and 4.5, respectively.

4.1 Performance characteristics of the schemes δ_f,δ_G and F_{SS}

In this section, attention is confined to the study of suboptimal schemes that based on posterior expectation of p_i(i=1, …..,k) using Monte Carlo simulation technique. The purely sequential schemes δ_f , which are constructed from the sampling values A_1, A_2, A_3, A_4, A_5 and A_6 in conjunction with the stopping rule DS and terminal decision rule DT. In the group sequential schemes δ_G, when the group size n=N/K, we will have the fixed suboptimal sampling size.

To assess the properties of these schemes we need to study the effect of the parameter δ_0 on the behavior of the schemes. The performance measures P(CS) , E(M) , $E(N_{(1)})$ and E(S) are increasing functions of δ_0 but the rate of increase is large for small value of δ_0 and small for large values of δ_0 . Therefore , an increase in the values of δ_0 has little effect on P(CS), but allowing a small decrease in P(CS) can be compensated for by a large reduction in E(M) and $E(N_{(1)})$ as it is clear from table (4.1).

Graphically, the effect of δ_0 on the performance measures of the schemes δ_f (A_1) and $\delta_f(A_4)$ are displayed in Figures (4.1- 4.4).

Table (4.1)

Performance characteristics of the schemes $\delta_f(A_1)$and $\delta_f(A_4)$ for N=150, different values of δ_o, for fixed p_i (i=1,2,3) , $\underset{\sim}{p} = (0.4,0.6,0.8)$ and for Uniform priors .

	P(CS)	E(M)	$E(N_{(1)})$	E(S)
δ_0	$\delta_f(A_1)$			
0.0	0.80	1.20	0	0.88
0.1	0.80	1.20	0	0.88
0.2	0.80	19.77	1.33	13.79
0.3	0.81	63.35	5.47	40.52
0.4	0.85	121.57	19.87	73.01
0.5	0.87	148.53	19.98	87.17
0.6	0.87	149.99	20.08	88.09
0.7	o.87	150	20.90	88.20
0.8	0.87	150	20.96	88.28
0.9	0.87	150	20.96	88.29
δ_0	$\delta_f(A_4)$			
0.0	0.79	1.20	0.0	o.88
0.1	0.80	1.20	0.0	o.88
0.2	0.81	3.97	0.21	2.77
0.3	0.82	26.12	2.22	18.18
0.4	0.84	104.85	4.07	79.85
0.5	0.84	144.61	4.95	110.08
0.6	0.84	149.91	4.96	114.05
0.7	0.84	150	5.29	114.07
0.8	0.84	150	5.30	114.10
0.9	0.84	150	5.83	114.98

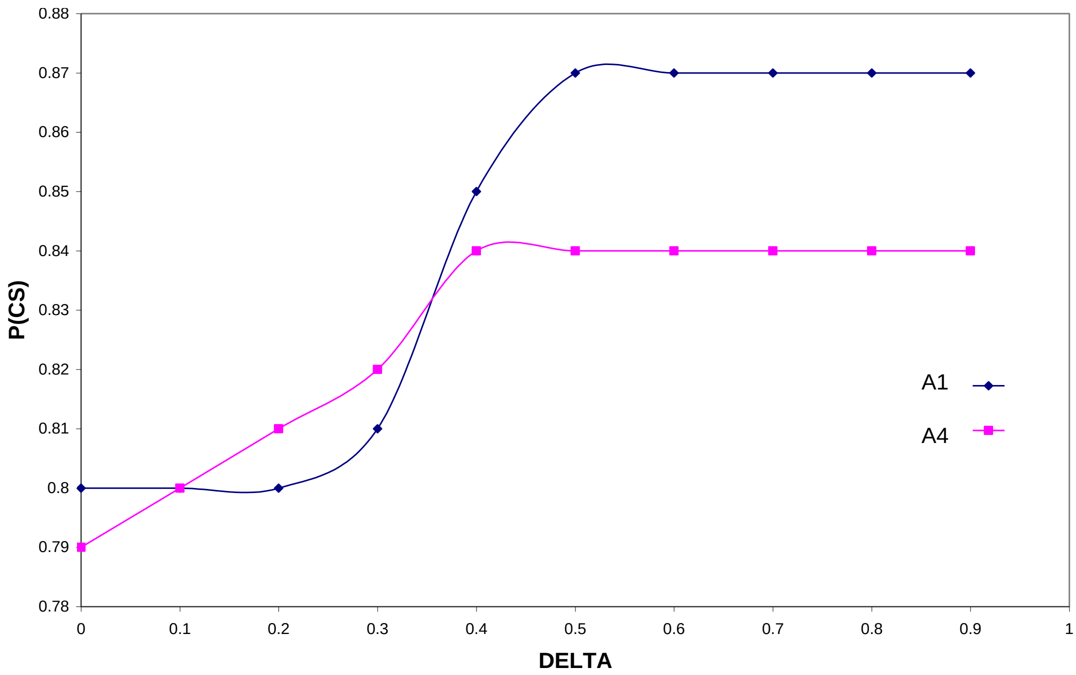


Fig. 4.1 P(CS) as a function of δ_0 for the schemes δ_f (A_1) and δ_f (A_4) when N=150 with fixed Pi's , where i=1,2,3 ,under uniform priors.

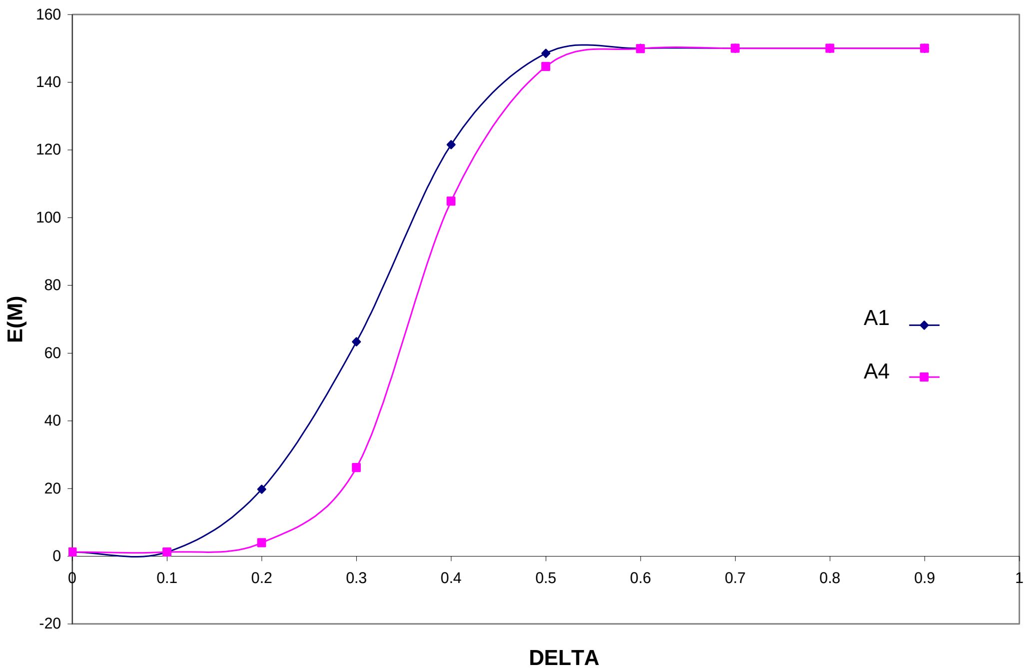


Fig 4.2 E(M) as a function of δ_0 for the schemes δ_f (A_1) and δ $_f$(A_4) when N =150 with fixed pi's ,where i =1,2,3 ,under uniform priors.

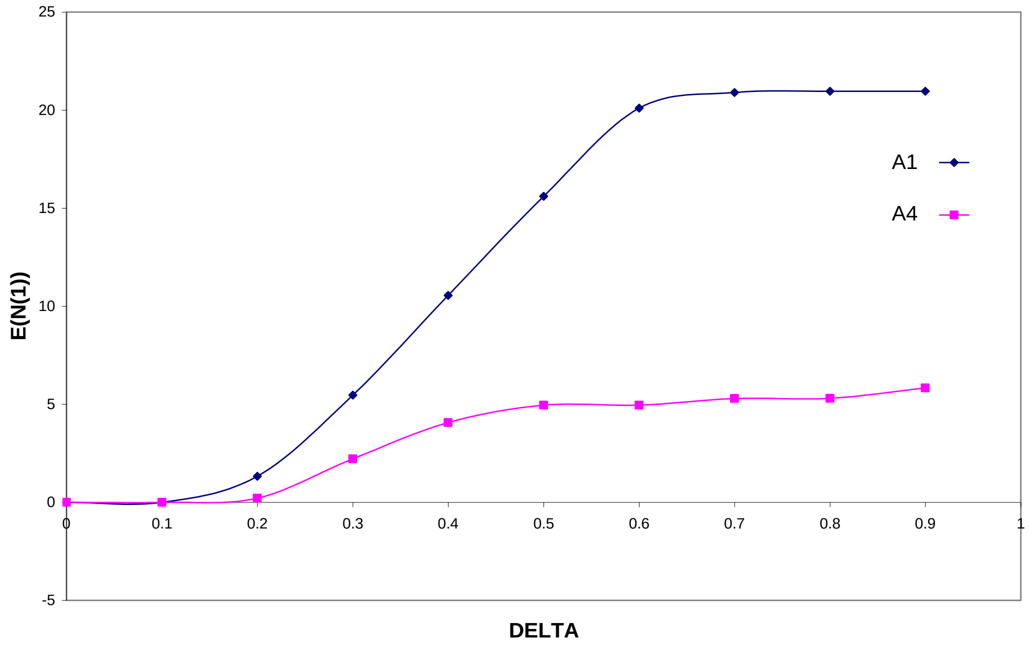


Fig 4.3 $E(N_{(1)})$ as a function of δ_0 for the schemes δ_f (A_1) and δ_f(A_4) when N =150 with fixed pi's ,where i =1,2,3 ,under uniform priors.

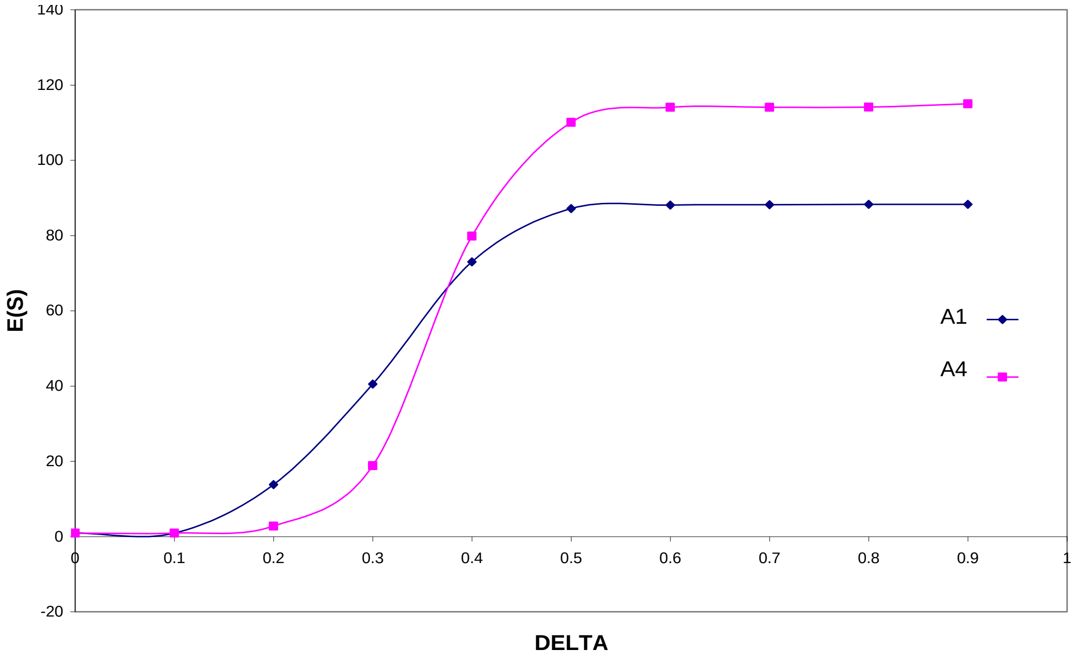


Fig 4.4 E(S) as a function of δ_0 for the schemes δ_f (A_1) and δ_f (A_4) when N =150 with fixed pi's, where i =1,2,3 ,under uniform priors.

We conclude that it is necessary to find a compromise between δ_0 which given earlier stopping resulting in smaller P(CS), and larger δ_0 giving later stopping with larger P(CS). In our particular examples, where P_i (i=1, ….,k) are fixed, the value δ_0= 0.4 is a good compromise if we wish to judge the overall performance of the schemes under all measures we considered.

If we took at the comparison between the sampling rules (A_1-A_6). There is a large differences between the groups (A_1- A_3) and (A_4-A_6) under all criteria P(CS), E(M), E($N_{(1)}$) and E(S) as in table (4.2). This table shows that the performance measures are increasing function of N. As a group the sampling rules (A_4-A_6) have the property of sampling from the population which has the greater posterior expected success probability and hence the property of stay on the winner. The group of the sampling rules (A_1-A_3) has the property of sampling form population that has the smaller posterior expected success probability and hence it has the property of stay on the loser. In general, the sampling rule A_2 performs better that others sampling rule within the group (A_1-A_3) and A_5 performs better than other sampling rules within the group (A_4-A_6). Therefore, these two sampling rules are chosen as the candidates representing these two groups for future comparisons.

Table (4.2)

Performance characteristics of the schemes $\delta_f(A_1)$, $\delta_f(A_2)$, $\delta_f(A_3)$, $\delta_f(A_4)$, $\delta_f(A_5)$, $\delta_f(A_6)$, N=30(30)150 for fixed p_i (i=1,2,3), $\underset{\sim}{p} = (0.2,0.6,0.8)$ under the uniform priors.

$\delta_f(A_1)$				
N	P(CS)	E(M)	$E(N_{(1)})$	E(S)
30	0.96	28.27	2.59	13.93
60	0.98	53.81	2.90	26.11
90	0.98	78.57	3.05	37.93
120	0.98	102.39	3.06	49.37
150	0.98	127.38	3.12	61.29
$\delta_f(A_2)$				
30	0.97	22.60	2.61	11.81
60	0.98	39.59	2.88	20.08
90	0.99	56.02	3.09	28.10
120	0.99	71.88	3.10	35.81
150	0.99	87.01	3.15	43.32
$\delta_f(A_3)$				
30	0.96	28.55	2.67	13.20
60	0.98	54.48	3.04	26.31
90	0.98	79.61	3.22	38.33
120	0.98	104.40	3.38	50.40
150	0.98	129.11	3.38	62.11

Table(4.2)

$\delta_f(A_4)$				
N	P(CS)	E(M)	$E(N_{(1)})$	E(S)
30	0.95	18.96	0.45	16.49
60	0.95	28.60	0.60	24.80
90	0.95	35.69	0.71	30.84
120	0.96	41.49	0.87	35.65
150	0.96	47.78	0.98	40.81
$\delta_f(A_5)$				
30	0.94	18.40	0.51	15.15.75
60	0.95	27.67	0.64	23.60
90	0.95	34.19	0.69	29.10
120	0.95	39.71	0.82	33.73
150	0.96	45.15	0.83	38.27
$\delta_f(A_6)$				
30	0.92	18.30	0.54	10.12
60	0.94	27.79	0.66	15.02
90	0.94	34.20	0.71	19.16
120	0.95	39.93	0.85	22.35
150	0.95	45.31	0.86	25.48

In group sequential schemes, n observations are taken from each population at each stage. A series of the simulation runs was carried out with N=450 and different n, where N=kn. It was decided in these cases the use δ_o=0.4 in the sampling and stopping rules. The results are presented in table (4.3). Varying the group size has little effect on P(CS) but earlier stopping is obtained by decreasing n, in judging the performance one must take into account that the smaller the number of groups the easier and cheaper the scheme would to implement.

In group sequential schemes there are two special cases which are of interest the first case n=1, called vector at a time sampling rule GS(VT), where on observation is taken from each population at each stage, the second case n=N/k, called Fixed sample size F_{SS}

Graphically, the effect of group size on the performance measures on the group sequential scheme displayed figures (4.5- 4.7).

Table (4.3)

Performance characteristics of δ_G for different group size n,δ_0=0.4, where N=450, for fixed p_i(i=1,2,3), $\underset{\sim}{p}$ =(0.1, 0.2, 0.3) under the uniform priors.

Group size	N	P(CS)	E(M)	E(S)
1	3	0.94	370.74	74.07
2	6	0.96	404.32	80.99
10	30	0.98	434.68	87.90
50	150	0.98	450	90.03
150	450	0.98	450	90.18

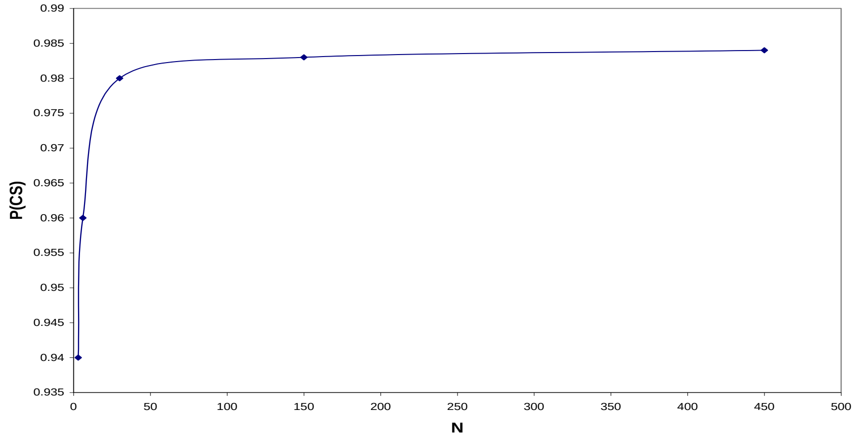


Fig.4.5 P(CS) for the scheme δ_G when N= 450 and δ_0 = 0.4 with fixed pi's, where i= 1,2,3 under uniform priors and different group size.

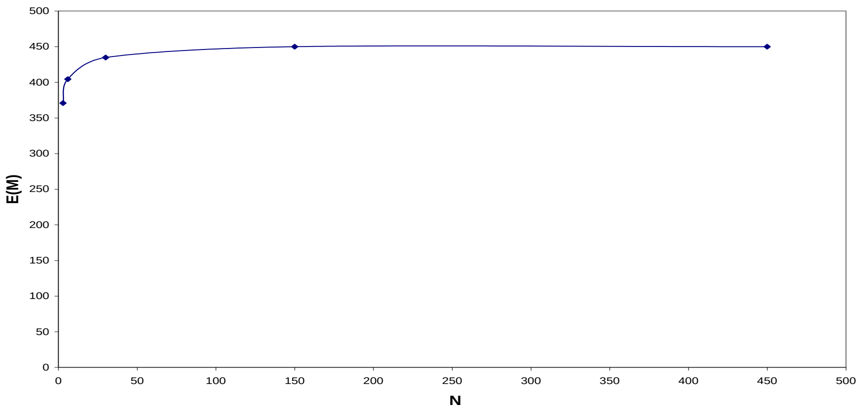


Fig. 4.6 E(M)for the scheme δ_G when N= 450 and δ_0 = 0.4 with fixed pi's, where i = 1,2,3, under uniform priors and different group size.

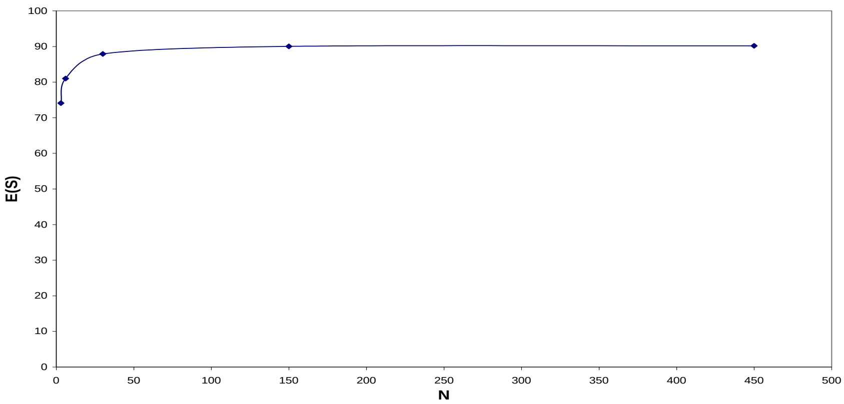


Fig. 4.7 E(S) for the scheme δ_G when N= 450 and δ_0 = 0.4 with fixed pi's, where i = 1,2,3, under uniform priors and different group size.

Next we discuss in details the MC estimates of the performance characteristics of the schemes $\delta_f(A_2)$, $\delta_f(A_5)$, GS(VT) and F_{SS}.

4.1.1 The MC estimates of P(CS)

we present some results, for fixed p_i .(i=1,…..,k), k=3,4,5and for small, equal and large p-values, in tables (4.4), (4.8), (4.12) and (4.16). In all tables and for small and large p-values we notice that when N is small the value of the performance measure P(CS) is changeable in all schemes, but these results performs uniformly better than others as far as P(CS) is concerned, but that is compensated for by using larger sample size.

In general ,the performance of schemes can be ordered as follows:

$$F_{SS} > GS(VT) > \delta_f(A_5) > \delta_f(A_2).$$

Also, the tables show that when p-values are equal. The performance P(CS)=$\frac{1}{k}$ for all cases k=3,4,5 and 9.

In general, the performance, in all schemes, is increasing function of N.

4.1.2 The MC estimate of E(M).

Tables (4.5), (4.9), (4.13) and (4.17) contain some results of performance E(M),for fixed p_i (i=1,…,k),where k=3,4,5and 9,respectivly. These show that when p-values are small the E(M) is the best in scheme $\delta_f(A_2)$. The performance can be ordering, interims of E(M), is as follows :

$$\delta_f(A_2) > GS(VT) > \delta_f(A_5) > F_{SS}$$

Also when p-values are equal the scheme $\delta_f(A_2)$ is the best scheme, the performance measure is ordered as:

$$\delta_f(A_2) > \delta_f(A_5) > GS(VT) > F_{SS}$$

Now, In these tables when *p-values* are large and p-values are small in tables (4.17). The scheme $\delta_f(A_5)$ is best under E(M). The ordered of the schemes is as follows:

$$\delta_f(A_5) > \delta_f(A_2) > GS(VT).$$

It is intuitively clear that the performance of the scheme F_{SS} as measured by E(M) is poor since E(M)=N.

In all tables and in all schemes the performance measure E(M) is increasing function of N.

4.1.3 The MC estimates of $E(N_{(1)})$

The value of $E(N_{(1)})$ for fixed p_i (i=1,......,k) , where k=3,4,5 and 9 populations, are presented in tables (4.6), (4.10), (4.14) and (4.18). It can be seen from these tables that the scheme $\delta_f(A_5)$ is better than the scheme $\delta_f(A_2)$. This means that the order of the schemes is

$$\delta_f(A_5)>\delta_f(A_2).$$

Also the ordered of schemes is same if we take the percentage ratio of $E(N_{(1)})$ for both schemes.

In generally, The performance $E(N_{(1)})$ is increasing function of N in all schemes . In the schemes GS(VT) and F_{SS}, we have $E(N_{(1)})= E(M)/k$. From the above interpretation we conclude that $\delta_f(A_5)$ is the best among $\delta_f(A_2)$, GS(VT) and F_{SS} as measured by the performance characteristic $E(N_{(1)})$.

4.1.4 The MC estimates of E(S)

Tables (4.7), (4.11), (4.15) and (4.19) contains some results of the performance characteristic E(S), for fixed p_i(i=1,……..,k), k=3,4,5 and 9 population, respectively. It can be observed from these results the measure E(S) is changeable for all schemes, but when be taken the percentage ratio of E(S). They show that the schemes $\delta_f(A_5)$ is the best among other schemes. The performance is order as:

$$\delta_f(A_5)> \delta_f(A_2)> GS(VT) \cong F_{SS},$$

where ($GS(VT) \cong F_{SS}$) means both schemes have roughly same performance. These order is for large and small p-values, when p-values are equal, the performance E(S) is equal in all tables has the percentage ratio is 1/2.

Generally, the performance measure E(S) is increasing function of N in all tables.

Table(4.4)

P(CS) for the schemes $\delta_f(A_2)$,$\delta_f(AS)$,GS(VT)and F_{SS} with δ_0=0.4, N=30(30) 300, under uniform priors for fixed p_i(i=1,2,3),

Scheme	N	$\underset{\sim}{p}=(0.15,0.25,0.35)$	$\underset{\sim}{p}=(0.5,0.5,0.5)$	$\underset{\sim}{p}=(0.7,0.8,0.9)$
A_2	30	0.70	0.30	0.73
	90	0.71	0.30	0.81
	150	0.71	0.33	0.83
	210	0.71	0.33	0.84
	300	0.71	0.34	0.84
A_5	30	0.64	0.32	0.89
	90	0.71	0.33	0.89
	150	0.72	0.35	0.89
	210	0.74	0.35	0.90
	300	0.75	0.35	0.91
$GS(V_T)$	30	0.61	0.34	0.66
	90	0.76	0.34	0.84
	150	0.82	0.34	0.92
	210	0.86	0.34	0.95
	300	0.89	0.34	0.98
F_{SS}	30	0.64	0.34	0.65
	90	0.79	0.34	0.84
	150	0.85	0.34	0.92
	210	0.90	0.34	0.95
	300	0.94	0.34	0.98

Table (4.5)

E(M) for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$ and GS(VT) with δ_0=0.4, for fixed $p_{i,}$(i=1,2,3), N=30(30) 210 300, under uniform priors .

Scheme	N	$\underset{\sim}{p}=(0.15,0.25,0.35)$	$\underset{\sim}{p}=(0.5,0.5,0.5)$	$\underset{\sim}{p}=(0.7,0.8,0.9)$
A_2	30	23.01	26.97	29.74
	90	61.18	75.01	89.12
	150	98.00	125.10	148.44
	210	133.85	174.40	207.71
	300	187.75	248.00	296.32
A_5	30	28.82	27.29	18.11
	90	85.42	79.77	3709
	150	141.98	133.49	54.48
	210	199.21	186.75	64.00
	300	283.00	263.22	79.97
GS(VT)	30	26.46	28.70	29.78
	90	75.00	81.79	89.09
	150	123.90	134.84	148.53
	210	172.89	190.92	207.90
	300	245.99	269.83	296.41

Table (4.6)

$E(N_{(1)})$ for the schemes $\delta_f(A_2)$ and $\delta_f(A_5)$ with δ_0=0.4, for fixed $p_{i,}$(i=1,2,3), N=30(30) 210 300, under uniform priors

Scheme	N	$\underset{\sim}{p}=(0.15,0.25,0.35)$	$\underset{\sim}{p}=(0.5,0.5,0.5)$	$\underset{\sim}{p}=(0.7,0.8,0.9)$
A_2	30	4.86	7.80	4.8
	90	8.14	22.47	17.12
	150	11.01	36.54	27.03
	210	13.80	50.79	33.80
	300	19.40	71.49	42.20
A_5	30	2.50	5.35	1.66
	90	6.89	17.66	4.83
	150	9.86	29.91	7.96
	210	10.87	42.54	10.78
	300	12.36	61.05	13.99

Table (4.7)

E(S) for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$,GS(VT) and F_{SS} with δ_0=0.4, for fixed $P_{i,}$(i=1,2,3), N=30(30) 300, under uniform priors .

Scheme	N	$\underset{\sim}{p} = (0.15, 0.25, 0.35)$	$\underset{\sim}{p} = (0.5, 0.5, 0.5)$	$\underset{\sim}{p} = (0.7, 0.8, 0.9)$
A_2	30	5.99	13.33	23.26
	90	15.90	38.24	71.52
	150	25.35	62.44	118.61
	210	35.34	87.68	165.42
	300	49.30	124.35	235.14
A_5	30	8.30	13.72	16.74
	90	26.2	40.01	33.36
	150	62.95	66.66	45.87
	210	62.95	93.40	56.74
	300	90.09	133.23	72.29
GS(VT)	30	6.62	14.06	23.80
	90	18.78	40.85	71.18
	150	43.26	67.36	118.77
	210	93.26	95.13	166.35
	300	61.41	139.60	237.17
F_{SS}	30	7.51	14.98	23.98
	90	22.51	44.98	71.94
	150	37.44	74.89	119.99
	210	52.56	105.11	168.09
	300	74.92	149.98	239.95

Table(4.8)

P(CS) for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$,GS(VT) and F_{SS} with δ_0=0.4, for fixed $p_{i,}$(i=1,2,3,4), N=40(40)280 400, under uniform priors .

Scheme	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
A_2	40	0.58	0.23	0.75
	120	0.58	0.24	0.80
	200	0.59	0.25	0.82
	280	0.59	0.30	0.83
	400	0.59	0.30	0.83
A_5	40	0.62	0.25	0.89
	120	0.66	0.25	0.89
	200	0.68	0.25	0.89
	280	0.68	0.25	0.91
	400	0.69	0.26	0.92
GS(VT)	40	0.58	0.25	0.66
	120	0.75	0.25	0.84
	200	0.82	0.25	0.92
	280	0.85	0.25	0.95
	400	0.88	0.25	0.98
F_{SS}	40	0.59	0.25	0.66
	120	0.77	0.25	0.84
	200	0.84	0.25	0.92
	280	0.88	0.25	0.95
	400	0.92	0.25	0.98

Table(4.9)

E(M) for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$ and GS(VT with δ_0=0.4, for fixed $p_{i,}$(i=1,…..,4), N=40(40)280 400, under uniform priors .

Scheme	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
A_2	40	31.51	36.74	39.69
	120	81.75	105.38	118.87
	200	128.95	173.88	197.88
	280	177.00	234.73	277.28
	400	247.02	349.10	395.07
A_5	40	37.71	37.96	23.93
	120	111.82	111.91	47.04
	200	184.92	186.88	65.12
	280	259.24	261.76	78.88
	400	367.53	370.93	100.80
GS(VT)	40	36.41	38.94	39.36
	120	104.37	115.03	119.62
	200	172.13	192.52	199.43
	280	241.93	268.75	279.00
	400	336.82	384.06	398.57

Table(4.10)

$E(N_{(1)})$ for the schemes $\delta_f(A_2)$ and $\delta_f(A_5$ with δ_0=0.4, for fixed $p_{i,}$(i=1,…..,4), N=40(40)280 400, under uniform priors

Scheme	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
A_2	40	3.55	7.83	4.23
	120	4.81	22.28	14.37
	200	5.42	37.60	25.21
	280	5.67	51.51	34.67
	400	6.20	72.25	49.89
A_5	40	2.44	5.07	0.27
	120	3.87	18.75	0.55
	200	3.88	33.77	0.87
	280	3.82	44.00	1.20
	400	4.21	67.1	1.50

Table(4.11)

E(S)for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$,GS(VT) and F_{SS} with δ_0=0.4, for fixed $p_{i,}$(i=1,…..,4), N=40(40)280 400, under uniform priors.

Scheme	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
A_2	40	10.20	18.33	31.51
	120	27.39	52.63	92.82
	200	43.73	86.88	153.71
	280	60.25	121.82	214.81
	400	84.74	172.19	306.09
A_5	40	14.04	18.99	20.61
	120	44.05	55.92	40.99
	200	71.31	93.40	57.64
	280	105.44	130.84	71.33
	400	150.10	185.68	91.33
GS(VT)	40	10.90	19.42	29.90
	120	31.30	57.53	89.71
	200	51.63	96.24	149.54
	280	72.75	134.20	299.24
	400	101.99	192.22	299.40
F_{SS}	40	12.01	19.96	29.95
	120	35.96	60.01	90.95
	200	60.04	99.97	149.91
	280	83.95	139.93	209.90
	400	120.09	200.18	300.01

Table(4.12)

P(CS) for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$,GS(VT) and F_{SS} with δ_0=0.4, for fixed $p_{i,}$(i=1,….,5), N=50(50)350 500, under uniform priors

Scheme	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
A_2	50	0.54	0.20	0.77
	150	0.55	0.20	0.80
	250	0.55	0.20	0.82
	350	0.55	0.20	0.82
	500	0.55	0.21	0.83
A_5	50	0.60	0.20	0.89
	150	0.63	0.20	0.89
	250	0.64	0.20	0.90
	350	0.64	0.21	0.90
	500	0.64	0.21	0.91
GS(VT)	50	0.56	0.20	0.63
	150	0.73	0.20	0.89
	250	0.81	0.20	0.91
	350	0.86	0.20	0.95
	500	0.90	0.20	0.98
F_{SS}	50	0.58	0.20	0.67
	150	0.75	0.20	0.84
	250	0.85	0.20	0.92
	350	0.88	0.20	0.95
	500	0.92	0.20	0.98

Table(4.13)

E(M)for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$ and GS(VT) with δ_0=0.4, for fixed $p_{i,}$(i=1,…….5), N=50(50)350 500, under uniform Priors.

Scheme	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)

A_2	50	47.53	47.95	27.70
	150	137.68	143.05	55.10
	250	227.09	236.85	75.99
	350	318.04	332.79	95.00
	500	452.40	473.10	122.02
A_5	50	47.53	47.95	27.70
	150	137.68	143.05	55.10
	250	227.09	236.85	75.99
	350	318.04	332.79	95.55
	500	452.40	473.10	122.02
GS(VT)	50	46.49	49.53	49.96
	150	135.00	147.24	149.86
	250	224.86	245.73	249.72
	350	314.51	343.38	349.21
	500	446.00	990.03	499.36

Table(4.14)

$E(N_{(1)})$for the schemes $\delta_f(A_2)$ and $\delta_f(A_5)$ with δ_0=0.4, for fixed $P_{i,}$(i=1,…….5), N=50(50)350 500, under uniform Priors.

Scheme	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)

A_2	50	2.57	7.71	3.26
	150	2.68	23.28	7.12
	250	2.80	36.89	12.45
	350	2.95	49.98	14.88
	500	3.14	75.53	20.56
A_5	50	1.78	5.51	0.89
	150	1.96	20.06	1.70
	250	1.96	34.42	2.00
	350	2.05	50.15	2.74
	500	2.06	67.74	2.80

Table(4.15)

E(S)for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$,GS(VT) and F_{SS} with δ_0=0.4, for fixed $p_{i,}$(i=1,…….5), N=50(50)350 (500), under uniform Priors

Scheme	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)

A_2	50	16.90	23.25	34.99
	150	47.50	67.91	105.02
	250	67.61	112.04	175.08
	350	106.20	154.97	244.97
	500	151.81	221.67	349.92
A_5	50	21.59	24.71	23.64
	150	66.99	72.23	48.01
	250	112.20	120.05	67.72
	350	158.90	167.54	84.23
	500	225.54	239.28	109.30
GS(VT)	50	16.56	24.71	34.94
	150	48.14	73.60	104.93
	250	79.53	122.86	179.82
	350	111.21	171.55	244.34
	500	158.21	245.05	349.51
F_{SS}	50	17.93	24.93	38.62
	150	52.42	79.97	114.16
	250	87.51	125.06	188.39
	350	122.46	174.93	263.25
	500	174.96	250.01	375.22

Table(4.16)

P(CS)for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$,GS(VT) and F_{SS} with δ_0=0.4, for fixed $p_{i,}$(i=1,…….9), N=90(90)630 (900), under uniform Priors.

Sampling rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45 ,0.55,0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.50.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5.0.6 ,0.7,0.8,0.9)

A_2	90	0.88	0.9	0.78
	270	0.91	0.11	0.81
	450	0.92	0.12	0.82
	630	0.93	0.12	0.82
	900	0.93	0.12	0.83
A_5	90	0.91	0.10	0.83
	270	0.91	0.11	0.83
	450	0.91	0.12	0.83
	630	0.92	0.12	0.83
	900	0.92	0.12	0.84
GS(VT)	90	0.67	0.11	0.62
	270	0.89	0.11	0.85
	450	0.95	0.11	0.92
	630	0.98	0.11	0.95
	900	0.99	0.11	0.98
F_{SS}	90	0.69	0.11	0.65
	270	0.89	0.11	0.85
	450	0.95	0.11	0.92
	630	0.98	0.11	0.95
	900	0.99	0.11	0.98

Table (4.17)

E(M) for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$ and GS(VT) with δ_0=0.4, for fixed $p_{i,}$(i=1,…….9), N=90(90)630 900, under uniform Priors.

Sampling rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45 ,0.55,0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.50.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5 0.6 ,0.7,0.8,0.9)

A_2	90	89.41	86.67	88.68
	270	267.68	254.22	263.35
	450	445.78	420.96	439.93
	630	622.65	586.53	613.17
	900	891.51	833.89	876.34
A_5	90	19.97	88.85	40.73
	270	28.63	266.74	87.13
	450	39.13	442.47	120.13
	630	45.94	617.66	154.08
	900	59.02	885.03	205.99
GS(VT)	90	90	89.95	89.98
	270	269.98	269.60	269.70
	450	449.88	449.57	449.63
	630	629.88	628.99	629.67
	900	899.75	898.64	899.93

Table(4.18)

$E(N_{(1)})$ for the schemes $\delta_f(A_2)$ and $\delta_f(A_5$ whit δ_0=0.4, for fixed p_i (i=1,…….9), N=90(90)630 900, under uniform Priors

Sampling rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45 ,0.55,0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.50.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5 0.6 ,0.7,0.8,0.9)

A_2	90	0.32	7.59	0.42
	270	0.33	23.31	0.42
	450	0.33	40.58	0.43
	630	0.33	54.49	0.43
	900	0.34	79.60	0.44
A_5	90	0.07	6.06	0.15
	270	0.07	23.25	0.15
	450	0.07	42.60	0.15
	630	0.07	61.16	0.15
	900	0.07	86.45	0.15

Table (4.19)

E(S) for the schemes $\delta_f(A_2)$, $\delta_f(A_5)$,GS(VT) and F_{SS} with δ_0=0.4, for fixed $p_{i,}$(i=1,…….9), N=90(90)630 900, under uniform Priors.

Sampling rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5

		,0.55,0.65,0.75,0.85,0.95)	0.5,0.5,0.5,0.5)	0.6 ,0.7,0.8,0.9)
A_2	90	73.26	43.27	66.98
	270	218.08	126.99	199.61
	450	363.02	210.20	334.35
	630	506.48	293.53	467.07
	900	726.23	416.52	665.72
A_5	90	18.00	44.38	34.37
	270	25.06	132.73	72.82
	450	33.29	221.13	99.71
	630	38.92	309.07	126.44
	900	49.50	442.20	169.39
GS(VT)	90	49.50	44.96	44.98
	270	148.96	134.72	134.76
	450	247.33	224.72	224.76
	630	346.36	314.59	314.94
	900	494.68	449.18	449.52
F_{SS}	90	49.48	44.96	44.94
	270	148.49	134.86	134.87
	450	247.38	224.90	224.83
	630	346.63	315.20	315
	900	494.93	449.80	449.84

4.2 Performance characteristics of the sampling rule BKR with the stopping rule BKS and modifications

The values of the measures were calculated from the results of Monte Carlo (MC) simulations with 100 000 trials for fixed values of p_i (i=1,….k).

In the following we discuss in details the performance of the sampling rules BKR, with stopping rules BKS, BKS_1, BKS_2, and BKS_3 and the terminal decision rule BKT.

4.2.1 The MC estimates of P(CS).

The simulation results of the performance measure P(CS) are presented in tables (4.20), (4.24), (4.28) and (4.32) for fixed p_i(i=1,…….,k) where k=3,4,5 and 9, respectively.In these tables, we note that the scheme (BKR,BKS_1) is best in terms of measure P(CS). However, when N increases the scheme (BKR,BKS_1) becomes equal to the scheme (BKR,BKS_3). The performance measure can be ordered as follows.

$BKS_1 > BKS_3 > BKS > BKS_2$ when N is small

$BKS_1 \cong BKS_3 > BKS > BKS_2$ when N is large

When p-values are equal, the performance measure P(CS) ≅1/k in all tables. In general, the performance measure is increasing function of N.

4.2.2 The MC estimates of E(M)

It is preferable to stop sampling from populations as soon as possible, that is the scheme with less E(M) is preferred.

Tables (4.21), (4.25), (4.29) and (4.33) display some results of E(M) for fixed $p_{i,}$(i=1,…….k) , where k=3,4,5 and 9, respectively. We can see from these tables that the scheme (BKR, BKS) is better than the scheme (BKR, BKS_1) and the scheme (BKR, BKS_2) is better than the scheme (BKR, BKS) in the measure E(M) . Also the scheme (BKR, BKS_3) is better than the scheme (BKR, BKS_1) and the scheme (BKR, BKS) is better than the scheme(BKR, BKS_3), Therefore, the scheme (BKR, BKS_2) is the best among other schemes so the order of the performance is as follows:

$$BKS_2 > BKS > BKS_3 > BKS_1$$

The performance in all schemes and in all tables is increasing function of N.

4.2.3 The MC estimates of $E(N_{(1)})$.

The values of the performance measure of $E(N_{(1)})$ are presented in tables (4.22), (4.26), (4.30) and (4.34) for fixed p_i(i=1,……,k), where k=3,4,5 and 9 populations, respectively . From these tables, we conclude that the scheme (BKR,BKS_2) is the best (lees value of $E(N_{(1)})$) among schemes. But by taking the percentage ratio of $E(N_{(1)})$. We note that in all tables the scheme (BKR,BKS_1) is best among other schemes. Also show that neither scheme (BKR, BKS_2) nor (BKR,BKS) is better than others. The order of $E(N_{(1)})$ is as follows.

$$BKS_1 > BKS_3 > [BKS \cong BKS_2]$$

The performance measure $E(N_{(1)})$ is increasing function of N.

4.2.4 The MC estimates of E(S).

Comparisons of tables (4.21) with (4.23), (4.25) with (4.27), (4.29) with (4.31) and (4.33) with (4.35), show that if E (M) is large then E(S) is large too. Then the scheme (BKR, BKS2) is the worst and the scheme (BKR, BKS1) is best. The performance ordering is as follows

$$BKS_1 > BKS_3 > BKS > BKS_2$$

But it is appears that the measure E(S) is changeable in all schemes specially when k=3 and 4 in both cases when p-values is small or large under taken percentage ratio of E(S)/E(M). However, neither the scheme

(BKR, BKS_1) nor the scheme (BKR, BKS_3) is superior in terms of measure E(S). In addition, we note that when p-values are equal that the percentage ratio equal to 1/2 under all schemes. Moreover, in all tables the measure E(S) is increasing function of N.

The conclusion to be drawn from these result and given in this section is that the scheme (BKR, BKS_1) and (BKR, BKS_3) should be used if P(CS), E(S) and $E(N_{(1)})$ are more important criteria, while the scheme (BKR,BKS_2) is preferred if E(M) is more interest. .

Table (4.20)

P(CS) of the schemes that are combination of the sampling rule BKR, the stopping rules BKS,BKS_1, BKS_2, BKS_3. In addition, terminal decision rules. BKT, where N=30(30)210 300, for fixed p_i (i=1, 2, 3), under uniform prior

Stopping rule	N	$\underset{\sim}{p}=(0.15,0.25,0.35)$	$\underset{\sim}{p}=(0.5,0.5,0.5)$	$\underset{\sim}{p}=(0.7,0.8,0.9)$
BKS	30	0.63	0.34	0.67
	90	0.78	0.34	0.87
	150	0.86	0.34	0.93
	210	0.90	0.34	0.96
	300	0.94	0.34	0.90
BKS_1	30	0.63	0.34	0.70
	90	0.80	0.34	0.87
	150	0.87	0.34	0.94
	210	0.91	0.34	0.96
	300	0.95	0.34	0.99
BKS_2	30	0.57	0.34	0.58
	90	0.73	0.34	0.82
	150	0.83	0.34	0.90
	210	0.88	0.34	0.94
	300	0.93	0.35	0.98
BKS_3	30	0.61	0.34	0.76
	90	0.78	0.34	0.87
	150	0.86	0.34	0.94
	210	0.90	0.34	0.96
	300	0.94	0.35	0.99

Table (4.21)

E (M) of the schemes that are combination of the sampling rule BKR, the stopping rules BKS,BKS_1, BKS_2, BKS_3. In addition, terminal decision

rules. BKT, for fixed $p_i(i=1,2,3)$, where N=30(30)210 300 under uniform prior.

Stopping rule	N	$\underset{\sim}{p}=(0.15,0.25,0.35)$	$\underset{\sim}{p}=(0.5,0.5,0.5)$	$\underset{\sim}{p}=(0.7,0.8,0.9)$
BKS	30	19.25	22.33	15.77
	90	67.18	76.47	52.34
	150	116.99	132.16	89.80
	210	166.83	188.76	127.50
	300	242.47	274.70	182.33
BKS_1	30	27.69	26.89	22.08
	90	84.91	84.85	69.65
	150	142.16	143.44	117.59
	210	199.41	202.35	165.28
	300	285.24	290.94	236.29
BKS_2	30	11.28	11.23	8.41
	90	41.36	53.73	36.97
	150	77.41	105.81	69.74
	210	114.84	158.75	104.62
	300	170.68	242.14	153.52
BKS_3	30	24.47	28.86	17.16
	90	77.42	77.38	61.97
	150	130.28	134.83	107.68
	210	183.21	192.50	153.35
	300	262.52	280.80	221.46

Table (4.22)

$E(N_{(1)})$ of the schemes that are combination of the sampling rule BKR, the stopping rules BKS, BKS_1, BKS_2, BKS_3. In addition, terminal decision

rules. BKT, for fixed $p_i(i=1,2,3)$, where N=30(30)210 300 under uniform prior.

Stopping rule	N	$\underset{\sim}{p}=(0.15,0.25,0.35)$	$\underset{\sim}{p}=(0.5,0.5,0.5)$	$\underset{\sim}{p}=(0.7,0.8,0.9)$
BKS	30	7.01	7.41	3.35
	90	22.49	23.50	9.54
	150	37.86	44.05	16.33
	210	53.18	62.85	23.21
	300	76.25	91.49	33.00
BKS_1	30	7.23	8.91	4.27
	90	22.07	28.23	12.47
	150	36.84	47.76	21.07
	210	51.60	67.41	29.62
	300	73.88	96.79	42.79
BKS_2	30	3.29	3.71	1.94
	90	11.89	17.89	6.90
	150	22.19	35.24	12.52
	210	33.20	52.82	19.02
	300	49.52	80.69	27.57
BKS_3	30	5.57	6.94	3.43
	90	19.34	25.70	11.12
	150	33.76	44.88	14.22
	210	48.23	64.11	27.57
	300	70.21	93.17	39.95

Table (4.23)

E(S) for the schemes that are combination of the sampling rule BKR, the stopping rules BKS,BKS_1, BKS_2, BKS_3. In addition, terminal decision

rules. BKT, for fixed $p_i(i=1,2,3)$, where N=30(30)210 300 under uniform prior.

Stopping rule	N	$\underset{\sim}{p}=(0.15,0.25,0.35)$	$\underset{\sim}{p}=(0.5,0.5,0.5)$	$\underset{\sim}{p}=(0.7,0.8,0.9)$
BKS	30	6.35	11.61	13.06
	90	20.06	38.21	43.72
	150	33.76	66.11	75.08
	210	47.39	94.34	106.58
	300	67.90	137.90	152.55
BKS_1	30	7.23	13.41	14.27
	90	22.07	42.40	52.87
	150	36.84	71.71	92.30
	210	51.6	101.18	129.62
	300	73.88	145.49	187.79
BKS_2	30	2.90	5.61	6.93
	90	10.69	26.84	30.84
	150	20	52.85	58.27
	210	29.73	79.27	87.47
	300	44.22	121.28	128.43
BKS_3	30	5.20	10.43	14.26
	90	17.42	38.67	51.83
	150	30.38	67.44	90.10
	210	43.29	96.23	128.28
	300	62.88	140.01	185.31

Table (4.24)

P(CS)for the schemes that are combination of the sampling rule BKR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3. In addition, the terminal decision rules. BKT, for fixed p_i(i=1,…,4), where N=40(40)280 400 under uniform prior

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	0.60	0.25	0.65
	120	0.76	0.25	0.85
	200	0.84	0.25	0.92
	280	0.88	0.25	0.95
	400	0.92	0.25	0.98
BKS_1	40	0.62	0.25	0.72
	120	0.79	0.25	0.90
	200	0.86	0.25	0.95
	280	0.90	0.26	0.98
	400	0.94	0.26	0.99
BKS_2	40	0.53	0.25	0.55
	120	0.73	0.25	0.82
	200	0.81	0.25	0.91
	280	0.87	0.25	0.95
	400	0.92	0.25	0.97
BKS_3	40	0.60	0.25	0.70
	120	0.78	0.25	0.89
	200	0.86	0.26	0.95
	280	0.90	0.26	0.97
	400	0.94	0.26	0.99

Table (4.25)

E(M) for the schemes that are combination of the sampling rule BKR, the stopping rules BKS,BKS_1, BKS_2 and $BKS_3$and the terminal decision rules. BKT, for fixed p_i(i=1,…,4), where N=40(40)280 400 under uniform prior

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	29.20	27.72	18.05
	120	94.07	97.74	59.32
	200	158.76	171.01	101.71
	280	224.08	245.83	144.23
	400	321.31	358.98	207.64
BKS_1	40	37.14	37.01	30.95
	120	113.59	115.27	96.06
	200	190.16	194.10	161.27
	280	266.73	273.22	255.98
	400	381.34	391.98	323.05
BkS_2	40	14.30	14.42	9.22
	120	57.79	73.33	43.54
	200	107.46	142.54	81.62
	280	158.29	213.44	119.39
	400	235.62	321.88	176.52
BKS_3	40	29.86	32.20	25.96
	120	99.38	109.09	88.36
	200	168.97	186.73	151.07
	280	240.46	264.73	213.16
	400	346.52	382.56	305.04

Table (4.26)

$E(N_{(1)})$ for the schemes that are combination of the sampling rules BKR, stopping rules BKS,BKS_1, BKS $_2$and BKS_3 and terminal decision rules. BKT, for fixed p_i(i=1,…,4), where N=40(40)280 400 under uniform prior

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	5.83	6.90	2.44
	120	18.87	24.38	7.05
	200	31.83	42.68	12.10
	280	44.99	61.44	17.32
	400	64.38	89.79	24.94
BKS_1	40	7.23	9.21	3.66
	120	22.47	28.78	11.20
	200	37.79	48.50	18.95
	280	53.18	68.21	26.69
	400	76.13	96.97	38.21
BKS_2	40	2.81	3.58	1.53
	120	11.42	18.33	5.21
	200	21.35	35.65	9.72
	280	31.51	53.35	14.21
	400	47.09	80.46	21.02
BKS_3	40	5.82	8.04	3.20
	120	19.66	27.20	10.46
	200	33.81	46.63	17.84
	280	48.01	66.13	25.27
	400	96.22	95.56	36.24

Table (4.27)

E(S) of the schemes that combinations of the sampling rule BKR, stopping rules BKS,BKS_1, BKS_2 and BKS_3 and terminal decision rules. BKT, for fixed p_i(i=1,…,4), where N=40(40)280 400 under uniform prior

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	9.27	13.23	14.42
	120	29.93	48.81	47.92
	200	50.48	85.43	82.19
	280	71.21	122.85	116.49
	400	102.38	179.64	107.66
BKS_1	40	13.49	19.22	24.98
	120	40.96	58.74	77.72
	200	68.50	98.45	130.43
	280	96.54	138.33	182.83
	400	138.39	197.97	261.39
BKS_2	40	4.57	7.20	4.46
	120	18.40	36.62	22.6
	200	34.24	71.27	42.96
	280	50.43	106.60	64.89
	400	75.23	161.04	97.86
BKS_3	40	15.84	18.03	20.87
	120	47.88	57.28	71.42
	200	79.70	96.59	122.15
	280	112.01	136.14	172.34
	400	160.47	195.68	246.74

Table (4.28)

P(CS) of the schemes that combinations of the sampling rule BKR, stopping rules BKS,BKS_1, BKS_2, BKS_3. In addition, terminal decision rules.

BKT, for fixed p_i(i=1,…,5), where N=50(50)350 500 under uniform prior

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	0.54	0.20	0.65
	150	0.75	0.20	0.85
	250	0.83	0.21	0.92
	350	0.89	0.21	0.95
	500	0.92	0.21	0.98
BKS_1	50	0.62	0.20	0.74
	150	0.79	0.21	0.91
	250	0.87	0.21	0.96
	350	0.91	0.21	0.98
	500	0.95	0.21	0.99
BkS_2	50	0.51	0.20	0.54
	150	0.71	0.20	0.82
	250	0.82	0.20	0.91
	350	0.87	0.20	0.95
	500	0.92	0.20	0.99
BKS_3	50	0.61	0.20	0.73
	150	0.79	0.20	0.91
	250	0.86	0.20	0.96
	350	0.91	0.21	0.98
	500	0.94	0.21	0.99

Table (4.29)

E(M) of the schemes that combinations of the sampling rules BKR, stopping rules BKS,BKS_1, BKS_2 and BKS_3 and terminal decision rules. BKT, for fixed p_i(i=1,…,5), where N=50(50)350 500 under uniform prior

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	31.99	32.51	90.00
	150	105.01	118.56	65.11
	250	178.66	209.26	111.45
	350	252.09	301.84	158.60
	500	362.18	441.05	227.51
BKS_1	50	46.31	47.13	39.66
	150	141.74	145.57	122.65
	250	237.07	244.52	205.20
	350	332.64	343.66	287.49
	500	475.44	492.54	410.42
BKS_2	50	16.43	17.51	11.10
	150	70.59	92.02	48.50
	250	130.39	177.82	89.86
	350	191.42	264.86	131.25
	500	281.97	399.92	193.70
BKS_3	50	39.96	43.06	34.63
	150	129.79	139.99	113.98
	250	220.02	238.03	193.26
	350	309.80	336.17	272.28
	500	445.65	484.08	389.33

Table (4.30)

$E(N_{(1)})$ of the schemes that combinations of the sampling rule BKR, stopping rules BKS,BKS_1, BKS_2 and BKS_3 and terminal decision rules. BKT, for fixed p_i(i=1,…,5), where N=50(50)350 500 under uniform prior

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	4.64	6.48	1.89
	150	15.24	23.68	5.68
	250	25.95	50.98	9.68
	350	36.67	69.68	13.94
	500	52.72	88.20	19.90
BKS_1	50	6.49	9.39	3.32
	150	20.27	29.04	10.36
	250	34.03	48.84	17.62
	350	48.00	68.62	24.78
	500	68.78	98.28	35.54
BKS_2	50	2.32	3.49	1.24
	150	10.13	18.39	4.15
	250	18.79	35.52	7.80
	350	27.66	52.89	11.44
	500	40.82	79.96	16.89
BKS_3	50	5.59	8.54	2.93
	150	18.60	27.88	9.66
	250	31.69	47.55	16.65
	350	44.79	67.16	23.44
	500	64.55	96.74	33.88

Table (4.31)

E(S) of the schemes that combinations of the sampling rule BKR, stopping rules BKS,BKS_1, $BKS_2$and BKS_3 and terminal decision rule. BKT, for fixed p_i(i=1,…,5), where N=50(50)350 500 under uniform prior

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	12.19	16.23	15.40
	150	40.14	59.25	50.82
	250	68.10	104.45	87.09
	350	96.17	151.07	123.80
	500	138.21	220.94	177.82
BKS_1	50	17.80	23.50	17.80
	150	54.31	72.74	54.31
	250	90.74	122.21	90.74
	350	127.35	171.98	127.35
	500	181.81	246.51	181.81
BKS_2	50	6.30	8.76	8.41
	150	26.98	45.94	37.83
	250	49.94	88.85	70.19
	350	73.26	132.38	103.53
	500	107.77	199.98	151.25
BKS_3	50	15.36	21.50	27.05
	150	49.64	61.91	89.13
	250	84.21	118.93	151.11
	350	118.63	168.24	212.86
	500	170.14	242.21	304.63

Table (4.32)

P(CS) for the schemes that combinations of the sampling rule BKR, stopping rules BKS,BKS_1, BKS_2 and BKS_3 and terminal decision rule BKT, for fixed p_i(i=1,…,9), where N=90 (90) 630 900 under uniform prior.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0 .5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5. 0.6, 0.7,0.8,0.9)
BKS	90	0.70	0.12	0.65
	270	0.90	0.12	0.85
	450	0.95	0.12	0.92
	630	0.98	0.12	0.95
	900	0.99	0.12	0.98
BKS_1	90	0.89	0.12	0.81
	270	0.99	0.12	0.95
	450	0.99	0.12	0.99
	630	1	0.12	0.99
	900	1	0.12	1
BKS_2	90	0.50	0.11	0.50
	270	0.77	0.12	0.75
	450	0.89	0.12	0.85
	630	0.95	0.12	0.92
	900	0.98	0.12	0.96
BKS_3	90	0.88	0.12	0.80
	270	0.99	0.12	0.95
	450	0.99	0.12	0.98
	630	1	0.12	0.99
	900	1	0.12	1

Table (4.33)

E(M) for the schemes that combinations of the sampling rule BKR, stopping rules BKS,BKS_1, BKS_2 and $BKS_3$and terminal decision rule BKT, for fixed p_i(i=1,…,9), where N=90 (90) 630 900 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5 ,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5. 0.6, 0.7,0.8,0.9)
BKS	90	21.91	50.07	25.21
	270	62.79	198.26	80.92
	450	103.22	356.48	138.95
	630	145.88	517.89	195.45
	900	208.05	766.35	281.84
BKS_1	90	68.10	87.35	74.88
	270	204.97	266.15	228.85
	450	340.99	445.18	382.59
	630	476.19	624.52	535.56
	900	681.44	893.80	764.75
BKS_2	90	7.95	16.40	9.15
	270	30.19	127.82	39.35
	450	52.48	264.32	73.75
	630	75.88	412.60	111.13
	900	110.97	641.22	164.54
BKS_3	90	61.42	84.29	68.85
	270	192.28	261.80	217.68
	450	322.35	440.03	365.72
	630	453.05	618.64	512.62
	900	649.45	886.67	733.95

Table (4.34)

$E(N_{(1)})$ for the schemes that combinations of the sampling rule BKR, stopping rules BKS,BKS_1, BKS_2 and BKS_3 and terminal decision rule BKT, for fixed p_i(i=1,…,9), where N=90 (90) 630 900 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5, 0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5. 0.6, 0.7,0.8,0.9)
BKS	90	0.78	5.53	1.04
	270	1.82	21.94	3.17
	450	2.91	39.44	5.47
	630	4.14	57.36	7.67
	900	5.87	85.03	11.03
BKS_1	90	1.78	9.65	2.67
	270	5.55	29.48	8.68
	450	9.42	49.35	14.72
	630	13.19	69.27	20.70
	900	18.99	99.04	29.67
BkS_2	90	0.40	1.80	0.49
	270	0.97	14.14	1.55
	450	1.53	29.28	2.87
	630	2.18	45.75	4.34
	900	3.14	71.04	6.42
BKS_3	90	1.64	9.28	2.48
	270	5.23	28.97	8.29
	450	8.91	48.70	14.06
	630	12.55	68.54	19.85
	900	18.13	98.28	28.54

Table (4.35)

E(S) for the schemes that combinations of the sampling rule BKR, stopping rules BKS,BKS_1, BKS_2 and BKS_3 and terminal decision rule BKT, where N=90 (90) 630 900 for fixed p_i(i=1,…,9), under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5, 0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5. 0.6, 0.7,0.8,0.9)
BKS	90	16.19	25.00	16.83
	270	48.83	99.09	55.17
	450	80.89	178.20	94.64
	630	114.09	259.00	133.35
	900	163.07	383.30	192.41
BKS_1	90	53.74	43.59	51.93
	270	161.43	132.99	156.92
	450	268.08	222.74	261.95
	630	374.22	312.40	366.14
	900	535.01	446.93	522.56
BKS_2	90	5.39	3.18	5.72
	270	22.91	63.87	26.66
	450	40.75	132.13	50.31
	630	59.21	206.39	75.81
	900	86.83	320.56	112.32
BKS_3	90	48.27	42.10	47.56
	270	151.32	130.83	149.06
	450	253.90	220.11	250.29
	630	356.04	309.38	350.33
	900	509.76	443.26	501.25

4.3 Performance characteristics of PWCR with BKS and modifications

The values of the measures were calculated from the results of MC simulations with 100 000 trials for fixed p-values. It should be noted that the measures are related to N. In the following, we discuss in details, the performance of the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS3 and the terminal decision rule BKT.

4.3.1 The MC estimates of P(CS)

Tables (4.36), (4.40), (4.44) and (4.48) give the performance measures P(CS) of designs that consist of the sampling rules PWCR with the stopping rules BKS, BKS_1, BKS_2 and BKS_3 for fixed p_i(i=1,……,k), where k=3,4,5 and 9 , respectively. For all schemes, P(CS) increases as N increases. The same tables also show that the measure P(CS) under the stopping rule BKS_1 is the best. However, when N increases the performance measure P(CS) has roughly the same values under the stopping rules BKS, BKS_1 and BKS_3. We also note that the scheme under the stopping rule BKS_2 has P(CS) less than others for small N and nearly the same for large N.

When the p-values are equal, we have the value of P (CS) ≅1/K.

4.3.2 The MC estimates of E (M)

Table (4.37), (4.41), (4.45) and (4.49) contain some results of E(M) for fixed p_i(i=1,……….,k) where k=3,4,5 and 9, respectively. These tables show that performance measure E (M) under the stopping rule BKS_2 is best than other schemes. The performance of the stopping rules for k=3 and 4 for small p-values can be ordered as follows:

$$BKS_2 > BKS_3 > BKS > BKS_1 .$$

And for others values as

$$BKS_2 > BKS > BKS_3 > BKS_1 .$$

The performance measure E(M) increases as N increases.

4.3.3 The MC estimates of E ($N_{(1)}$)

Tables (4.38), (4.42), (4.46) and (4.50) presented some numerical results on the performance measure $E(N_{(1)})$, where $p_i(i=1,\ldots..,k)$ are fixed, and k=3,4,5 and 9, respectively. The results in the tables indicate the superiority of the stopping rule BKS_2 over other stopping rules if the performance measure E ($N_{(1)}$) is used.

Based on the results given in the tables, the percentage ratio of $E(N_{(1)})/E(M)$ shows that the stopping rule BKS_1 is best than other stopping rules. Moreover, the order of the performance is as follows:

$$BKS_1 > BKS_3 > BKS > BKS_2 .$$

However, the tables show that if p-values are equal, the ratio has roughly the same values and is approximately equal to 1/k. In general, the ratio decreases as N increases.

4.3.4 The MC estimates of E(S)

Some results of E(S) are presented in tables (4.39), (4.43), (4.47) and (4.51) for fixed $P_i(i=1,\ldots..,k)$, k=3,4,5, and 9, respectively. If E (M) is taken into account then the scheme under the stopping rule, BKS_2 yields small values of E(S).

It would be reasonable to compare the schemes in terms of the percentage ratio E(S)/E(M) which indicates the superiority of the scheme under the stopping rule BKS. According to this ratio we can order the performance of the stopping rules under PWCR as follows:

$$BKS_1 > BKS_3 > BKS > BKS_2$$

This is true when p-values are small or large; however, when p-values are equal, the ratio has roughly equaled to 1/2. In general, the ratio is decreasing function of N.

From this discussion, we can conclude that the schemes (PWCR, BKS_1) and (PWCR, BKS_3) should be used if P (CS), E ($N_{(1)}$) and E(s) is more important while the scheme (PWCR, BKS_2) is the more suitable for use if we are interested in reducing E(M).

Table (4.36)

P(CS) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3) where N=30 (30) 210 300 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35)	$\underset{\sim}{p}$ = (0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.7,0.8,0.9
BKS	30	0.63	0.34	0.64
	90	0.78	0.34	0.82
	150	0.85	0.34	0.90
	210	0.89	0.34	0.93
	300	0.93	0.34	0.97
BKS_1	30	0.63	0.34	0.67
	90	0.78	0.34	0.84
	150	0.85	0.34	0.90
	210	0.89	0.34	0.95
	300	0.93	0.35	0.97
BKS_2	30	0.57	0.33	0.56
	90	0.74	0.33	0.80
	150	0.82	0.34	0.88
	210	0.88	0.34	0.93
	300	0.92	0.34	0.96
BKS_3	30	0.61	0.34	0.65
	90	0.77	0.34	0.84
	150	0.84	0.34	0.91
	210	0.89	0.34	0.94
	300	0.93	0.34	0.97

Table (4.37)

E(M)for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3) ,where N=30 (30) 210 300 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35)	$\underset{\sim}{p}$ = (0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.7,0.8,0.9)
BKS	30	25.92	23.89	16.59
	90	80.57	79.19	55.48
	150	134.53	136.80	93.75
	210	188.61	193.76	131.50
	300	268.81	280.56	188.05
BKS_1	30	27.75	27.03	22.58
	90	84.91	84.87	71.23
	150	142.31	143.33	119.63
	210	199.60	202.26	167.74
	300	285.37	290.70	238.74
BKS_2	30	12.37	12.48	8.62
	90	43.34	55.06	39.62
	150	79.37	106.61	73.76
	210	116.90	160.38	108.58
	300	173.39	242.15	159.18
BKS_3	30	20.11	21.60	17.80
	90	67.93	77.25	64.01
	150	117.78	139.39	110.03
	210	167.49	192.36	156.44
	300	243.06	279.22	223.85

Table (4.38)

$E(N_{(1)})$for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3) ,where N=30 (30) 210 300 under uniform prior.

Stopping rule	N	$\underset{\sim}{p} = (0.15,0.25,0.35)$	$\underset{\sim}{p} = (0.5,0.5,0.5)$	$\underset{\sim}{p} = (0.7,0.8,0.9)$
BKS	30	7.65	7.91	3.75
	90	23.57	26.35	11.10
	150	39.20	45.25	18.09
	210	55.02	64.53	25.02
	300	78.24	93.35	35.26
BKS1	30	5.99	8.97	4.79
	90	20.40	28.21	13.65
	150	36.40	47.67	22.58
	210	52.85	67.36	31.17
	300	78.38	96.68	44.32
BKS_2	30	3.71	4.14	2.11
	90	12.75	18.31	8.06
	150	23.22	35.47	14.43
	210	34.18	53.40	21.06
	300	50.54	80.57	29.93
BKS_3	30	5.96	7.20	3.94
	90	19.88	25.66	12.39
	150	34.42	44.73	20.75
	210	48.84	64.07	29.27
	300	70.77	92.91	41.46

Table (4.39)

E(S) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3), where N=30 (30) 210 300 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35)	$\underset{\sim}{p}$ = (0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.7,0.8,0.9)
BKS	30	5.58	11.16	12.38
	90	17.95	39.57	40.39
	150	30.38	67.93	68.92
	210	42.92	96.79	97.91
	300	61.55	140.20	141.58
BKS_1	30	7.15	13.51	18.69
	90	22.03	42.44	59.28
	150	36.80	71.56	99.77
	210	51.73	101.14	139.86
	300	73.96	145.29	199.37
BKS_2	30	3.19	6.25	7.07
	90	11.18	27.50	32.72
	150	20.51	53.26	61.29
	210	30.17	80.18	90.44
	300	44.77	121.20	132.83
BKS_3	30	6.96	10.83	14.70
	90	20.89	38.61	53.26
	150	34.80	67.11	91.78
	210	48.87	96.07	30.53
	300	69.50	139.56	186.97

Table (4.40)

P(CS) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1 and BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,...,4), where N=40 (40) 280 400 under uniform prior.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	0.61	0.25	0.61
	120	0.76	0.25	0.80
	200	0.83	0.25	0.88
	280	0.89	0.25	0.92
	400	0.93	0.25	0.96
BKS_1	40	0.58	0.25	0.66
	120	0.76	0.25	0.84
	200	0.83	0.26	0.92
	280	0.88	0.26	0.95
	400	0.92	0.26	0.98
BKS_2	40	0.54	0.25	0.52
	120	0.72	0.25	0.77
	200	0.81	0.25	0.87
	280	0.86	0.25	0.92
	400	0.92	0.25	0.96
BKS_3	40	0.59	0.25	0.64
	120	0.75	0.25	0.84
	200	0.82	0.25	0.92
	280	0.88	0.25	0.95
	400	0.92	0.26	0.97

Table (4.41)

E(M)for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,...,4) ,where N=40 (40) 280 400 under uniform prior.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	33.03	30.89	19.43
	120	101.53	104.34	64.18
	200	168.97	179.84	108.32
	280	236.10	256.29	151.96
	400	335.62	371.55	217.09
BKS_1	40	37.16	37.02	31.47
	120	113.54	75.87	97.73
	200	190.23	154.34	163.61
	280	266.47	233.20	228.52
	400	381.36	325.87	325.87
BKS_2	40	16.70	16.48	10.03
	120	60.78	74.93	46.98
	200	110.47	142.28	84.90
	280	160.75	213.29	125.69
	400	236.98	320.24	189.71
BKS_3	40	30.47	16.48	10.03
	120	99.76	74.93	46.98
	200	170.25	142.28	85.90
	280	240.42	213.29	125.69
	400	236.98	320.24	189.71

Table (4.42)

$E(N_{(1)})$for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,...,4) ,where N=40 (40) 280 400 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	6.73	7.65	2.99
	120	20.54	26.03	8.45
	200	34.01	44.85	13.75
	280	47.67	63.97	19.00
	400	67.48	92.72	26.86
BKS_1	40	7.53	9.15	9.24
	120	22.82	28.70	12.35
	200	38.27	48.31	29.27
	280	53.37	68.06	28.05
	400	76.44	97.70	39.55
BKS_2	40	3.48	4.09	1.72
	120	12.35	18.66	6.44
	200	22.40	35.43	11.16
	280	32.52	53.23	15.91
	400	47.70	79.82	22.78
BKS_3	40	6.20	8.08	3.75
	120	20.11	27.12	11.17
	200	34.29	46.38	19.10
	280	48.23	65.97	26.50
	400	69.48	95.29	37.46

Table (4.43)

E(S) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,...,4) ,where N=40 (40) 280 400 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	10.47	15.43	15.33
	120	32.17	52.11	51.36
	200	53.79	89.20	86.96
	280	74.98	128.23	122.18
	400	106.76	185.85	174.80
BKS_1	40	11.76	18.50	95.11
	120	36.06	57.40	78.52
	200	60.47	96.95	121.66
	280	87.73	136.37	184.18
	400	121.20	195.69	262.82
BKS_2	40	5.24	8.23	7.83
	120	19.27	37.40	37.44
	200	34.93	71.06	68.81
	280	50.94	106.59	100.91
	400	75.35	160.04	147.00
BKS_3	40	9.61	16.25	21.15
	120	31.63	54.28	72.57
	200	54.11	93.08	123.47
	280	76.44	132.07	173.72
	400	110.32	190.80	249.16

Table (4.44)

P(CS) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,...,5), where N=50 (50) 350 500 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	0.59	0.20	0.60
	150	0.75	0.20	0.79
	250	0.82	0.21	0.87
	350	0.86	0.21	0.92
	500	0.91	0.21	0.95
BKS_1	50	0.58	0.20	0.67
	150	0.75	0.20	0.85
	250	0.82	0.21	0.92
	350	0.87	0.21	0.95
	500	0.91	0.21	0.98
BKS_2	50	0.51	0.20	0.50
	150	0.71	0.20	0.76
	250	0.80	0.20	0.86
	350	0.85	0.21	0.91
	500	0.90	0.21	0.95
BKS_3	50	0.57	0.20	0.66
	150	0.75	0.20	0.85
	250	0.82	0.21	0.92
	350	0.87	0.21	0.95
	500	0.90	0.21	0.98

Table (4.45)

E(M) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,...,5) ,where N=50 (50) 350 500 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	37.36	37.36	21.68
	150	115.28	128.79	71.33
	250	230.36	222.91	120.30
	350	306.44	318.20	168.18
	500	381.62	462.06	239.21
BKS_1	50	46.35	47.00	40.26
	150	141.65	145.20	123.94
	250	237.24	244.00	207.41
	350	332.63	343.07	290.16
	500	476.02	491.70	413.91
BKS_2	50	19.83	19.76	11.24
	150	74.01	93.14	52.23
	250	133.09	176.71	95.50
	350	194.20	262.27	138.41
	500	285.08	396.33	200.25
BKS_3	50	40.52	43.05	35.28
	150	124.66	139.37	115.83
	250	220.18	237.00	196.25
	350	310.17	335.31	275.16
	500	445.83	482.28	393.73

Table (4.46)

$E(N_{(1)})$ for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,...,5) ,where N=50 (50) 350 500 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	5.55	7.40	2.37
	150	16.39	25.71	6.81
	250	27.99	44.47	11.15
	350	39.28	63.41	15.27
	500	55.71	92.23	21.48
BKS_1	50	6.82	9.35	3.80
	150	20.69	28.91	11.31
	250	34.60	48.68	18.84
	350	48.33	68.48	25.83
	500	69.25	98.10	36.65
BKS_2	50	3.04	3.92	1.42
	150	10.95	18.91	5.15
	250	19.63	35.20	9.05
	350	28.40	52.28	12.81
	500	41.63	79.14	18.12
BKS_3	50	6.01	8.53	3.40
	150	18.95	27.81	10.60
	250	32.27	47.28	17.70
	350	45.01	66.81	24.56
	500	64.87	96.15	43.95

Table (4.47)

E(S) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,...,5) ,where N=50 (50) 350 500 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	14.16	18.67	16.44
	150	43.90	64.32	54.96
	250	73.20	111.45	93.14
	350	102.29	159.21	130.55
	500	145.55	231.08	86.04
BKS_1	50	17.63	23.49	31.02
	150	54.03	72.55	96.28
	250	90.46	121.92	161.33
	350	127.09	171.63	226.16
	500	181.67	245.89	322.60
BKS_2	50	7.46	9.89	8.36
	150	28.07	46.56	40.04
	250	50.52	88.33	73.72
	350	73.98	131.22	107.16
	500	108.57	198.24	156.01
BKS_3	50	15.36	21.51	27.09
	150	49.43	69.69	89.98
	250	83.85	118.44	152.53
	350	118.48	167.79	214.40
	500	170.10	241.19	306.81

Table (4.48)

P(CS) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,......,9) ,where N=90 (90) 630 900 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	0.61	0.12	0.58
	270	0.82	0.12	0.78
	450	0.90	0.12	0.87
	630	0.94	0.12	0.91
	900	0.96	0.12	0.93
BKS_1	90	0.78	0.12	0.71
	270	0.94	0.12	0.89
	450	0.98	0.12	0.95
	630	0.99	0.12	0.97
	900	1	0.12	0.99
BKS_2	90	0.56	0.11	0.56
	270	0.67	0.11	0.73
	450	0.80	0.11	0.81
	630	0.87	0.11	0.86
	900	0.93	0.12	0.90
BKS_3	90	0.77	0.12	0.70
	270	0.94	0.12	0.89
	450	0.98	0.12	0.95
	630	0.99	0.12	0.97
	900	1	0.12	0.99

Table (4.49)

E(M) for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2, BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,......,9) ,where N=90 (90) 630 900 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5, 0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	23.92	59.91	28.96
	270	71.85	221.57	91.84
	450	117.80	387.68	153.48
	630	162.40	559.09	213.77
	900	228.46	816.07	301.33
BKS_1	90	70.49	87.08	76.03
	270	210.59	265.59	231.81
	450	347.15	444.42	385.68
	630	484.75	623.67	539.27
	900	688.96	892.57	768.46
BKS_2	90	7.74	14.97	24.50
	270	33.61	113.83	61.49
	450	59.73	237.66	101.72
	630	87.44	373.34	138.33
	900	126.79	587.98	178.16
BKS_3	90	63.29	83.75	69.81
	270	153.58	260.80	220.63
	450	256	438.49	369.57
	630	359.12	616.79	516.97
	900	512.76	889.67	738.37

Table (4.50)

$E(N_{(1)})$ for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,......,9) ,where N=90 (90) 630 900 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	1.02	6.57	1.39
	270	2.45	24.53	3.90
	450	3.80	42.84	6.32
	630	5.01	61.85	8.73
	900	6.97	90.54	12.14
BKS_1	90	2.30	9.36	3.16
	270	5.30	29.37	9.38
	450	10.14	49.06	15.29
	630	14.09	69.21	21.45
	900	19.76	98.99	30.27
BKS_2	90	0.45	1.64	1.19
	270	1.32	12.57	2.72
	450	2.09	26.28	4.30
	630	2.91	41.23	5.76
	900	4.03	65.09	7.32
BKS_3	90	2.1	9.20	2.93
	270	5.98	28.87	8.93
	450	9.66	48.43	14.71
	630	13.44	68.34	20.65
	900	18.89	98.09	29.11

Table (4.51)

E(S)for the schemes that are combinations of the sampling rule PWCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,......,9) ,where N=90 (90) 630 900 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5. 0.6, 0.7,0.8,0.9)
BKS	90	16.92	29.93	18.51
	270	53.96	110.68	61.10
	450	89.74	193.93	102.98
	630	124.63	279.69	144.00
	900	176.31	408.18	203.81
BKS_1	90	53.96	43.46	50.90
	270	163.04	132.73	156.65
	450	270.28	222.26	262.09
	630	377.66	311.86	366.47
	900	537.91	446.34	523.11
BKS_2	90	5.62	7.47	15.59
	270	24.23	56.88	40.48
	450	44.62	118.82	67.61
	630	65.97	186.75	92.81
	900	96.68	294.10	119.80
BKS_3	90	47.74	41.79	46.62
	270	153.58	130.31	149.02
	450	256	219.28	250.90
	630	359.12	308.40	351.04
	900	512.76	442.38	502.29

4.4 Performance characteristics of PCWR with BKS and modifications.

In the following we discuss in details the performance characteristics of the scheme that are combination of the sampling rule PCWR, The stopping rule BKS with some modification and the terminal decision rule BKT.

4.4.1 The MC estimate of P(CS)

The simulation results of performance measure P(CS) are presented in tables (4.52), (4.56), (4.60)and (4.64) for fixed p_i(i=1,….,k), where k=3,4,5 and 9, respectively. For all schemes the measure P(CS) is increasing function of N. It is clear from these tables that the values of P(CS) is changeable for all scheme, but as N increase P(CS) has roughly same values under all schemes .However , when p-values are equal the measure P(CS) has nearly same values which are equal to 1/k in all schemes .

4.4.2 The MC estimates of E (M)

In tables (4.53) ,(4.57),(4.61) and (4.65) ,we presented some simulation results , for fixed pi (i=1,…,k) ,where k=3,4,5.and 9, respectively .Under the performance measure E(M) has the same order as in section 4.3.2. Also has the same properties for the versions values of p_i's .Indeed BKS_2 shows the superiority over the other stopping rules in terms of E (M).

4.4.3 The MC estimates of E ($N_{(1)}$).

Tables (4.54), (4.58), (4.62) and (4.66) contain some results of performance measures E ($N_{(1)}$). It is a parent from these tables that the scheme under the stopping rule BKS_2 is the best among others in terms of

$E(N_{(1)})$, but the percentage ratio of $E(N_{(1)})/E(M)$ is used for comparison, we note that the scheme under the stopping rule BKS_1 is the best . The general order of performance E ($N_{(1)}$) under this ratio will be as :

$$BKS_1 > BKS_3 > BKS > BKS_2.$$

In general, the ratio is decreasing function of N.

4.4.4: The MC estimates of E(S)

The values of performance of E(S) are presented the tables (4.55), (4.59), (4.63) and (4.67). As usually, if E (M) is large so E(S) is large. Therefore, the performance E(S) has the same order of E (M). That is

$$BKS_1 > BKS_3 > BKS > BKS_2.$$

This order remains as the same when percentage ratio taken. The ratio is decreasing function of N, while the E(S) is an increasing function of N.

The conclusion to be drawn the results given in this section is that the schemes (PCWR, BKS_1) and (PCWR, BKS_3) should be used if P (CS), $E(N_{(1)})$ and E(S) is more important criterion, while the scheme (PCWR, BKS_2) is preferred if E(M) is of more interest.

Tale (4.52)

P(CS)for the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3),where N=30(30) 210 300 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35)	$\underset{\sim}{p}$ = (0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.7,0.8,0.9)
BKS		0.61	0.34	0.68
	90	0.77	0.34	0.86
	150	0.85	0.34	0.92
	210	0.89	0.34	0.96
	300	0.93	0.34	0.98
BKS_1	30	0.76	0.34	0.69
	90	0.91	0.34	0.87
	150	0.96	0.34	0.94
	210	0.98	0.34	0.96
	300	0.99	0.34	0.99
BKS_2	30	0.62	0.33	0.66
	90	0.81	0.33	0.86
	150	0.90	0.34	0.93
	210	0.99	0.34	0.96
	300	0.97	0.34	0.98
BKS_3	30	0.71	0.33	0.70
	90	0.89	0.34	0.87
	150	0.95	0.34	0.94
	210	0.98	0.34	0.96
	300	0.99	0.34	0.98

Table (4.53)

E(M)for the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3),where N=30(30) 210 300 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35)	$\underset{\sim}{p}$ = (0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.7,0.8,0.9)
BKS	30	27.49	24.78	24.17
	90	87.35	80.73	74.39
	150	147.72	138.04	124.43
	210	207.89	195.72	174.08
	300	298.36	282.92	248.23
BKS_1	30	27.71	28.42	82.73
	90	83.53	86.86	84.98
	150	139.36	145.73	141.11
	210	194.89	204.58	197.31
	300	278.96	293.80	281.33
BKS_2	30	12.67	15.22	20.59
	90	41.85	63.22	68.65
	150	71.61	117.33	116.49
	210	102.11	172.40	163.98
	300	147.03	256.46	235.43
BKS_3	30	20.50	24.41	26.95
	90	65.25	81.37	82.55
	150	110.05	139.31	137.78
	210	155.63	197.65	182.99
	300	222.83	285.32	275.58

Table (4.54)

$E(N_{(1)})$for the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3),where N=30(30) 210 300 under uniform priors.

Stopping rule	N	$\underset{\sim}{p} = (0.15,0.25,0.35)$	$\underset{\sim}{p} = (0.5,0.5,0.5)$	$\underset{\sim}{p} = (0.7,0.8,0.9)$
BKS	30	5.95	8.257	7.19
	90	17.96	26.90	21.85
	150	30.03	45.97	36.45
	210	42.14	65.21	50.88
	300	60.18	94.29	72.59
BKS_1	30	5.96	9.47	8.50
	90	17.10	28.93	24.90
	150	28.34	48.57	41.31
	210	39.44	68.24	57.67
	300	56.08	97.92	82.23
BKS_2	30	2.98	5.07	6.17
	90	8.86	21.05	20.14
	150	14.77	39.08	34.12
	210	20.85	57.41	47.98
	300	29.89	85.46	68.80
BKS_3	30	4.56	8.12	7.98
	90	13.51	27.08	24.19
	150	22.50	46.40	40.37
	210	31.59	65.86	56.44
	300	44.96	95.07	80.55

Table (4.55)

E(S)for the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3),where N=30(30) 210 300 under uniform priors

Stopping rule	N	$\underset{\sim}{p} = (0.15,0.25,0.35)$	$\underset{\sim}{p} = (0.5,0.5,0.5)$	$\underset{\sim}{p} = (0.7,0.8,0.9)$
BKS	30	7.41	7.49	17.52
	90	24.06	24.06	60.08
	150	40.83	40.83	100.46
	210	57.42	57.42	140.67
	300	82.50	82.50	200.76
BKS_1	30	7.56	14.20	23.21
	90	23.03	43.39	68.60
	150	38.46	72.78	113.98
	210	53.86	102.48	159.51
	300	76.90	147.06	227.53
BKS_2	30	3.41	7.61	16.62
	90	11.49	31.63	55.42
	150	19.68	58.60	94.10
	210	28.2	86.29	132.50
	300	40.6	128.18	190.27
BKS_3	30	5.58	12.20	21.77
	90	17.95	40.66	66.66
	150	30.38	69.53	111.31
	210	42.92	98.88	156.04
	300	61.55	142.79	222.87

Table (4.56)

P(CS) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 andBKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,4),where N=40(40) 280 400 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	0.69	0.26	0.67
	120	0.89	0.26	0.86
	200	0.95	0.26	0.93
	280	0.97	0.26	0.96
	400	0.99	0.26	0.98
BKS_1	40	0.71	0.26	0.69
	120	0.89	0.26	0.89
	200	0.95	0.26	0.94
	280	0.97	0.26	0.97
	400	0.99	0.26	0.99
BKS_2	40	0.59	0.25	0.64
	120	0.81	0.25	0.86
	200	0.89	0.25	0.93
	280	0.92	0.26	0.96
	400	0.95	0.26	0.98
BKS_3	40	0.70	0.26	0.69
	120	0.88	0.26	0.88
	200	0.95	0.26	0.94
	280	0.97	0.26	0.97
	400	0.99	0.26	0.99

Table (4.57)

E(M) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,4),where N=40(40) 280 400 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	34.39	32.49	30.64
	120	107.93	106.85	93.18
	200	180.68	128.80	155.90
	280	253.69	259.82	217.92
	400	363.21	375.94	310.63
BKS_1	40	35.57	38.86	38.91
	120	112.87	117.47	114.81
	200	187.95	196.45	190.68
	280	263.16	275.72	266.33
	400	375.92	394.90	379.82
BKS_2	40	17.02	21.70	26.53
	120	57.18	87.72	86.32
	200	98.82	158.97	145.89
	280	139.92	232.7	205.39
	400	202.48	343.35	293.88
BKS_3	40	31.26	35.71	37.39
	120	98.11	113.08	112.30
	200	165.51	191.04	187.23
	280	232.52	269.55	261.72
	400	333.15	387.83	373.74

Table (4.58)

E($N_{(1)}$) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,4),where N=40(40) 280 400 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	4.83	8.12	6.34
	120	14.08	26.60	18.82
	200	23.23	45.60	31.37
	280	32.24	64.91	43.77
	400	45.97	93.95	62.24
BKS_1	40	5.18	9.68	7.98
	120	14.62	29.34	23.18
	200	24.01	49.06	38.35
	280	33.36	86.87	53.55
	400	47.48	98.80	76.17
BKS_2	40	2.26	5.40	5.50
	120	7.67	21.91	17.48
	200	12.90	39.70	29.30
	280	17.90	58.14	41.27
	400	25.83	85.73	59.01
BKS_3	40	4.44	8.90	7.69
	120	12.80	28.24	22.60
	200	21.18	47.65	37.64
	280	29.50	67.35	52.50
	400	42.12	96.89	75.93

Table (4.59)

E(S) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,4),where N=40 (40) 280 400 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35 ,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	11.56	16.20	23.41
	120	36.63	53.44	71.31
	200	61.53	91.41	119.40
	280	86.59	129.98	167.01
	400	128.86	187.96	238.03
BKS_1	40	12.61	19.37	29.76
	120	38.36	58.79	87.85
	200	64.05	98.29	146.12
	280	89.98	137.97	204.13
	400	128.24	197.43	291.05
BKS_2	40	5.59	10.81	20.26
	120	19.37	43.76	66.07
	200	33.60	79.48	111.75
	280	47.56	116.32	157.37
	400	69.70	171.76	225.32
BKS_3	40	10.47	17.80	28.60
	120	33.29	56.55	86.03
	200	56.39	95.53	143.42
	280	79.42	134.84	200.59
	400	113.66	193.84	286.37

Table (4.60)

P(CS) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,5),where N=50 (50) 350 500 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.5)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	0.69	0.20	0.67
	150	0.85	0.20	0.86
	250	0.93	0.20	0.93
	350	0.96	0.20	0.96
	500	0.98	0.20	0.98
BKS_1	50	0.69	0.21	0.68
	150	0.87	0.21	0.89
	250	0.93	0.21	0.95
	350	0.96	0.21	0.97
	500	0.98	0.21	0.99
BKS_2	50	0.69	0.20	0.65
	150	0.87	0.20	0.86
	250	0.93	0.20	0.92
	350	0.96	0.20	0.96
	500	0.98	0.20	0.98
BKS_3	50	0.69	0.20	0.70
	150	0.86	0.20	0.89
	250	0.93	0.20	0.95
	350	0.96	0.20	0.98
	500	0.98	0.20	0.99

Table (4.61)

E(M) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed $p_i(i=1,\ldots,5)$,where N=50 (50) 350 500 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	38.58	40.03	36.23
	150	120.56	132.43	109.43
	250	201.90	227.26	182.15
	350	284.54	323.37	254.88
	500	360.48	468.12	362.73
BKS_1	50	47.60	49.19	49.08
	150	142.37	147.97	144.63
	250	237.25	247.15	240.08
	350	331.67	346.41	335.52
	500	474.01	495.57	478.66
BKS_2	50	21.28	27.81	31.49
	150	72.30	110.76	100.96
	250	124.84	200.10	170.79
	350	178.19	291.15	239.92
	500	255.74	481.04	343.68
BKS_3	50	42.44	46.48	47.55
	150	131.29	143.93	141.88
	250	220.16	242.21	236.31
	350	308.46	340.76	330.41
	500	441.82	489.03	471.84

Table (4.62)

$E(N_{(1)})$ of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,5),where N=50 (50) 350 500 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	3.89	7.99	5.45
	150	10.97	26.46	15.93
	250	17.97	45.43	26.24
	350	25.02	64.64	36.71
	500	35.48	93.51	52.08
BKS_1	50	4.65	9.82	7.28
	150	12.78	29.59	20.96
	250	20.91	49.41	34.59
	350	29.06	69.22	48.21
	500	41.26	99.07	68.69
BKS_2	50	2.41	5.54	4.78
	150	6.82	22.56	14.70
	250	11.29	39.99	24.73
	350	15.86	58.19	34.60
	500	22.42	86.11	49.36
BKS_3	50	4.23	9.27	7.06
	150	11.83	28.75	20.54
	250	19.48	48.39	34.04
	350	26.97	68.08	47.44
	500	38.43	97.77	67.74

Table (4.63)

E(S)of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,5),where N=50 (50) 350 500 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	15.37	19.99	26.24
	150	48.78	66.17	79.53
	250	82.05	113.71	132.52
	350	115.57	161.59	185.70
	500	165.20	234.08	264.17
BKS_1	50	19.04	24.26	35.62
	150	57.62	73.97	105.20
	250	96.48	123.67	174.72
	350	134.73	173.07	244.54
	500	192.68	247.83	348.53
BKS_2	50	8.33	13.87	22.80
	150	29.09	55.38	73.32
	250	50.57	100.03	124.27
	350	72.35	145.63	174.78
	500	103.83	215.60	250.30
BKS_3	50	16.94	23.22	34.51
	150	53.13	71.90	103.20
	250	89.57	121.19	172.02
	350	125.35	170.25	240.78
	500	179.68	244.51	343.62

Table (4.64)

P(CS) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,9),where N=90 (90) 630 900 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	0.71	0.12	0.67
	270	0.91	0.12	0.87
	450	0.96	0.12	0.94
	630	0.98	0.12	0.96
	900	0.99	0.12	0.98
BKS_1	90	0.77	0.12	0.74
	270	0.95	0.12	0.93
	450	0.99	0.12	0.98
	630	1	0.12	0.99
	900	1	0.12	1
BKS_2	90	0.67	0.12	0.63
	270	0.89	0.12	0.86
	450	0.96	0.12	0.93
	630	0.98	0.12	0.96
	900	0.99	0.12	0.98
BKS_3	90	0.79	0.12	0.75
	270	0.96	0.12	0.93
	450	0.99	0.12	0.97
	630	1	0.12	0.99
	900	1	0.12	1

Table (4.65)

E(M) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,9),where N=90 (90) 630 900 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5, 0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	51.97	68.55	50.37
	270	150.55	232.96	150.95
	450	248.25	402.33	251.11
	630	345.23	573.84	351.05
	900	490.94	832.84	501.02
BKS_1	90	89.32	90.46	88.76
	270	263.09	269.41	262.32
	450	436.76	448.61	435.69
	630	610.59	628.14	609.42
	900	870.93	897.37	869.60
BKS_2	90	46.33	51.86	43.31
	270	140.38	200.16	132.51
	450	233.42	358.72	221.83
	630	326.54	522.98	311.07
	900	466.01	772.14	444.32
BKS_3	90	87.79	88.23	86.99
	270	260.58	266.22	258.97
	450	432.97	444.74	430.82
	630	605.55	623.55	602.73
	900	864.30	892.28	860.25

Table (4.66)

$E(N_{(1)})$ of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,9),where N=90 (90) 630 900 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	2.30	7.59	1.93
	270	5.29	25.85	4.18
	450	8.31	44.64	6.40
	630	11.19	63.68	8.61
	900	15.62	92.47	11.97
BKS_1	90	3.44	10.30	2.77
	270	8.71	29.85	6.64
	450	13.97	49.79	10.50
	630	19.26	69.70	14.29
	900	27.07	99.64	20.09
BKS_2	90	2.13	5.75	1.75
	270	4.97	22.21	3.72
	450	7.82	39.83	5.76
	630	10.61	58.01	7.69
	900	14.81	85.70	10.70
BKS_3	90	3.4	9.76	2.75
	270	8.59	29.53	6.55
	450	13.86	49.34	10.36
	630	19.07	69.20	14.21
	900	26.90	99.06	19.91

Table (4.67)

E(S) of the schemes that are combinations of the sampling rule PCWR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,….,9),where N=90 (90) 630 900 under uniform priors

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	33.79	34.23	30.72
	270	99.88	116.59	94.40
	450	165.49	201.16	157.74
	630	230.44	286.98	220.95
	900	328.37	416.30	315.96
BKS_1	90	58.86	45.20	55.04
	270	175.48	134.78	165.09
	450	292.04	224.22	274.63
	630	408.79	313.99	384.69
	900	583.64	448.64	549.62
BKS_2	90	30.07	25.89	26.22
	270	93.15	100.10	82.73
	450	155.65	179.40	139.36
	630	218.06	261.56	195.73
	900	311.78	386.26	280.12
BKS_3	90	57.83	44.06	53.86
	270	173.85	133.12	162.83
	450	289.56	222.32	271.77
	630	405.36	311.68	380.46
	900	579.22	446.15	543.48

4.5 Performance characteristics of GS (VT) with BKS and modifications.

In this section, we discuss the performance characteristics of the schemes that are combinations of the sampling rule GS (VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and terminal decision rule BKT.

The values of the performance measure E ($N_{(1)}$) do not appear in these tables since $E(N_{(1)})=E(M)/K$.

4.5.1 The *MC* estimates of P(CS)

Tables(4.68), (4.71), (4.74) and (4.77) display some results of P(CS) ,for fixed $p_i(i=1,……,k)$,where k=3,4,5 and 9, respectively. They show that the performance measure P (CS) has the same behavior as in the last section 6.3.1.

4.5.2 The MC estimates of E (M)

The results presented in the tables (4.69), (4.72), (4.75) and (4.78) show also the same behavior in section 4.3.2 in terms of E(M).

4.5.2 The *MC* estimates of E(S)

Tables (4.70), (4.73),(4.76) and (4.79) contains some results of E(S), for fixed $P_i(i=1,…….,k)$, where k=3,4,5 and 9, respectively. We notice from these tables that the performance measure E(S) has the same order as in section 4.3.4.

we can suggest the use of the schemes (GS(VT), BKS_1) and (GS(VT), BKS_3) if we are interested in the performance measure P(CS) and E(S) and the scheme (GS(VT), BKS_2) , if the performance E(M) is of more importance.

Table (4.68)

P(CS) for the schemes that are combinations of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3),where N=30 (30) 210 300 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35)	$\underset{\sim}{p}$ = (0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.7,0.8,0.9)
BKS	30	0.63	0.34	0.68
	90	0.79	0.34	0.85
	150	0.86	0.34	0.92
	210	0.90	0.34	0.95
	300	0.94	0.34	0.98
BKS_1	30	0.63	0.34	0.67
	90	0.74	0.34	0.85
	150	0.86	0.34	0.92
	210	0.90	0.34	0.95
	300	0.94	0.34	0.98
BKS_2	30	0.58	0.34	0.66
	90	0.75	0.34	0.84
	150	0.84	0.34	0.92
	210	0.89	0.34	0.95
	300	0.93	0.34	0.98
BKS_3	30	0.62	0.34	0.73
	90	0.72	0.34	0.91
	150	0.85	0.34	0.96
	210	0.90	0.34	0.98
	300	0.94	0.34	0.99

Tale (4.69)

E(M) for the schemes that are combinations of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3),where N=30 (30) 210 300 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35)	$\underset{\sim}{p}$ = (0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.7,0.8,0.9)
BKS	30	26.12	26.85	27.07
	90	80.79	83.45	82.02
	150	135.79	141.52	136.80
	210	190.72	199.96	191.27
	300	272.46	287.90	273.44
BKS_1	30	29.47	29.47	29.79
	90	87.63	88.66	88.07
	150	145.75	148.20	146.28
	210	203.96	207.48	204.28
	300	291.14	296.82	291.50
BKS_2	30	13.90	18.67	23.36
	90	50.71	71.70	77.63
	150	92.08	127.95	131.17
	210	136.90	184.62	184.64
	300	200.75	270.06	264.48
BKS_3	30	23.45	26.90	28.92
	90	74.42	84.80	86.53
	150	127.72	143.66	144.12
	210	180.51	202.54	201.74
	300	260.11	290.99	288.16

Tale (4.70)

E(S) for the schemes that are combinations of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,2,3),where N=30 (30) 210 300 under uniform priors .

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35)	$\underset{\sim}{p}$ = (0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.7,0.8,0.9)
BKS	30	6.51	13.28	21.67
	90	20..21	41.68	65.58
	150	33.97	70.79	109.43
	210	47.66	99.94	152.93
	300	68.42	143.93	218.73
BKS_1	30	7.35	14.79	23.36
	90	21.93	44.30	70.44
	150	36.49	74.04	116.98
	210	50.92	103.71	163.37
	300	72.76	148.45	233.3
BKS_2	30	3.49	9.34	18.7
	90	12.65	35.81	62.05
	150	23.01	63.96	104.97
	210	34.15	92.28	147.7
	300	50.11	135.20	211.63
BKS_3	30	5.86	13.94	23.13
	90	18.58	42.55	69.19
	150	31.93	71.85	115.3
	210	45.18	101.24	161.39
	300	65.09	145.49	230.56

Tale (4.71)

P(CS) for the schemes that are combinations of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,4),where N=40 (40) 280 400 under uniform .

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	0.59	0.25	0.65
	120	0.76	0.25	0.85
	200	0.84	0.25	0.92
	280	0.88	0.25	0.96
	400	0.92	0.25	0.98
BKS_1	40	0.59	0.25	0.66
	120	0.77	0.25	0.85
	200	0.84	0.25	0.92
	280	0.89	0.25	0.96
	400	0.93	0.25	0.98
BKS_2	40	0.55	0.55	0.55
	120	0.74	0.74	0.74
	200	0.83	0.83	0.83
	280	0.87	0.87	0.87
	400	0.92	0.92	0.92
BKS_3	40	0.59	0.25	0.65
	120	0.76	0.25	0.85
	200	0.84	0.26	0.92
	280	0.88	0.26	0.96
	400	0.93	0.26	0.98

Tale (4.72)

E(M) for the schemes that are combinations of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,4),where N=40 (40) 280 400 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	34.91	35.96	36.40
	120	107.85	112.42	109.42
	200	180.82	182.51	182.51
	280	274.34	268.21	255.38
	400	363.93	385.91	364.38
BKS_1	40	39.77	39.93	39.98
	120	118.02	119.31	118.49
	200	196.17	198.80	196.65
	280	254.06	278.29	274.69
	400	391.54	397.80	391.86
BKS_2	40	21.85	27.30	31.88
	120	80.09	99.60	103.92
	200	142.42	174.78	175.12
	280	204.60	250.78	246.35
	400	299.00	366.20	352.91
BKS_3	40	35.46	28.25	39.02
	120	109.53	116.45	117.05
	200	184.01	195.43	194.74
	280	258.72	274.33	272.29
	400	370.97	393.23	388.45

Table (4.73)

E(S) for the schemes that are combinations of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,4),where N=40 (40) 280 400 under uniform priors.

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.6,0.7,0.8,0.9)
BKS	40	10.44	17.95	27.30
	120	32.39	56.18	82.01
	200	54.19	99.92	136.89
	280	76.80	134.05	191.50
	400	109.21	193.06	273.39
BKS_1	40	11.92	19.92	30.00
	120	35.44	59.60	88.79
	200	85.78	99.26	147.38
	280	82.61	139.04	205.98
	400	117.38	148.87	293.87
BKS_2	40	6.54	13.64	23.94
	120	24.00	49.75	77.92
	200	43.79	87.35	131.31
	280	61.38	125.34	183.74
	400	89.11	183.28	264.85
BKS_3	40	10.59	19.08	29.26
	120	32.85	58.19	87.71
	200	55.10	97.60	145.90
	280	77.55	137.10	204.18
	400	111.38	196.64	291.37

Table (4.74)

P(CS) for the schemes that are combinations of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,5),where N=50 (50) 350 500 under uniform priors .

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	0.59	0.20	0.66
	150	0.75	0.20	0.85
	250	0.83	0.20	0.92
	350	0.88	0.20	0.95
	500	0.92	0.21	0.98
BKS_1	50	0.59	0.20	0.66
	150	0.75	0.20	0.85
	250	0.83	0.21	0.92
	350	0.87	0.21	0.95
	500	0.92	0.21	0.98
BKS_2	50	0.55	0.20	0.63
	150	0.74	0.20	0.84
	250	0.82	0.20	0.92
	350	0.88	0.20	0.95
	500	0.92	0.20	0.98
BKS_3	50	0.58	0.20	0.64
	150	0.76	0.20	0.84
	250	0.83	0.20	0.92
	350	0.88	0.21	0.95
	500	0.92	0.21	0.98

Table (4.75)

E(M) for the schemes that are combinations of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,5),where N=50 (50) 350 500 under uniform priors .

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	43.92	45.42	45.49
	150	135.00	141.37	136.77
	250	226.29	238.85	228.17
	350	317.28	336.67	319.13
	500	475.19	483.83	455.25
BKS_1	50	49.93	49.99	50
	150	148.40	149.68	148.91
	250	246.62	249.29	247.06
	350	344.78	348.91	345.20
	500	491.69	498.44	492.27
BKS_2	50	30.47	35.92	40.12
	150	110.67	127.14	129.88
	250	182.33	221.63	218.88
	350	274.65	317.48	307.56
	500	397.76	461.40	440.72
BKS_3	50	47.34	48.95	49.76
	150	142.76	147.32	147.48
	250	238.38	246.11	245.16
	350	334.03	345.31	342.70
	500	477.94	494.26	488.95

Table (4.76)

E(S) for the schemes that are combination of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,5),where N=50 (50) 350 500 under uniform priors .

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.5,0.6,0.7,0.8,0.9)
BKS	50	15.35	22.67	31.87
	150	47.29	70.66	95.66
	250	70.06	119.31	159.64
	350	111.21	168.51	223.46
	500	158.82	241.90	318.49
BKS_1	50	24.93	24.93	35.00
	150	17.43	74.77	104.13
	250	87.26	124.56	172.92
	350	120.82	174.61	241.68
	500	172.14	249.31	344.62
BKS_2	50	10.76	17.93	28.10
	150	38.70	63.51	90.84
	250	67.21	110.66	153.12
	350	96.12	158.82	215.31
	500	139.26	230.57	308.33
BKS_3	50	16.52	24.94	34.84
	150	50.00	103.15	103.15
	250	83.43	171.61	171.61
	350	117.03	239.94	239.94
	500	167.23	342.23	342.23

Table (4.77)

P(CS) for the schemes that are combination of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,9),where N=90 (90) 630 900 under uniform priors .

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	0.69	0.12	0.65
	270	0.90	0.12	0.85
	450	0.95	0.12	0.92
	630	0.98	0.12	0.95
	900	0.99	0.12	0.98
BKS_1	90	0.69	0.11	0.65
	270	0.90	0.12	0.85
	450	0.95	0.12	0.92
	630	0.98	0.12	0.95
	900	0.99	0.12	0.98
BKS_2	90	0.67	0.12	0.63
	270	0.89	0.12	0.84
	450	0.95	0.12	0.92
	630	0.98	0.12	0.95
	900	0.99	0.12	0.98
BKS_3	90	0.70	0.12	0.65
	270	0.90	0.12	0.85
	450	0.95	0.12	0.92
	630	0.98	0.12	0.95
	900	0.99	0.12	0.98

Table (4.78)

E(M) for the schemes that are combination of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,9),where N=90 (90) 630 900 under uniform priors .

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	83.10	83.20	82.10
	270	247.25	257.28	246.24
	450	411.12	433.07	410.30
	630	574.56	609.90	571.54
	900	820.33	875.62	820.40
BKS_1	90	90	90	90
	270	269.99	270	269.95
	450	449.27	449.98	448.95
	630	627.21	629.90	627.02
	900	894.04	899.73	894.13
BKS_2	90	74.37	70.15	72.50
	270	236.62	235.85	233.49
	450	397.99	407.79	394.21
	630	558.67	580.76	554.19
	900	800.17	842.47	794.10
BKS_3	90	90	89.88	89.97
	270	269.30	269.49	268.87
	450	447.66	448.82	446.80
	630	624.92	628.17	624.39
	900	891.44	897.21	890.74

Table (4.79)

E(S) for the schemes that are combination of the sampling rule GS(VT), the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT , for fixed p_i(i=1,…..,9),where N=90 (90) 630 900 under uniform priors .

Stopping rule	N	$\underset{\sim}{p}$ = (0.15,0.25,0.35,0.45,0.55, 0.65,0.75,0.85,0.95)	$\underset{\sim}{p}$ = (0.5,0.5,0.5,0.5,0.5,0.5, 0.5,0.5,0.5)	$\underset{\sim}{p}$ = (0.1,0.2,0.3,0.4,0.5, 0.6, 0.7,0.8,0.9)
BKS	90	45.68	41.51	41.04
	270	135.97	128.55	123.11
	450	226.15	216.68	205.12
	630	315.98	304.84	287.22
	900	451.52	437.88	410.36
BKS_1	90	49.43	44.93	44.99
	270	148.35	134.86	134.87
	450	247.13	225.15	224.49
	630	344.80	314.81	313.37
	900	491.77	449.90	447.18
BKS_2	90	40.91	35.03	36.27
	270	130.08	117.91	116.65
	450	218.97	203.90	197.12
	630	307.25	290.40	277.19
	900	440.08	421.36	397.02
BKS_3	90	49.44	44.93	44.96
	270	148.00	134.61	134.24
	450	246.27	224.36	223.50
	630	343.66	313.94	312.08
	900	490.38	448.64	445.51

We have discussed stopping rules effect on the sampling rule and deduced the best of the stopping rule .In this section we discuss the best sampling rules.

We select some tables, where k=9 to investigate this case ,the other tables have the same behavior.

4.6 The Sampling rules BKR, PWCR, PCWR, and GS(VT) under the stopping rule BKS

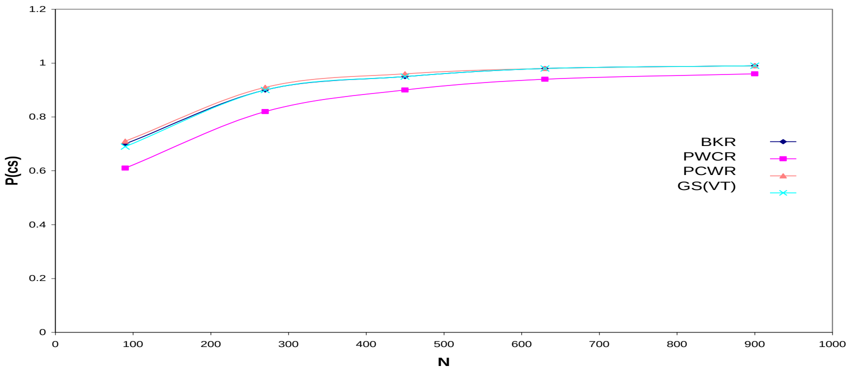
Figures (4.8-4.13) presents some numerical results on the performance characteristics, for fixed p_i(i=1,…..,9).

In fig. (4.8a), (4.8b) the performance measure P(CS) is the best under the sampling rule PWCR and we can show that this performance is increasing function N but in Fig.(4.8b) the measure P(CS)≈1/k in all schemes.

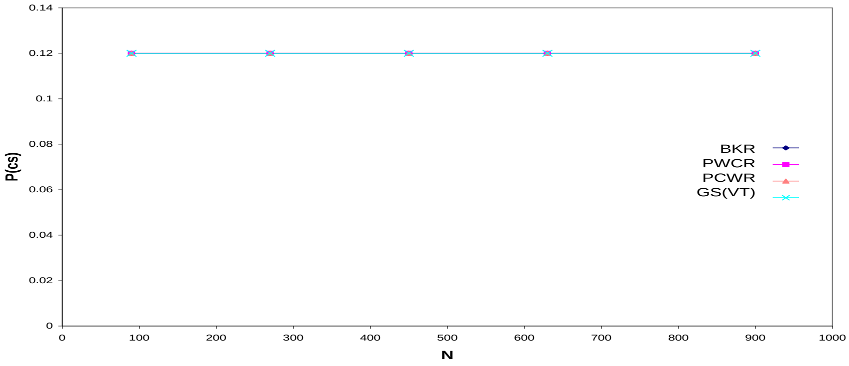
The fig.(4.9) show that the performance measure E(M) is best under the sampling rule BKR, and worst under the sampling rule GS(VT).

From the fig.(4.10), it is clear that the measure $E(N_{(1)})$ is always the best (less values of $E(N_{(1)})$)using the sampling rule BKR. Furthermore little differences can be noted among the sampling rules if percentage ratio $E(N_{(1)})/E(M)$ is considered. However, it is clear that the sampling rule BKR and PCWR are the best alternatively, see Fig. (4.11). Also we can notice that from fig.(4.11b) $E(N_{(1)})\approx 1/k$ when p-values are equal, and this ratio is decreasing function of N.

As E(M) increases, E(S) also increases. Since GS(VT) is the worse as E(M) (the highest observations) hence GS(VT) being the best as E(S), but sometimes BKR or PCWR values become the best ,this happens when the percentage ratio is take and this ratio is increasing function of N. see fig.(4.12) and (4.13).



(a)



(b)

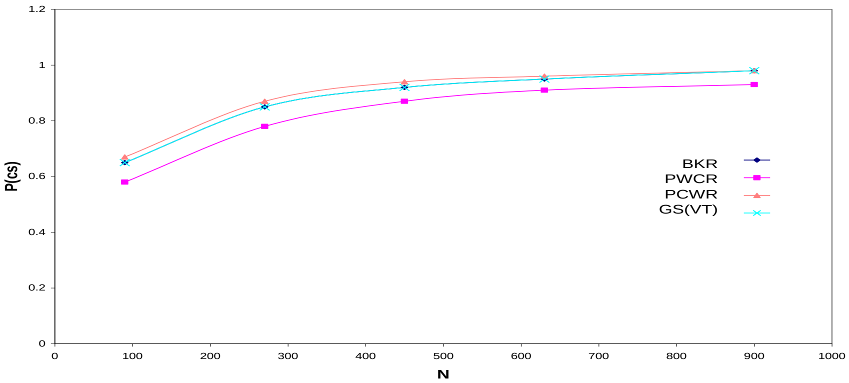
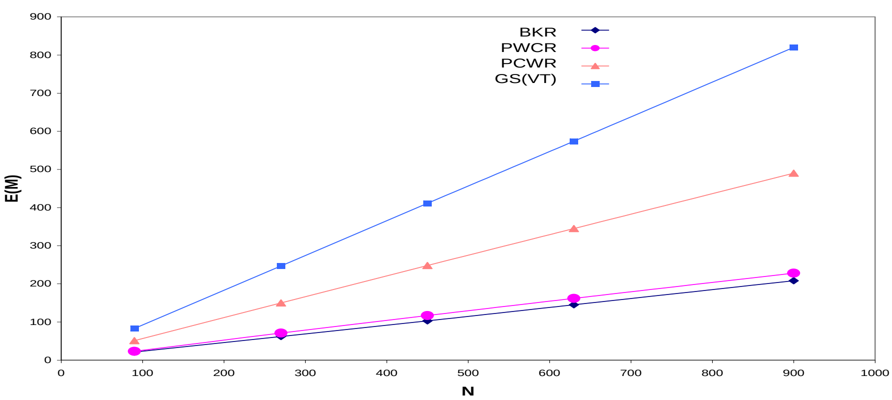
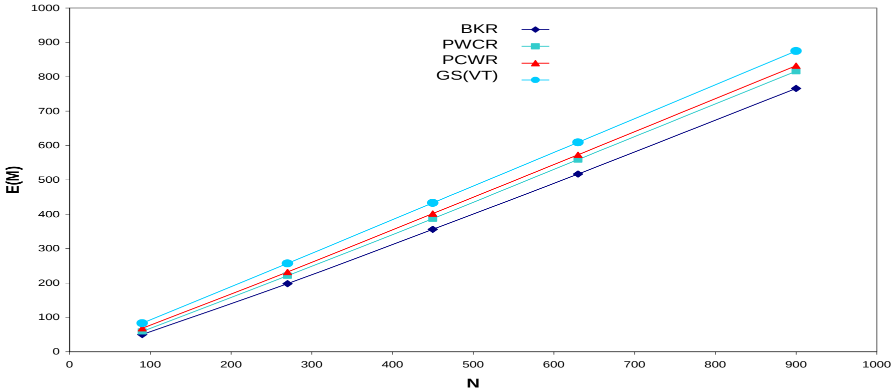


Fig 4.8

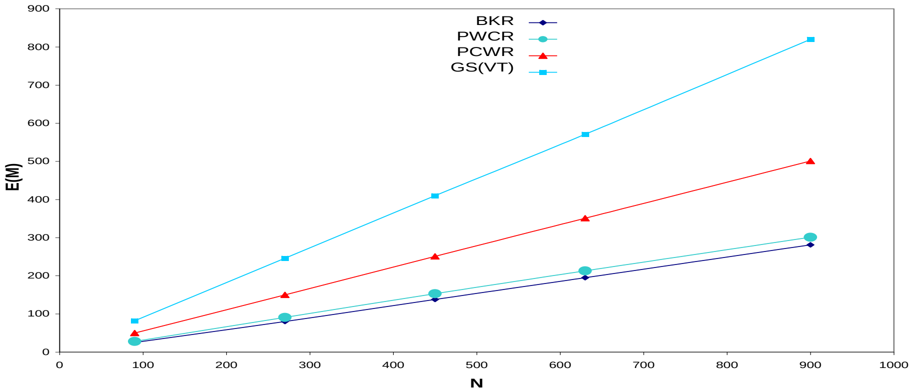
P(CS) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5, 0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4,0.5,0.6,0.7, 0.8,0.9).



(a)

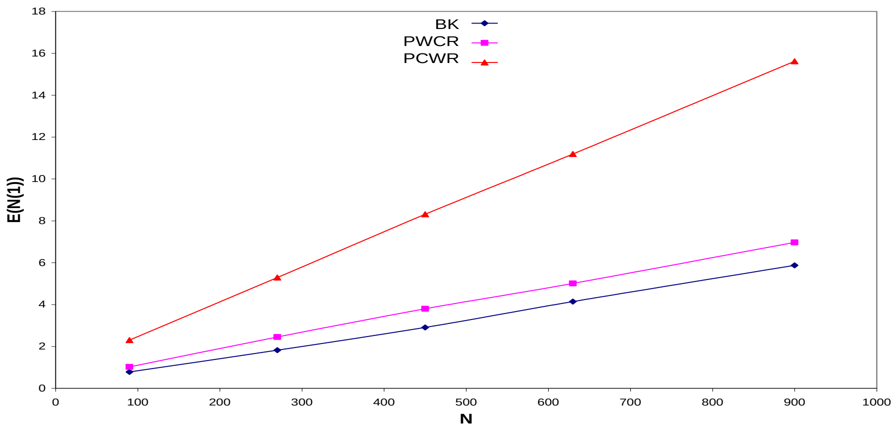


(b)

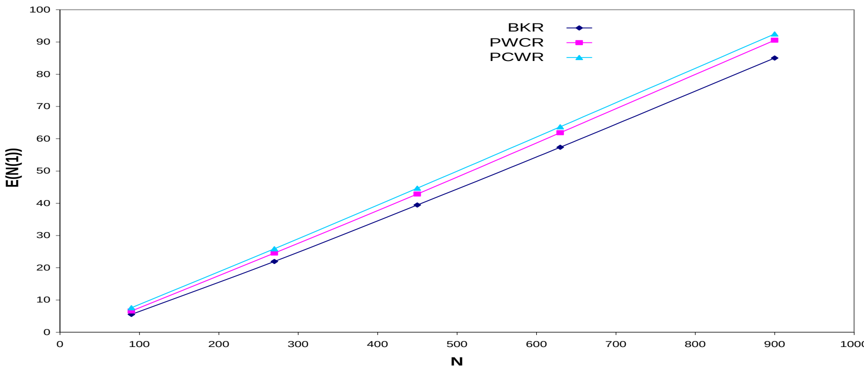


(c)

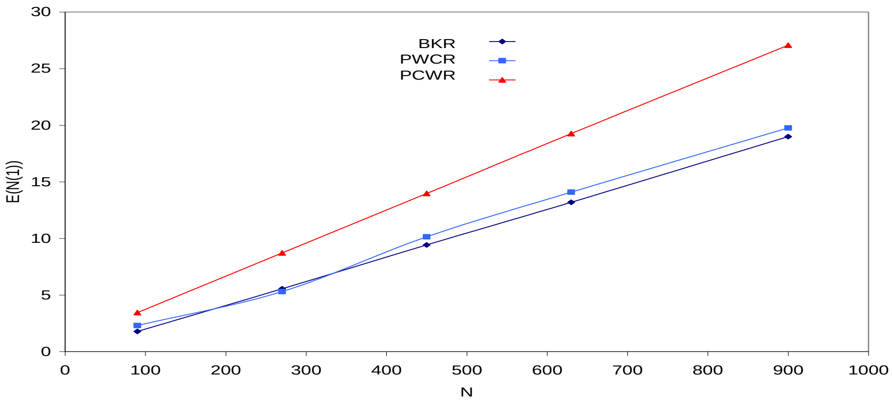
Fig. 4.9 E(M) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5, 0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4,0.5,0.6,0.7, 0.8, 0.9) .



(a)

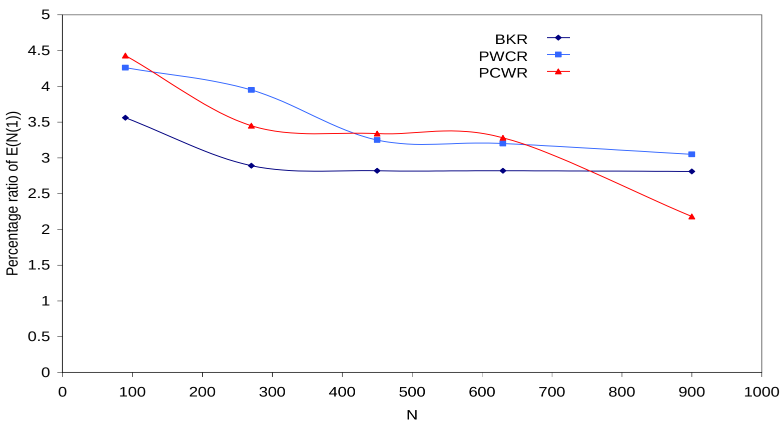


(b)

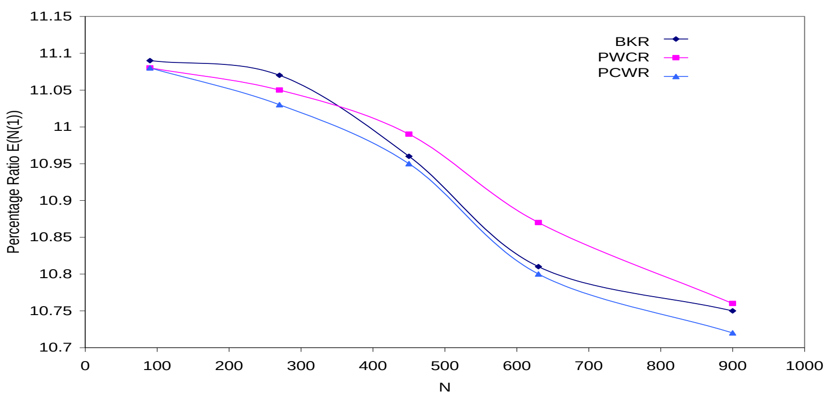


(c)

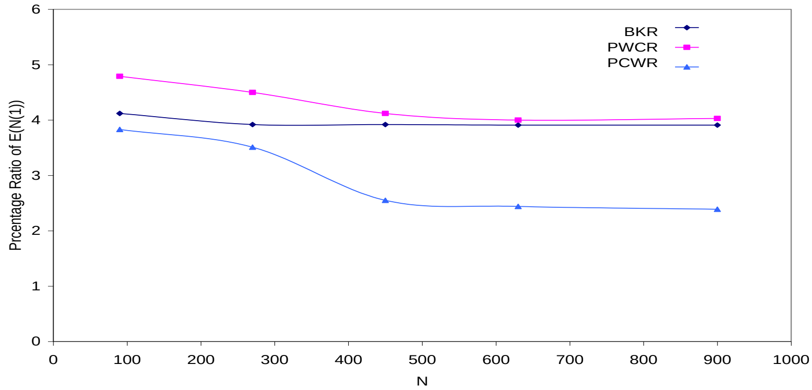
Fig .4.10 E(N(1)) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4 , 0.5,0.6,0.7,0.8,0.9) .



(a)

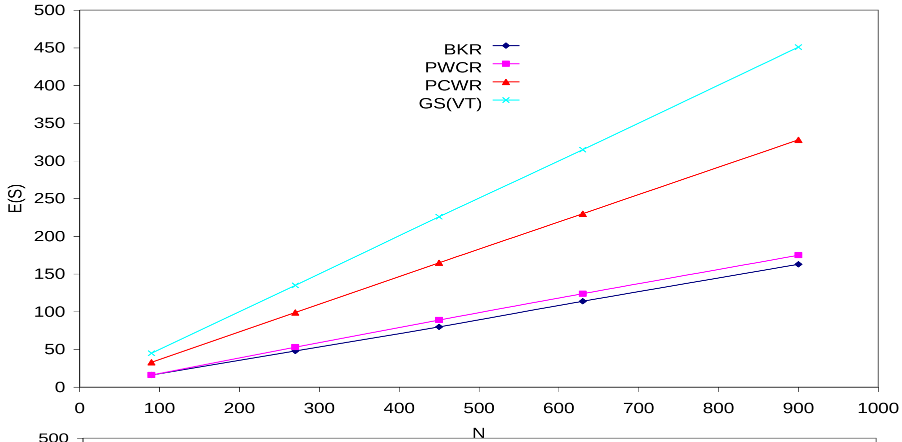


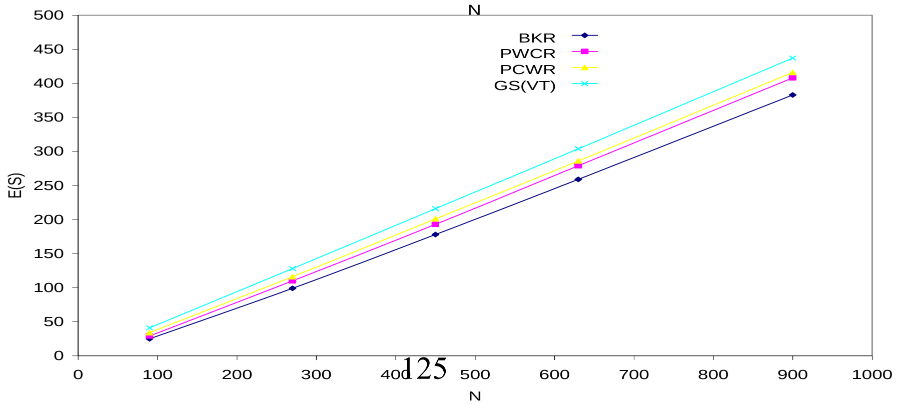
(b)



(c)

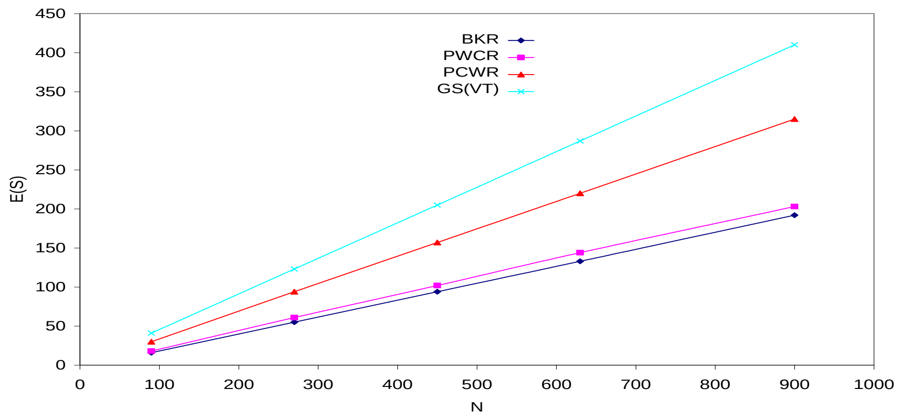
Fig 4.11 Percentage ratio of E(N(1)) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS and terminal decision rule BKT, where N= 90(90) 360 900 under uniform priors ,for fixed $p_i(i=1,\ldots,9)$, when p-values equals for (a) (0.15,0.25,0.35,0.45,0.55,0.65,0.75 ,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2 ,0.3,0.4,0.5,0.6,0.7,0.8,0.9).





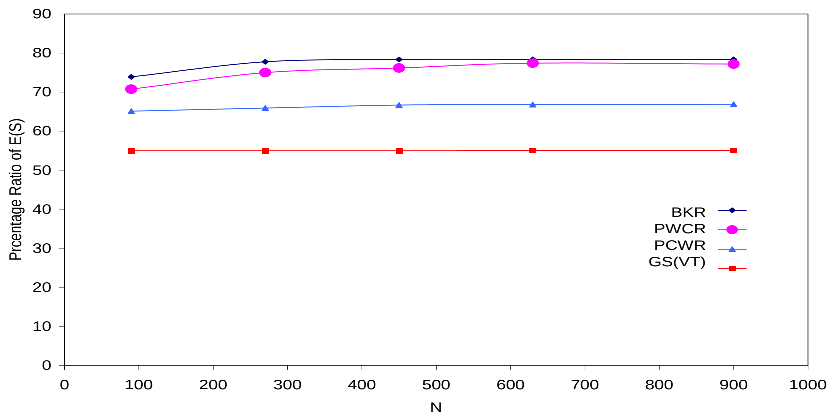
(a)

(b)

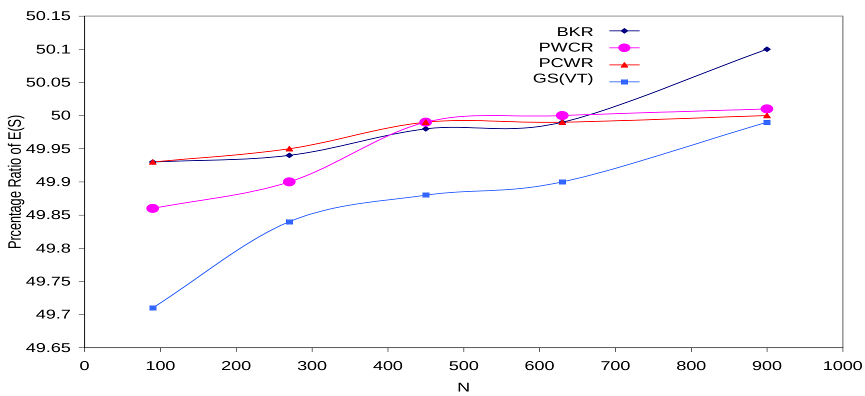


(c)

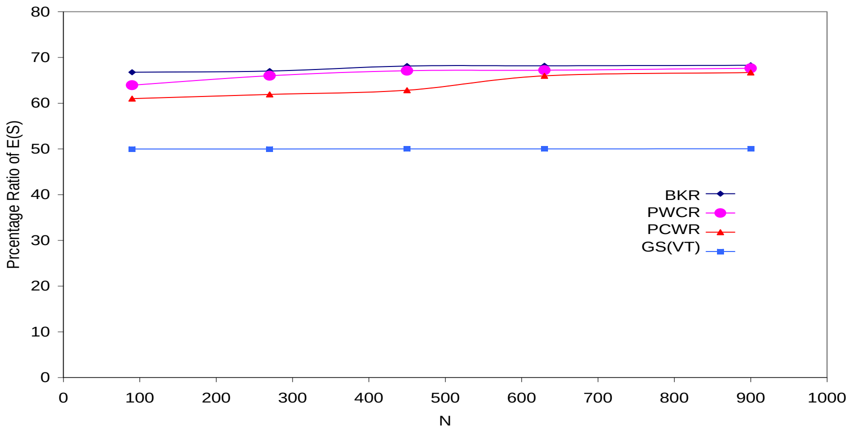
Fig 4.12 E(S) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5 ,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4,0.5,0.6,0.7, 0.8,0. 9).



(a)



(b)



(c)

Fig 4.13 Percentage ratio of E(S) for the schemes that are combination of the sampling rules BKR,PWCR,PCWR and GS(VT),the stopping rule BKS and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9).

4.7 The Sampling rules BKR, PWCR, PCWR and GS(VT) under the stopping rule BKS_1.

Figures (4.14 – 4.19) involve some results of the performance characteristics of the sampling rule BKR, PWCR, PWR and GS(VT) in conjunction with the stopping rules BKS_1 for fixed p_i (i=1, ……,9).

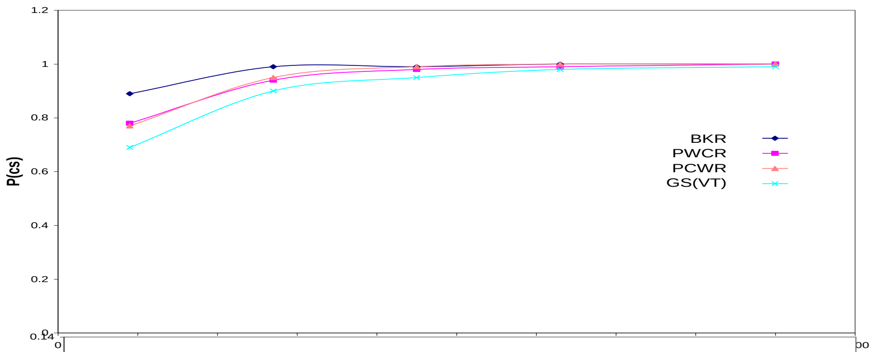
From fig.(4.14) it is notable that the values of the performance measures P(CS) is the best under BKR. It can be show that when we increase N the all sampling rules has roughly the same values.

The performance E(M) is the best under the sampling rules PWCR and BKR alternatively as in fig.(4.15) and from fig.(4.15b) it is clear that E(M) has roughly the same value in all sampling rules.

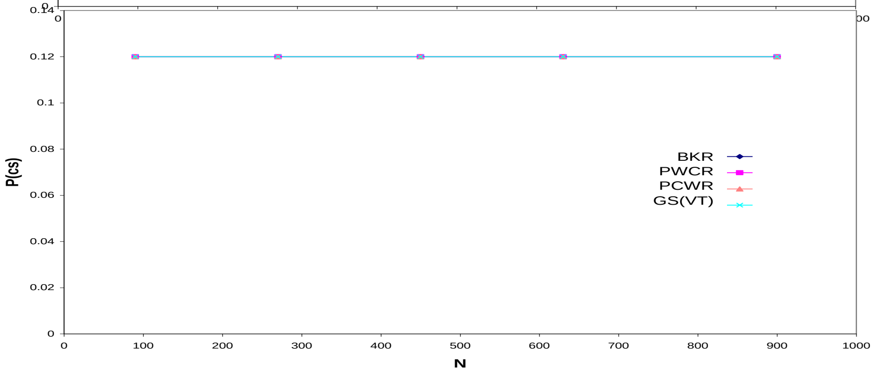
It is clear from fig.(4.16) the performance measure $E(N_{(1)})$ is changeable but it can be show from fig.(4.17) that the sampling rules BKR or PWCR are the best after taking the percentage ratio of $E(N_{(1)})/E(M)$, while we note that the ratio $E(N_{(1)}) \approx 1/k$ when p-values are equals. Moreover, this ratio is decreasing function of N.

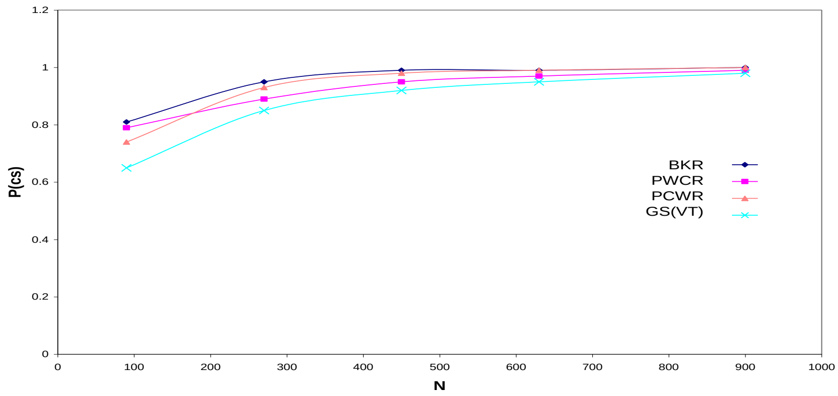
It is clear from fig.(4.18) that E(S) is the best under either the sampling rules BKR or PWCR , also if we take the percentage ratio. And appears the ratio of $E(S) \approx 0.5$ when p-values are equals as in fig. (4.19) and it can show that this ratio is increasing function of N.

(a)



(b)





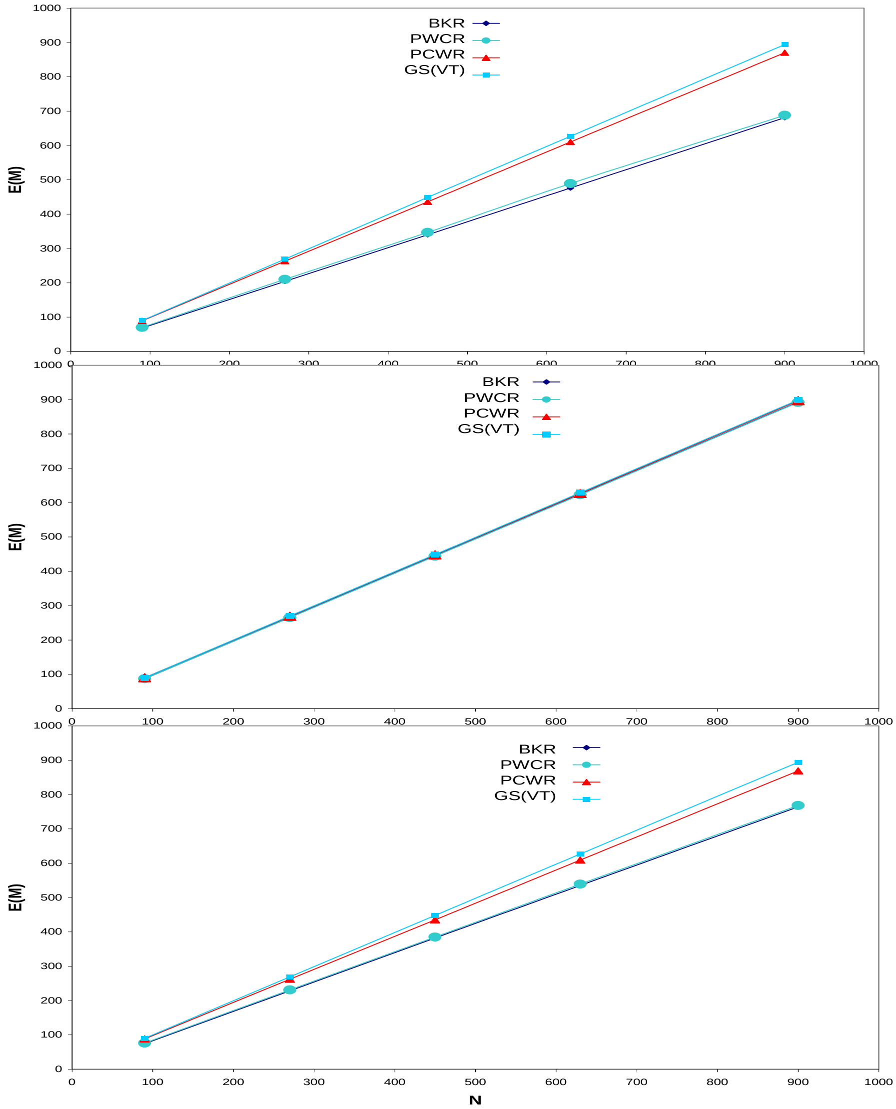
(c)

Fig 4.14

P(CS) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS1 and terminal decision rule BKT,where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b)(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c)(0.1,0.2,0.3,0.4,0.5 ,0.6,0.7,0.8,0.9).

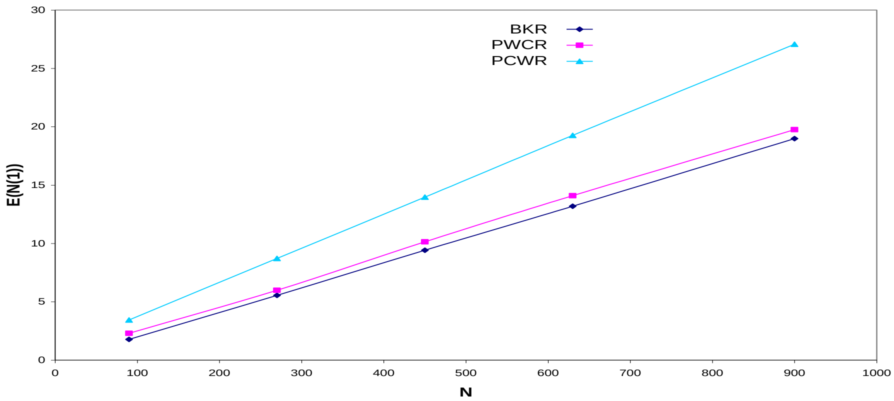
(a)

(b)

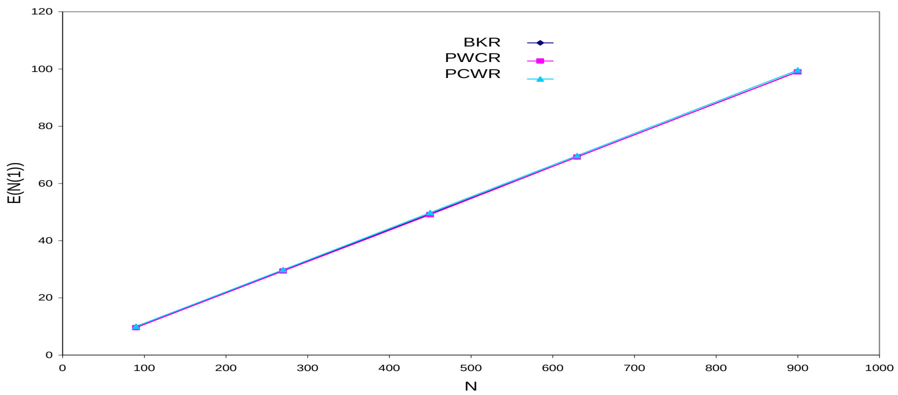


(c)

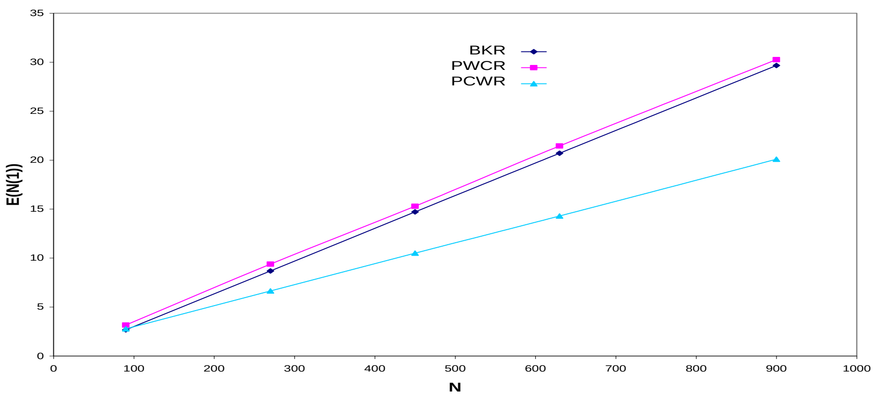
Fig 4.15 E(M) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS1 and terminal decision rule BKT,where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b)(0.5,0.5,0.5 ,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8 ,0.9).



(a)



(b)

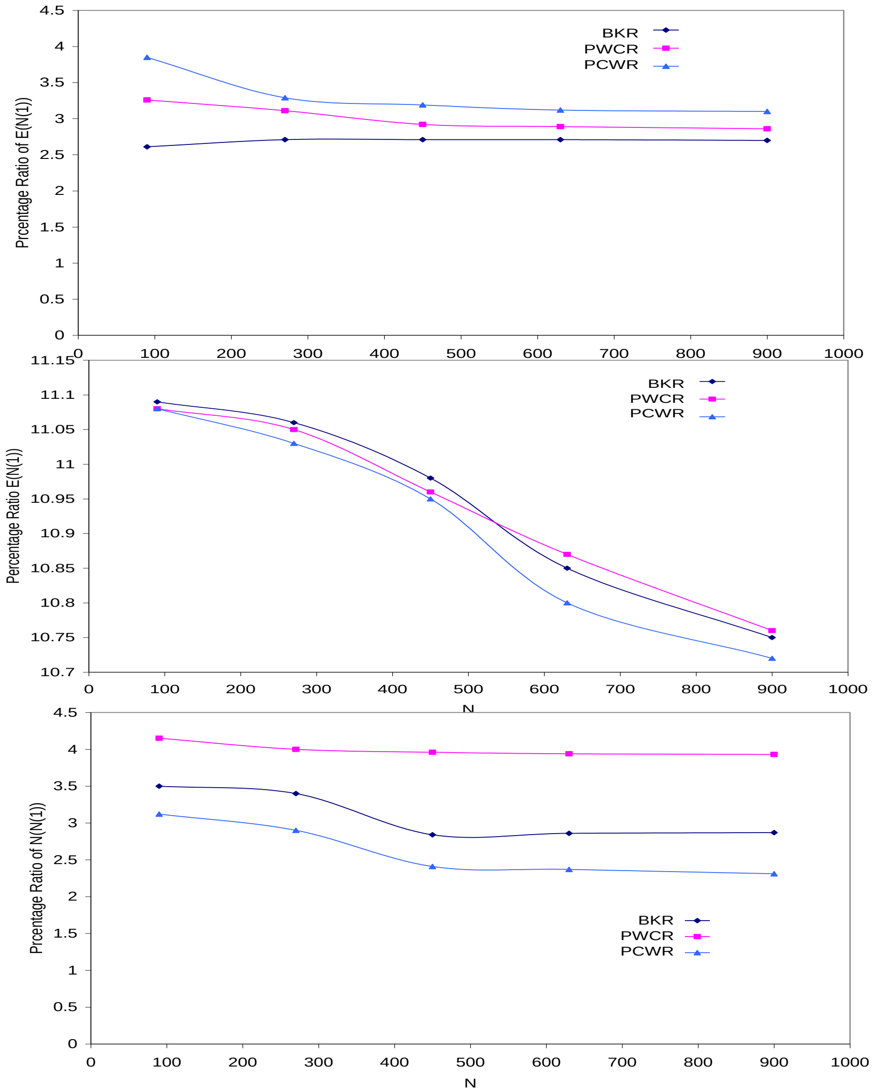


(c)

Fig 4.16 E(N(1)) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT),the stopping rule BKS1 and terminal decision rule BKT,where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a) (0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c)(0.1,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9).

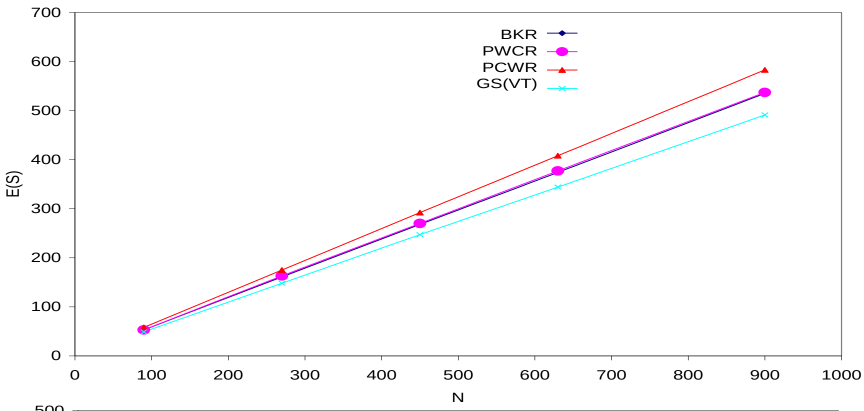
(a)

(b)

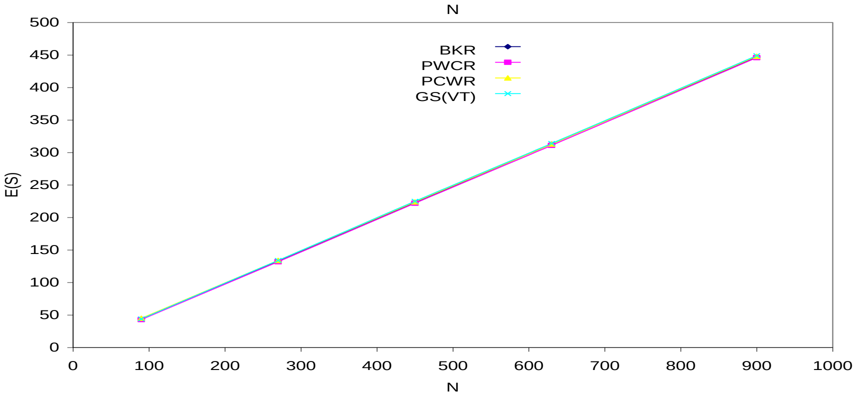


(c)

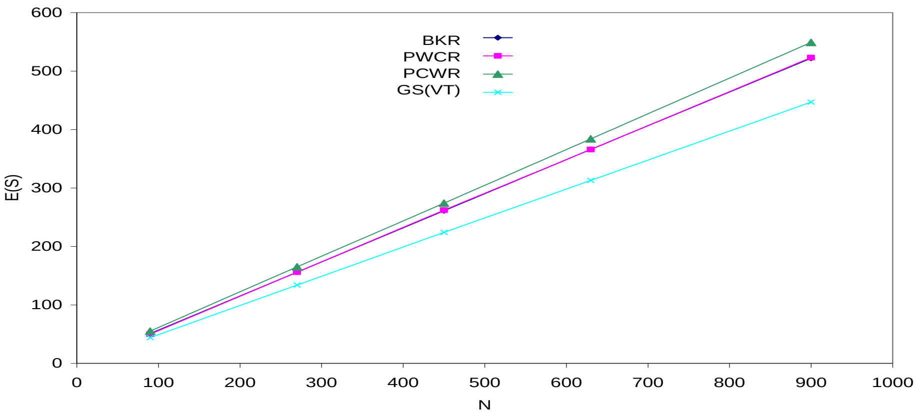
Fig 4.17 Percentage ratio of E(N(1)) for the schemes that are combination of the sampling rules BKR,PWCR,PCWR and GS(VT),the stopping rule BKS1 and terminal decision rule BKT,where N= 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,…..,9), when p-values equals for (a)(0.15,0.25,0.35,0.45,0.55,0.65,0.75, 0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)(c) (0.1,0.2 0.3,0.4,0.5,0.6,0.7,0.8,0.9).



(a)

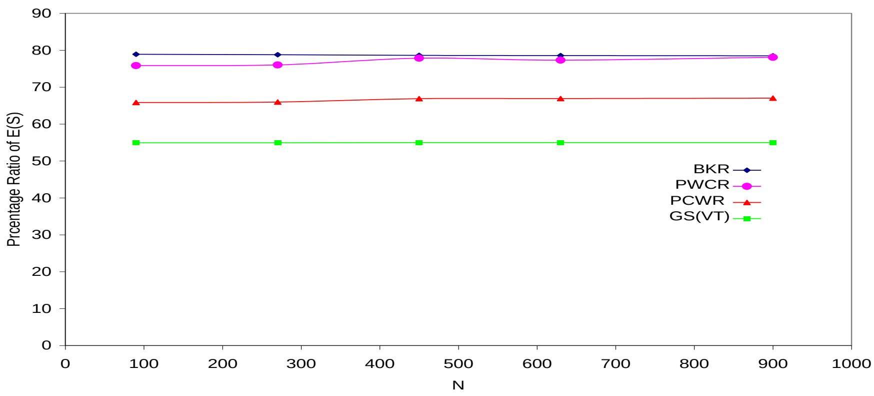


(b)

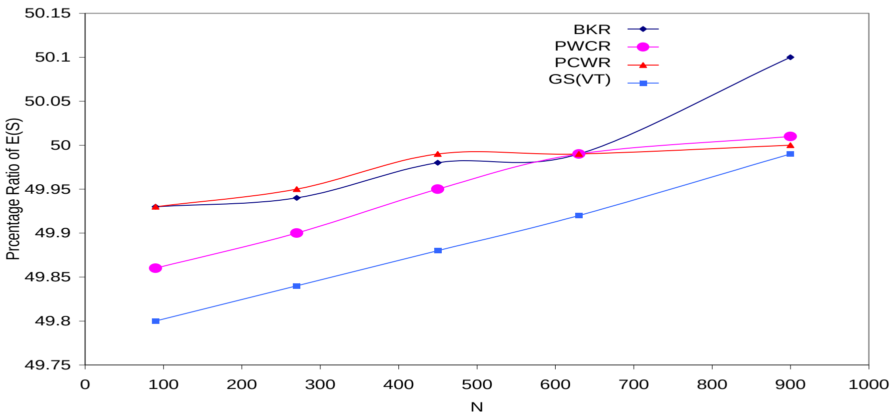


(c)

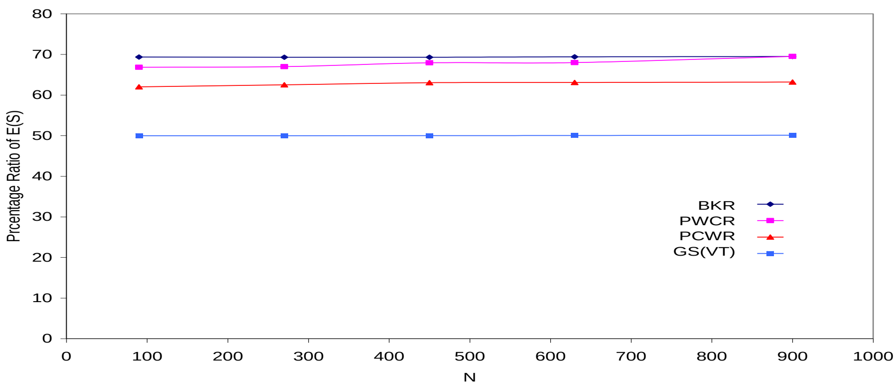
Fig 4.18 E(S) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS1 and terminal decision rule BKT,where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,…,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5, ,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4,0.5,0.6,0.7 ,0.8,0.9).



(a)



(b)



(c)

Fig 4.19 Percentage ratio of E(S) for the schemes that are combination of the sampling rules BKR , PWCR, PCWR and GS(VT),the stopping rule BKS1 and terminal decision rule BKT, where N= 90(90)360 900 under uniform priors ,for fixed $p_i(i=1,\ldots,9)$, when p-values equals for (a)(0.15,0.25,0.35 0.45,0.55,0.65 ,0.75,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9).

4.8 The sampling rules BKR, PWCR, PCWR and GS(VT) under the stopping rule BKS_2

Some numerical results concerning the performance of the sampling rules BKR, PWCR, PCWR and GS(VT) in conjunction with the stopping rule BKS_2 and terminal decision rule BKT for fixed p_i (i=1,…,9) presented in figures (4.20-4.25).

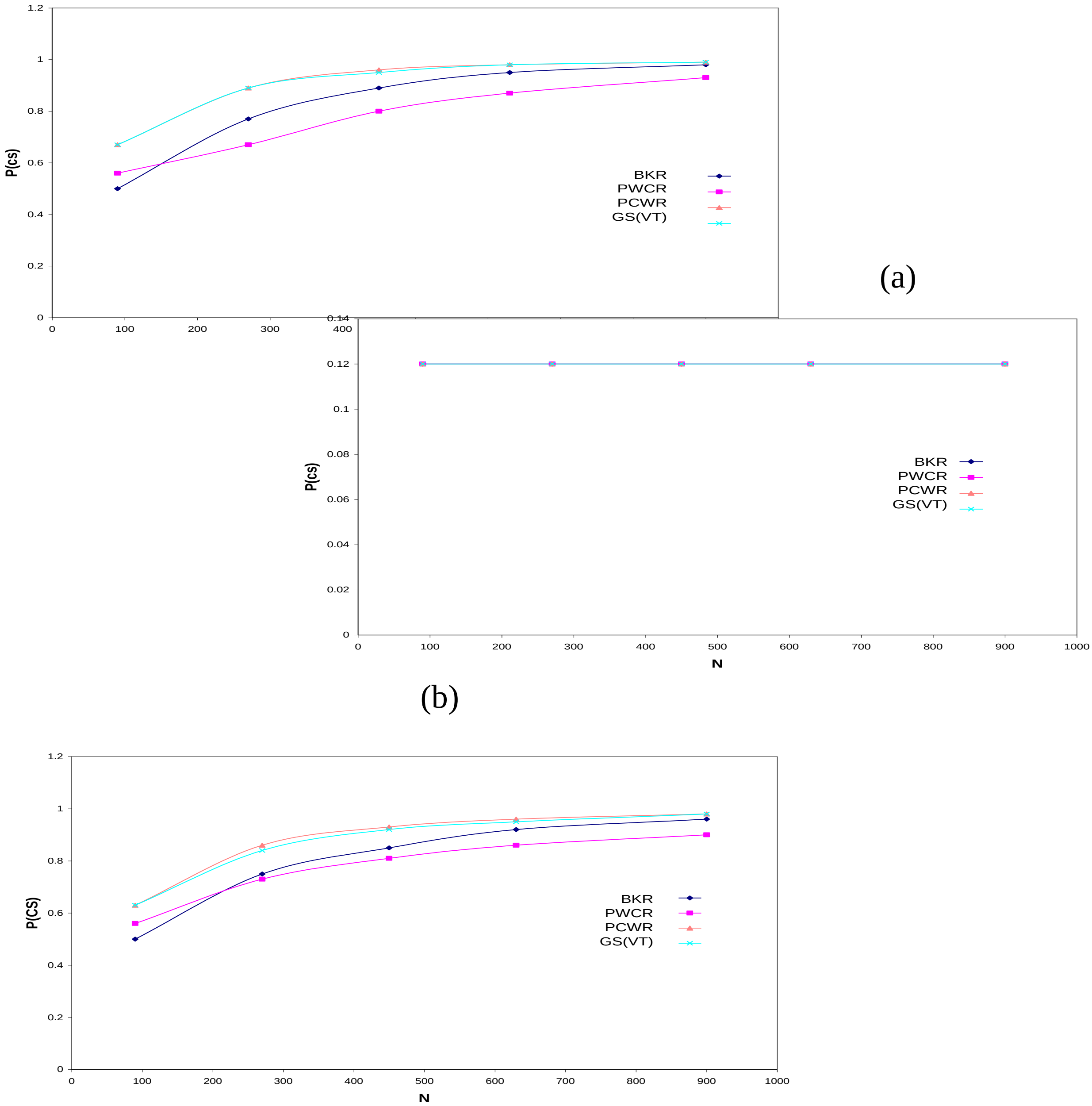
It can be observed from these figures that all the performance characteristics are increasing function of N.

We found in fig.(4.20) the value of the performance measure P(CS) is the best for sampling rule PWCR.

From the results of the fig.(4.21), it can be observed that the measures E(M) is the best either under the sampling rule BKR or PWCR and the value of E(M) is the worse under the sampling rule GS(VT).

Fig.(4.22) shows that either the sampling rule BKR or PWCR is the best for the performance measure $E(N_{(1)})$, but $E(N_{(1)})$ is the best under the sampling rules BKR and PCWR alternatively when we take the percentage ratio of $E(N_{(1)})$ and this ratio is decreasing function of N. (see fig.(4.23))

We note that, E(S) increases as E(M) increases. Therefore the sampling rule GS(VT) is the best in this case. However BKR is the best when we use the percentage ratio (see fig.s(4.24) and (4.25)).

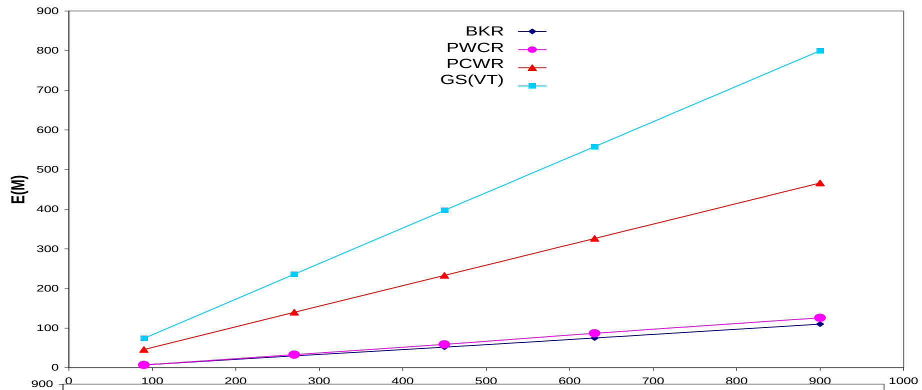


(a)

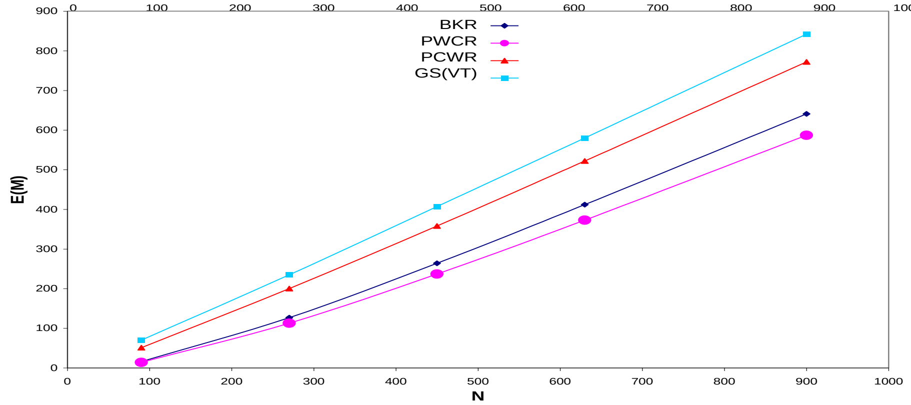
(b)

(c)

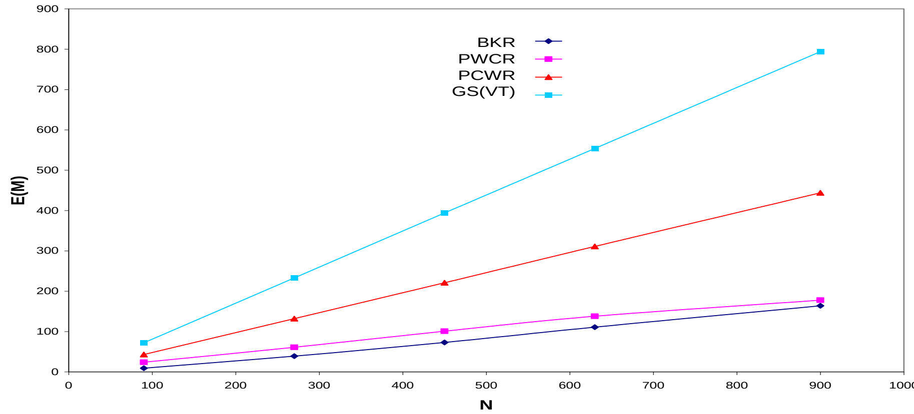
Fig. 4.20 P(CS) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS2 and terminal decision rule BKT,where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for(a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b)(0.5,0.5 ,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c)(0.1,0.2,0.3,0.4,0.5,0.6,0.7 ,0.8,0.9).



(a)

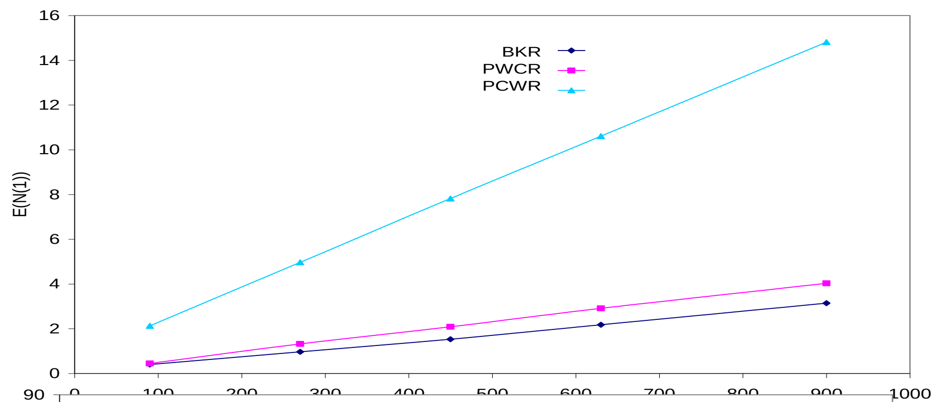


(b)

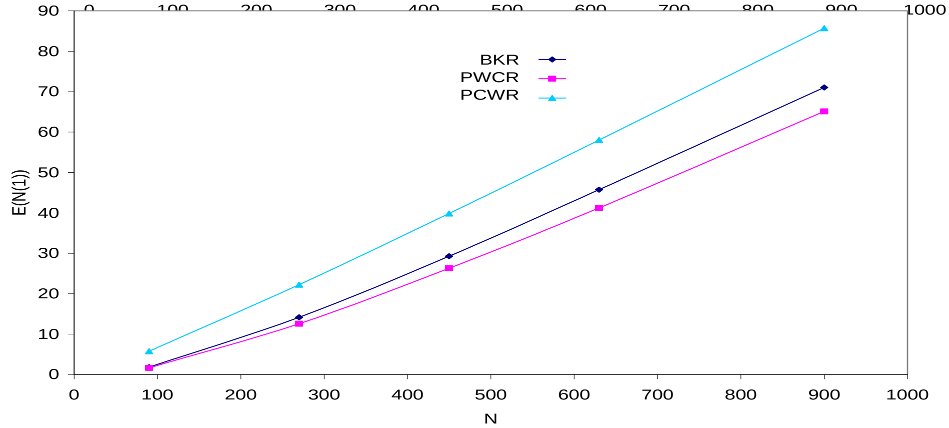


(c)

Fig. 4.21 E(M) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS2 and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1, …,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b)(0.5,0.5, 0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c)(0.1,0.2,0.3,0.4,0.5 ,0.6,0.7,0.8, 0.9).



(a)



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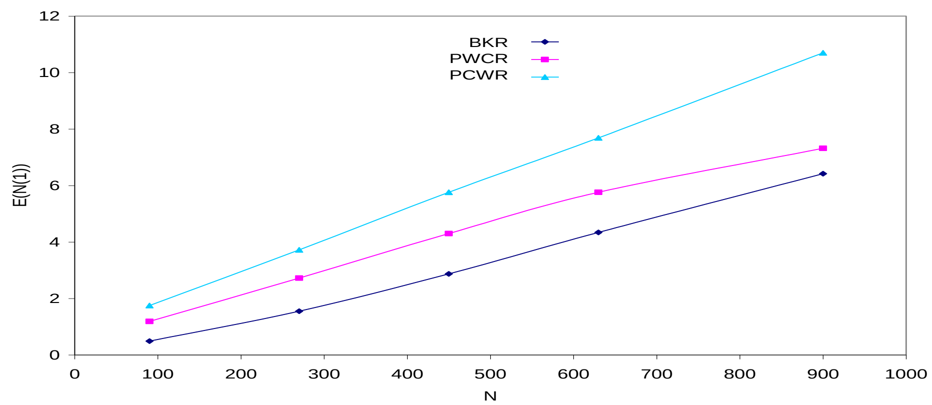
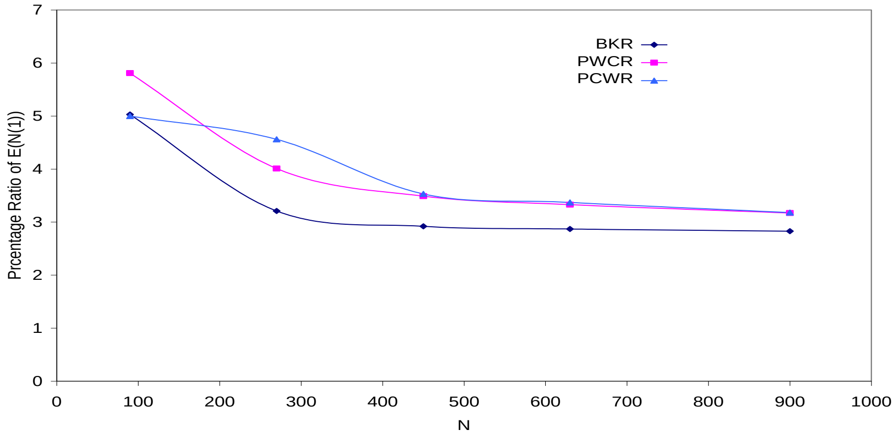


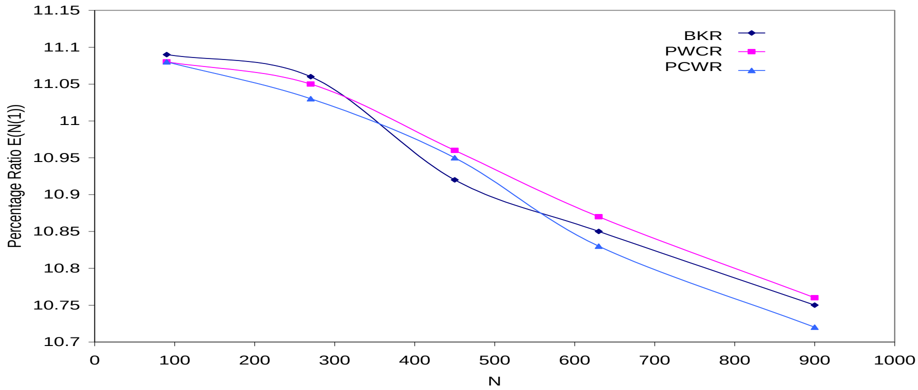
(c)

Fig .4. 22

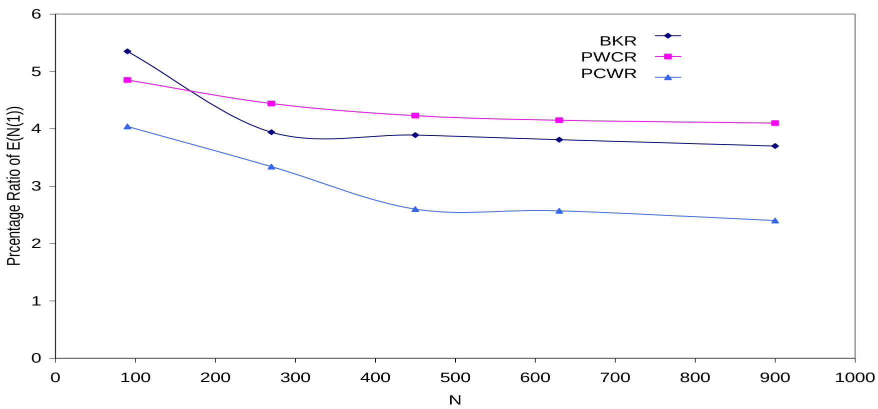
E(N(1)) for the schemes that are combination of the sampling rules BKR,PWCR,PCWR and GS(VT) ,the stopping rule BKS2 and terminal decision rule BK, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9).



(a)

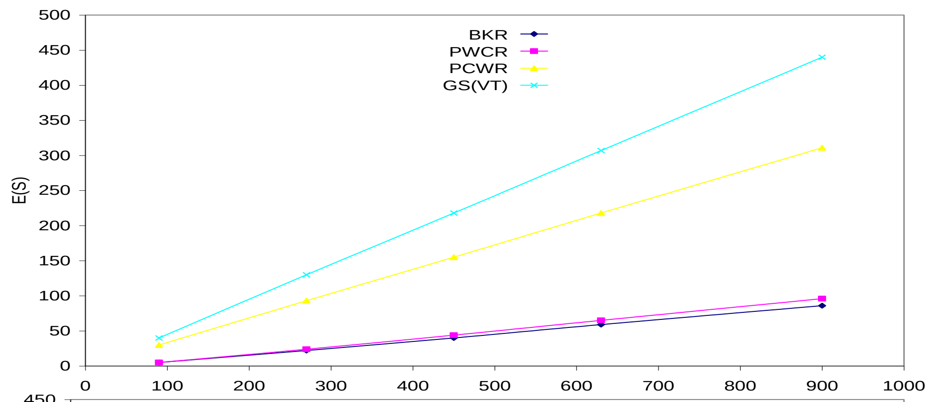


(b)

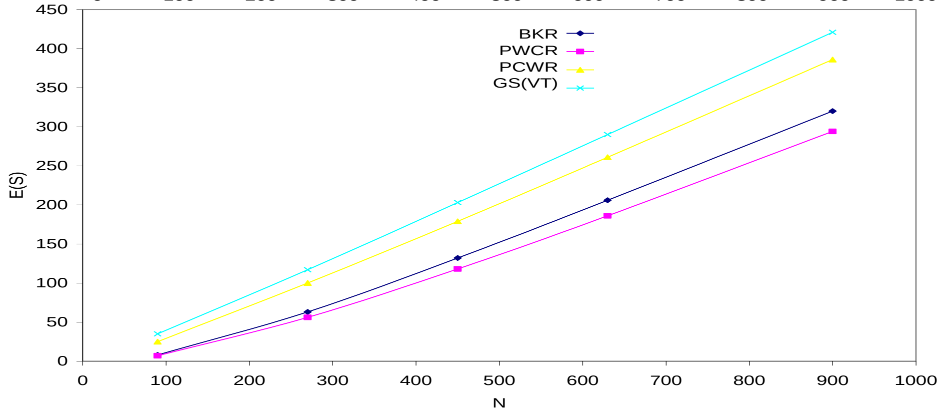


(c)

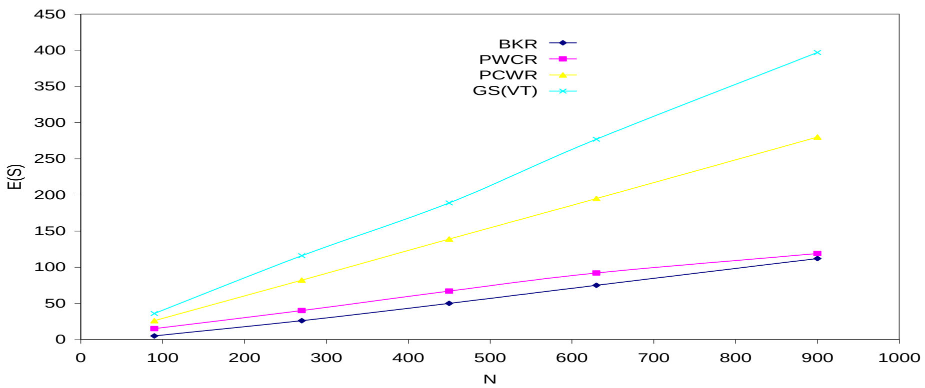
Fig.4.23 Percentage ratio of E(N(1)) for the schemes that are combination of the sampling rules BKR,PWCR,PCWR and GS(VT),the stopping rule BKS2 and terminal decision rule BKT, where N= 90(90)360 900 under uniform priors ,for fixed p_i(i=1,…..,9) when p-values equals for(a) (0.15,0.25,0.35,0.45,0.55,0.65,0.75, 0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)(c) (0.1,0.2, 0.3,0.4,0.5,0.6,0.7,0.8,0.9).



(a)

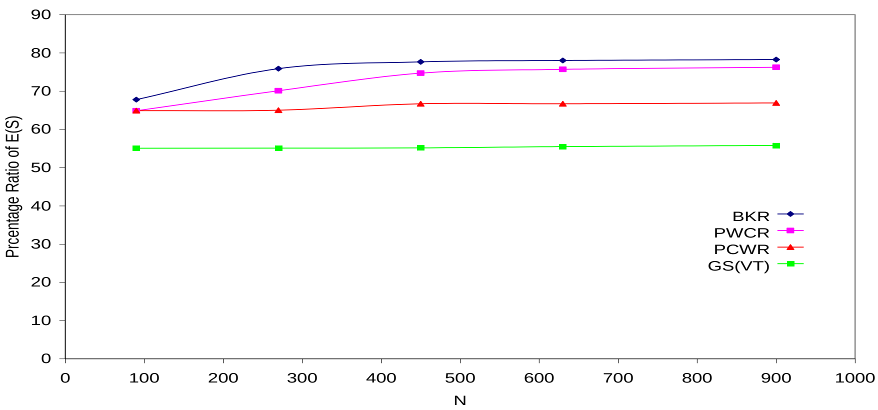


(b)

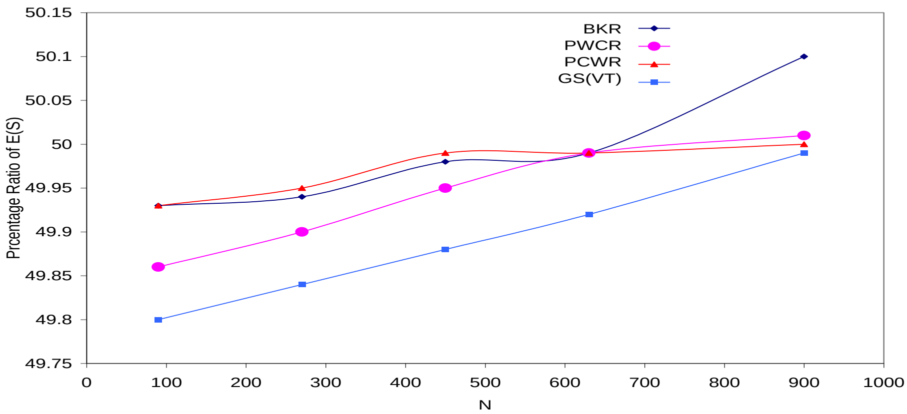


(c)

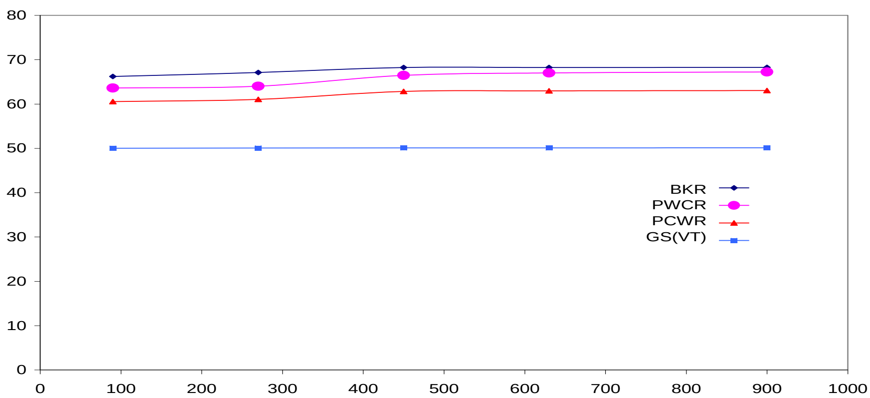
Fig. 4.24 E(S) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT),the stopping rule BKS2 and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for(a)(0.15,0.25,0.35,0.45,0.55,0.65,0.75,0.85,0.95) (b)(0.5,0.5 ,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4,0.5,0.6,0.7, 0.8, 0.9).



(a)



(b)



(c)

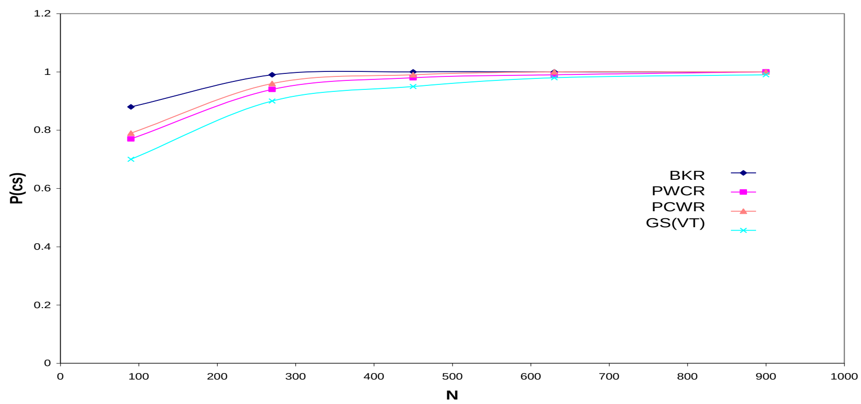
Fig.4.25 Percentage ratio of E(S) for the schemes that are combination of the sampling rulesBKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS2 and terminal decision rule BKT ,where N= 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,….,9) ,when p-values equals for (a)(0.15,0.25,0.35,0.45,0.55,0.65,0.75, 0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1, 0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9).

4.9: The sampling rules BKR, PWCR, PCWR and GS(VT) under the stopping rule BKS_3.

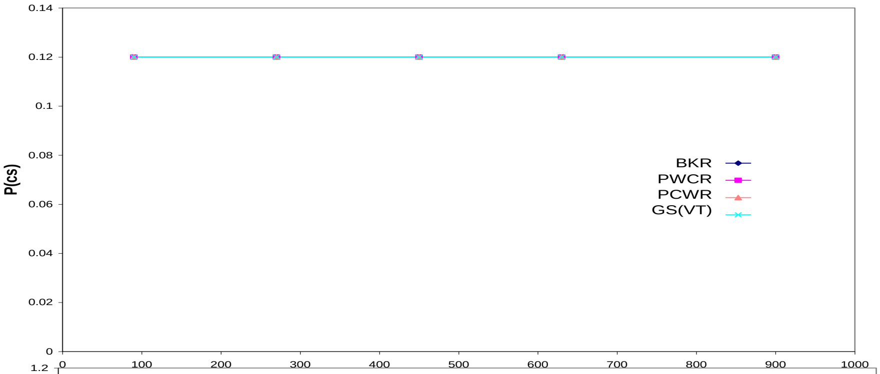
It can be observed from fig.(4.26) that the sampling rule BKR is the best under the performance measure P(CS), also the performance P(CS)≈1/k for all sampling rules when p-values are equals.

E(M) (less values of E(M))is the best under the sampling rule BKR and PWCR alternatively, (see fig. (4.27)).

From fig.(4.28), it is clear that the measure $E(N_{(1)})$ is the best under either sampling rule PCWR or BKR, $E(N_{(1)})$ approximately equals in all sampling rules, and this appears clearly when we take percentage ratio of $(EN_{(1)})$. (see fig.(4.29)). It is note worthy that E(S) is the best under the sampling rule PCWR, but it appears that in sometimes the sampling rules BKR and PCWR being the best when we use the percentage ratio . (see fig.(4.30) and (4.31)).



(a)



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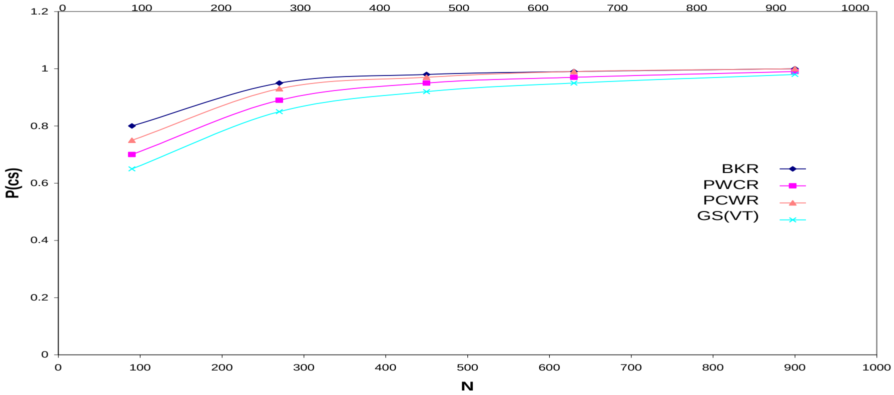
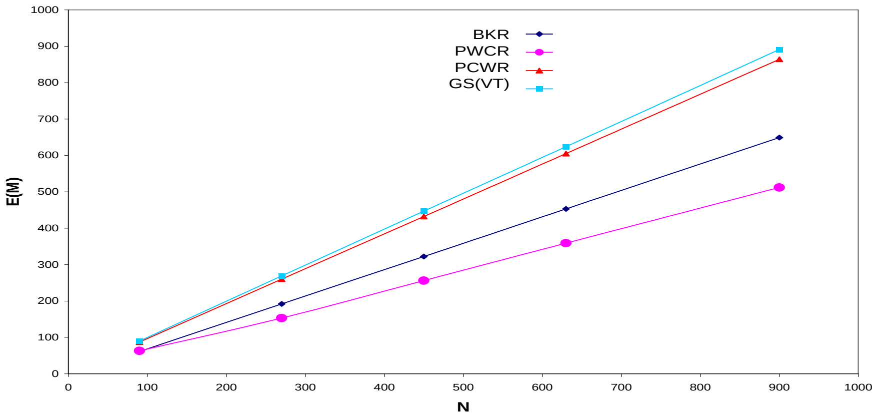
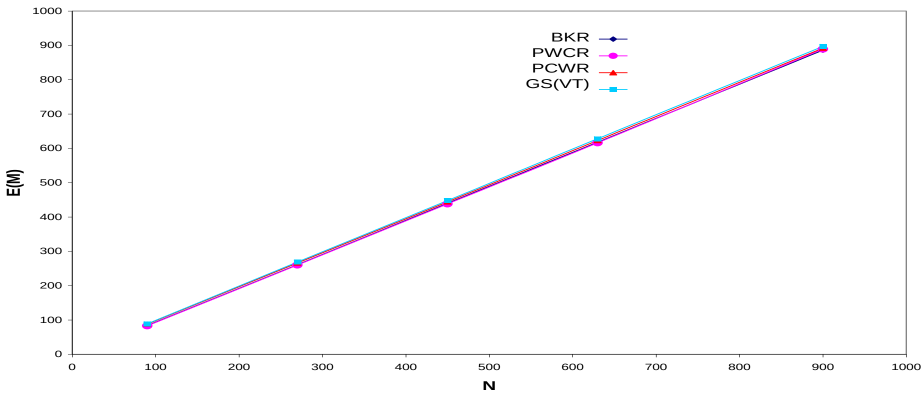


(c)

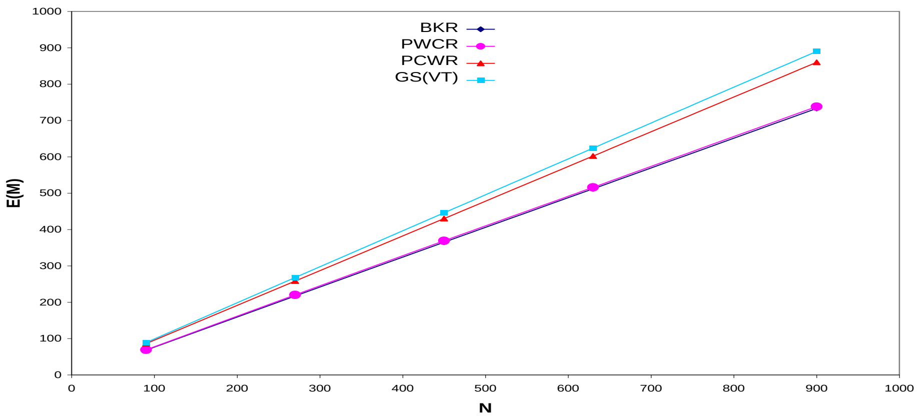
Fig. 4.26
P(CS) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS3 and terminal decision rule BKT,where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4,0.5 , 0.6, 0.7, 0.8, 0.9) .



(a)

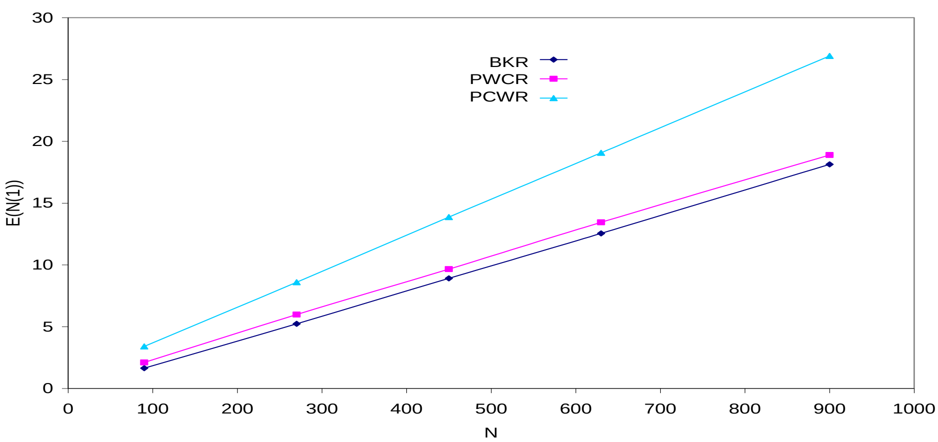


(b)



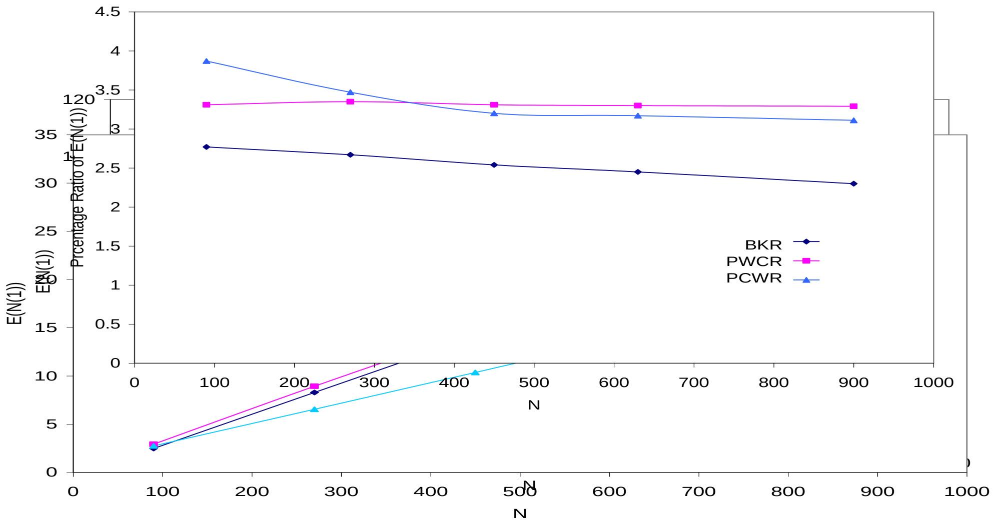
(c)

Fig. 4.27 E(M) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS3 and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85 , 0.95) (b) (0.5,0.5 ,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3 ,0.4,0.5,0.6,0.7, 0.8 ,0.9).

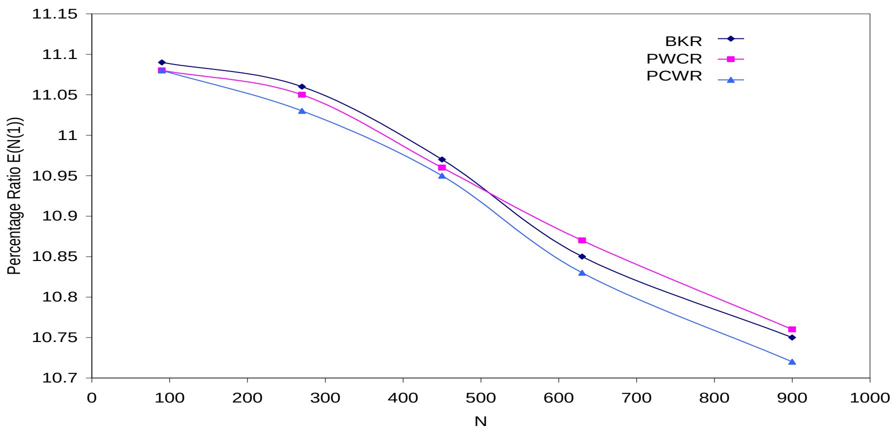


(a)
(b)
(c)

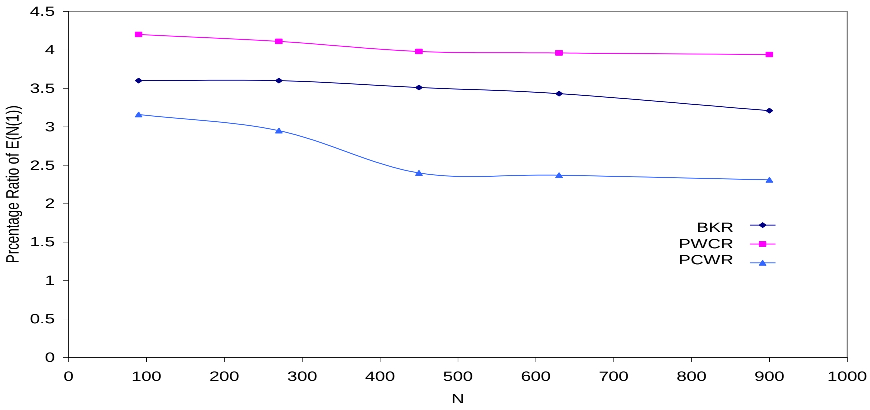
Fig 4.28 E($N_{(1)}$) for the schemes that are combination of the sampling rules BKR,PWCR,PCWR and GS(VT) ,the stopping rule BKS3 and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9).



(a)

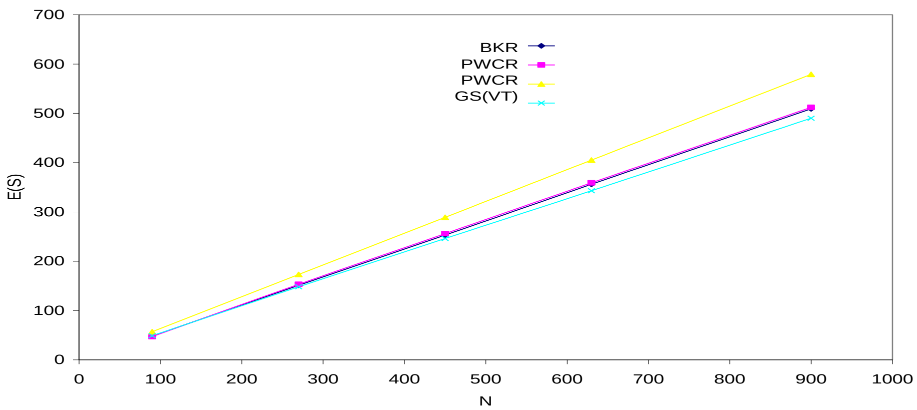


(b)

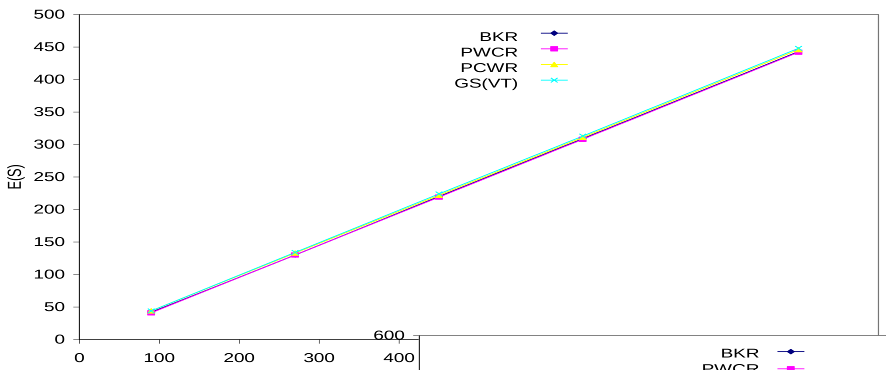


(c)

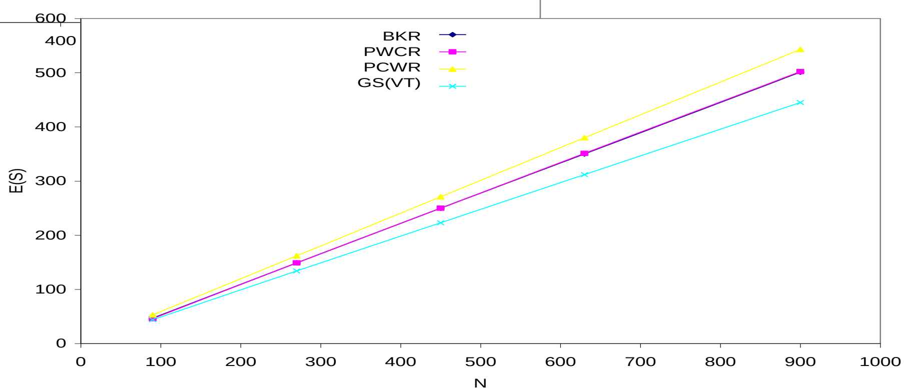
Fig.4.29 Percentage ratio of E(N(1)) for the schemes that are combination of the sampling rules BKR,PWCR,PCWR and GS(VT) ,the stopping rule BKS3 and terminal decision rule BKT, where N=90(90) 360 900 under uniform priors, for fixed p_i(i=1,….,9), when p-values equals for (a) (0.15,0.25,0.35, 0.45,0.55,0.65,0.75 ,0.85,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1, 0.2,03,0.4,0.5,0.6,0.7,0.8,0.9).



(a)

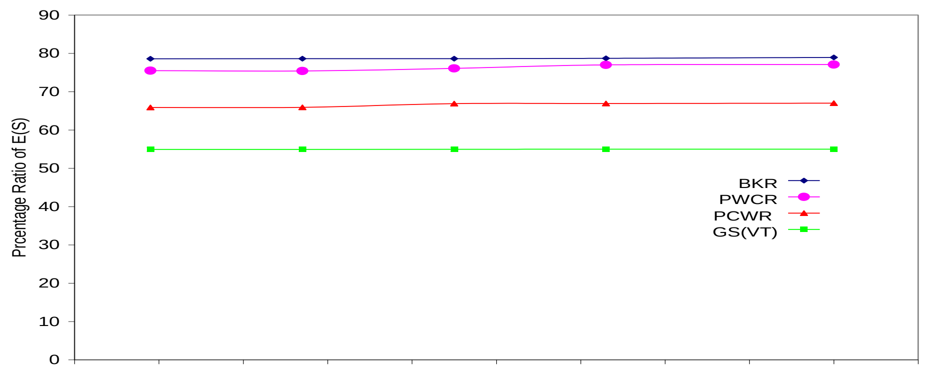


(b)

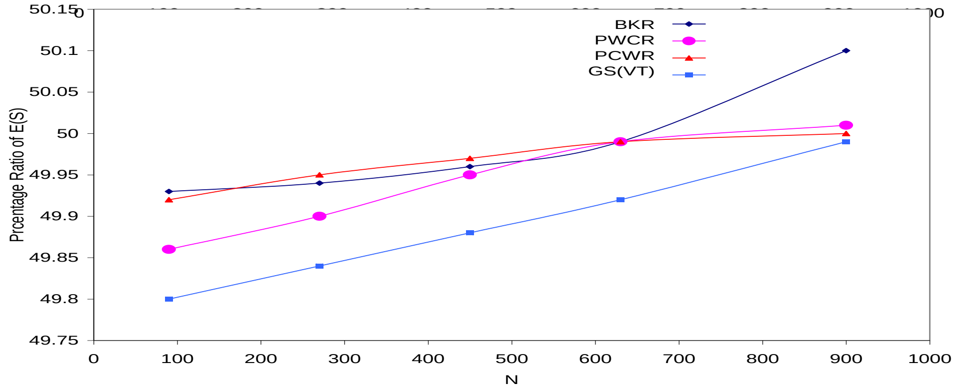


(c)

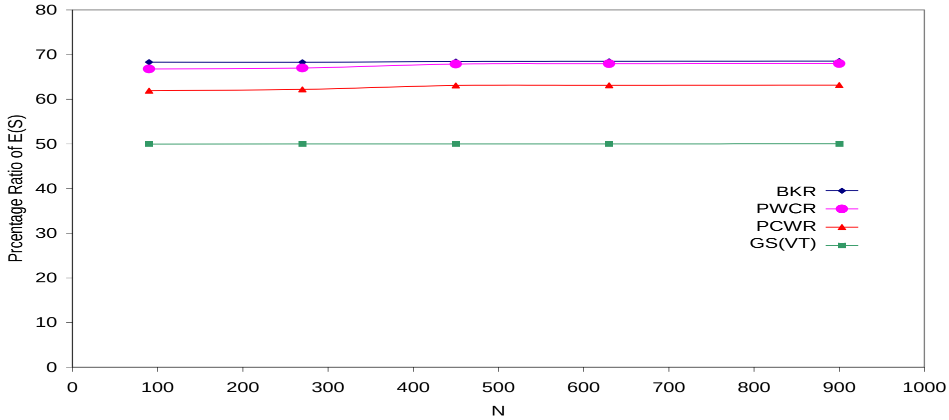
Fig. 4.30 E(S) for the schemes that are combination of the sampling rules BKR ,PWCR,PCWR and GS(VT) ,the stopping rule BKS3 and terminal decision rule BKT, where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,……,9),when p-values equals for (a)(0.15,0.25,0.35, 0.45,0.55,0.65,0.75,0.85,0.95)(b)(0.5,0.5, 0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c) (0.1,0.2,0.3,0.4,0.5,0.6,0.7, 0.8,0.9).



(a)



(b)



(c)

Fig. 4.31 Percentage ratio of E(S) for the schemes that are combination of the sampling rules BKR,PWCR,PCWR and GS(VT),the stopping rule BKS and terminal decision rule BKT,where N = 90(90) 360 900 under uniform priors ,for fixed p_i(i=1,…,9),when p-values equals for (a)(0.15,0.25,0.35 0.45,0.55,0.65,0.75 ,0.85 ,0.95) (b) (0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5) (c)(0.1,0.2,0.3 , 0.4,0.5,0.6,0.7,0.8,0.9).

4.10 Comparison of sampling rule BKR and BKR*

In this section we compare the sampling rule BKR with the sampling rule BKR* using the stopping rules BKS, BKS, BKS_2 and BKS_3 and same terminal decision rule BKT, where the effect of these sampling rules are of interest.

Table (4.80) give the performance characteristics of the designs that consist of the sampling rules BKR and BKR* in conjunction with the stopping rules BKS, BKS_1, BKS_2 and BKS_3 for fixed p_i(i=1,2,3). For all designs P(CS) increases as N increases.

It is clear that the performance measure P(CS) is best under the schemes (BKR, BKS_1) and (BKR, BKS_3), but the schemes (BKR*, BKS_1) and (BKR*, BKS_3) is the worst than others.

As we have shown before that the scheme (BKR, BKS_2) is the best scheme under the measure E(M). We can also confirm that the scheme (BKR*, BKS_2) is the best scheme among those schemes that use the same sampling rule. It can be observed from table (4.80) that the measure E(M)=11.02 for the scheme which uses the sampling rule BKR, while E(M)=12.15 for the scheme that uses the sampling rule BKR* this means that the scheme uses sampling rule BKR is better than the scheme using sampling rule BKR*.

Also the sampling rules BKR has the smallest values of the performance measure $E(N_{(1)})$ with respect to the sampling rule BKR^*. When we take the percentage ratio of $E(N_{(1)})/ E(M)$.

It is clears that the schemes (BKR, BKS_1) and (BKR, BKS_3) are the best schemes, Also the scheme (BKR^*, BKS_1) and (BKR^*, BKS_3) is the best, but the values of ratio using sampling rule BKR^* is smaller than the value using the sampling rule BKR.

We can observe that the measure E(S) under the sampling rule BKR^* is better than that under the sampling rule BKR. However, using the percentage ratio of E(S)/ E(M). We note that the value of ratio under the sampling rule BKR is better under the sampling rule BKR^*.

Table (4.80)

Comparison of the performance characteristics for selection procedure that are combinations of the sampling rules BKR and BKR*, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT, when N=30, 300 and $\underset{\sim}{p} = (0.3, 0.4, 0.5)$.

Sampling rule	Stopping rule	N	P(CS)	E(M)	$E(N_{(1)})$	E(S
BKR	BKS	30	0.60	20.07	5.044	9.34
	BKS_1		0.63	26.96	7.46	11.11
	BKS_2		0.55	11.02	3.11	4.52
	BKS_3		0.60	20.04	5.54	8.25
	BKS	300	0.92	253.32	70.78	147.05
	BKS_1		0.92	281.26	78.91	115.69
	BKS_2		0.92	190.60	53.28	78.28
	BKS_3		0.92	253.41	70.87	104.31
BKR^*	BKS	30	0.61	19.91	5.14	7.66
	BKS_1		0.49	29.70	7.30	11.29

	BKS_2		0.56	12.15	3.27	4.71
	BKS_3		0.50	26.23	6.60	10.00
	BKS	300	0.93	233.72	59.70	89.73
	BKS_1		0.60	229.37	75.91	114.60
	BKS_2		0.93	226.50	57.92	86.81
	BKS_3		0.61	297.21	75.20	113.71

4.11 Comparison of sampling rule PWCR and PLCR

Table (4.81) presents some results about the scheme that are combinations of the sampling rules PWCR and PLCR, with the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT.

Under the performance measure P(CS) the schemes PWCR , BKS_1 and (PWCR, BKS_3) are the best and the schemes (PLCR, BKS) and (PLR, BKS_2) are better than the schemes (PWCR, BKS) and (PWCR, BKS_2).

It is clear that all schemes that use the sampling rule PWCR are better than the schemes that use the sampling rule PLCR. The order of the performance under the stopping rules is the same under both sampling rules.

This table shows that the performance measure $E(N_{(1)})$ small differences between the sampling rule PWCR and PLCR. However, taking the percentage ratio of $E(N_{(1)})/E(M)$, it is clear that all schemes using the sampling rule PLCR the measure $E(N_{(1)})$ is the best among all that schemes use the sampling rules PWCR.

The table shows that as E(M) increases, E(S) is also increases. Hence, under E(S) all schemes that use sampling rule PLCR are better than the scheme using sampling rule PWCR. But PWCR is the best ,by use the percentage ratio of (E(S)/E(M)).

Table (4.81)

Comparison of the performance characteristics for selection procdure that are combinations of the sampling rules PWCR and PLCR, the stopping rules BKS, BKS_1, BKS_2 and BKS_3 and the terminal decision rule BKT, where N=30, 300 and p_1=0.3, p_2=0.4 and p_3=0.5.

Sampling rule	Stopping rule	N	P(CS)	E(M)	$E(N_{(1)})$	E(S)
PWCR	BKS	30	0.59	24.25	6.93	9.94
	BKS_1		0.60	27.04	7.69	11.05
	BKS_2		0.55	12.31	3.59	5.02
	BKS_3		0.59	20.88	5.98	8.55
	BKS	300	0.91	259.36	72.95	106.54
	BKS_1		0.91	281.48	78.93	115.61
	BKS_2		0.91	192.80	54.24	79.12
	BKS_3		0.91	253.79	71.18	104.24
PLCR	BKS	30	0.60	24.24	5.77	9.77
	BKS_1		0.45	28.95	7.97	11.07

	BKS_2		0.55	13.91	3.81	5.39
	BKS_3		0.45	24.47	6.37	5.89
	BKS	300	0.92	238.19	81.17	61.17
	BKS_1		0.49	296.89	75.77	115.63
	BKS_2		0.91	237.90	61.94	91.34
	BKS_3		0.49	289.39	73.97	110.88

4.12 Conclusions

The most important of the concluded points of this thesis are:

- For the schemes that are combination of the sampling rules $\delta_f(A2)$,$\delta_f(A5)$,F_{SS} and δ_G,the stopping rule ST and terminal decision rule DT.

  The scheme (F_{SS},ST,DT)is the best for the performance measure P(CS),and the schemes($\delta_f(A2)$,ST,DT) and ($\delta_f(A5)$,ST,DT) are preferred if E(M) is more interest. We can suggest the use of the scheme ($\delta_f(A5)$,ST,DT) if we are interested in the performance measures $E(N_{(1)})$ and E(S).

- The study conclusions of the effect stopping rules on the sampling rules, we can summers that on all sampling rules the stopping rules BKS1 and BKS3 is the best for the performance measures P(CS),$E(N_{(1)})$ and E(S) ,and the stopping rules BKS2 is preferred for E(M),sometimes BKS is the best for E(M).

Also from the study of the effect sampling rules, we conclude that the sampling rules BKR, PWCR and PCWR are the best for all performance characteristics.

Thus , we can conclude that , the schemes(BKR,BKS1,BKT) and (BKR,BKS3,BKT) are prefer for the measure P(CS),and the schemes (BKR,BKS2,BKT) and(PWCR,BKS2,BKT) are the more suitable for use if we are interested in reducing E(M). The schemes (BKR,BKS1,BKT) ,(BKR,BKS3,BKT) , (PWCR,BKS1,BKT) and (PCWR,BKS3,BKT) should be used if $E(N_{(1)})$ and E(S) are more important .

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