

OPTIMUM DESIGN OF CONTROL DEVICES FOR SAFE SEEPAGE UNDER HYDRAULIC STRUCTURES

A Thesis

**Submitted to the College of Engineering
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fulfillment of the requirements for
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By

Waked Hameed Hassan

Al-Musawi

التصميم الامثل لوسائل السيطرة لتسرب أمن اسفل المنشآت الهيدروليكية

الخلاصة

مشاكل التسرب اسفل المنشآت الهيدروليكية تعالج بطرق عملية مختلفة. فكرة هذا البحث هي ايجاد التصميم الامثل لوسائل السيطرة المستخدمة في تقليل التسرب اسفل المنشآت الهيدروليكية.

استخدمت طريقة العناصر المحددة (Finite elements) لتحليل التسرب خلال تربة الاساس النفاذة تحت المنشآت الهيدروليكية المزودة بوسائل سيطرة على التسرب هي الطبقة الصماء (Blanket) والقاطع الرأسي والمرشح. ولقد تم دراسة تأثير اطوال ومواقع هذه الوسائل في تقليل التسرب اسفل ارضية المنشآت الهيدروليكية وايجاد الافضل من بينها، وذلك من خلال مقارنة مجموعة منحنيات توزيع ضغط الاصعاد تحت ارضية المنشأ (Uplift pressure) وتوزيع قيم الانحدار الهيدروليكي عند مؤخرة المنشأ (Exit gradient).

طبق نموذج الامثلية على منشأ افتراضي (Hypothetical case) واستخدمت طريقة (Lagrange Multiplier) لحل مسألة الامثلية ، وذلك لايجاد المعالجات الاقل كلفة مع المحافظة على ضغط اصعاد أمين وانحدار هيدروليكي امين عند مؤخرة المنشأ.

دللت نتائج البحث على حدوث اقتصاد كبير في الكلفة من خلال استخدام نظام الامثلية . كما بينت نتائج التحليل على ان استخدام الطبقة الصماء في المقدمة يقلل من ضغط الاصعاد والانحدار الهيدروليكي عند المؤخرة بينما استخدامها في مؤخرة المنشأ يزيدهما . كذلك فإن استخدام القاطع الرأسي في المقدمة فعال جدا في تقليل ضغط الاصعاد، في حين ان استخدامه في المؤخرة سيكون فعالا جدا في تقليل الانحدار الهيدروليكي عند المؤخرة .وقد بينت النتائج أيضا ان استخدام المرشح فعال جدا في تقليل ضغط الاصعاد والانحدار الهيدروليكي عند المؤخرة في نفس الوقت.

لقد بين البحث ان اكثر المعالجات اقتصادية هو عند استخدام المرشح مع المنشأ الهيدروليكي في حين ان اكثرها كلفة يكون عند استخدام قاطع رأسي في مؤخرة المنشأ.

acknowledgment

All the thanks for GOD who lightened my way during the critical times.

Sincere thanks go in particular to my supervisors Asst. Prof. Dr. Abdul-Hadi A. Al-Delewy and Asst. Prof. Dr. Abdul-Hassan K. Shukur for their valuable guidance and constructive suggestions during the preparation of this thesis.

It is with pleasure, the author would like to thank the Dean of the College of Engineering, the Head and the staff of the Civil Engineering Department in University of Babylon for Their co-operation and assistance.

Finally, love and gratitude go to my family for their encouragement and interest in seeing this thesis completed.



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٢٠٠٢

abstract

Problems of seepage under hydraulic structures could be tackled by use of a variety of practical measures. This research aimed at determining the optimal design of control devices in order to decrease the seepage under the hydraulic structures.

The finite – element method has been used to analyze seepage through porous media below hydraulic structures with blanket, cut-off, and filter trench as seepage control devices. The effect of length and location of the control devices have been investigated to determine their optimum length and location. A set of curves has been obtained showing the uplift pressure distribution and exit gradient.

The formulated optimization model has been applied to a hypothetical case study. The optimization problem has been solved by the Lagrange-multiplier method to find the minimum costs of control devices with safe exit gradient and uplift pressure.

The results of analysis have indicated that the use of an upstream blanket will reduce the uplift pressure and exit gradient. However, the use of a downstream blanket would increase the uplift pressure and exit gradient. Moreover, the use an upstream cut-off was very effective in decreasing the uplift pressure, while the downstream cutoff was very effective in decreasing the exit gradient. However, the use of a filter trench was very effective in decreasing both uplift pressure and exit gradient.

For a hydraulic structure with different control devices, the minimum total cost could be achieved when using a filter trench, while the maximum total cost was when a downstream cut-off is used.

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LIST OF symbols

Symbol	Definition	Dimension
A	area of the problem domain	L^2
a	area of floor base	L^2
b	length of the floor of the hydraulic structure	L
b_1	length of the upstream blanket	L
b_2	length of the downstream blanket	L
c	distance from upstream to the filter trench	L
c_1	cost of one cubic meter of the floor base material	unit cost
c_2	cost of one square meter of cut-off	unit cost
c_3	cost of one square meter of upstream blanket	unit cost
c_4	cost of one cubic meter of the filter trench	unit cost
d	depth of the cut-off	L
d_1	depth of the upstream cut-off	L
d_2	depth of the downstream cut-off	L
H^*	different ratio of piezometric head	-
H	different heads between upstream and downstream sides	L
h	pressure head (piezometric head) at any point in the problem domain	L
h_1	piezometric head at upstream	L

Symbol	Definition	Dimension
h_r	piezometric head at downstream	L
i	hydraulic gradient	-
I_c	critical exit gradient	-
I_e	exit gradient	-
[J]	Jacobian matrix	-
k_x	hydraulic conductivity in x-direction	L/T
k_y	hydraulic conductivity in y-direction	L/T
L_x, L_y	direction cosines	-
[N_i]	shape function matrix of element	-
n_e	number of element	-
Re	Reynolds's number	-
S	distance along the flow line	L
S_1	previous boundaries	-
S_r	impervious boundaries	-
T	depth of impervious layer	L
t	thickness of floor base	L
u	velocity in x-direction	L/T
v	velocity in y-direction	L/T
v_s	discharge velocity through the porous media	L/T
ω	velocity in z-direction	L/T
x, y	coordinate axes in the real region	-
\bar{x}	distance along downstream bed from toe of structure	L
w	width of filter trench	L
Z	objective function (total cost)	unit cost
z	depth of filter trench	L
ζ, η	local coordinates	-

CERTIFICATION

We certify that this thesis titled “**Optimum design of control devices for safe seepage under hydraulic structures**”, was prepared by “**Waked Hameed Hassan Al-Musawi**” under our supervision at Babylon University in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

Signature :

Name: Dr. Abdul-Hadi A. AL-Delewy

Date:

Signature :

Name: Dr. Abdul-Hassan K. Shukur

Date:

ABSTRACT

The coupling of the hydraulic structures design problems of a confined aquifer system with an optimization model has been presented in this research. The objective is to determine the optimal design of control devices in order to decrease the seepage under the hydraulic structures.

The present research employs the finite – element method to analyze seepage flow through porous media below hydraulic structures with blanket, cut-off and filter trench as seepage control devices. The effect of length and locations of the control devices were investigated to determine its optimum length and locations. Also, a set of curves is obtained showing the uplift pressure distribution and exit gradient.

The formulated optimization model is applied to a hypothetical case study; and solve the optimization problem by the Lagrange-multiplier method to found the minimum costs of control devices with safe exit gradient (used FS = ξ) and uplift pressure.

The research indicated the following:

1. Remarkable savings in materials and cost can be ensured by the use of the optimization procedure.
2. Use the upstream blanket reduced the uplift pressure and exit gradient, while the use of downstream blanket increase the uplift pressure and exit gradient under the hydraulic structures.
3. Use the upstream cut-off is very effect to decrease the uplift pressure and exit gradient, while the downstream cutoff is very effect to decrease the exit gradient under the hydraulic structures. Use filter trench very effect to decrease both uplift pressure and exit gradient.

ξ. For a hydraulic structure with different control devices, the minimum total cost could be given when used the filter trench, while the maximum total cost when used downstream cut-ff.

Appendix A

COMPUTER PROGRAM

A-1 General.

The next paragraph contains an explanation of the different parts of the (F.E.M) computer program used in this thesis.

A-2 The F.E.M Program.

The main program consists of number of basic subroutines as shown in Fig. (A-1). The basic subroutines are:

1- Input subroutine.

This subroutine inputs all the data required for the solution of the problem by dividing it into:

- a. Control data.
- b. Geometric data.
- c. Boundary condition.
- d. Input (θ , K_y , K_x) for each element as shown in Fig.(A-2).

This subroutine calls for another subroutines as follows:

• **Autmg Subroutine.**

This subroutine is used for generation of the geometric data required for the solution of the problem and reduced the required data .for the problem as shown in Fig.(A-3).

• **Check4 Subroutine.**

The goal of this subroutine calculates the front width of the front subroutine and checks the coordinates of the nodes as shown in figure. (A-4).

• **Gaussq Subroutine.**

The function of this subroutine is to generate the sampling points ($4*4$ Gauss points) and weighting factors according to the order of the numerical integration as shown in figure (A-5).

ϒ- Stifps Subroutine.

The objective of this subroutine calculates all the elements stiffness matrix as show in figure (A-ϖ). This subroutine calls for another subroutine as follows:

a) SFrϒ Subroutine,

This subroutine calculates the shape functions and their derivatives, as shown in figure (A-ϗ).

b) Jacobϒ subroutine.

This subroutine calculates the Cartesian coordinates of the Gauss points. The Jacobian matrix, and the inverse of the Jacobian matrix, and the Cartesian derivative of the shape function, as shown in figure (A-⊗)

c) Bmatps Subroutine

This subroutine calculates the matrix [**B**], as shown in figure (A-⊘)

d) DBE Subroutine.

This subroutine multiplies the matrix [**D**] (the matrix of the soil hydraulic conductivity coefficient) by [**B**] matrix as shown in figure (A-⊙)

ϓ- Front Subroutine.

This subroutine accomplish the assembly and deleting process of the element stiffness matrix and applying the B.C and the inverse substitution then find out the value of variables in the different nodes as shown in figure (A-⊚).

ϔ- Gradag Subroutine.

This subroutine calculates the hydraulic gradient in the x and y directions, and find out the resultant of hydraulic gradient and its direction in Gauss points for each element after knowing the piezometric head at the nodes, as shown in figure (A-⊛).

⊘- Gradan Subroutine.

This subroutine calculates the hydraulic gradient and its direction but at nodal points, as shown in figure. (A-⊜).

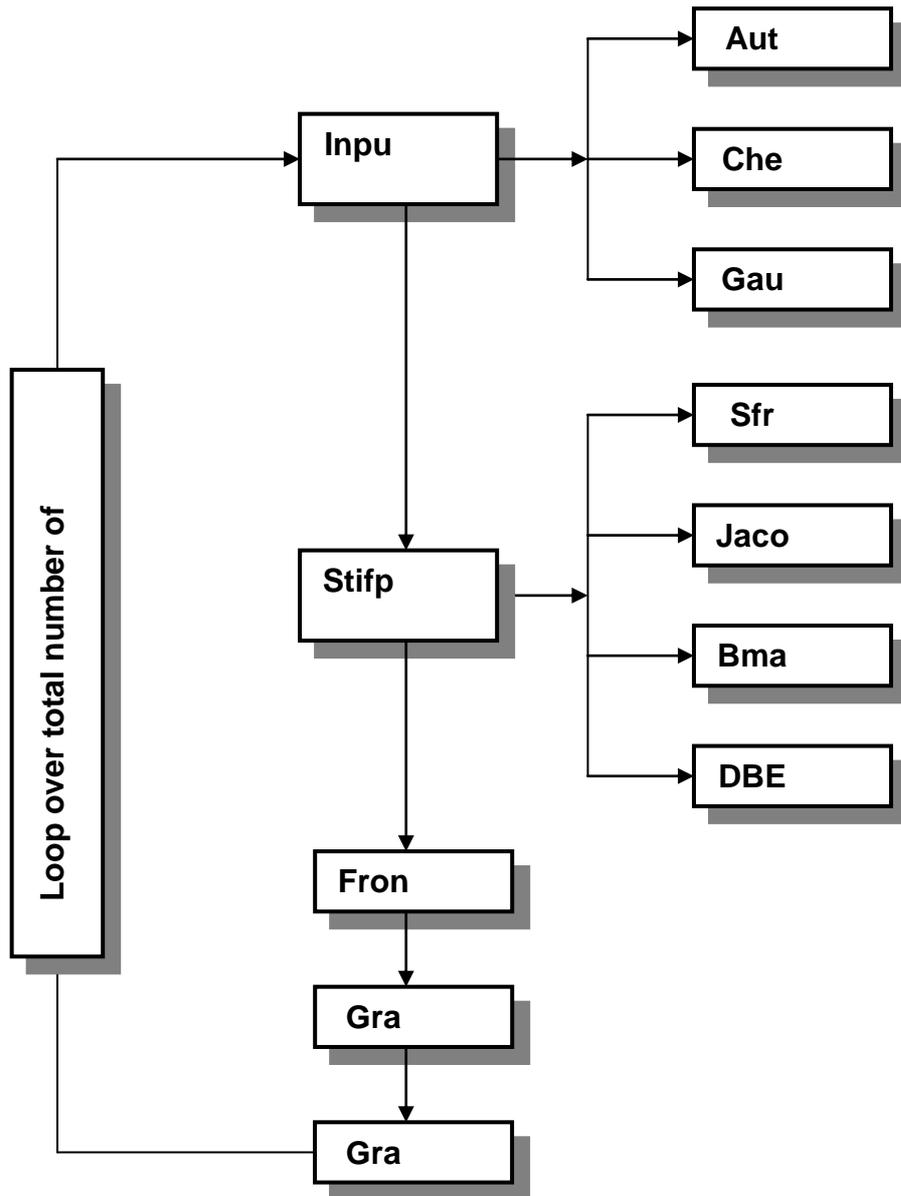


Fig.(A-1): General flowchart for the F.E.M computer program

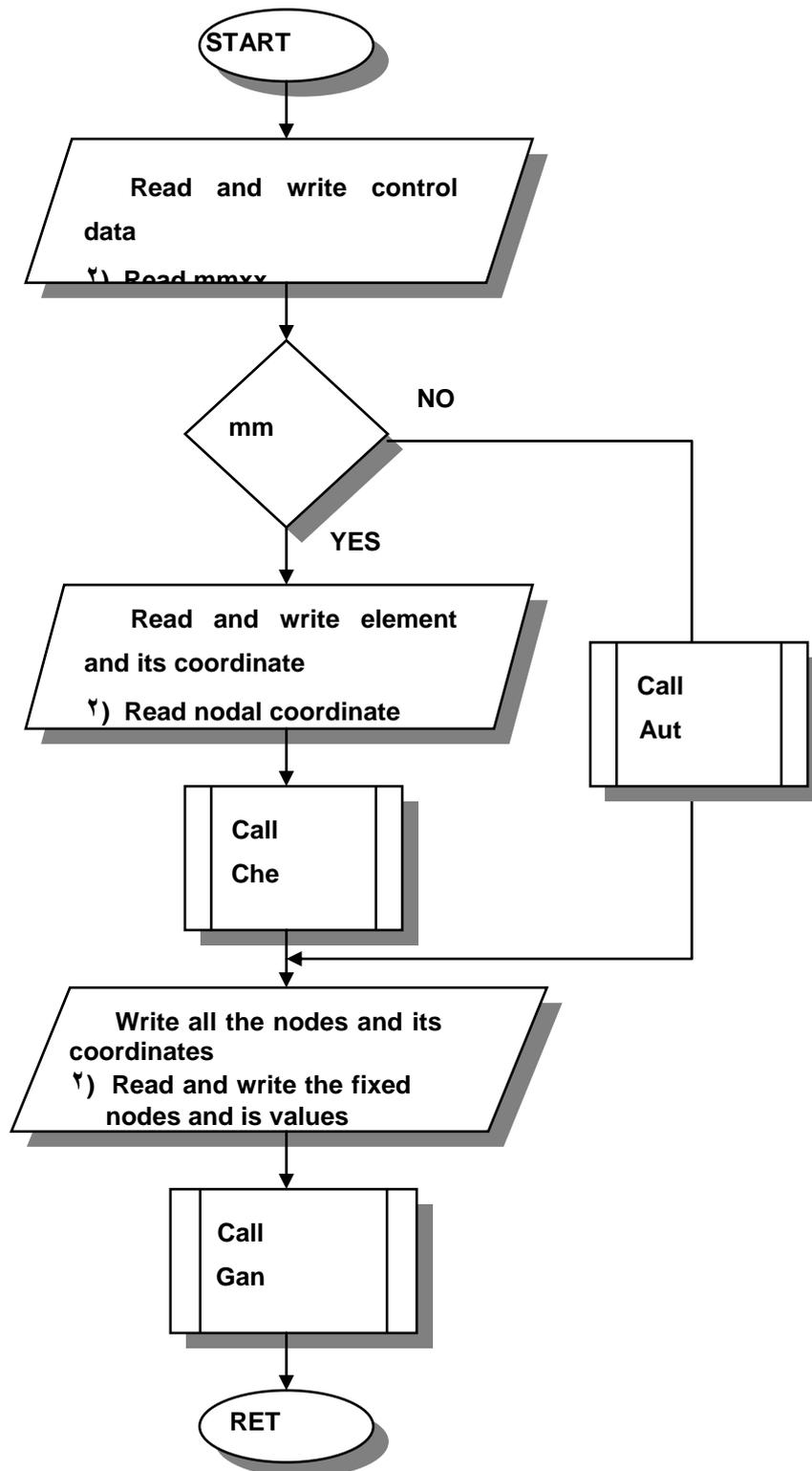


Fig.(A-2): Input subroutine flowchart

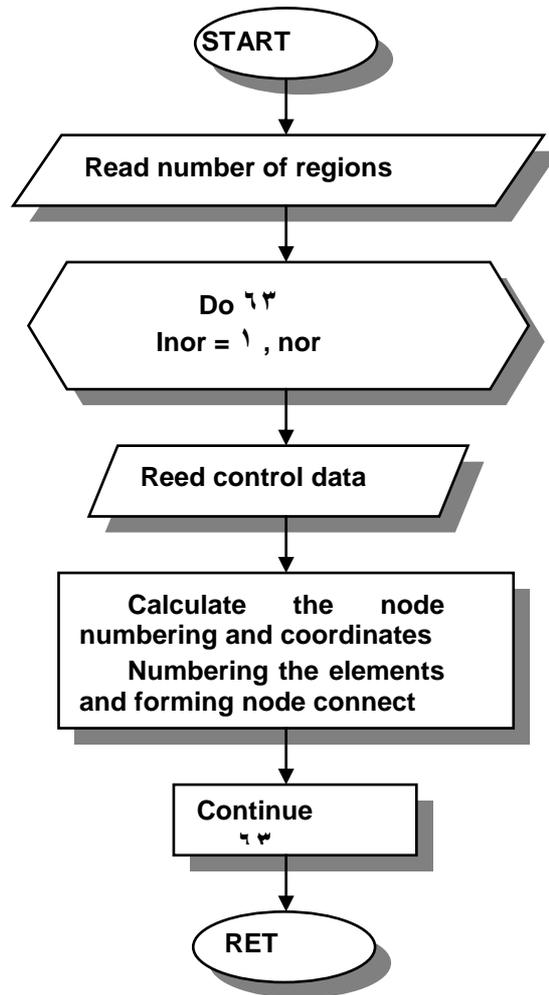


Fig.(A-٣): Autmg subroutine flowchart

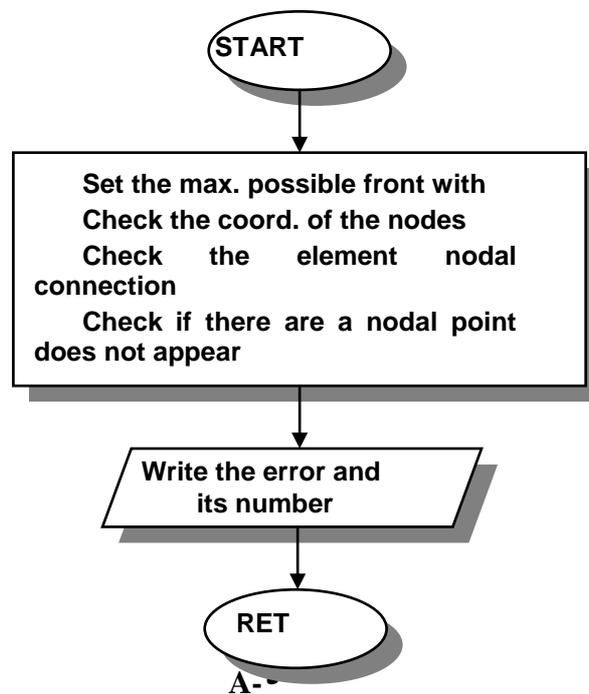


Fig.(A-4): Check γ subroutine flowchart

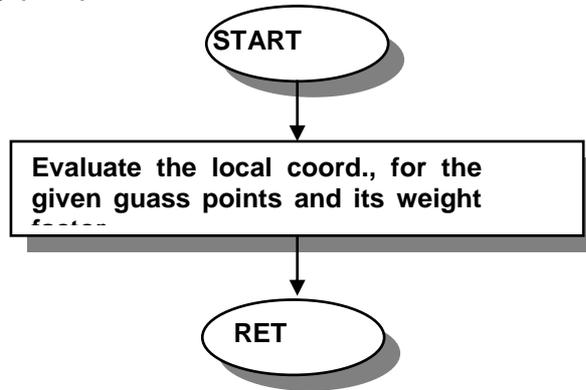


Fig.(A-5): Gassq subroutine flowchart

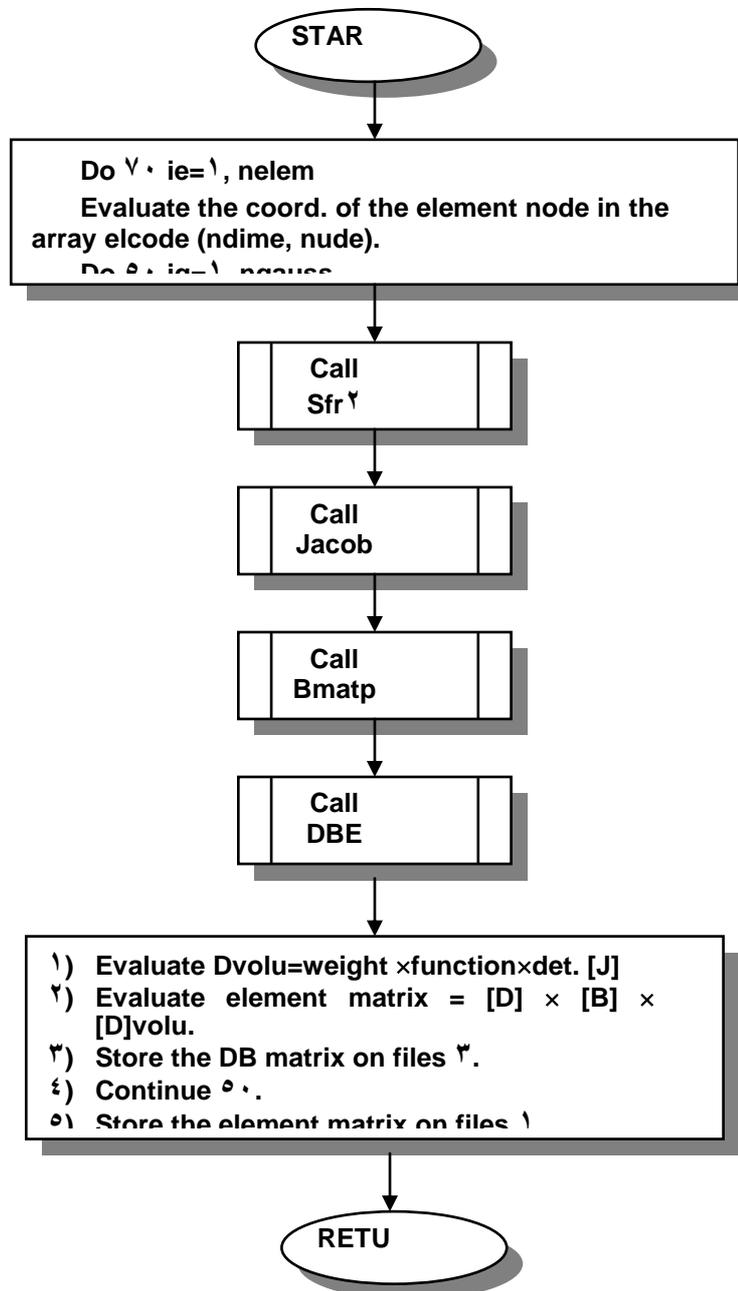


Fig.(A-٦): Sfifps subroutine flowchart.

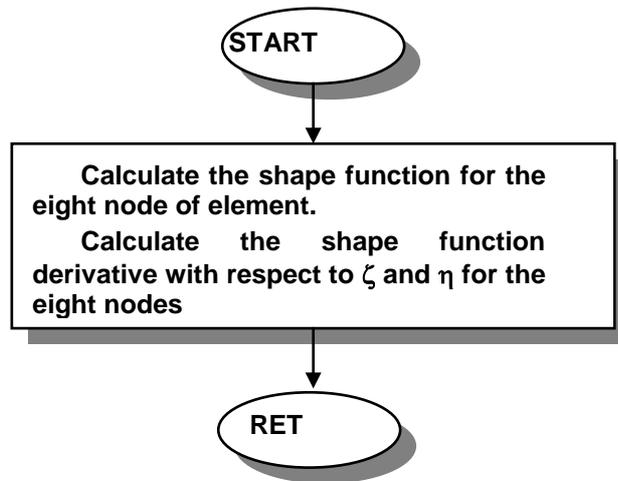


Fig.(A-٧): Ffr٧ subroutine flowchart

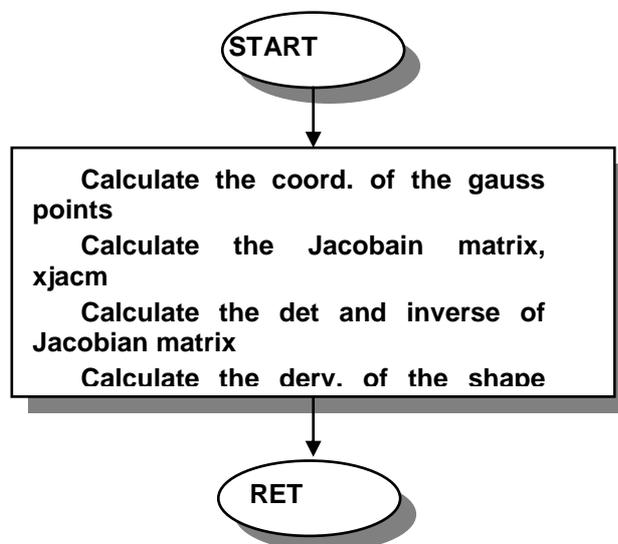


Fig.(A-٨): Jacob٧ subroutine flowchart

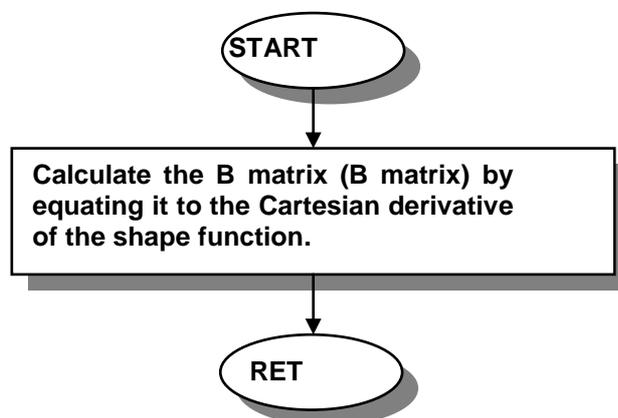


Fig.(A-9): Bmatps subroutine flowchart

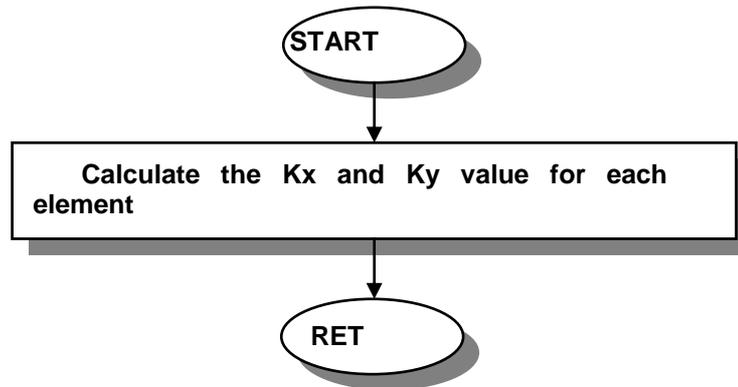


Fig.(A-10): DBE subroutine flowchart

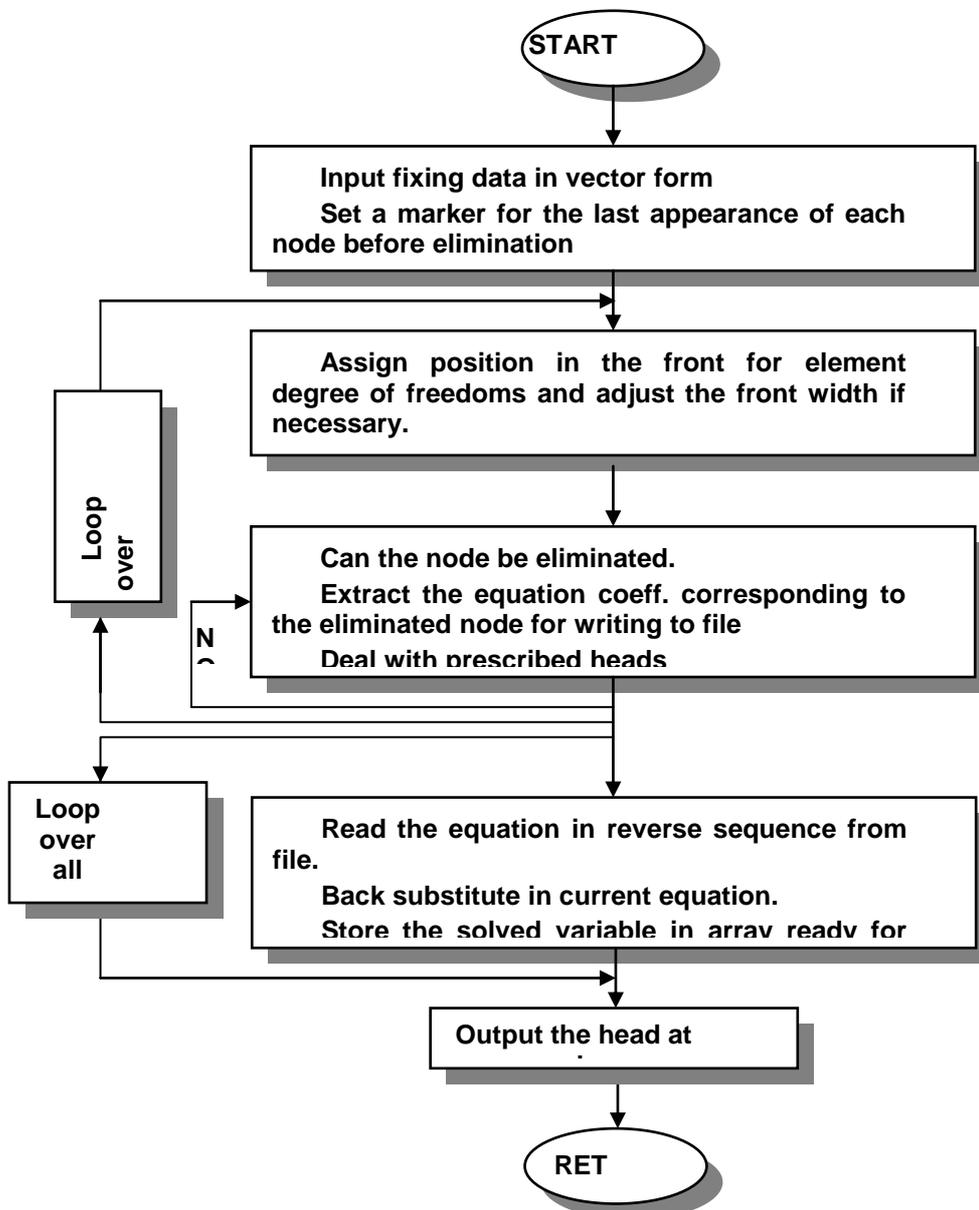


Fig.(A-11): Flowchart

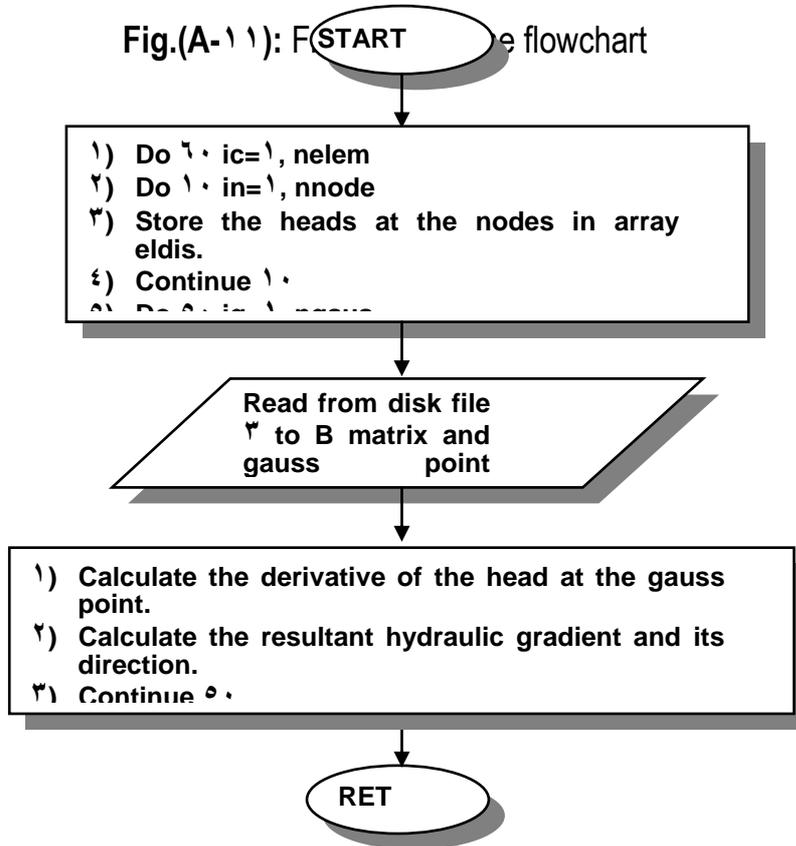


Fig.(A-12): Gradag subroutine flowchart

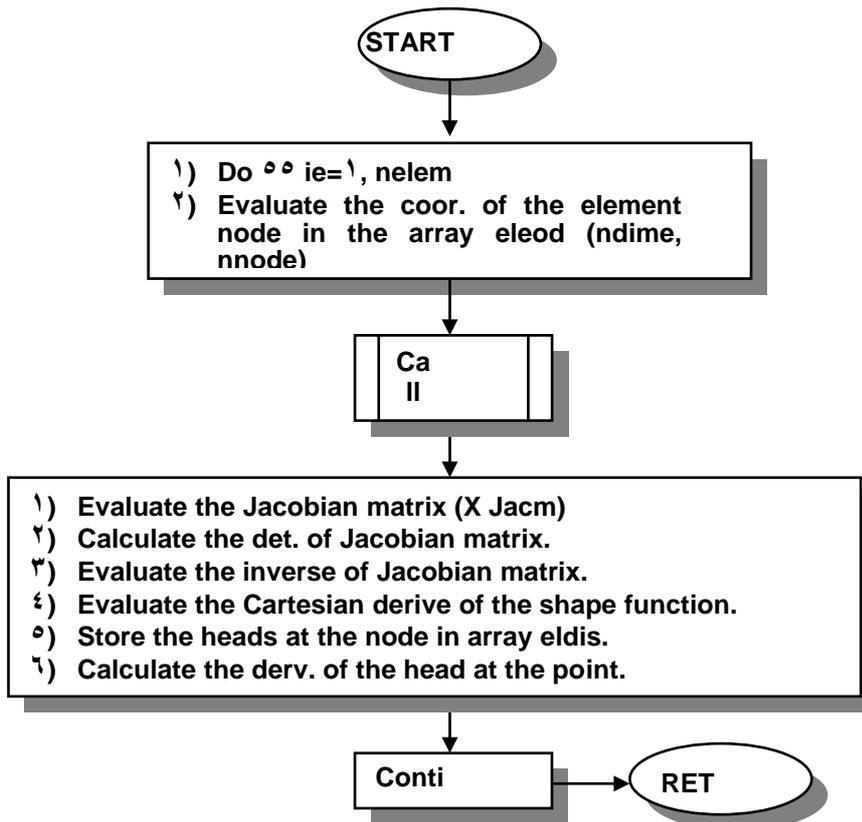


Fig.(A-۱۳): Gradan subroutine flowchart

CHAPTER ONE

INTRODUCTION

1.1: General

Optimum use of water nowadays cannot be overemphasized. Hydraulic structures are a specific type of engineering structures designed and executed in order to utilize it to control water and ensure the aforementioned objective. The hydraulic structures represent the important part of any flow network. Examples of such structures are dams, regulators, weirs, ... etc. The basic aim of these structures is to control the flow discharge and water levels.

The foundation of any hydraulic structure should be given the greatest importance in analysis and design as compared with other parts of the structure, because failure in the foundation would destroy the whole structure.

One of the most important problems that causes damage to hydraulic structures is seepage under the foundations, which occurs due to the difference in water level between the upstream and downstream sides of the structures. The water seeping underneath the hydraulic structure endangers the stability of the structure and may cause failure.

Hydraulic structures may either be found on an impervious solid rock foundation or on pervious foundation. The problem of seepage becomes more serious with the latter case, particularly when the pervious foundation is of a fine texture.

Water seeping under the base of a hydraulic structure starts from the upstream side and tries to emerge at the downstream end of the impervious

floor. If the exit gradient is greater than the critical value for the foundation, a phenomenon called piping may occur due to the progressive washing and removal of the fines of the subsoil [Terzaghi and Peck (1967)].

The uplift force which occurs as a result of the water seeping below the structure exerts an uplift pressure on the floor of the structure. If this pressure is not counterbalanced by the weight of the floor, the structure may fail by rupture of a part of the floor.

The problems of piping and uplift are practically tackled through a variety of methods of seepage control, aiming at ensuring the safety of the respective structure and at the same time saving the possibly-seeping water. The common provisions in this respect are:

1. Upstream blanket.
2. Upstream or / and downstream cut-offs.
3. Subsurface drain on the downstream side.
4. Filter trench on the downstream side.
5. Weep holes, or pressure relief wells on the downstream side.

The use of these devices in controlling the seepage under hydraulic structures depends on the nature of foundation, the type of structure, and the cost of the devices to be used.

1.2: Objectives of the Research

The main objectives of this research can be summarized as follows:

1. Evaluating the effect of using different seepage control devices on the uplift pressure and exit gradient.
2. Finding the optimum design (hydraulic and economical) for the control devices in order to decrease the seepage under hydraulic structures.
3. Study the effect of using different combinations of seepage control devices on the uplift pressure and exit gradient.

۱.۳: Scope of the Thesis

To meet the above mentioned objectives, the present research is divided into the following tasks:

۱. A review of literature concerning the subjects involved is presented in Chapter Two.
۲. In Chapter Three some of the basic principles of flow through porous media are clarified and also the basic and relevant aspects of the finite element theory are presented.
۳. Formulation of the optimization model is presented in Chapter Four.
۴. Analyses of the results are displayed in Chapter Five.
۵. Conclusions obstructed from the research are presented in Chapter Six. The recommendations for future studies are contained therein too.

CHAPTER TWO

REVIEW OF LITERATURE

२.१: The Seepage Problem

Seepage under hydraulic structures and the uplift pressure associated with it comprise a very important class of confined-flow problems. Consequently, the control of seepage problems in hydraulic structures found on permeable soils determines the final design and dimensions of these structures.

There are different methods to find the peizometric head distribution under hydraulic structures from which the hydraulic gradient is computed. Examples of such methods are empirical, analytic, and numerical methods.

२.१.१: Empirical Methods

According to Bligh's creep method {quoted in [Garg (१९९८)]}, the percolating water follows the outline of the base of the foundation of the hydraulic structure. In other wards, water creeps along the bottom contour of the structure.

The length of the path thus traversed by water is called the length of the creep, and the method has been called the line-of-creep method. Further, it is assumed in this theory that loss of head is proportional to the creep length.

According to Bligh, the safety against piping can be ensured by providing sufficient creep length, given by:

$$L = C \times H \quad \text{.....}(२-१)$$

where:

L = length of the path;

C = Bligh's coefficient for the respective soil, which depends on type of soil.

H = head difference between upstream and downstream sides.

Different values of (C) for different types of soils are given in Table (۲-۱).

Table (۲-۱): Values of Bligh's Safe Hydraulic Gradients [After Garg, (۱۹۹۸)].

Soil No.	Type of soil	Bligh's coeff., C	Safe hydraulic gradient should be less than
۱	Fine micaceous sand	۱۵	$۱ / ۱۵$
۲	Coarse grained sand	۱۲	$۱ / ۱۲$
۳	Sand mixed with boulder and gravel, and for loam soil	۵ to ۹	$۱/۵$ to $۱ / ۹$
۴	Light sand and mud	۸	$۱ / ۸$

In ۱۹۳۴, Professor Lane introduced his empirical "weighted – creep" theory [Leliavsky (۱۹۷۹)]. In this theory it is assumed that the line of flow will follow the line of contact between a dam and its foundation. However, the vertical contact is considered more effective than the horizontal contact, and the coefficient (C) is different from that given by Bligh. After a study of the design of a large number of dams in the United States and of the failure of some others, Lane recommended that a unit horizontal length of contact be considered to be one third as effective as a unit length of vertical contact. That is:

$$L_w = \frac{1}{3}L_h + L_v \quad \text{.....(۲-۲)}$$

where:

L_w = weighted creep-length;

L_h = sum of all horizontal contacts and all the sloping contacts less than (۴۵°);

L_v = sum of all vertical contacts and all the sloping contacts greater than (ξ°).

To ensure safety against piping, (L_w) must not be less than ($C_1 H$), in which (H) is the difference of water levels on the upstream and the downstream and (C_1) is an empirical coefficient (ranges between 1.0 to 1.5 , depending on the nature of the soil) [Rao, and Abduil Khader (1964)].

By the Khosla's method {quoted in [Grishin (1982)]}, Khosla proved that seeping water through permeable soils follows parabolic streamlines and not along the underside profile of the impervious floor as envisaged by Bligh. He proved that the flow of seeping water takes place according to Laplace equation.

Khosla solved the actual profile of the hydraulic structure by an empirical method known as the method of independent variables. According to this method, the actual complex profile is broken into a number of simple profiles known as elementary profiles. Each elementary profile is independently amenable to mathematical treatment and, thus, it is treated independently.

Khosla's theory is rather complex and needs many requirements to be applicable. The exit gradient at the downstream of the hydraulic structure from Khosla's method can be calculated from the Equation:

$$I_e = \frac{H}{d} \cdot \frac{1}{\pi \sqrt{\lambda}} \quad \dots\dots(2-3)$$

where;

I_e = exit gradient;

H = head difference between upstream and downstream of the hydraulic structure;

d = depth of downstream cut-off;

$$\lambda = \frac{1 + \sqrt{1 + \alpha^2}}{2}$$

$$\alpha = \frac{b}{d}$$

b = length of the hydraulic structure floor.

Polubarinove-Kochina (1902) {quoted in [Polubarinova-Kochina (1962)]} obtained the complete solution for an impervious structure resting on the surface of a flow domain of infinite depth with a horizontal floor, by using the velocity components and complex velocity transformation at the boundaries of the flow region. The magnitude and distribution of the exit gradient along the distance downstream from the hydraulic structure is computed as follows:

$$I_e = \frac{H}{\pi \sqrt{\bar{x}^2 - (b/2)^2}} \quad \bar{x} > b/2 \quad \dots\dots(2-4)$$

where:

\bar{x} = distance from the center of the hydraulic structure floor along the downstream direction; other parameter are as defined before.

There are many methods in which a derived relationship to calculate the magnitude of the hydraulic gradient can be obtained; however, their applications are limited because of the complexity and limitations involved. Some of these methods have been presented in detail in [Harr (1962)].

2.1.2: Analytical Methods

Bennett (1946) introduced a mathematical analysis of seepage through natural or artificial blankets of relatively impervious soil overlying a pervious foundation.

Chawla (1976) determined the effect of a vertical drain (the width equal to zero) of any dimension located anywhere, with a cut – off at the end of a flat floor foundation.

A solution has been obtained using conformal mapping. His results indicate that the uplift pressure reduces considerably along the structure with the provision of drain even for very small lengths; the uplift pressure, in general, decreases on the downstream side as the drain is moved from upstream to downstream.

Chawla (1976) has also obtained an exact solution using conformal mapping for seepage below a flat floor with two cut-offs and a horizontal intermediate filter (the depth equal to zero) located anywhere between the two cut-offs. His results indicate that the uplift pressure and the exit gradient decrease with decrease in the depth of pervious strata, and the filter increases the stability of the structure and increases the stability of soil particles against piping.

Since the real dimension of the filter trench is two-dimension, hence these analytical results do not represent the real situation.

2.1.3: Numerical Methods

As a result of the difficulties which were met through empirical and analytical methods, it was resorted to numerical methods. Such methods provide obtaining the required results with a good accuracy such that they are well comparable to the results of the analytical solutions.

The two common numerical methods applied to flow through porous media are the finite-differences method and the finite – elements method. For certain complex problems, the finite element method is preferable to the finite difference method for the following reasons [Khsaf (1998)]:

١. In the finite-element method, anisotropy and non-homogeneity are taken into account quite easily in comparison with the finite-difference method.
٢. The boundary conditions are easily handled by the finite element method whereas special formulas must be developed for each condition with the finite difference method.
٣. In the finite element method there is no restriction for the mesh size where the size of each element can be varied independent of other elements. For example, small elements may be used in areas of rapid changes whereas large elements may be used where these changes are less severe.

Therefore, considering accuracy and simplicity at the same time, the finite element method is considered to be the best in solving different real-life problems.

Conner and Brebbia, (١٩٧٦) solved the seepage problem under a hydraulic structure with two piles resisting on isotropic soil by using finite element method in order to find the distribution of pressure under the base of the structure.

Cechi and Mancino (١٩٧٨) and Zienkiewicz (١٩٨٢) have given application examples of simple cases for seepage under hydraulic structures by using triangular grid of finite element to find the head distribution under the structures.

Nassir (١٩٨٤) has applied the finite element method in a wider range. He analyzed the seepage problems under hydraulic structures by using different shapes of the finite elements in order to choose the best solution that gives the most exact results. He also studied the effect of an inclined sheet pile on the piezometric head distribution and exit gradient. The

method gave good results as concerning the optimum angle of the inclined sheet pile and its position. His results led to a relationship for the exit gradient which is the same as the Khosla's formula, Eq. (3-3), that is:

$$I_e = \frac{H}{\pi d} \frac{1}{\sqrt{b/d}} \quad \text{.....(3-5)}$$

where all the symbols are as defined before.

Hatab (1987) and Ijam and Hatab (1993) have applied the finite element method in the analysis of seepage through homogeneous and isotropic porous media below hydraulic structures with horizontal filters. They also studied the stability of the structures with vertical drains. As a result, useful design information was presented. The results indicated the advantage of using a filter in reducing the uplift pressure and hydraulic gradient.

Ijam and Nassir (1988) used the finite element method to study seepage under hydraulic structures for isotropic soil with two cut-offs. Their results presented a set of uplift pressure distribution curves, whereas the efficiency of the cut-offs was defined. Another relationship was also obtained to calculate the factor of safety against failure due to heave.

Ihsan (1989) studied the use of the finite element method in the analysis of seepage under hydraulic structures in homogeneous and isotropic soils, once with a filter trench and another with pressure relief wells. The results were presented in the form of curves showing the distribution of the piezometric head and the hydraulic gradient. The results were good as compared with analytical and experimental results.

Khsaf (1998) used the finite element method to analyze seepage in isotropic, anisotropic, homogeneous and non-homogeneous soil foundations underneath hydraulic structures provided with flow control

devices. He studied a hydraulic structure with an upstream and / or downstream cut-off(s) with various permeable-layer thickness and various depths and locations of cut-offs. He found that the location, number and depth of the cut-offs have a noticeable effect in reduction the uplift pressure.

۲.۲: The Optimization Problem

Optimization is a mathematical technique that selects the best solution from a set of feasible alternatives. Many studies were used the optimization model in hydraulic structures problems. Some of them are as given below:

Gill (۱۹۸۰) used the theoretical ideas presented by Bennet (۱۹۴۶) criteria for optimal design of rectangular, triangular and trapezoidal blankets. It is shown that a triangular blanket is more effective, hence more economical than rectangular and trapezoidal blankets, in reducing under seepage and uplift force. The uplift force is reduced in the same proportion as the seepage.

Jawad (۱۹۹۶) presented the coupling of the hydraulic behavior of an unconfined aquifer system with an optimization model.

Hamed (۱۹۹۶) used Rosenbrock constrained optimization and Sequential Unconstraint Minimization Technique methods in order to solve the non-linear programming problem. He found the optimum design of barrage floor (area of concrete and reinforcement).

CHAPTER THREE

THEORETICAL BACKGROUND

3.1: The basic equation of flow through a porous medium

The flow medium for seeping water under hydraulic structures is a saturated porous soil. Groundwater flow depends on many factors such as soil properties, fluid characteristics, flow geometry, time, and boundary conditions. The most important properties for such a medium to be taken into consideration are porosity and permeability. It is possible to analyze groundwater flow from knowing the flow properties which are represented by the velocity, pressure, and temperature; these properties represent the dependant variables which vary with both space and time.

In study the flow of groundwater through porous media, two matters concerning the problem must be specified in advance. The first is the type of flow through the porous media; the second is the boundary conditions of the flow domain. Seepage beneath hydraulic structures can be considered as a steady confined flow [Harr and Deen (1961)].

Flow through saturated porous media is generally governed by Darcy's law [Harr (1962)]:

$$V_s = K i \quad \text{.....(3-1)}$$

where:

V_s = discharge velocity through the porous medium; (L/T);

K = hydraulic conductivity, (L²/L².T=L/T);

i = hydraulic gradient = $- dh / ds$;

h = piezometric head = $(p / \gamma_w) + z$, (L);

p = hydrostatic pressure, (F / L²);

γ_w = unit weight of water, (F/L³);

z = elevation head, (L);

S = distance along the flow line, (L).

Equation (3-1) represented the linear relationship between the discharge velocity and the hydraulic gradient.

Darcy's law is valid when the special form of the Reynolds number (Re) for sub-surface flow is equal to or less than unity [Harr (1962)]. That is:

$$Re = \frac{V d \rho}{\mu} \leq 1.0 \quad \dots\dots(3-2)$$

where:

V = flow velocity; (L/T);

d = average diameter of soil particles, (L);

ρ = fluid density, (F T³ / L³);

μ = coefficient of viscosity, (FT / L²).

Fortunately, most of practical seepage under hydraulic structures is laminar, (i.e., ($Re < 1$) [Grishin (1967)].

3.2: The general equation of flow

The components of the seepage velocity through porous media according to the general form of Darcy's Law for three-dimensional flow are [Freeze and Cherry, (1979)]

$$u = -k_x \frac{\partial h}{\partial x} \quad \dots\dots(3-3a)$$

$$v = -k_y \frac{\partial h}{\partial y} \quad \dots\dots(3-3b)$$

$$w = -k_z \frac{\partial h}{\partial z} \quad \dots\dots(3-3c)$$

where:

u, v, ω = velocity components in the x -, y -, and z -direction, respectively, (L/T);

k_x, k_y, k_z = hydraulic conductivity in the x -, y -, and z - directions, respectively; (L/T).

The continuity equation for three-dimensional and incompressible flow is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} = 0 \quad \text{.....}(\text{3-}\xi)$$

Substituting Darcy's Law, Eq.(3-3), in Eq.(3-ξ) results in:

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial h}{\partial z} \right) = 0 \quad \text{.....}(\text{3-}\omicron)$$

For a homogenous and isotropic soil, the hydraulic conductivity is equal in all directions, that is:

$$k_x = k_y = k_z = k$$

Thus, Eq.(3-omicron) could be written as:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{.....}(\text{3-}\upsilon)$$

Equation (3-υ) is the Laplace equation; it is similar to Laplace equation of velocity potential for ideal fluid flow.

Most of the problems of seepage control analysis beneath hydraulic structures represent two-dimensional flow. Hence, Eq.(3-υ) for two-dimensional flow becomes:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad \text{.....}(\text{3-}\eta)$$

Consequently, for conditions of steady-state, laminar flow, the seepage pattern can be completely determined by solving Eq.(3-8), subject to the boundary conditions of the flow domain.

3.3: Boundary conditions

The boundary conditions should be specified before starting the solution. For the steady state of a confined flow the boundary conditions are defined as follows:

3.3.1: Reservoir boundaries

The water depth (h_o) above such boundaries are always known; so, the pressure (p) at any point on these boundaries would be:

$$p = \gamma_w h_o \quad \text{.....(3-8)}$$

Therefore, the piezometric head distribution along the reservoir boundaries (S_r) is constant; that is:

$$h = h_o = \frac{p}{\gamma_w} + z \quad \text{.....(3-9)}$$

For this reason, all the reservoir boundaries are equipotential lines.

3.3.2: Impervious boundaries

At impervious boundaries, (S_r), the water cannot seep through the surface, so that the velocity component normal to the boundary, (V_n), must be equal to zero. That is:

$$V_n = k_x \left(\frac{\partial h}{\partial x} \right) L_x + k_y \left(\frac{\partial h}{\partial y} \right) L_y = 0 \quad \text{.....(3-10)}$$

where (L_x) and (L_y) are the direction cosines of the normal vector on the surface with the directions (x) and (y), respectively.

These boundaries represent streamlines of constant stream functions.

Figure (3-1) illustrates the boundary conditions of a typical problem of seepage under a hydraulic structure.

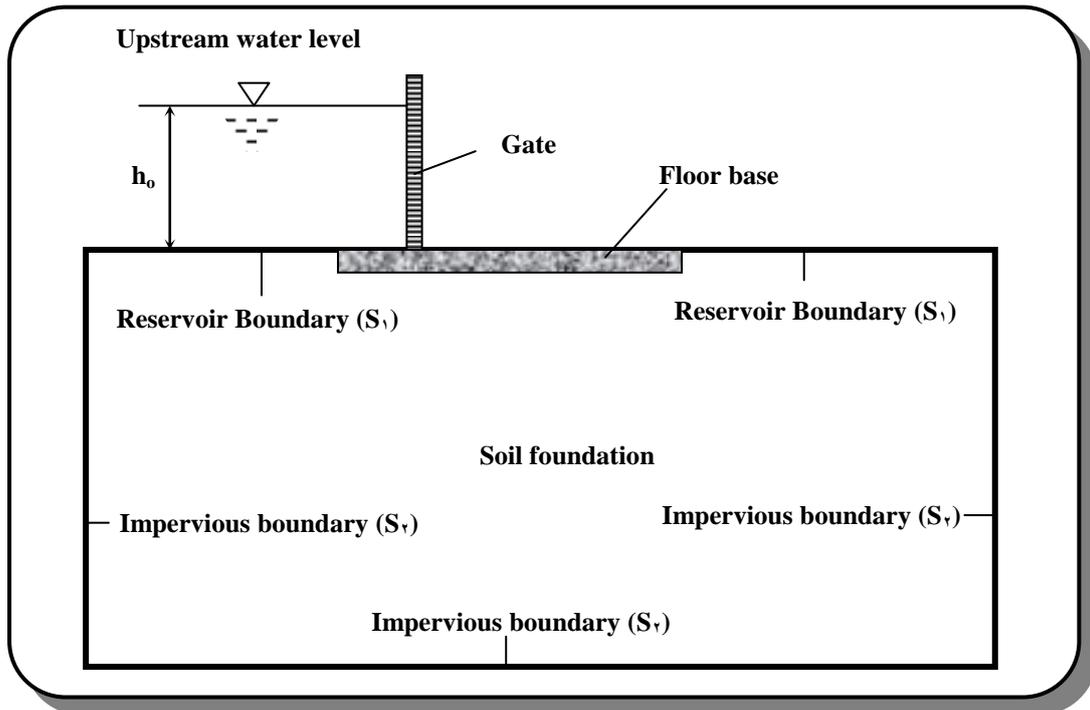


Fig.(۳.۱): Boundary conditions of a typical problem of seepage under a hydraulic structure.

3.4: The finite-elements formulation

A mathematical model consists of a set of differential equations that are known to govern the problem under consideration. For a realistic situation, it is usually necessary to solve the mathematical model approximately by using numerical techniques. Numerical models have been the favored types of models for studying groundwater flow by either the finite-element method or the finite-difference method [Herbert and Mary, (1982)].

In the previous parts, review of the basic principles of the seepage flow through porous media has been reviewed. The governing equation for three-dimensional flow (3-D) has one dependent variable, the piezometric head (h), and three independent variables x, y and z.

This chapter presents the principles of the finite element method and the finite element formulations for two dimensional flow through porous media using Galerkin weighted residual method.

The finite element method is a very powerful and relatively modern computation tool. It requires the use of digital computer because of the large number of computations involved [Stas (1986)].

The basic idea of the finite element method is to discretize the problem domain to sub-domains or finite elements. These elements may be one, two, or three-dimensional and jointed to each other by nodes existing on element boundaries. The nodes are regarded as part of the element. After the discretization process, the behavior of the field variable on each element is represented approximately by a continuous function depending on nodal values of the field variable as follows:

$$h^e = \sum_{i=1}^n N_i h_i \quad \dots (3-11)$$

where:

h^e = approximate solution for piezometric head distribution in the element (e), (L);

N = shape function of the element (e);

h_i = nodal values of head, of the element (e), (L);

n = number of nodes in the element (e).

It is possible to write Equation (3-11) in matrix form as follows [Zienkiewicz, (1966)]:

$$h^e = [N_i] \{h_i\} \quad \dots(3-12)$$

where;

$[N_i]$ = shape function matrix;

$\{h_i\}$ = vector matrix of nodal values.

The approximate solution for head variation, h , over the whole domain is given as follows:

$$h = \sum_{e=1}^{n_e} h^e = \sum_{e=1}^{n_e} \sum_i^n N_i h_i \quad \dots(3-13a)$$

or

$$h = \sum_{e=1}^{n_e} [N_i] \{h_i\} \quad \dots(3-13b)$$

where (n_e) is the total number of elements in the problem domain.

3.4.1: The weighted residual method

The weighted residual method is a technique which can be used to obtain approximate solutions to linear and non-linear differential equations. If this method is used, the finite – element equations can be derived directly from the governing differential equation of the problem [Rao (1982)].

If (A) is a problem domain, and (h) is the field variable then the governing equation can be written as follows:

$$F(h) = \cdot \quad \text{in } A \quad \dots(\text{3-14})$$

If the approximate solution as (h_a) then, by substituting it in Eq.(3-14), the equation does not equal zero, but there is a residual (R):

$$F(h_a) = R \neq 0 \quad \dots(\text{3-15})$$

The best solution is the one which makes this residual a minimum or maintains it small at all points of the domain. In order to reach this aim, Eq.(3-15) should be integrated on the problem domain after weighting by a certain function and should equal zero as follows:

$$\int_A W_j R \, dA = 0 \quad \dots(\text{3-16a})$$

or

$$\sum_1^{n_e} \int_{A^e} W_j R^e \, dA = 0 \quad \dots(\text{3-16b})$$

where:

W_j = weighted function;

R^e = element residual.

There are different approaches which can be used, depending on the choice of the weighted function. The best is the one which is known as Galerkin technique where the weighted function is taken equal to the shape function (i.e., $W_j = N_j$) [Seglind (1976)].

3.4.2: The Galerkin principle

The Galerkin principle is applied to derive the elements matrix. From equation (3-16):

$$h^e = \sum_{i=1}^n N_i h_i \quad \dots\dots(\text{3-17})$$

where (h_i) is the value of the piezometric head in node (i).

For a two-dimensional flow, the general equation for seepage in porous media, Eq.(3-6), becomes:

$$\frac{\partial}{\partial x} \left[k_x \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[k_y \frac{\partial h}{\partial y} \right] = 0 \quad \dots\dots(\text{3-18})$$

Substituting Eq.(3-17) in Eq.(3-18) gives:

$$\frac{\partial}{\partial x} \left[k_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i h_i \right] + \frac{\partial}{\partial y} \left[k_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i h_i \right] = R^e \neq 0 \quad \dots\dots(\text{3-19})$$

Now, by applying Galerkin principle and substituting Eq.(3-19) in Eq.(3-16b) yields:

$$\sum_1^{ne} \left[\int_{A^e} N_j^e \left[\frac{\partial}{\partial x} \left(k_x \frac{\partial}{\partial x} \sum_{i=1}^n N_i h_i \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial}{\partial y} \sum_{i=1}^n N_i h_i \right) \right] dA \right] = 0 \quad (\text{3-20})$$

where:

$$dA = dx .dy; \quad (j=1, 2, \dots, n)$$

n = number of nodes for each element.

To reduce continuity requirements for the shape function, (N), from (C^1 -continuity) to (C^0 -continuity), integration by parts with Green's theorem is applied to the second order derivatives terms, where (C^1) and (C^0) are the continuity for the shape function for the first and zero stage, respectively [Burnett, (1987)].

Accordingly, the first term of Eq.(3-20) will be:

$$\int_{A^e} N_j^e \frac{\partial}{\partial x} \left(k_x \frac{\partial}{\partial x} \sum_1^n N_i h_i \right) dA = \int_S N_j^e k_x \frac{\partial}{\partial x} \sum_1^n N_i h_i dy - \int_{A^e} \frac{\partial N_j^e}{\partial x} k_x \frac{\partial}{\partial x} \sum_1^n N_i h_i dA$$

.....(3-21)

The second term of Eq. (3-20) will be:

$$\int_{A^e} N_j^e \frac{\partial}{\partial y} \left(k_y \frac{\partial}{\partial y} \sum_1^n N_i h_i \right) dA = \int_S N_j^e k_y \frac{\partial}{\partial y} \sum_1^n N_i h_i dx - \int_{A^e} \frac{\partial N_j^e}{\partial y} k_y \frac{\partial}{\partial y} \sum_1^n N_i h_i dA$$

.....(3-22)

Substituting Eqs.(3-21) and (3-22) in Eq.(3-20) results in:

$$\sum_1^{n_e} \left[\int_{A^e} - \left(\frac{\partial N_j}{\partial x} k_x \frac{\partial}{\partial x} \sum_1^n N_i h_i + \frac{\partial N_j}{\partial y} k_y \frac{\partial}{\partial y} \sum_1^n N_i h_i \right) dA \right] + \int_S N_j k_n \frac{\partial}{\partial n} \sum_1^n N_i h_i ds$$

.....(3-23)

where $(S = S_1^e + S_2^e)$ represents the surface boundaries of the element.

The boundary conditions are:

$$\text{\textcircled{1}}: (h=h_0) \quad \dots(3-24a)$$

on (S_1) , which represents the reservoir boundaries; and

$$\text{\textcircled{2}}: \left[k_x \frac{\partial h}{\partial x} L_x + k_y \frac{\partial h}{\partial y} L_y \right] = 0 \quad \dots(3-24b)$$

on (S_2) , which represents the impermeable boundaries.

By applying the finite-elements method to Eq.(3-24b), it becomes:

$$k_x \frac{\partial}{\partial x} \sum_1^n N_i h_i L_x + k_y \frac{\partial}{\partial y} \sum_1^n N_i h_i L_y = R^e = 0 \quad \dots(3-25)$$

where (R^e) is the element boundary residual.

Using the Galerkin weighted residual method, Eq.(3-25) becomes:

$$\sum_1^{n_e} \left[\int_{S_2^e} \left(N_j k_x \frac{\partial}{\partial x} \sum_1^n N_i h_i L_x + N_j k_y \frac{\partial}{\partial y} \sum_1^n N_i h_i L_y \right) ds \right] = 0 \dots (\text{3-26})$$

where: $dx = L_x ds$, and $dy = L_y ds$

Multiplying Eq.(3-23) by (-1), then adding it to Eq.(3-26) gives:

$$\sum_1^{n_e} \left[\int_{A^e} \left(\frac{\partial N_j}{\partial x} k_x \frac{\partial}{\partial x} \sum_1^n N_i h_i + \frac{\partial N_j}{\partial y} k_y \frac{\partial}{\partial y} \sum_1^n N_i h_i \right) dx dy - \int_{S_1^e} \left(N_j k_x \frac{\partial}{\partial x} \sum_1^n N_i h_i L_x + N_j k_y \frac{\partial}{\partial y} \sum_1^n N_i h_i L_y \right) ds \right] = 0 \dots (\text{3-27})$$

and in matrix form:

$$\sum_1^{n_e} [K^e] \{h_i\} = 0 \dots (\text{3-28})$$

where $[K^e]$ represents the element matrix:

$$[K^e] = \int_{A^e} [B^e]^T [D^e] [B^e] dx dy = 0 \dots (\text{3-29})$$

where:

$$[B^e] = \begin{bmatrix} \frac{\partial N_1^e}{\partial x} & \frac{\partial N_2^e}{\partial x} & \dots & \frac{\partial N_n^e}{\partial x} \\ \frac{\partial N_1^e}{\partial y} & \frac{\partial N_2^e}{\partial y} & \dots & \frac{\partial N_n^e}{\partial y} \end{bmatrix}$$

$$[D^e] = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

and from assemblage:

$$[K] \{h\} = \dots (\text{3-30})$$

where, $[K]$ is the global matrix $= \sum [K^e]$

The assembled equation, Eq. (3-30), is solved using a frontal solution because of its efficiency in computers storage requirement [Irons (1970)].

3.9: The Isoparametric quadratic element

The isoparametric concept allows any arbitrary geometry to be closely approximated, thereby minimizing any error associated with modeling the geometry and without resorting to the use of fine mesh along the boundaries. Linear elements have straight edges all round while quadratic elements have edges varying as parabolas. For curved boundaries it is thus more desirable to use a quadratic element which is known as “higher order isoparametric element”. The basic idea under lying the isoparametric element is to use the same shape or geometry as well as the field variable (representing the piezometric head in this research) within the element. In the present work two-dimensional quadratic isoparametric elements are used which have eight nodes, as shown in Fig.(3-2).

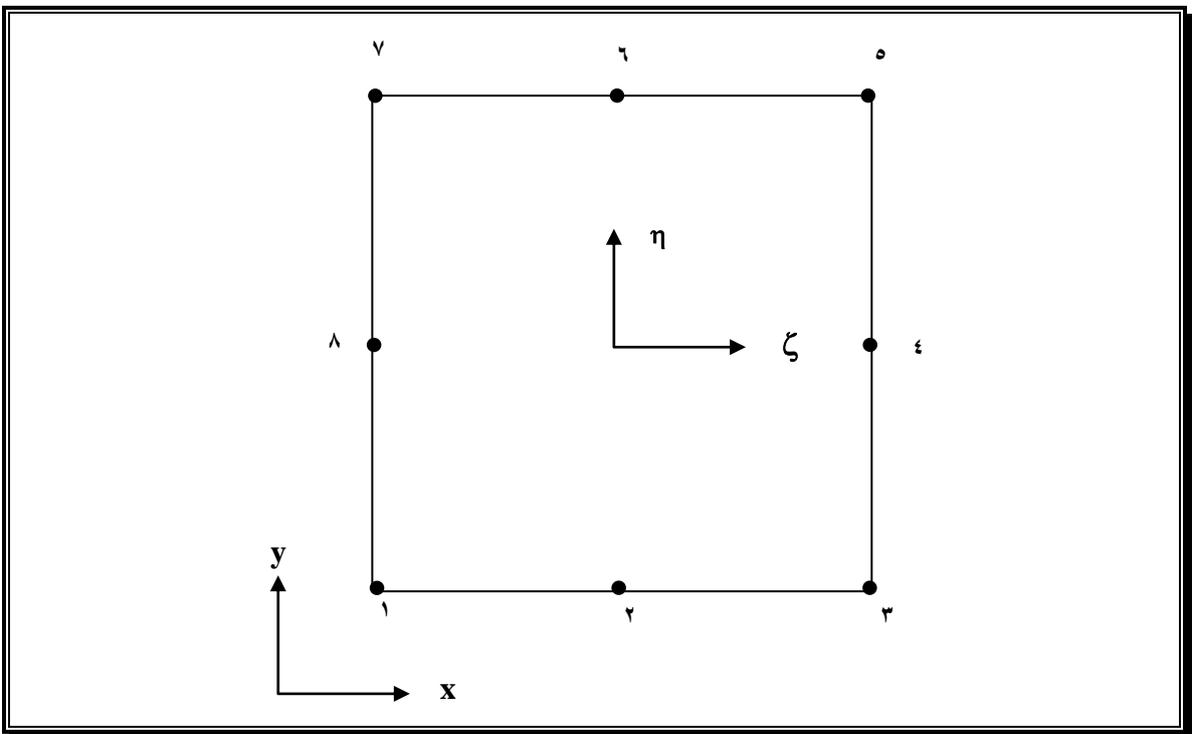


Fig.(3-2): Quadratic isoparametric element. [After (Al-Hashamie 2001)].

The shape function of the isoparametric quadratic finite-element with respect to the local coordinates (ζ) and (η), as given in [Cheung and Yeo, (1999)], are as follows:

For mid-side nodes with ($\zeta = \pm 1$):

$$\left. \begin{aligned} N_2 &= \frac{1}{2}(1 - \zeta^2)(1 - \eta) \\ N_6 &= \frac{1}{2}(1 - \zeta^2)(1 + \eta) \end{aligned} \right\} \dots (3-31)$$

For mid-side nodes with ($\eta = \pm 1$):

$$\left. \begin{aligned} N_4 &= \frac{1}{2}(1 + \zeta)(1 - \eta^2) \\ N_8 &= \frac{1}{2}(1 - \zeta)(1 - \eta^2) \end{aligned} \right\} \dots (3-32)$$

For nodes at the corners, $\eta = \pm 1, \zeta = \pm 1$:

$$\left. \begin{aligned}
N_1 &= \frac{-1}{4}(1-\zeta)(1-\eta)(1+\zeta+\eta) \\
N_3 &= \frac{1}{4}(1+\zeta)(1-\eta)(\zeta-\eta-1) \\
N_5 &= \frac{1}{4}(1+\zeta)(1+\eta)(\zeta+\eta-1) \\
N_7 &= \frac{1}{4}(1-\zeta)(1+\eta)(-\zeta+\eta-1)
\end{aligned} \right\} \dots(\text{3-33})$$

The derivatives of the shape function with respect to (ζ) and (η) , which are needed in the element matrix, can be easily obtained from Eqs.(3-31), (3-32) and (3-33).

The matrix $[B]$ in Eq.(3-29) has the derivatives of the shape function with respect to the coordinates (x) and (y) . However, the derivatives of the shape function of the finite-element must be for local coordinates (ζ) and (η) . Hence, it is necessary to find out a relationship joining the derivatives in local coordinates and in Cartesian coordinates. Such a relationship would be:

$$\frac{\partial N}{\partial \zeta} = \frac{\partial N}{\partial x} \cdot \frac{\partial x}{\partial \zeta} + \frac{\partial N}{\partial y} \cdot \frac{\partial y}{\partial \zeta} \quad \dots(\text{3-34a})$$

$$\frac{\partial N}{\partial \eta} = \frac{\partial N}{\partial x} \cdot \frac{\partial x}{\partial \eta} + \frac{\partial N}{\partial y} \cdot \frac{\partial y}{\partial \eta} \quad \dots(\text{3-34b})$$

which, by matrix form, is:

$$\begin{bmatrix} \frac{\partial N_i}{\partial \zeta} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = [J] \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} \quad \dots(\text{3-35})$$

where [J] is the Jacobian matrix.

For the isoparametric element, the coordinates (X) and (Y) inside the element as indicated in [Smith, (1998)], is:

$$X = \sum_{i=1}^n N_i X_i = X(\zeta, \eta) \quad \dots(3-36a)$$

$$Y = \sum_{i=1}^n N_i Y_i = Y(\zeta, \eta) \quad \dots(3-36b)$$

where:

n = total number of nodes in each element;

X_i, Y_i = coordinates of the nodes.

The Jacobian matrix can be obtained by differentiating Eqs.(3-36) with respect to (ζ) and (η) coordinates:

$$[J] = \begin{bmatrix} \frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \dots & \frac{\partial N_8}{\partial \zeta} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \dots & \frac{\partial N_8}{\partial \eta} \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_8 & y_8 \end{bmatrix} \quad \dots(3-37)$$

or

$$[J] = \begin{bmatrix} \frac{\partial \sum N_i x_i}{\partial \zeta} & \frac{\partial \sum N_i y_i}{\partial \zeta} \\ \frac{\partial \sum N_i x_i}{\partial \eta} & \frac{\partial \sum N_i y_i}{\partial \eta} \end{bmatrix} \quad \dots(3-38)$$

To compute the derivation of the shape function with respect to (x,y), it is easy to find the inverse of the Jacobian matrix from Eq. (3-38), that is:

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \zeta} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} \quad \dots(\text{3-39})$$

To compute the transformation between the two systems it is also necessary to express the element area ($dx.dy$) in Eq.(3-39) as follows [Zienkiewicz (1966)]:

$$dA = dx dy = \det.[J] d\zeta d\eta \quad \dots(\text{3-40})$$

Thus, Eq.(3-39) may be written as:

$$K_{ij} = \int_{A^e} [B^e]^T [D^e] [B^e] \det.[J] d\zeta d\eta \quad \dots(\text{3-41})$$

3.6: Computation of the hydraulic gradient

From the value of the piezometric head at the nodes, it is possible to compute the hydraulic gradient directly without the need for transformation.

$$h = \sum N_i h_i \quad i = 1, 2, \dots, 8$$

$$h = N_1 h_1 + N_2 h_2 + \dots + N_8 h_8 \quad \dots(\text{3-42})$$

Therefore; $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$ can be written as follows:

$$\left. \begin{aligned} \frac{\partial h}{\partial x} &= \frac{\partial N_1}{\partial x} h_1 + \frac{\partial N_2}{\partial x} h_2 + \dots + \frac{\partial N_8}{\partial x} h_8 \\ \frac{\partial h}{\partial y} &= \frac{\partial N_1}{\partial y} h_1 + \frac{\partial N_2}{\partial y} h_2 + \dots + \frac{\partial N_8}{\partial y} h_8 \end{aligned} \right\} \dots(\text{r-}\xi\text{r})$$

In matrix form, the above equation becomes:

$$\left\{ \begin{array}{l} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \end{array} \right\} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \dots & \frac{\partial N_8}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \dots & \frac{\partial N_8}{\partial y} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_8 \end{bmatrix} \dots(\text{r-}\xi\xi)$$

or

$$\left\{ \begin{array}{l} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \end{array} \right\} = [J]^{-1} \begin{bmatrix} \frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \dots & \frac{\partial N_8}{\partial \zeta} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \dots & \frac{\partial N_8}{\partial \eta} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_8 \end{bmatrix} \dots(\text{r-}\xi\circ)$$

The solution of Eq.(r-ξ°) will give the hydraulic gradient in the x- and y-directions.

CHAPTER FOUR

FORMULATION OF THE OPTIMIZATION PROBLEM

4.1: General

The purpose of optimization is to find the best possible solution among the many potential solutions satisfying the chosen criteria. Designers often base their designs on the minimum cost as an objective, taking into account mainly the costs of foundation, safety and serviceability.

A general mathematical model of the optimization problem can be represented in the following form:

A certain function (Z), called the objective function,

$$Z = f \{x_i\} \quad i=1, 2, \dots, n \quad \dots\dots(4-1)$$

which is usually the expected benefit (or the involved cost), involves (n) design variables $\{x\}$. Such a function is to be maximized (or minimized) subject to certain equality or inequality constraints in their general forms:

$$g_i \{x_i\} = b_i \quad i=1, 2, \dots, I \quad \dots\dots(4-2)$$

$$q_j \{x_j\} \geq b_j \quad j=1, 2, \dots, J \quad \dots\dots(4-3)$$

The constrain reflects the design and functional requirements. The vector $\{x\}$ of the design variables will have optimum values when the objective function reaches its optimum value.

ॡ.ॢ: Methods of optimization

In the last three decades, most of the development in the field of optimization theory and methods have occurred due to the explosive growth of large computers.

The available methods of optimization can be subdivided into three categories as briefly discussed below.

ॡ.ॢ.ॠ: The linear programming (LP)

The main characteristic of a linear programming (LP) problem is that the objective function and all the constraints form linear relationships with the design variables. Although a relatively small proportion of structural design problems can be directly formulated as (LP) problems, the method is widely used.

Several algorithms are available to solve LP-problems. However, the simplex method, developed by C.B. Dantzig in ॠॡॡॢ, is the most widely used [Dantzig (ॠॡॢ)]. Some other methods are available such as graphical methods, revised simplex method and transportation method [Phillips *et al.*, (ॠॡॣ)].

ॡ.ॢ.ॡ: The non-linear programming methods (NLP)

If the objective function or any of the constraints is non-linear the optimization problem is termed a non-linear problem. Such a formulation is more important than (LP) since most of real-world design problems are in fact non-linear.

There are a large number of algorithms and techniques for solving NLP- problems. Some of these are briefly reviewed here in after.

4.2.2.1: The analytical approach

In this approach the problem is represented by a number of mathematical relationships from which certain equations can be developed which aid in the search for an optimum. The methods of this approach usually require the use of differential calculus and the optimum solution is theoretically found exactly [Gallagher and Zienkiewicz (1973)]. The most familiar methods underlying this approach are:

a. Differential calculus

This method is generally used to solve simple unconstrained NLP-problems by using the laws of differential calculus to find the optimum solution.

b. Lagrange multiplier

This method is used to solve constrained non-linear optimization. More detailed definitions and explanation of this method will be given later.

4.2.2.2: The numerical approach

In this approach, a near optimum solution is automatically generated in an iterative manner. An initial guess is used as a starting point for search of better solutions. The search continues until no further improvement in the objective function is possible or until a certain convergence criterion is satisfied, which indicates that the optimum solution has been achieved within the desired accuracy.

The two distinguished methods in these respects are the direct-search method and the gradient – search method.

a. Direct search

Here, the search proceeds with the evaluation of the objective function only in an iterative manner until a local or an approximate optimum solution is reached. The step sizes and the direction of moves at each iteration represent the main features of this approach. Many methods are proposed as standard algorithms such as the pattern search of Hooke and Jeeves, the complex method, the Fletcher and Powell method, and the Rosenbrock method [Bunday (1984)].

b. Gradient search

The basic idea, here, is to evaluate the gradient of the objective function at a point and utilize it to improve and accelerate the search. The acceleration is gained by finding the direction of the steepest descent, following this direction until no further improvement is possible, then the direction is changed again. The search continues until the gradient becomes zero indicating that the optimum solution is reached.

The most widely used method following this approach is the Sequential Unconstraint Minimization Technique, Rosen method and Fletcher and Powell method. These methods require smaller number of iterations than the direct search methods.

4.2.3: Dynamic programming

Dynamic programming is used to solve special types of optimization problems which involve multistage decision processes. The optimization of each stage will affect the next stage and the final optimum decision is ensured to be the sum of all the optimization decision of all stages.

The method is very powerful and used to solve continuous and discrete non-linear programming problems [Slaby (1987)].

ξ.۳: Design variables

The design variables are taken as follows [see Fig.(ξ-۱)]:

۱. Difference head (H): (۳, ξ, ۶, ۸, ۱۰, ۱۲ and ۱۴ m).
۲. Length of floor (b): $۳ \leq b < ۱۰$ (m).
۳. Thickness of the floor base (t): Calculated by Eq. (ξ-۱).
۴. Length of upstream blanket (b_۱): Calculate from the floor length value; in order to avoid the uplift pressure and critical exit gradient.
۵. Depth of the upstream cut-off (d_۱): ۰ – ۱۲; step ۰.۵ m.
۶. Depth of the downstream cut-off (d_۲): ۰ – ۱۲; step ۰.۵ m.
۷. Filter trench: with filter and without filter.

ξ.۴: The optimization model

ξ.۴.۱ The objective function

The cost objective function (Z) of the present research involves the cost of both floor and any control device. Such a function is formulated as follows:

$$Z = c_1 a + c_2 d_1 + c_3 d_2 + c_4 b_1 + c_5 (w.z) \quad \dots\dots(\xi-۴)$$

where:

c_1 = cost of one cubic meter of the floor base material, (۱۰^۳ I.D./m^۳);

a = area of floor base, (m^۲);

$c_۲$ = cost of one square meter of cut-off, (۱۰^۳ I.D. / m^۲);

$d_۱, d_۲$ = depths of upstream and downstream cut-offs, respectively, (m);

$c_۳$ = cost of one square meter of upstream blanket, (۱۰^۳ I.D. / m^۲);

$b_۱$ = length of upstream blanket, (m);

$c_۴$ = cost of one cubic meter of the filter trench, (۱۰^۳ I.D. / m^۳);

w = width of filter trench, (m); and

z = depth of filter trench, (m).

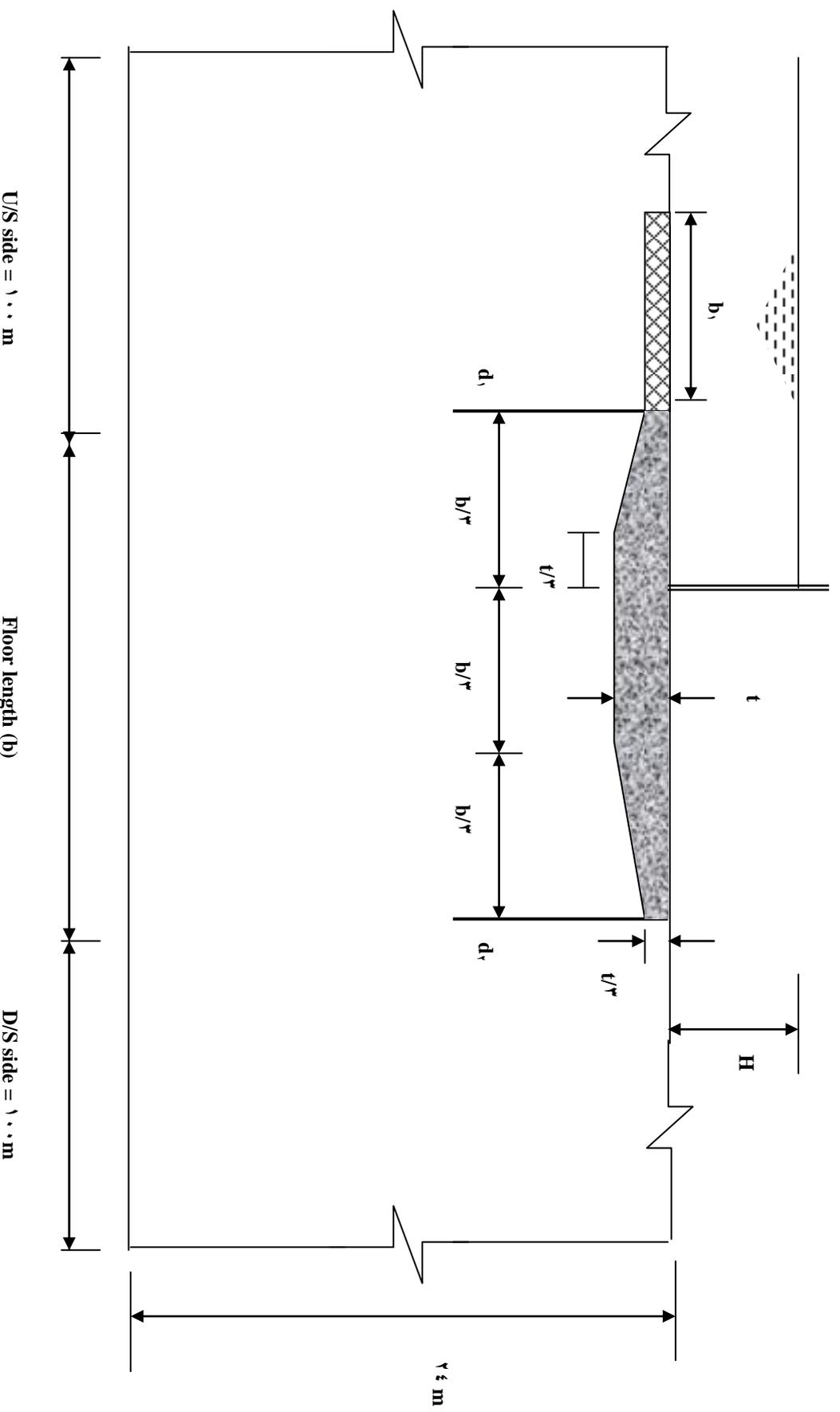


Fig.(4-1): Illustrative sketch of a hypothetical case study.

4.4.2: Constraints

The objective function is minimum and subject to the following constraints:

1. The relationships between the length of any control device used and the length of floor base enough to find the safe exit gradient (used 0.20 in this research), for certain differential head (H).
2. The floor thickness, with any control device used, shall be limited to a certain value to help in counteracting the uplift water pressure.

This requirement is stated as follows:

For equilibrium

Uplift pressure = Downward pressure

$$\gamma_w(h+t) = \gamma_w \cdot G_c \cdot t$$

or

$$t = \frac{h}{G_c - 1} \quad \dots\dots(4-5)$$

where:

t = thickness of floor base under the gates , (L);

h = head at any point under the hydraulic structure, (L);

G_c = specific gravity of concrete (the floor material)=2.4; and

γ_w = unit weight of water, (F/L³).

3. The cut-off depth should be equal or less than (1.2 m).

4.4.3: Method of optimization

The method of optimization employed in this research is the Lagrange multiplier method. The procedure is to convert the problem into unconstrained one by multiplying each of the constraint equations by $(\lambda_1, \lambda_2, \dots, \lambda_m)$, respectively, and adding them to the objective function to obtain:

$$L = F(\{x\}) + \sum_{i=1}^m \lambda_i g_i(\{x\}) + \sum_{j=i+1}^m \lambda_j \{q_j(\{x\}) - \theta_j^2\} \quad \dots\dots(4-7)$$

where (m) denotes the total number of constraint equations and $(\lambda_1, \lambda_2, \dots, \lambda_m)$ are called Lagrange multipliers. The above equation is often called the (Lagrangian) and is denoted by (L) . Taking the $(m+n)$ (where n is total number of variables) derivatives of the new objective function (L) and setting them equal to zero, gives:

$$\frac{\partial L}{\partial x_j} = \frac{\partial F}{\partial X_j} + \sum_{i=1}^m \lambda_i \frac{\partial g_i}{\partial x_i} - \sum_{j=i+1}^m \lambda_j \frac{\partial q_j}{\partial x_j} = 0 \quad \dots\dots(4-8)$$

and

$$\frac{\partial L}{\partial \lambda_i} = g_i = 0, \frac{\partial L}{\partial \theta_j} = 2 \lambda_j \theta_j = 0 \quad \dots\dots(4-9)$$

Solving Eqs. (4.8) and (4.9) simultaneously gives the required solution [Dimitri (1982)].

CHAPTER FIVE

ANALYSIS OF THE RESULTS

5.1: General

In this chapter different cases were analyzed to investigate the effect of length and location of control devices (blanket, cut-off, and filter trench) on uplift pressure and exit gradient by using the finite-element method. Figure (5-1) shows the discretization mesh of the finite – element such of the hydraulic structure with an upstream blanket (b_1), downstream blanket (b_2), upstream cut-off (d_1), downstream cut-off (d_2), and filter trench.

In order to determine the optimum design of control devices, many cases were examined using the Lagrange multiplier method and cost calculation curves with various values of active head (H).

For the generation of the results of this part the difference ratio of piezometric head is taken into consideration, i.e., that is:

$$H^* = \frac{h - h_2}{h_1 - h_2} \quad \text{.....(5-1)}$$

where:

$$H = h_1 - h_2$$

H^* = ratio of head difference;

h = piezometric head at any point in the flow domain, (L);

h_1, h_2 = piezometric head upstream (U/S) and downstream (D/S) of the structure, respectively, (L).

Consequently, (H^*) equals (one) for the upstream bed and (zero) for the downstream bed.

๑.๒: Effect of different seepage control devices on the uplift pressure and exit gradient

In this section the effect of length and location of blanket, cut-off and filter trench on uplift pressure and exit gradient are analyzed.

๑.๒.๑: The horizontal blanket

In the design of hydraulic structures which are built on pervious foundations, there is a trend toward the control of seepage through the foundation by constructing an upstream blanket connected to the impervious section of the structure.

Blanket is usually used when cut-offs of bed rock or to an impervious layer are not practicable because of excessive depth, they are also in conjunction with partial cut-offs [Gill, ๑๙๙๐]

With an assumed a homogenous isotropic foundation, the effect of the length of upstream blanket (b_1) is shown in Fig.(๑-๒). The figure indicates that the uplift pressure decreases with increasing the upstream length of the blanket. This is an expected result because the blanket will increase the horizontal length of the path of seepage under the hydraulic structure.

Figure (๑-๓) illustrates the distribution of the uplift pressure under the floor with different lengths of downstream blanket (b_2). It has been noted that the value of uplift pressure is greater in the case of using a blanket than that without a blanket, other parameters being unchanged. This is attributed to the fact that the streamlines are directed more towards the floor of the hydraulic structure, leading to an increase in the uplift pressure. Also, shown in this figure that the uplift pressure increases with the increase of the downstream length of blanket.

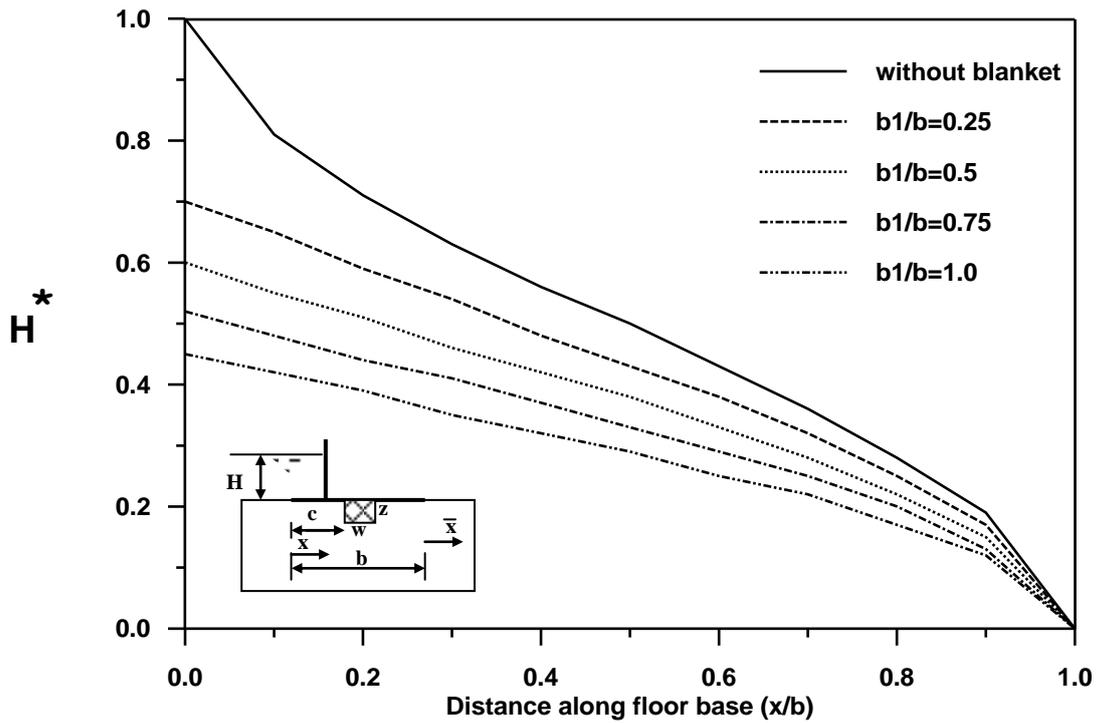


Fig.(9-2): Uplift pressure distribution under a hydraulic structure with various lengths of U/S blanket.

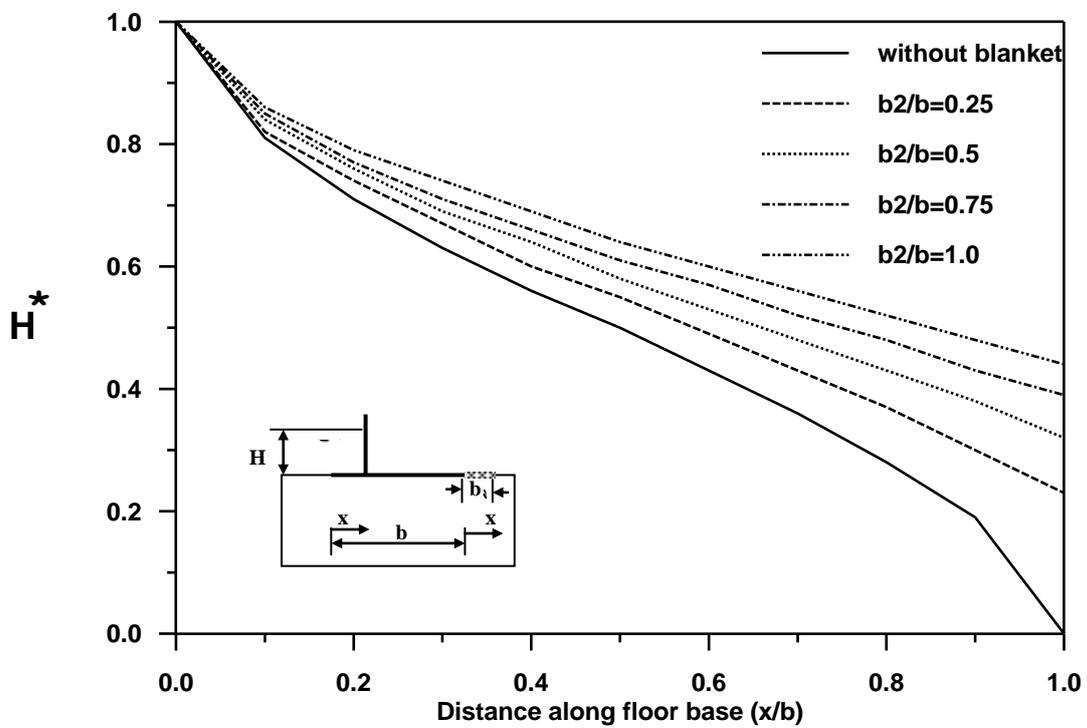


Fig.(9-3): Uplift pressure distribution under a hydraulic structure with various lengths of D/S blanket.

Figure (3-4) illustrates the distribution of the exit gradient along the distance downstream of the hydraulic structure for different lengths of the upstream blanket. It has been noted from this figure that the exit gradient decreases with increasing the upstream length of the blanket (b_1). This is due to the fact that increasing the upstream length of the blanket lengthens the length of the path of the flow under the hydraulic structure which increases the head loss.

The effect of the downstream blanket on exit gradient is shown in Fig.(3-5). It has been noted that the exit gradient increases with increasing the length of downstream blanket (b_2).

Figure (3-6) shows the effect of using on upstream or downstream blanket on the uplift pressure. It has been noted that the uplift pressure. It has been noted that the uplift pressure is decreased when using an upstream blanket, as compared with the case without blanket, other parameters being unchanged. Also, the uplift pressure increases with using a downstream blanket. The same aforementioned effect can be noticed on the exit gradient and as it is shown in Fig.(3-7).

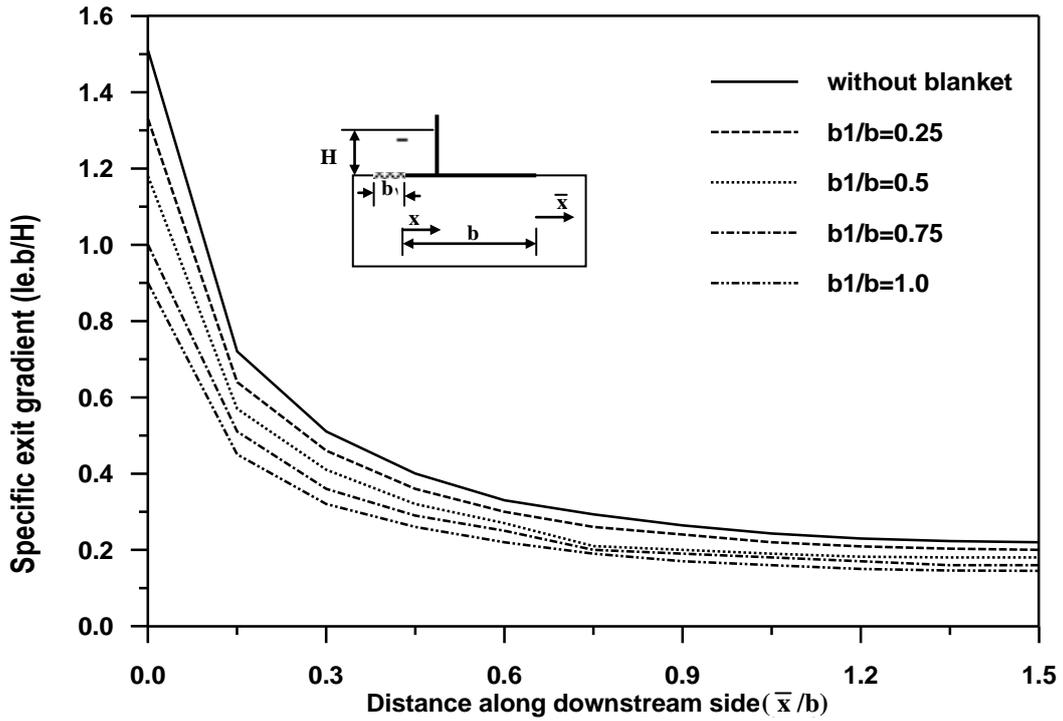


Fig.($\phi-\xi$):Exit gradient distribution downstream of a hydraulic structure with various lengths of U/S blanket (b_1).

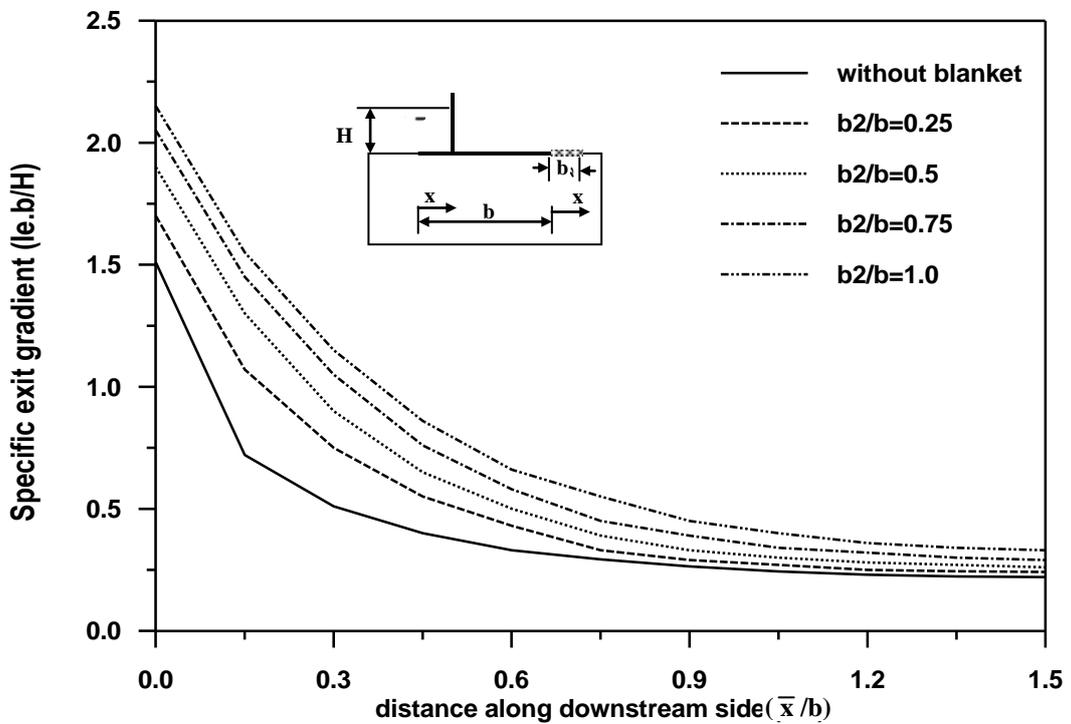


Fig.($\phi-\theta$):Exit gradient distribution downstream of a hydraulic structure with various length of D/S blanket (b_2).

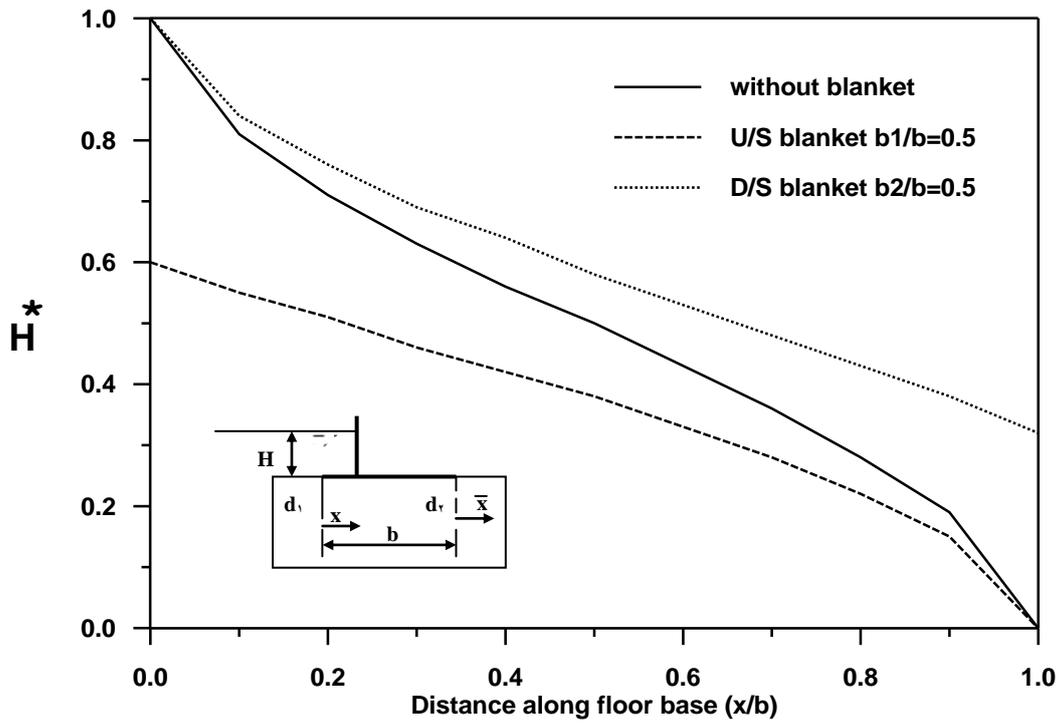


Fig.(٤-٧): Effect of using U/S or D/S blanket on the uplift pressure.

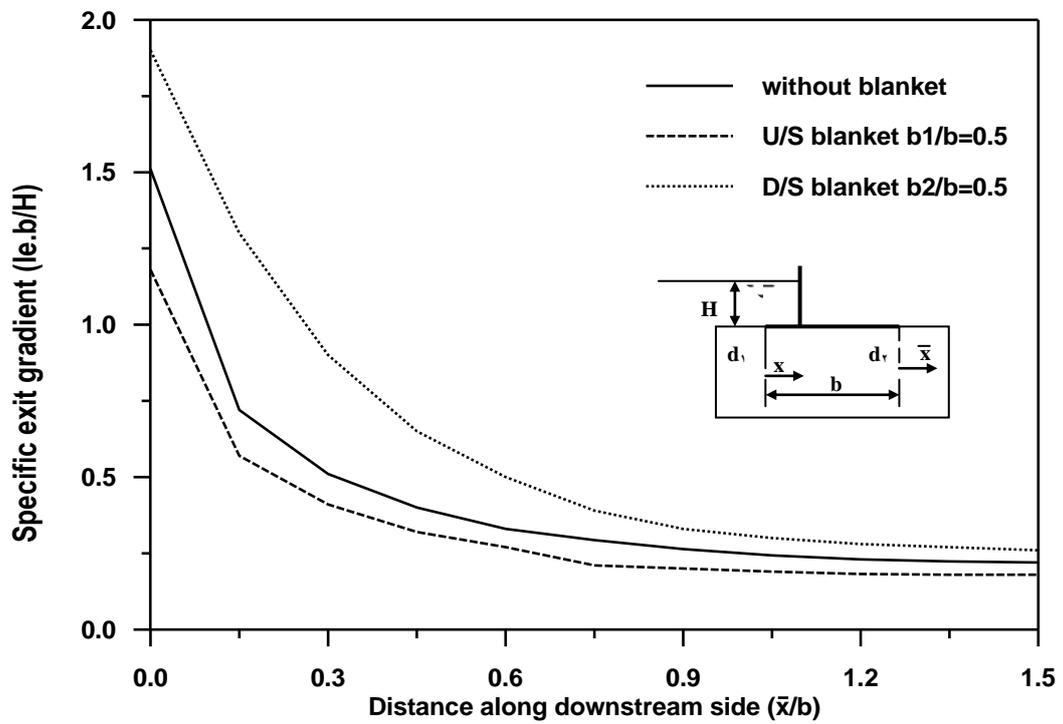


Fig.(٤-٧): Effect of using U/S or D/S blanket on the exit gradient.

๑.๒.๒: The cut-offs

A cut-off is a device which can be used to reduce or prevent the seepage of water through the previous foundation. In most hydraulic structures, cut-offs are used upstream and / or downstream the structure. The location and length of cut-off depend on the type of foundation soil.

The action of the cut-off is similar to that of an obstruction in a pipe; the flow is reduced because of the loss of head due to the obstruction and increases the vertical path of seepage. If a cut-off extends through the foundation to the impervious material below, it is called a complete cut-off. If it penetrates only a part of the pervious foundation, it is known as partial cut-off .

The cut-offs may be any form of the following:

- i. Cut-off trenches back filled with impervious material.
- ii. Masonry or concrete cut-offs.
- iii. Sheet piling, mostly made of concrete or steel.
- iv. Grout curtain, a barrier developed by grouting cement deep below the foundation.

Before choosing the type of cut-off to be used, it would be appropriate if foundation conditions are fully investigated. If seepage is moderate, sheet pile treatment may be adequate. If seepage is very heavy the foundation is pervious for large depths, masonry or concrete cut-off is found more appropriate [Khsaf, ๑๙๙๗].

Grout curtain is a sort of preventive measure. It is adopted mostly after construction when excessive seepage is observed and has to be controlled.

Figures (๑-๗) through (๑-๑๑) illustrate the effect of the location of cut-offs on uplift pressure with different depths (d) under the floor of the hydraulic structure. It can be seen from these figures that when there is a cut-off at the upstream, the uplift pressure is decreased along the base of

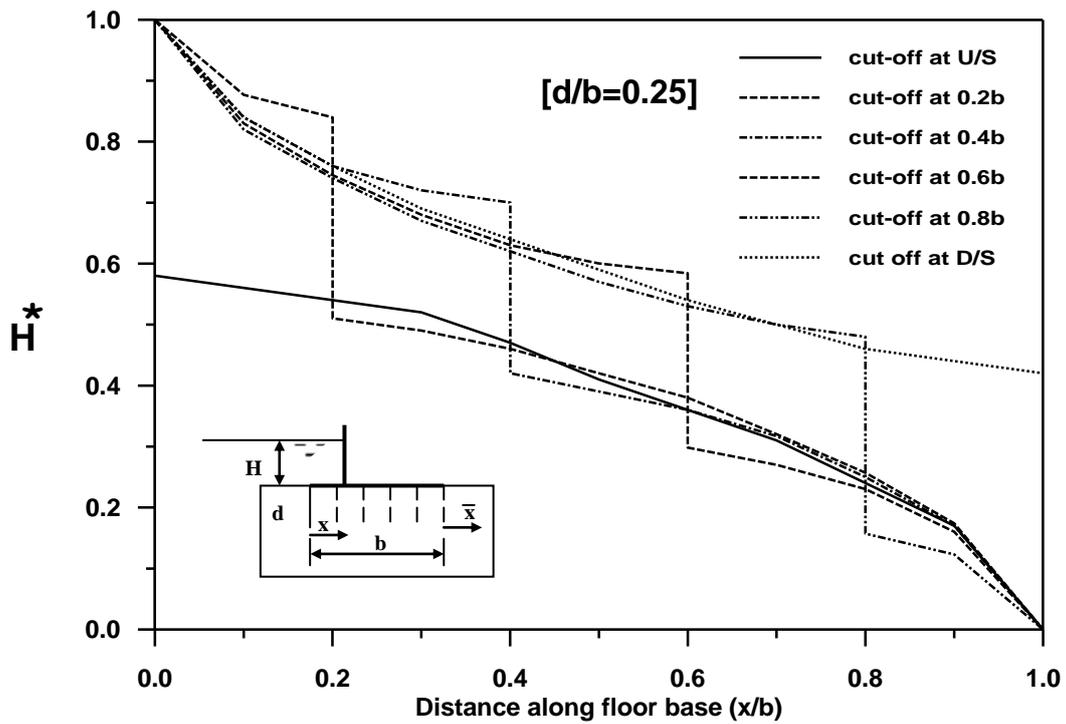


Fig.(8-8):Uplift pressure distribution under a hydraulic structure with various locations of cut-off,($d/b=0.25$).

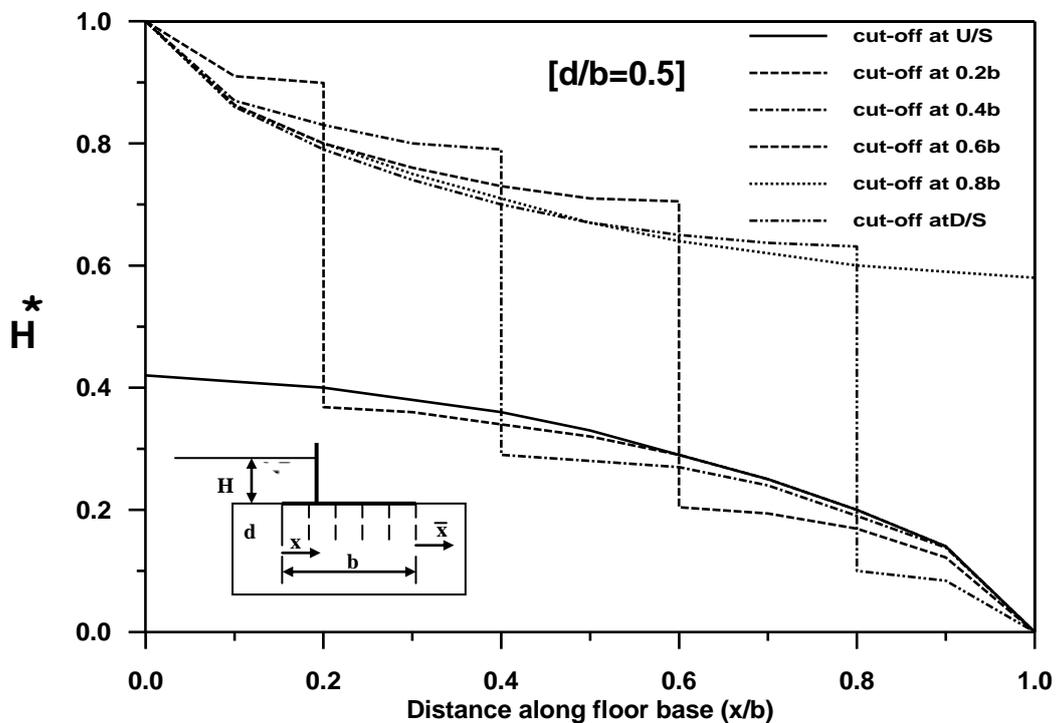


Fig.(8-9):Uplift pressure distribution under hydraulic structure with various locations of cut-off,($d/b=0.5$).

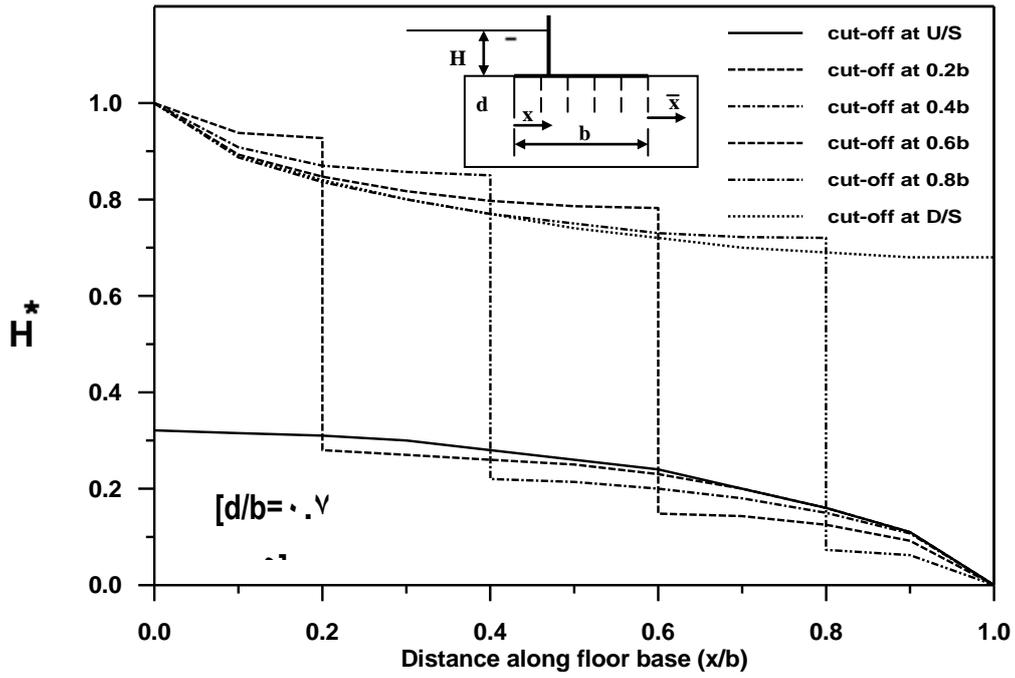


Fig.(9-10): Uplift pressure distribution under a hydraulic structure with various locations of cut-off, ($d/b = 0.7$).

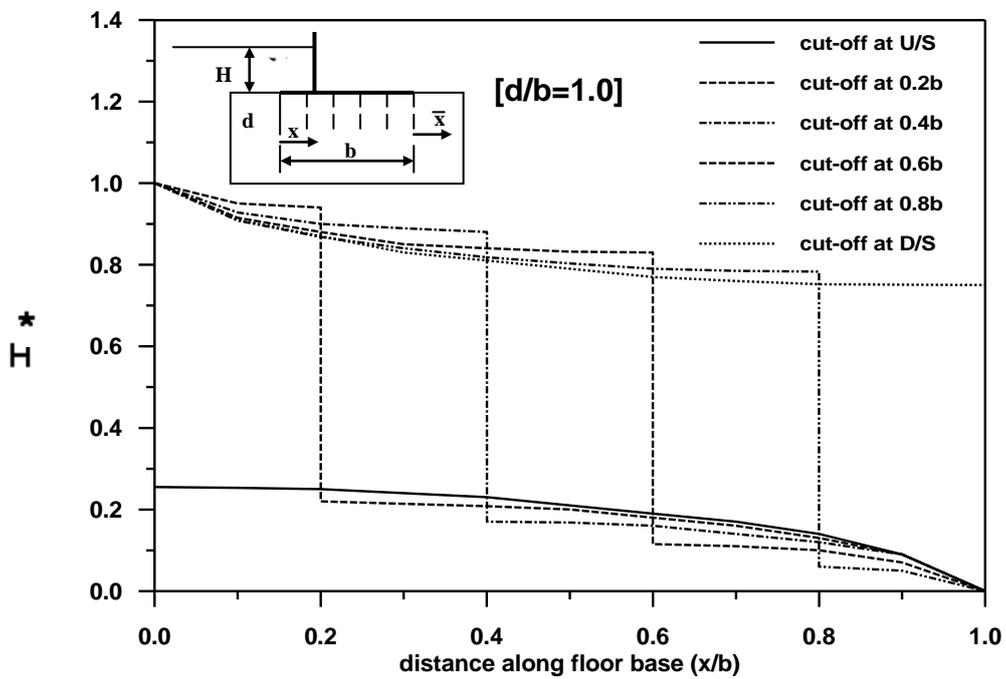


Fig.(9-11): Uplift pressure distribution under a hydraulic structure with various locations of cut-off, ($d/b = 1.0$).

floor. Also, the uplift distribution decreases with increasing the depth of cut-off. This is because the cut-off causes an increase in the length of creep, which increases the head loss. Whenever a cut-off is located a drop in the uplift pressure at that location is observed as expected.

It is clear from Figs.(2-12) through (2-15) that the exit gradient decreases with the distance downstream of the hydraulic structure. Also, it is shown that the magnitude of the exit gradient decrease when the cut-off moves from upstream to downstream of the hydraulic structure. This is because the cut-off decreases the seeping velocity causing the exit gradient to decrease as well.

In most practical cases two cut-offs are used, one at each end of the floor for seepage control. Thus, such a case worth the analysis.

Figures (2-16) illustrates the distribution of the uplift pressure for different lengths of cut-offs. The figure shows that the uplift pressure starts decreasing compared with the case where no cut-off is used ; this behavior is reversed beyond the point where $x = 0.5b$.

The effect of using two end cut-offs (U/D) on the exit gradient is shown in Fig.(2-17). It has been noted that the exit gradient decreases with increasing the depths of cut-offs (d).

Figures (2-18) and (2-19) show a comparison between the use of upstream cut-off, downstream cut-off, or two cut-offs at upstream and downstream (U/D). It has been noted from Fig.(2-18) that the upstream cut-off is most effective to reduce the uplift pressure. However, Fig.(2-19) indicates that the downstream cut-off is more effective than the upstream cut-off in terms of the reduction of the exit gradient. Moreover, using the two cut-offs at the ends is more effective than using the downstream cut-off only.

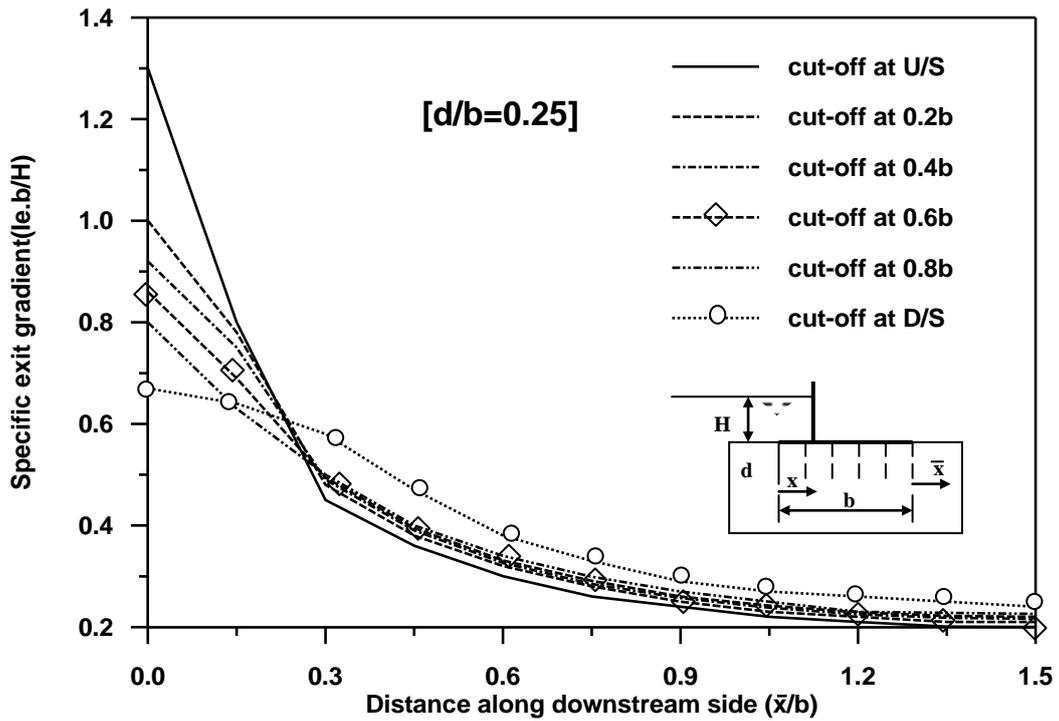


Fig.(9-12): Exit gradient distribution downstream of a hydraulic structure with various locations of cut-off, ($d/b=0.25$).

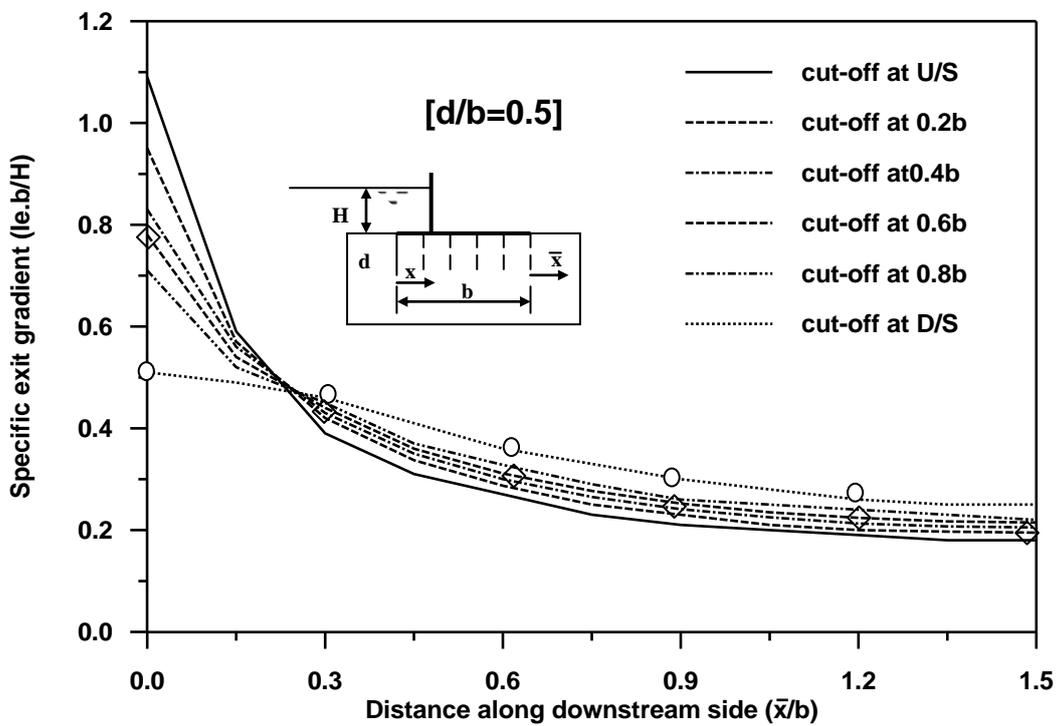


Fig.(9-13): Exit gradient distribution downstream of a hydraulic structure with various locations of cut-off, ($d/b=0.5$).

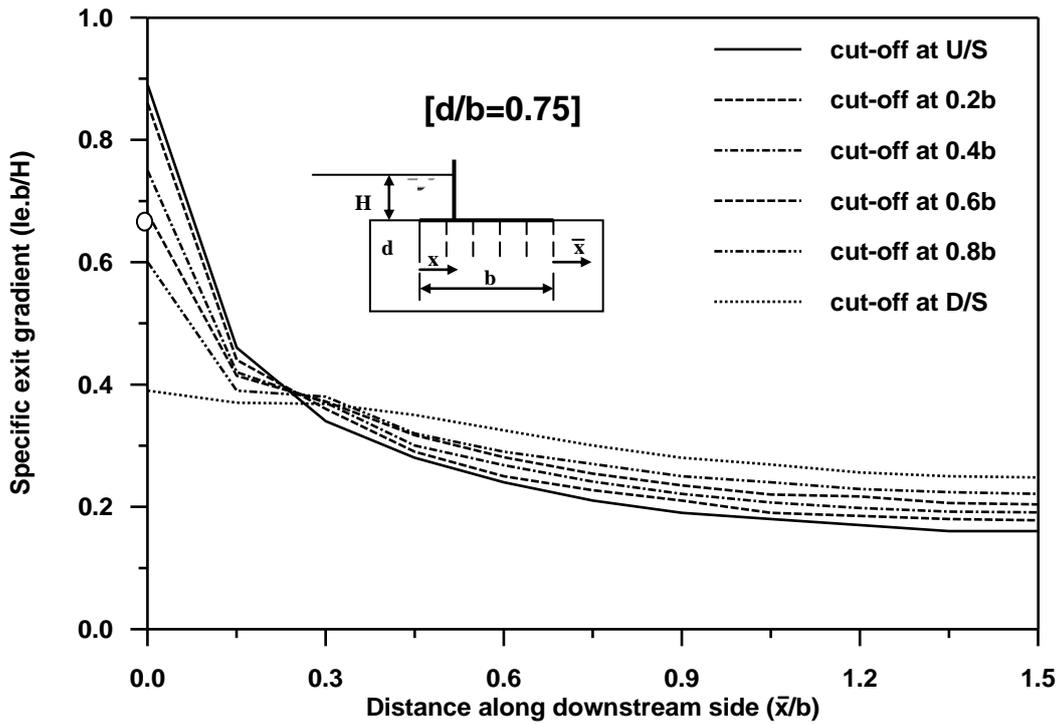


Fig.(9-14): Exit gradient distribution downstream of a hydraulic structure with various locations of cut-off, ($d/b=0.75$).

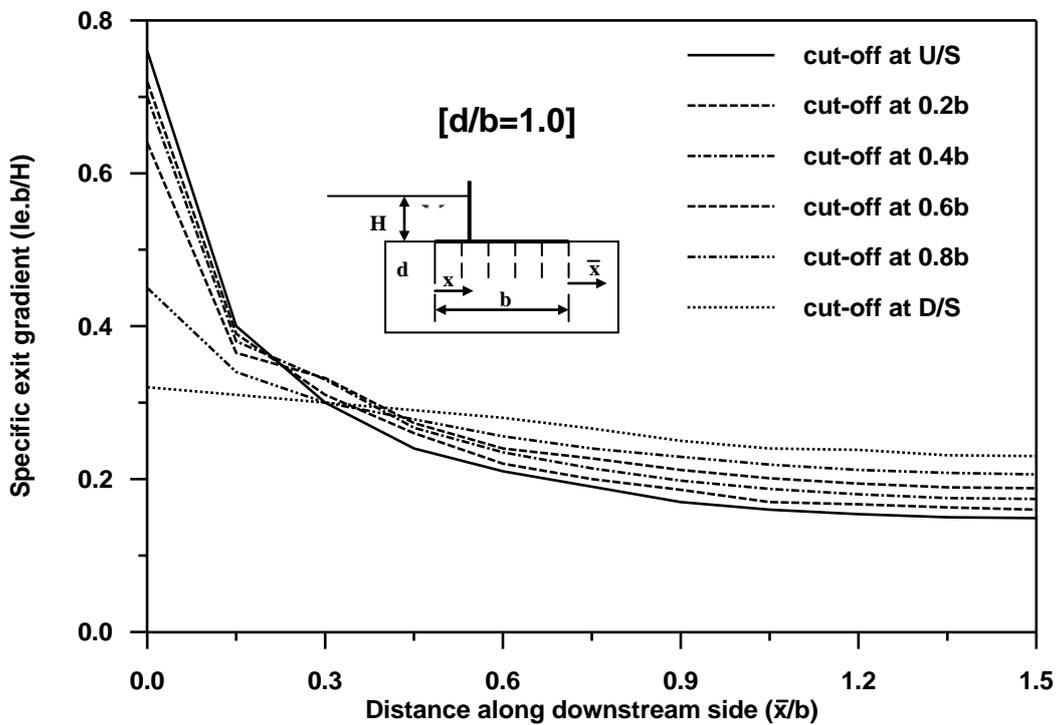


Fig.(9-15): Exit gradient distribution downstream of a hydraulic structure with various locations of cut-off, ($d/b=1.0$).

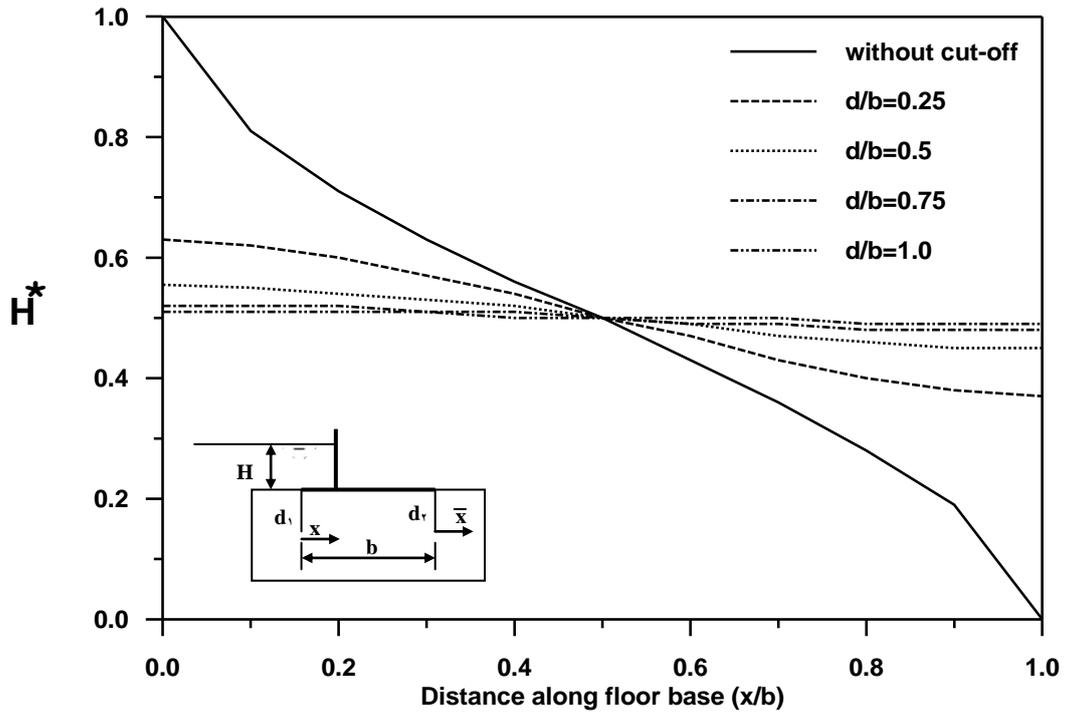


Fig.(9-16): Uplift pressure distribution under a hydraulic structure with various depths of U/D cut-offs.

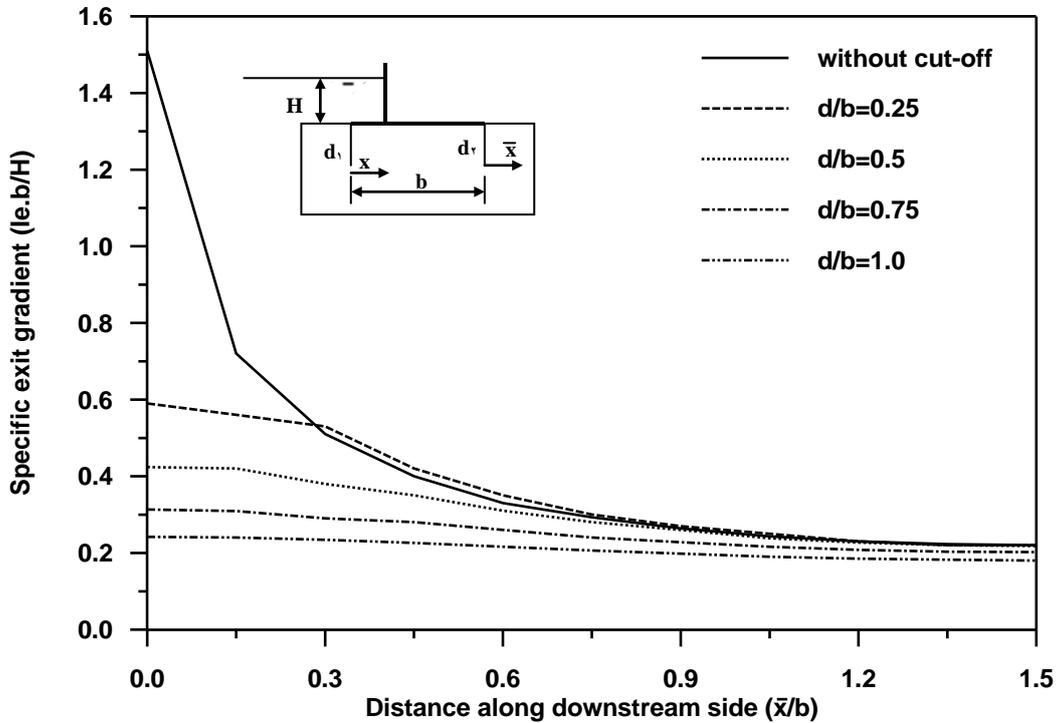


Fig.(9-17): Exit gradient distribution downstream of a hydraulic structure with various depths of U/D cut-offs.

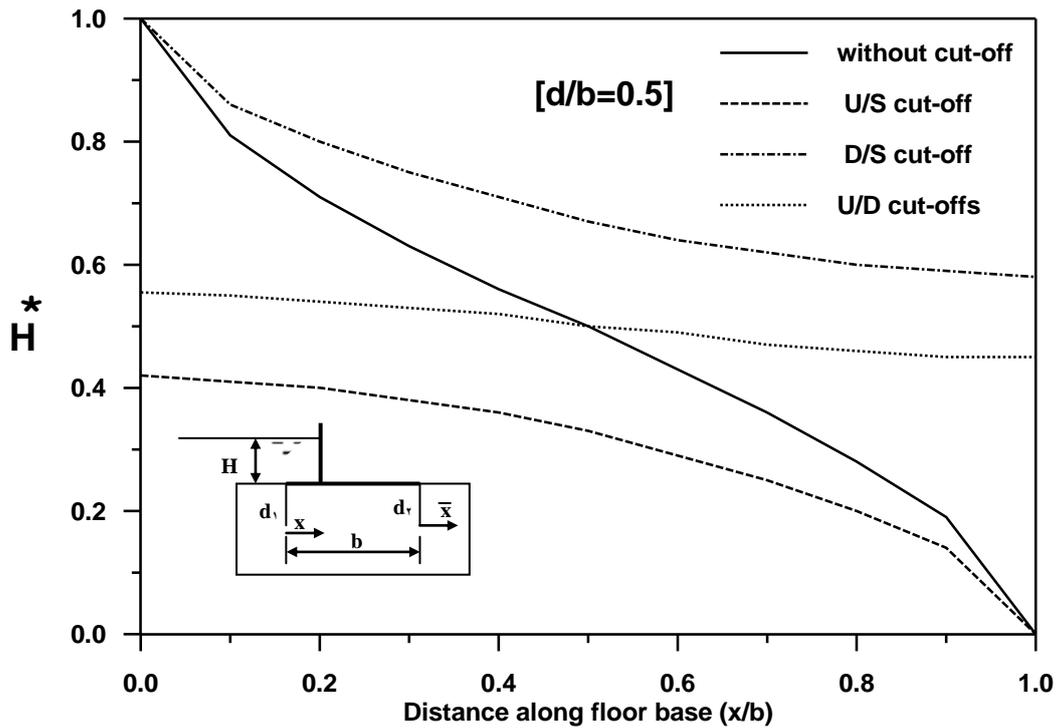


Fig.(9-18):Effect of using U/S, D/S and U/D cut-offs on the uplift pressure distribution under a hydraulic structure, ($d/b = 0.5$).

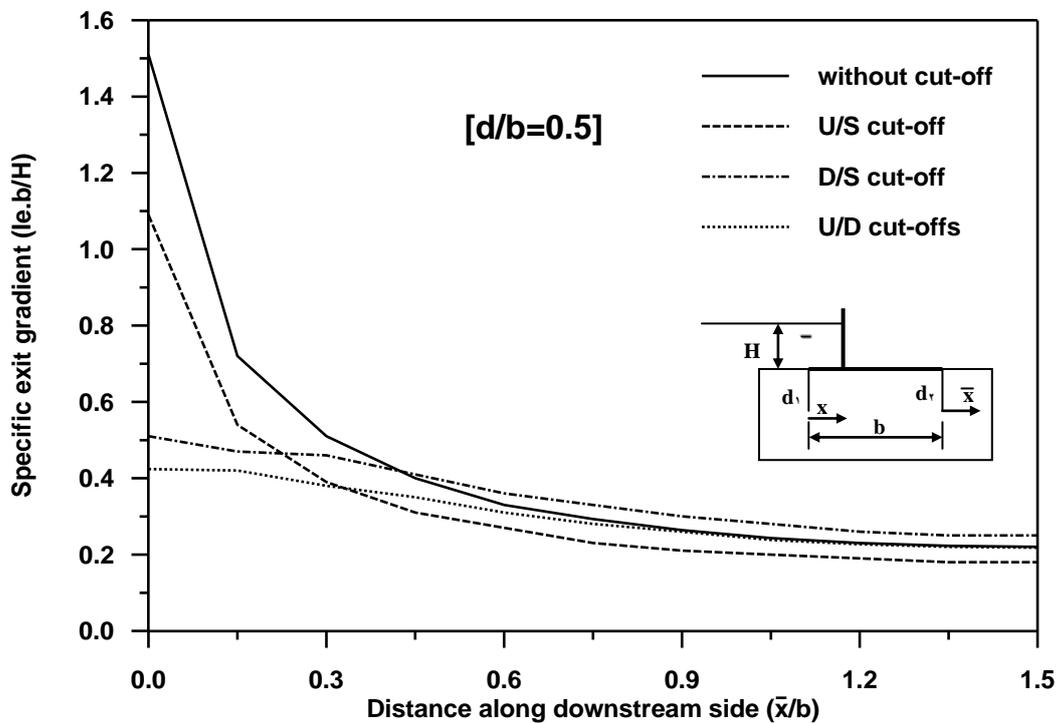


Fig.(9-19):Effect of using U/S, D/S and U/D cut-offs on the exit gradient distribution downstream of a hydraulic structure, ($d/b = 0.5$).

๑.๒.๓: The filter trench

A filter trench is simply an open trench vertical or horizontal which penetrates the impervious layer and relieves the uplift pressures in the underlying pervious soil. The filter trench is located anywhere in the downstream apron with certain width and depth.

The filter trench should be designed in such away that all seeping water through the hydraulic structure is effectively drained off. The filter consists of more than one layer. According to the (U.S.B.R., ๑๙๖๐) the following four main requirements should be satisfied:

- a. Filter material should be fine and property graded so that the voids in the filter are small and thus prevent base material from entering the filter.
- b. The filter material should be coarse and pervious in relation to base material. This aspect facilitates rapid removal of seeping water without building-up any seepage force within the filter.
- c. The filter material should be coarser than the perforations or openings in the drain pipes, so the filter material is not lost in the drains. The openings or perforations n the pipe drain should be adequate to admit all the seeping water safely.
- d. The thickness of filter material should be sufficient to provide a good distribution of all particle sizes throughout the filter. The thickness should also be adequate to provide safety against piping.

The piezometric head in the filter trench will be equal tot he piezometric head in the downstream [Chawla, ๑๙๗๖]. This means that (H^*) [as shown in Fig.(๑-๑)] equals (zero).

Figure (๑-๒๐) shows the finite element results for the uplift pressure distribution along the floor with different locations of filter trench [(c) in

Fig.(๑-๑)]. From this figure indicates that the uplift pressure decreases on the downstream side of the trench and increase on the upstream side as the filter trench is moved from upstream to downstream. Therefore, the filter trench must be located somewhere downstream of the location of the gates of the structure to get minimum total uplift pressure exerted on the downstream floor after the gates; the stability of this part of the structure is the most critical in practical design.

Figure (๑-๒๑) shows the exit gradient distribution with different locations of filter. It has been noted from this figure that the exit gradient decreases when moving the filter from upstream to downstream.

Figures (๑-๒๒) and (๑-๒๓) show the uplift pressure distribution along the floor for various width (w) and depth (z) of filter, respectively. From these figures it is obvious that the uplift pressure upstream and downstream of the filter trench decreases as the width and depth of filter trench increase.

The effect of filter trench width on the exit gradient is shown in Fig.(๑-๒๔). It has been noted that the exit gradient decreases as the width of the trench increases.

Figure (๑-๒๕) shows the effect of filter trench depth on the exit gradient. From this figure it has been noted that the exit gradient decreases as the depth of the filter trench increases.

Figure (๑-๒๖) illustrates the distribution of the uplift pressure along the floor for various (width/depth) ratio of filter trench with the same sectional area of filter. It has been noted that the uplift pressure decreases with increasing (w/z) ratio for the same sectional area of filter. Figure(๑-๒๗) shows the effect of different (w/z) ratio of filter on the exit gradient with the same sectional area of filter. It has been noted from this figure that the exit gradient decreases with increasing the (w/z)ratio.

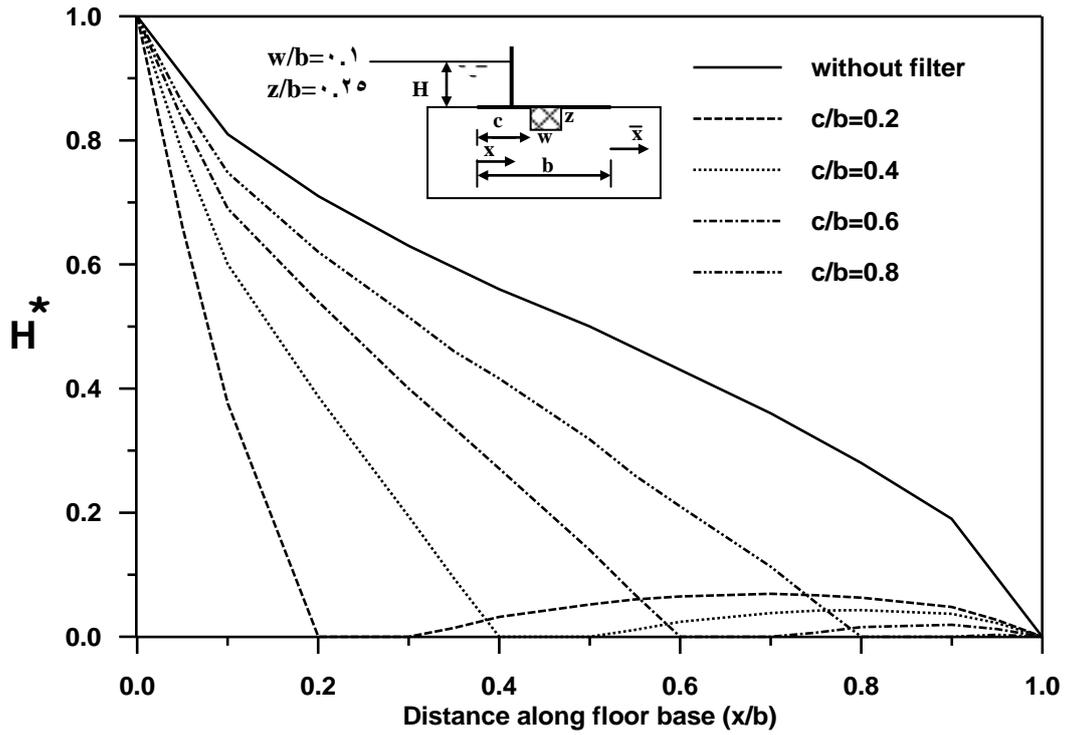


Fig.(9-20): Uplift pressure distribution under a hydraulic structure with various locations of filter trench.

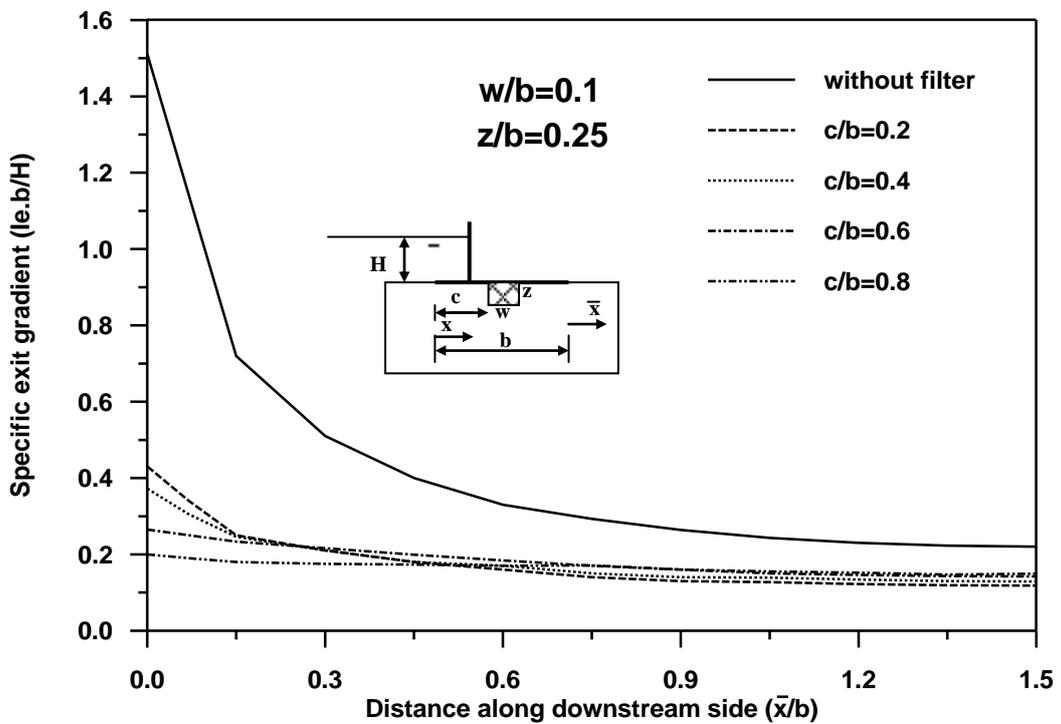


Fig.(9-21): Exit gradient distribution downstream of a hydraulic structure with various locations of filter trench.

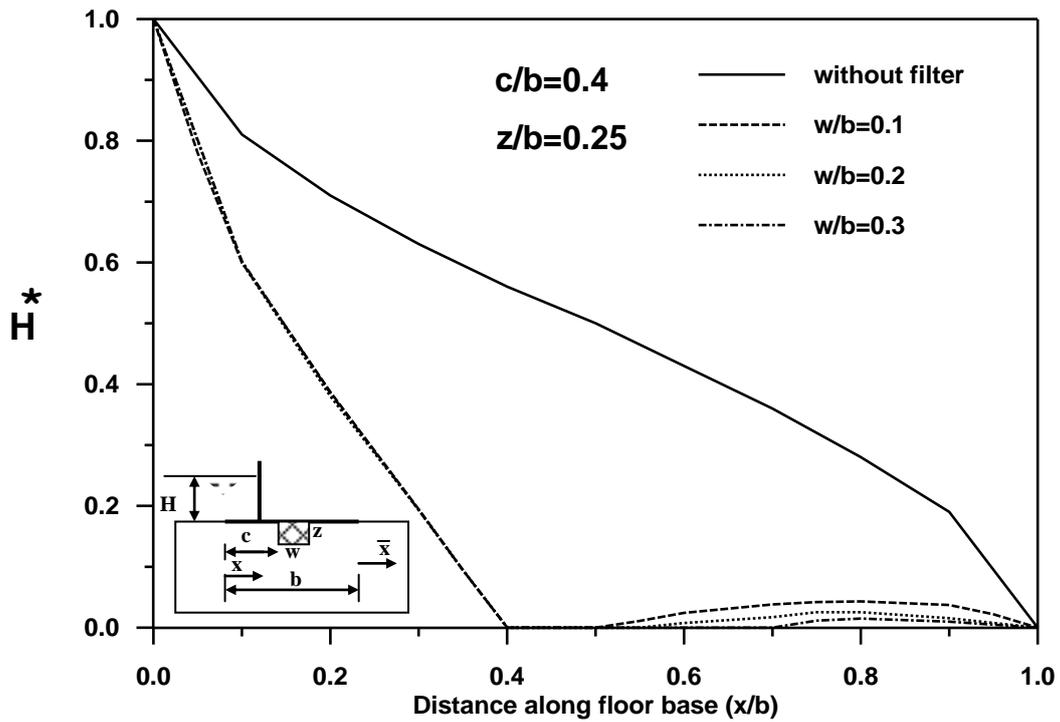


Fig.(9-22): Uplift pressure distribution under a hydraulic structure with various widths of filter trench.

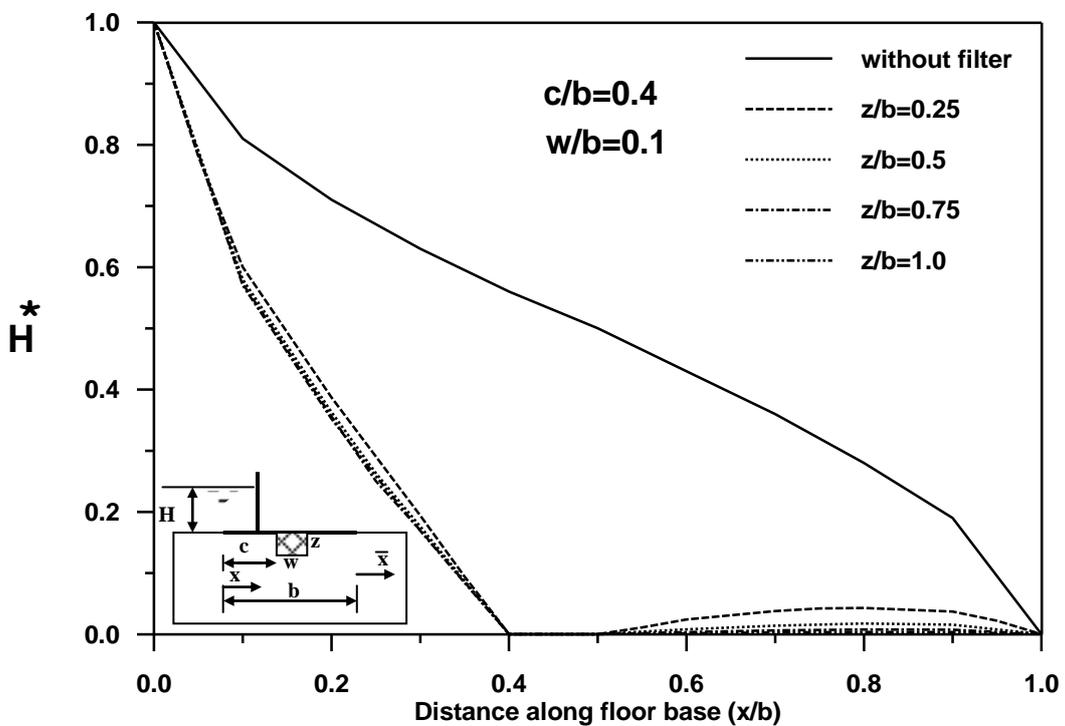


Fig.(9-23): Uplift pressure distribution under a hydraulic structure with various depths of filter trench.

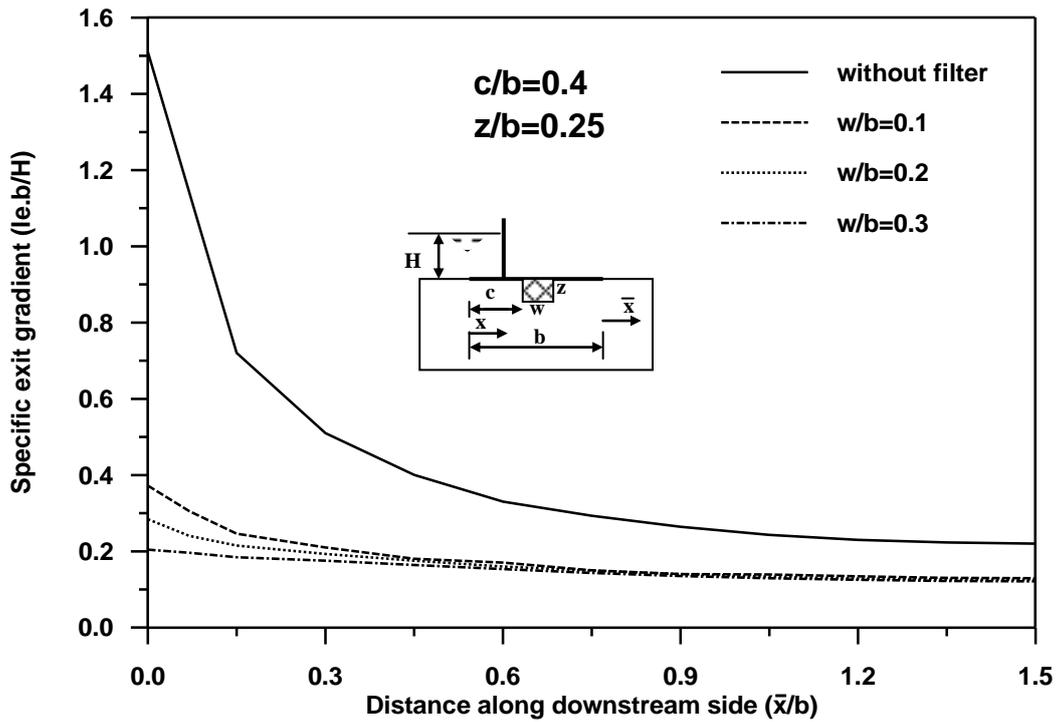


Fig.(9-24): Exit gradient distribution downstream of a hydraulic structure with various widths of filter trench.

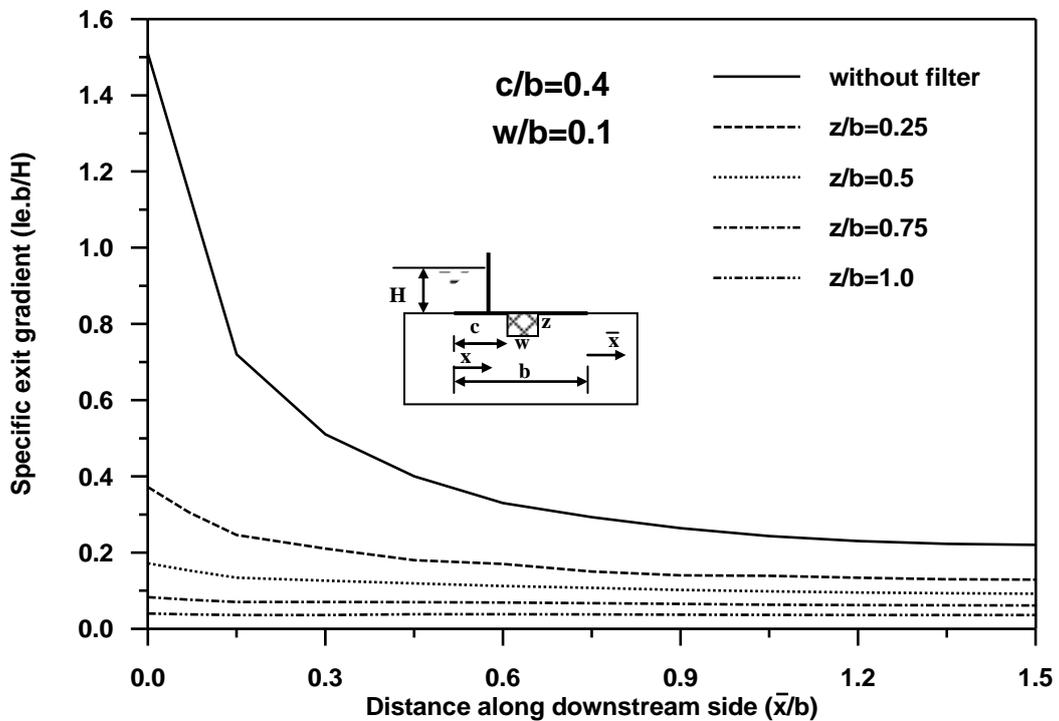


Fig.(9-25): Exit gradient distribution downstream of a hydraulic structure with various depths of filter trench.

3.3: The optimum design

In this research, the optimization process considers the hypothetical case shown in Fig.(3-1) in order to calculate the optimum design to decrease the seepage under it, by used different control devices.

The hydraulic structure would be safe when the existing exit gradient (I_e) is less than the critical gradient (I_c).

The existing exit gradient is defined as in Eq.(3-10) whereas the critical exit gradient is defined as:

$$I_c = \frac{G_s - 1}{1 + e} = \frac{\bar{\gamma}}{\gamma_w} \quad \text{.....(3-1)}$$

where:

I_e = exit gradient;

I_c = critical exit gradient;

G_s = specific gravity of the soil;

e = voids ratio of the soil;

$\bar{\gamma}$ = submerged unit – weight of the soil;

γ_w = unit weight of water.

Practical safety is ensured when

$$F_s = \frac{I_c}{I_e} > 1 \quad \text{.....(3-2)}$$

where (Fs) = factor of safety against piping.

Harza (1930) [quoted in Janice, 1976]: indicated that a factor of safety between (3) to (4) is enough for the safety of the structure. A factor of safety of (FS=4) has been chosen for use in this research.

๑.๓.๑: Floor length

The effect of floor length (b) on the exit gradient with different values of the head difference (H) is shown in Fig.(๑-๒๘). It has been noted that the exit gradient decreases as the length of the floor increases. From this figure could be fined the safe length of floor with any head difference applied.

The relationship between the cost of floor and its length (b) for each head difference is linear; cost increases with the increase of the floor length, as shown in Fig.(๑-๒๙).

๑.๓.๒ Upstream blanket

The effect of upstream and downstream blankets on uplift pressure and exit gradient has been discussed in article (๑.๒.๑). It has been found that the upstream blanket is very effective in decreasing the uplift pressure and exit grandaunt whereas the downstream blanket results in increasing the uplift pressure and exit gradient when used.

Figure (๑-๓๐) illustrates the relationship between upstream blanket length (b_1) and floor length (b) with various values of head difference, for a safe exit gradient of ($F_s = \xi$). It has been noted that the length of upstream blanket could be decrease when increase the length of floor. Also, the length of upstream blanket (b_1) increase with increase head difference (H) with same floor length.

Figure (๑-๓๑) shows the total costs (z) for upstream blanket length (b_1) and floor length (b) with safe exit gradient.

Figure (๑-๓๑) could be used to find the necessary upstream blanket length (b_1) for any floor length (b).

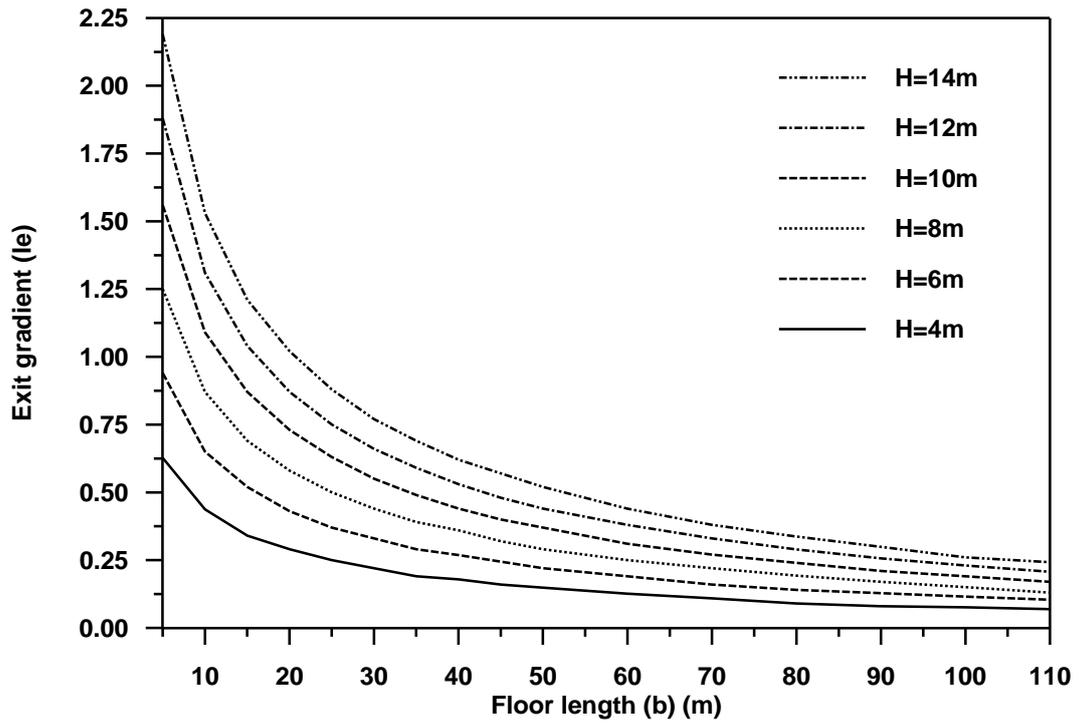


Fig.(5-26):Effect of floor length (b) on the exit gradient for various head difference (H).

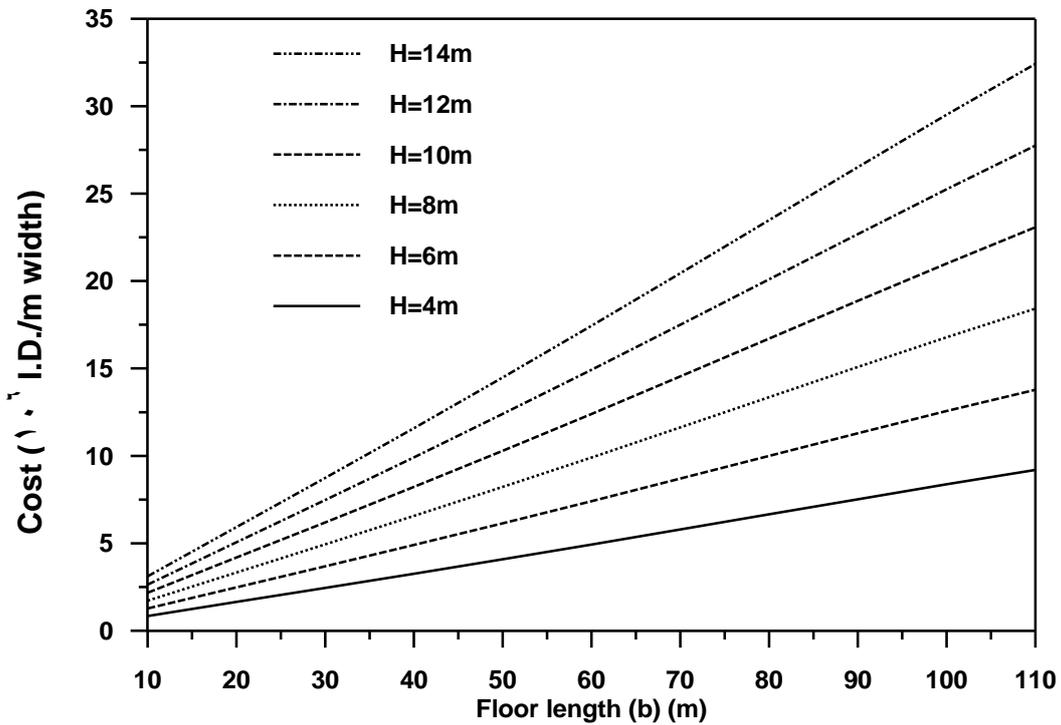
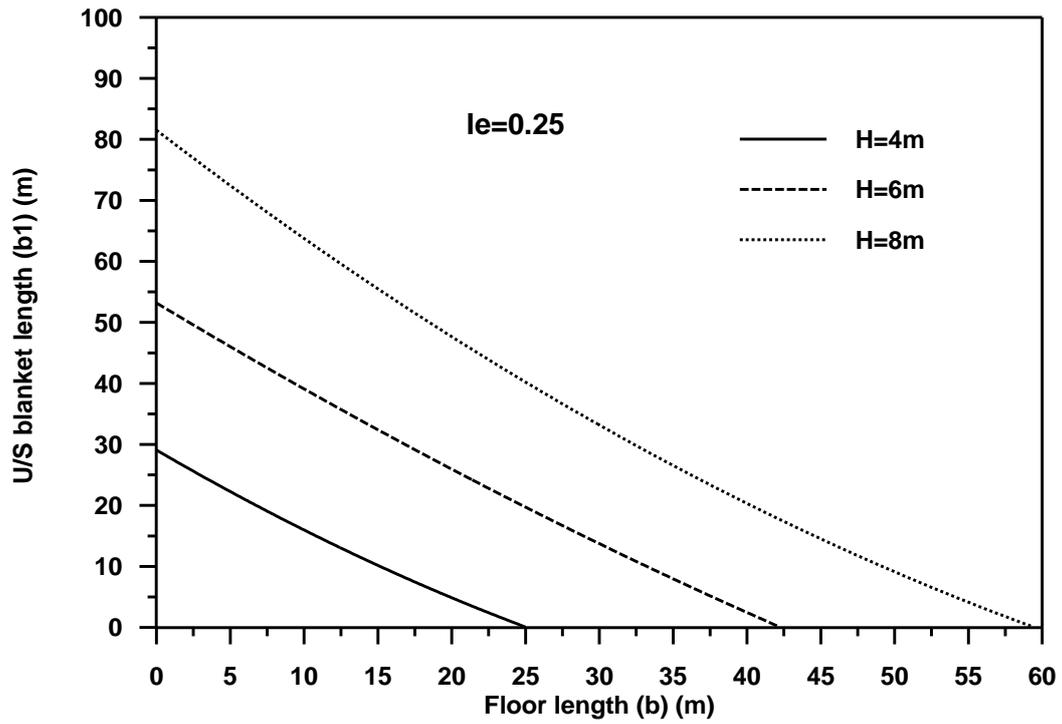
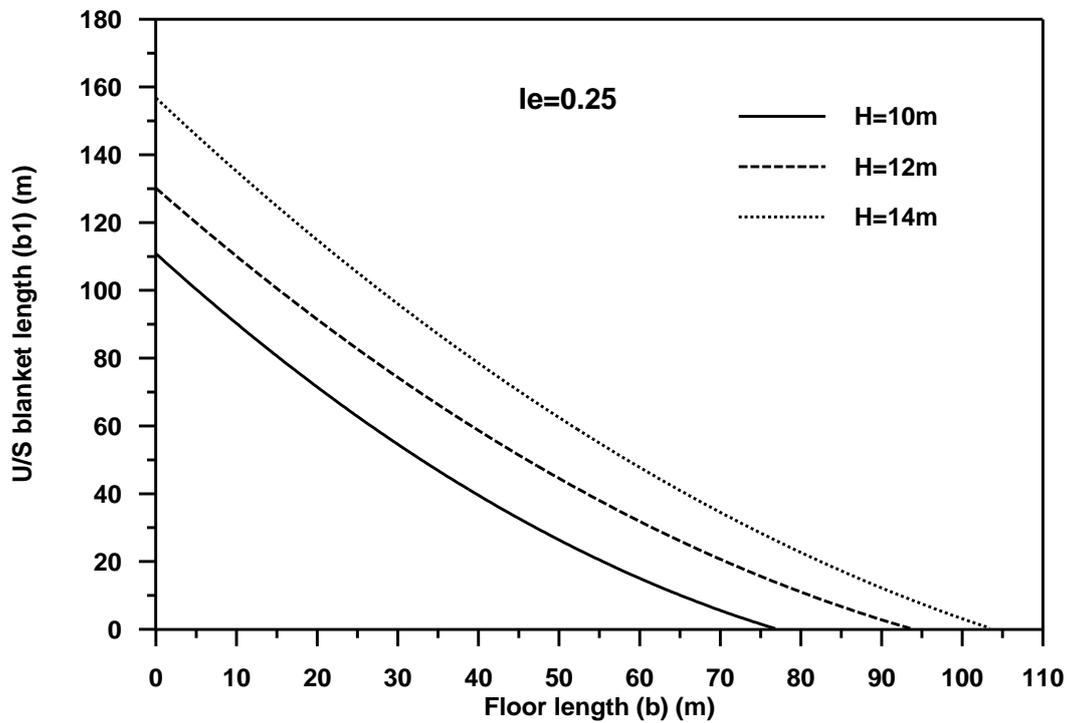


Fig.(5-27):Cost of floor for various head difference.

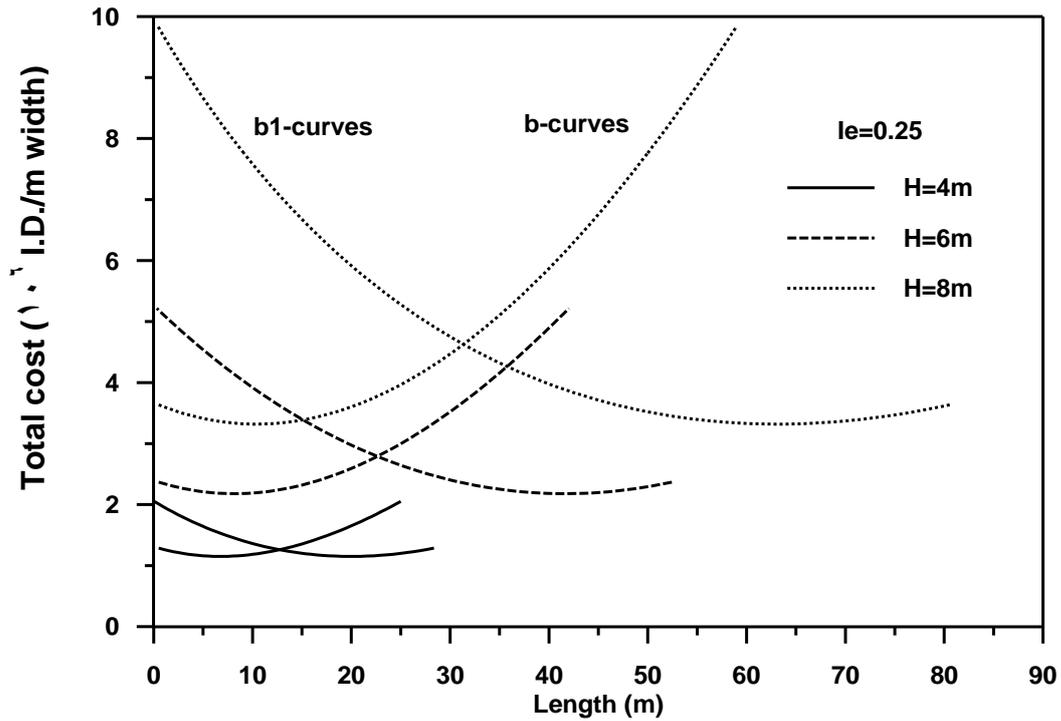


[a]: For ($H=4, 6, \text{ and } 8m$)

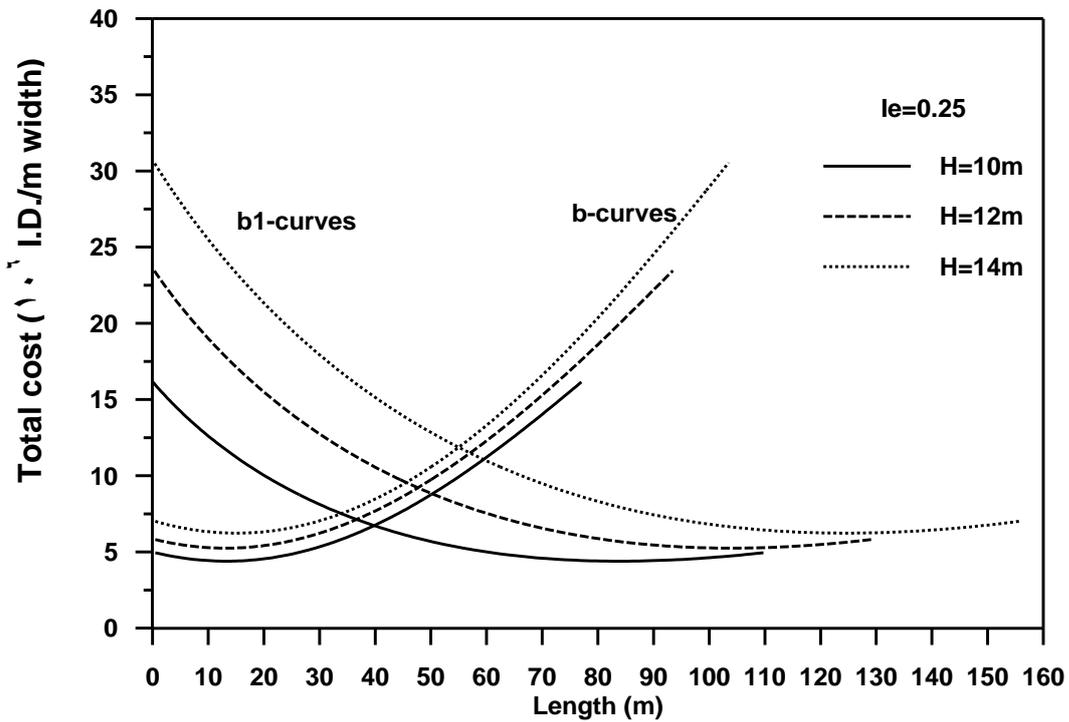


[b]: For ($H=10, 12, \text{ and } 14m$)

Fig.(9-28):Relation between U/S blanket length(b_1) and floor length(b) for safe exit gradient($le=0.25$).



[a]: For (H=4, 6, and 8 m)



[b]: For (H=10, 12, and 14 m)

Fig.(9-29): Total cost for U/S blanket length(b_1) and floor length(b), with safe exit gradient.

Moreover, the total cost is decreased with increased upstream blanket length (b_1), but it increases with increased floor length (b) for all the considered values of head difference (H).

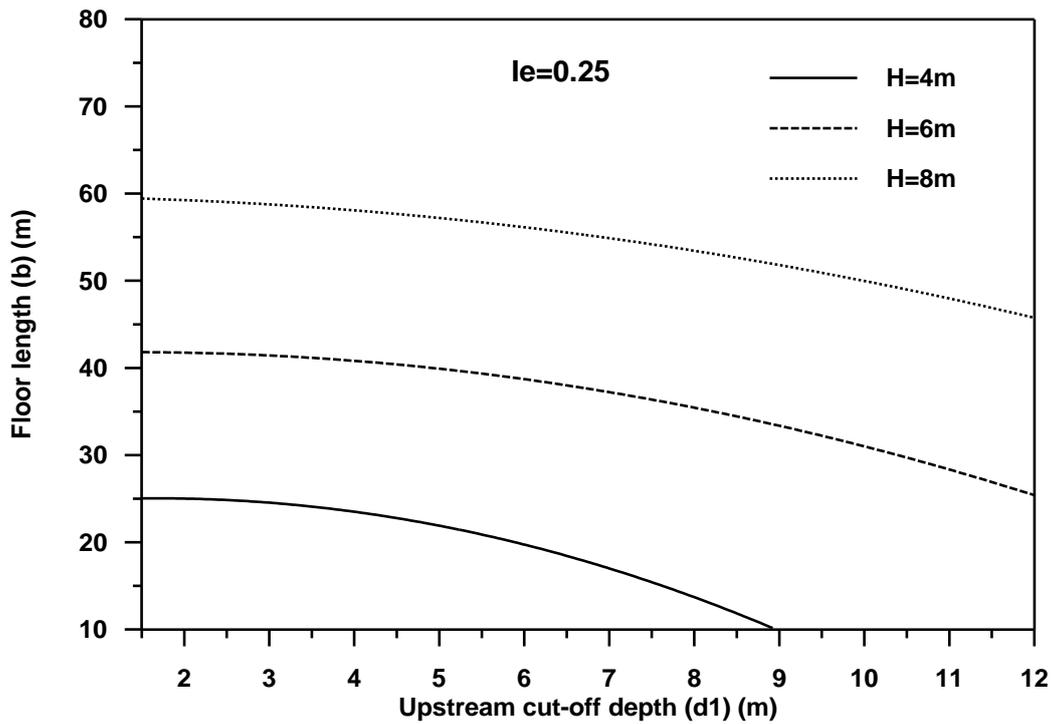
٥.٣.٣: Upstream cut-off

The effect of the upstream cut-off on uplift pressure and exit gradient has been discussed in Article (٥.٢.٢). It has been found that the upstream cut-off is very important provided under the hydraulic structures in order to decreasing the uplift pressure, i.e., (decreasing the thickness floor allowable (t)).

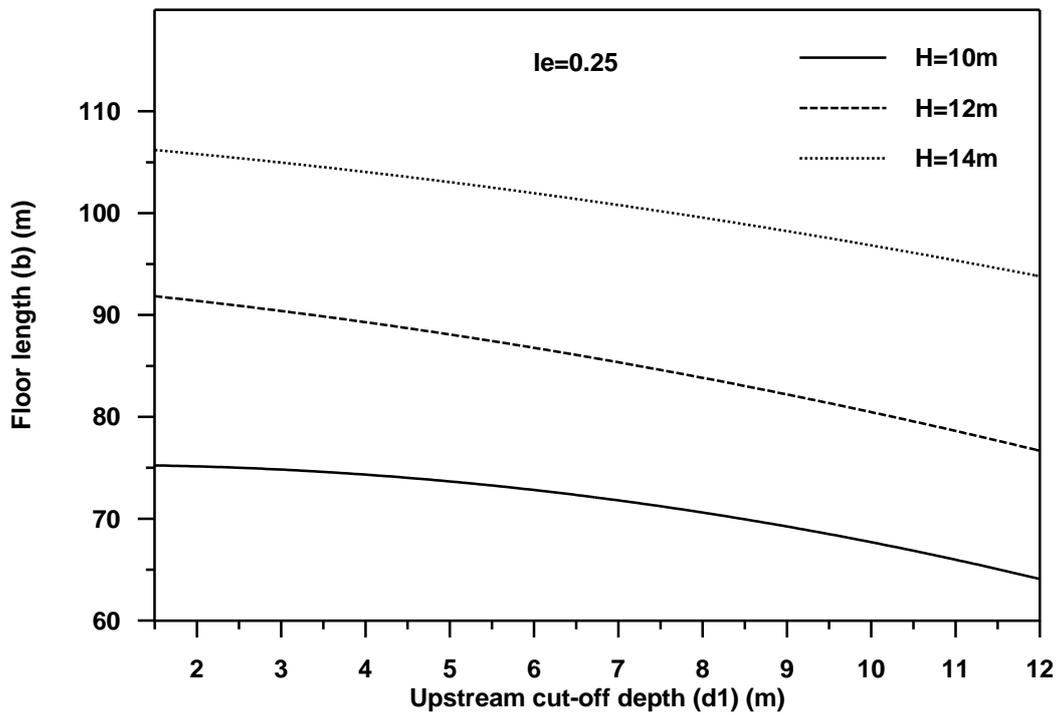
Figure (٥-٣٢) shows the relationship between the upstream cut-off depth (d_1) and the length of floor (b) with various values of head difference, for a safe exit gradient of ($FS=\xi$). All feasible solutions for upstream cut-off depth and floor length shows in this figure.

Figures (٥-٣٣) through (٥-٣٨) illustrate the total cost (z) for both the upstream cut-off and the floor, for the selected values of head difference. Each figure gives the depth of upstream cut-off, length of floor, and their total cost for safe exit gradient and uplift pressure for certain value of (H).

From these figures can be seen, that the total cost decrease with increases depth of upstream cut-off and decrease length of floor. Also, it has been noted that the minimum total cost gives when used maximum upstream cut-off depth for all the selected values of head difference.



[a]: For (H= 4 , 6 , and 8 m)



[b]: For (H= 10 , 12 , and 14 m)

Fig.(3-30):Relation between the U/S cut-off depth(d1) and the floor length(b), for safe exit gradient.

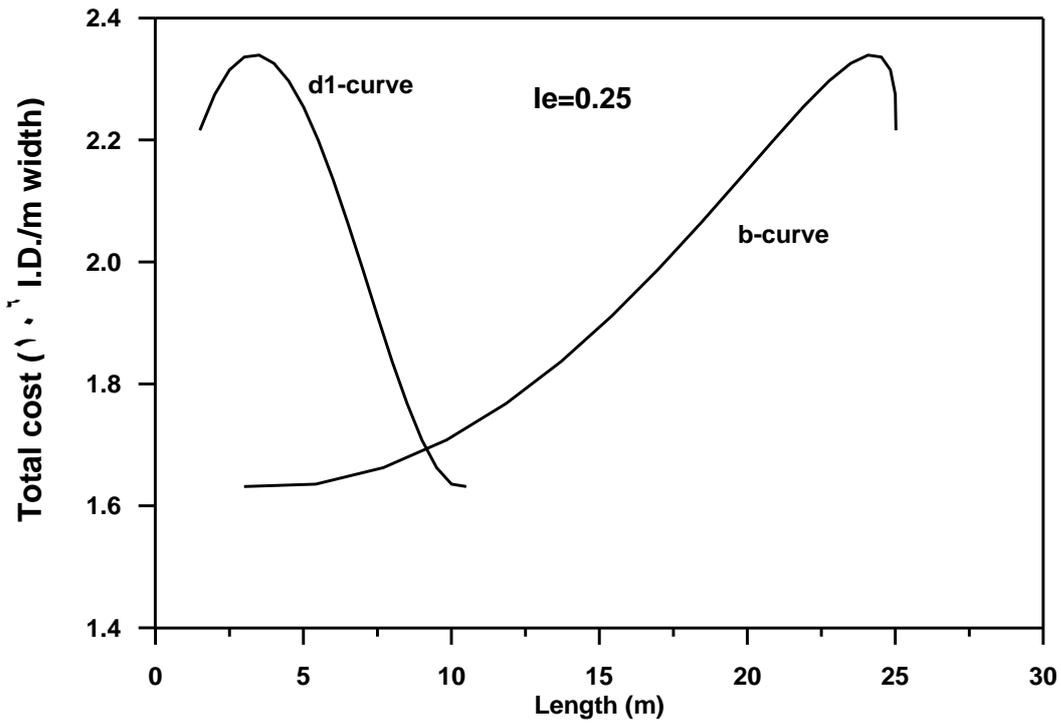


Fig.(9-31): Total cost for both the U/S cut-off (d1) and the floor (b), with safe exit gradient, ($H=4\text{m}$).

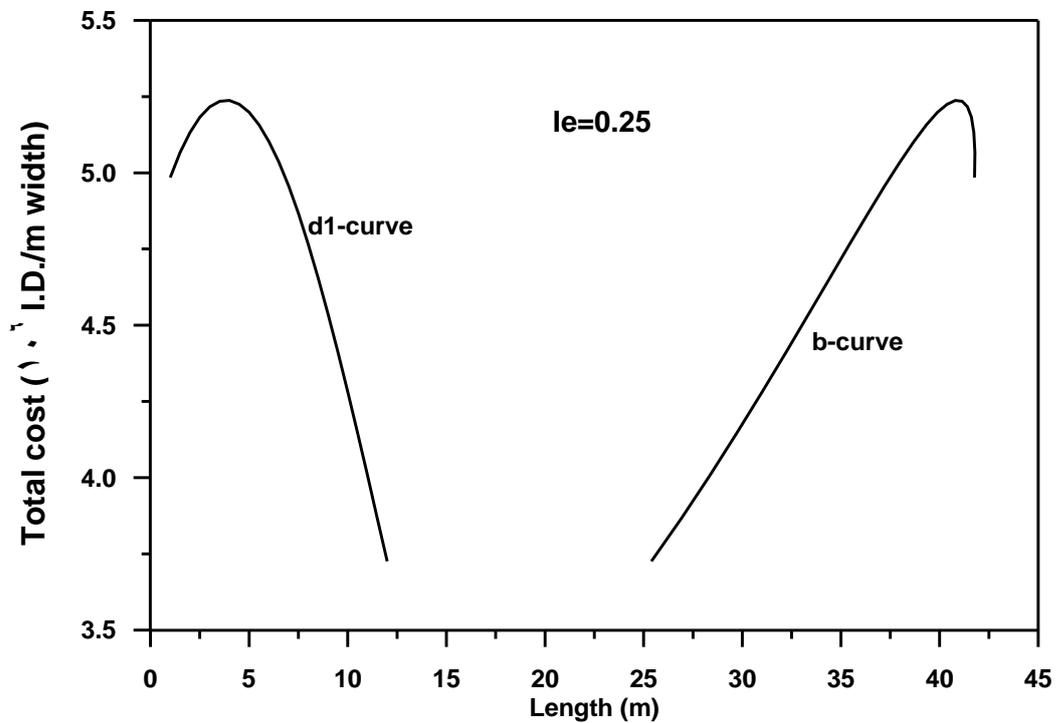


Fig.(9-32): Total cost for both the U/S cut-off (d1) and the floor (b), with safe exit gradient, ($H=6\text{m}$).

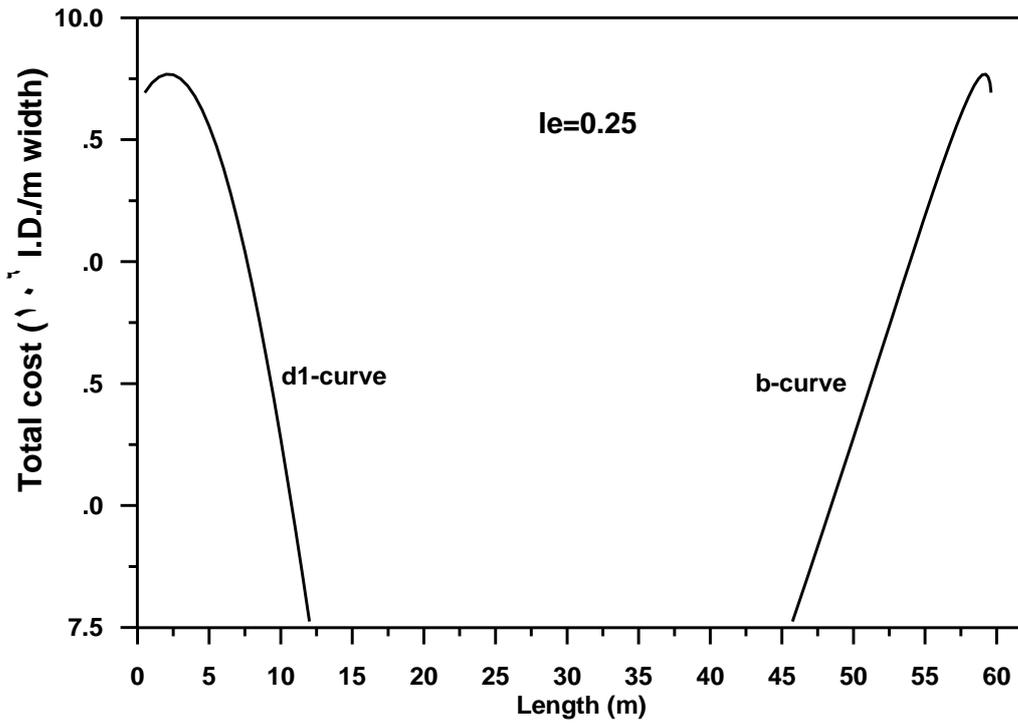


Fig.(9-33): Total cost for both the U/S cut-off (d) and the floor (b), with safe exit gradient, ($H=1m$).

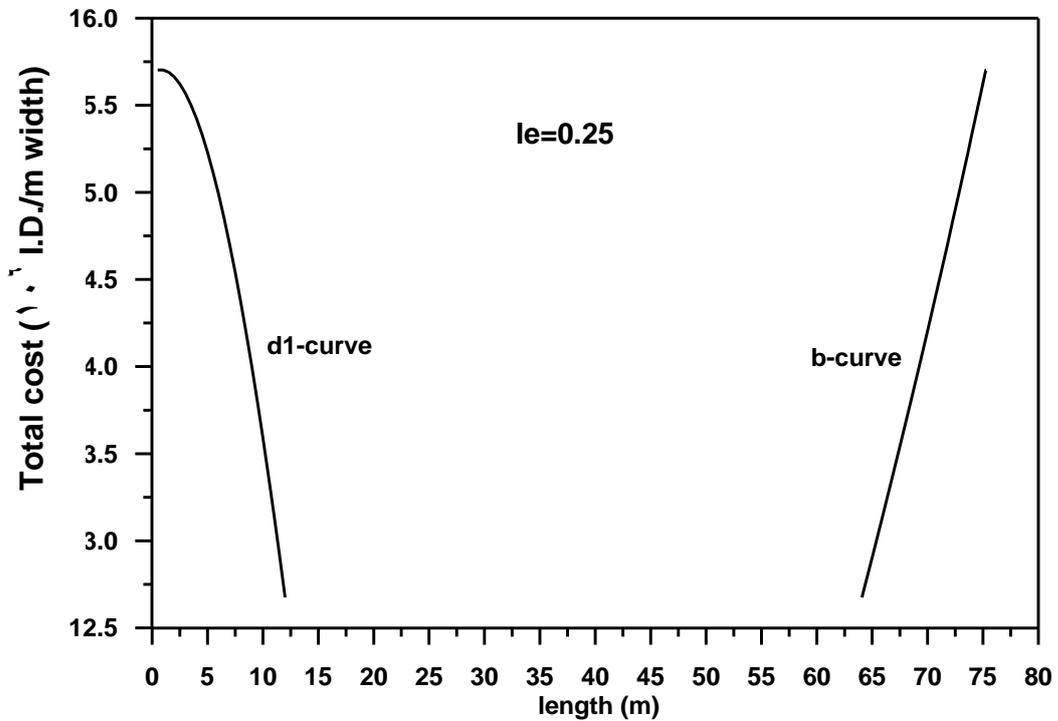


Fig.(9-34): Total cost for both the U/S cut-off (d) and the floor (b), with safe exit gradient ($H=1.5m$).

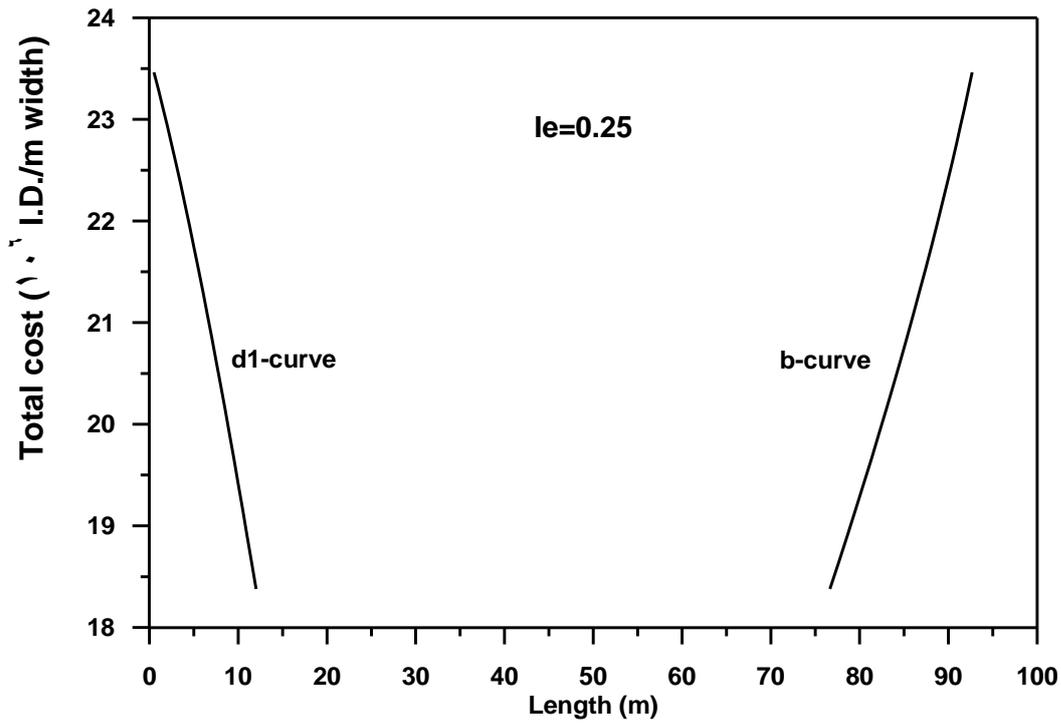


Fig.(9-35): Total cost for both the U/S cut-off (d1) and the floor (b), with safe exit gradient, (H=12m).

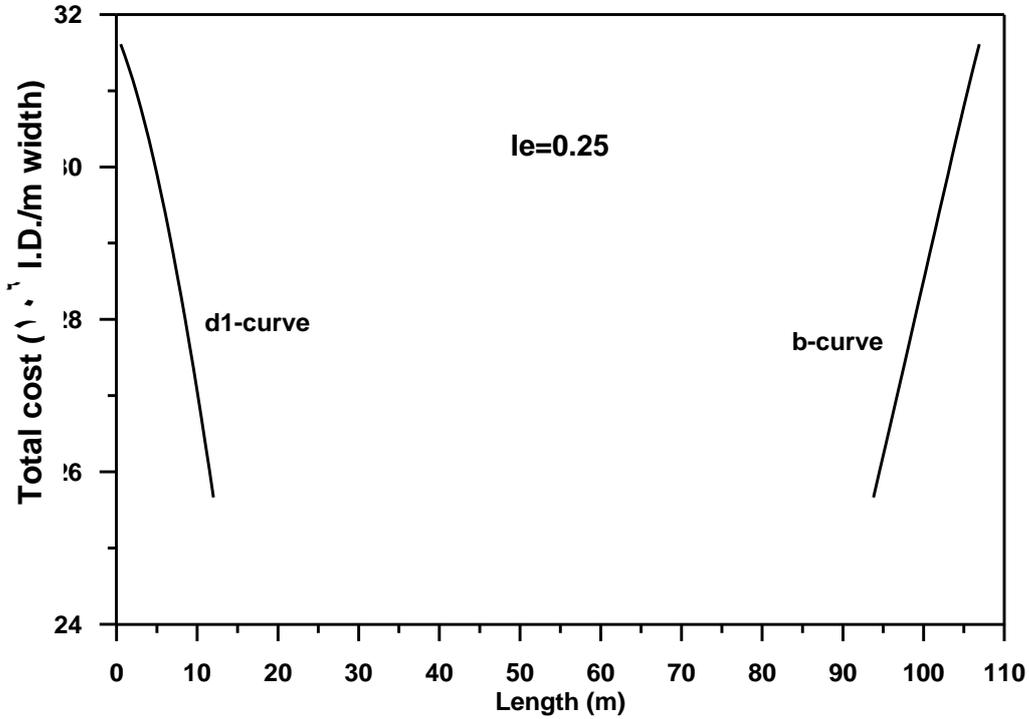


Fig.(9-36): Total cost for both the U/S cut-off (d1) and the floor (b), with safe exit gradient, (H=15m).

٥.٣.٤: Downstream cut-off

The downstream cut-off is more effective than upstream cut-off in terms of the reduction of the exit gradient, as has been shown in Article (٥.٢.٢).

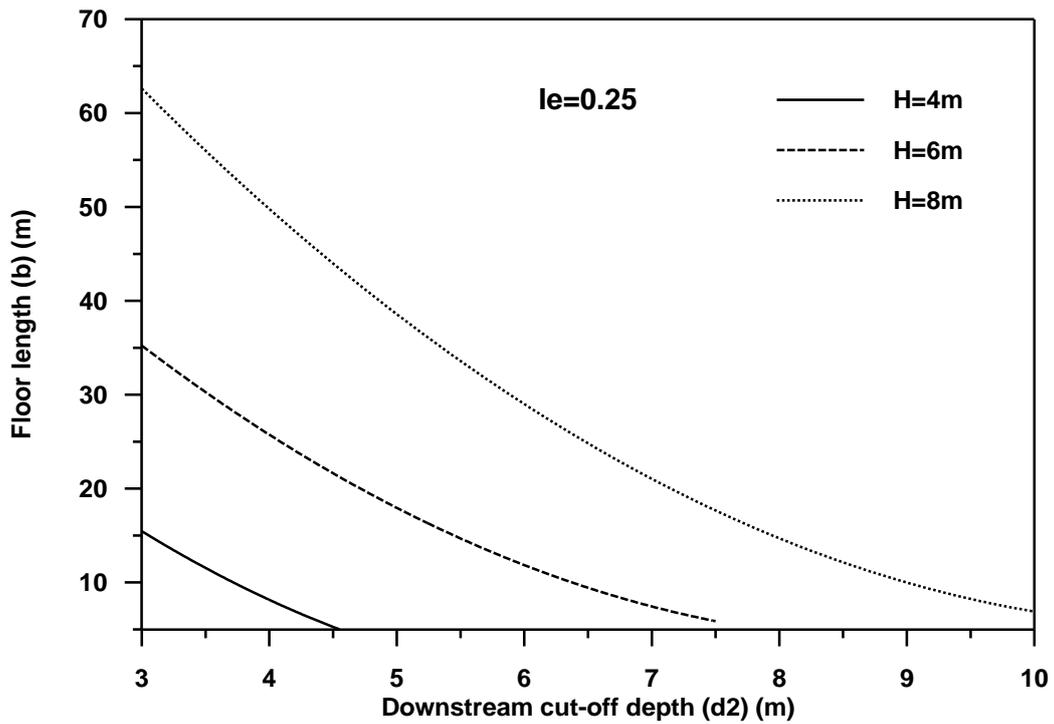
Figure (٥-٣٩) illustrates the relationship between the downstream cut-off depth (d_v) and the length of floor (b) for safe exit gradient, for the considered head difference (H). The figure indicates that the depth of downstream cut-off could be decreased when the floor length is increased.

Figure (٥-٤٠) illustrates the total cost (z) for both downstream cut-off and the floor, for the considered head difference. It can be noted that the total cost was decreased with the increase of the downstream cut-off depth whereas it increases with the increase of floor length. The figure could be used to find the necessary downstream cut-off depth (d_v) for any floor length (b).

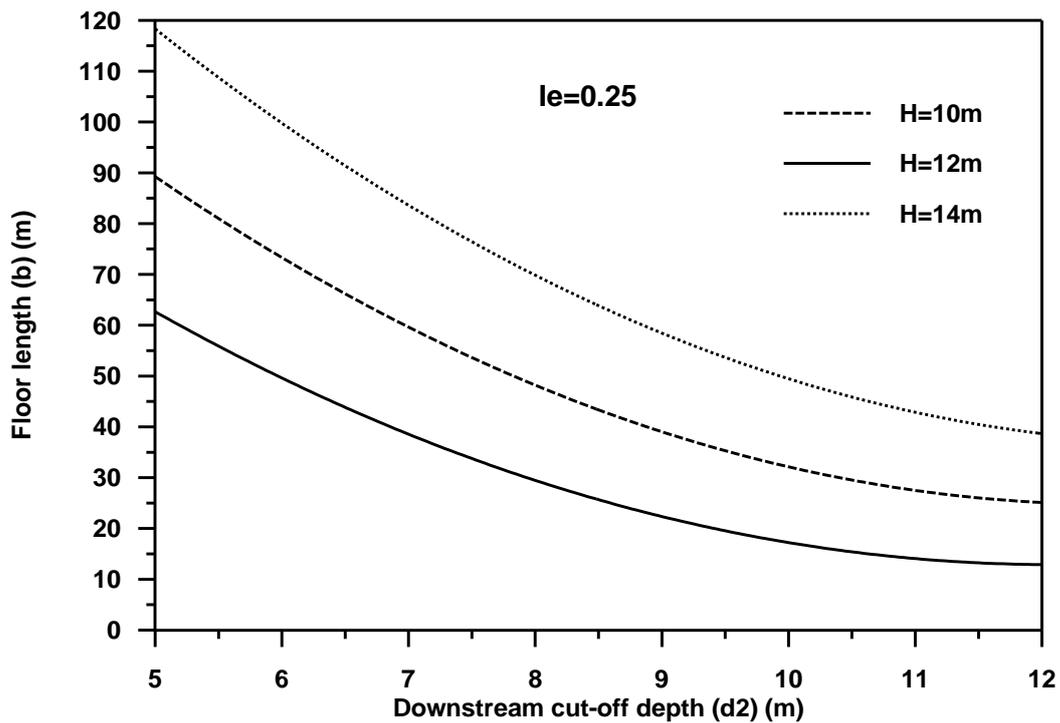
٥.٣.٥ Upstream and downstream cut-offs (U/D) under the hydraulic structure

Figures (٥-٤١) through (٥-٤٦) illustrate the relationship between downstream cut-off depth (d_v) and the length of floor (b) with various upstream cut-off depths (d_u), for safe exit gradient and the considered head difference (H).

It has been noted that the downstream cut-off depth was decreased with the increase of the floor length (b) for any upstream cut-off depth. Also, it was decreased with increase the upstream cut-off depth for same floor length value. Figures (٥-٤٧) through (٥-٥٢) illustrate the total costs of all feasible solutions for upstream, downstream depths and the floor length with various head difference (H). It has been noted that the magnitude of the total cost was decrease with increase downstream cut-off depth and it was increase with increase floor length for same value of upstream cut-off depth.

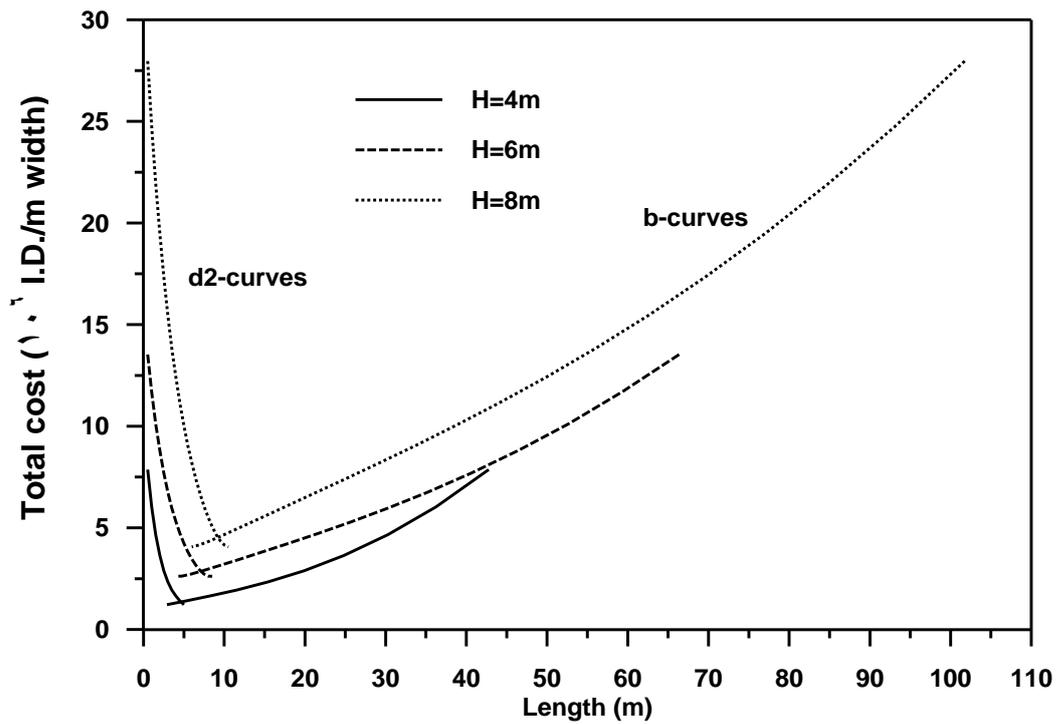


[a]: For (H= 4 , 6 , and 8 m)

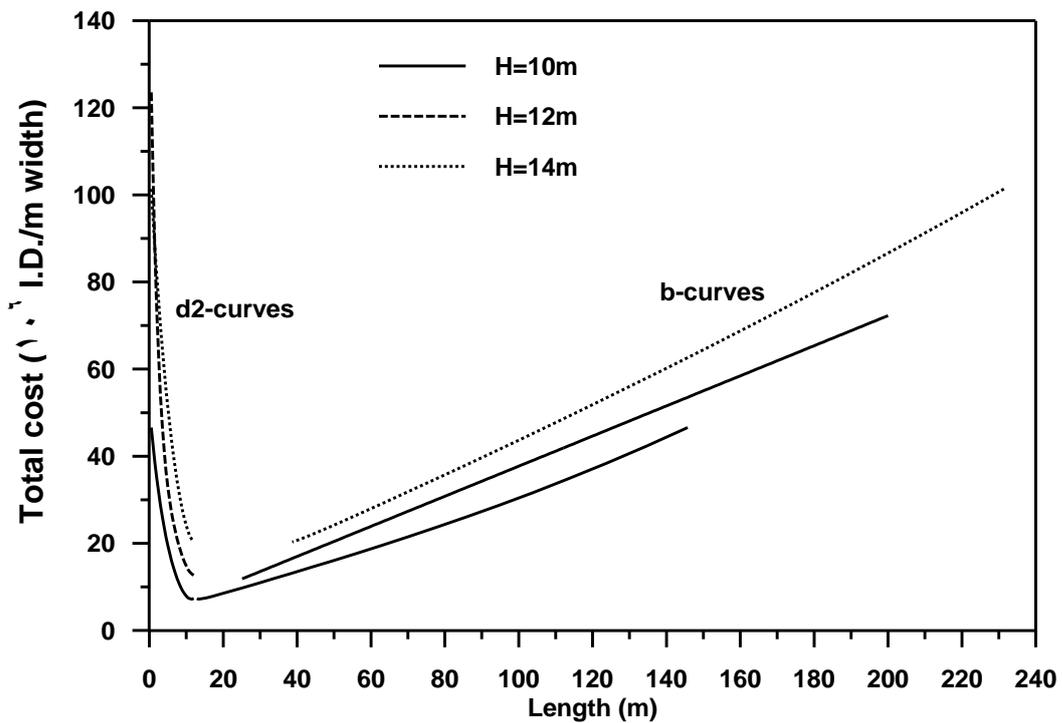


[b]: For (H= 10 , 12 , and 14 m)

Fig.(9-34):Relation between the D/S cut-off depth(d_2) and the floor length(b) for safe exit gradient.



[a]:For ($H= 4, 6, \text{ and } 8 \text{ m}$)



[b]:For ($H= 10, 12, \text{ and } 14 \text{ m}$)

Fig.(9-38):Total cost for both D/S cut-off (d₂) and the floor (b),with safe exit gradient.

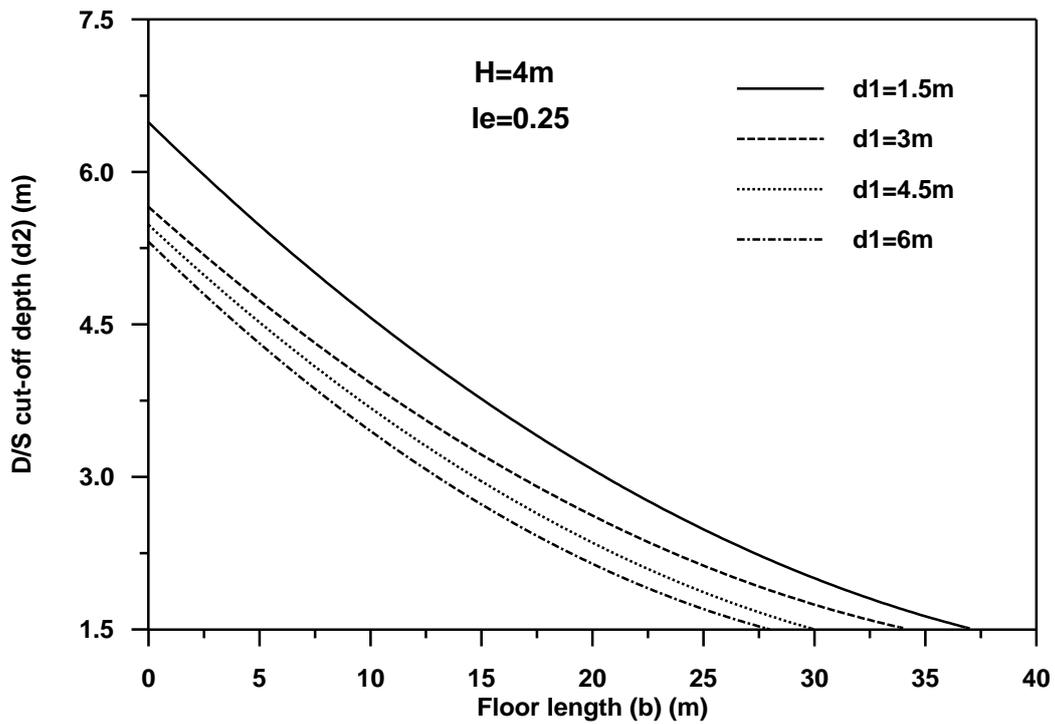


Fig.(٥-٣٩): Relation between D/S cut-off depth(d_2) and the floor length(b) with various depths of U/S cut-off(d_1), for safe exit gradient, ($H=4$ m).

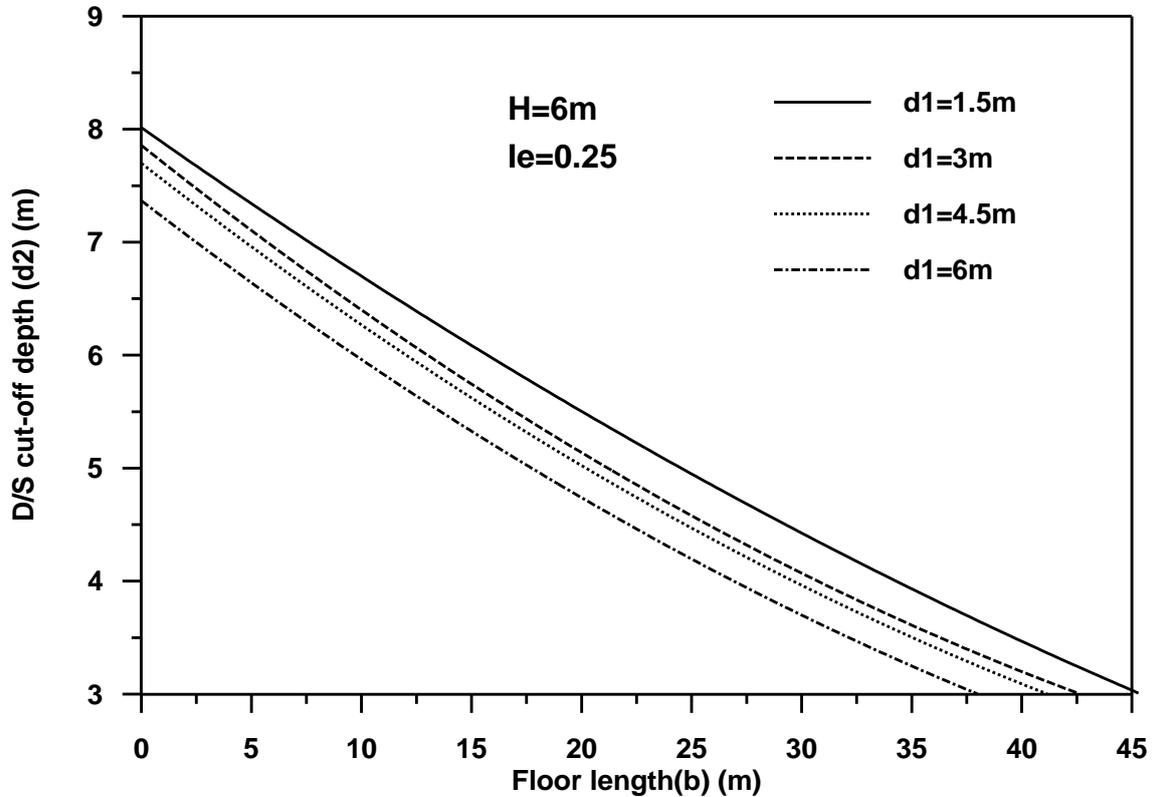


Fig.(٥-٤٠): Relation between D/S cut-off depth(d_2) and the floor length(b) with various depths of U/S cut-off(d_1), for safe exit gradient, ($H=6$ m).

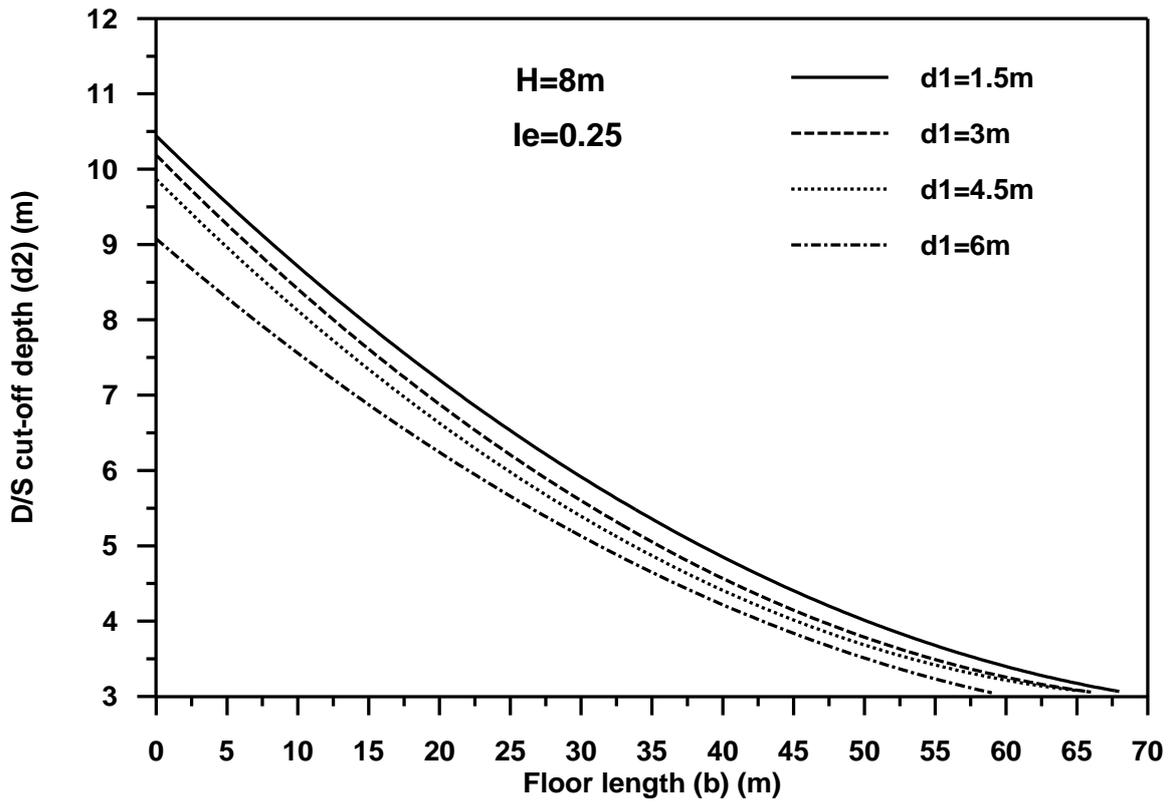
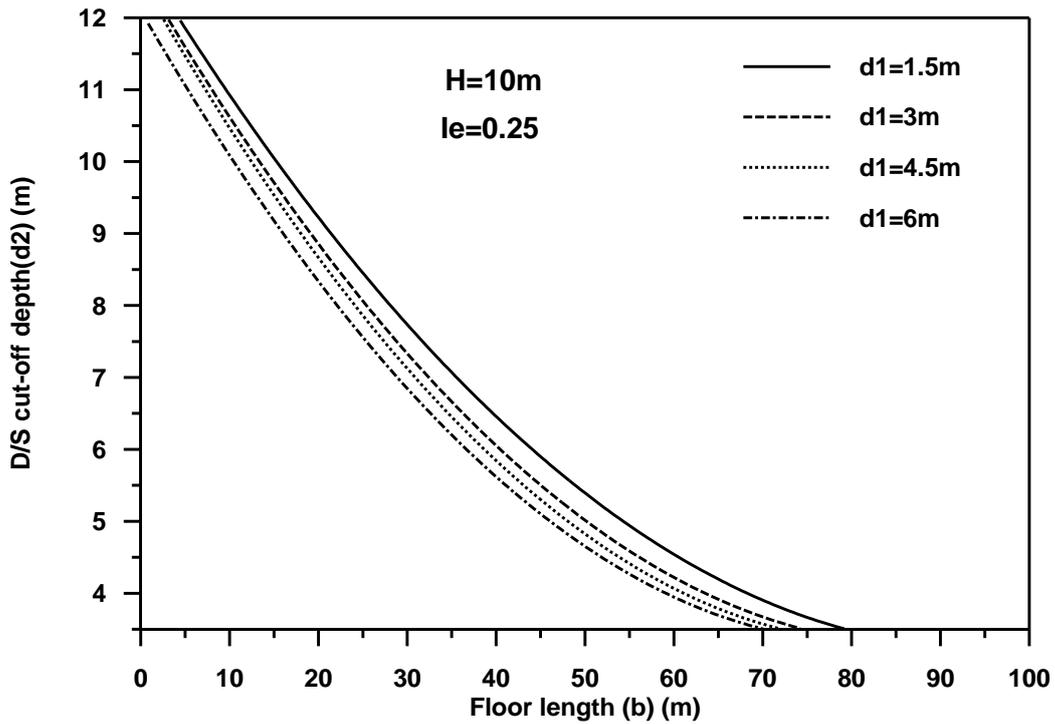
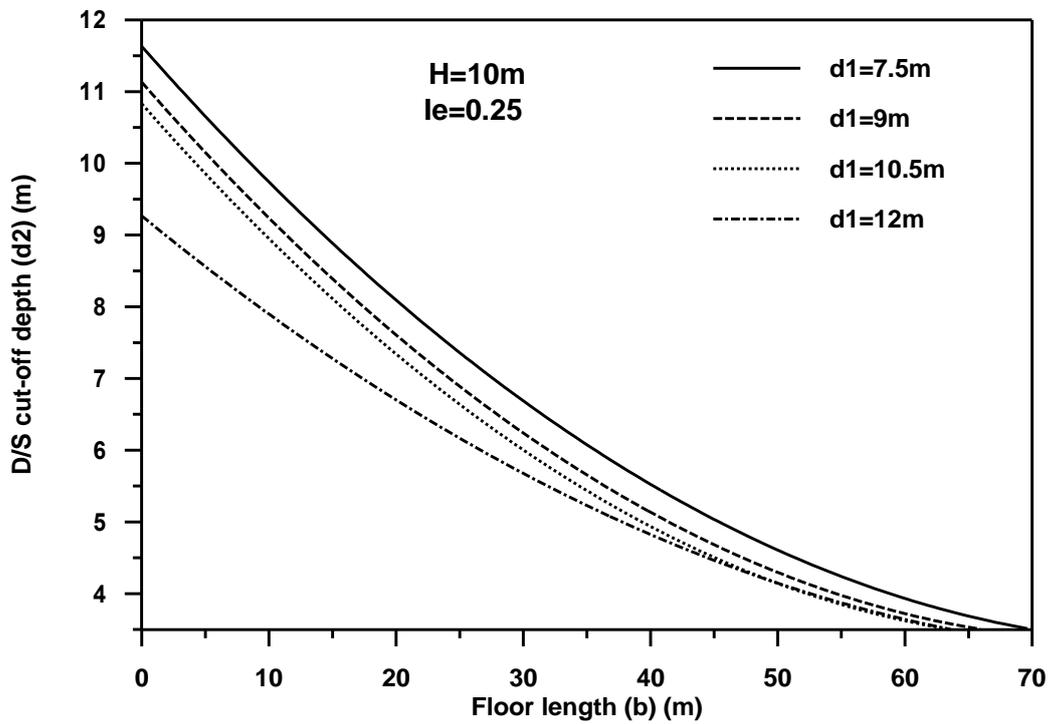


Fig.(9-41):Relation between the D/S cut-off depth(d_2) and the floor length(b) with various depths of U/S cut-off(d_1), for safe exit gradient, ($H=8m$).

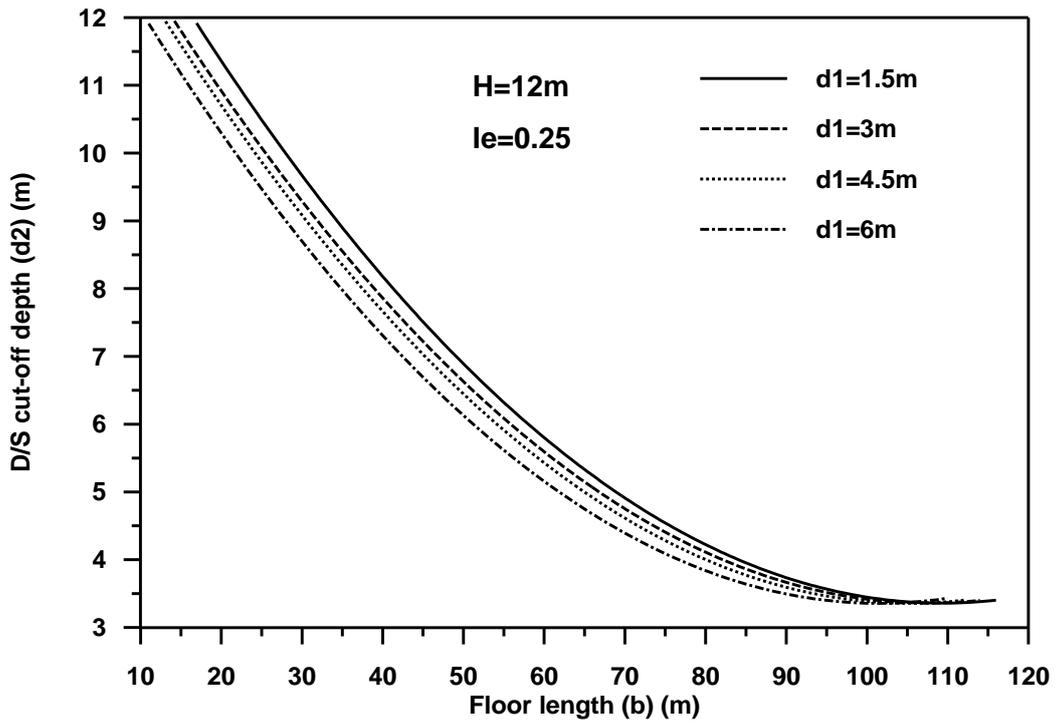


[a]: For ($d_1 = 1.5, 3, 4.5, \text{ and } 6 \text{ m}$)

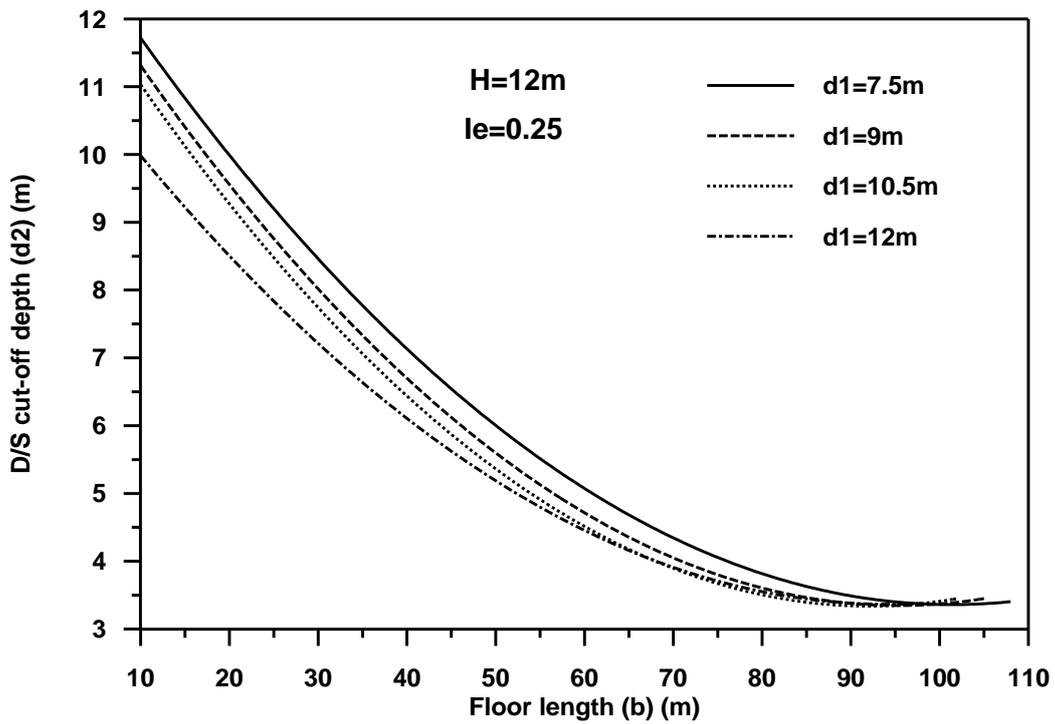


[b]: For ($d_1 = 7.5, 9, 10.5, \text{ and } 12 \text{ m}$)

Fig.(5-42):Relation between D/S cut-off depth(d_2) and the floor length(b) with various depths of U/S cut-off (d_1),for safe exit gradient,($H=10 \text{ m}$).

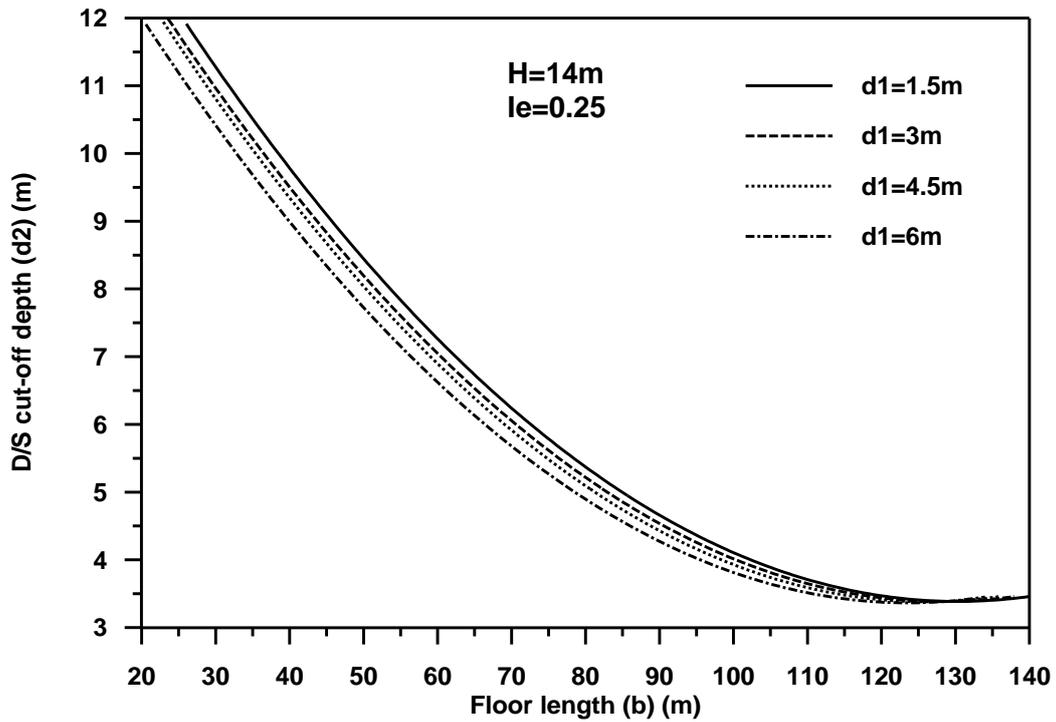


[a]: For ($d_1 = 1.5, 3, 4.5, \text{ and } 6$ m)

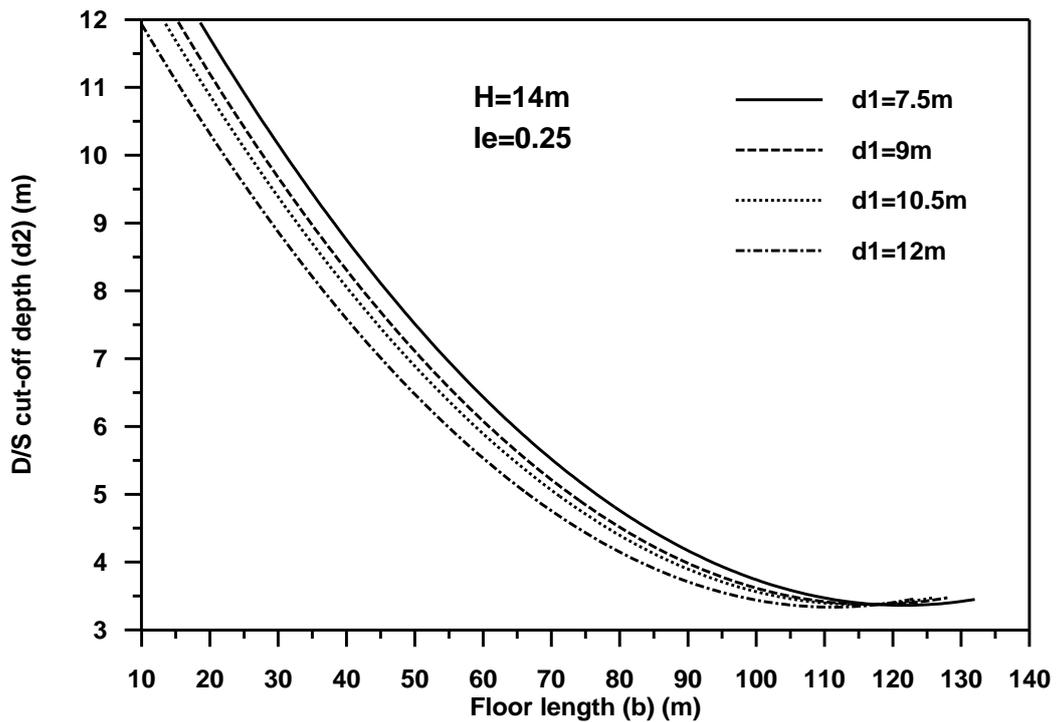


[b]: For ($d_1 = 7.5, 9, 10.5, \text{ and } 12$ m)

Fig.(9-43): Relation between D/S cut-off depth(d_2) and the floor length(b) with various depths of U/S cut-off(d_1), for safe exit gradient, ($H=12$ m).



[a]: For ($d_1 = 1.5, 3, 4.5, \text{ and } 6\text{ m}$)



[b]: For ($d_1 = 7.5, 9, 10.5, \text{ and } 12\text{ m}$)

Fig.(9-44): Relation between D/S cut-off depth(d_2) and the floor length(b) with various depths of U/S cut-off(d_1), for safe exit gradient, ($H=14\text{m}$).

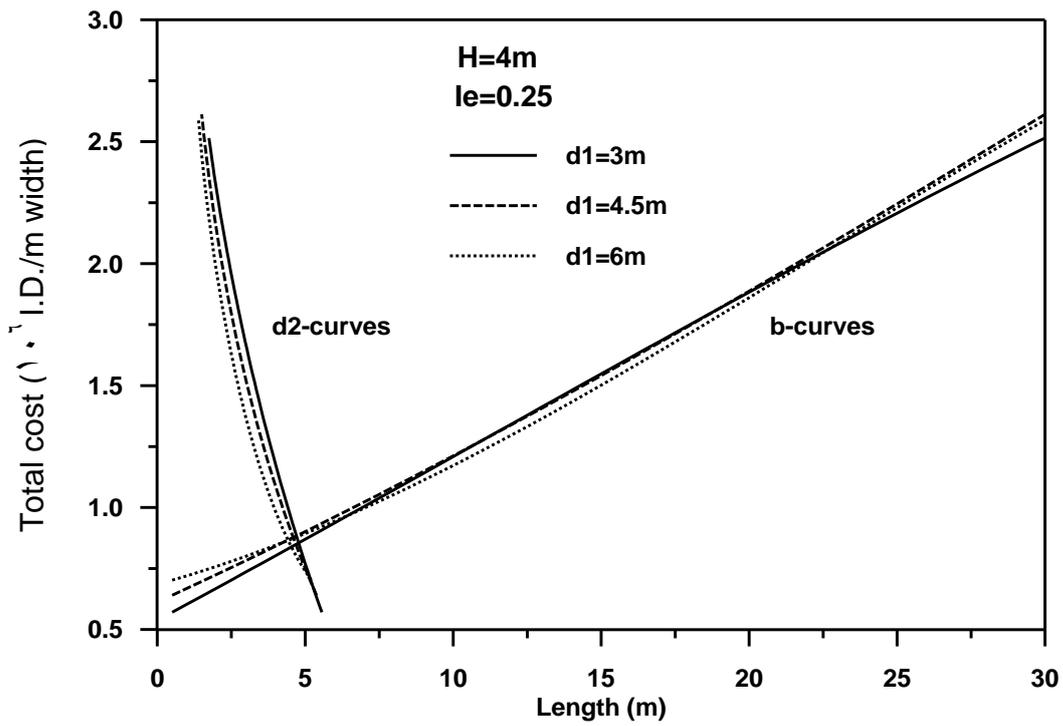


Fig.(9-45): Total cost of using U/D cut-offs with safe exit gradient, (H=4m).

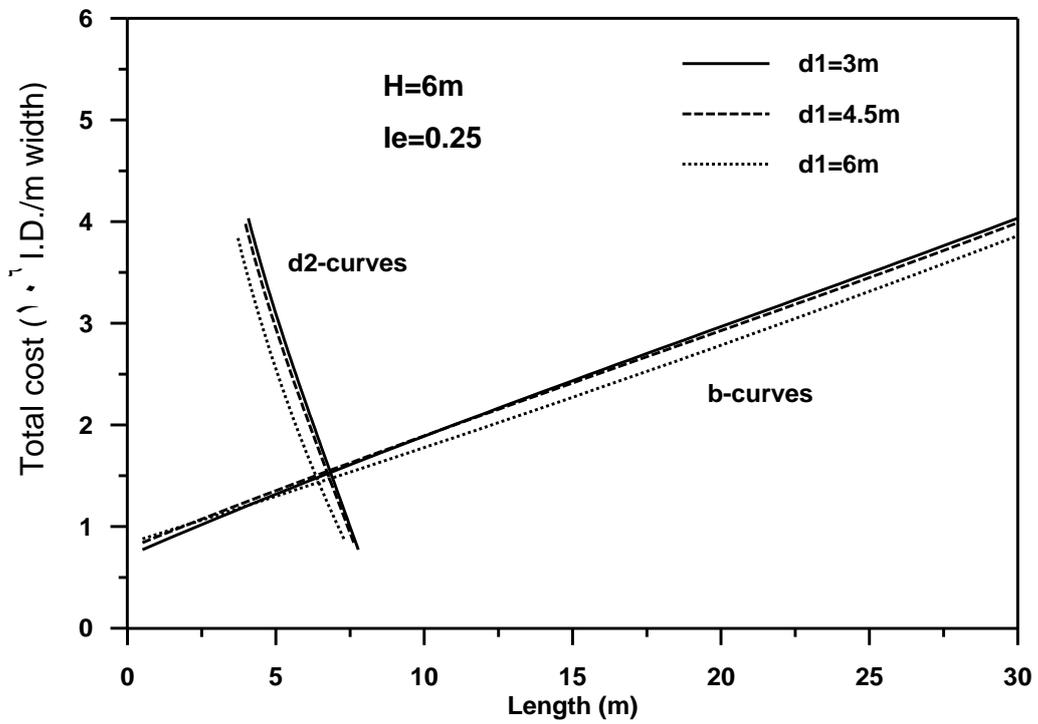


Fig.(9-46): Total cost of using U/D cut-offs with safe exit gradient, (H=6m).

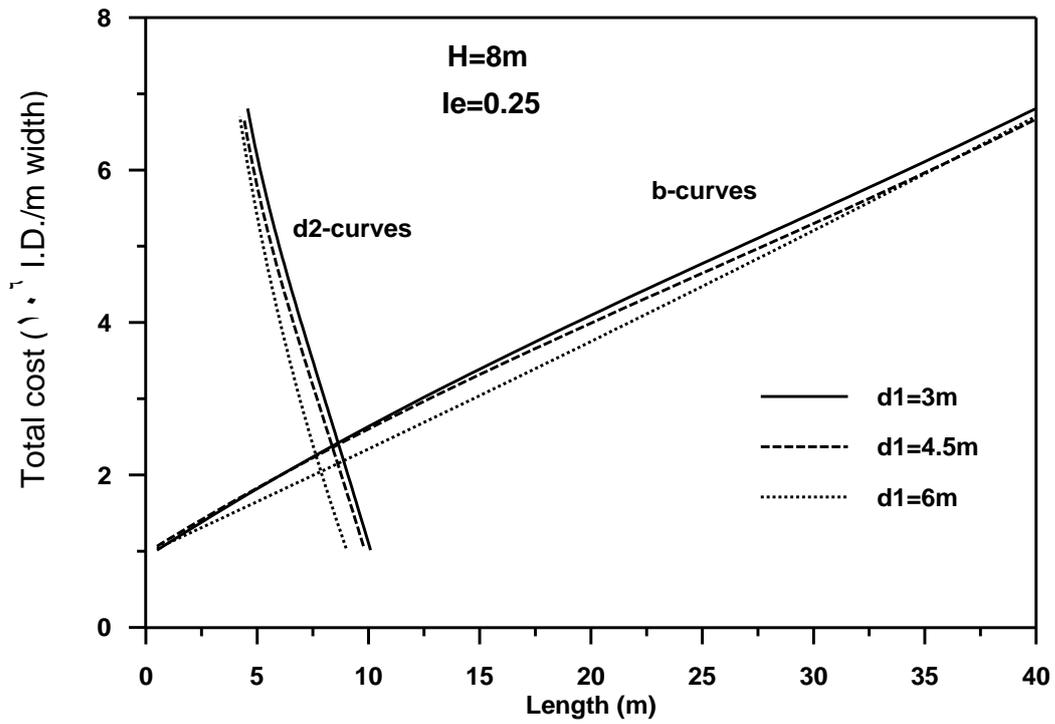


Fig.(9-47): Total cost of using U/D cut-offs with safe exit gradient, ($H=8$ m)

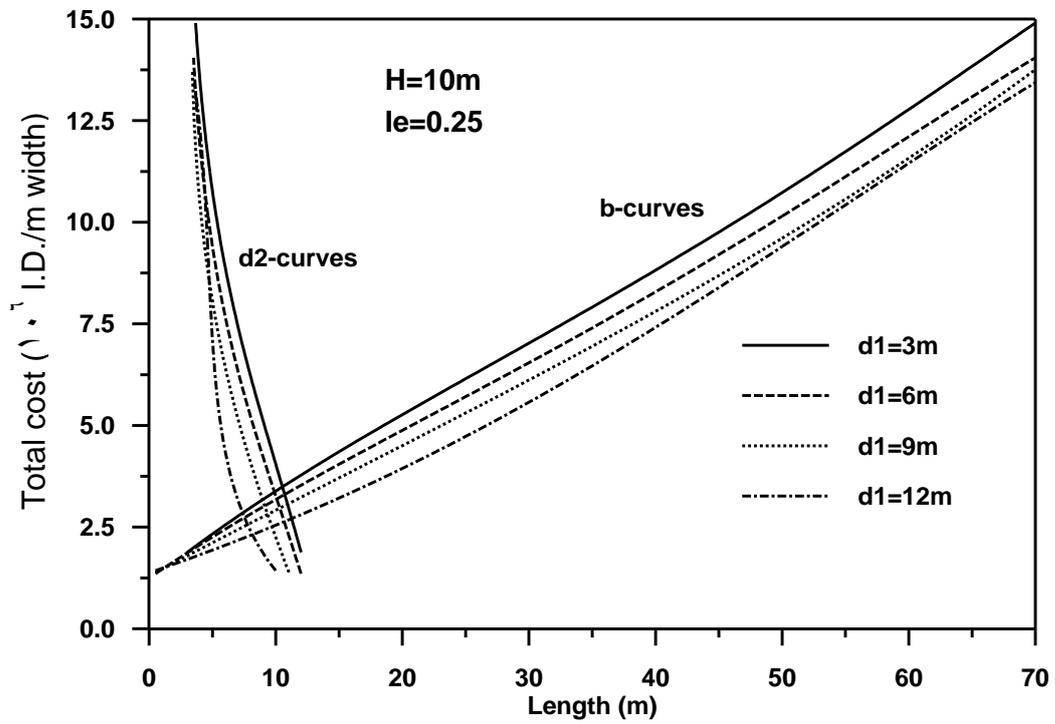


Fig.(9-48): Total cost of using U/D cut-offs with safe exit gradient, ($H=10$ m)

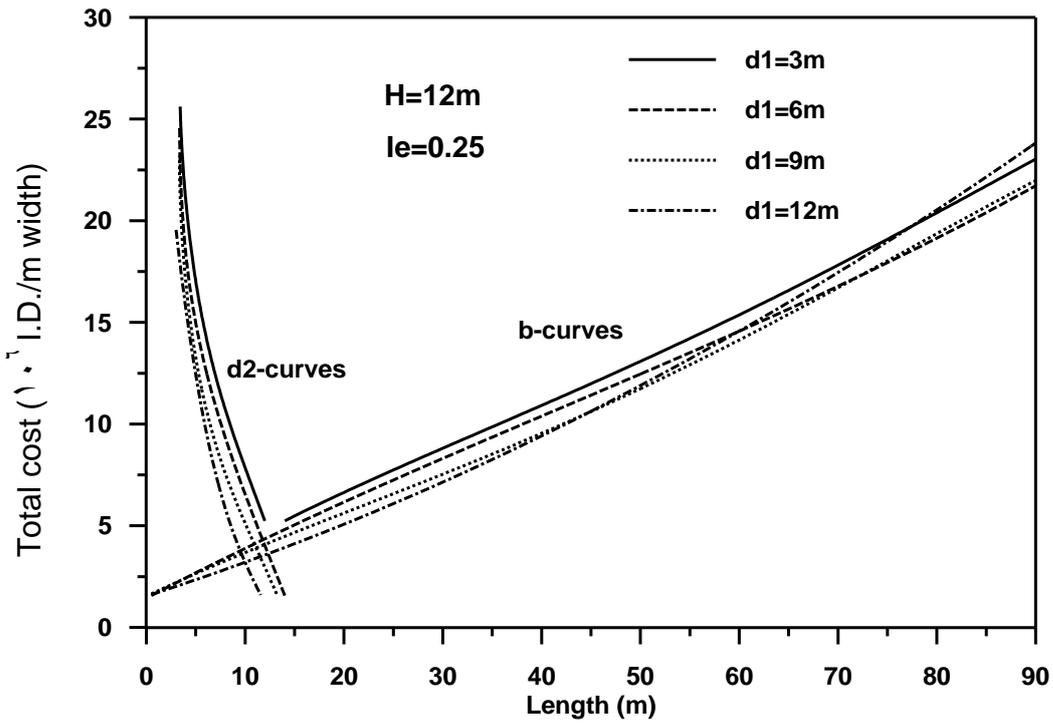


Fig.(9-49): Total cost of using U/D cut-offs with safe exit gradient, (H=12m).

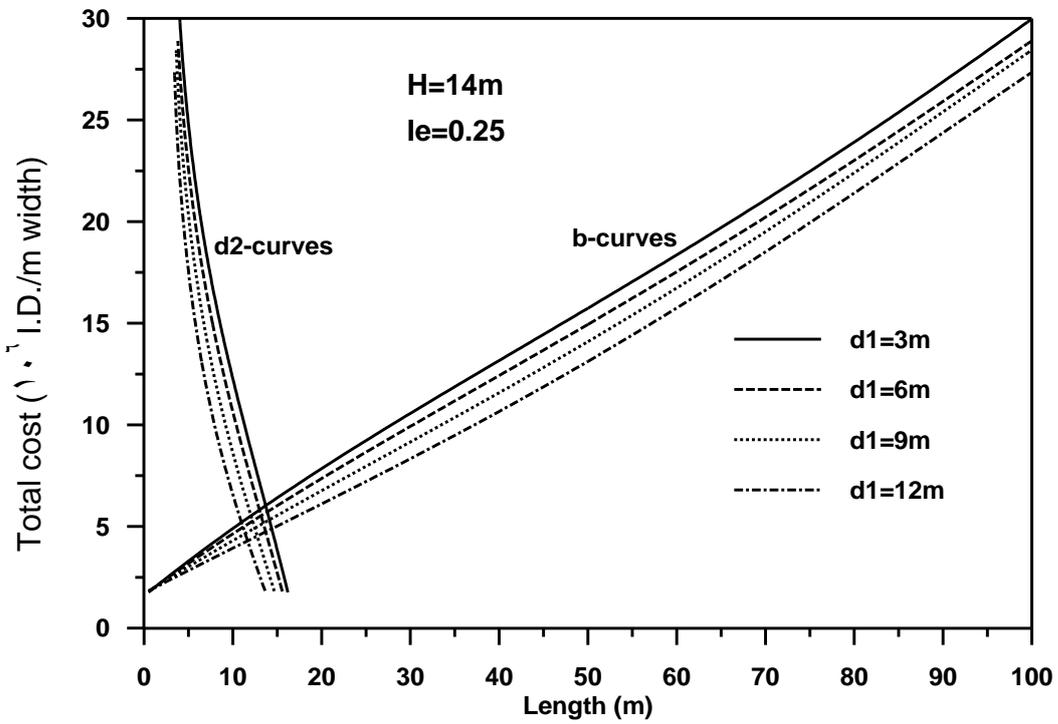


Fig.(9-50): Total cost of using U/D cut-offs with safe exit gradient, (H=14m).

٥.٤: Optimum design of the control system

For each head difference value (H), an optimum design (minimum total cost) could be found from the feasible solutions.

Table (٥-١) shows the optimum design of control system with various head difference (H). It has been noted from this Table that the filter trench gives a more economical cost than the upstream blanket, upstream cut-off and downstream cut-off. This is to be expected because the effect of the filter trench in decrease the uplift pressure and exit gradient with low cost of it.

Figure (٥-٥٣a) and (٥-٥٣b) show the flow chart of the optimization program.

Table (٥-١): Optimum design of control devices (Lagrange-multiplier method).

Diff. head (H) m	Floor length (b) in m	U/S blanket (b_1) in m	U/S cut-off (d_1) in m	D/S cut-off (d_2) in m	Total cost $\times 10^6$ in I.D. /m	
					With filter trench	Without filter trench
٤	٣.٠	٠	٣	٥	١.٥٥٠٤	١.٢٥٠٣
٦	٨.٥	٠	٣.٥	٦.٥	١.٧٨٩٤	٢.١٩٧٣
٨	١٥	٠	٤.٥	٧	٢.٧٤٣٧	٣.٤٢٦١
١٠	٢٠	٠	٦	٨.٥	٣.٣٩٣٦	٥.٢٨١٠
١٢	٣٢	٠	٩	٩.٥	٤.٢٢٢٦	٧.١٩٧٨
١٤	٤٠	٠	١٠	١١	٥.٦٦٩٣	١١.٤٥٨١

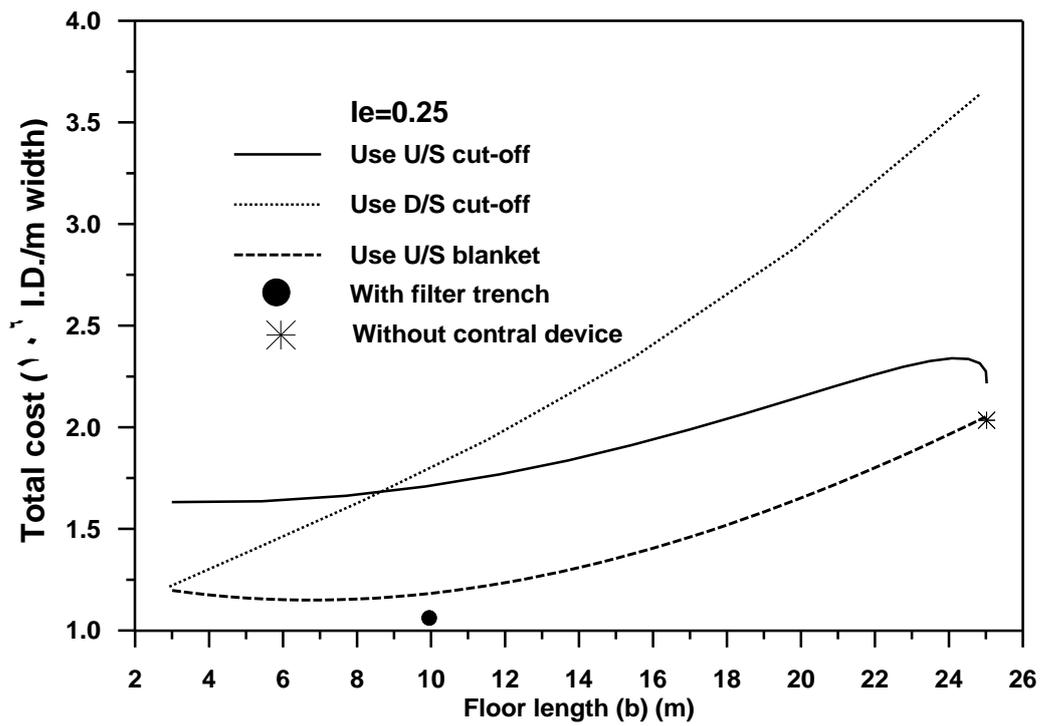
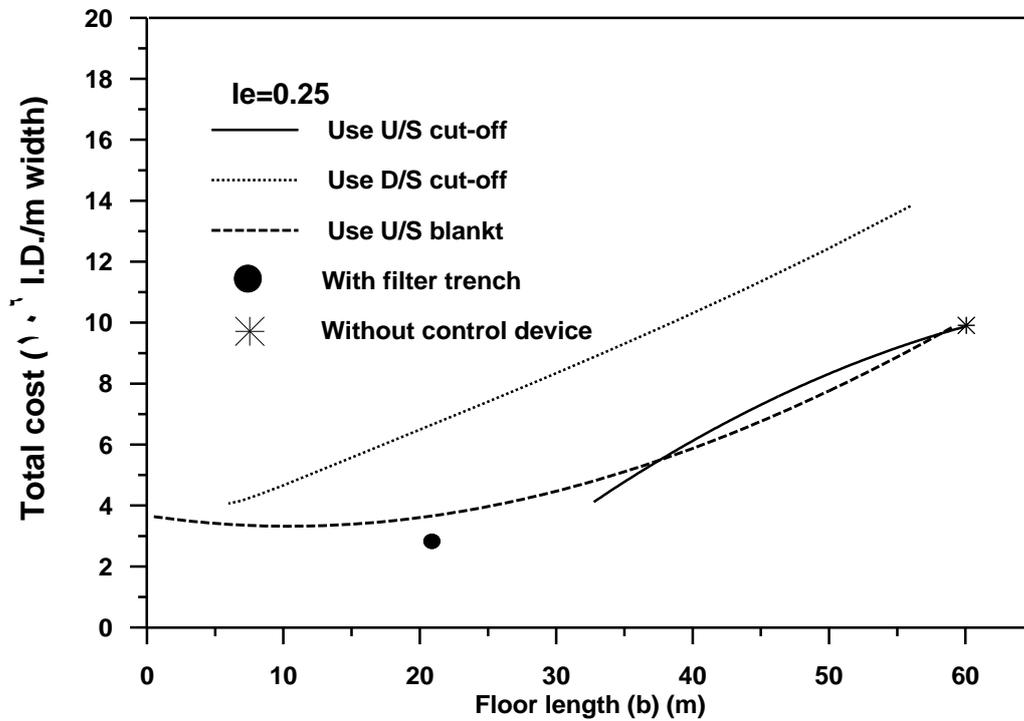


Fig. (9-9): Total cost of using different control devices with floor base, for safe exit gradient, ($H=1\text{ m}$).



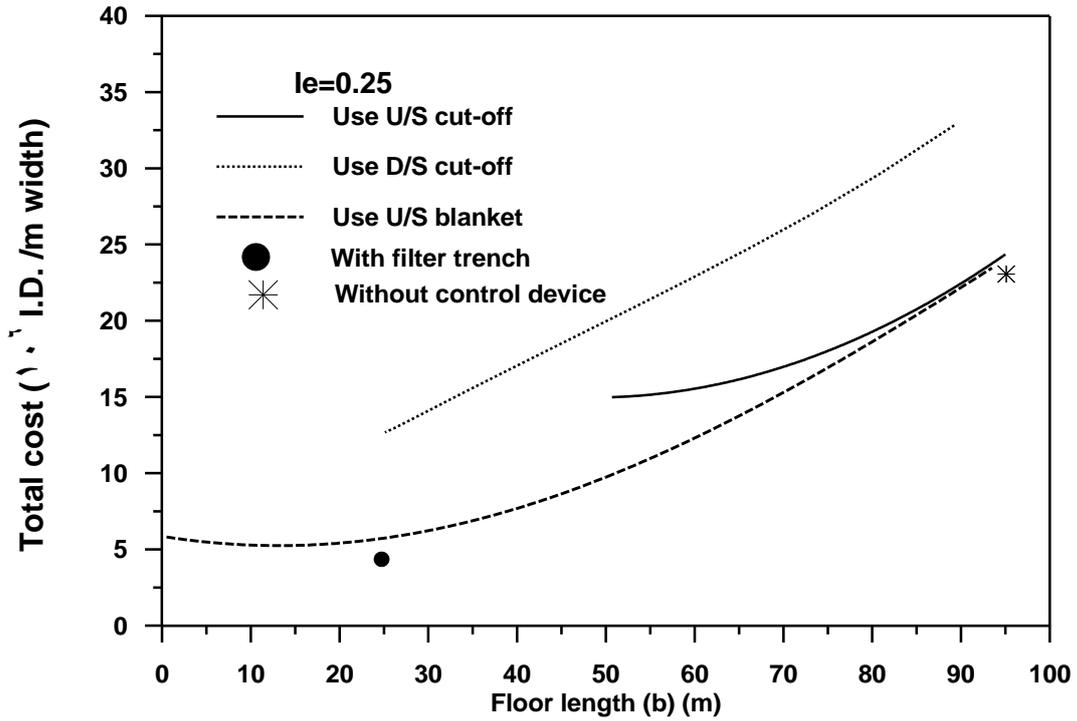
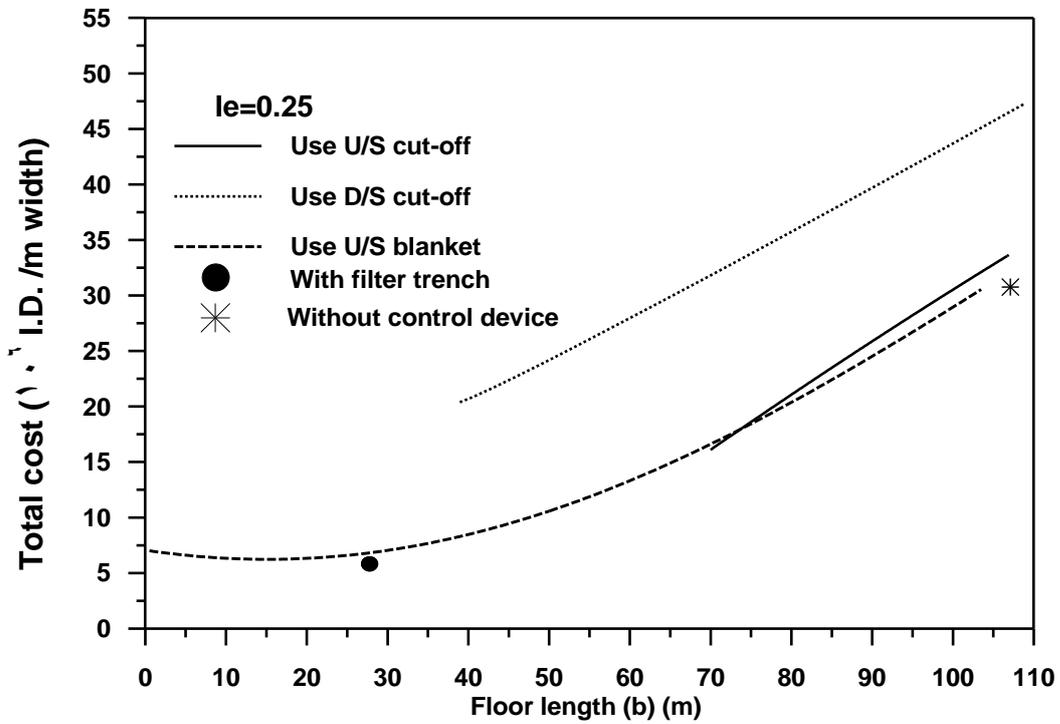


Fig.(9-99):Total cost of using different control devices with floor base,for safe exit gradient ,(H=1.7m).



Figures (2-21) through (2-26) as well as Table (2-1) show that the filter trench gives a more economical cost than The saving in cost between used the upstream blanket, upstream cut-off, downstream cut-off and filter trench are shown in Table (2-2). It is clear from this table that remarkable saving in the cost can be ensured by the use of the optimization procedure.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

In this research, the finite-element method has been used to analyze seepage through porous media under hydraulic structures that are provided with blanket, cut-offs, and filter trench as seepage control devices, and found on homogeneous isotropic soil foundation. The Lagrange multiplier method has been used to find the optimum design (hydraulic and economical) for the control devices.

6.1: Conclusions

The following conclusions have been abstracted:

1. The uplift pressure decreases with the increase in upstream length of the blanket; the same holds true with the exit gradient. However, the uplift pressure and exit gradient increases when using a downstream blanket.
2. Using a cut-off in various locations produces significant effect in reducing the uplift pressure values; that is, whenever a cut-off is located, a drop in the uplift pressure values at that location occurred. Also, the magnitude of the exit gradient decreases as the cut-off moves in location from upstream to downstream sides.
3. The downstream cut-off is more effective than upstream cut-off in terms of the reduction of the exit gradient.
4. The pressure head upstream and downstream of the filter trench decreases as the width or depth of filter increases; this remains true with

the exit gradient. Also, the exit gradient decreases as the filter trench is moving from upstream to downstream.

୧. The uplift pressure and exit gradient are decrease with increasing the (w/z) ratio for same sectional area of filter.
୧. For a hydraulic structure with upstream blanket of various lengths, a blanket of (zero) length gives the maximum floor cost for all applied head difference.
୧. For a hydraulic structure with upstream cut-off of various depths, the total cost decrease with increasing the depth of the upstream cut-off and decreasing the length of the floor. The minimum total cost is attained when the maximum upstream cut-off depth is used. This remains true regardless of using a downstream cut-off.
୧. For a hydraulic structure with different control devices, the minimum total cost could be achieved when a filter trench is used.
୧. Using the upstream cut-off is more economical than the downstream cut-off.

୧.୧: Recommendations for further work

The following recommendations are suggested for further research:

୧. Extension of the present analysis to include structures with various soil stratification including soil anisotropy.
୧. Applying the present analysis to practical cases studies.
୧. Using the finite-element method to include three-dimensional seepage problems through the soil under hydraulic structures.

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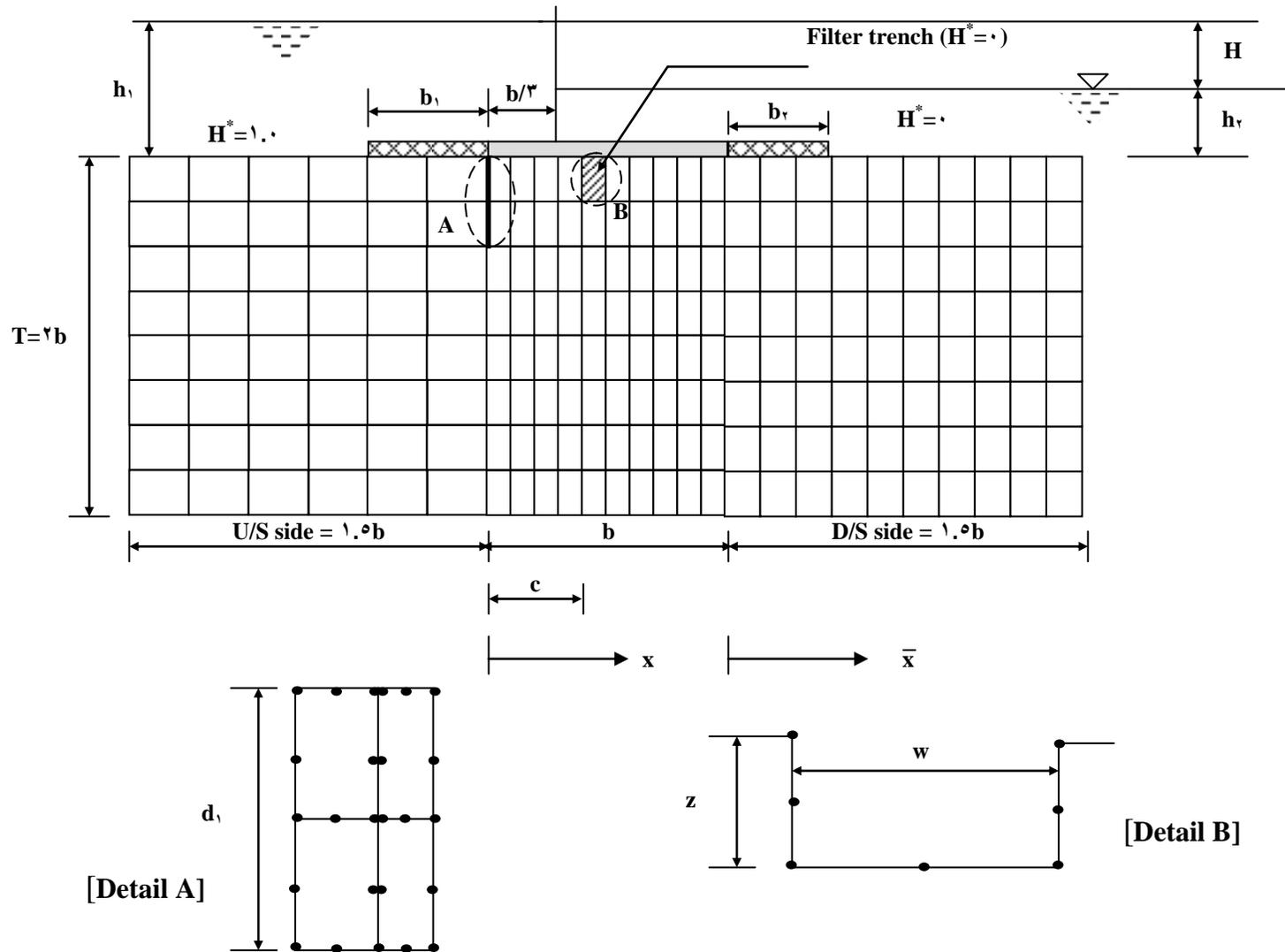


Fig.(9-1): The finite element mesh discretization of a hydraulic structure with U/D blankets, U/S cut-off, and filter trench.