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ذو الحجة ١٤٢٦ هـ   كانون الثاني ٢٠٠٦ م

بسم الله الرحمن الرحيم

ذلكَ مبْلغهُم من العِلمْ إن ربَك هو أعْلم بمن ضلّ عن سبيله وهو أعْلم بمن إهتدى

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Using Some Mathematical Models in Reliability Systems

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by

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Supervised by

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الخلاصة

في هذه الرسالة تم دراسة وإيجاد معولية بعض الأنظمة ،مثل المتوالية ، المتوازية ، المتوالية المتوازية ، المتوازية المتوالية و الأنظمة المعقدة باستخدام عدة طرق منها

Path tracing method, reduction to series element , event space, minimal cat , composite and decomposition method.

لقد تم إيجاد بعض المقارنات بين هذه الطرق لغرض الوصول إلى أفضل معولية كذلك تم دراسة تخصيص المعولية على مركبات بعض الأنظمة وذلك عندما نرغب بتحديد معولية معينة ينبغي الوصول إليها في النظام المعني ، سواء كان ذلك النظام هو متوالي أو نظام معقد .وتضمنت الرسالة دراسة تطبيق نماذج ماركوف على الأنظمة التي لا تخضع للصيانة ، وذلك من خلال تكوين منظومة من المعادلات الخطية وحلها باستخدام تحويل لابلاس ومعكوسه.

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Special thanks to my mentors in courses for their interest and consideration.

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Zahir

Abstract

In this thesis , we study some mathematical models in reliability systems , such as series , parallel , series – parallel , parallel – series and complex systems by using some methods such as ; path tracing method, reduction to series element method , minimal cut method , event space method , composite method and decomposition method . A comparison was done between those methods to get the best reliability.

Reliability allocation had been studied .The aim of it is to establish a goal or objective for the reliability of each component. Two methods had been used to find the reliability allocation for series and complex systems. This study also discuss the reliability for non – maintenance depending on the application of Markov and non – Markov models

List of Contents

Research paper

The following study based on the work done in the thesis have been accepted for publication,

"Reliability methods for solving complex systems", journal of Babylon University, Babylon, to appear, ٢٠٠٥.

Supervisor certificate

I certify that this thesis was prepared under my supervision in the department of mathematics, College of Education, at Babylon University as a partial fulfillment of the requirements for the degree of Master of science in Mathematics.

Signature:
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Date: / / ٢٠٠٦

In view of the available recommendations, I forward this thesis for debate by the examining committee.

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Date: / / ٢٠٠٦

Certification

We certify that we have read this thesis as examining committee examined the student in its contents and that in our opinion it is adequate for the partial fulfillment of the requirements for the degree of Master of Science in Mathematics.

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Approved by the University Committee on Graduate Studies.

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Dean of the College of Education

Date: / /٢٠٠٦

To my Father …

To those who seek the truth …

To those who believe in the endless power of knowledge …

١.١ Introduction

The Reliability is defined as the probability that a component, device, equipment, or system will perform its intended function for specified period of time under a given set of conditions. Committees and laboratories studied different problems, such as maintenance problems, failure of equipments and components. During the decades following of the war II many research laboratories and universities initiated and mathematicians interested in the study of life testing and reliability problems. There are many researchers focused on the study of different reliability systems. Like the series, the parallel, the series – parallel, the parallel – series systems, and the complex systems that enters in different life fields. Since there are different available systems and because of their importance, the researchers attempted to use more than one method to solve these complex systems, and adopt the best method through making a comparison among them. Furthermore there are researchers focused on the study of Markov model to help them in the calculation of different reliability systems. The Markov model assumes that the future is independent of the past given the present. In Markov models the random variable is indexed by time, which can be either discrete or continuous. ***Sandlar*, (١٩٦٣), [٧]** studied the calculation of reliability for the series system and for the parallel system without maintenance. Also the calculation of mean time to failure for these selected items is done by determining the transition matrix P=[p_{ij}] for each model holds the relations p_{ij} ≥ ١, i,j =١,٢,٣,…

and solving the obtained equations by using Laplace transformations and its inverse. ***Bhat,* (١٩٧٢), [٥]** studied stochastic process and classified it into two types depending on parameter space, and state space and these are Markov chain and Markov process **[١٣], [١٦], [٢١].**

***Sandlar,* [٧]** evaluate the non- Markovian processes and discuss the series and parallel systems when there is unconstant failure rate.

***Govil, (١٩٨٣),* [٦]** searching the series and parallel systems, which consist of n components [٨**], [١٥].**

***Govil,* [٦]** studied the series – parallel system and the parallel – series system **[١٩]**.

***Srinath,* (١٩٨٥), [٨]** experienced a K- out of n system, and this system requires at least K elements being active for the system to be active **[١٤] , [١٧].**

***Srinath,* [٨]** proposed three methods to solve the complex systems. These methods are path tracing method, reduction to series element and composite method and he gave some examples to illustrate these methods. ***Al-Ali and Subrie,*(١٩٩٩), [٤]** used a path set method to calculate the reliability of a power plant which is considered as a complete. ***The eMagazine for the Reliability Professional,* (٢٠٠٤), [٢٠]** studied the complex systems and used methods such as event space method, decomposition method and minimal cut method to find the reliability of them . ***Al-Ali and Hassan, (*٢٠٠٥),[٢]** studied a certain type of complex systems by using some of the common (useful) methods, and doing some comparisons among these methods to reach the best reliability for complex systems.

Srinath, **[٨]** considered the problem of reliability allocation of the series system . The reliability allocation means the allocation of the required reliability for individual components to attain the specified system reliability and it is the converse of calculating the system reliability. He used two methods to calculate the reliability allocation. ***Subrie,*** **(١٩٩٦), [٩]** used one single method to calculate the reliability allocation of the complex system. This project includes two methods to solve the complex systems and the series systems. ***Yalaoui, Chu and Chatelet,*** **(٢٠٠٣),[٢٢]** studied the reliability allocation in the series- parallel system.

Suri and Aggarwal, **(١٩٨٠), [١٠]** evaluate the reliability of three different models for computer programs. Such reliability evaluations are quite useful in testing algorithms for these programs. They are derived and discussed depending on the nature of logic flow of the program.

Al-Ali, **(١٩٨٨), [١]** searched two unit repairable system subject to random shocks, the operating units are put under two types of repair due to their failure, failure time of the operating unit is exponential distributed and then the researcher obtain the mean time to system failure and draw its graph against shock rates.

Al-Ali **(١٩٨٨), [٣]** studied two systems each consisting of one unit, the unit subject to random shocks which occurs at random times:-

System ١: without maintenance.

System ٢: with maintenance.

he found the mean time to system failure for these systems.

Danielsson and Olsson, **(١٩٩٨), [١٢]** showed that it is possible to specify the requirement for at least one quality aspect, the reliability, then we have full control over these requirement through the whole development process and even during the operational phase.

Beranek, **(٢٠٠١), [١١]** provided a guidance for assessing the reliability of mechanical and electrical systems of navigation locks and dams.

Abid, **(٢٠٠٢),[٢٥]** introduced a general algorithm to determine the optimum reliability system K out of n.

١.٢. Some definitions and General concepts

The terminology to follow is very important in creating and analyzing reliability block diagrams [٦], [١٧],[٢٣].

Definition (١.١) Probability (P (t))

Probability is a numerical measure of the likelihood of an event relative to a set of alternative events. For example, there is a ٥٠٪ probability of observing heads relative to observing tails when flipping a coin (assuming a fair or unbiased coin).

Probability Properties

The probability of an event A is expressed as P (A), and has the following properties:

١- **٠ ≤ P (A) ≤١,**

٢- **P (A) =١- P (A^c),**

٣- **P (Φ) =٠,**

٤- $\sum P(A) = 1$

Definition (١.٢) Mutually Exclusive Events

Two events A and B are defined as being mutually exclusive if it is impossible for them to occur simultaneously ($A \cap B = \Phi$).

In such cases, the expression for the union of these two events reduces to:

$$P\ (A \cup B) = P\ (A) + P\ (B) \qquad \text{… (١.١)}$$

Since the probability of the intersection of these events is zero.

Definition (1.3) Conditional Probability

Conditional probability of two events, A and B, is defined as the probability of one of these events occurring knowing that the other event has already occurred. The expression below denotes the probability of an occurring A given that B has already occurred.

$$P\ (A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad \text{… (١.٢)}$$

Note that knowing that event B has occurred reduces the sample space.

Definition (1.4) Independent Events

If knowing B gives no information about A, then the events are said to be independent and the conditional probability expression reduces to:

$$P\ (A / B) = P\ (A) \qquad \text{… (١.٣)}$$

From the definition of conditional probability, eq. (1.2) can be written as:

$$P\ (A \cap B) = P\ (A / B)\ P\ (B) \qquad \text{… (١.٤)}$$

Since events A and B are independent, the expression reduces to:

$$P\ (A \cap B) = P\ (A)\ P\ (B) \qquad \text{… (1.5)}$$

Definition (١.٥) probability density function p.d.f.

If X is a continuous random variable, then the *probability density function*, *p.d.f*, of X is a function, $f(x)$, such that for two numbers, a and b with $a \leq b$:

$$P(a \leq X \leq b) = \int_a^b f(x)dx \quad \text{and } f(x) \geq ٠\text{, for all } x \qquad \ldots (١.٦)$$

Definition (١.٦) Cumulative distribution function c.d.f

The cumulative distribution function, *c.d.f*, is a function, $F(x)$, of a random variable X, and is defined for a number x by:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(s)\,ds \qquad \ldots (١.٧)$$

Where s is a dummy integration variable.

The *c.d.f* is used to measure the probability that the item in question will fail before the associated time value, t, and is also called ***unreliability***.

The mathematical relationship between the *P.d.f* and *c.d.f* is given by:

$$F(x) = \int_{-\infty}^{x} f(s)ds$$

Conversely:

$$f(x) = \frac{d(F(x))}{dx} \qquad \ldots (١.٨)$$

The value of the *c.d.f.* at *x* is the area under the probability density function up to *x*, if so chosen. It should also be pointed out that the total area under the *P.d.f* is always equal to ١, or mathematically,

$$\int_{-\infty}^{\infty} f(x)dx = 1 \qquad \dots (١.٩)$$

Definition (١.٧) component

A piece of equipment or portion of a system viewed as an independent entity for evaluation, i.e. its reliability does not influence the reliability of another component.

Definition (١.٨) System

An orderly arrangement of components that interact among themselves and with external components, other systems, and human operators to perform some intended function.

Definition (١. ٩) Reliability function R (t)

It is the probability that the system survives from some specified period of time. This may be expressed in terms of random variable T the time to system failure:

$$R(t) = P\{T > t\} \qquad \dots (١.١٠)$$

≡ Probability that the system operates without failure for length of time.

The probability of survival or reliability R (t) at time t , has the following properties:

i) ٠ ≤ R (t) ≤١

ii) R (٠) =١; the device is assumed to be working properly at time t =٠ , and R(∞); no device can work for ever without failure.

iii) R (t) in general is a decreasing function of time t.

Let F (t) = P{T ≤ t} … (١.١١)

≡Probability that the failure takes places at time less than or equal to t.

Since a system that does not fail for T ≤ t must fail at same T > t, we have

R (t) =١-F (t) … (١.١٢)

Or equivalent

R (t) $=1-\int_0^t f(u)\,du$ … (١.١٣)

Or R (t) $=\int_t^\infty f(u)\,du$ … (١.١٤)

Definition (1.10) Failure Rate

The failure rate is the probability that a failure per unit time occurs in the interval, say [t, t +Δ t], given that a failure has not occurred before t. That is, the failure rate is the rate at which failures occur in [t, t +Δ t],

$$\text{Failure rate} \equiv \frac{P\lfloor t \le T < t + \Delta t | T > t \rfloor}{\Delta t}$$

$$= \frac{P[(t \le T < t + \Delta t) \cap (T > t)]}{\Delta t P[T > t]}$$

$$= \frac{F(t + \Delta t) - F(t)}{\Delta t R(t)} \qquad \dots (١.١٥)$$

Definition (١.١١) Hazard Rate h (t)

The hazard rate is defined as the limit of the failure rate as the interval approaches zero, that is, Δ t → ٠. Thus, we obtain ***the hazard rate*** at time t as

$$h(t) = \lim_{\Delta t \to 0} \frac{F(t + \Delta t) - F(t)}{\Delta t R(t)} = \frac{f(t)}{R(t)} \qquad \dots (١.١٦)$$

we can explain the hazard rate in the following figure.

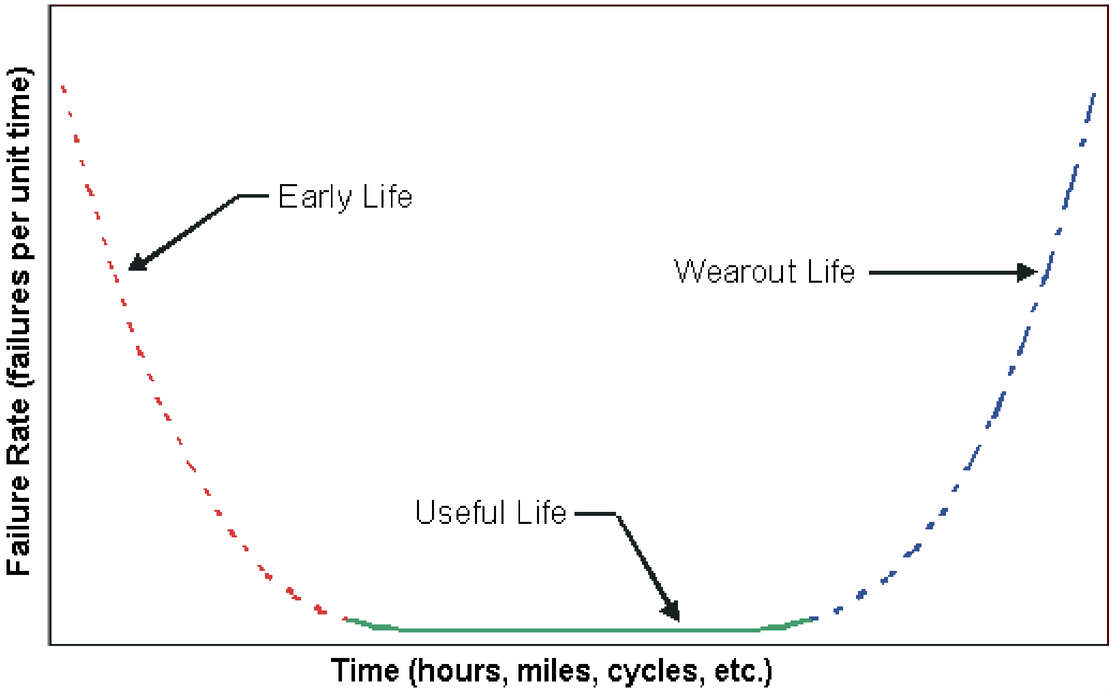


Figure (١.١)

The hazard rate is an instantaneous rate of failure at time t, given that the system survives up to t. In particular, the quantity h (t) dt represents the probability that a system of age t will fail in the small interval t to t + Δ t.

The hazard rate will change over the lifetime of a system. The curve above is typical for many systems and components. It may divide into three distinct regions:

١. **Early life** or **Burn-in** or debugging or infant mortality phase – early failures due to material or manufacturing defect or improper design.

٢. **Useful life** or **normal operating** or chance failures phase - failures caused by sudden stress of extreme conditions.

٣. **Wear-out life** – **fatigue failures** caused due to wearing out of components. These failures occur if the system is not maintained properly or not maintained at all and frequency of such failures increases rapidly with time.

Definition (١.١٢) Mean Time to Failure (MTTF)

In addition to the functions R (t) and h (t), a third useful reliability measure for non-repairable components is the mean time to failure (MTTF), or expected life [٢٣].

$$MTTF = E(t) = \int_0^\infty t f(t)\,dt \qquad \ldots (١.١٧)$$

Or in terms of the reliability function,

$$MTTF = \int_0^\infty R(t)dt \qquad \ldots (١.١٨)$$

Example (١): -

Consider a component with a constant hazard rate h (t) =λ.

R (t)=e$^{-\lambda t}$*, F (t)=* ١*- e*$^{-\lambda t}$*, and f (t)=*λ *e*$^{-\lambda t}$

$$MTTF = \int_0^\infty \exp(-\lambda t)dt = 1/\lambda$$

In most applications, the reliability of a non-repairable component can be described by a single parameter, such as the reliability R or the mean time to failure MTTF.

Definition (1.13) Mean Time between Failures

The Mean Time between Failures ***(MTBF)*** is a concept, which is frequently used in reliability work. It is defined to be the 'average' or 'expected' lifetime of an item. Alternative notation is μ or E (T). Then, from the definition of a mean or expected value,

$$MTBF = \int_0^\infty tf(t)dt \qquad \ldots (١.١٩)$$

Alternatively

$$MTBF = \int_0^\infty [1 - F(t)]dt \qquad \ldots (١.٢٠)$$

We most commonly express the mean time between failures in terms of the reliability function, namely

$$MTBF = \int_0^\infty R(t)\, dt$$

١.٢.١ Relations between f (t), F (t), R (t) and h (t)

The functions f (t), F (t), R (t) and h (t) can be transformed with one another [٢٣].

For example:

$$h(t) = \frac{dF(t)}{dt}\frac{1}{R(t)} \qquad \text{Since} \quad f(t) = \frac{dF(t)}{dt}$$

From (1.16) we know that $\frac{dR(t)}{R(t)} = -h(t)dt$, which yields

$$dF(t) = -\frac{dR(t)}{dt} \qquad \dots (١.٢١)$$

Then by integration,

$$\ln R(t) = -\int_0^t h(x)dx + c, \quad R(0) = 1(c = 0) \qquad \dots (١.٢٢)$$

And taking the exponential of both sides of the equation, one obtains the fundamental equation relating reliability a failure rate:

$$R(t) = \exp\left[-\int_0^t h(x)dx\right] \qquad \dots (١.٢٣)$$

Finally, by differentiation, we can write f (t) in terms of h (t),

$$f(t) = h(t)\exp\left[-\int_0^t h(x)dx\right] \qquad \dots (١.٢٤)$$

١. ٢. ٢ Life time following an exponential distribution

The exponential distribution is a useful model of the time (or length) between failures in situations where the failures are happening at random and at a known rate, λ. The p.d.f. is then given by

$f(t) = \lambda e^{-\lambda t}$ for $t>٠$, We can find associated functions of exponential distribution**.**

(i) Then the *c.d.f* is

$$F(t)=P(T\leq t)$$
$$=\int_0^t f(t)dt$$
$$=\int_0^t \lambda \exp[-\lambda t]dt$$

i.e. $F(t) = ١ - e^{-\lambda t}$ …(١.٢٥)

(ii)Reliability Function

Then since

$R(t) = ١ - F(t)$

$= ١-[١ - e^{-\lambda t}]$

i.e. $R(t) = e^{-\lambda t}$ … (١.٢٦)

(iii) Also, the *hazard function* is given by

$$h(t) = f(t)/R(t)$$

So in this case

$$h(t) = \lambda e^{-\lambda t} / e^{-\lambda t} \quad \text{…(١.٢٧)}$$

$h(t) = \lambda$ (i.e. a constant)

(iv) The *mean time between failures* (MTBF) is

$$MTBF = \int_0^\infty R(t)dt$$
$$= \int_0^\infty \exp[-\lambda t]\,dt$$
$$= \frac{1}{\lambda} \quad \text{… (١.٢٨)}$$

Example (٢): ***-(Assuming constant failure rates)***

(i) A system has a failure rate of ٢×١٠^{-٣} failures/hours. What is the mean time between failures?

(ii) A component has a failure rate of $٥ .١٠^{-٦}$ hours. What is the probability that the component will still be working after ١٠,٠٠٠ hours of operation?

(iii) Assume that a guided missile has a true MTBF of 2 hours and constant failure rate.

What is the probability of the missile failing to complete a two-hour mission?

Solution *(Using time units of ١ hour)*

(i) λ=٠.٠٠٢ failures/hour

Then the MTBF = $١/\lambda=١/٠.٠٠٢=٥٠٠$ hour

(ii) $\lambda=٥\times ١٠^{-٦}$

We know that $R(t) = e^{-\lambda t}$

Then $R(١٠٠٠٠) = e^{-(٥x١٠^{-٦})(١٠٠٠٠)}$

$= e^{-٠.٠٥} = ٠.٩٥١$

(iii) MTBF = ٢ hours $\rightarrow \lambda$ = ٠.٥ per hour

$P(X<٢) = F(٢)$ where $F(t) = ١ - e^{-\lambda t}$

Then $F(٢) = ١ - e^{-(١/٢)\times ٢} = ٠.٦٣٢$.

٢.١ Introduction

In chapter one we discuss some reliability concepts like hazard function, mean time to failure, mean time between failure …etc. and we reviewed most of the work which has done on this subject. In this chapter we study the systems reliability. This need a complete knowledge of the physical structure of the system which may vary from small or moderate to large that involves many components, it also necessitates sufficient acquaintance with the nature of the system in the event of failure of subsystem.

This chapter contains four sections. Section two deals with series system, parallel, series–parallel, parallel–series, Combination of series and parallel and K– out of - n system. Section three complex systems are studied with some methods for calculating the reliability of complex systems and we have done comparison with them.

The last section is concerned with reliability allocation for series and complex systems.

٢. ٢. SYSTEMS RELIABILITY

Suppose that we have to calculate the Reliability of a system made up of several components. The total reliability can be calculated by calculating the reliability of each individual component, and combining these individual reliabilities [٦],[٨], [١٩]. The ways by which the system elements can be combined are the following:-

١– In series.

٢– In parallel.

٣– In series – parallel.

٤– In parallel – series.

٥– Combination of series and parallel.

٦- K-out of-n Parallel Configuration.

٢. ٢. ١. Series Systems

Consider a system of n components connected in series so that the system will only work (i.e. a signal will pass from I to O) if all of the components work.

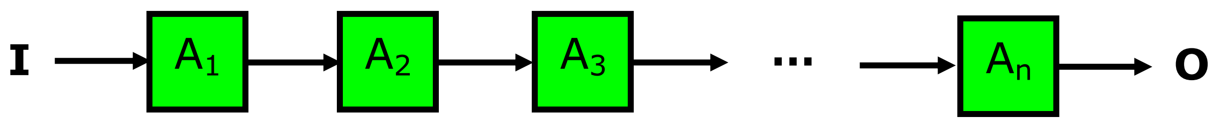


Figure (٢.١)

If the components fail independent of each other it is easy to show that if $R_{١}$, $R_{٢}$, …, R_{n} are the reliabilities of the individual components, then the reliability of the system is given by:

$$R_{SYSTEM} = R_{١}\times R_{٢}\times \dots \times R_{n} \qquad \dots (٢.١)$$

Example (١): -

Consider a system of ٣ components connected in series, each component having a constant failure rate. (In other words, the components have exponential lifetimes).

These rates for components A, B and C are ٠.٢, ٠.٤ and ٠.٥ per ١٠,٠٠٠ hours respectively. Thus we have for constant failure rate λ, the reliability

$$R(t) = e^{-\lambda t}.$$

Thus for component A, $R_A = e^{-٠.٢t}$

Similarly, $R_B = e^{-٠.٤t}$ and $R_C = e^{-٠.٥t}$.

Hence the reliability of the system is:

$$R(t) = e^{-٠.٢t} \times e^{-٠.٤t} \times e^{-٠.٥t}$$

$$= e^{-١.١t}.$$

Then, for example, the probability that the system is still working after ٢٠,٠٠٠ hours = R (٢) = $e^{-٢.٢}$ = ٠.١١٠٨.

٢. ٢. ٢. Parallel Systems

If n components are connected in parallel so that the system works (signal from I to O) as long as at least one of the components works.

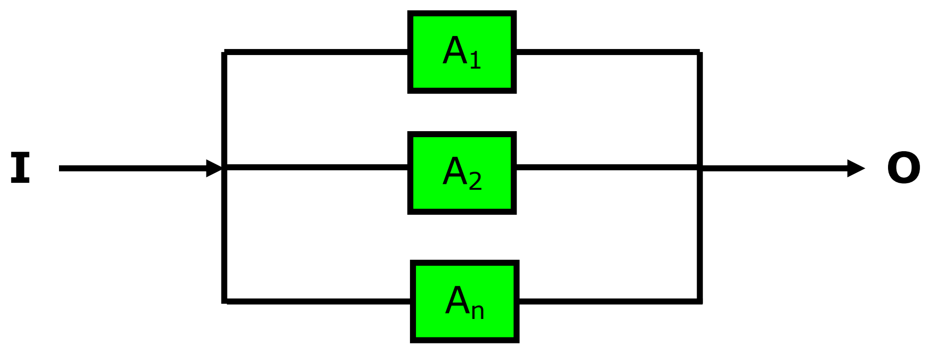


Figure (٢.٢)

The reliability of the system (again assuming independent failures) is then:

R_{SYSTEM} = ١- P (all fail)

=١ – [P ($A_{١}$ fails) ×P ($A_{٢}$ fails) × … × P (A_n fails)]

= ١ - (١ – $R_{١}$) (١ – $R_{٢}$) … (١ – R_n)

$$R_{SYSTEM} = 1 - \prod_{i=1}^{n} (1 - R_i) \qquad \ldots (٢.٢)$$

Example (٢): -

Component A with a constant failure rate of ١.٥ per ١٠٠٠ hrs and component B with a constant failure rate of ٢ per ١٠٠٠ hrs are connected in parallel. Find the overall reliability of this system. (Note that components A and B have exponential lifetimes).

Solution:

For the individual components, assuming a constant failure rate

$R_A(t) = e^{-١.٥t}$

$R_B(t) = e^{-٢t}$

Then if these components are connected in parallel,

$R_{system}(t)$ = ١ – [(١ – R_A (t)) × (١ – R_B (t))]

= ١ – [(١ – $e^{-١.٥t}$) (١ – $e^{-٢t}$)]

= $e^{-١.٥t} + e^{-٢t} - e^{-٣.٥t}$

Then, for example, the probability that the system is still working after ١٠٠٠ hours is

R (١) = $e^{-١.٥(١)} + e^{-٢(١)} - e^{-٣.٥(١)}$

= ٠.٣٢٨٣

i.e. there is a ٣٣٪ chance that such a system will still be working after ١٠٠٠ hours.

٢. ٢. ٣. Series - Parallel systems

The reliability of a system comprising n component connected in parallel redundancy, is

$$R_P = ١ – (١ – R)^n \qquad \ldots (٢.٣)$$

Where R is the reliability of an individual component.

If we place n sets in parallel, where each set has m components connected in series, we get the reliability of such a system as

$$R_p = ١- (١- R)^n$$

Where $R_s = \prod_{i=1}^{m} r_i$, and r_i is the reliability of the i th component in series. Thus,

$$R_S = ١- (١- \prod_{i=1}^{m} r_i)^n \qquad \ldots (٢.٤)$$

The series – parallel configuration is shown in fig (٢.٣).

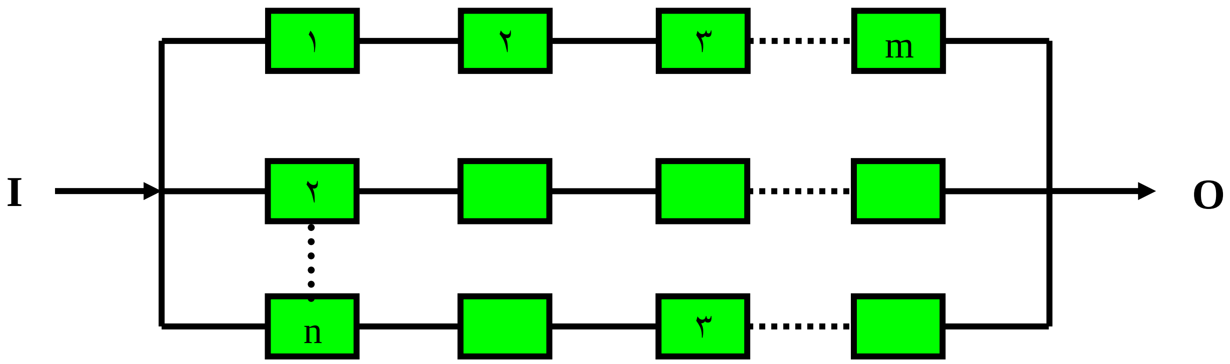


Fig (٢.٣) series – parallel system.

Example (٣): -

Compute the reliability of the system for the connection given in fig (٢.٤).

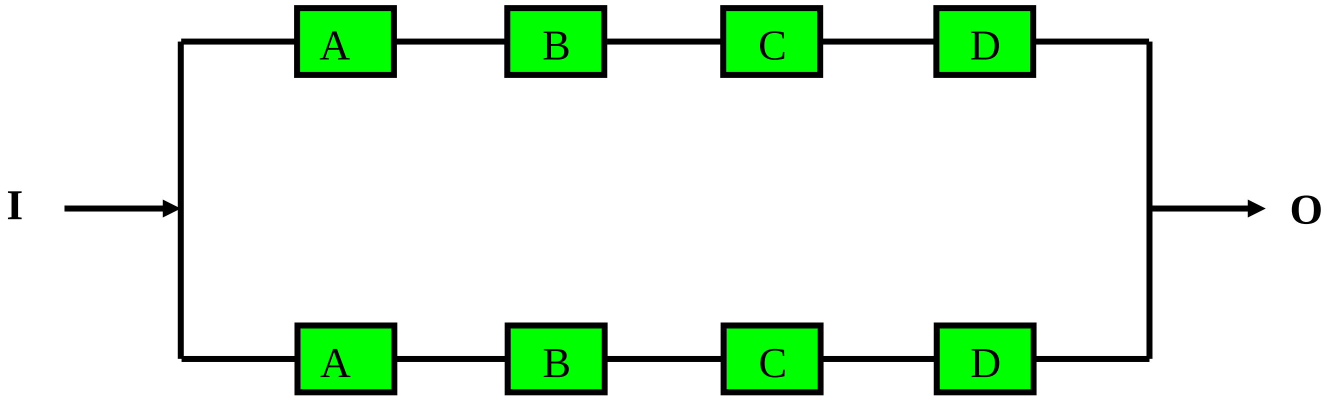


Figure (٢.٤) series – parallel system when m = ٤ and n = ٢.

The reliability of A, B, C and D are ٠.٩٥, ٠.٩٩ ٠.٩٠ and ٠.٩٦ respectively.

Making use of equation (٢.٧), we get the system reliability for m = ٤ and n = ٢.

$$R_S = ١ - (١ - \prod_{i=1}^{4} r_i)^{٢}$$

$$= ١ - [١ - (٠.٩٥ \times ٠.٩٩ \times ٠.٩٠ \times ٠.٩٦)]^{٢}$$

$$= ١ - (١ - ٠.٨١٣)^{٢}$$

$$= ٠.٩٦٥٠ \text{ or } ٩٦.٥٠\%$$

٢. ٢. ٤. Parallel – Series systems

The reliability of n components connected in parallel is given by R_P

$$R_P = ١ - (١ - R)^n \qquad \ldots (٢.٥)$$

Where R is the reliability of an individual component.

If m such sets are connected in series, where each set consists of n component in parallel, then the reliability of the system is given by

$$R_S = [\,١ - (\,١ - R)^n]^m \qquad \ldots (٢.٦)$$

The parallel – series configuration is depicted in fig (٢.٥).

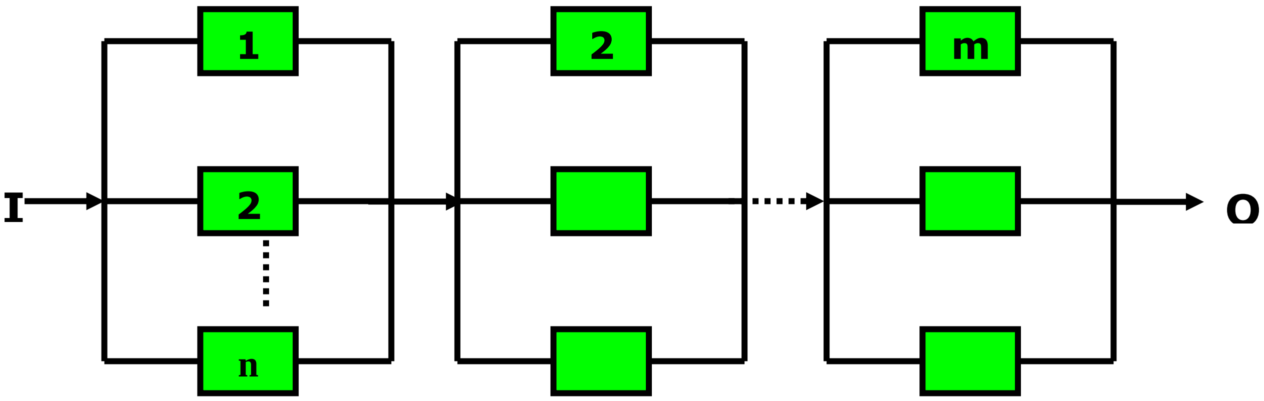


Fig (٢.٥) parallel – series system

Example (٤): -

Compute the reliability of the system for the connections given in fig (٢.٦):

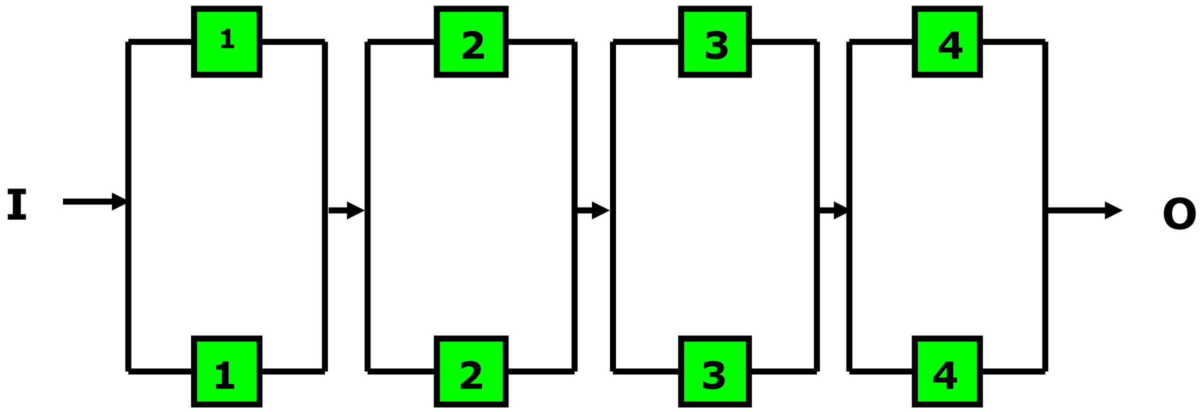


Fig (٢.٦) parallel – series system

Assuming the reliability of each component is ٠.٩٥

Here, we are given n=٢, m= ٤, using eq. (٢.٦), we get

$$R_S = [١ - (١ - R)^{n}]^{m}$$

$$= [١ - (١ - ٠.٩٥)^{٢}]^{٤}$$

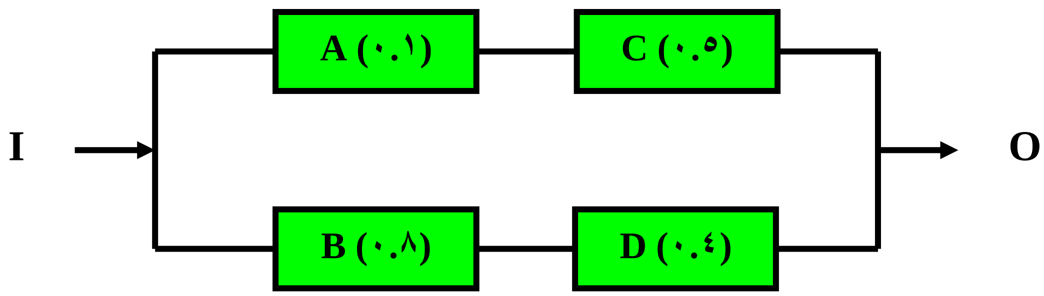
$$= (٠.٩٩٧٥)^{٤} = ٠.٩٩٠٠ = ٩٩\%$$

٢. ٢. ٥. *Mixed Systems*

To evaluate reliability of a system comprising both parallel and series systems, divide the system into ‘series only’ and ‘parallel only’ subsystems. Find the reliability of each subsystem as above then combine in a suitable manner?

Example (٥): -

Find the reliability of the system shown where the number in brackets indicates the constant failure rates per year for each of the components.



Solution:

Consider the two branches (AC) and (BD) first. These are both series ‘subsystems’.

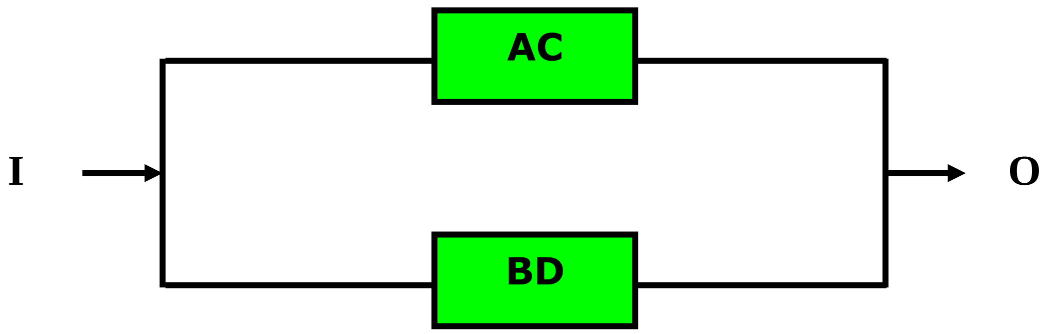
Then Reliability of top branch =

$$R_{top}(t) = R_A(t) \times R_C(t) = e^{-٠.١t} \times e^{-٠.٥t} = e^{-٠.٦t}$$

Reliability of bottom branch =

$$R_{bottom}(t) = R_B(t) \times R_D(t) = e^{-٠.٨t} \times e^{-٠.٤t} = e^{-١.٢t}$$

We can then consider these two subsystems (AC) and (BD) as part of a larger system, namely



And for this parallel system

$$R_{system}(t) = ١ - [(١ - R_{AC}(t)) \times (١ - R_{BD}(t))]$$

$$= ١ - [(١ - e^{-٠.٦t}) \times (١ - e^{-١.٢t})]$$

$$= e^{-٠.٦t} + e^{-١.٢t} - e^{-١.٨t}$$

Then, if for example we wish to find the probability that the system will still be operational after ٦ months, R (٠.٥), we have

$$R(٠.٥) = e^{-٠.٦(٠.٥)} + e^{-١.٢(٠.٥)} - e^{-١.٨(٠.٥)}$$

$$= ٠.٨٨٣$$

(i.e. there is an ٨٨٪ chance that the mixed system under consideration will still is operational after ٦ months).

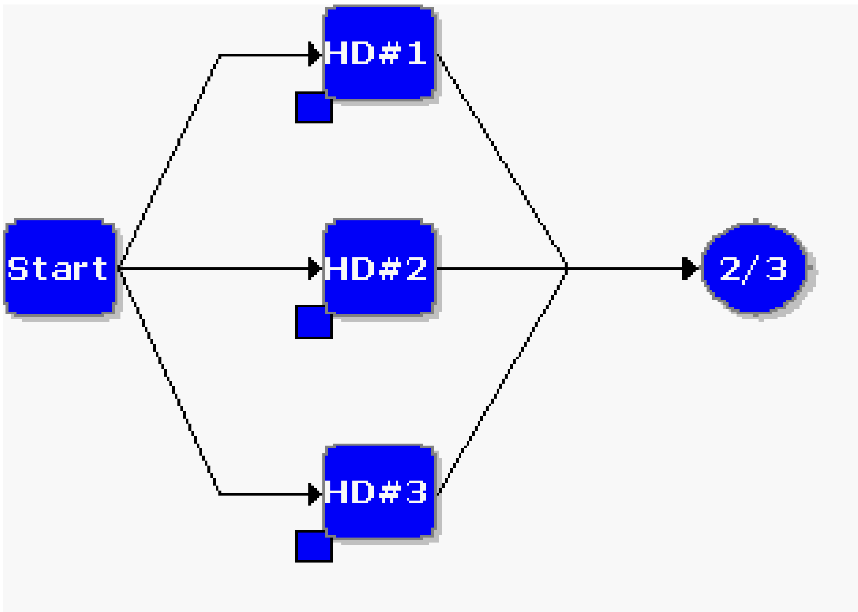
٢. ٢. ٦. K- out of - n Parallel Configuration

The k-out of-n configuration is a special case of parallel redundancy. This type of configuration requires that at least k components succeed out of the total n parallel components for the system to succeed. For example, consider an airplane that has four engines. Furthermore, suppose that the design of the airplane is such that at least two engines are required to function for the aircraft to remain airborne. This means that the engines are reliability-wise in a k-out of n configuration, where k = ٢ and n = ٤. More specifically, they are in a ٢-out of -٤ configurations.

Even though we classified the k-out of-n configuration as a special case of parallel redundancy, it can also be viewed as a general configuration type. As the number of units required keeping the system functioning approaches the total number of units in the system, the system's behavior tends towards that of a series system. If the number of units required is equal to the number of units in the system, it is a series system. In other words, a series system of statistically independent components is an n-out of-n system and a parallel system of statistically independent components is a ١-out of-n system [١٨].

Example (٦): -

Three hard drives in a computer system are configured reliability-wise in parallel. At least two of them must function in order for the computer to work properly. Each hard drive is of the same size and speed, but they are made by different manufacturers and have different reliabilities. The reliability of HD #١ is ٠.٩, HD #٢ is ٠.٨٨ and HD #٣ is ٠.٨٥, all at the same mission time



Solution:

Since at least two hard drives must be functioning at all times, only one failure is allowed. This is a ٢-out of-٣ configuration.

The following operational combinations are possible for a system success:

١- All ٣ hard drives operate.

٢- HD #١ fails, while HDs #٢ and #٣ continue to operate.

٣- HD #٢ fails, while HDs #١ and #٣ continue to operate.

٤- HD #٣ fails, while HDs #١ and #٢ continue to operate.

The probability of success for the system (reliability) can now be expressed as:

$P_S = R_١R_٢R_٣ + (١ – R_١) R_٢R_٣ + R_١ (١- R_٢) R_٣+ R_١R_٢ (١-R_٣)$

This equation for the reliability of the system can be reduced to:

$$R_S = R_١R_٢ + R_٢R_٣ + R_١ R_٣ - ٢ R_١R_٢ R_٣ \qquad \ldots (٢.٧)$$

Or:

$$R_S = ٩٥.٨٦\%$$

If all three hard drives had the same reliability, R, then the equation for the reliability of the system could be further reduced to:

$$R_S = ٣ R^٢ – ٢ R^٣ \qquad \ldots (٢.٨)$$

٢. ٣. Complex Systems

In many cases, it is not easy to recognize which components are in series and which are in parallel, in a complex system. The network shown in Figure (٢.٨) is a good example of such a complex system, [٢٠].

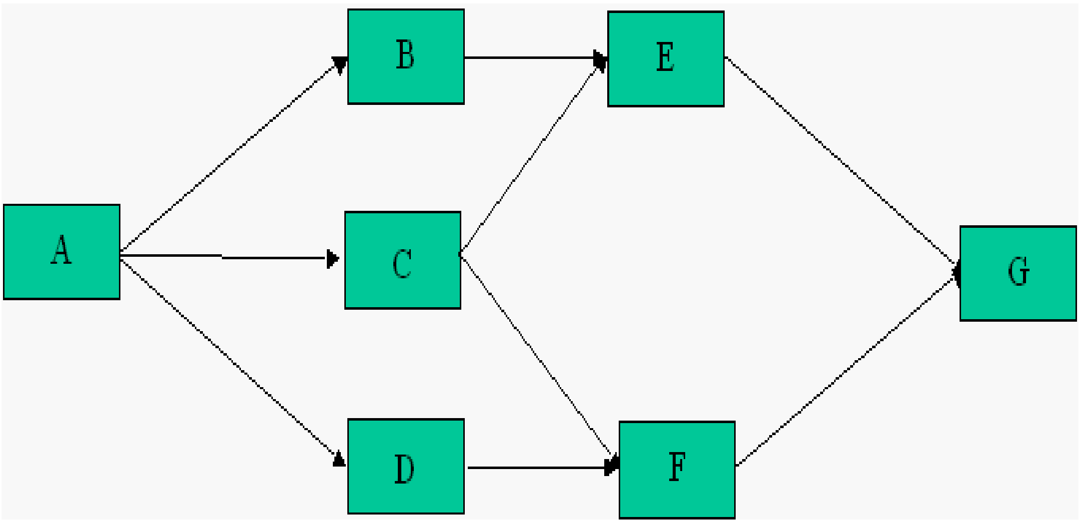


Figure (٢.٨): Example of a complex system.

The system in Figure (٢.٨) cannot be broken down into a group of series and parallel systems. This is primarily due to the fact that component C has two paths leading away from it, whereas B and D have only one. Several methods exist for obtaining the reliability of a complex system including:

١- The path-tracing method.

٢- Reduction to series elements.

٣- Minimal cut method.

٤- The decomposition method.

٥- The event space method.

٦- Composite method.

٢. ٣. ١. Path-Tracing Method

With the path-tracing method, every path from a starting point to an ending point is considered. Since system success involves having at least one path available from one end of the reliability block diagram to the other, as long as at least one path from the beginning to the end of the path is available, then the system has not failed. One could consider the reliability block diagram to be a plumbing schematic. If a component in the system fails, the "water"' can no longer flow through it. As long as there is at least one path for the "water" to flow from the start to the end of the system, the system is successful. This method involves identifying all of the paths the "water" could take and calculating the reliability of the path based on the components that lie along that path.

The reliability of the system is simply the probability of the union of these paths. In order to maintain consistency of the analysis, starting and ending blocks for the system must be defined.

Now we solve figure (٢.٨) by using this method

$R_{s١} = R_A.R_B.R_E.R_G$

$R_{s٢} = R_A.R_C.R_E.R_G$

$R_{s٣} = R_A.R_C.R_F.R_G$

$R_{s٤} = R_A.R_D.R_F.R_G$

$$R_S = 1 - \prod_{i=1}^{4}(1 - R_{si})$$

$= ١ - [(١ - R_{S١})(١ - R_{S٢})(١ - R_{S٣})(١ - R_{S٤})]$ … (٢.٩)

We obtain

$R_S(t) = R_A.R_E.R_D.R_G + R_A.R_B.R_E.R_G + R_A.R_C.R_E.R_G$

$-R_A.R_C.R_B.R_D.R_G - R_A.R_C.R_E.R_D.R_G - R_A.R_C.R_E.R_B.R_G$

$+ R_A.R_B.R_C.R_D.R_E.R_G + R_A.R_F.R_D.R_G + R_A.R_F.R_B.R_G$

$+ R_A.R_F.R_C.R_G - R_A.R_F.R_D.R_B.R_G - R_A.R_F.R_D.R_C.R_G$

$-R_A.R_F.R_C.R_B.R_G + R_A.R_F.R_D.R_C.R_G - R_A.R_E.R_F.R_D.R_G$

$-R_A. R_E R_F. R_B. R_G - R_A. R_E R_F. R_C. R_G$

$+R_A.R_E.R_F.R_B.R_D.R_G + R_A.R_E.R_F.R_C.R_D.R_G$

$+R_A.R_E.R_F.R_B.R_C.R_G$

$$- R_A.R_E.R_F.R_B.R_C.R_D.R_G \qquad \ldots (٢.١٠)$$

Let $R_A=R_B=\ldots=R_G=٠.٩$, with independent units we get

$$R_S = R^{٤} + R^{٤} + R^{٤} - R^{٥} - R^{٥} - R^{٥} + R^{٦} + R^{٤} + R^{٤}$$

$$+ R^{٤} - R^{٥} - R^{٥} - R^{٥} + R^{٥} - R^{٥} - R^{٥} - R^{٥}$$

$$+ R^{٦} + R^{٦} + R^{٦} - R^{٧}$$

$$R_S = ٦\, R^{٤} - ٨\, R^{٥} + ٤\, R^{٦} - R^{٧} \qquad \ldots (٢.١١)$$

And we substitute (٠.٩) in eq. (٢.١١) we get

$$R_S = ٠.٨٦$$

٢. ٣. ٢. Reduction to series elements

In this method we systematically replace each parallel path by an equivalent single path, and ultimately reduce the given system to one consisting of only series element, [٩].

Step ١: The parallel elements E, F will be first replaced by an equivalent series element. ***Say P ١***

$$R_{P١}(t) = ١ - [(١-R_E)\,(١-R_F)]$$

$$= R_E+R_F - R_E.R_F \qquad \ldots (٢.١٢)$$

The system has now been modified to another system is shown in fig. (٢.٩). Blew

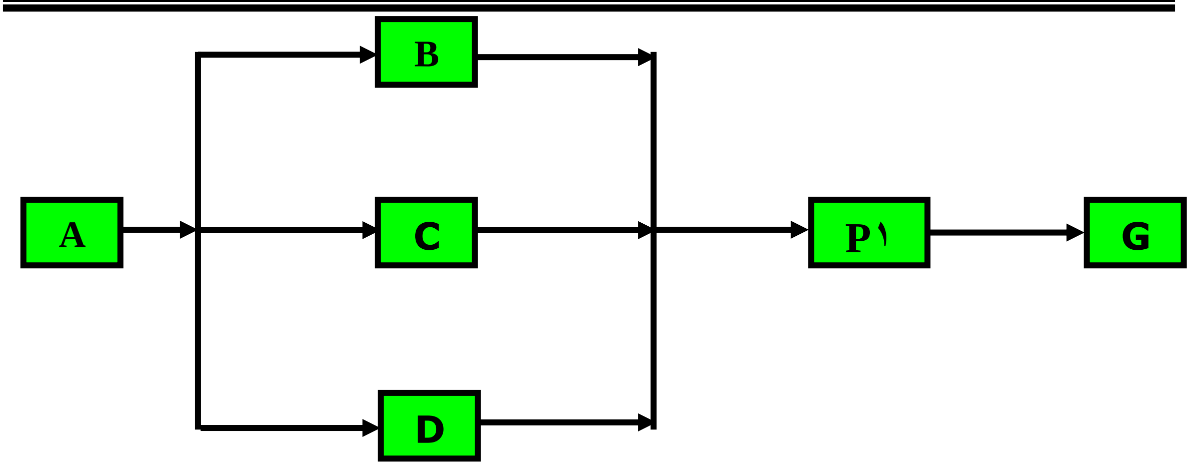


Figure (٢.٩)

Step (٢): parallel elements B, C, D can be replaced by an equivalent element whose reliability is obtained from the rule that. **Say P٢**

$$R_{P٢}(t) = ١ - [(١-R_B)(١-R_C)(١-R_D)]$$

$$= R_D+R_B+R_C - R_B.R_D - R_C.R_D - R_B.R_C + R_B.R_C.R_D \quad \ldots (٢.١٣)$$

The system has now been reduced to a system contains series elements as:

Fig (٢.١٠)

Step ٣:

The system reliability of Fig (٢.١٠) is

$$R_S = R_A.R_B.R_E.R_G + R_A.R_C.R_E.R_G + R_A.R_D.R_F.R_G$$

$$+R_A.R_C.R_F.R_G - R_A.R_C.R_D.R_F.R_G - R_A.R_C.R_E.R_F.R_G$$

$$- R_A.R_B.R_D.R_E.R_F.R_G - R_A.R_B.R_C.R_D.R_E.R_G$$

$$+ R_A.R_B.R_C.R_D.R_E.R_F.R_G \qquad \ldots (٢.١٤)$$

If we take $R_A=R_B=\ldots=R_G=٠.٩$, with independent identical units we get

$$R_S\,(t) = R^{٤} + R^{٤} + R^{٤} + R^{٤} - R^{٥} - R^{٥} - R^{٦} - R^{٦} + R^{٧}$$

$$= ٤\,R^{٤} - ٢\,R^{٥} - ٢\,R^{٦} + R^{٧} \qquad \ldots (٢.١٥)$$

$$= ٠.٨٥$$

٢. ٣. ٣. Minimal cut method

A cut is a set of components such that when the component in the cut is removed from the system, i.e. there is no path from one terminal to other, the minimal cut method is a powerful tool in analyzing systems that are connected in form of a bridge. When we apply this method to fig. (٢.٨), we obtain.

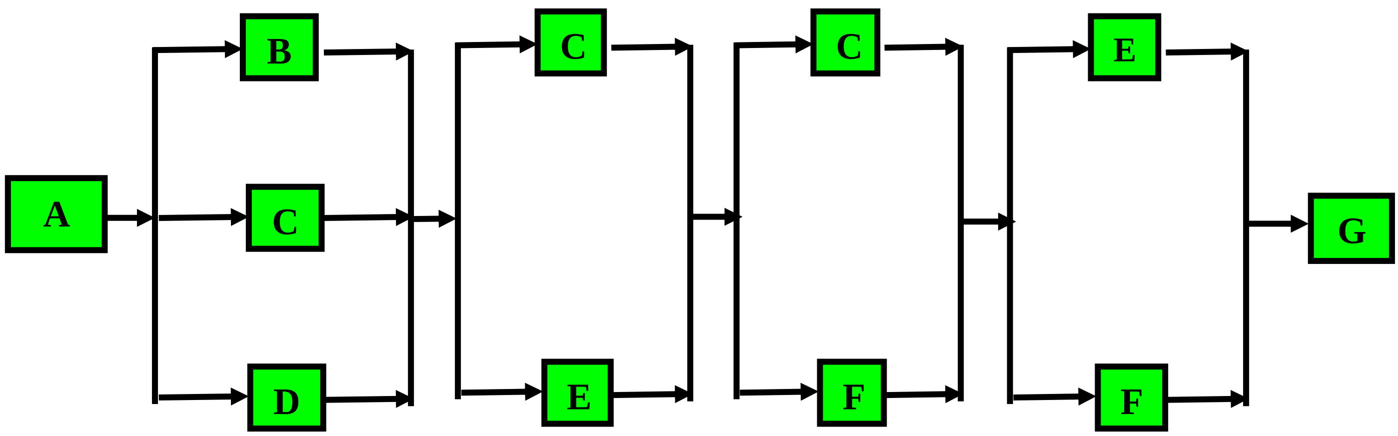


Fig (٢.١١)

$$R_{P١}\,(t) = ١ - [(١-R_B)\,(١-R_C)\,(١-R_D)]$$

$$R_{P٢}\,(t) = ١ - [(١-R_C)\,(١-R_E)]$$

$$R_{P٣}\,(t) = ١ - [(١-R_C)\,(١-R_F)]$$

$R_{P٤}(t) = ١ - [(١-R_E)(١-R_F)]$

$R_S(t) = R_A.R_C.R_E.R_G + R_A.R_C.R_F.R_G - R_A.R_C.R_E.R_F.R_G$

$+ R_A.R_B.R_E.R_F.R_G - R_A.R_B.R_C.R_E.R_F.R_G$

$+ R_A.R_D.R_E.R_F.R_G - R_A.R_D.R_C.R_E.R_F.R_G$

$- R_A.R_B.R_D.R_E.R_F.R_G + R_A.R_B.R_C.R_D.R_E.R_F.R_G$ … (٢.١٦)

If we take $R_A=R_B=\ldots=R_G=$٠.٩, with independent identical units we get

$R_S(t) = ٢R^{٤} + R^{٥} - ٣R^{٦} + R^{٧}$ … (٢.١٧)

= ٠.٧٨

٢. ٣. ٤. Decomposition Method

The decomposition method is an application of the law of total probability. It involves choosing a "key" component and then calculating the reliability of the system twice: once as if the key component failed (R = ٠) and once as if the key component succeeded (R = ١). These two probabilities are then combined to obtain the reliability of the system, since at any given time the key component will be failed or operating. Using probability theory, the equation is:

$$R_S = P(s \cap A) + P(s \cap \overline{A})$$

$$= P(s|A)P(A) + P(s|\overline{A})P(\overline{A}) \quad \ldots (٢.١٨)$$

Example (٧): -

Consider three units in series.

A is the event of Unit ١ success.

B is the event of Unit ٢ success.

C is the event of Unit ٣ success.

S is the event of system success.

First, select a "key" component for the system. Selecting Unit ١, the probability of success of the system is:

$$R_S = P(S|A)P(A) + P(S|\overline{A})P(\overline{A})$$

If Unit ١ is success, then P (S| A) = $R_٢$ $R_٣$

That is, if Unit ١ is operating, the probability of the success of the system is the probability of Units ٢ and ٣ succeeding.

If Unit ١ fails, then:

$$P(s \mid \overline{A}) = 0$$

That is, if Unit ١ is not operating, the system has failed since a series system requires all of the components to be operating for the system to operate. Thus the reliability of the system is:

R_S= $R_٢$ $R_٣$ P (A) = $R_١ R_٢ R_٣$

Example (٨): -

Consider the following system:

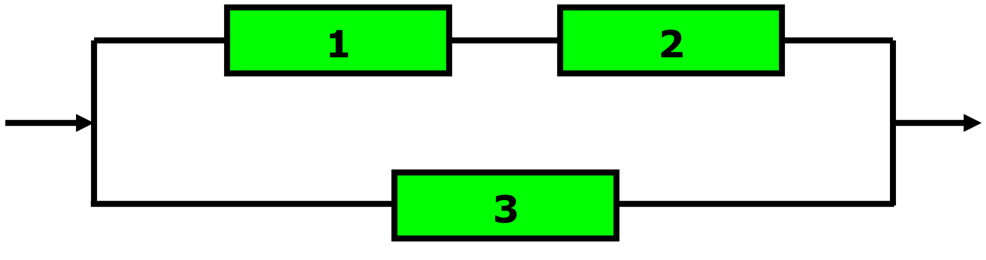


Figure (٢.١٢)

A is the event of Unit ١ success.

B is the event of Unit ٢ success.

C is the event of Unit ٣ success.

S is the event of system success

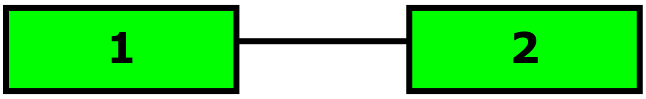
Selecting Unit ٣ as the "key" component, the system reliability is:

$$R_S = P(S/C)P(C) + P(S/\overline{C})P(\overline{C})$$

If Unit ٣ success, then P (S| C) =١

That is, since Unit ٣ represents half of the parallel section of the system, then as long as it is operating, the entire system operates.

If Unit ٣ fails, then the system is reduced to:



$$P(s\,|\,\overline{C}) = R_1R_2$$

The reliability of the system is given by:

$$R_S = P(C) + R_1R_2P(\overline{C}) = R_3 + R_1R_2(1 - R_3)$$

Or:

R_S=R٣ + R١ R٢ - R١ R٢ R٣ … (٢.١٩)

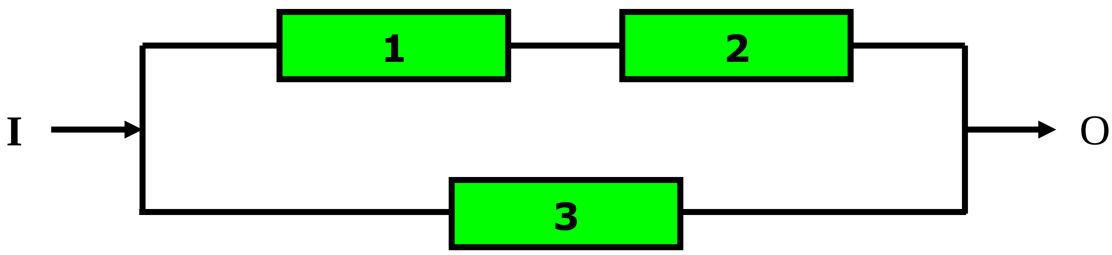
٢. ٣. ٥. *Event Space Method*

The event space method is an application of the mutually exclusive event axiom. All mutually exclusive events are determined and those that result in system success are considered.

The reliability of the system is simply the probability of the union of all mutually exclusive events that yield a system success. Similarly, the unreliability is the probability of the union of all mutually exclusive events that yield a system failure. This is illustrated in the following example.

Example (٩): -

Consider the following system, with reliabilities $R_{١}$, $R_{٢}$ and $R_{٣}$ for a given time:



A is the event of Unit ١ success.

B is the event of Unit ٢ success.

C is the event of Unit ٣ success.

The mutually exclusive system events are:

$X_{١} \equiv ABC$, all units succeed.

$X_{٢} \equiv \bar{A}BC$, unit ١ fail and ٢, ٣ succeed

$X_{٣} \equiv A\bar{B}C$, unit ٢ fail and ١, ٣ succeed

$X_{٤} \equiv AB\bar{C}$, unit ٣ fail and ١, ٢ succeed

$X_{٥} \equiv \overline{AB}C$, unit ١, ٢ fail and ٣ succeed

$X_{٦} \equiv \bar{A}B\bar{C}$, unit ١, ٣ fail and ٢ succeed

$X_{٧} \equiv A\overline{BC}$, unit ٢, ٣ fail and ١ succeed

$X_{٨} \equiv \overline{ABC}$, all units fail.

System events $X_{٦}$, $X_{٧}$ and $X_{٨}$ result in system failure. Thus the probability of failure of the system is:

$$P_f = P(X_6 \bigcup X_7 \bigcup X_8) \qquad \dots (٢.٢٠)$$

Since events $X_{٦}$, $X_{٧}$ and $X_{٨}$ are mutually exclusive, then:

$$P_f = P(X_{٦}) + P(X_{٧}) + P(X_{٨}) \qquad \dots (٢.٢١)$$

And:

$$P(X_6) = P(\overline{A}B\overline{C}) = (1-R_1)(R_2)(1-R_3)$$
$$P(X_7) = P(A\overline{B}\overline{C}) = (R_1)(1-R_2)(1-R_3)$$
$$P(X_8) = P(\overline{A}\overline{B}\overline{C}) = (1-R_1)(1-R_2)(1-R_3)$$

Combining terms yields:

$$P_f(t) = ١ - R_{١} R_{٢} - R_{٣} + R_{١} R_{٢} R_{٣} \qquad \dots (٢.٢٢)$$

Since:

$$R_S(t) = ١ - P_f$$

Then:

$$R_S(t) = R_{١} R_{٢} + R_{٣} - R_{١} R_{٢} R_{٣}$$

This is of course the same result as the one obtained previously using the decomposition method.

If $R_{١}$ = ٩٩.٥٪, $R_{٢}$= ٩٨.٧٪ and $R_{٣}$= ٩٧.٣٪, then:

$R_S(t)$ = (٠.٩٩٥) (٠.٩٨٧) + (٠.٩٧٣) - (٠.٩٩٥) (٠.٩٨٧) (٠.٩٧٣)

= ٠.٩٩٩٥١٥٧٥٥ or R_S =٩٩.٩٥٪

٢. ٣. ٦. *Composite method*

This method is similar to the path tracing method, but it does not list the entire parallel path. The probability factor is derived in a comprehensive manner starting from the input end. The series elements are assigned “and” and the parallel elements “or” in setting up the expressions for the probability factor.

The system in the example (٩) has elements A and B in series connected to the input end. Next comes element in parallel. Hence,

P (S) = P [(A and B) or C]

Assuming that the elements are independent, we have

P (S) = P {(A or C) and (B or C)}

P (S) ={P (A) or P(C)} ×{ P (B) or P(C)}

={P (A) + P(C) - P (A) P(C)}× {P (B) + P(C) - P (B) P(C)}

= P (A) P (B)+ P (C) P (B)- P (A) P (B) P (C)+ P (A) P (C)

+ P (C) - P (A) P (C) - P (A) P (B) P (C) - P (B) P (C)

+ P (A) P (B) P (C)

= P (A) P (B) + P(C) - P (A) P (B) P(C) … (٢.٢٣)

Or

$R_S (t) = R_١ R_٢ + R_٣ - R_١ R_٢ R_٣$

This is of course the same result as in equation (٢.١٩).

If $R_١$ = ٩٩.٥٪, $R_٢$= ٩٨.٧٪ and $R_٣$= ٩٧.٣٪, then:

R_S (t) = (٠.٩٩٥) (٠.٩٨٧) + (٠.٩٧٣) - (٠.٩٩٥) (٠.٩٨٧) (٠.٩٧٣)

$= ٠.٩٩٩٥١٥٧٥٥$

And we can solve previous example by using the first three methods as follows:

١- *by using path tracing method*

Solution:

Assume starting and ending blocks that cannot fail, as shown next.

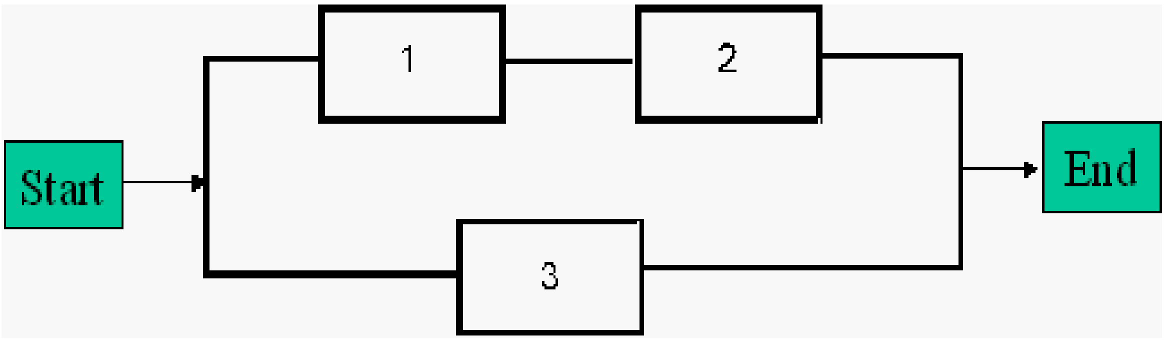


Figure (٢.١٣)

The paths for this system are:

$$X_{١} = ١,\ ٢ \text{ and } X_{٢} = ٣$$

The probability of success of the system is given by:

$$P\,(X_{١} \cup X_{٢}) = P\,(X_{١}) + P\,(X_{٢}) - P\,(X_{١} \cap X_{٢})$$

$$= P\,(١,\ ٢) + P\,(٣) - P\,(١,\ ٢,\ ٣)$$

Or:

$$R_S\,(t) = R_{١}\,R_{٢} + R_{٣} - R_{١}R_{٢}R_{٣}$$

This is of course the same result as in equation (٢.١٩).

٢- ***Reduction to series element***

Solution:

In this method we systematically replace each parallel path by an equivalent single path, and ultimately reduce the given system to one consisting of only series element. We shall assume the elements to be independent

Step ١:

Accordingly, elements A, B in fig (٢.١٢) are first replaced by an equivalent single element, say s١. Using the probability rules, we get

P (s١) =P (A and B) = P (A). P (B) … (٢.٢٤)

Or $R_{s١}=R_A.R_B$

Step ٢

The given system has now been modified to that shown in fig (٢.١٤)

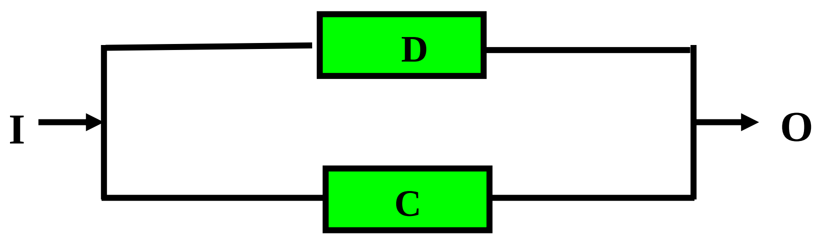


Figure (٢.١٤)

Now $R_{s٢}=R_{C}$, there fore, the reliability of the system is

R_S (t) = $1-\prod_{i=1}^{2}(1-(R_{si}(t))$

$=١-[(١-R_{s١})(١-R_{s٢})]$

$=R_{s١}+R_{s٢}-R_{s١}.R_{s٢}$ … (٢.٢٥)

And we substitute Rs١, Rs٢ by equivalent elements we obtain

R_S (t) = R_A. R_B + R_C – R_A. R_B. R_C … (٢.٢٦)

This is of course the same result as in equation (٢.١٩)

٣-*Minimal cut method*

When we apply this method to fig (٢.١٢) we get

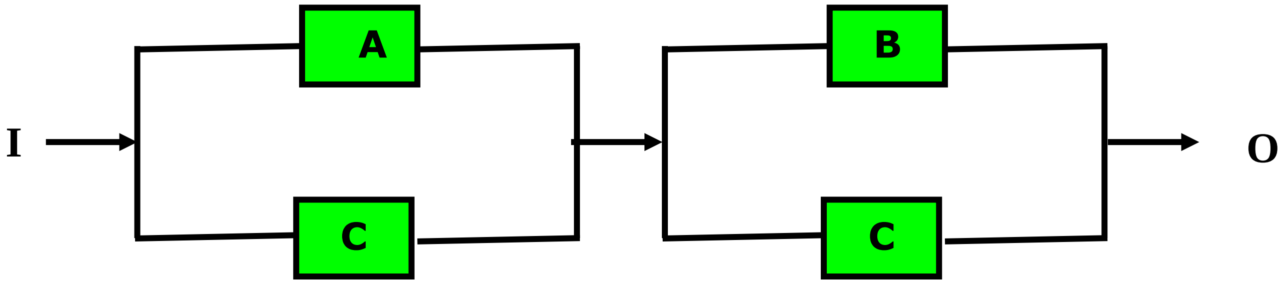


Fig (٢.١٥)

$R_{p١}$ (t) = ١-[(١-R_A) (١-R_C)]⇒ $R_{p١}$ = R_A + R_C - R_A. R_C … (٢.٢٧)

$R_{p٢}$ (t) = ١-[(١-R_B) (١-R_C)] ⇒ $R_{p٢}$ = R_B + R_C - R_B. R_C … (٢.٢٨)

$$R_S(t)=\prod_{i=1}^{n} R_{pi}(t)=(R_A+R_C-R_A.R_C)(R_B+R_C-R_B.R_C) \quad \text{… (٢.٢٩)}$$

Now we solve the equation (٢.٢٩) we get

$$R_S(t)=\prod_{i=1}^{n} R_{pi}(t)=(R_A.R_B+R_C-R_A.R_BR_C)$$

This same result as in equation (٢.١٩).

٢.٤. *Reliability Allocation*

Reliability allocation is the process by which the failure allowance for a system is allocated in some logical manner to its, sub-systems and elements. The purpose of reliability allocation is to establish a goal or objective for the reliability of each component so that the manufacturers can have an idea of the performance required of this product.

The reliability factor of a system is known or is specified on the basis of the overall mission required .If the system comprises many elements and units, we must have method to determine the reliability factor for each of them.

Consider a system consisting of n components (dependent or independent) with individual reliability factors $R_١,R_٢,R_٣,\ldots R_n$. The system reliability R (t) is a function of these components reliability. Thus,

$$R(t) = f(R_١, R_٢, R_٣,\ldots, R_n) \qquad \ldots (٢.٣٠)$$

The problem now is to determine the values of $R_١,R_٢,R_٣,\ldots,R_n$ for a given value of R(t) .This problem will not have a unique answer , at the same time , the value of $R_١, R_٢, R_٣,\ldots, R_n$ cannot be altogether arbitrary . The problem can be viewed from two aspects. First, if we allocate the reliability requirement in some logical manner among the n component, a goal will be set for the manufacturer to product components with these reliability factors. Second, if the state of art is such that the reliability R_i of a particular component i cannot possibly be improved upon, and if the allocated reliability factor is higher.

In our project we shall discuss two methods by which the reliability can be logically apportioned among the consistent element of a system. The problem will not be treated in its most general form as stated in eq. (٢.٣٠), but in a slightly different manner. The reliability or the predicted failure probability of each component is obtained from its failure – data analysis. If the failure allowance specified for the system is apportioned among the components in some equitable manner [٨].

٢.٤.١. Reliability allocation for a series system

We discuss the reliability allocation for series system by using two methods:-

١- First method

The system consisting of n components connected in series. The principle adopted in this method for subdividing the system failure allowance is that the failure allowance of each component is directly proportional to the predicted probability of failure. This rule based on the assumption that the components exhibit a constant failure rate, and if λ_i is the failure rate of component i, i= ١,…. , n then we can use the approximation :

$R_S(t) = \exp(-\lambda_S t) \approx ١-\lambda_S t$ … (٢.٣١)

Where $\lambda_S= \sum_{i=1}^{n} \lambda_i$, λ_S is termed the predicted system failure rate. λ^*s is the specified system failure rate which is assumed to be less than λs. To compute the required failure rate for component i which is denoted by λ^*_i, i = ١,٢,…,n , we use the following equation :

$\lambda^*_i t = (\lambda_i t/\lambda_S t).\ \lambda^*_S t$ … (٢.٣٢)

So the reliability goal for the component i is

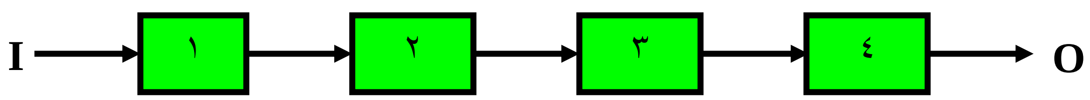
$R^*_i(t) = \exp(-\lambda^*_i t) \approx ١-\lambda^*_i t$ … (٢.٣٣)

Example (١٠): -

A system is composed of ٤ units connected in series .the failure rates for these units are as follows:

$\lambda_١ =$ ٠.٠٠٢, $\lambda_٢ =$ ٠.٠٠٣

$\lambda_٣ =$ ٠.٠٠٤, $\lambda_٤ =$ ٠.٠٠٧



It is desired that the maximum failure rate for the system be $\lambda^{*}s = ٠.٠١$:

(i) Allocate this among the four units.

(ii) Allocate reliabilities to the four units.

Solution (i):

The sum of the unit failure rate is

$$\lambda_s = \lambda_{١} + \lambda_{٢} + \lambda_{٣} + \lambda_{٤} \qquad \ldots (٢.٣٤)$$

$$= ٠.٠٠٢ + ٠.٠٠٣ + ٠.٠٠٤ + ٠.٠٠٧ = ٠.٠١٦$$

Hence, the allocated unit failure rates are

$$\lambda^{*}_{١} = (٠.٠١/٠.٠١٦) \times ٠.٠٠٢ = ٠.٠٠١٢٥$$

$$\lambda^{*}_{٢} = (٠.٠١/٠.٠١٦) \times ٠.٠٠٣ = ٠.٠٠١٨٨$$

$$\lambda^{*}_{٣} = (٠.٠١/٠.٠١٦) \times ٠.٠٠٤ = ٠.٠٠٢٥٠$$

$$\lambda^{*}_{٤} = (٠.٠١/٠.٠١٦) \times ٠.٠٠٧ = ٠.٠٠٤٣٨$$

(ii) If $\lambda^{*}_{١}$, $\lambda^{*}_{٢}$, $\lambda^{*}_{٣}$ and $\lambda^{*}_{٤}$ are allocated failure rates, we get

$$R^{*}_{s}(t) = e^{-\lambda^{*}_{١} t}\, e^{-\lambda^{*}_{٢} t}\, e^{-\lambda^{*}_{٣} t}\, e^{-\lambda^{*}_{٤} t} \qquad \ldots (٢.٣٥)$$

$$\lambda^{*}_{١} = (\lambda s/\lambda^{*}s)\, \lambda_{١},\ \lambda^{*}_{٢} = (\lambda s/\lambda^{*}s)\, \lambda_{٢}$$

$$\lambda^{*}_{٣} = (\lambda s/\lambda^{*}s)\, \lambda_{٣},\ \lambda^{*}_{٤} = (\lambda s/\lambda^{*}s)\, \lambda_{٤}$$

Hence, the allocated reliabilities are

$$R^{*}_{١}(t) = e^{-(\lambda s/\lambda^{*}s)\, \lambda_{١}}; \qquad \ldots (٢.٣٦)$$

$$R^{*}_{٢}(t) = e^{-(\lambda s/\lambda^{*}s)\, \lambda_{٢}}; \qquad \ldots (٢.٣٧)$$

$$R^{*}_{٣}(t) = e^{-(\lambda s/\lambda^{*}s)\, \lambda_{٣}}; \qquad \ldots (٢.٣٨)$$

$$R^*_{٤}(t) = e^{-(\lambda s/\lambda^* s)\,\lambda_{٤}}; \qquad \ldots (٢.٣٩)$$

$$R^*_{١}(t) = e^{-(٠٠.٠١/٠.٠١٦)\ ٠.٠٠٢} = ٠.٩٩٨٧٥$$

$$R^*_{٢}(t) = e^{-(٠٠.٠١/٠.٠١٦)\ ٠.٠٠٣} = ٠.٩٩٨١٣$$

$$R^*_{٣}(t) = e^{-(٠٠.٠١/٠.٠١٦)\ ٠.٠٠٤} = ٠.٩٩٧٥٠$$

$$R^*_{٤}(t) = e^{-(٠٠.٠١/٠.٠١٦)\ ٠.٠٠٧} = ٠.٩٩٦٥٣$$

٢- ***Second method***

In this method (***known as the minimum effort method***) the reliability of only those components is increased that have a value less than a certain figure. Further, the reliability of each of those components is raised to a common value and not apportioned in the sense in which it was done in first method.

Let the system consist of n component connected in series, and let $R_{١}$, $R_{٢}$, $R_{٣}$,…, R_n be their individual reliabilities .The system reliability will be

$$Rs\ (t) = R_{١} \times R_{٢} \times R_{٣} \times \ldots \times R_n$$

Let R^*_S (t) be the desired reliability for the system, and assume that R^*_S (t) is greater than R_S .we have now to increase the reliability of a minimum number of components to a value such that the new reliability of the system becomes R^*s (t). To achieve this we proceed as follows:

(a) The known reliabilities of the component are arranged in an ascending order. Having done this assume that

$$R_{١} \le R_{٢} \le R_{٣} \le \ldots \le R\ n \qquad \ldots (٢.٤٠)$$

(b) The reliabilities of the first K components in eq.(٢.٤٠) are increased to the same value R_{0}.

The reliabilities $R_{k+١}$, $R_{k+٢}$,…, R_0 at the remaining component are left unchanged. For determining k and R_0 the first is to calculate the value of

$$\left[\frac{R_S^*}{R_2\times R_3\times\ldots\times R_n}\right]^{1/1} \qquad \ldots\ (٢.٤١)$$

This is compared with $R_{١}$ .IF

$$\left[\frac{R_S^*}{R_2\times R_3\times\ldots\times R_n}\right]^{1/1} > R_{١} \qquad \ldots\ (٢.٤٢)$$

Then $R_{١}$ will have to be increased to the value R_0. The next step is to calculate the value of

$$\left[\frac{R_S^*}{R_3\times R_4\times\ldots\times R_n}\right]^{1/2} \qquad \ldots\ (٢.٤٣)$$

This is compared with $R_{٢}$ .If

$$\left[\frac{R_S^*}{R_3\times R_4\times\ldots\times R_n}\right]^{1/2} > R_{٢} \qquad \ldots\ (٢.٤٤)$$

Then $R_{٢}$ will have to be increased to the value R_0. This procedure is continued till.

$$\left[\frac{R_S^*}{R_{k+1}\times R_{k+2}\ldots\times R_n}\right]^{1/k} > R_k \qquad \ldots\ (٢.٤٥)$$

Then R_k will also have to be increased to the value R_0. In general, k is the maximum value of j such that

$$\left[\frac{R_S^*}{\prod_{i=j+1}^{n+1} R_i}\right]^{1/j} > R_J \qquad \dots (٢.٤٦)$$

Where $R_{n+١}$ .The reason for introducing $R_{n+١}$ is to include the possibility that in a system it may be necessary to raise the reliabilities $R_١$, $R_٢$, $R_٣$,…,R_n of all the n components to common higher value , whose maximum value will be at most equal to $R_{n+١}$=١.

Example (١١): -

Four units are connected in series, with reliabilities $R_١$=٠.٨٥, $R_٢$=٠.٩, $R_٣$=٠.٨, $R_٤$=٠.٩٥. Calculate the system reliability. If the reliability is to be increased to a value of ٠.٦٥, how should this is apportioned among the four units according to the minimum effort method?

Solution:

The system reliability Rs for these units in series is

Rs (t) = $R_١$. $R_٢$. $R_٣$. $R_٤$

= ٠.٨٥ × ٠.٩ ×٠.٨ × ٠.٩٥

= ٠.٥٨١٤

The value of R_S^* is ٠.٦٥. Following the procedure explained earlier, we have

$$\left[\frac{R_S^*}{R_2 \times R_3 \times R_4 \times R_{n+1}}\right]^{1/1} = \frac{0.65}{0.684} = ٠.٩٥$$

Since ٠.٩٥ > ٠.٨٥, the value of $R_١$ will have to be increased to R_0. Next

$$\left[\frac{R_S^*}{R_3 \times R_4 \times R_{n+1}}\right]^{1/2} \quad \left(\frac{0.65}{0.76}\right)^{1/2}$$

= = ٠.٩٣

Since ٠.٩٣ > ٠.٩ the value of R٢ will have to be increased to $R_{0.}$

Next

$$\left[\frac{R_S^*}{R_4 \times R_{n+1}}\right]^{1/3} = \left(\frac{0.65}{0.95}\right)^{1/3} = ٠.٨٨١$$

Since ٠.٨٨١ > ٠.٨ the value of R٣ will have to be increased to $R_{0.}$ Next

$$\left[\frac{R_S^*}{R_{n+1}}\right]^{1/4} = \left[\frac{0.65}{1.00}\right]^{1/4} = ٠.٨٩$$

Since ٠.٨٩ is less than ٠.٩٥. The value of R٤ will be left unaltered. From (٢.٤٥), we see that K =٣ and the value of R_0 is obtained from

$$R_0 = \left[\frac{R_S^*}{R_4 \times R_{n+1}}\right]^{1/3} = \left(\frac{0.65}{0.95}\right)^{1/3} = ٠.٨٨١$$

This means that the reliability of the first unit is to be increased from its present value of ٠.٨٥ to ٠.٨٨١; the reliability of the second unit is to be increased from ٠.٩ to ٠.٨٨١; and the reliability of the third unit is to be increased from ٠.٨ to ٠.٨٨١ and the reliability of the fourth unit will continue to be ٠.٩٥.the reliability of the system will then be

$$R_S^* = R_1^* \times R_2^* \times R_3^* \times R_4^*$$

= ٠.٨٨١ × ٠.٨٨١ × ٠.٨٨١ × ٠.٩٥ ≈ ٠.٦٥

٢.٤.٢. Reliability allocation for a complex system

Similarly, we can find the reliability allocation for complex system by using previous methods [٩].

Example (١٢): -

A system consists of six elements connected as shown in fig (٢.١٦) the predicted reliabilities of the components for a ١٠-hour period are also shown.

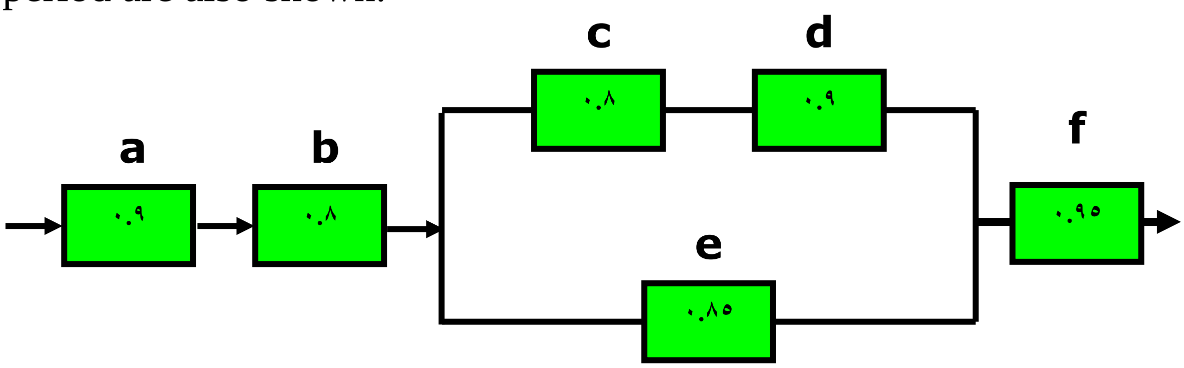


Fig (٢.١٦)

١- Convert the given system into an equivalent system consisting of only series elements.

٢- Calculate the system reliability.

٣- If the system reliability is to be improved to a value of ٠.٨٥ for a ١٠-hour period, determine the reliability goal of each component. Assume constant failure rate of all elements ***without using the approximation R (t) ≈ ١-λ t***

٤- Solve problem by using minimum effort method.

Solution:

١- For calculating the system reliability, the given system converted into an equivalent system consisting of only series components, the components c,d,e mentioned in the reduction method

R g (t) = Rc Rd + Re- Rc Rd Re (by probability rules)

= (٠.٨) (٠.٩) + (٠.٨٥) - (٠.٨) (٠.٩) (٠.٨٥) =٠.٩٥٨

Now the given system has been reduced to a series system as shown in fig. (٢.١٧)

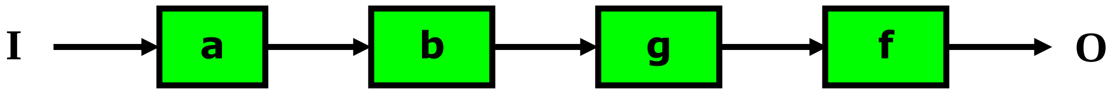


Fig. (٢.١٧)

We have, the reliability of the system is

$R_a(t) = \exp(-\lambda_a t) =$ ٠.٩

$R_b(t) = \exp(-\lambda_b t) =$ ٠.٨

$R_g(t) = \exp(-\lambda_g t) =$ ٠.٩٥٨

And $R_f(t) = \exp(-\lambda_f t) =$ ٠.٩٥

The value of λt would have been

$\lambda_a t =$ ٠.١

$\lambda_b t =$ ٠.٢

$\lambda_g t =$ ٠.٠٤٢

And $\lambda_f t =$ ٠.٠٥

٢- Thus the system reliability is given by

$R_S(t) = R_a(t) \times R_b(t) \times R_g(t) \times R_f(t)$

$= \exp[-(\lambda_a t + \lambda_b t + \lambda_g t + \lambda_f t)]$

$= \exp(-\lambda_s t)$

Where $\lambda_S t$ = ٠.١+ ٠.٢+ ٠.٠٤٢ + ٠.٠٥= ٠.٣٩٢ (from eq. ٢.٣٤)

$$R_S(t) = \exp(-٠.٣٩٢) = ٠.٦٧٥$$

٣- The desired reliability $R^*_s(t) = ٠.٨٥$ for the system, there fore

$$R^*_S(t) = e^{-\lambda^*_S t} = ٠.٨٥ \Rightarrow \lambda^*_s t = ٠.١٦٢$$

$$R^*_S(t) = e^{-\lambda^*_a t}\, e^{-\lambda^*_b t}\, e^{-\lambda^*_g t}\, e^{-\lambda^*_f t}$$

$$\lambda^* a = (\lambda^* s/\lambda s)\, \lambda a,\ \lambda^* b = (\lambda^* s/\lambda s)\, \lambda b$$

$$\lambda^* g = (\lambda^* s/\lambda s)\, \lambda g,\ \lambda^* f = (\lambda^* s/\lambda s)\, \lambda f$$

Hence, the allocated reliabilities are

$$R^*_a(t) = e^{-(\lambda^* s/\lambda s)\,\lambda a};$$

$$R^*_b(t) = e^{-(\lambda^* s/\lambda s)\,\lambda b};$$

$$R^*_g(t) = e^{-(\lambda^* s/\lambda s)\,\lambda g};$$

$$R^*_f(t) = e^{-(\lambda^* s/\lambda s)\,\lambda f};$$

$$R^*_a(t) = e^{-(٠.١٦٢/٠.٣٩٢)\ ٠.١} = ٠.٩٥٩$$

$$R^*_b(t) = e^{-(٠.١٦٢/٠.٣٩٢)\ ٠.٢} = ٠.٩٢٠$$

$$R^*_g(t) = e^{-(٠.١٦٢/٠.٣٩٢)\ ٠.٠٤٢} = ٠.٩٨٢$$

And $$R^*_f(t) = e^{-(٠.١٦٢/٠.٣٩٢)\ ٠.٠٥} = ٠.٩٧٩$$

٤- To solve this problem by using ***minimum effort*** we follow this step

(a) From (٢.٤٠) we arranged the known reliabilities of the component in an ascending order. Having done this assume that

$R_{١} \leq R_{٢} \leq R_{٣} \leq \ldots \leq R_n$

$٠.٨ \leq ٠.٩ \leq ٠.٩٥ \leq ٠.٩٥٨$

(b) The value of R^{*}_{s} (t) is ٠.٨٥ . Following the procedure explained earlier, we have

$$\left[\frac{R_S^*}{R_2 \times R_3 \times R_4 \times R_{n+1}}\right]^{1/1} = \frac{0.85}{0.819} \approx ١.٠٠$$

Since ١.٠٣> ٠.٨, the value of $R_{١}$ will have to be increased to $R_{0.}$ Next

$$\left[\frac{R_S^*}{R_3 \times R_4 \times R_{n+1}}\right]^{1/2} = \left(\frac{0.85}{0.910}\right)^{1/2} = ٠.٩٦٦$$

Since ٠.٩٦٦ > ٠.٩ the value of $R_{٢}$ will have to be increased to $R_{0.}$ Next

$$\left[\frac{R_S^*}{R_4 \times R_{n+1}}\right]^{1/3} = \left(\frac{0.85}{0.958}\right)^{1/3} = ٠.٩٦٠$$

Since ٠.٩٦ > ٠.٩٥ the value of $R_{٣}$ will have to be increased to $R_{0.}$ Next

$$\left[\frac{R_S^*}{R_{n+1}}\right]^{1/4} = \left[\frac{0.85}{1.00}\right]^{1/4} = ٠.٩٦٠$$

Since ٠.٩٦٠> ٠.٩٥٨. The value of $R_{٤}$ will have to be increased to R_0, we see that K = ٤ and the value of R_0 is obtained from

$$R_0 = \left[\frac{R_S^*}{R_{n+1}}\right]^{1/4} = \left(\frac{0.85}{1.00}\right)^{1/4} = ٠.٩٦٠$$

$$R_S^* = R_a^* \times R_b^* \times R_g^* \times R_f^*$$

$$= ٠.٩٦٠ \times ٠.٩٦٠ \times ٠.٩٦٠ \times ٠.٩٦٠ = ٠.٨٥$$

٢.٥ Conclusion

In this chapter, we study six – method to find the reliability of complex system, path-tracing method, reduction to series element, minimal cut method, the decomposition method, the event space method and composite method.

We concluded

١- ***In the figure (٢.٨)*** the reliability in the first method as in eq. (٢.١١) is the largest than the reliability of the second and third method as an eq. (٢.١٥) and (٢.١٧). While the reliability in ***figure (٢.١٢)*** is equality for all methods.

٢- ***In the figure (٢.٨)*** the system reliability is bounded between the upper bound as in eq. (٢.١١) and lower bound as in eq. (٢.١٧). But the reliability value in fig. (٢.١٢) is similar.

٣- The system reliability is depending on the way with which the elements of the system are connected **.**

٣.١ Introduction

In the previous chapter, we studied system reliability for different models and discussed some methods of the complex systems reliability. Also some comparisons have been done to select the optimal way to find the reliability of complex system. In this chapter, we will give some basic concepts from stochastic process, as Markov chain and Markov process, which will be used in this work.

This chapter contains four sections. Section two deals with some concepts on Markov models. In section three reliability models of non-maintained system are studied. Also in this section we shall be concerned with developing the reliability functions and expected time to system failure for the series and parallel system configurations.

The last section is concerned with non-Markovian processes.

٣.٢. Some concepts on a Markov models

In this section, we will present some concepts in Markov models, needed for calculating the reliability system; most of those definitions are taken from [١٩], [٢١],[٢٣].

Definition (3.1) stochastic process

It is a function whose values are random variables. The classification of a random process depends on different quantities [١٦]:

١– State space

٢– Index (time) parameter

١- *State Space*

– The set of possible values (states) that X (t) might take on.

– If there are finite states => discrete-state process or chain

– If there is a continuous interval => continuous process.

٢- *Index (Time) Parameter*

- If the times at which changes may take place are finite or countably infinite , then we say we have a discrete-(time) parameter process.
- If the changes may occur anywhere within a finite or infinite interval on the time axis, then we say we have a continuous-parameter process.

There are many kinds of stochastic processes. Two of the most important distinguishing characteristics of a random process such as ***Markov chain and Markov process***

<u>Definition (٣.٢) Markov Chain</u>

It is a stochastic process with a discrete state space and time space [١٦].

A set of random variables forms a Markov chain if the probability of the next state $S_{(n+١)}$ depends only on the current state $S_{(n)}$, and not on any previous states.

i.e. $S_{n+١}$ depends only on S_n and not on any X_i , $١\leq i < n$

States must be

١– Mutually exclusive.

٢– Let P_i (t) = Probability of being in state S_i at time t.

When $\sum_{\forall i} P_i(t) = 1$

Definition (٣.٣) Markov process

It is a stochastic process with a discrete - state space and continuous time. In this section, we will be concerned with the discrete-state continuous – time model. i.e. the Markov process.

Any Markov model is defined by the set of probabilities P_{ij} where P_{ij} is the probability of transition from any state i to any state j. One of the most important features of the Markov process is that the transition probability P_{ij} depends only on states i and j and is completely independent of all past states except the last one.

٣.٢.١. The transition probability matrix

The transition probabilities can be arranged as transition probability matrix P = ($p_{i,j}$)

$$\begin{array}{ll} & \text{Final state} \longrightarrow \\ P = & \begin{pmatrix} p_{0,0} & p_{0,1} & p_{0,2} & \cdots \\ p_{1,0} & p_{1,1} & p_{1,2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \Bigg\downarrow \text{Initial state} \end{array}$$

- The row i contains the transition probabilities from state i to other states.

Since the system always goes to some state, the sum of the row probabilities is ١.

- A matrix with non-negative elements such that the sum of each row equals ١ is called a stochastic matrix.
- One can easily show that the product of two stochastic matrices is a stochastic matrix.

٣. ٢. ٢. Laplace Transform

The Laplace transform is important in solving differential equations [٦]. The Laplace transform of a time function f (t) is defined by the integral

$$L\{f(t)\} = f^*(s) = \int_0^\infty f(t) e^{-st}\, dt, \mathrm{Re}(s) > 0 \qquad \ldots (٣.١)$$

Where L represents the Laplace transform.

The utility of the Laplace transform is that it reduces ordinary constant coefficient linear differential equations to algebraic equations in s which are easily solved .The solution in terms of **s** is then converted back to a time function by an inverse transform procedure. In this section, we give a few examples to show the transformation, [٢٤], [٢٦].

Example (١):

Find the Laplace transform of the exponential function f (t) =e $^{-at}$

Solution:

$$L\{f(t)\} = f^*(s) = \int_0^\infty e^{-at} e^{-st}\, dt = \int_0^\infty e^{-(a+s)} dt$$

$$= -\frac{e^{-(a+s)t}}{s+a}\Big|_0^\infty = \frac{1}{s+a} \qquad for \quad s > -a$$

Example (٢):

Find the Laplace transform of the function f (t) =١

Solution:

$$L\{f(t)\} = f^*(s) = \int_0^\infty 1.e^{-st}\,dt$$

$$= \frac{e^{-st}}{-s}\Big|_0^\infty = \frac{1}{s} \quad for \quad s > 0$$

Example (٣):

Find the Laplace transform of the function $\frac{df(t)}{dt}$

$$L\left\{\frac{df(t)}{dt}\right\} = f^*(s) = \int_0^\infty \frac{df(t)}{dt}.e^{-st}\,dt$$

$$= \int_0^\infty e^{-st}\,df(t)$$

This integral can be evaluated in parts. Let d y = d f (t) and u = e $^{-st}$ On integrating, we get

$$L\left\{\frac{df(t)}{dt}\right\} = e^{-st} f(t)\Big|_0^\infty + s\int_0^\infty f(t)e^{-st}\,dt$$

The value of e^{-St} f (t) at the upper limit becomes zero as

$$\lim_{t\to\infty} e^{-st} f(t) = 0$$

At the lower of e^{-st} f (t) we get f (٠), the initial value of the function .The second part of the integral is the Laplace transform of the function itself. Thus,

$$L\left\{\frac{df(t)}{dt}\right\} = s\, f^*(s) - f(0) \qquad \text{… (٣.٢)}$$

Example (٤):

Solve the following differential equations

$$\frac{d\,p_0(t)}{dt} = -\lambda\, p_0(t) \qquad \ldots (٣.٣)$$

$$\frac{d\,p_1(t)}{dt} = \lambda\, p_0(t) - \lambda\, p_1(t) \qquad \ldots (٣.٤)$$

Solution:

Taking the Laplace transform of eqns. (٣.٣) and (٣.٤) we have

$$s\,P_0^*(s) - P_0(0) = -\lambda\, P_0^*(s)$$

$$s\,P_1^*(s) - P_1(0) = \lambda\, P_0^*(s) - \lambda\, P_1^*(s)$$

Here,

$$P_0(0) = 1 \quad and \quad P_1(0) = 0.$$

Thus,

$$s\,P_0^*(s) = -\lambda\, P_0^*(s) + 1 \qquad \ldots (٣.٥)$$

$$s\,P_1^*(s) = \lambda\, P_0^*(s) - \lambda\, P_1^*(s) \qquad \ldots (٣.٦)$$

Solving eqns. (٣.٥) and (٣.٦) simultaneously, we obtain

$$P_0^*(s) = \left(\frac{1}{s+\lambda}\right) \qquad \ldots (٣.٧)$$

$$P_1^*(s) = \left(\frac{\lambda}{(s+\lambda)^2}\right) \qquad \ldots (٣.٨)$$

On inversion, we get the time dependent functions

$P_{٠}(t) = e^{-\lambda t}$ … (٣.٩)

$P_{١}(t) = (\lambda t)\, e^{-\lambda t}$ … (٣.١٠)

These are the terms of the Poisson distribution for n =٠ and n =١.

٣. ٢. ٣. Poisson process

In discussing the Markov process, one of the examples, which is great importance, is the Poisson process. In this section, we shall derive the Poisson distribution as the governing probability law for a Poisson process [٧].

In a Poisson process, we are interested in the number of occurrences in time, the probability of each occurrence in small time (Δ t) being a constant. The probability density function is given by

$$P(X=x)=\frac{(\lambda t)^{x}e^{-\lambda t}}{x!} \quad ; x=١, ٢,٣\ldots \qquad \ldots (٣.١١)$$

The basic assumptions that are necessary in driving a Poisson process model are as follows:

١- The probability that a transition occurs from the state of n occurrences to state of (n+١) occurrences in time Δ t is λ Δ t.

٢- Each occurrence is independent of all other occurrences.

٣- The transition of two or more occurrences in interval Δ t is negligible.

In view of these assumption, a transition matrix P, which describes this failure process for an interval t, t+ Δ t, can be developed .For example, if at time t the system is in state ٠, the conditional probability of remaining in that state is ١- λ Δ t .The probability of a transition to state ١ is λ Δt. the probability of transition to states higher than ١ are ٠(Δ t) from assumption

٢.Similarly if the system is in state ١ at time t , then it can remain in that state during t ,t +Δ t with probability ١- λ Δ t , or transit to state ٢ with probability λ Δ t. notice that the system cannot return to state ٠ unless repairs are made on failed equipments. Thus we can write a transition matrix for each of these conditions. It should be noted that the sum of each row of matrix is always unity. Since the system must be in one or another state.

$$p = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ . \\ . \\ . \\ n-1 \\ n \end{array} \begin{array}{c} \begin{array}{cccccc} ٠ & ١ & ٢ & \dots & n-١ & n \end{array} \\ \begin{pmatrix} 1-\lambda & \lambda & 0 & \dots & 0 & 0 \\ 0 & 1-\lambda & \lambda & \dots & 0 & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & \dots & 1-\lambda & \lambda \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix} \end{array}$$

We can write the probabilities of being in each state directly as:

$P_{٠}(t+\Delta t) = P_{٠}(t)(١-\lambda\Delta t) \quad + ٠(\Delta t)$

$P_{١}(t+\Delta t) = P_{٠}(t)\,\lambda\Delta t + P_{١}(١-\lambda\Delta t) \quad + ٠(\Delta t)$

$P_{n}(t+\Delta t) = P_{n-١}(t)\,\lambda\Delta t + P_{n}(t) \quad + ٠(\Delta t)$

This formulation leads to our system of differential equations. To see this let us examine $P_{١}$ (t+ Δ t). Expanding we have:

$P_{١}(t+\Delta t) = P_{٠}(t)\,\lambda\Delta t + P_{١}(t) - P_{١}(t)\,\lambda\Delta t + ٠(\Delta t)$

From the definition of a derivative we have

$$\frac{d\,p_1(t)}{dt} = \frac{p_1(t+\Delta t)-p_1(t)}{\Delta t} = \lambda\,p_0(t) - \lambda\,p_1(t)$$

Similarly, the other equations are found to be:

$$P_٠'(t) + \lambda P_٠(t) \qquad = ٠$$

$$P_١'(t) - \lambda P_٠(t) + \lambda P_١(t) \qquad = ٠$$

$$P_n'(t) - \lambda P_{n-١}(t) \qquad = ٠$$

If we assume that at t=٠ all equipment's are in working order, then the initial conditions are simply:

$$P_٠(٠) = ١,\ P_١(٠) = ٠,\ P_٢(٠) = ٠, \ldots, P_n(٠) = ٠,$$

And by using the Laplace transformation we shall first solve for $P_٠(t)$, then for the remaining states. Applying the definition of the Laplace transform, we have the first equation

$$s P_٠(s) - ١ + \lambda P_٠(s) = ٠$$

So that

$$P_٠(s) = ١/(s+\lambda)$$

To find $P_٠(t)$ we must take the inverse transform of the expression for $P_٠(s)$.

We find

$$L^{-١}\{P_٠(s)\} = f(t) = e^{-\lambda t}$$

Thus, the probability of no failures up to a time t is simply the negative exponential function. With the solution to $P_٠(t)$ it is easy to find $P_١(t)$, $P_٢(t)$ and so forth. For example

$$P_١'(t) = \lambda P_١(t) + \lambda e^{-\lambda t}$$

And transforming the expression

$$sP_1(s) + \lambda P_1(s) = \frac{\lambda}{s+\lambda}$$

And

$$P_1(s)=\frac{\lambda}{(s+\lambda)^2}$$

So that

$$P_{١}(t) = L^{-١}\{P_{١}(s)\}= \lambda t\, e^{-\lambda t}$$

Continuing in this fashion we note that in general

$$P_n(s)=\frac{\lambda^n}{(s+\lambda)^{n+1}}$$

Whose inverse is

$$P_n(t)=\frac{(\lambda t)^n e^{-\lambda t}}{n!} \quad \text{, For n = ١, ٢,٣,...} \qquad \text{...(٣.١٢)}$$

This is the Poisson density function.

٣. ٣. Reliability models of non – maintained system

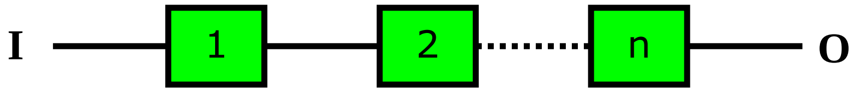
For non – maintained system we shall define the reliability function R (t) as the probability distribution which gives the probability that the system will not fail within a given interval of operating the probability time (٠,t). In this chapter we shall be concerned with developing the reliability functions and expected time to system failure for the following system configurations [٧].

١- ***Series configuration.***

٢- ***Parallel configuration.***

٣. ٣. ١. Series configuration

The simplest arrangement of a number of equipment is a series configuration .In this type of configuration all n equipment must be operating for the system to be operating.



Series configuration

In order to define the reliability induced by this kind of process, we need only consider the state where all the equipment are operating. Thus, the probability that it had been operating at time t, and that none of the n equipment's failed in the interval t, t+dt.Therefore, we make the following statements and assumptions:

١- The system is failed when any one of the n equipment fails.

٢- The failure probability of each equipment is independent of the remaining n-١ equipments.

٣- The probability that any one equipment will fail in the interval t, t+dt, given that it was operating at time t, is λdt

We then have

$$P_{٠}(t+dt) = P_{٠}(t)(١-n\lambda\, dt) + ٠dt$$

And

$$P'_{٠}(t) + n\lambda P_{٠}(t) = ٠$$

Solving by Laplace transforms as in the previous examples, we find

$$P_0(s) = \left(\frac{1}{s+n\lambda}\right) \qquad \dots (٣.١٣)$$

And they're fore

$$P_{٠}(t) = e^{-n\lambda t} \qquad \dots (٣.١٤)$$

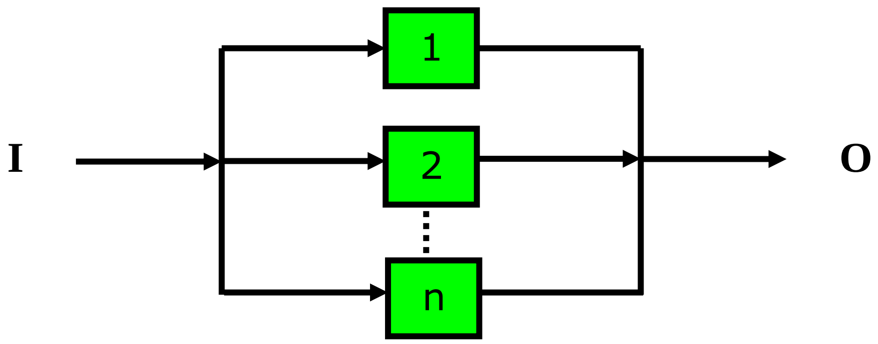
If we let $R=e^{-\lambda t}$, the reliability function for a series configuration can simply be written as

$$R(t) = R^{n} \qquad \ldots(٣.١٥)$$

And we noticed that this low is similarly to find the reliability system in eq. (٢.١).

٣. ٣. ٢. Parallel configuration

In this configuration the assumation is that all n equipments operate simultaneously and that each equipment has the same failure rate. For the system to be in an operable state at a time t, the system can be in any state other than n .



Parallel configuration

The transition matrix in this case is of the form:

$$
\begin{array}{cc}
 & \begin{array}{ccccc} ٠ & ١ & ٢ & \ldots & ٠ \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ . \\ . \\ . \\ n \end{array} &
P = \begin{pmatrix}
1-n\lambda & n\lambda & 0 & \ldots & 0 \\
0 & 1-(n-1)\lambda & (n-1)\lambda & \ldots & 0 \\
0 & 0 & 1-(n-2)\lambda & \ldots & 0 \\
. & . & . & \ldots & . \\
. & . & . & \ldots & . \\
. & . & . & . & . \\
0 & 0 & 0 & \ldots & 1
\end{pmatrix}
\end{array}
$$

For example let us consider a system consists of two equipment where the equipments are connected in parallel and it has the same failure rate besides the two equipment operates simultaneously

The state space is ٠, ١ and ٢ representing the number of equipment failed. The system is failed when it researches state ٢. The transition matrix in this case is of the form:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} ٠ & ١ & ٢ \end{array} \\ \begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ 0 & 1-\lambda & \lambda \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

To develop the system of differential equations we must first enumerate the probabilities of being in each state at t+ dt. These are

$$p_0(t+dt) = p_0(1-2\lambda dt) \qquad +0dt$$
$$p_1(t+dt) = p_0 2\lambda dt + p_1(t)(1-\lambda dt) \quad +0dt$$
$$p_2(t+dt) = \qquad p_1(t)\lambda dt \qquad +0dt$$

From which we obtain

$$\frac{d\,p_0(t)}{dt} = -2\lambda\, p_0(t)$$
$$\frac{d\,p_1(t)}{dt} = 2\lambda\, p_0(t) - \lambda\, p_1(t)$$
$$\frac{d\,p_2(t)}{dt} = \qquad \lambda\, p_1(t)$$

With initial conditions

$$p_0(0)=1, \quad p_1(0)=0, \quad p_2(0)=0$$

Taking the Laplace transform we have

$$
\begin{aligned}
(s+2\lambda)\, p_0(s) \qquad\qquad &= 1 \\
-2\lambda\, p_0(s)+(s+\lambda)\, p_1(s) \qquad &= 0 \\
-\lambda\, p_1(s)+s\, p_2(s) &= 0
\end{aligned}
$$

And

$$p_0(S)=\frac{1}{s+2\lambda}$$

$$p_1(S)=\frac{2\lambda}{(s+2\lambda)(s+\lambda)}=\frac{2}{(s+\lambda)}-\frac{2}{(s+2\lambda)}$$

$$p_2(S)=\frac{2\lambda^2}{s(s+2\lambda)(s+\lambda)}$$

And since

$$R(t)=p_0(t)+p_1(t) \qquad \dots (٣.١٦)$$

We have

$$R\ (t) = ٢e^{-\lambda t} - e^{-٢\lambda t} \qquad \dots (٣.١٧)$$

In general case where n equipments are operating in parallel, the reliability function is

$$R(t)=1-\left[1-e^{-H(t)}\right]^n \qquad \dots (٣.١٨)$$

Where

$$H(t)=\int_0^t h(t)\,dt$$

Or $$R\ (t) = ١- (\ ١- e^{-\lambda t})^{\ n} \qquad \dots(٣.١٩)$$

٣. ٣. ٣. MTTF of series configuration

When n equipments are connected in a series arrangement, the system fails when any one equipment fails. Applying the definition (١.١٢), the mean time to reach state ١ when starting state ٠, is:

$$m_{01}=\int_0^\infty tf(t)dt=\int_0^\infty R(t)dt \qquad \dots (٣.٢٠)$$

From equation (٣.١٤)

$$m_{01} = \int_0^\infty e^{-n\lambda t} dt \qquad \dots (٣.٢١)$$

$$m_{01} = \frac{1}{n\lambda}$$

If each equipment has a different failure rate,

$$m_{01} = \frac{1}{\sum_{i=1}^{n} \lambda_i}$$

٣. ٣. ٤. MTTF of parallel configuration

When n equipment are connected in a parallel arrangement, the system operate when any one equipment operate. Applying the definition (١.١٢), the mean time to reach state n when starting state ٠, is:

$$m_{01} = \int_0^\infty t f(t) dt = \int_0^\infty R(t) dt \qquad \dots (٣.٢٢)$$

From equation (٣.١٨)

$$m_{0n} = \int_0^\infty (1-(1-e^{-\lambda t})^n)\, dt \qquad \dots (٣.٢٣)$$

We can make use from the binomial formula $(a + b)^n$ to solve the term $(١ - e^{-\lambda t})^n$ that is

$$(1-e^{-\lambda t})^n = 1^n + n(-e^{-\lambda t}) + \frac{n(n-1)}{1.2}(-e^{-\lambda t})^2 + \frac{n(n-1)(n-2)}{1.2.3}(-e^{-\lambda t})^3$$
$$+ \cdots + n(-e^{-\lambda t})^{n-1} + (-e^{-\lambda t})^n$$

Suppose that the system have two equipment connected in parallel , so we obtain

$$(1-e^{-\lambda t})^2 = 1+2(-e^{-\lambda t})+\frac{2(2-1)}{1.2}(-e^{-\lambda t})^2$$
$$= 1-2e^{-\lambda t}+e^{-2\lambda t}$$

Now from eq. (٣.٢٣) we get

$$\int_0^\infty (1-(1-e^{-\lambda t})^2)dt = \int_0^\infty \left[1-(1-2e^{-\lambda t}+e^{-2\lambda t})\right]dt$$

$$= \int_0^\infty \left[2e^{-\lambda t}-e^{-2\lambda t})\right]dt$$
$$= \frac{2}{\lambda}-\frac{1}{2\lambda}=\frac{3}{2\lambda} \qquad \ldots (٣.٢٤)$$

٣.٤ NON – MARKOVIAN PROCESSES

It will be recalled that the basic assumption made with regard to Markov processes was that they were characterized by constant transition probabilities. This implies that the intervene distribution of times to failure is exponentially distributed and we say therefore that equipments fail at a constant rate. Thus knowledge of the system at a time t uniquely determines its future stochastic behavior. Now if we assume a more general form of the failure distribution function, F (t), we define a non – Markovian processes we shall soon see, it is often possible to treat a stochastic process of the non – Markovian type by reducing it to a Markovian process.

To define a non- Markov process completely we must specify more information than is required in a Markov process. In the Markov formulation we did not concern ourselves with how the system got to a particular state since its present state determines the future behavior of the system. This is a consequence of the constant failure rate assumption [٧].

Example (٥): -

In a two equipment redundant system where each equipment has the failure distributed ١-$e^{-\lambda t}$, there is no need to know how long each equipment has been operating, since their probability of failure in t, t+dt , given that they were operating at t, is $\lambda\ dt$, whether they had just been replaced or had been operating for some time. However, if this probability is related to time *h(t) dt* we must also have information as to how long ago the equipment was replaced. Thus, if at a time t one equipment had been operating for t hours and the second had failed and been replaced at t-T, their respective conditional failure probabilities in *t, t + dt* are *h (t)* and *h (t-T) dt*, respectively, there are many cases where the equipment failure distributions are other than exponential, but the transition process can be treated in the Markov sense by increasing the number of states, each being described by a constant transition rate.

Example (٦):

Consider a single–equipment system with F (t)=١- $e^{-\lambda t}$ - $\lambda t\ e^{-\lambda t}$, the gamma distribution. Suppose we assume that the equipment goes through two exponential phases each of length ١/λ.

This is reasonable since $\int_0^\infty e^{-\lambda t} dt = 1/\lambda$ and $\int_0^\infty \lambda t e^{-\lambda t} dt = 1/\lambda$

Now we define three states:

State ٠: the system is operating in the first phase.

State ١: the system is operating in the second phase.

State ٢: the system is failed.

This formulation leads to the transition matrix.

$$
\begin{array}{cc}
 & \begin{array}{ccc} ٠ & ١ & ٢ \end{array} \\
P = \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-\lambda & \lambda & 0 \\ 0 & 1-\lambda & \lambda \\ 0 & 0 & 1 \end{pmatrix}
\end{array}
$$

To develop the system of differential equations we must first enumerate the probabilities of being in each state at t + Δt. These are

$P_o (t + \Delta t) = P_o (t) (1 - \lambda \Delta t) \qquad + o(\Delta t)$

$P_1 (t + \Delta t) = P_o (t) \lambda \Delta t + P_1 (1 - \lambda \Delta t) \qquad + o(\Delta t)$

$P_2 (t + \Delta t) = P_1 (t) \lambda \Delta t + P_2 (t) \qquad + o(\Delta t)$

From the definition of derivative we have

$$\frac{d\, p_0(t)}{dt} = -\lambda\, p_0(t)$$

$$\frac{d\, p_1(t)}{dt} = \lambda\, p_0(t) - \lambda\, p_1(t)$$

The solutions of these equations are simply R (t)=P٠ (t)+ P١ (t), R (t) = $e^{-\lambda t}$ (١+λ t), as in eq. (٣.٩) and (٣.١٠).

In the case where the equipments are operating in parallel and equipment has a conditional failure rate h (t) =λ t, we have the following matrix of differential equations (The transition matrix) in this case is of the form:

$$
\begin{array}{cc}
 & \begin{array}{ccc} ٠ & ١ & ٢ \end{array} \\
P= \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-2\lambda t & 2\lambda t & 0 \\ 0 & 1-\lambda t & \lambda t \\ 0 & 0 & 1 \end{pmatrix}
\end{array}
$$

The state space is ٠,١ and ٢ representing the number of equipment failed. The system is failed when it researches state ٢.

To develop the system of differential equations we must first enumerate the probabilities of being in each state at t + dt. These are:

$$p_0(t+dt)=p_0(t)(1-2\lambda tdt) \qquad +0dt$$
$$p_1(t+dt)= \; p_0(t)2\lambda tdt+p_1(t)(1-\lambda tdt) \qquad +0dt$$
$$p_2(t+dt)= \qquad p_1(t)\lambda tdt \qquad +0dt$$

From which we obtain

$$\frac{d\,p_0(t)}{dt}=-2\lambda t\,p_0(t)$$
$$\frac{d\,p_1(t)}{dt}=2\lambda t\,p_0(t)-\lambda t\,p_1(t)$$
$$\frac{d\,p_2(t)}{dt}= \qquad \lambda t\,p_1(t)$$

With initial conditions

$$p_0(0)=1,\;\; p_1(0)=0,\;\; p_2(0)=0$$

Taking the Laplace transform we have

$$(s+2\lambda t)\,p_0(s) \qquad =1$$
$$-2\lambda t\,p_0(s)+(s+\lambda t)\,p_1(s) \qquad =0$$
$$-\lambda t\,p_1(s)+s\,p_2(s) \qquad =0$$

And

$$p_0(S)=\frac{1}{s+2\lambda t}$$
$$p_1(S)=\frac{2\lambda t}{(s+2\lambda t)(s+\lambda t)}=\frac{2}{(s+\lambda t)}-\frac{2}{(s+2\lambda t)}$$

And since

$$R(t)=p_0(t)+p_1(t)$$

By substituting h (t) = λt , in eq. (٣.١٨)

We have

$$R(t) = 2e^{-1/2\lambda t^2} - e^{-\lambda t^2} \quad \ldots (٣.٢٥)$$

٣.٥ Conclusions

١- Notice that the reliability value (for series system) that can be obtained from (٣.١٥) is similar to that which can be achieved by the eq. (٢.١) .

٢- Notice that the reliability value (for parallel system) that can be obtained by eq. (٣.١٩) is similar to that calculated by the eq. (٢.٤) .

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