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Effect of Chamfering on the Stability of the Plain Cylindrical Hydrodynamic Journal Bearing

A Thesis

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By

Lekaa' Abbass Fadhil Ibrahim

Al-Joubori

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نيل درجة ماجستير علوم في الهندسة الميكانيكية
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﴿بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ﴾

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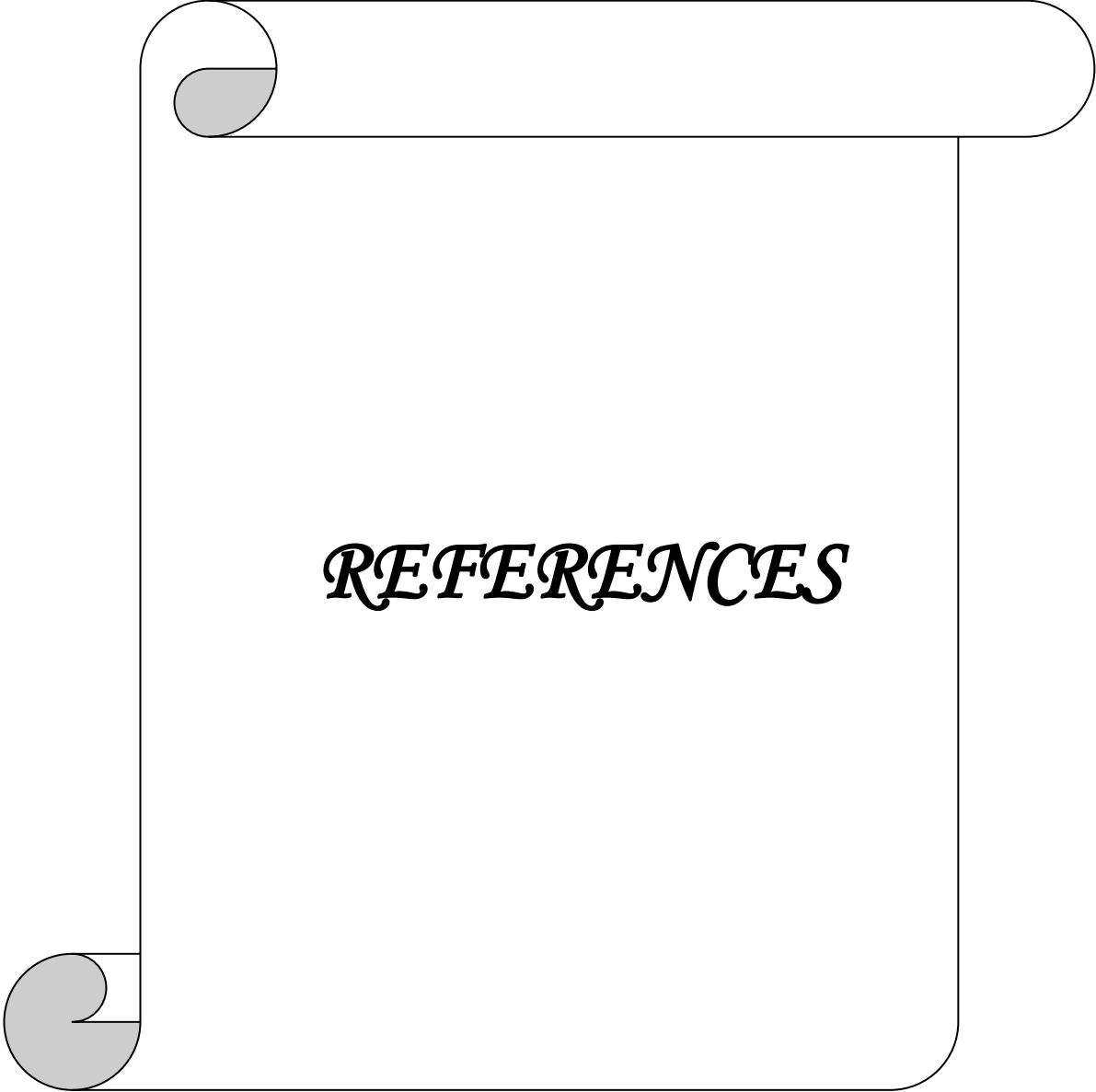
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NOMENCLATURE

The following symbols are generally used throughout the text:-

Symbol	Description	Units
C	Journal- Bearing Radial Clearance	m
$C_{i,j}(i,j=x,y)$	Damping Coefficients	N.s/m
$\bar{C}_{i,j}(i,j=x,y)$	Dimensionless Damping Coefficients($\bar{C}_{i,j}=C_{i,j} * C * \omega / F_0$)	-
C_{xx}, C_{yy}	Direct Damping Coefficients in X,Y Coordinates	N.s/m
C_{xy}, C_{yx}	Cross-Coupled Damping Coefficients in X,Y Coordinates	N.s/m
$\bar{C}_{xx}, \bar{C}_{yy}$	Dimensionless Direct Damping Coefficients in X,Y Coordinates	-
$\bar{C}_{xy}, \bar{C}_{yx}$	Dimensionless Cross-Coupled Damping Coefficients in X,Y Coordinates	-
$C_{\varepsilon\varepsilon}, C_{\phi\phi}$	Direct Damping Coefficients in Rotating Coordinates (r,t)	N.s/m
$C_{\varepsilon\phi}, C_{\phi\varepsilon}$	Cross-Coupled Damping Coefficients in Rotating Coordinates (r,t)	N.s/m
$\bar{C}_{\varepsilon\varepsilon}, \bar{C}_{\phi\phi}$	Dimensionless Direct Damping Coefficients in Rotating Coordinates (r,t)	-
$\bar{C}_{\varepsilon\phi}, \bar{C}_{\phi\varepsilon}$	Dimensionless Cross-Coupled Damping Coefficients in Rotating Coordinates (r,t)	-
D	Journal Diameter	m
e	Eccentricity, ($e=C*\varepsilon$)	m
e_0	Steady-State Eccentricity, ($e_0=C*\varepsilon_0$)	m
e_x, e_y	Position of Journal Center	m
e_{x0}, e_{y0}	Static Equilibrium Position of Journal Center	m

F_{ε}	Fluid film force component in radial coordinate	N
F_{ϕ}	Fluid film force component in tangential coordinate	N
$_{\varepsilon}\overline{F}$	Dimensionless Fluid film force component in radial coordinate	-
$_{\phi}\overline{F}$	Dimensionless Fluid film force component in tangential coordinate	-
F_x	Bearing Reaction (Restoring)Force in Vertical Direction	N
F_y	Bearing Reaction (Restoring)Force in Horizontal Direction	N
$_x\overline{F}$	Dimensionless Bearing Reaction (Restoring)Force in Vertical Direction	-
$\overline{F}_y$	Dimensionless Bearing Reaction (Restoring)Force in Horizontal Direction	-
F_o	Load Carrying Capacity	N
$_o\overline{F}$	Dimensionless Load Carrying Capacity	-
F_{xo}	Bearing Reaction Equilibrium Force in Vertical Direction	N
F_{yo}	Bearing Reaction Equilibrium Force in Horizontal Direction	N
$f_{\varepsilon o}$	Static Radial Force at the equilibrium position Using Short Bearing Theory	N
$f_{\phi o}$	Static Tangential Force at the equilibrium position Using Short Bearing Theory	N
h	Oil-Film Thickness	m
h_o	Zeroth-Order Perturbed Oil-Film Thickness	m
hc	Oil-Film Thickness in Chamfer Regim	m

$K_{i,j}(i,j=x,y)$	Stiffness Coefficients	N/m
$\bar{K}_{i,j}(i,j=x,y)$	Dimensionless Stiffness Coefficients ($\bar{K}_{i,j}=K_{i,j} * C/F_o$)	-
K_{xx}, K_{yy}	Direct Stiffness Coefficients in X,Y Coordinates	N/m
K_{xy}, K_{yx}	Cross-Coupled Stiffness Coefficients in X,Y Coordinates	N/m
$\bar{K}_{xx}, \bar{K}_{yy}$	Dimensionless Direct Stiffness Coefficients in X,Y Coordinates	-
$\bar{K}_{xy}, \bar{K}_{yx}$	Dimensionless Cross-Coupled Stiffness Coefficients in X,Y Coordinates	-
$K_{\varepsilon\varepsilon}, K_{\phi\phi}$	Direct Stiffness Coefficients in Rotating Coordinates (r,t)	N/m
$K_{\varepsilon\phi}, K_{\phi\varepsilon}$	Cross-Coupled Stiffness Coefficients in Rotating Coordinates (r,t)	N/m
$\bar{K}_{\varepsilon\varepsilon}, \bar{K}_{\phi\phi}$	Dimensionless Direct Stiffness Coefficients in Rotating Coordinates (r,t)	-
$\bar{K}_{\varepsilon\phi}, \bar{K}_{\phi\varepsilon}$	Dimensionless Cross-Coupled Stiffness Coefficients in Rotating Coordinates (r,t)	-
L	Bearing length	m
L _c	Chamfer length	m
N _j	Journal Rotational Speed	r.p.m
N ₁	Number of Nodes Around the Bearing Circumferential	-
N ₂ , N _L	Number of Nodes Around the Bearing Length	-
N _c	Number of Nodes Around the Chamfered Bearing Length	-
o _b	Bearing center	-
o _j	Geometrical center of Journal	

$(P)_{i,j}$	Oil- Film Pressure distribution	N/m^2
P_o	Zeroth-Order ,Equilibrium Oil- Film Pressure distribution	N/m^2
ΔP_1	First-Order ,Perturbed Oil- Film Pressure distribution	N/m^2
P_x, P_y	First-Order ,Perturbed Oil- Film Pressure distribution Due to Perturbation in Displacement	N/m^2
$P_x^\bullet, P_y^\bullet$	First-Order ,Perturbed Oil- Film Pressure distribution Due to Perturbation in Velocity	N/m^2
P_c	Oil- Film Pressure distribution due to Surface Tension	N/m^2
P_{cav}	Oil- Film Pressure distribution due to Cavitation Zone	
q	Side-Leakage Flow Rate	m^3/s
Q	Dimensionless Side-Leakage Flow Rate	-
R_f	Over-Relaxation Factor	-
S	Reference Sommerfeld Number, $[S=((\mu\omega LR)/(\pi F_o))*(R/C)^2]$	
t	Time	s
r,t	Rotating Radial and tangential coordinate	-
W	Total Resultant Force	N
W_x	External Force Applied in Vertical Direction	N
W_y	External Force Applied in Horizontal Direction	N
X,Y,Z	Cartesian Coordinates Directions (X in the Load Line Direction, $X=R*\theta$) (Y Normal to Vertical Load Direction) (Z Axial Direction)	-

$\bar{Z}$	Dimensionless Coordinate in Axial Direction	-
Z_c	Axial Coordinate Direction in Chamfer Zone	-
$\bar{Z}_c$	Dimensionless Coordinate in Axial Direction for the Chamfer	

Greek Symbols

Symbol	Description	Units
Δh_1	First- Order Perturbed Oil-Film Thickness	m
$\Delta\theta$	Increment in θ -Direction	rad.
ΔZ	Increment in Z-Direction	m
$\Delta\bar{Z}$	Dimensionless Increment in Z-Direction	-
ΔZ_c	Increment in Z-Direction for Chamfer Zone	m
$\Delta\bar{Z}_c$	Dimensionless Increment in Z-Direction for Chamfer Zone	-
Δx	Mesh Dimension, Perturbation Displacement in Vertical Journal Position	m
Δy	Mesh Dimension, Perturbation Displacement in Horizontal Journal Position	m
$\Delta x^\cdot$	Mesh Dimension, Perturbation Velocity in Vertical Journal Position	m/s
$\Delta y^\cdot$	Mesh Dimension, Perturbation Velocity in Journal Position	m/s
Δe	Journal Center Radial Displacement in the Radial Horizontal Direction	m
$\Delta\phi$	Journal Center Tangential Displacement in the Tangential Direction	rad

$\Delta e^{\cdot}$	Journal Center Radial Velocities in the Radial Direction	m
$\Delta \phi^{\cdot}$	Journal Center Tangential Velocities in the Tangential Direction	rad
σ	Modified Sommerfeld Number($\sigma=S*\pi*(L/D)^2$)	-
μ	Lubricant Absolute Viscosity	Pa.sec
ε	Eccentricity Ratio, ($\varepsilon=e/C$)	m
ε_o	Steady State Eccentricity Ratio, ($\varepsilon_o=e_o/C$)	m
θ	Angular Coordinate, Circumferential Direction.	degree
θ_c	Cavitation Angle	degree
Θ	Bearing Angle	degree
α	Chamfer Angle	degree
ϕ	Attitude Angle	degree
ϕ_o	Steady State Attitude Angle	degree
σ	Surface Tension	N m ⁻¹
ω	Journal Angular speed	rad/s.
$\overline{\omega}_r$	Threshold Speed Instability	rad/s.

Subscripts

b	Referring to Bearing
c	Referring to Chamfer
i,j	=1 referred for X Direction =2 referred for Y Direction
j	Referring to journal

i,j	Grid Number in Axial, Circumferential Direction, Respectively
r,t	Referring to Polar Coordinates
x,y	Referring to Dynamic Journal Center Displacement in Cartesian Coordinates
$\dot{x},\dot{y}$	Referring to Dynamic Journal Center Velocity in Cartesian Coordinates
ε,ϕ	Referring to Dynamic Journal Center Displacement in Rotating Coordinates
ε_0,ϕ_0	Referring to Static Journal Center Displacement in Rotating Coordinates
$\dot{\varepsilon},\dot{\phi}$	Referring to Dynamic Journal Center Velocity in Rotating Coordinates
$0,1$	Referring to Static ,Dynamic State
$+,-$	Referring to Positive ,Negative Perturbed Values

Superscripts

—	Dimensionless Quantity
n	Iteration Number
.	Time Derivation

ABSTRACT

The stability and the dynamic characteristics of chamfer journal bearing under hydrodynamic lubrication condition are theoretically analyzed. An isothermal finite bearing theory was adopted during this analysis. The effect of chamfer which was machined circumferentially at both sides of the bearing inner surface is studied on the stability characteristics. It was claimed that chamfer minimizing or almost preventing the side leakage flow rate in lightly loaded journal bearings.

A non linear transient stability analysis was used in this work which is represented by Reynolds equation with squeeze term has been solved to evaluate the dynamic coefficient which describe the dynamic behavior of the bearing. It is well known that eight coefficients, four stiffness coefficient and four damping coefficients are used to completely describe the dynamic behavior of the chamfered journal bearings. Perturbation technique was used to treat the non linear Reynolds equation in order to specify the dynamic coefficients of such bearing.

A finite difference technique has been adopted through the solution. The Reynolds equation has been transformed into a discrete form and direct iterative procedure was used to evaluate the pressure distribution through the oil film the iterative loops are stopped when the error reach a certain value. A suitable computer program has been designed in (FORTRAN-90) and used in solving the governing equations of the problem. The computer program was verified by comparing the results obtained in this work with that obtained by solving the problem of short bearing theory. The bearing critical speed has been evaluated from the obtained dynamic coefficients.

The effect of chamfer on the static characteristics has been studied. It was found that the load carrying capacity is increased while attitude angle is decreased due to the existence of the chamfer at both ends of the bearing. It was found also that the attitude angle decreased by increasing the chamfer length. The effect of chamfer on the dynamic characteristics showing that the effect of chamfer angle has a little effect on the this dynamic characteristics for lower values of (L/D) ratio and high eccentricity ratio. This effect appear as the chamfer length increases and (L/D) increases value of (1.5) ,but it is still has little effect.

The direct stiffness coefficients decrease and the cross coupled stiffness increase with chamfer machined at both ends of the bearing. It was found also that the cross coupled stiffness found to be slightly decrease with increasing the chamfer length (Lc). The direct damping coefficient shows an decrease with increasing the chamfer angle for higher values of (L/D) ratio.

As the value of (L/D) increases the dynamic coefficients seem to be lower ,for example direct stiffness coefficient ($\bar{K}_{xx}$) decrease 18%, from (L/D=0.25 to 0.5) and decrease 23%, from (L/D=0.5 to 1.5).

The effect of chamfer on the stability has been studied. It was found that by introducing a chamfer at both ends of the bearing, some decrease in stability line that mean the curve tend to the stability zone by (10%) and then enhances the stability.

الخلاصة

يتضمن هذا البحث دراسة نظرية للخصائص الديناميكية و الاستقرار للمساند المشطوبة النهايات ذات التزيب الهيدروديناميكي. تم اعتماد نظرية الركائز المحددة الطول و التي تعمل تحت ظروف بثوت درجة الحرارة لغرض إجراء هذه الدراسة. تم دراسة إضافة تأثير الشطب الذي يُشغل بشكل محيطي في كلتا نهايتي السطح الداخلي للركيزة مما يؤدي إلى تقليل أو تقريباً منع معدل تدفق التسرب الجانبية في حالة المساند المحورية بأحمال خفيفة نسبياً.

لغرض عمل هذا المطلب تم استخدام تحليل الاستقرار العابر اللاخطي في هذا البحث و الذي يتمثل بمعادلة رينولدز بوجود تأثير العصر ثم حل هذه المعادلة وذلك لإيجاد المعاملات الديناميكية و التي تصف التصرف الديناميكي للركيزة.

من المعروف إن هناك ثمانية معاملات ديناميكية, أربعة منها لوصف الجساء والأربعة الأخرى لوصف التضائل (الإخماد) والتي تستعمل بالكامل لوصف التصرف الديناميكي للمساند المحورية ذا النهاية المشطوبة.

لقد تم اعتماد تقنية الفروقات المحددة لحل المعادلات الحاكمة. تم تحويل معادلة رينولدز إلى صيغ مجزأة و بإجراء التكرار المباشر يتم استخراج توزيع الضغط في غشاء الزيت إلى حلقات تكرارية و تتوقف عند وصول نسبة الخطأ إلى قيمة محددة.

تم إعداد برنامج مناسب مصمم بلغة فورتران (90) لحل المعادلات الحاكمة لهذه المسألة و النتائج المتحصلة من برنامج الحاسوب تم إثباتها مع النتائج المألوفة المتوفرة من تطبيق نظرية المسند القصير.

من خلال استخراج المعاملات الديناميكية تم إيجاد السرعة الحرجة للمحور

الدوار.

تم دراسة تأثير الشطب على الخصائص السكونية و اتضح انه بإضافة الشطب لكلتا نهايتي الركيزة فان قابلية التحميل قد زادت مع نقصان زاوية الوضع . و تم إيجاد بأنه كلما ازدادت طول المنطقة المشطوبة كلما قلّت زاوية الوضع.

وتم دراسة تأثير الشطب على الخصائص الديناميكية موضحة بأنه لزاوية الشطب تأثيراً قليلاً على هذه الخصائص لقيم نسب (L/D) قليلة و نسب اللاتمرکز عالية. هذا التأثير يبدأ بالوضوح كلما ازدادت طول المنطقة المشطوبة و مع زيادة نسبة (L/D) بقيمة (1.5) ولكنها لا تزال ذات تأثير قليل.

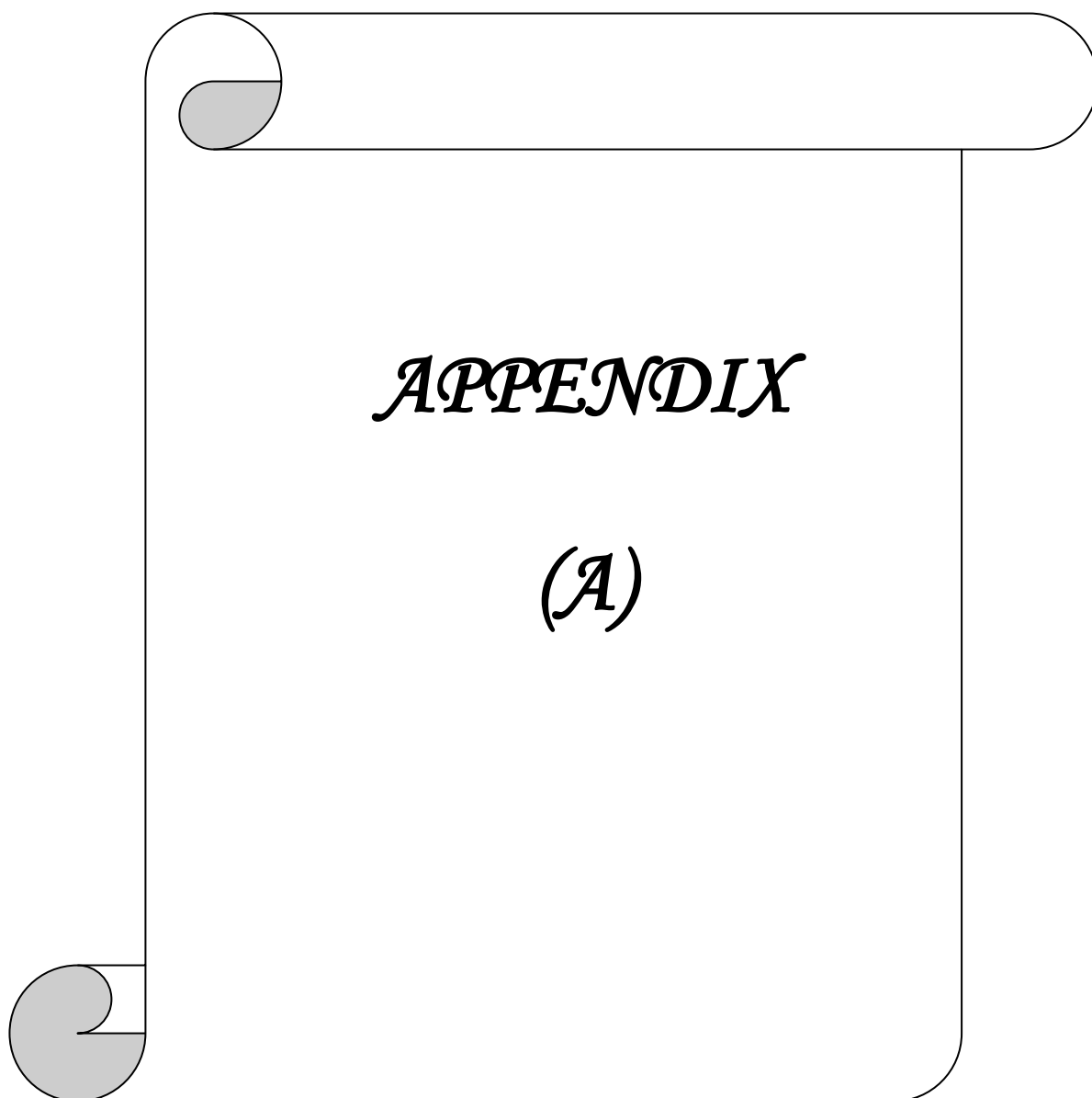
إن معاملات الجساءة المباشرة التأثير تقل بينما تزداد معاملات الجساءة المقرونة (المزدوجة) التأثير و ذلك بإضافة الشطب في كلتا نهايتي الركيزة .

وكذلك تم إيجاد إن معاملات الجساءة المقرونة (المزدوجة) التأثير تقل بعض الشيء مع زيادة طول المنطقة المشطوبة.

أظهرت معاملات التضاؤل المباشرة نقصان مع زيادة زاوية الشطب لقيم نسب (L/D) عالية .

كلما ازدادت نسبة (L/D) أدى ذلك إلى نقصان في المعاملات الديناميكية , مثلاً ($\bar{K}_{xx}$) تقل بنسبة (18%) من نسبة (L/D) = (0.25 إلى 0.5) وتقل بنسبة (23%) من نسبة (L/D) = (0.5 إلى 1.0) .

وكذلك تم دراسة تأثير الشطب على الأستقرارية , تم إيجاد انه بإضافة الشطب لكلتا نهايتي الركيزة فان ذلك سيحسن من الأستقرارية بجعل المنحني يميل إلى الأستقرارية أي يقل بنسبة (10%) .



Appendix A

Definition of Dynamic Force Coefficients in Fluid Film Bearings

Fluid film bearing $(K_{ij})_{i,j=x,y}$ and $(C_{ij})_{i,j=x,y}$ stiffness and damping force coefficients are defined as;

$$K_{ij} = -\frac{\partial F_i}{\partial x_j}, \quad C_{ij} = -\frac{\partial F_i}{\partial \dot{x}_j} \quad ij = x, y \quad \dots(A.1)$$

For example, $K_{xy} = -\partial F_x / \partial Y$ corresponds to a stiffness produced by a fluid force in the X direction due to a journal static displacement in the Y direction. By definition, this coefficient is evaluated at the equilibrium Position with other journal center displacements and velocities equal to zero. The negative sign in the definition assures that a positive magnitude stiffness coefficient corresponds to a restorative force. The coefficients are known as the direct stiffness terms, while the coefficients (K_{xx}, K_{yy}) are known as the direct stiffness terms, while the coefficients (K_{xy}, K_{yx}) are referred as cross-coupled. Figure(3-3) provides a pictorial representation of the bearing force coefficients as physical parameters. Note that the defined force coefficients allow the representation of the dynamic fluid film bearing forces in terms of the fundamental mechanical parameters $\{K, C\}$.

However; this does not mean that these coefficients must be accordance with conventional knowledge. For example, the viscous damping coefficients may be negative, i.e. non-dissipative or the stiffness coefficients non-restorative.

Fluid film force coefficients in the radial and tangential directions (r,t) can also be defined. Thus, the radial and tangential fluid film forces are expressed as;

$$F_r = F_{ro} + \frac{\partial F_r}{\partial e} \Delta e + \frac{\partial F_r}{e_o \partial \phi} e_o \Delta \phi + \frac{\partial F_r}{\partial \dot{e}} \Delta \dot{e} + \frac{\partial F_r}{e_o \partial \dot{\phi}} e_o \Delta \dot{\phi} \quad \dots\dots(A.2)$$

$$= F_{ro} - K_{rr} \Delta e - K_{rt} e_o \Delta \phi - C_{rr} \Delta \dot{e} - C_{rt} e_o \Delta \dot{\phi}$$

$$F_t = F_{to} + \frac{\partial F_t}{\partial e} \Delta e + \frac{\partial F_t}{e_o \partial \phi} e_o \Delta \phi + \frac{\partial F_t}{\partial \dot{e}} \Delta \dot{e} + \frac{\partial F_t}{e_o \partial \dot{\phi}} e_o \Delta \dot{\phi} \quad \dots\dots(A.3)$$

$$= F_{to} - K_{tr} \Delta e - K_{tt} e_o \Delta \phi - C_{tr} \Delta \dot{e} - C_{tt} e_o \Delta \dot{\phi}$$

Note that $\{\Delta \dot{e}, e_o \Delta \dot{\phi}\}$ are the journal radial and tangential velocities in the (r, t) coordinate system.

The relationship between the force coefficients in both coordinate system is easily determined from following equation;

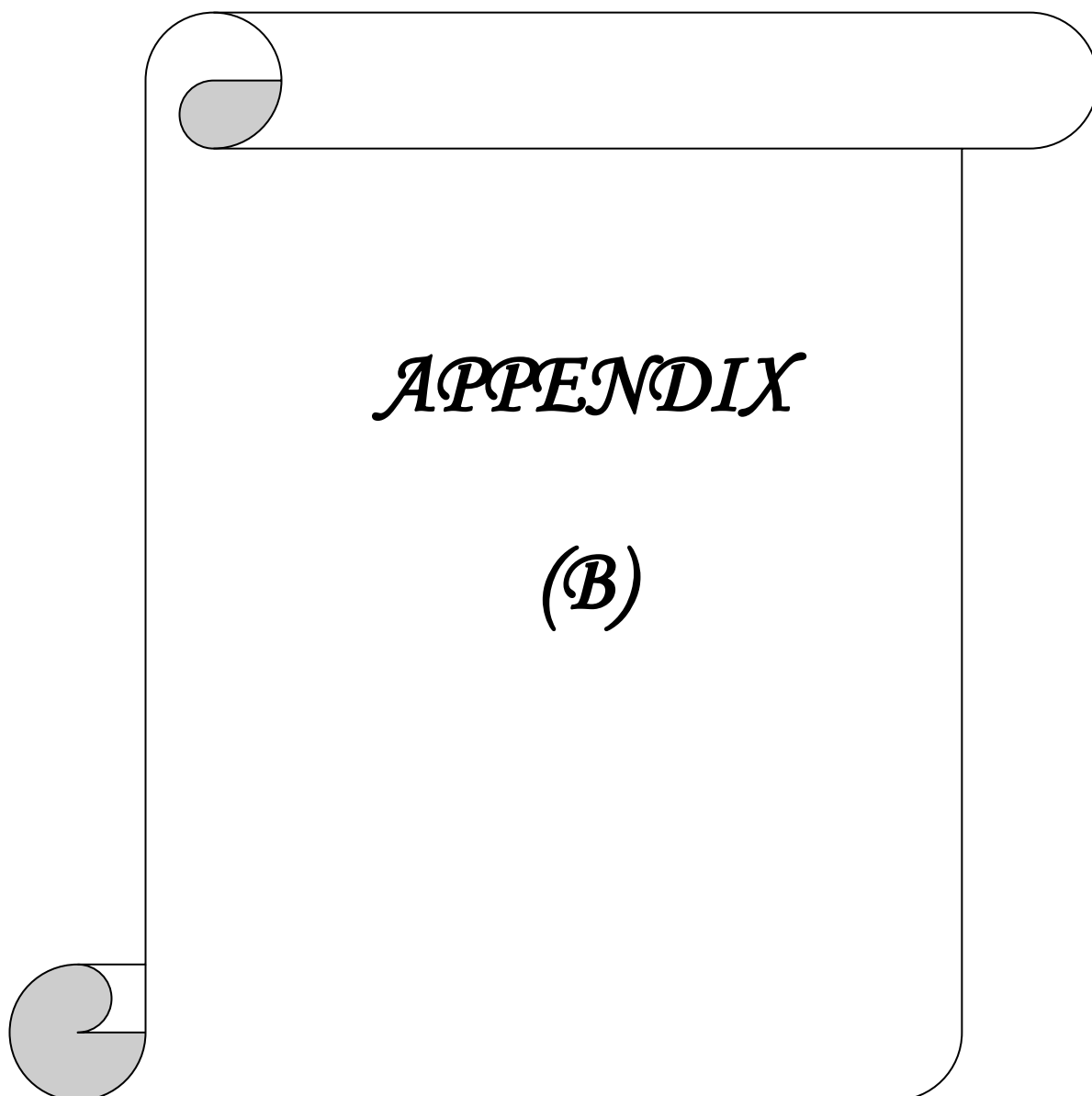
$$\begin{pmatrix} \Delta e_x \\ \Delta e_y \end{pmatrix} = \begin{pmatrix} \cos \phi_o & -\sin \phi_o \\ \sin \phi_o & \cos \phi_o \end{pmatrix} \begin{pmatrix} \Delta e(t) \\ e_o \Delta \phi(t) \end{pmatrix} \quad \dots\dots\dots(A.4)$$

$$\begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix} = \begin{pmatrix} \cos \phi_o & -\sin \phi_o \\ \sin \phi_o & \cos \phi_o \end{pmatrix} \begin{pmatrix} K_{\varepsilon\varepsilon} & K_{\varepsilon\phi} \\ K_{\phi\varepsilon} & K_{\phi\phi} \end{pmatrix} \begin{pmatrix} \cos \phi_o & \sin \phi_o \\ -\sin \phi_o & \cos \phi_o \end{pmatrix} \quad \dots\dots\dots(A.5)$$

$$\begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix} = \begin{pmatrix} \cos \phi_o & -\sin \phi_o \\ \sin \phi_o & \cos \phi_o \end{pmatrix} \begin{pmatrix} C_{\varepsilon\varepsilon} & C_{\varepsilon\phi} \\ C_{\phi\varepsilon} & C_{\phi\phi} \end{pmatrix} \begin{pmatrix} \cos \phi_o & \sin \phi_o \\ -\sin \phi_o & \cos \phi_o \end{pmatrix} \quad \dots\dots\dots(A.6)$$

Substitution of the force coefficients definition (A-1) into equation (4-42) gives the following equation;

$$\begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix} = \begin{bmatrix} F_{x_o} \\ F_{y_o} \end{bmatrix} - \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} - \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{pmatrix} \Delta \dot{x} \\ \Delta \dot{y} \end{pmatrix} \quad \dots\dots(A.7)$$



Appendix B

Force coefficient for the short length journal bearing

The force coefficient in dimensionless form according to the definition is;

$$\bar{K}_{ij} = K_{i,j} \frac{C}{F_o}; \bar{C}_{ij} = C_{ij} \frac{C\omega}{F_o} \quad i,j=X,Y \quad \dots\dots(B.1)$$

Where (F_o) is the static load applied on bearing (in the X direction). [Note that the total load $W=2F_o$ is shared by two bearings in a symmetric rotor mount]

By knowing that the total load is given by following equation such as came in Ref.[2]:-

$$F_o = (\mu\omega(L/C)^2 LR) / (4\sigma) \quad \dots\dots(B.2)$$

Where (σ) is the modified Sommerfeld Number is given by ;

$$\frac{\mu\omega LR}{4F_o} \left(\frac{L}{C} \right)^2 = \sigma = \frac{(1-\varepsilon^2)^2}{\varepsilon \sqrt{\{16\varepsilon^2 + \pi^2(1-\varepsilon^2)^{3/2}\}}} \quad \dots\dots(B.3)$$

Using the following definitions by accomplishing the static radial and tangential forces at the equilibrium position as;

$$f_{\varepsilon o} = -\frac{W_{\varepsilon o}}{F_o} = \cos \phi_o = \frac{4\sigma\varepsilon^2}{(1-\varepsilon^2)^2} \quad \dots\dots(B.4)$$

$$f_{\phi o} = -\frac{W_{\phi o}}{F_o} = \sin \phi_o = \frac{\pi\sigma\varepsilon}{(1-\varepsilon^2)^{3/2}} \quad \dots\dots(B.5)$$

$$\phi_o = \tan^{-1} \left[\pm \frac{f_{\phi o}}{f_{\varepsilon o}} \right] \quad \dots\dots(B.6)$$

Then the dimensionless force coefficients in the (ε, ϕ) coordinate system

$$\text{become;} \quad \bar{K}_{\varepsilon} = f_{\varepsilon o} \frac{2(1+\varepsilon^2)}{\varepsilon(1-\varepsilon^2)} \quad \dots\dots(B.7)$$

$$\bar{K}_{\phi} = f_{\phi o} \frac{1}{\varepsilon} \quad \dots\dots(B.8)$$

$$\bar{K}_{\phi\epsilon} = -f_{\phi o} \frac{(1+2\epsilon^2)}{\epsilon(1-\epsilon^2)} \quad \text{.....(B.9)}$$

$$\bar{K}_{\phi\phi} = f_{\epsilon o} \frac{1}{\epsilon} \quad \text{.....(B.10)}$$

$$\bar{C}_{\epsilon\epsilon} = f_{\phi o} \frac{2(1+2\epsilon^2)}{\epsilon(1-\epsilon^2)} \quad \text{.....(B.11)}$$

$$\bar{C}_{\epsilon\phi} = \bar{C}_{\phi\epsilon} = -f_{\epsilon o} \frac{2}{\epsilon} \quad \text{.....(B.12)}$$

$$\bar{C}_{\phi\phi} = f_{\phi o} \frac{2}{\epsilon} \quad \text{.....(B.13)}$$

Whilst the force coefficients in the (X,Y) coordinate are easily obtained using the matrix transformation given by equation (A-7). And after a lengthy algebraic procedure can be obtained ;

$$\bar{K}_{XX} = K_{XX} \frac{C}{F_o} = \frac{f_{\epsilon o}}{\epsilon(1-\epsilon^2)} \{f_{\epsilon o}^2 + 1 + 2\epsilon^2\} \quad \text{.....(B.14)}$$

$$\bar{K}_{YY} = K_{YY} \frac{C}{F_o} = \frac{f_{\epsilon o}}{\epsilon(1-\epsilon^2)} \{f_{\epsilon o}^2 + 1 - \epsilon^2\} \quad \text{.....(B.15)}$$

$$\bar{K}_{YX} = K_{YX} \frac{C}{F_o} = \frac{f_{\phi o}}{\epsilon(1-\epsilon^2)} \{f_{\epsilon o}^2 - 1 + \epsilon^2\} \quad \text{.....(B.16)}$$

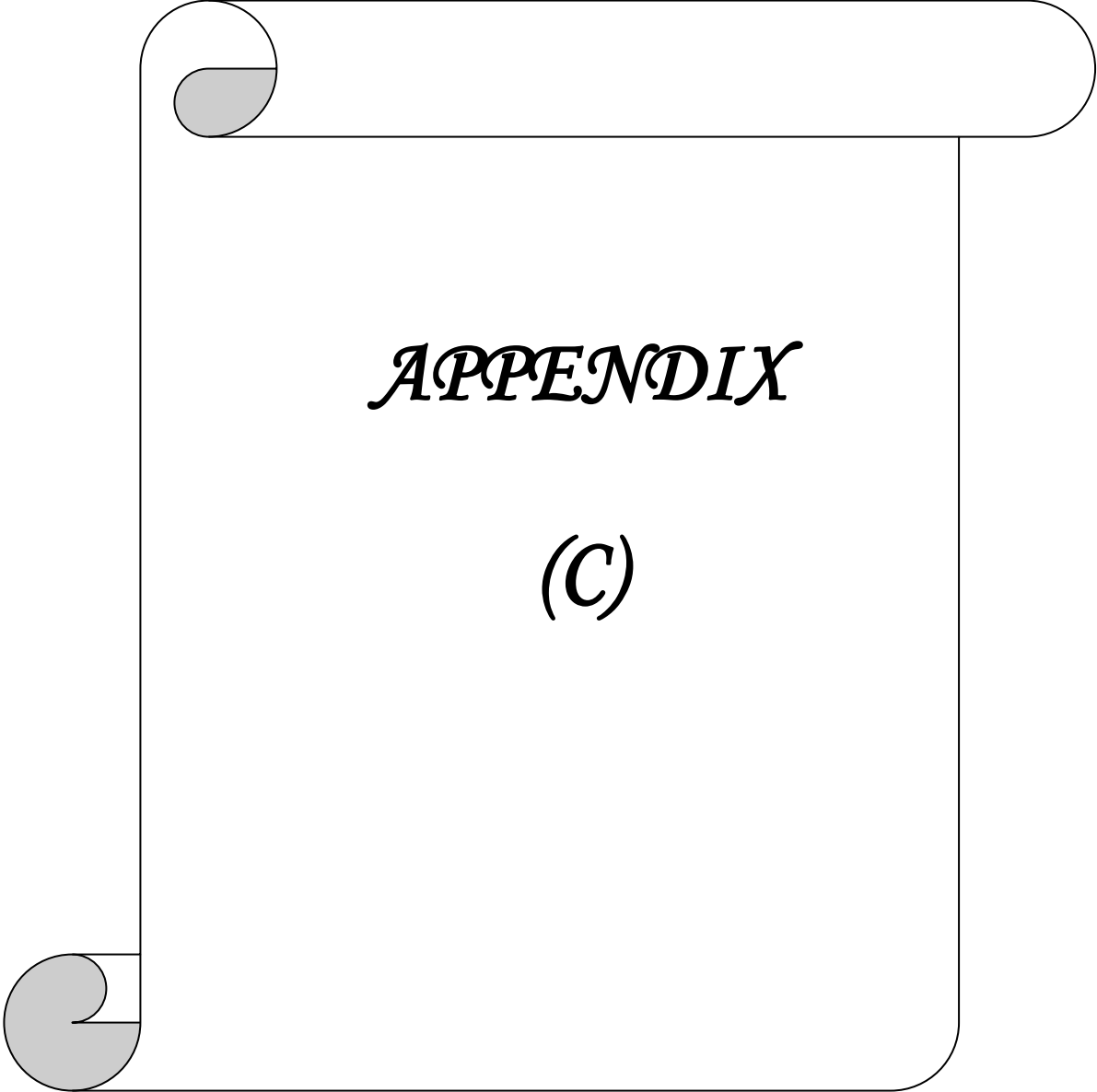
$$\bar{K}_{XY} = K_{XY} \frac{C}{F_o} = \frac{f_{\phi o}}{\epsilon(1-\epsilon^2)} \{f_{\epsilon o}^2 + 1 + 2\epsilon^2\} \quad \text{.....(B.17)}$$

$$\bar{C}_{XX} = C_{XX} \frac{C\omega}{F_o} = \frac{2.f_{\phi o}}{\epsilon(1-\epsilon^2)} \{[2 + \epsilon^2] f_{\epsilon o}^2 + 1 - \epsilon^2\} \quad \text{.....(B.18)}$$

$$\bar{C}_{YY} = C_{YY} \frac{C\omega}{F_o} = \frac{2.f_{\phi o}}{\epsilon(1-\epsilon^2)} \{[2 + \epsilon^2] f_{\phi o}^2 - 1 + \epsilon^2\} \quad \text{.....(B.19)}$$

$$\bar{C}_{XY} = C_{XY} \frac{C\omega}{F_o} = \frac{2.f_{\epsilon o}}{\epsilon(1-\epsilon^2)} \{[2 + \epsilon^2] f_{\phi o}^2 - 1 + \epsilon^2\} \quad \text{.....(B.20)}$$

$$\bar{C}_{YX} = C_{YX} \frac{C\omega}{F_o} = \frac{2.f_{\epsilon o}}{\epsilon(1-\epsilon^2)} \{[2 + \epsilon^2] f_{\phi o}^2 - 1 + \epsilon^2\} \quad \text{.....(B.21)}$$



APPENDIX

(C)

Appendix C

Gauss –Seidel Method

The equation used to calculate the pressure field in the lubricant film is equation (4.10), which can be written as follows:-

$$P_{i,j}^n = \left[\frac{A1_{i,j} P_{i+1,j}^n + A2_{i,j} P_{i-1,j}^{n+1} + A3_{i,j} P_{i,j+1}^{n-1} + A4_{i,j} P_{i,j-1}^n - A5_{i,j}}{A6_{i,j}} \right] \dots\dots(C.1)$$

Where, n is the iteration number. This equation is presented in a form for ascending order of (i), and (j).

One method usually used to accelerate the rate of convergence of the iterative process, is to consider the residual, which is the right - hand side of equation (B- 1) less ($P_{i,j}^{n-1}$) multiplied by a relaxation factor (R_f), and add to the current value of ($P_{i,j}^{n-1}$), to yield the following, [11,12];

$$P_{i,j}^n = (1 - R_f) P_{i,j}^{n-1} + R_f \left[\frac{A1_{i,j} P_{i+1,j}^n + A2_{i,j} P_{i-1,j}^{n+1} + A3_{i,j} P_{i,j+1}^{n-1} + A4_{i,j} P_{i,j-1}^n - A5_{i,j}}{A6_{i,j}} \right] \dots\dots(C.2)$$

Where, $0 < R_f < 2$

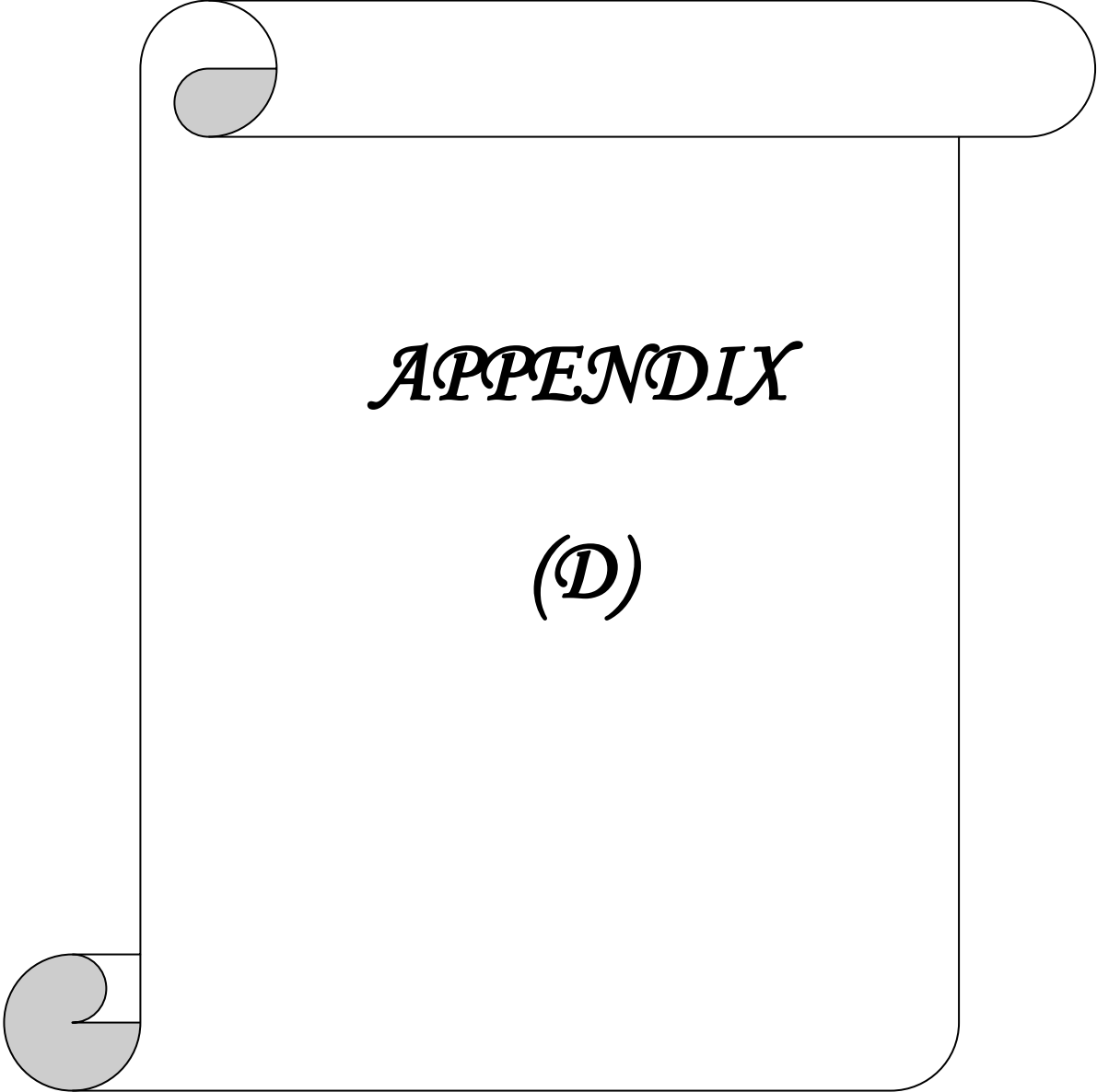
For, $0 < R_f < 1$, the scheme is termed successive under — relaxation, for $1 < R_f < 2$, it is termed successive over - relaxation, and for $R_f = 1$, it is reduced to the ordinary Gauss — Seidel method. The successive over relaxation scheme is the most widely used iterative method for solving the Reynolds' equation for incompressible fluid [36,47].

To terminate the iterative process, a convergence criterion is chosen, so To give the following ,[37,46,36];

$$\text{Error} < 10^{-5} \dots\dots(C.3)$$

Where, the error can be evaluated as;

$$\text{Error} = \frac{\sum \sum |P_{i,j}^n - P_{i,j}^{n-1}|}{\sum \sum |P_{i,j}^n|} \quad \text{.....(C.4)}$$



APPENDIX

(D)

Appendix D

Rotor Dynamic Coefficients

The rotor dynamic stiffness and damping coefficients for the Reynolds equation with common finite difference equation (4.9) are computed numerically and compared with the analytical values calculated by Lund [4] using the short bearing theory.

Rotor dynamic coefficients are computed numerically using a perturbation analysis the procedure is as follows: -

1. Reynolds equation (4.9) is solved by applying finite difference scheme. Grid size of 100 x 52 along the circumferential and axial directions, respectively to obtain pressure P for a given unperturbed film thickness equation (3.2).
2. Once the converged pressure field is obtained for the given steady state oil film thickness equation (3.2). This converged pressure field is integrated over the bearing area to obtain hydrodynamic fluid forces $\bar{F}_\varepsilon$ and $\bar{F}_\phi$ in the radial and the tangential directions, respectively. The load carrying capacity F and attitude angle are obtained from equations (4.33) and (4.34), respectively.
3. Once the steady state position ε_0 and ϕ_0 are obtained. In order to obtain stiffness coefficients, first the oil film is perturbed in a positive radial direction by $\Delta\varepsilon$, as given by equation (D.1) and steps 1 and 2 were repeated to obtain the hydrodynamic fluid forces $\bar{F}_\varepsilon$ and $\bar{F}_\phi$ in the radial and the tangential directions, respectively. Then the film is perturbed in the negative radial direction by $\Delta\varepsilon$ as given by equation (D.2).

And steps land 2 are repeated to obtain the hydrodynamic fluid forces $\bar{F}_\varepsilon$

and $\bar{F}_\phi$ in the radial and the directions, respectively. The stiffness coefficients

$\bar{K}_\varepsilon$ and $\bar{K}_{\phi\varepsilon}$ are calculated as given by equation (D.3).
.....(D.6)

For notations used in equation (D.3), refer to equation (4.42).

$$\bar{h}_+ = \bar{h}_0 + \Delta h = 1 + (\varepsilon_0 + \Delta\varepsilon) \cos(\theta) \quad \text{.....(D.1)}$$

$$\bar{h}_- = \bar{h}_0 - \Delta h = 1 + (\varepsilon_0 - \Delta\varepsilon) \cos(\theta) \quad \text{.....(D.2)}$$

$$\bar{K}_\varepsilon = \frac{\Delta \bar{F}_\varepsilon(\varepsilon_0 + \Delta\varepsilon, \phi_0, \varepsilon = 0, \phi = 0) - \Delta \bar{F}_\varepsilon(\varepsilon_0 - \Delta\varepsilon, \phi_0, \varepsilon = 0, \phi = 0)}{2\Delta\varepsilon} \quad \text{.....(D.3)}$$

$$\bar{K}_{\phi\varepsilon} = \frac{\Delta \bar{F}_\phi(\varepsilon_0 + \Delta\varepsilon, \phi_0, \varepsilon = 0, \phi = 0) - \Delta \bar{F}_\phi(\varepsilon_0 - \Delta\varepsilon, \phi_0, \varepsilon = 0, \phi = 0)}{2\Delta\varepsilon}$$

4. In order to obtain the other two stiffness coefficients, first the oil film is perturbed in a positive direction by $\Delta\phi$, as given by the equation (D.4)

and steps 1 and 2 are repeated to obtain the hydrodynamic fluid forces $\bar{F}_\varepsilon$

and $\bar{F}_\phi$ in the radial and the tangential directions, respectively. Then, the film is perturbed in the negative tangential direction by $\Delta\phi$, as given by equation (D.5) and steps 1 and 2 are repeated to obtain the hydrodynamic

fluid forces $\bar{F}_\varepsilon$ and $\bar{F}_\phi$ in the radial and the tangential directions, respectively. The stiffness coefficients $\bar{K}_{\varepsilon\phi}$ and $\bar{K}_{\phi\phi}$ were calculated as given by equation (D.6).

$$\bar{h}_+ = \bar{h}_0 + \Delta h = 1 + \varepsilon_0 \cos(\theta + \Delta\phi) \quad \text{.....(D.4)}$$

$$\bar{h}_- = \bar{h}_0 - \Delta h = 1 + \varepsilon_0 \cos(\theta - \Delta\phi) \quad \text{.....(D.5)}$$

$$\bar{K}_{\phi\phi} = \frac{\Delta \bar{F}_{\phi}(\varepsilon_o, \phi + \Delta\phi, \varepsilon = 0, \phi = 0) - \Delta \bar{F}_{\phi}(\varepsilon_o, \phi - \Delta\phi, \varepsilon = 0, \phi = 0)}{2\varepsilon_o \Delta\phi} \dots\dots(D.6)$$

$$\bar{K}_{\varepsilon\phi} = \frac{\Delta \bar{F}_{\varepsilon}(\varepsilon_o, \phi + \Delta\phi, \varepsilon = 0, \phi = 0) - \Delta \bar{F}_{\varepsilon}(\varepsilon_o, \phi - \Delta\phi, \varepsilon = 0, \phi = 0)}{2\varepsilon_o \Delta\phi}$$

is very important to mention that the trigonometric sine and cosine functions of $\Delta\phi$ in equations (D.4) and (D.5) are expanded using the Taylor series expansion as given by equations (D.7) and (D.8).

$$\sin\Delta\phi = \Delta\phi - \frac{(\Delta\phi)^3}{3f} + \frac{(\Delta\phi)^5}{5f} - \frac{(\Delta\phi)^7}{7f} + \dots\dots\dots(D.7)$$

$$\cos\Delta\phi = 1 - \frac{(\Delta\phi)^2}{2f} + \frac{(\Delta\phi)^4}{4f} - \frac{(\Delta\phi)^6}{6f} + \dots\dots\dots(D.8)$$

5. In order to obtain the damping coefficients the squeeze film term is added to the steady state pressure equation (3.1). The squeeze term is the time derivative of film thickness and for equation (D.1) and (D.2) is given by equation (D.9) and (D.10), respectively:

$$\left(\frac{d\bar{h}_+}{dt} \right) = \Delta\varepsilon \cos(\theta) \dots\dots\dots(D.9)$$

$$\left(\frac{d\bar{h}_-}{dt} \right) = -\Delta\varepsilon \cos(\theta) \dots\dots\dots(D.10)$$

To compute $\bar{C}_{\varepsilon}$ and $\bar{C}_{\phi\varepsilon}$, first the film thickness is perturbed with the positive radial velocity of $\Delta\dot{\varepsilon}$ and steps 1 and 2 are repeated to obtain the hydrodynamic fluid forces $\bar{F}_{\varepsilon}$ and $\bar{F}_{\phi}$ in the radial and the tangential directions, respectively. Then the film thickness is perturbed with a negative radial velocity of $\Delta\dot{\varepsilon}$ and steps 1 and 2 are repeated to obtain the

hydrodynamic fluid forces $\bar{F}_\varepsilon$ and $\bar{F}_\phi$ in the radial and the tangential directions, respectively. The damping coefficients are calculated using equation (D.11);

$$C_{\varepsilon}^{-} = \frac{\Delta F_{\varepsilon}^{-}(\varepsilon_o, \phi_o, \dot{\varepsilon} = \Delta \dot{\varepsilon}, \phi = 0) - \Delta F_{\varepsilon}^{-}(\varepsilon_o, \phi_o, \dot{\varepsilon} = -\Delta \dot{\varepsilon}, \phi = 0)}{2\Delta \varepsilon} \dots\dots(D.11)$$

$$C_{\phi\varepsilon}^{-} = \frac{\Delta F_{\phi}^{-}(\varepsilon_o, \phi_o, \dot{\varepsilon} = \Delta \dot{\varepsilon}, \phi = 0) - \Delta F_{\phi}^{-}(\varepsilon_o, \phi_o, \dot{\varepsilon} = -\Delta \dot{\varepsilon}, \phi = 0)}{2\Delta \varepsilon}$$

6. The time derivative or squeeze term for the film thickness given by equations (D.4) and (D.5) reduces to:

$$\left(\frac{d \bar{h}_+}{dt} \right) = \varepsilon_o \sin(\theta) \Delta \phi \dots\dots(D.12)$$

$$\left(\frac{d \bar{h}_+}{dt} \right) = \varepsilon_o \sin(\theta) \Delta \phi \dots\dots(D.13)$$

To compute $\bar{C}_{\varepsilon\phi}$ and $\bar{C}_{\phi\phi}$, first the film thickness is perturbed with a positive tangent velocity of $\varepsilon_o \Delta \dot{\phi}$ and steps 1 and 2 are repeated to obtain the hydrodynamic fluid forces $\bar{F}_\varepsilon$ and $\bar{F}_\phi$ in the radial and the tangential directions, respectively. Then the film thickness is perturbed with a negative radial velocity of $\varepsilon_o \Delta \dot{\phi}$ and steps 1 and 2 were repeated to obtain the hydrodynamic fluid forces $\bar{F}_\varepsilon$ and $\bar{F}_\phi$ in the radial and the tangential directions, respectively. The damping coefficients are calculated using equation (D.14):

$$C_{\varepsilon}^{-} = \frac{\Delta F_{\varepsilon}^{-}(\varepsilon_o, \phi_o, \varepsilon = 0, \dot{\phi} = \Delta \dot{\phi}) - \Delta F_{\varepsilon}^{-}(\varepsilon_o, \phi_o, \varepsilon = -0, \dot{\phi} = -\Delta \dot{\phi})}{2\varepsilon_o \Delta \dot{\phi}} \dots\dots(D.14)$$

$$C_{\phi\phi}^{-} = \frac{\Delta F_{\phi}^{-}(\varepsilon_o, \phi_o, \varepsilon = 0, \dot{\phi} = \Delta \dot{\phi}) - \Delta F_{\phi}^{-}(\varepsilon_o, \phi_o, \varepsilon = -0, \dot{\phi} = -\Delta \dot{\phi})}{2\varepsilon_o \Delta \dot{\phi}}$$

It is important to mention that for the calculation of damping coefficients only first order terms of Taylor series had been considered, while neglecting the higher order terms, hence $\cos(\Delta\phi) \approx 1$, and $\sin(\Delta\phi) \approx \Delta\phi$ for small amplitude motion.

The perturbation sizes Δ ranging from $\pm 1 \times 10^{-3}$ to $\pm 1 \times 10^{-6}$ were found to be reasonable for all of eight rotor dynamic coefficients.

CERTIFICATION

We certify, as an examining committee, that we have read this thesis, entitled 'Effect of Chamfering on the Stability of the Plain Cylindrical Hydrodynamic Journal Bearing', presented by "Leḡaa' Abbass Fadhiḡ Ibrahim" and examined the candidate in its contents and in our opinion it meets the standard of a thesis for the degree of Master of Science in Mechanical Engineering.

Lecturer Dr. Basim A. Abbass
(Supervisor)

Date / /

Asst. Prof. Dr. Alaa' M. Hussein
(Supervisor)

Date / /

Asst. Prof. Dr. Ahmed A. Ali
(Member)

Date / /

Asst. Prof. Dr. Hatem H. Obeid
(Member)

Date / /

Prof. Dr. Hussein J. Al-Alkawi
(Chairman)

Date / /

Approval of The Mechanical Engineering
Department
Head of The Mechanical Engineering
Department
Asst. Prof. Dr. Adil A. Alwan

Approval of The Deanery of
The College of Engineering
Dean of The College
Asst. Prof. Dr. Salah
Tawfeeq Al-Bazzaz

SUPERVISOR CERTIFICATE

We certify that this thesis entitled 'Effect of Chamfering on the Stability of the Plain Cylindrical Hydrodynamic Journal Bearing' was prepared under our supervision at the Mechanical Engineering Department, University of Babylon, Babylon –Iraq , as a partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering.

Signature

Lecturer

Dr. Basim A. Abass

Al-Makhsosi

(Supervisor)

Date / /

Signature

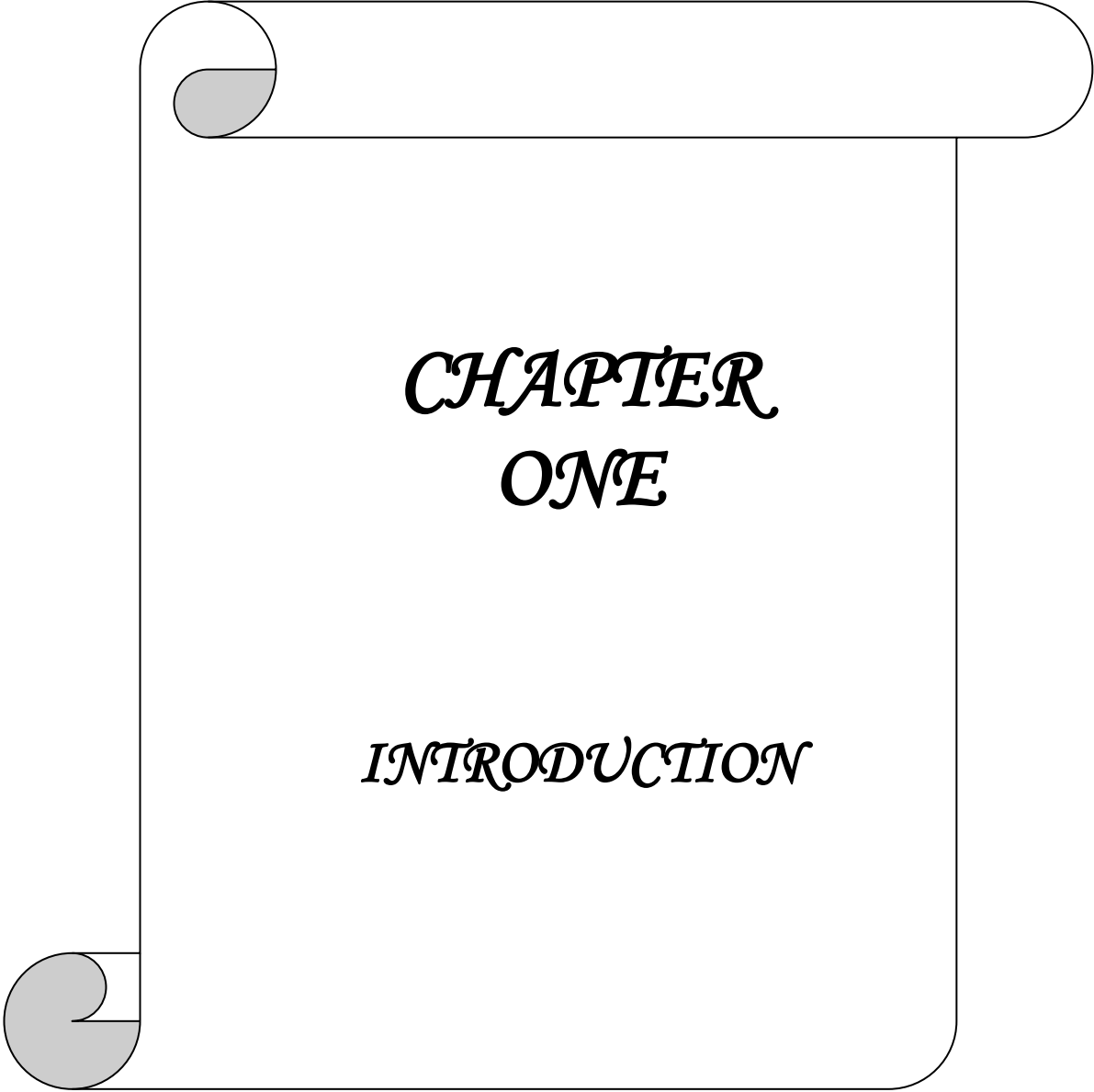
Asst.Prof.

Dr. Alaa' M. Hussan

AL-Jesany

(Supervisor)

Date / /



CHAPTER ONE

INTRODUCTION

Chapter One

Introduction

1.1. General:-

A fluid film bearing may be defined as a bearing in which the mating surfaces are completely separated by a layer of fluid lubricant . A widely used bearing type is the plain journal bearing with application in compressors, turbines ,pumps , electric motors and electric generators. Journal bearings consist of two cylinders rotating relative to each other. The outer cylinder (bearing), almost stationary, and the inner cylinder (shaft) rotating with an angular velocity about its center as shown later in chapter (3).

In a journal bearing, pressure is generated in thin lubricant oil film that separates the shaft and bushing thus preventing metal to metal contact. The high pressure fluid film in the clearance between the journal and the bearing due to rotation of the journal provides the hydrodynamic film lubrication and the load capacity to the bearing. The displacement of the shaft center relative to the bearing center known as eccentricity. The main primary advantages of fluid film bearings over rolling element bearings are their superior ability to absorb energy to dampen vibrations and their long life due to the absence of rolling contact stresses. The damping is very important in many types of rotating machines where the fluid film bearings are often the primary source of the energy absorption needed to control vibration.

Journal bearing which is used to support radial loads under high speed operating conditions are classified as plain journal bearings, axial groove bearings, pressure dam bearings, multilobe journal bearings, tilting pad journal bearings and hybrid hydrodynamic hydrostatic bearings. Some bearing geometry are used to suspect the bearing stability.

1.2. Chamfered Journal Bearing:-

It was claimed that a chamfered journal bearing has a minimum side leakage flow when operating under lightly loads. The above conclusion was obtained by **Somer Nacy (1997),[1]**. The common application of this type is in fans, ventilators, mixers, blenders, and all the application that working under lightly loads.

However this work was executed to study the chamfer effect on the steady state characteristics of the journal bearing. The present work represents an attempt to study the effect of a chamfer machined circumferentially at both sides of the bearing on the stability characteristics of the journal bearing. For this purpose the governing equations of the problem with appropriate boundary conditions mainly at the chamfer locations, since the pressure is not ambient are solved to determine the dynamic characteristics of the bearing and to evaluate the critical speed of the bearing. The problem was solved for a finite length journal bearing lubricated with incompressible isoviscous lubricant with constant oil film temperature (isothermal case).

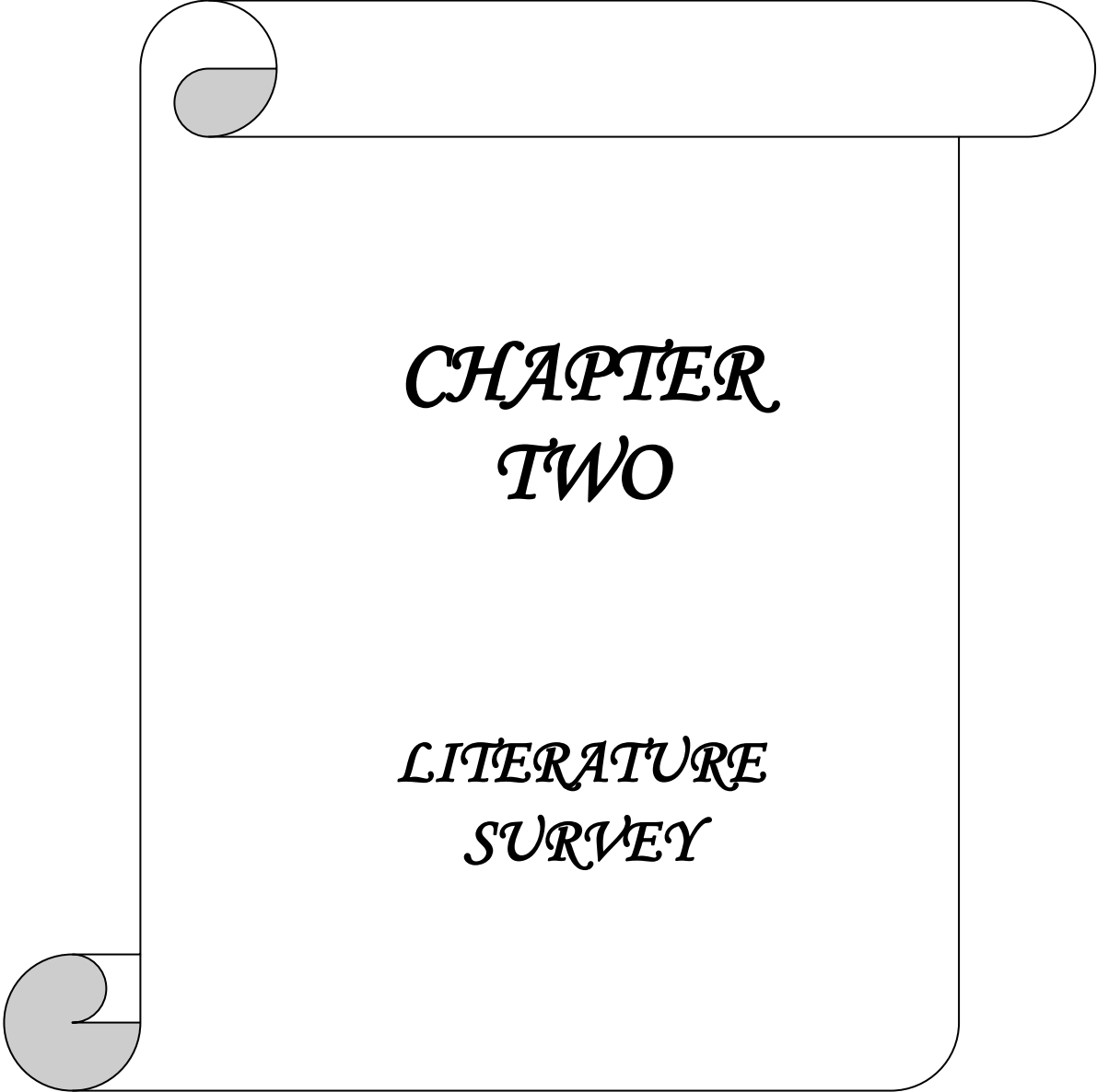
The Reynolds equation has been deriving through a common finite difference technique that transformed it into a discrete form and direct iterative scheme with successive over relaxation procedure was adopted to solve the problem numerically in order to find the oil film pressure (zeroth and first order) after a suitable perturbation technique.

A nonlinear transient stability analysis has been considered during this work. Perturbation technique was adopted to evaluate the eight dynamic coefficients (four stiffness coefficient and four damping coefficients) which completely describe the dynamic behavior of the bearing. A suitable computer program has been prepared and used in solving the governing equations of the problem. The computer program was verified by comparing the results obtained in this work with that obtained by solving the problem of infinitely short bearing.

1.3. Objectives of The Present Work:-

The main objectives of the present work are;

1. Study the effect of chamfer (angle and length) machined circumferentially at both ends of the bearing on the stability characteristics of the journal bearing.
2. Deriving the governing equations of the problem and solving it using finite difference method. For this purpose a computer program has been designed in Fortran-90 to solve these equations.
3. Study the effect of different parameters, namely (L/D) ratio on the effect of chamfer and ,therefore on the dynamic characteristics of such bearing.



CHAPTER TWO

LITERATURE SURVEY

Chapter Two

Literature Survey

2.1. Chamfered Journal Bearing:-

Journal bearing with a chamfer machined circumferentially at both sides of the bearing inner surface was found to be a new concept for minimizing or almost preventing the side-leakage flow rate in lightly loaded journal bearings. This concept eliminates the need for sealing and collecting the oil leakage. The circumferential Flow is found to be sufficient to achieve cooling of the bearing as referenced by **Somer M.,(1997),[1]**.

2.2. Dynamic Fluid Film Forces Coefficients

The review of literatures related to the present work can be shown as follows:

J. W. Lund (1987),[4], briefly reviewed the development of the concept of dynamic coefficients for fluid film journal bearing .Methods for computing the spring and damping coefficients were described and their use in rotor dynamics calculations (unbalance response, stability) was discussed .The limitations imposed by nonlinearities on the application of the coefficients was illustrated by examples . The emphasis is on experimental measurements of the bearing coefficients and establishing more uniform agreement with calculated values and specific assumptions were made in fluid film bearing analysis could be important. The need for some improvements or refinements in the theory seem indicated, but the basic calculation method appeared sound. The stability analysis applied to a rigidRotor which was symmetric and supported on two identical bearings and rank various bearing types for stability on the basis of the critical mass the value of measurement stiffness of bearing. The elliptical whirl orbit Calculated on the basis of the eight coefficients belonging to indicated

static Equilibrium position, was compared to whirl orbit obtained from a true Nonlinear solution and found close agreement.

V. Meruane, and R. Pascual, (2008),[5], proposed a framework to the numerical identification of nonlinear fluid film bearing parameters from large journal orbital motion (20–60% of the bearing clearance). They defined Nonlinear coefficients by a third order Taylor expansion of bearing reaction forces and they evaluated through a least mean square in time domain technique. The journal response was obtained from a computational fluid dynamic (CFD) model of a plain journal bearing on high dynamic loading conditions. The model considered fluid–structure interaction between the fluid flow and the journal. The case in study considered a laboratory test rig. Results indicated that nonlinear coefficients had an important effect on stiffness and damping. It was found a change on nonlinear behavior occurred when the Oil Whirl phenomenon starts, which it is not seen in classical linear models.

Jia Jeng,(2007),[6], studied how to predict the lateral vibration characteristics of the full-size rotor-bearing system by using the scale rotor-bearing model and the associated scaling laws, such that one might significantly reduce the time and money required by the experiments. A systematic theory and technique in there aspect was provided by concerning the use of scaling laws and scale rotor-bearing model for predicting free and forced vibration characteristics of full-size rotor-bearing system. This would be provide by derivng first from the equations of motion of the last two systems to this end the scaling laws for all the relevant parameters regarding the full-size rotor-bearing system and its scale model (such as, sizes of the constituent members, spring constants and damping coefficients of the supports, natural frequencies (of free vibration) and forced vibration displacements, etc.). Next, he determined with the theory of dimensional analysis the scaling factors for all relevant parameters of the above mentioned two systems. Now, based on all of the above-mentioned

relevant parameters of the full-size rotor-bearing system, one may determine the corresponding parameters of the scale rotor-bearing model by using the associated scaling factors and predict the vibration characteristics of the full-size rotor-bearing system by using the corresponding ones of the scale rotor-bearing model. Finally, he investigated the influence of constituent-member sizes on the similarity of natural frequencies (of free vibrations) and the influence of spring constants, damping coefficients and external loads on the similarity of dynamic responses (of forced vibrations).

Koichi M., and Shinya K.,(2007),[7], proposed many types of clearance configuration to solve this instability problem ,that whirl instability occurs at higher rotating speeds for a full circular fluid-film journal bearing. A clearance configuration of fluid-film journal bearings was optimized in a sense of enhancing the stability of the full circular bearing at high rotational speeds. Chosen a performance index as the sum of the squared whirl frequency ratios over a wide range of eccentricity ratios, and a Fourier series was used to represent an arbitrary clearance configuration of fluid-film bearings, "then formulated an optimization problem to find the Fourier coefficients to minimize the index. The designed bearing had a clearance configuration similar to that of an offset two-lobe bearing, for smaller length to-diameter ratios. It was shown that the designed bearing could not destabilize the Jeffcott rotor at any high rotating speed for a wide range of eccentricity ratio. The load capacity of the designed bearings was nearly in the same magnitude as that of the full circular bearing for smaller length-to-diameter ratios. The whirl-frequency ratios of the designed bearing were very sensitive to truncating higher terms of the Fourier series for some eccentricity ratio. The designed bearings successfully, enhance the stability of a full circular bearing and were free from the whirl instability.

Malenovsky ,E.,(2007),[8], used the wave equation as a new presented approach and basic mathematical model, which is compiled using the Navier-

Stokes and continuity equations. He used for some cases this mathematical model is the same as the Reynolds equation. He showed the composition of the differential equations for ideal and real, compressible and incompressible liquids and presented a totally new approach for its solution. It allows the determine of the liquid films added coefficients matrices by using only one parameter, in this case on the shaft center position. In the most general case includes the following added coefficients matrices: mass, damping and stiffness. His new approach to the differential equations solution is called the “complete solution”, which also includes other time dependent parts. They are not taken into account in contemporary approaches. The Laplace transformation, which allows the analysis of both the frequency or time domain, is the base of this new approach. The reverse Laplace transformation includes time dependent parts, which represent the “memory effects”. These effects haven’t, up to date, been taken into account in analyses using the Reynolds equation.

Castro ,H. F.,(2007),[9],used linear models and synchronous response which were generally adequate to describe and analyze rotors supported in hydrodynamic bearings. Hence, stiffness and damping coefficients could an provide a good model for a considerable range of situations. However, there were some cases where this kind of model was not sufficient to describe the dynamic behavior of rotor-bearing-system. Besides, unstable motion occurred due to precessional orbits at the rotor-bearing-system. This instability was called “oil whirl” or “oil whip”. The oil whirl phenomenon occurred when the journal bearings were lightly loaded and the shaft was whirling at a frequency close to one-half of rotor speed. It remained until the rotor speed reached twice the natural frequency of the system. The oil whip phenomenon occurred when the rotor speed reached about twice the natural frequency of the system and didn’t disappear even if the rotor speed increased. Its frequency and vibration mode corresponds to the first critical speed and did not vary much with changing speed of the rotating shaft. It was in full agreement with all analysis and results

presented . The rotor speed and the system stiffness were important parameters in the system fluid-induced instability. So, these parameters could determine if the system was stable or not. A flexible rotor with a central disc was modeled and unbalance excitation was assumed. A non-linear hydrodynamic model was considered for short bearing and laminar flow. The variation of the speed rotational was also taken into account, so that the instability effect could be observed in the simulation results. The effects of unbalance and rotor arrangement were verified. He intended to verify the consistence of an oil film non-linear model for the simulation of the rotor in run-up acceleration.

L. Roy and P. L. Choudhury,2007,[10], studied the steady state and dynamic characteristics including whirl instability of journal bearings with three axial groove, one located at the top and another two 90 degrees clockwise and anticlockwise from the top, from which oil was supplied from one end of the bearing. A linear pressure drop was assumed in the groove and the steady state and dynamic characteristics were obtained theoretically. They solved the Reynolds equation numerically by finite difference method satisfying the appropriate boundary conditions and found the dynamic behaviors in terms of stiffness and damping coefficients of fluid film and stability using a first-order perturbation method. It had been shown that both load capacity and stability improve for smaller groove angle was stability improved at higher eccentricity ratio.

Rao, J.,S. and Saravanakumar , M.(2006),[11], adopted a finite volume scheme using Fluent platform to determine the stiffness and damping of plain seals used in high speed cryogenic pumps. The film pressures obtained were integrated circumferentially and axially to determine the radial and transverse forces and from there the stiffness and damping coefficients. The results were compared with the analysis using Hires coefficients and shown good agreement.

Forrai, L., (2006), [12], deal, with the stability analysis of self-excited bending no vibrations of linear isotropic rotoring-bearing systems with internal damping using the finite element method. The rotor system consisted of uniform circular Rayleigh shafts with both internal viscous and hysteretic damping, symmetric rigid disks, and discrete isotropic undamped bearings. They presented complex form by combining the sensitivity analysis and the eigenvalue problem of the rotor dynamics equations, they proved theoretically that the whirling motion of the rotor system became unstable at all speeds beyond the threshold speed of instability. Furthermore, they found that the rotor stability was always improved by increasing internal viscous damping, whereas increasing internal hysteretic damping would result in a reduction of the threshold speed of instability. They also shown that the stability threshold speed of the rotor is less than the first critical speed. They gave Numerical examples to confirm the validity of the theoretical results, and to study the influence of internal damping on the rotor stability.

Yuan K., and Ping-chen S., (2006), [13], developed simple identified algorithm of bearing dynamic coefficients in flexible rotor systems. In practical operation, the sensor of unbalance Responses could not be mounted at bearing position in systems. On basis of mechanical impedance technique, the measured unbalance responses on two nodes of shaft at adjacent bearing position were employed to determine the unbalance responses on the node of shaft at bearings position. It would overcome the limitation of sensors mounting and reduce the estimated error. Moreover, the experimental accuracy of bearing dynamic coefficients was also verified by the comparison between rotor kit and estimated system model of the frequency response functions.

B. B., Maharathi, (2005),[14], developed a complex transfer matrix method for analyzing the steady-state response of linear rotor-bearing systems in the frequency domain. He derived the transfer matrix of the shaft segment by

considering the state variables of the shaft in, a continuous system sense, to give the most general formulation. In his analysis, all the influencing parameters of the shaft, disk, and bearings are included. A three-disks-rotor-bearings system had been used as the physical model to demonstrate the effectiveness of this matrix formulation for the evaluation of the dynamic characteristics of any rotor-bearing system. In order to establish the accuracy of this technique, experiments were also conducted on the same rotor-disks model under various bearing conditions as used in the numerical analysis. The numerical results evaluated by various researchers were also compared with these experimental results.

Raja Shekar Balupari, (2004),[15], analyzed bearing systems involved the prediction of their static and dynamic characteristics. The capability to compute the dynamic characteristics for hydrodynamic bearings had been added to Bearing Design System (BRGDS), a finite element program developed by Dr. R.W. Stephenson, and the results obtained were validated. In this software, a standard finite element implementation of the Reynolds equation was used to model the land region of the bearing with pressure degrees of freedom. The assumptions of incompressible flow, constant viscosity, and no fluid inertia terms are made. He integrated the pressure solution to give the bearing load, and the stiffness and damping characteristics were calculated by a perturbation method. The static and dynamic characteristics of 60°, 120° and 180° partial bearings were verified and compared for a length to diameter (L/D) ratio of 0.5. A comparison had also been obtained for the 120° bearing with L/D ratios of 0.5, 0.75 and 1.0. A 360°-journal bearing was verified for an L/D ratio of 0.5 and also compared to an L/D ratio of 1.0. The results were in good agreement with other verified results. The effect of providing lubricant to the recesses had been shown for a 120° hybrid hydrostatic bearing with a single and double recess.

Omidreza Ebrat and Nickolas Vlahopoulos, (2004), [16], presented a detailed journal bearing analysis for accurate evaluation of film dynamic characteristics. The new formulation is based on a local perturbation of the oil film at each computational node that captures the important effects of Journal misalignment and bearing structural deformation in rotor dynamic and engine applications. The algorithm is an extension to the classical approach of evaluating film dynamic characteristics based on journal eccentricity perturbation. In this paper was the governing equation for oil film pressure, stiffness, and damping characteristics had been Discretized using finite difference formulation and was validated with numerical results. The journal misalignment and bearing structural deformation effects could be very important in accurately assessing the crankshaft-block dynamic interaction of an internal combustion (I.C) engine or rotor dynamic behavior.

Julio Gomez-Mancilla, Valeri N., (2004), [17], presented new closed-form expressions for calculating the linear stability thresholds for rigid and flexible Jeffcott systems and the imbalance response for a rotor supported on a hybrid bearing. For typical bearings characteristics, expressions yield stability thresholds practically equal to those reported by Lund (1966). The hybrid bearing design had a single injection port whose location was so chosen to stabilize the bearing performance and to reduce the steady equilibrium attitude angle. Rotor dynamics coefficients graphs for conventional and pressurized bearings, as functions of bearing equilibrium eccentricity and/or Sommerfeld number, were presented. Using the rotor dynamics coefficients into the expressions for the corresponding velocity thresholds and the imbalance response, the system stability and vibration performances were estimated and analyzed. When comparing the Jeffcott flexible shaft supported on two journal bearings of the conventional type with the hybrid type, the results show a clear superiority of the pressurized design as far as stability behavior was concerned. Specifically for cases of flexible shafts with similar characteristics to those used

in industry, the analysis showed that this design yields velocity thresholds 25%-40% higher compared to the conventional circular ones. Also this bearing displayed nonlinear feeding pressure behavior, and it was capable of reducing the synchronous vibration amplitude in most speed ranges, except around the critical speed; moreover, for certain Jeffcott configurations the amplitude reduction could be substantial.

Thomas Staubi and Matthias Bissig, (2001), [18], tested a procedure to determine rotor dynamic coefficients on the basis of numerical , three-dimensional flow simulations in the labyrinth seals and the rotor side space of a pump using a commercial CFD-code. Choosing an adequate, rotating coordinate system made steady calculations possible. Rotor dynamic coefficients could be identified from the regression of calculated forces. A good agreement between numerical and experimental results had been obtained and would be proved with the bulk-flow theory of Childs. Furthermore, during this work demonstrating the proposed method for calculation of rotor dynamic coefficients could be applied to very general cases. Any movement of the axis of rotation of the impeller-shaft relative to its casing affected the rotor deflection or vibration and contributions of rotor dynamic forces of this vibration could be arise from seals, the rotor side space, the flow through the impeller, leakage flows, or the flow in the bearings themselves. So rotor dynamic coefficients were requiring as input data for prediction of rotor vibration.

Itzhak Green and Roger M. Barnsby,(2001),[19], presented a numerical solution for the dynamic analysis of gas lubricated non-contacting mechanical face seals having a single grounded flexibly mounted stator. They solved Seal dynamics in axial and angular modes of motion. Both the Reynolds equation and the equations of motion were arranged into a single state space form, allowing the fluid film lubrication and the dynamics to be solved simultaneously. The resulting set of equations is solved using a high-order

multi-step ordinary differential equation solver, yielding a complete simulation for the seal dynamic behavior. Examples of seal motion were given in detailed transient responses. They investigated the stability threshold to gauge the influence of seal parameters such as inertia, speed, coning, and the direction of sealed pressure drops. The results showed two modes of instability: (1) When the inertia effect was larger than a critical value, the natural response of the seal grew monotonically in a half-frequency-whirl mode. (2) When the seal coning was less than some critical value in an outside pressurized seal, the minimum film thickness diminishes because of hydrostatic instability, and face contact occurred. Conversely, an inside pressurized seal was shown to be hydrostatically stable and had a superior dynamic response at any coning.

Abd Alla El-Marhomy,(1999),[20], presented a general method of analysis based on Liapunov's direct method for studying the behavior of the nonlinear system of differential equations governing the motion of a rotor-bearing system in the neighborhood of its equilibrium point. He adopted a model comprised of an axially symmetric rigid appendage attached at an arbitrary location along a non-uniform spinning shaft mounted on two dissimilar eight component end bearings to develop stability criteria involving different system parameters. He provided the stability boundaries and presented it graphically in terms of system non-dimensional parameters were typical examples of the types of design information available to engineers through the equations. Among the results reached he demonstrated the roles played by (1) bearing mass, (2) appendage mass and dimensions, (3) bearing principal stiffness and damping coefficient, and (4) bearing cross-coupling stiffness and damping coefficients in affecting the nature of system whirl stability.

J. F. Tu, M. Corless and M. J. Gehrich, (1998), [21], study the operating performance of a hydrodynamic journal bearing and used wheel spindle for submicron precision cylindrical grinding. In particular, the spindle of interest

was a wheel spindle for a large cylindrical grinder, operated in the horizontal mode. The weight of the spindle shaft/ wheel assembly functions as the primary load. They examined the effects of lubricant, speed, and temperature on the spindle running accuracy, thermal drift, dynamic characteristics, and stability. First they began with a discussion of the practical methods developed to assess spindle running accuracy followed by a description of the experimental apparatus and related signal processing. Experimental results were then presented regarding how lubricant, temperature growth, and speed affect the Steady-state temperature, center position, and spindle error motion. The results were compared with classical theoretical results.

A. Z. Szeri and H. J. Kim,(1994),[22], described user's manual of the software package CAD JOB. With graphics user interface, the designer selects, or lets the program select, the major design parameters of a journal bearing, corresponding to desired working conditions. The program then calculated the performance characteristics of the bearing. Calculations could be performed either in the isothermal or in the non-isothermal (i.e., thermo hydrodynamic or THD) mode. The numerical results and their graphical representation might be viewed on the screen, as well as being stored on the hard disk in file DES*.Dat. Should the primary decimal prove to be unsatisfactory, CADJOB provided the designer with the tools necessary to repeatedly modify the desire1 and inspect each modification until acceptable solution was obtained.

T.kato and Y.hori, (1988),[23], proposed a fast method for calculating the dynamic coefficients of the finite width journal bearings under quasi Reynolds boundary condition in which the trailing boundary line was given by a straight line and the continuity of bulk flow across this line is ensured. They were found that the calculated linear stiffness and damping coefficients agree well with the data of Lund and Thomson. The time required by this calculation was very small compare with the time required by ordinary (FDM),which

suggests possibility of real time journal bearing designs without using the data base of the dynamic coefficients.

P.K lit and J.W. hind, (1986), [24], calculated the dynamic coefficients of a journal bearing, using a solution by variational approach equivalent of the solution by Reynolds equation. A perturbation method is applied to find the individual dynamic coefficients .The finite element method is used in the numerical evaluation of the equations. The flow is assumed to be laminar and the lubricant is Newtonian .Allowance is made for viscosity-temperature dependency in circumferential and axial directions. This suggested that film forces could be obtained very efficiently if the pressure expanded as a series of arbitrary orthonormal functions and the matrix from of the Reynolds equation i.e. infinite number of dimensional linear equation system is derived without an approximation and thus this method was shown to be identical with the Raleigh Ritz's method i.e. the finite element method with one element.

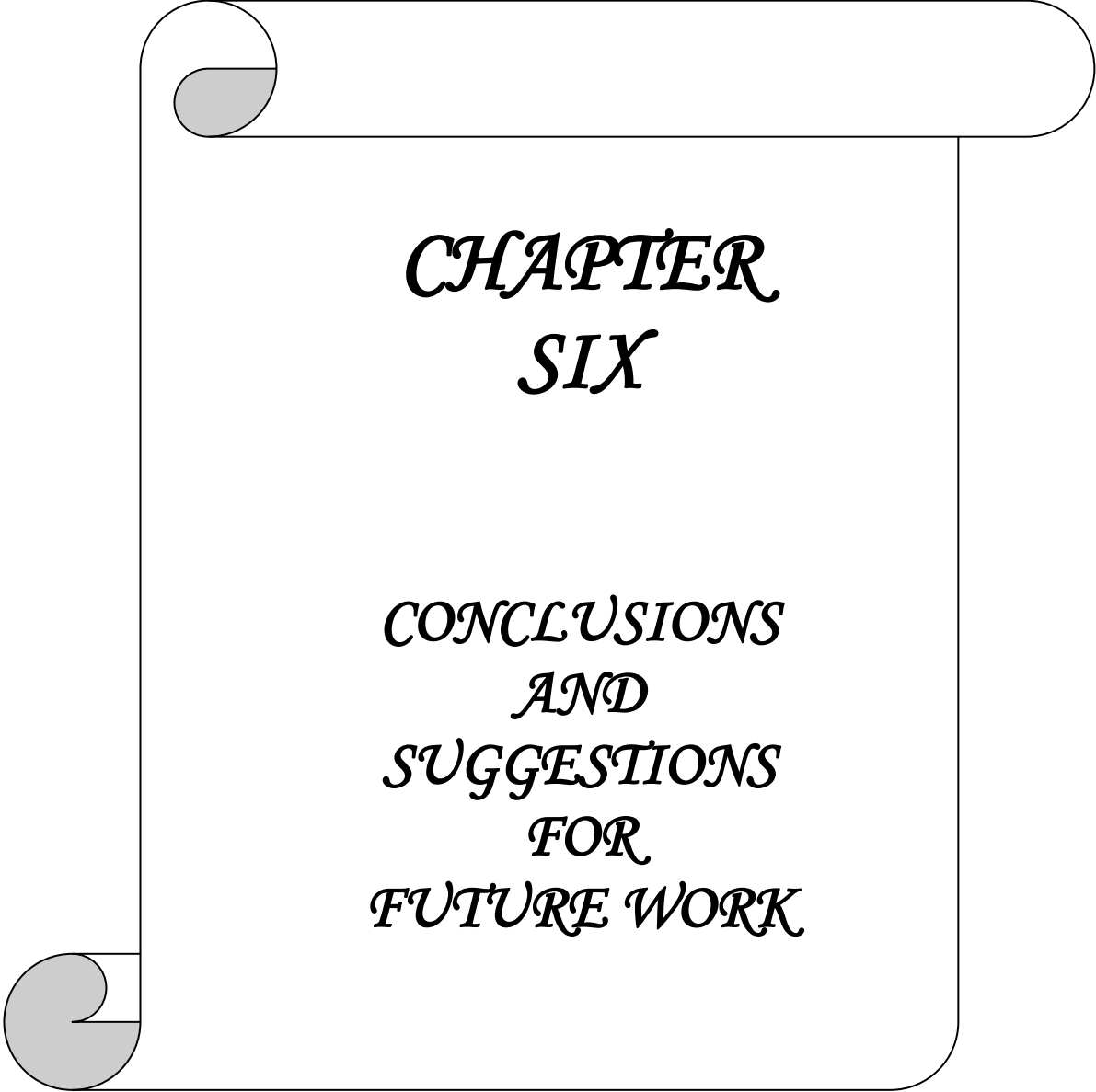
Craighead, D. Dowson, (1980), [25], studied theoretically the influence of thermal effects, shaft misalignment and non-laminar lubricant flow upon the dynamic behavior of fluid film bearing. However, it was difficult to appreciate the influence these coefficients might had merely through their inspection .Thus the effect of the factors under consideration might have been determined by using the eight linearized coefficients in studies of bearing stability and of the behavior of two-bearing rotor systems. They clearly recognized the contribution of the bearing characteristics to the stability and response behaviour of a shaft - bearing system .They might be use to examine the stability of a single bearing with an associated mass (the so-called critical mass analysis). Also a study of the effect of variable viscosity could be presented .Eight solutions of the governing Reynolds equation in a thin lubricating film were required to determine these coefficients by a conventional finite difference method.

D. W. Parkins, (1979), [26], investigated the nonlinear characteristics of the eight coefficient theoretically and experimentally which specify the Lateral flexibility of a hydrodynamic journal bearing .Coefficient calculations Allowed viscosity to vary with temperature and pressure. Examined a range of positive and negative displacements and velocities .Using realistic criteria measured coefficient non- linearity was found to be significant at eccentricity ratios greater than 0.78, Theory adequately predicted some "zero" values but not gradients .An improvement in the coefficient prediction may depend on the inclusion of some previous history dependent factors.

2.3. Final Remarks

It seems from the above review that there is a little work related to the performance of chamfered journal bearing working under different conditions and also it effect to reduce the side leakage flow rate. Also, there is nothing (to the our knowledge) about the effect of chamfering into the rotor dynamic forces coefficients.

The present work represents the attempt to study the effect of chamfering into the stability of the plain cylindrical hydrodynamic journal bearing for $L/D=0.25,0.5,1.5$ under different working conditions, namely different chamfer length and angle. Hence, the purpose of this work is to analyze the dynamic performances of chamfered journal bearing.



CHAPTER SIX

*CONCLUSIONS
AND
SUGGESTIONS
FOR
FUTURE WORK*

CHAPTER SIX

“Conclusions and Suggestions for Future work”

6.1. Conclusions:-

In view of the theoretical results obtained from the solution of the governing equations which describe the effect of chamfer machined at both end of the plain hydrodynamic journal bearing on the stiffness coefficients, damping coefficients and whirl instability threshold for journal bearings, the following concluding remarks can be considered during this work:-

1. Chamfer angle seems to have a little effect on the dynamic characteristics of the chamfered bearing ,for lower values of (L/D) ratio the maximum decrease found to be (7%) for $\bar{K}_{xx}$ at high eccentricity ratio.
2. The following effects of increasing (L/D) ratio on the dynamic characteristics have been noticed, stiffness coefficients tend to decrease while the damping coefficients tend to increase.
3. The load carrying capacity is increased while attitude angle is decreased for a chamfered journal bearing when compare with that without chamfer .
4. The maximum decrease in attitude angle was found to be (4%) when increasing the chamfer length.
5. The direct stiffness coefficients found to be decrease while the cross-coupled stiffness increases with the existence of chamfer machined at both sides of the journal bearing.
6. The Cross coupled damping found to be slightly decrease with increasing the chamfer length (Lc). The maximum decrease found to be (5%).
7. The direct damping coefficient shows an increase with increasing the chamfer angle for higher values of (L/D) ratio. The maximum increase was found about (15%).

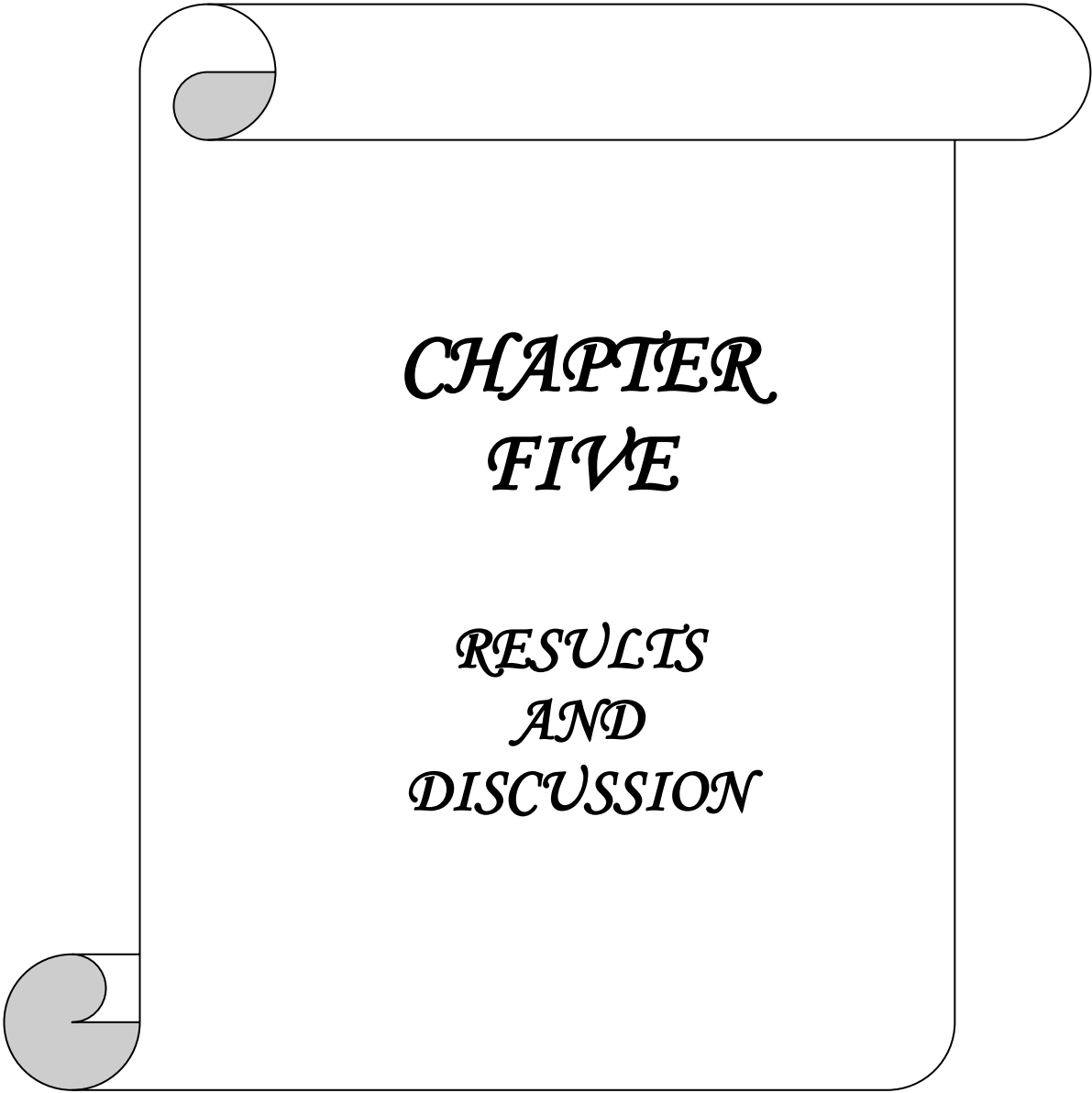
8. The direct damping coefficient $\bar{C}_{xx}$ shows an decrease with increasing the chamfer length at high eccentricity ratio. This maximum decrease was found (25%).
9. A decrease in stability line, that mean the curve tend to the stability zone when a chamfer has been machined at both ends of the bearing inner surface ,this difference of about (10%).
10. The side leakage flow rate tends to be almost negligible when compared with that of plain bearing for large Sommerfeld number.
11. An increase in chamfer length might reduced the side leakage flow rate at large Sommerfeld number and vise versus .
12. An increase in (L/D) ratio might reduced the side leakage flow rate at lower Sommerfeld number.

6.2. Suggestions for Future Work:-

The chamfered journal bearing is not known well yet as shown from the literature survey. Thus, much more work is required on this type of the bearings to reveal all its performance characteristics, to make it more realistic to be used by the designers. Based on this study, the following recommendations can be suggested as a future work:-

1. Study the effect of inlet viscosity on the dynamic behavior of the chamfer bearing.
2. Study the bearing performance when the effect of surface roughness is taken into consideration on the dynamic behavior of chamfered journal bearing.
3. Study the non Newtonian effect on the dynamic behavior of the chamfer bearing.
4. Study the effect of chamfer machined at both ends of the misaligned journal bearing on both steady state and dynamic characteristics.
5. Study the bearing performance with Bi-phase (liquid –solid)lubricant.

6. Study the dynamic characteristics of the chamfered bearing using elasto-hydrodynamic lubrication theory.
7. Study the bearing performance under superlaminar and turbulent regimes.
8. Study the bearing performance with gas lubricant.



CHAPTER FIVE

RESULTS AND DISCUSSION

Chapter Five

“Results and Discussion”

5.1. Verification of the computer programs.

The computer program prepared to solve the governing equations of the present work has been verified for steady state and dynamic characteristics of the bearing. Figure (5-1) shows a comparison between the attitude angle obtained in the present work with obtained by Lund[4]. It is clear that there is a good agreement between the results. The dimensionless static force components ($\bar{F}_\varepsilon$ and $\bar{F}_\phi$) obtained in the present work have been compared with that obtained in Lund[4] as shown in figure (5-2). It can be attributed from the figure that the result obtained from the present computer program are in a good agreement with that obtained by Lund.

The deviation in results for high Values of eccentricity ratio can be explained if we know that Lund used the exact solution of short bearing while a numerical solution was used in the present work. The total load obtained in the present work has been compared with that obtained by Lund[4] as shown in figure (5-3). It can be shown from the figure that the result obtained in this work are reasonable in comparison with that obtained by Lund[4]. Some of the dynamic coefficients which describe the dynamic characteristics have been presented in figures (5-4) and (5-5). The direct damping coefficient ($\bar{C}_{\phi\phi}$) and the cross coupled damping coefficient ($\bar{C}_{\varepsilon\phi}$) obtained in the present work has been compared with that obtained by Sumit [3]. It is clear that the Results are in a good agreement.

5.2. Bearing characteristics:-

Oil flow rate through the bearing decreases with increasing the Sommerfeld number as shown in figure (5-6). It can be notice from this figure that the oil flow rate decreases when chamfer has been machined at both ends for high values of Sommerfeld number (for lightly loaded bearing). This is attributed to the effect of surface tension effect at both ends which plays a big role in prohibiting lubricant flow to the outside, thus maintains a minimum side leakage.

Total static force of the bearing with and without chamfer has shown in figure (5-7). It is clear that the total force shows higher values for the chamfer bearing than that of non-chamfer bearing, If the journal eccentricity increased the load carrying capacity increases due to the decrease of the oil film thickness. Since the oil leakage decreases with the existence of chamfer machined at both end of the bearing ,hence the load increases in this case. The load components show an increase with increasing the eccentricity ratio as shown in figure (5-8) this again must be due to increase static force for the chamfered bearing as mentioned above .

Figure (5-9) shows that the attitude angle of the bearing increases with increasing the Sommerfeld number in other meaning nearly centered operation ,that mean the journal eccentricity vector is nearly orthogonal to the applied load. It can also be shown that attitude angle for the chamfered bearing show a lower values than that obtained in the bearing with unchamfered bearing that must the reverse effect of the chamfer on the load carrying capacity. The maximum decrease of attitude angle is (10%) occurs at high Sommerfeld number. The attitude angle slightly decreases with increasing the chamfer length $[L_c]$ as shown in figure (5-10). The maximum decrease in attitude angle has been calculated and found to be (4%).

5.3. Dynamic Characteristics of The Bearing:

Figures (5-11) to (5-14) shows a comparison between the four stiffness coefficients (two direct and two cross coupled coefficients) for a chamfer and non chamfer bearings. It can be deduced from figure (5-11) that the direct stiffness ($\bar{K}_{xx}$) shows a decrease due to the existence of chamfer machined at both end of the journal bearing in comparison with the bearing without chamfer. This figure shows that if the journal eccentricity increased the stiffness coefficients decreased this due to the effect of the dynamic reaction forces.

The direct stiffness ($\bar{K}_{yy}$) decreases with increasing the eccentricity ratio as shown in figure (5-12) this also due to the increasing the fluid film forces. From this figure it could be shown that the stiffness coefficient decrease in the case of chamfered bearing. Figure(5-13) shows that the cross coupled stiffness coefficient ($\bar{K}_{xy}$) become higher for the chamfered bearing when compared with that obtained in non chamfer bearing .Also the cross coupled stiffness ($\bar{K}_{yx}$) show an increase for chamfered bearing than that obtained for non-chamfer bearing as shown in figure (5-14). The negative sign of this coefficients can be interpreted as beneficial for bearing stability.

Figure (5-15) explains the effect of the chamfer angle on the stiffness coefficient. It is clear that the direct stiffness ($\bar{K}_{xx}$) decreases as the angle of chamfer increases .Also it seems from this figure, that the angle of the chamfer has no considerable effect on the stiffness coefficients spatially at the lower values of eccentricity ratios.

The maximum decrease in direct stiffness coefficient (at high eccentricity ratio $\epsilon=0.9$) was calculated and found to be (7%) . The damping coefficient show a slightly decrease with increasing the angle of the chamfer as show in fig(5-16) spatially at higher values of eccentricity ratio . The maximum decrease in direct stiffness ($\bar{C}_{xx}$) was calculated and found to be (6%) . The effect of chamfer length on the damping coefficient can be shown in figures (5-17) and (5-18) . It can be deduced from figure (5-17) that the chamfer length has a little

effect on the direct damping coefficient ($\bar{C}_{\phi\phi}$). The cross coupled damping ($\bar{C}_{\varepsilon\phi}$) found to be slightly decreases with in creasing the chamfer length (L_c) as shown in figure (5-18). The maximum decrease in damping coefficient has calculated and found to be (5%).

A set of curves represent the stiffness of the bearing against the eccentricity ratios for a new value of chamfer length can be shown in figures (5-19) to (5-22). It is clear from these figures that the chamfer angle has a slight effect on the stiffness coefficients but this effect become more apparent when the chamfer length increase.

Another set of curves represent the damping of the bearing against the eccentricity ratios for a new value of chamfer length can be shown in figures (5-23) to (5-26). It is clear from these figures that the chamfer angle has a slight effect on the damping coefficients but this effect also become more apparent when the chamfer length increase.

It can be inspected that the direct damping ($\bar{C}_{xx}$) show a decrease with increasing the chamfer angle as shown in figure (5-23) this is attributed to the increase in oil leakage of the bearing in this case. These figures insure the reliability of the results obtained before.

Also from the comparison between these curves with figure (5-16) it can be found that the ($\bar{C}_{xx}$) shows a decrease of about (25%) at high Sommerfeld number when chamfer length increased this is attributed to the increase in oil leakage of the bearing when chamfer length increased spatially at at high Sommerfeld number.

An investigation to the effect of chamfer angle on the dynamic coefficients of the bearing for higher value of (L/D) ratio ($L/D= 1.5$) has been shown in figures from (5-27) to (5-33). It is clear that the stiffness coefficients have been unaffected by increasing the angle of chamfer as shown in figures (5-27) to (5-30). It can be noticed from the figure (5-27) that if (L/D) increased ($\bar{K}_{xx}$) decreased by (30%) this fact is if the value of (D) increase then the stiffness is increase, the bearing are assumed to be a rigid support.

The direct damping coefficient ($\bar{C}_{xx}$) shows an increase with increasing the chamfer angle as shown in figure (5-31) . This is can be attributed to the decrease in side leakage flow with increasing the value of (L/D) ratio .The maximum increase was calculated and found to be (15%). It can be also noticed that if the value of (L/D) ratio increased the value of ($\bar{C}_{xx}$) increased by (20%) this also attributed to the decrease in side leakage flow with increasing the value of (L/D) ratio and that provide a sufficient lubricant oil film to carry the load .

The cross coupled damping coefficients have been drawn against the eccentricity ratio. As shown in figure (5-32) . For low eccentricity ratio the cross coupled damping coefficients show a decrease with increasing the chamfer angle while an increase in damping coefficients for high values of eccentricity ratios this could be attributed due to the cross effect of damping and it relation to the side leakage flow.

The effect of chamfer angle on damping coefficients can be shown in figure (5-34) . It can be deduced from this figure that the direct damping coefficient show an increase with decreasing the chamfer angle the maximum decrease found to be (8.5%) .

5.4. *Stability Zone*

The critical speed of the bearing has been evaluated and drawn against the eccentricity ratio the Stability working zone of the bearing can be shown from figures (5-35) to (5-37) ,when the journal eccentricity approaches the clearance for large loads, low speeds or light lubricant viscosity and it is aligned with the load vector. For small loads, high speeds or large viscosities (large Sommerfeld numbers) the journal travels towards the bearing center and its position is orthogonal to the applied load. This peculiar behavior is the source of rotordynamic instability as shown in stability chart. Figure (5-35) shown a slight decrease in stability curve thus tends to the stability region when a chamfer has been machined at both end of the bearing. This is can be attributed to the

decrease in damping coefficient and the negative value of ($\bar{K}_{yx}$) which can be interpreted as beneficial for bearing stability as remembered in figure (5-14).

An increase in stability curve can be shown or tend to the unstable region when the chamfer angle has been increased specially for lower values of eccentricity ratio which can be attributed the decreased in damping coefficient shown in figure (5-36) with chamfer angle increasing. It can also shown that the stability curve or tend to the stable region decreases for higher values of the eccentricity ratio since the oil flow rate increases at higher values of eccentricity ratio (lower value of Sommerfeld number) .

Increasing the chamfer length cause a decrease in stability curve or tend to the stable region as shown in figure (5-37). This is attributed to the ($\bar{K}_{yx}$) coefficient in this case with increase (L_c) as noticed from the comparison between the curves (5-14) and (5-21).

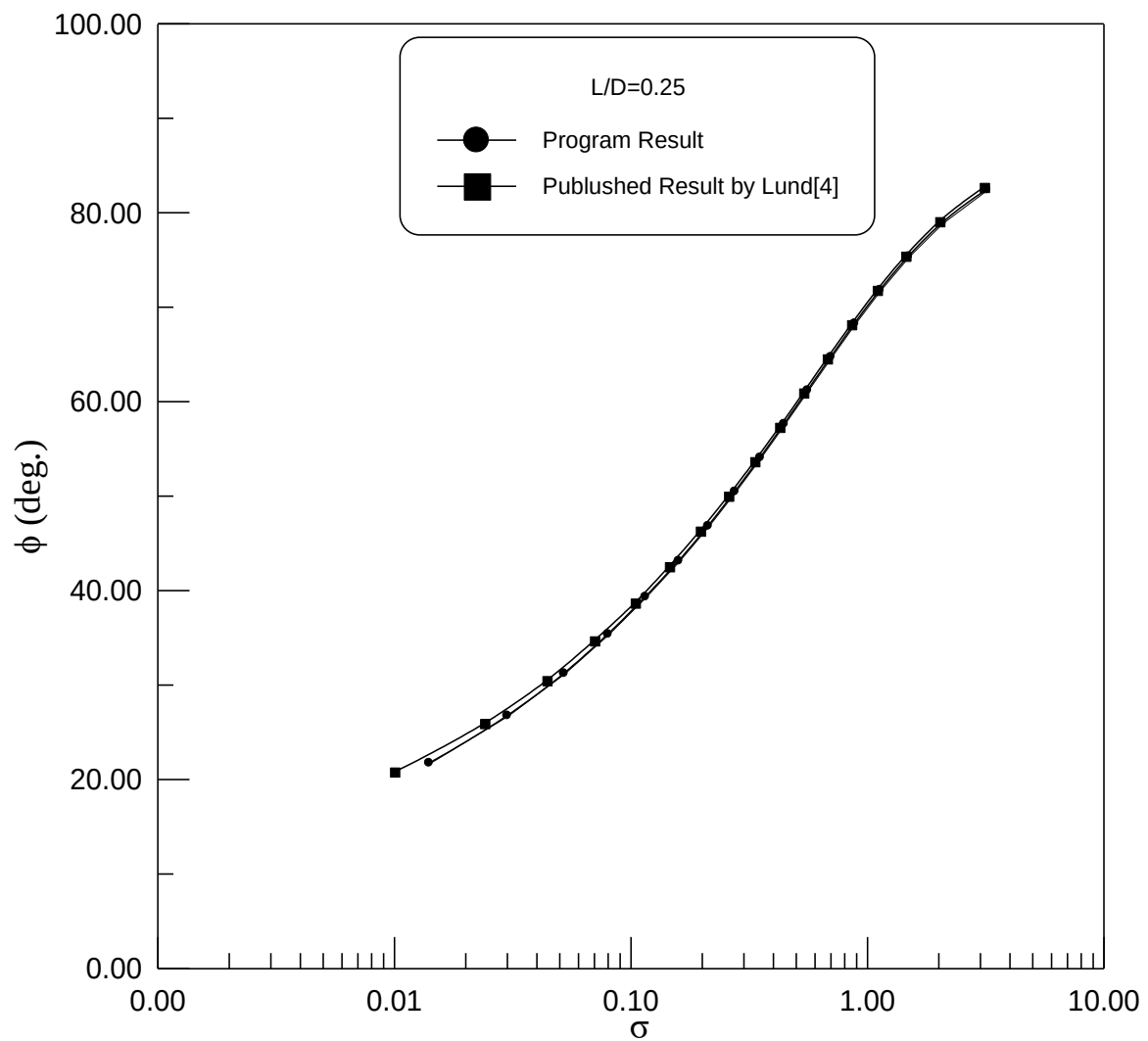


Figure (5-1), Comparison Between Published and Program Results for Attitude Angle verse Modified Sommerfeld number.

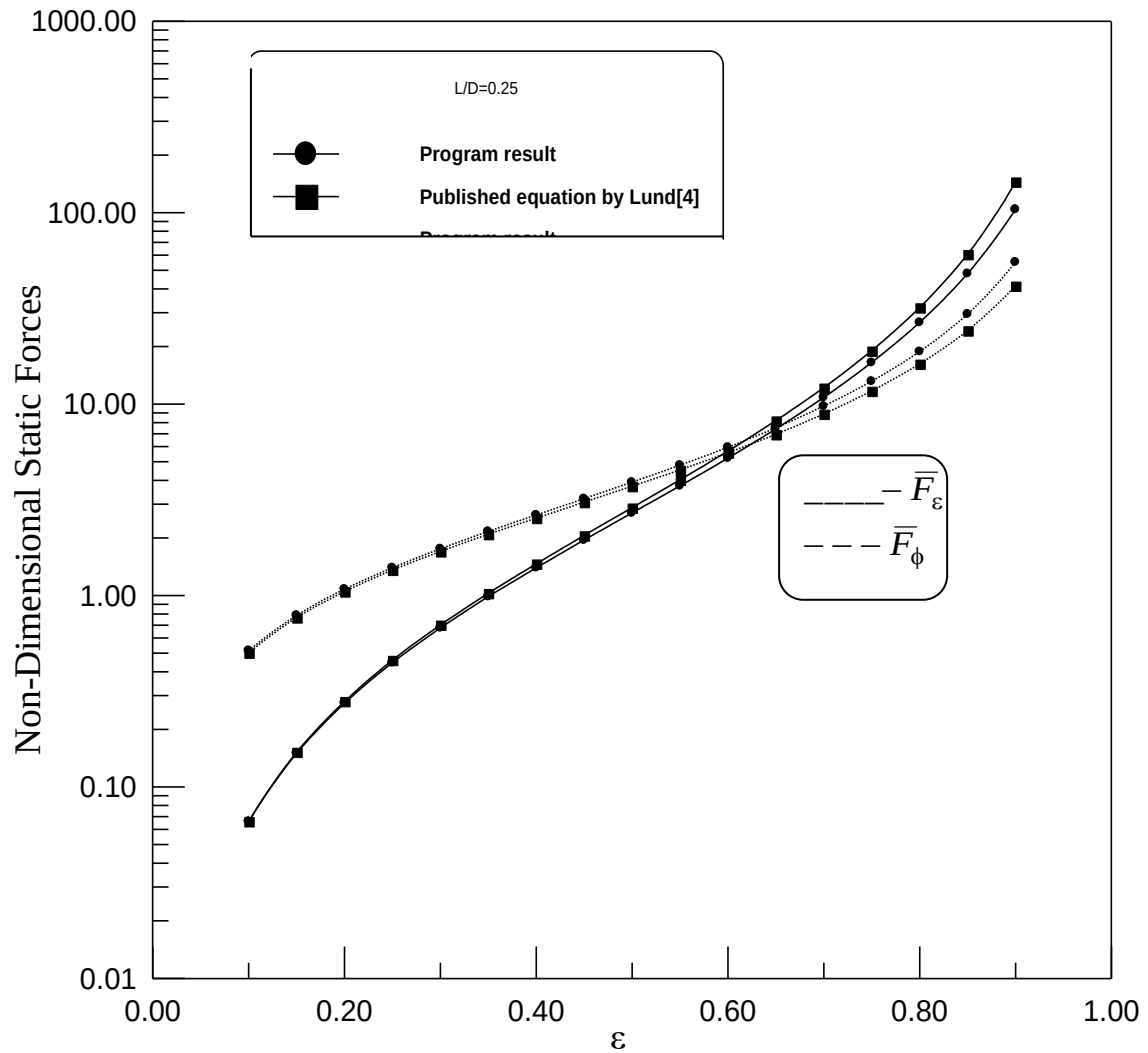


Figure (5-2), Comparison Between Published and Program Results for $\bar{F}_\varepsilon, \bar{F}_\phi$ verse eccentricity ratio.

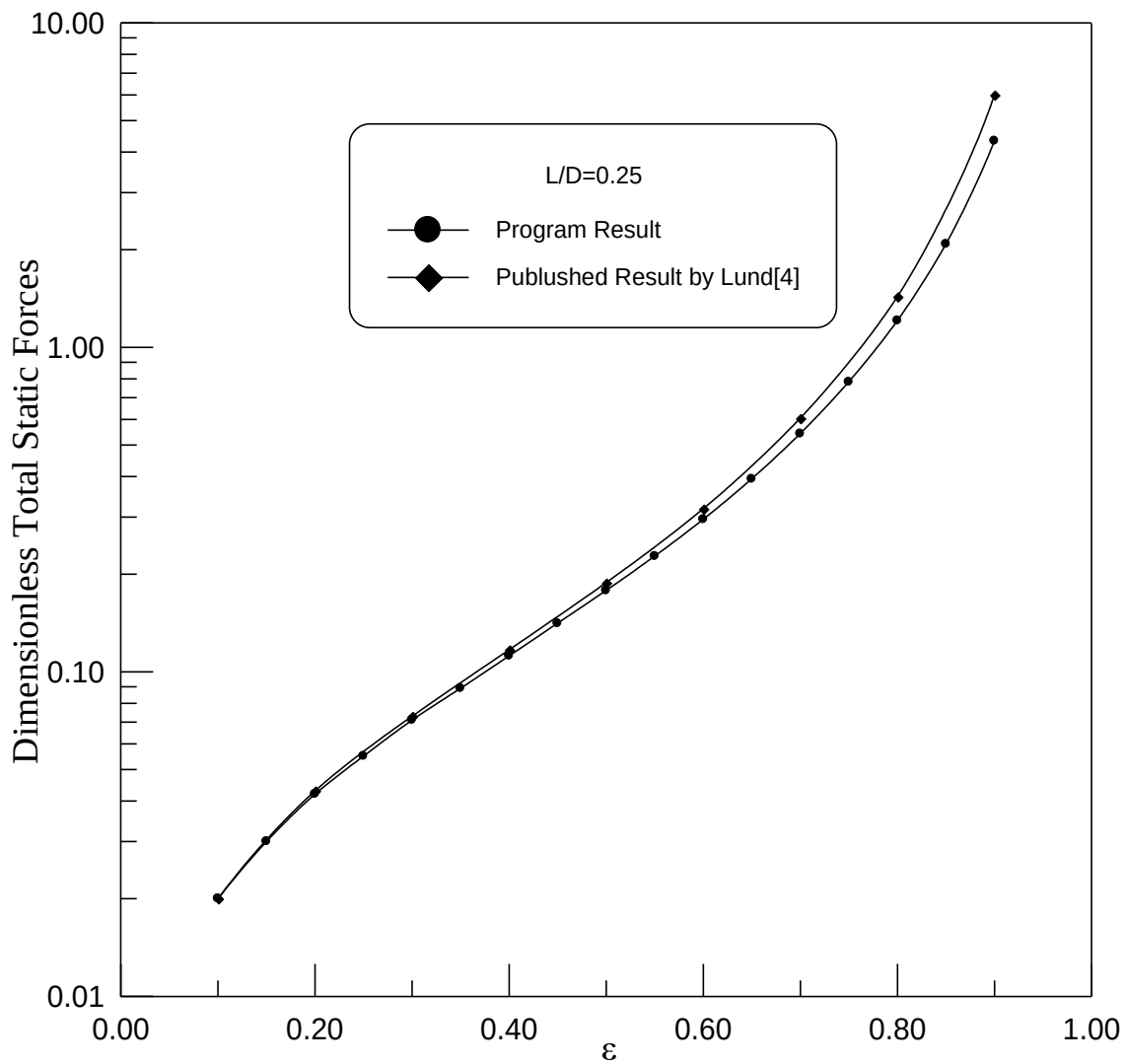


Figure (5-3), Comparison Between Published and Program Results for Dimensionless total Static Forces verse Eccentricity ratio.

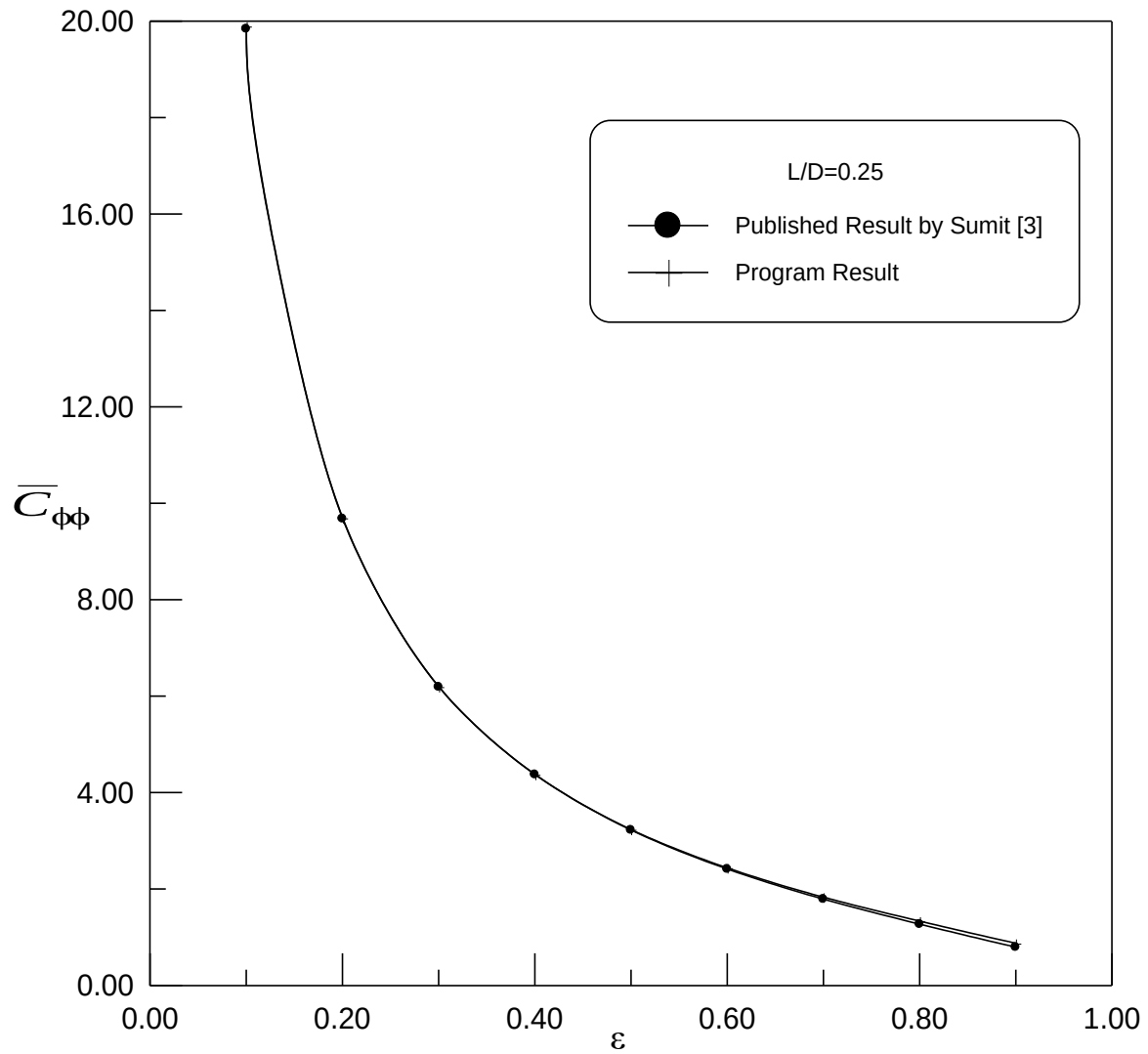


Figure (5-4), Verification Between Published Results and Program Results for Direct Damping Coefficient Versus Eccentricity ratio.

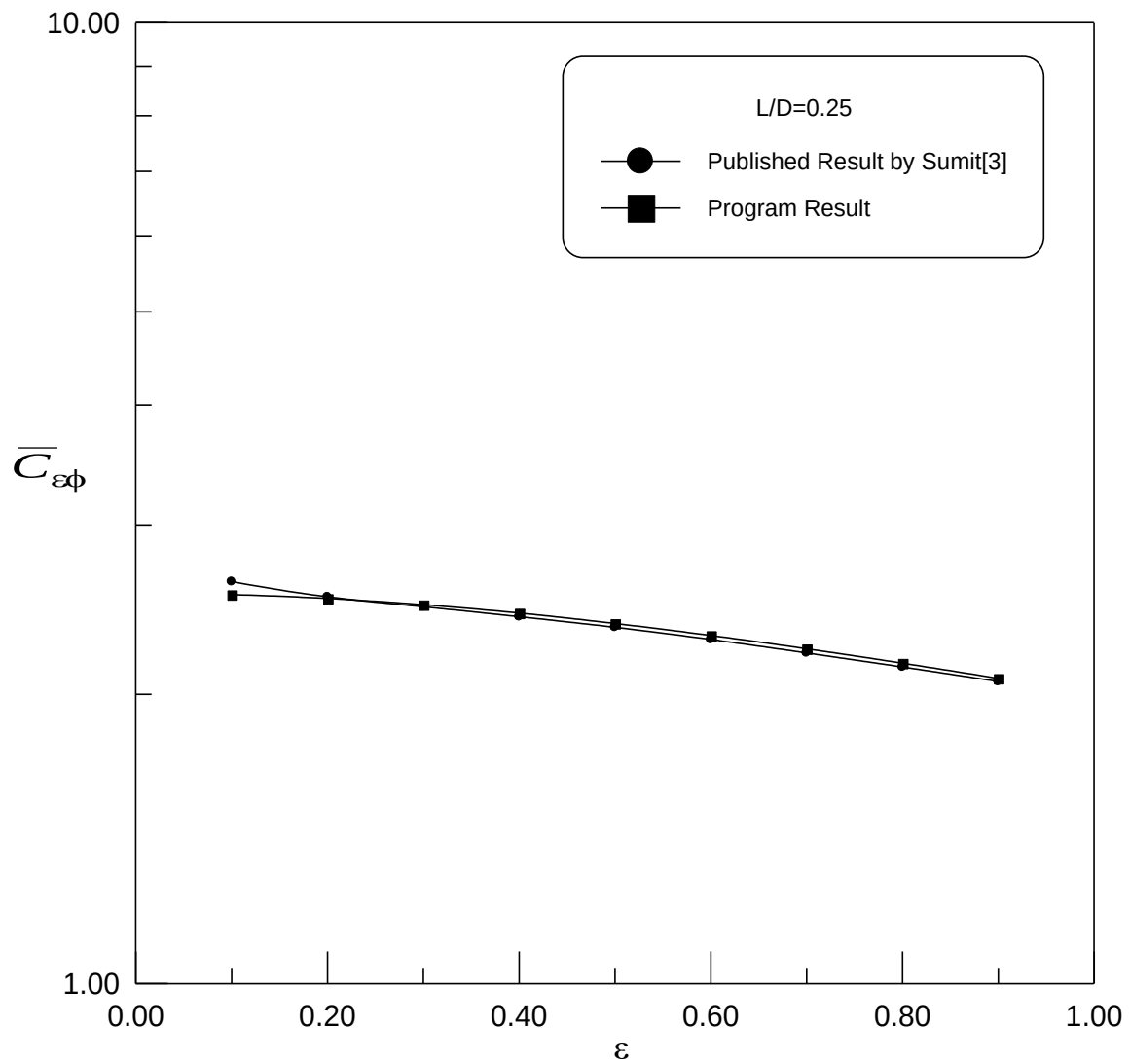


Figure (5-5), Verification Between Published Results and Program Results for Cross Coupled Damping Coefficient Versus Eccentricity ratio.

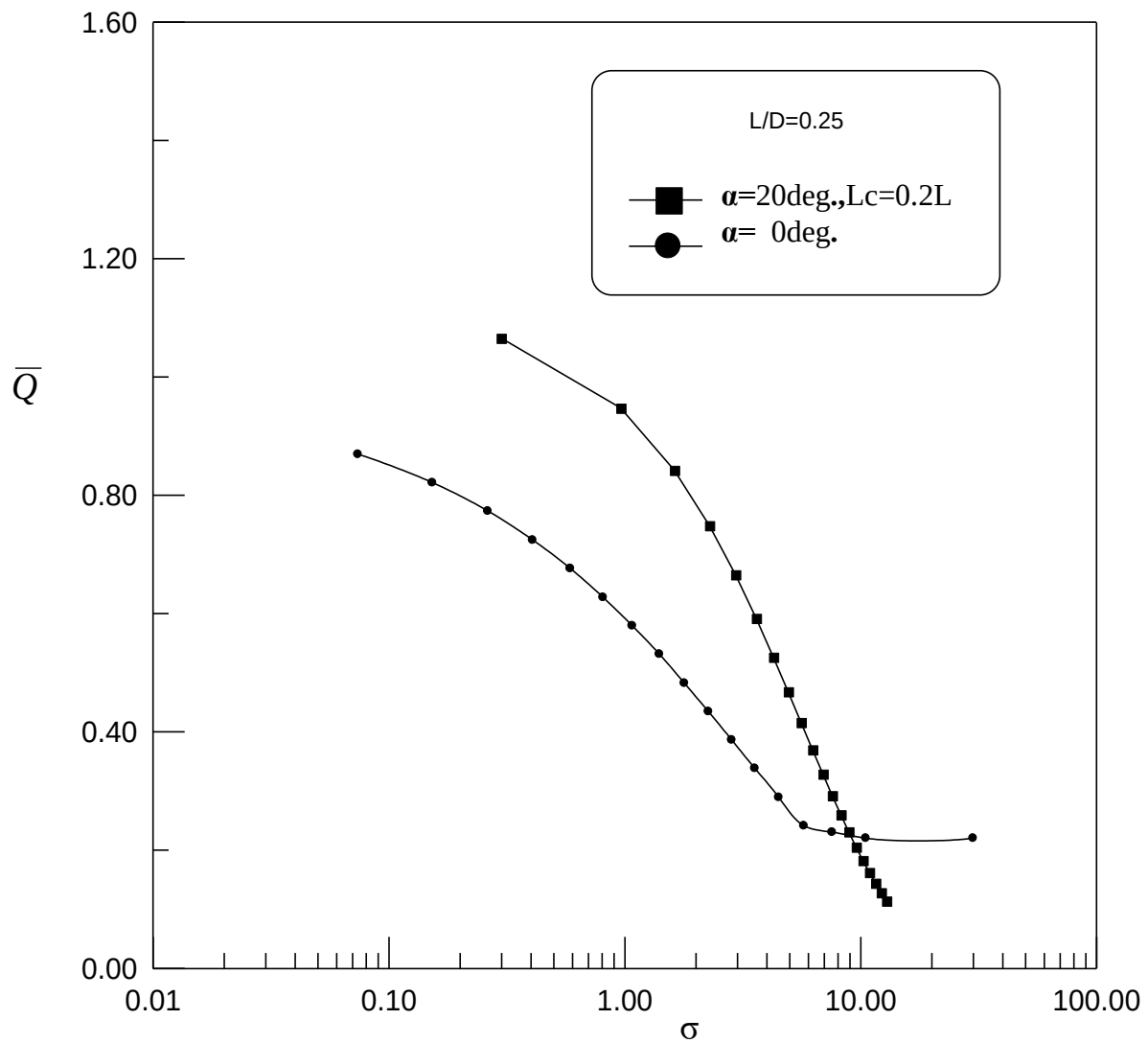


Figure (5-6), Comparison between oil flow rate of Chamfered and non Chamfered bearing .

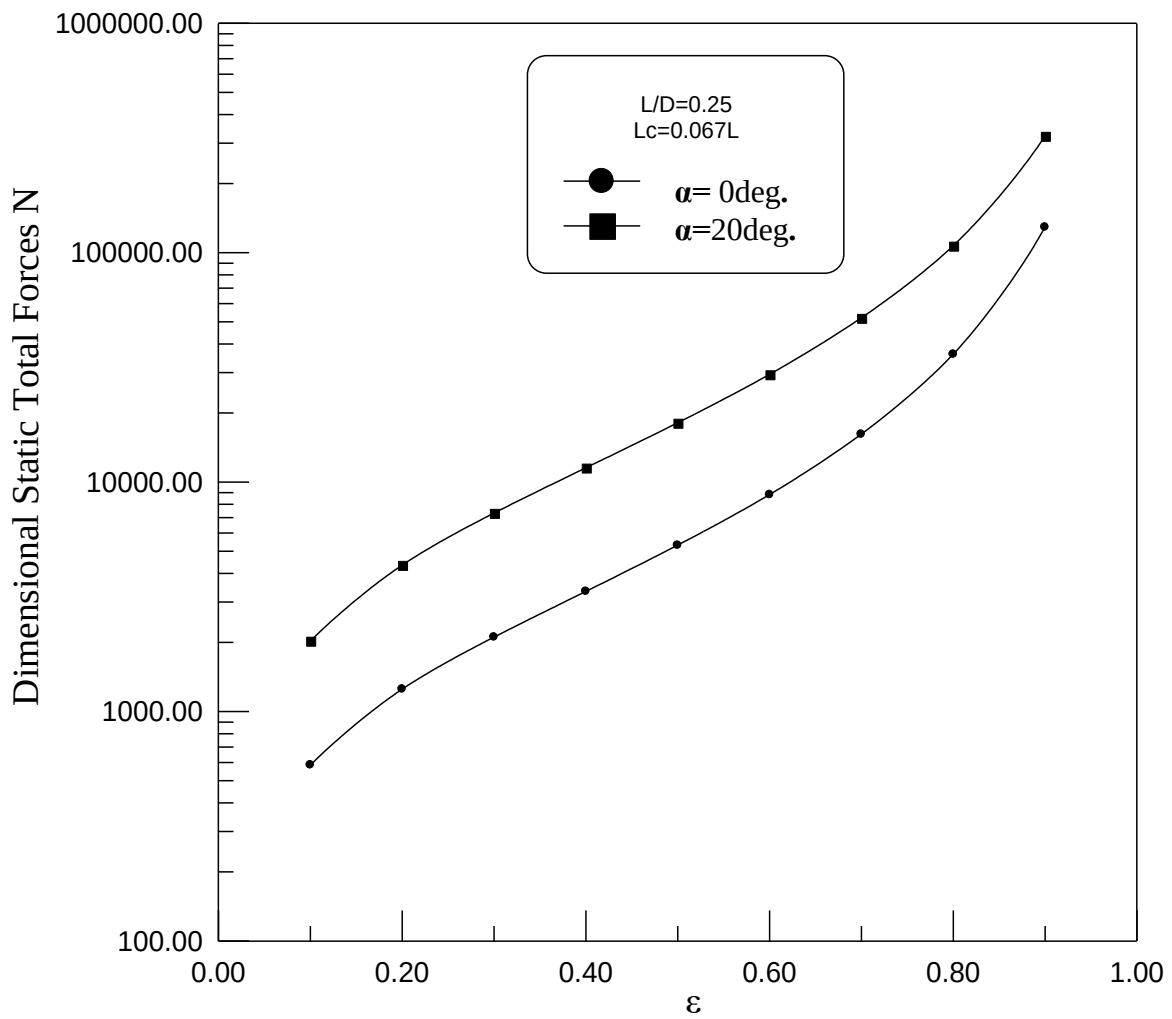


Figure (5-7), Comparison between the Static Total Forces of Chamfered and non Chamfered bearing.

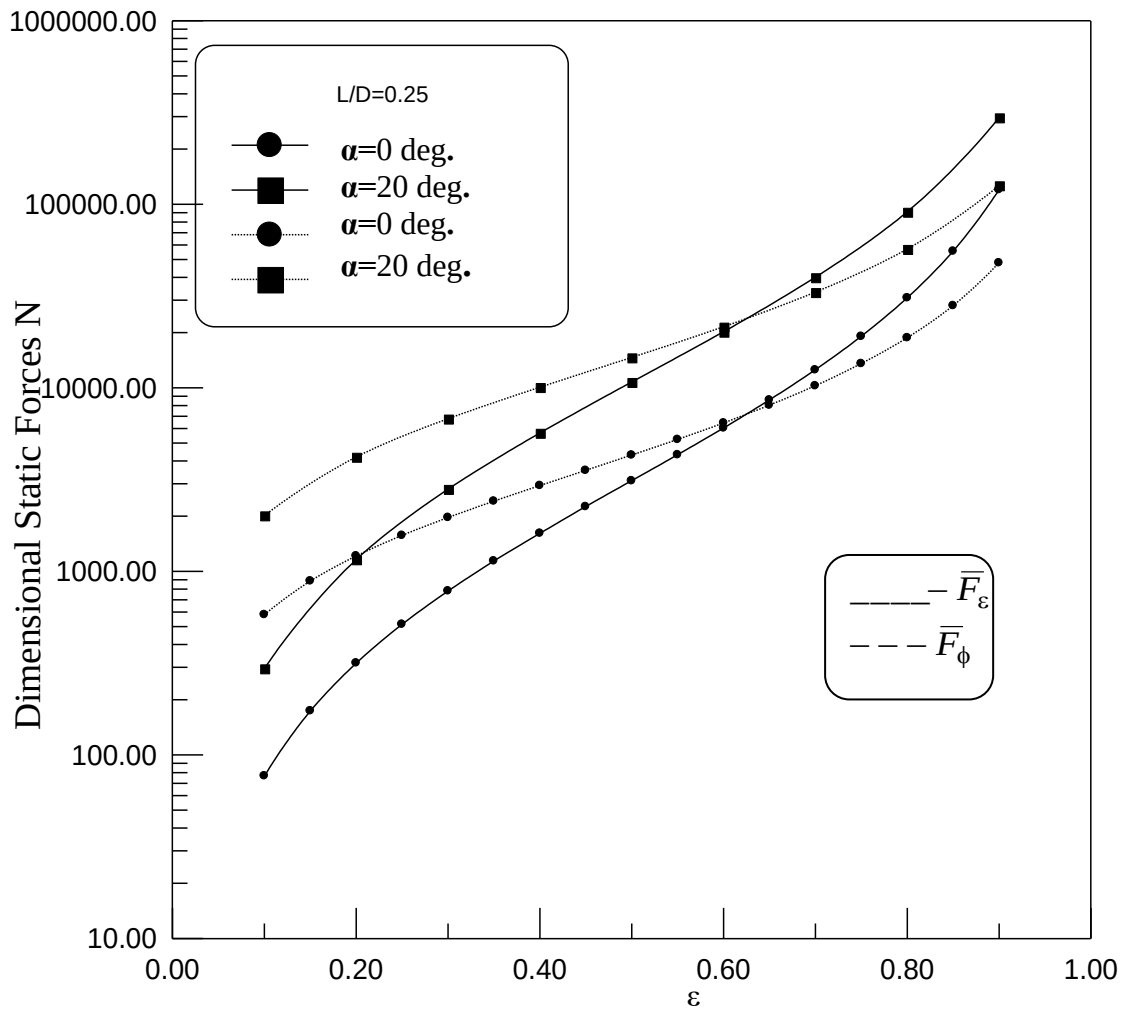


Figure (5-8),Comparison between the Static Forces obtained from Chamfered bearing with that obtained from non Chamfered bearing .

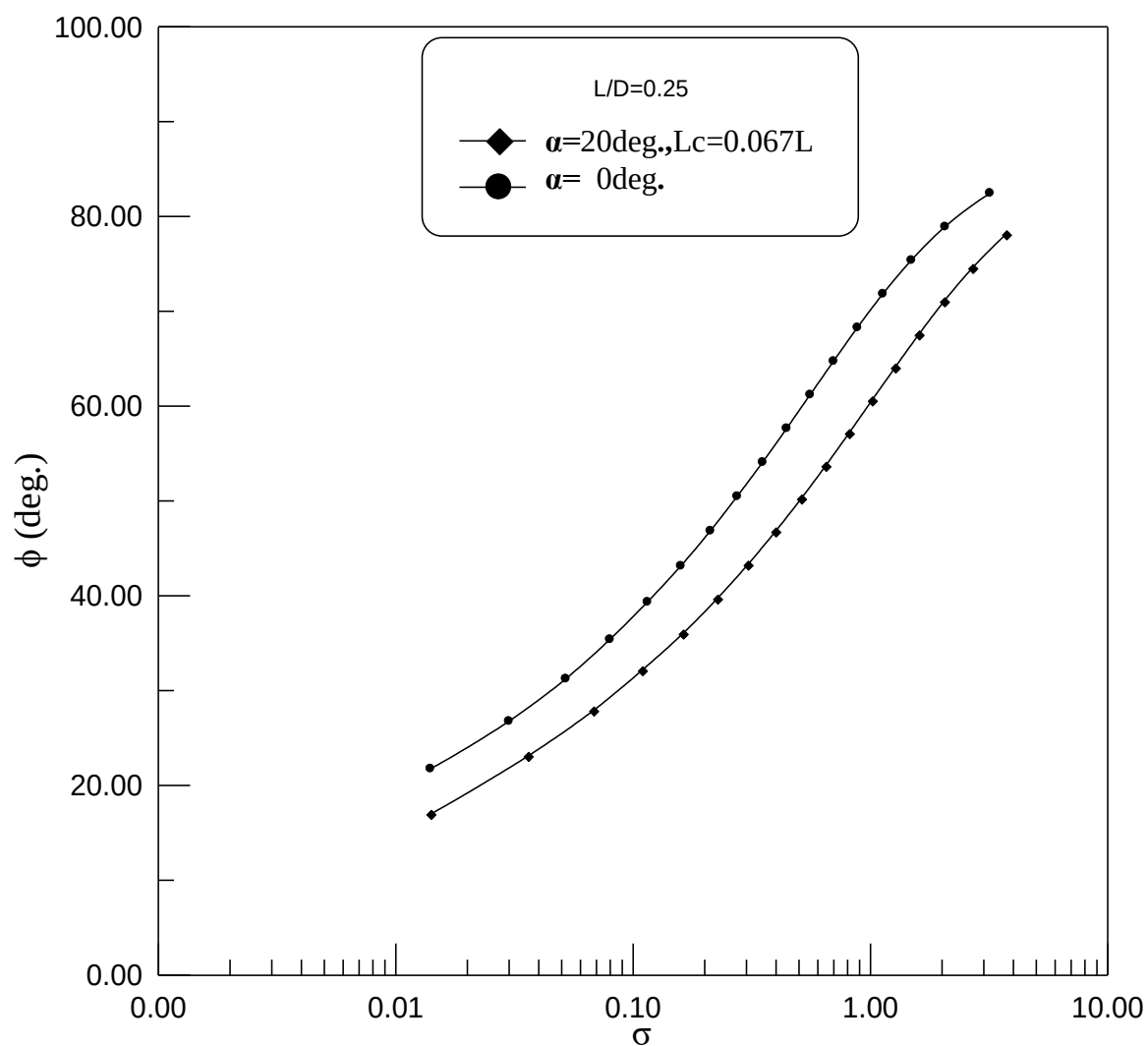


Figure (5-9), Comparison between the attitude angle of Chamfered and non Chamfered bearing.

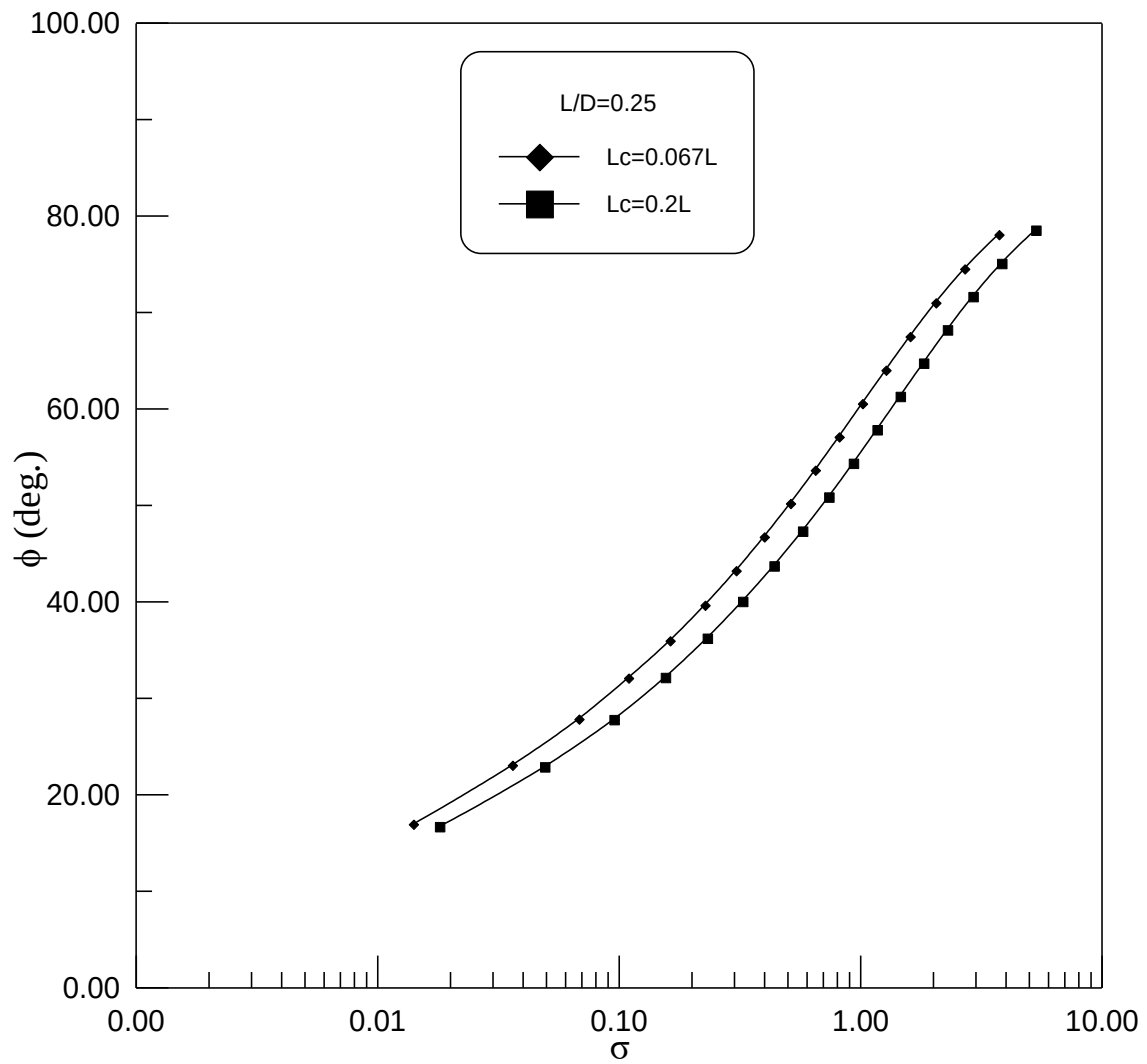


Figure (5-10), Comparison between the attitude angle of Chamfered and non Chamfered bearing for different Chamfered length.

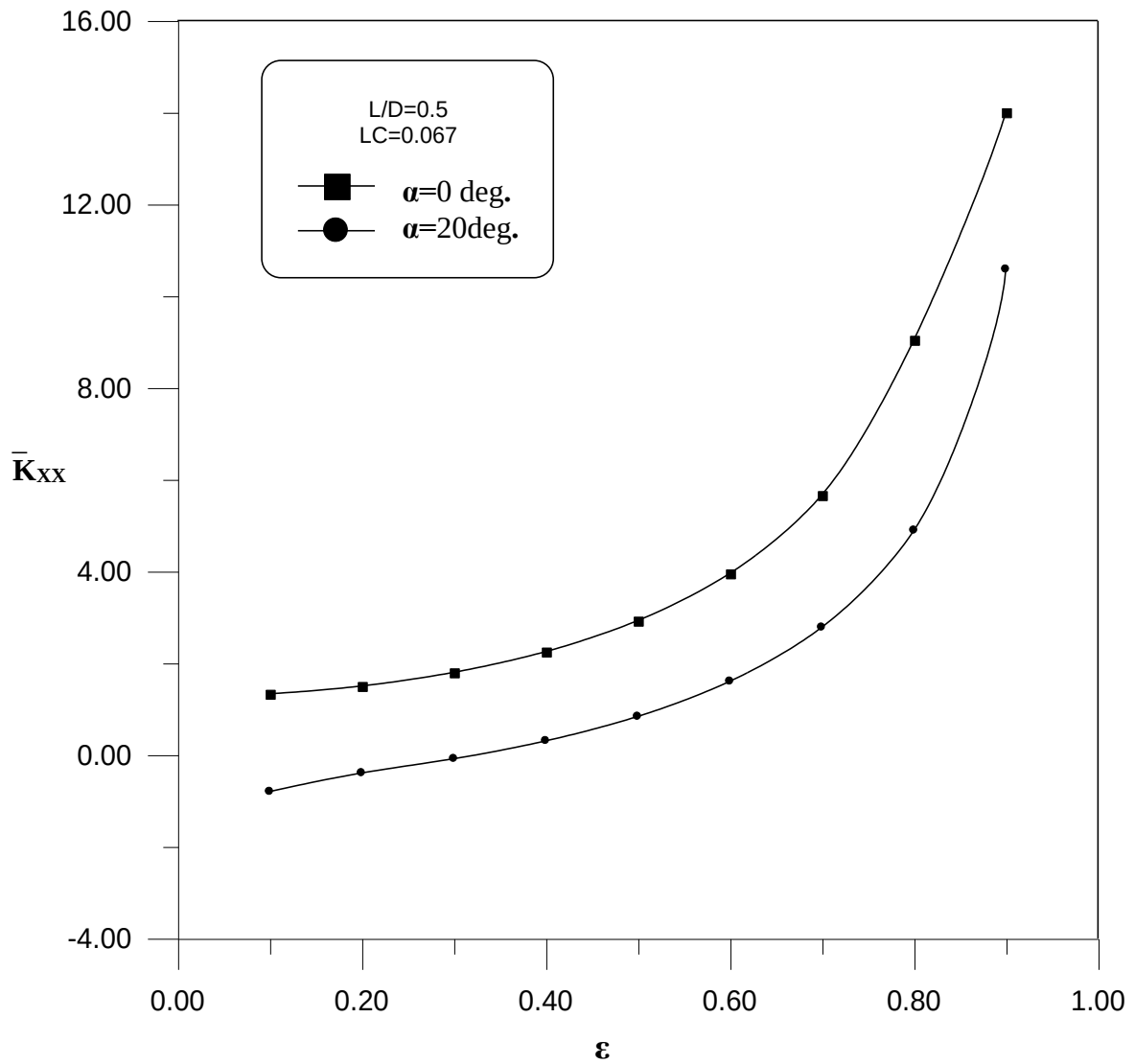


Figure (5-11). Comparison between direct stiffness $\bar{K}_{xx}$ coefficient of chamfered and non-chamfered bearing.

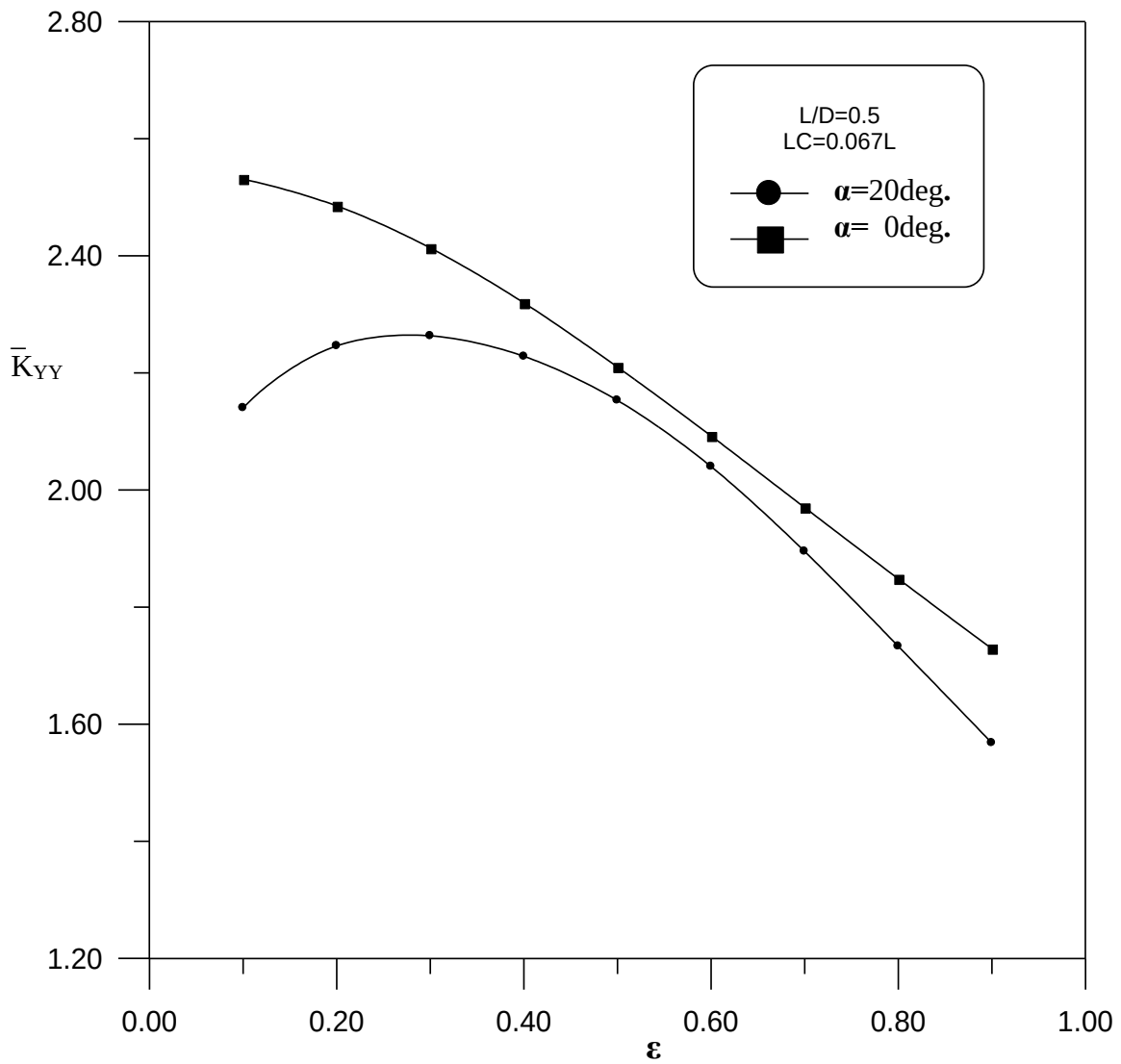


Figure (5-12), Comparison between Direct Stiffness Coefficient of $\bar{K}_{YY}$ Chamfered and non Chamfered bearing .

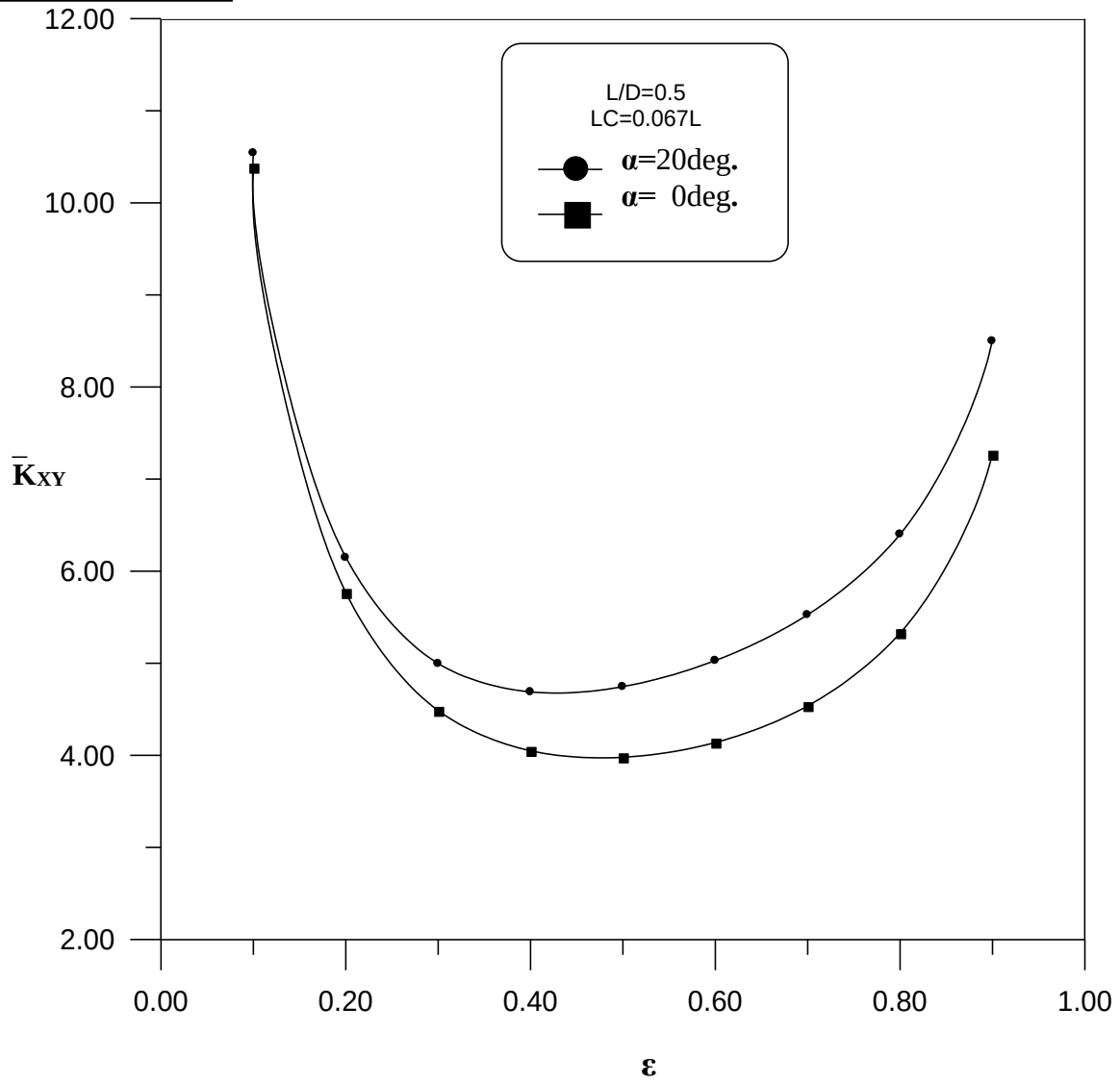


Figure (5-13). Comparison between cross coupled stiffness $\bar{K}_{XY}$ coefficient of chamfered and non-chamfered bearing.

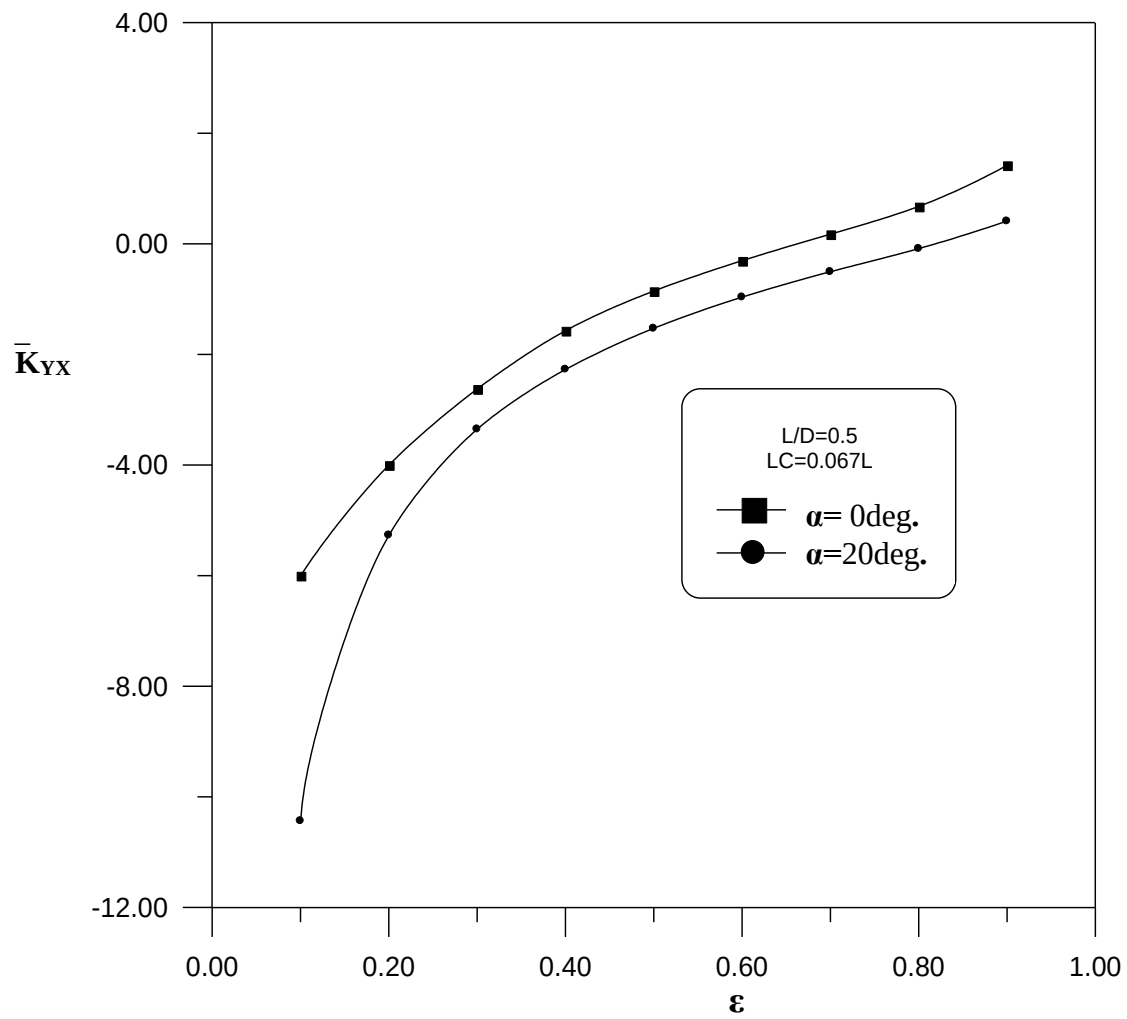


Figure (5-14), Comparison between Cross Coupled Stiffness $\bar{K}_{YX}$ of Chamfered and non Chamfered bearing .

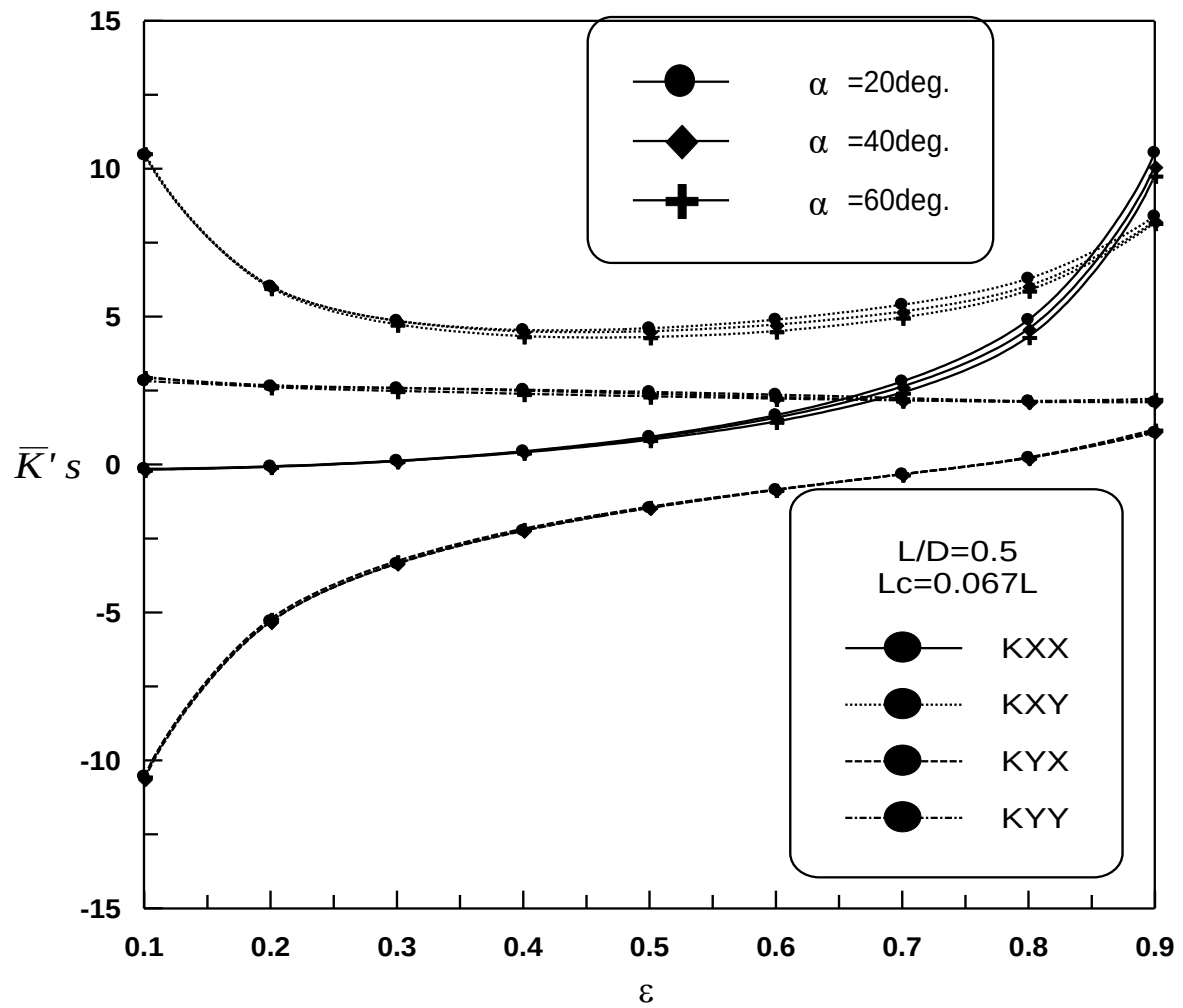


Figure (5-15),Stiffness Coefficients of Chamfered Bearing for Different Chamfer Angle .

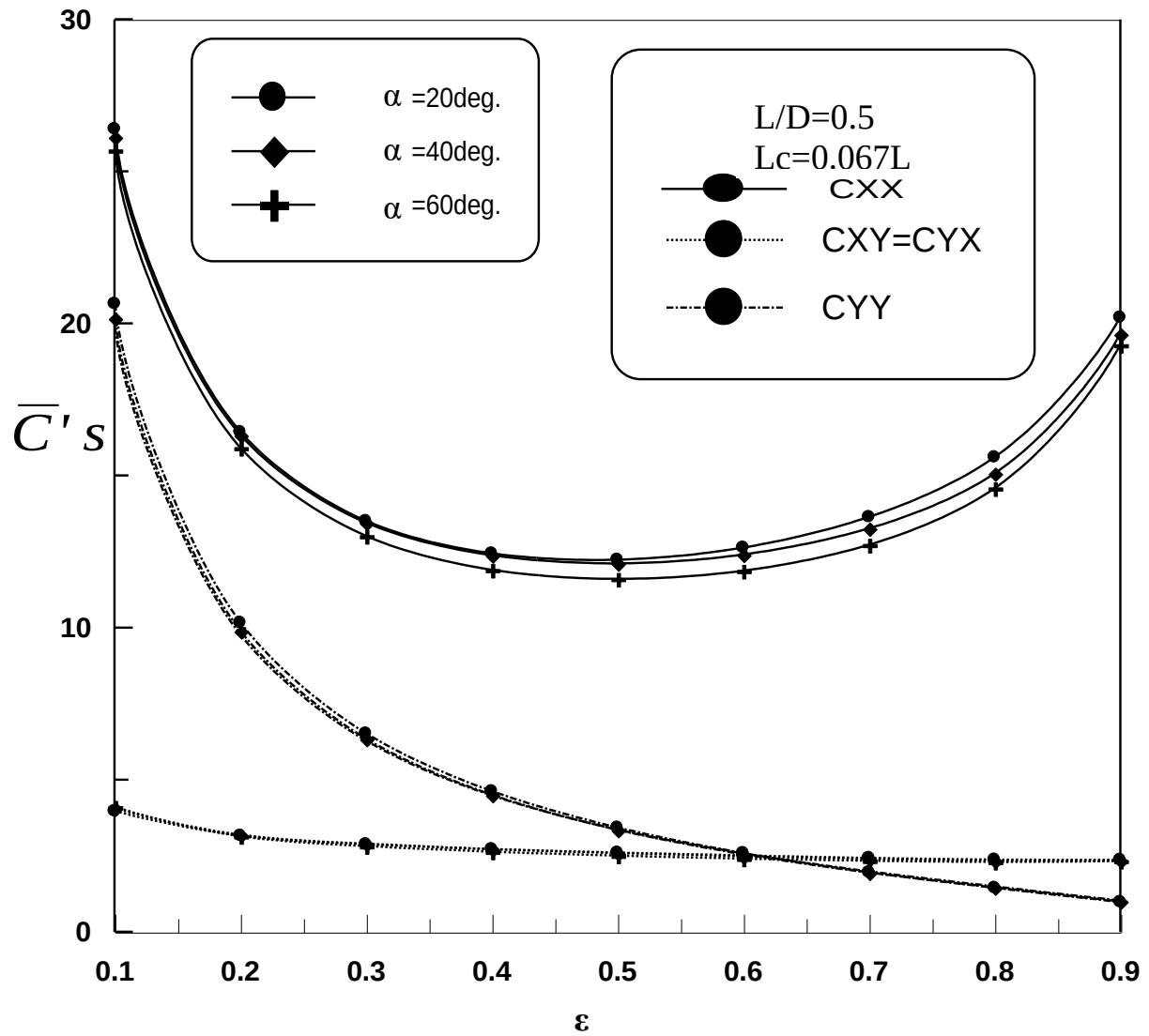


Figure (5-16), Damping Coefficients of Chamfered Bearing for Different Chamfer Angle .

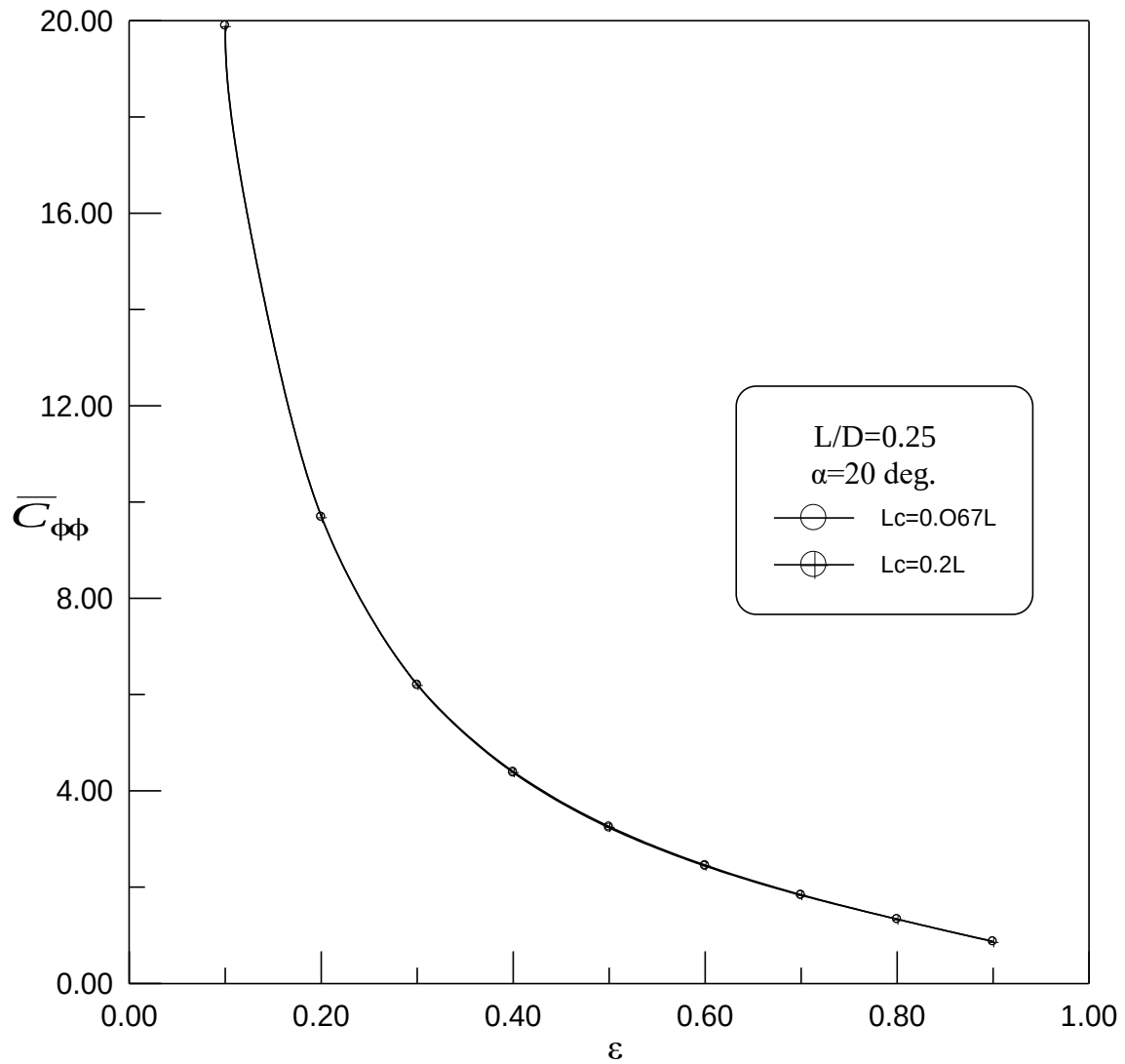


Figure (5-17), Effect of Chamfer length on Direct Damping Coefficient for the Chamfer bearing .

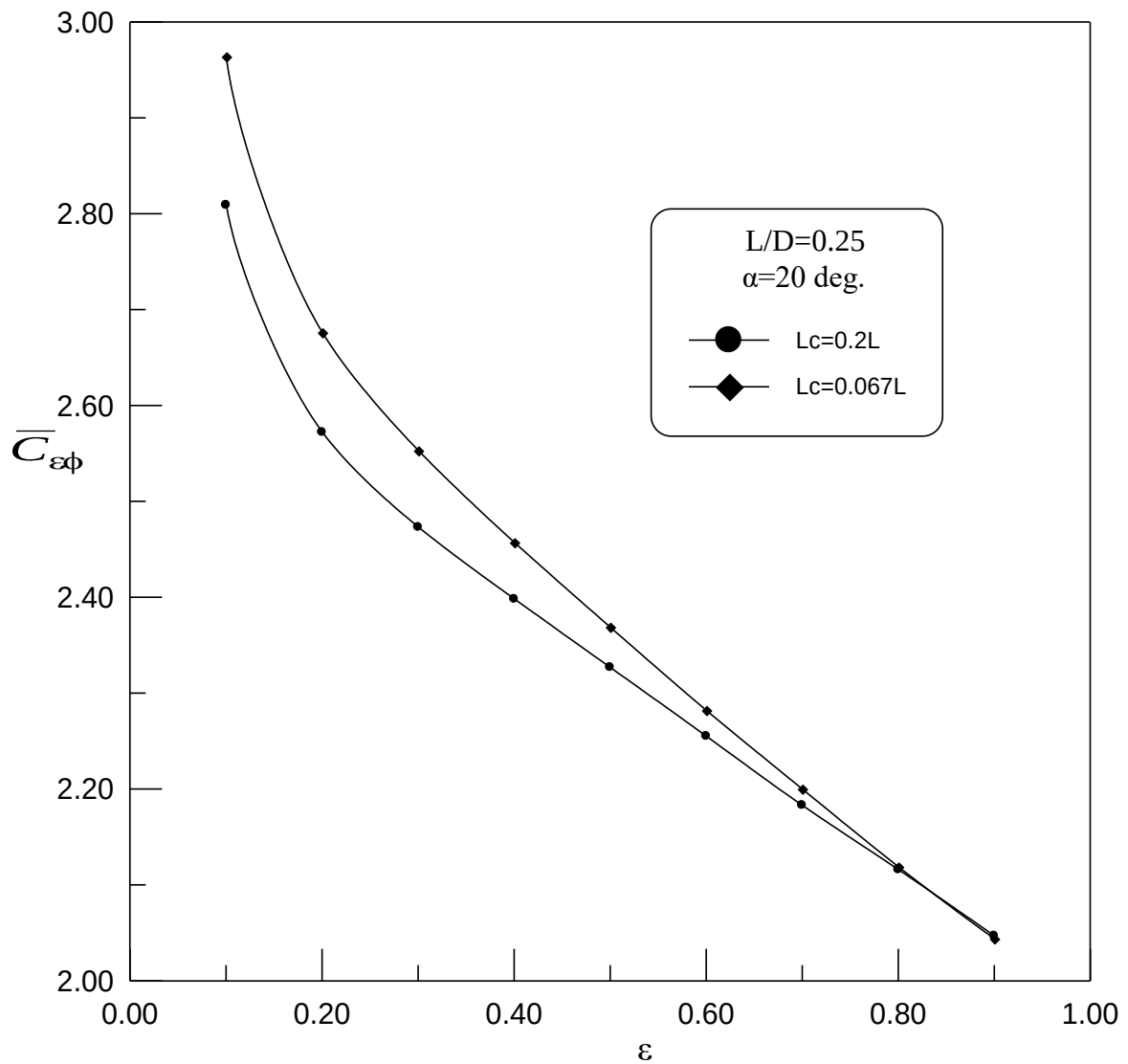


Figure (5-18), Effect of Chamfer length on cross coupling Damping Coefficient.

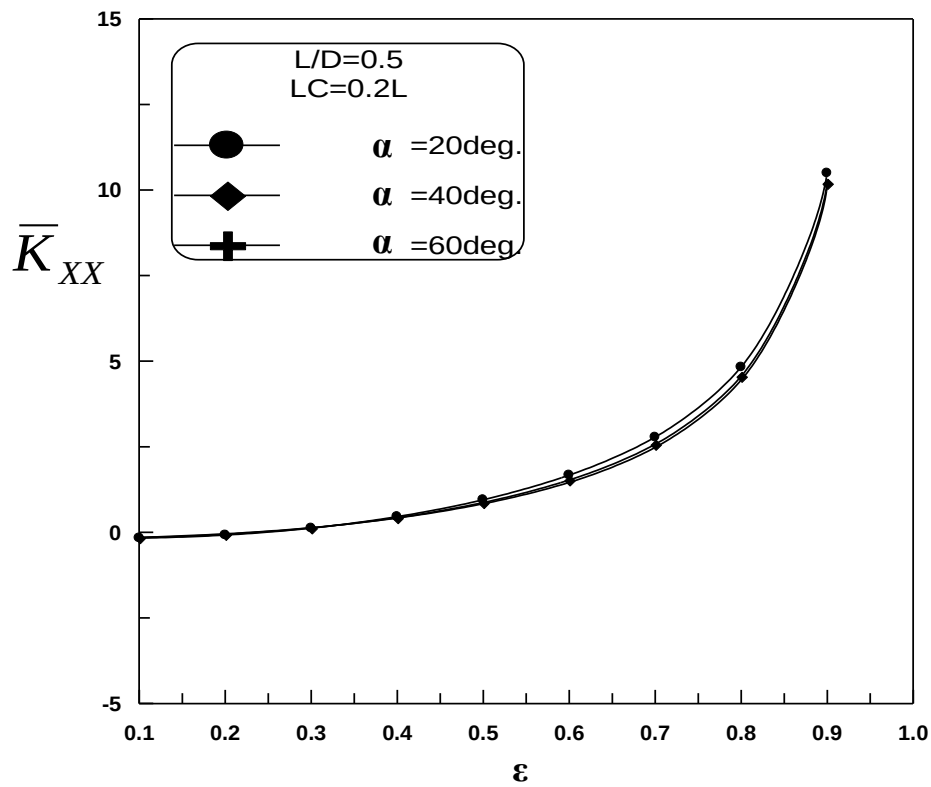


Figure (5-19),Effect of Chamfer Angle on Direct stiffness Coefficient.

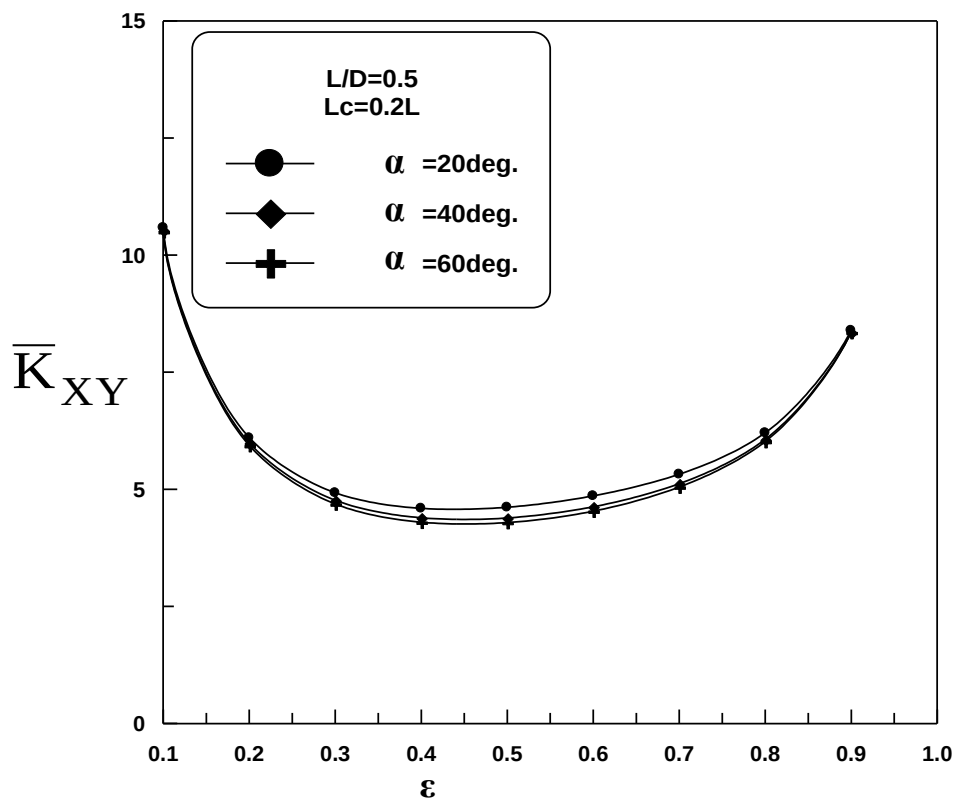


Figure (5-20), Effect of Chamfer Angle on Cross- Coupled stiffness Coefficient.

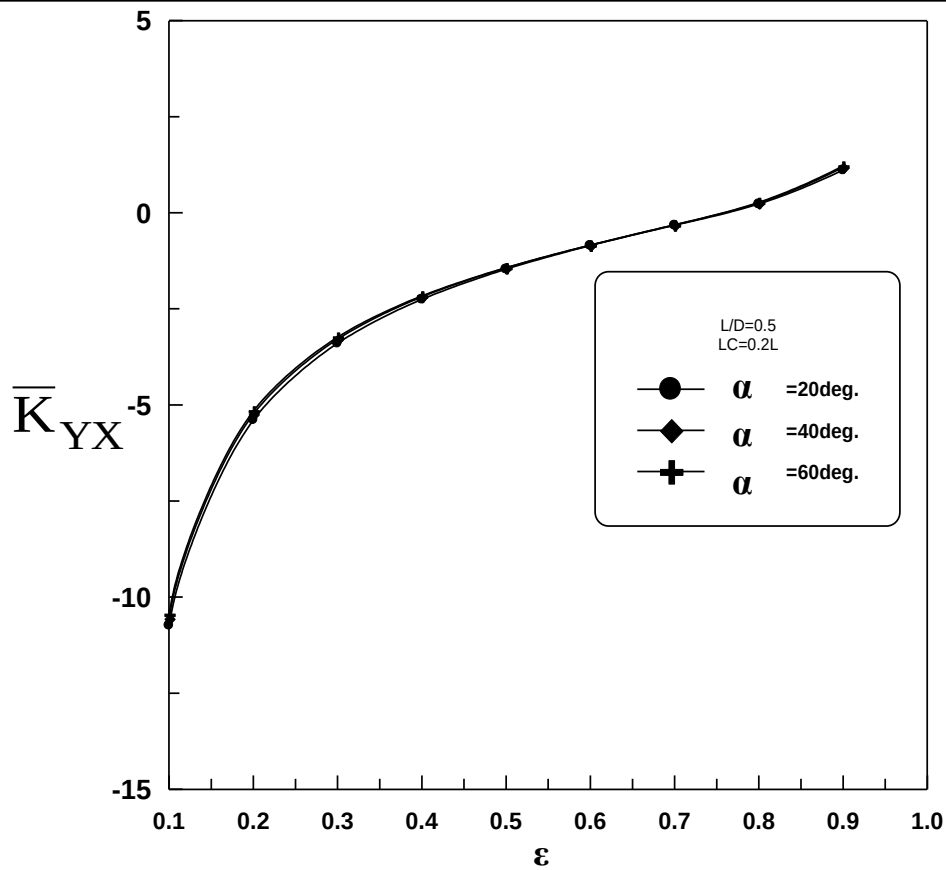


Figure (5-21), Effect of Chamfer Angle on Cross- Coupled stiffness Coefficient.

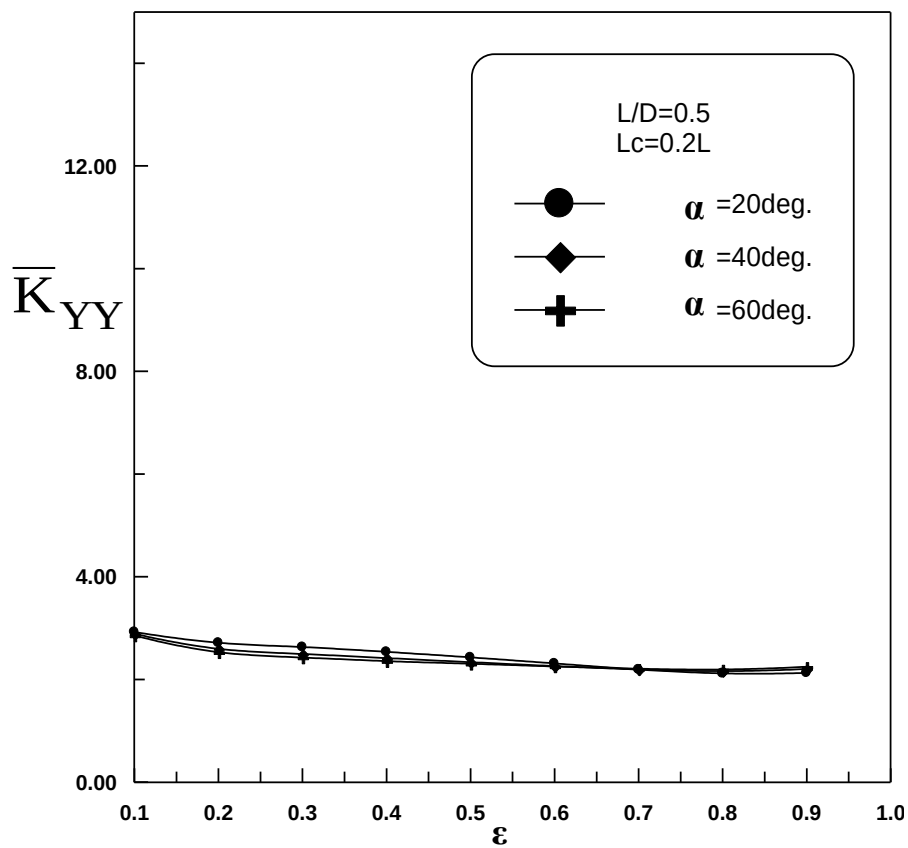


Figure (5-22), Effect of Chamfer Angle on Direct stiffness Coefficient.

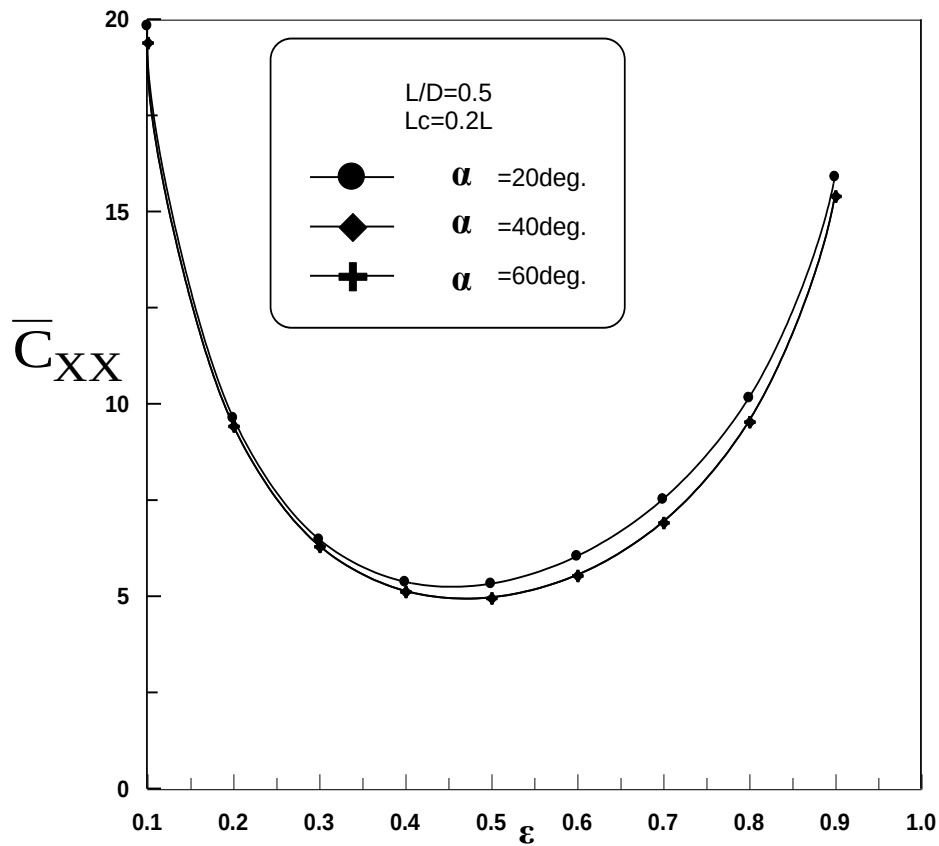


Figure (5-23), Effect of Chamfer Angle on Direct Damping Coefficient.

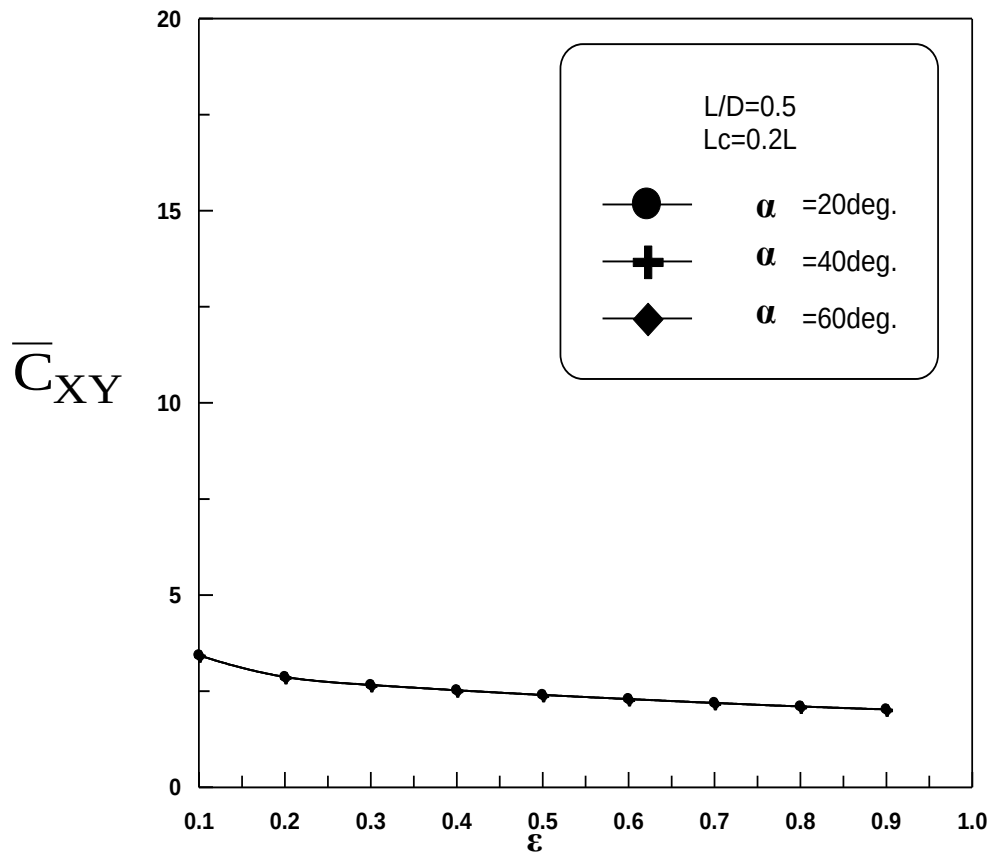


Figure (5-24), Effect of Chamfer Angle on Cross- Coupled Damping Coefficient.

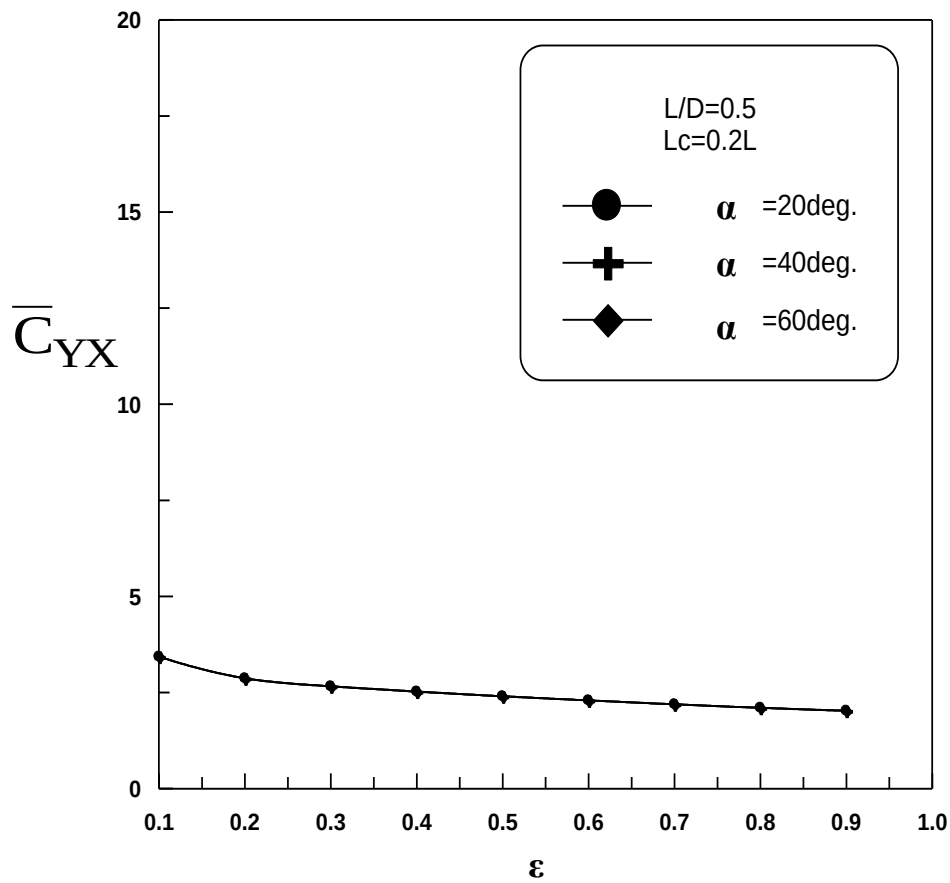


Figure (5-25), Effect of Chamfer Angle on Cross- Coupled Damping Coefficient.

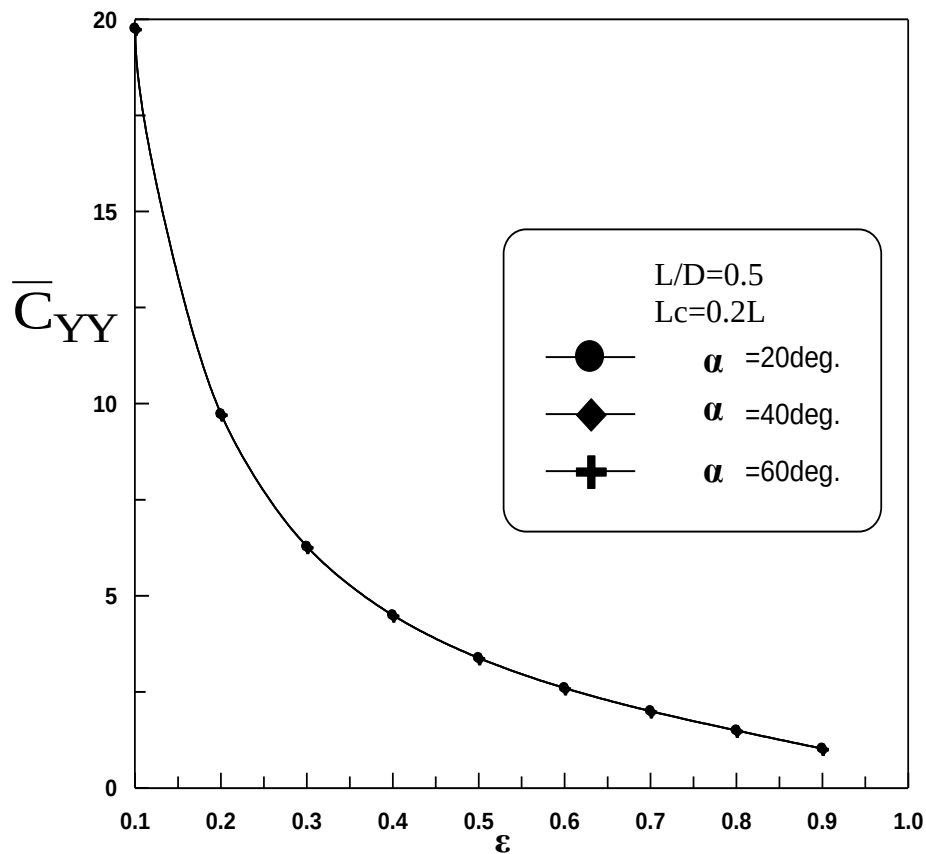


Figure (5-26), Effect of Chamfer Angle on Direct Damping Coefficient.

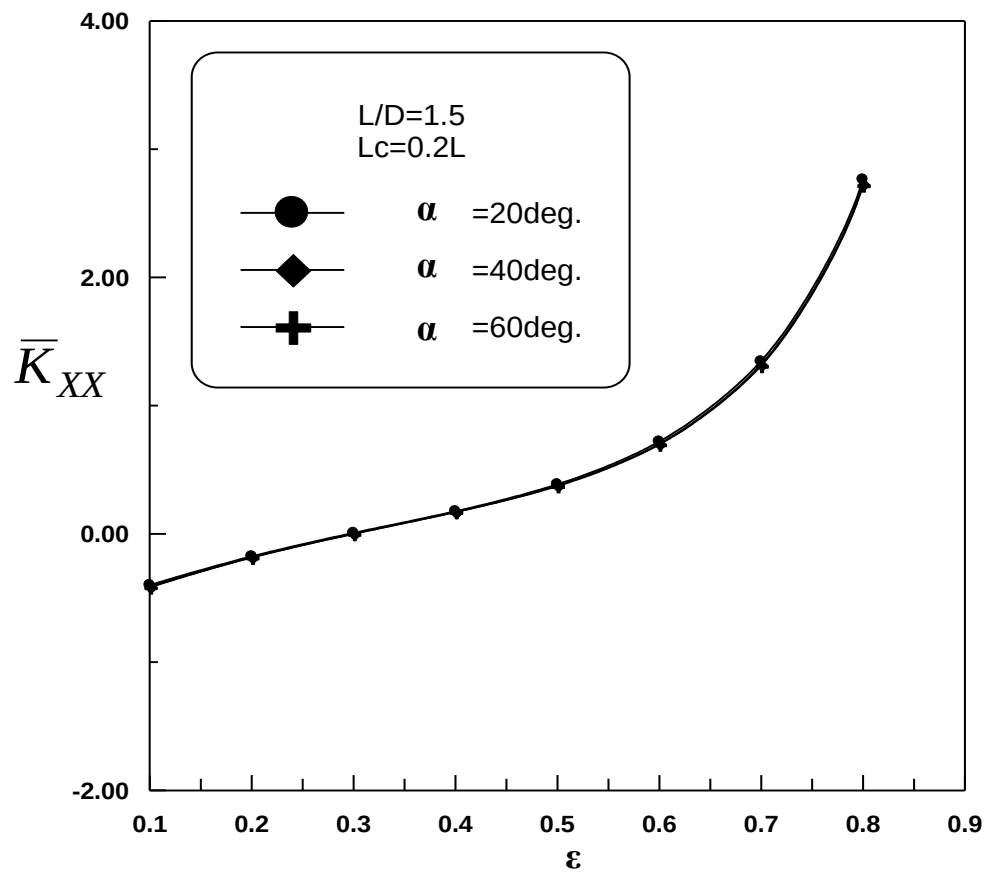


Figure (5-27), Effect of Chamfer Angle on Direct stiffness Coefficient for Different (L/D).

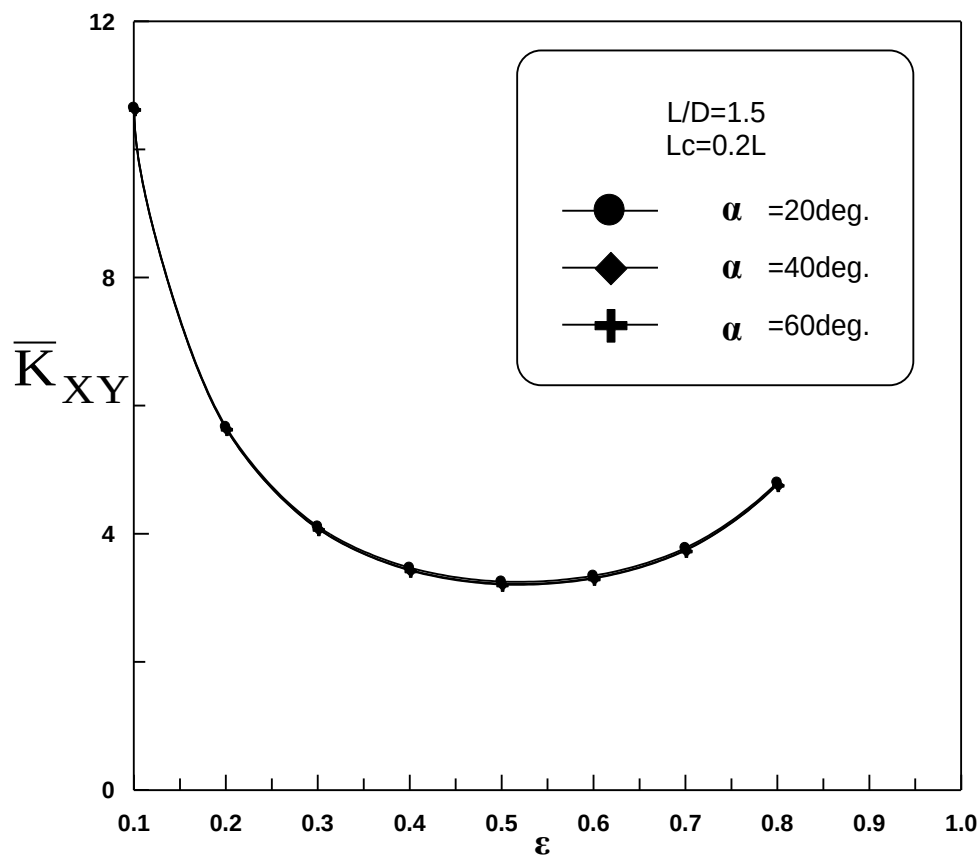


Figure (5-28), Effect of Chamfer Angle on Cross- Coupled stiffness Coefficient for Different (L/D).

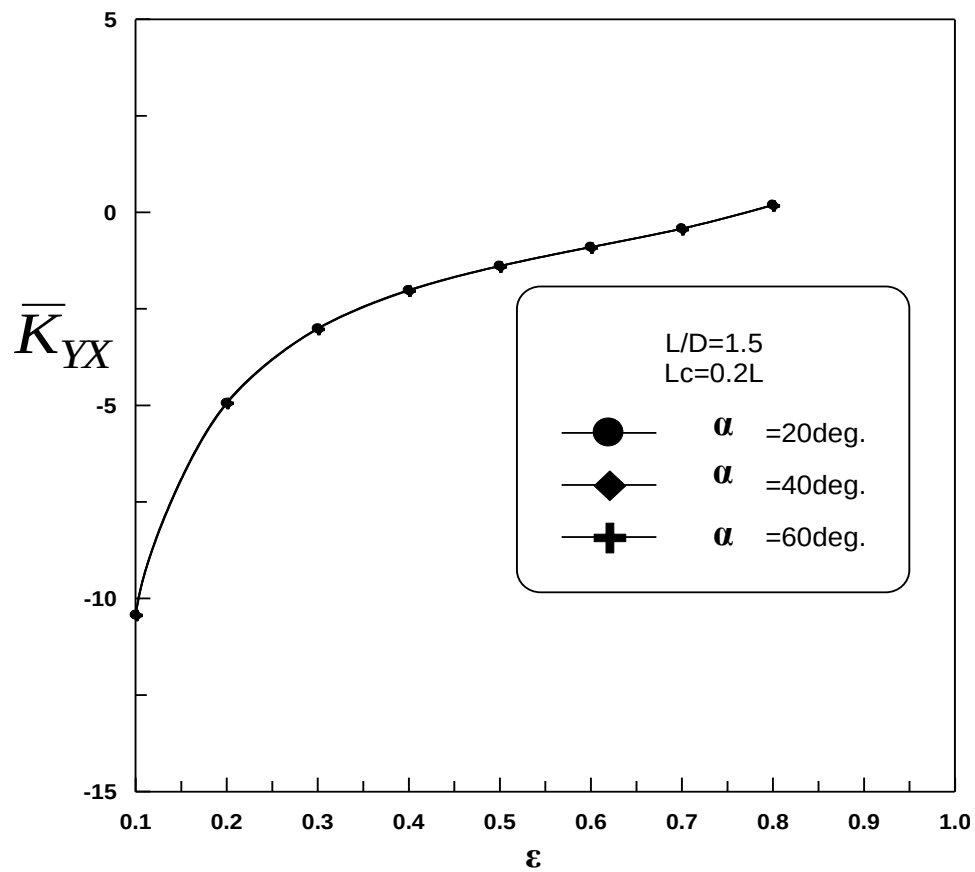


Figure (5-29), Effect of Chamfer Angle on Cross- Coupled stiffness Coefficient for Different (L/D).

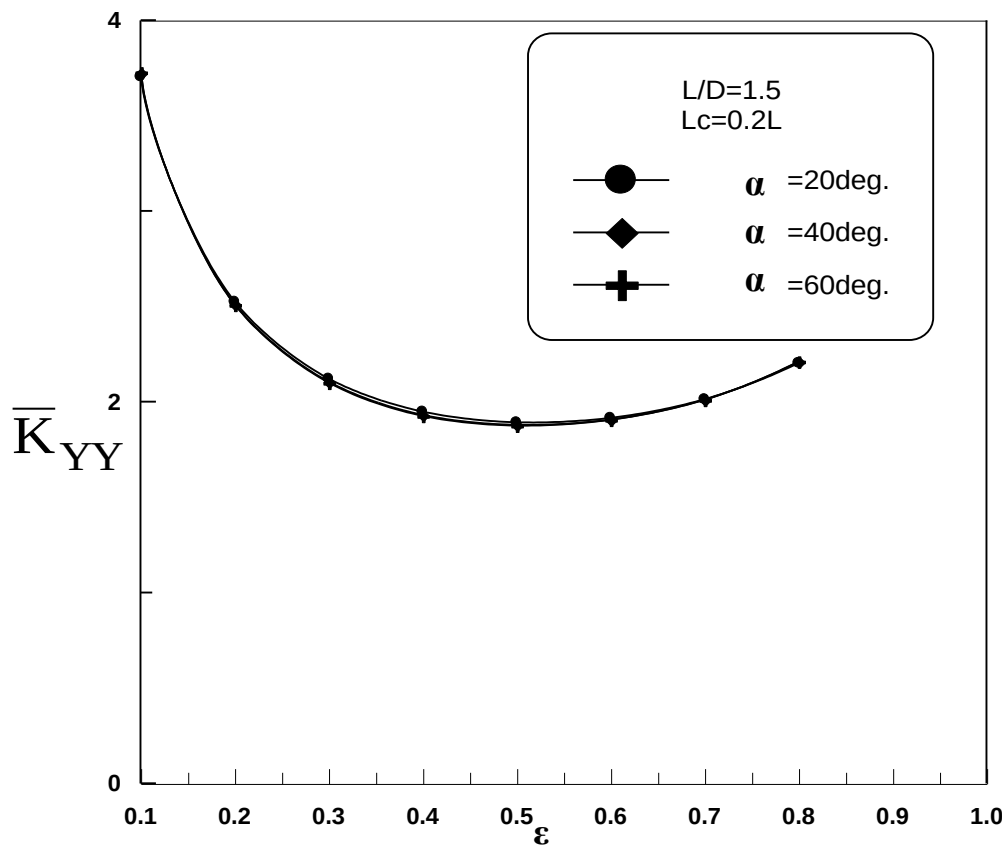


Figure (5-30), Effect of Chamfer Angle on direct stiffness Coefficient for Different (L/D).

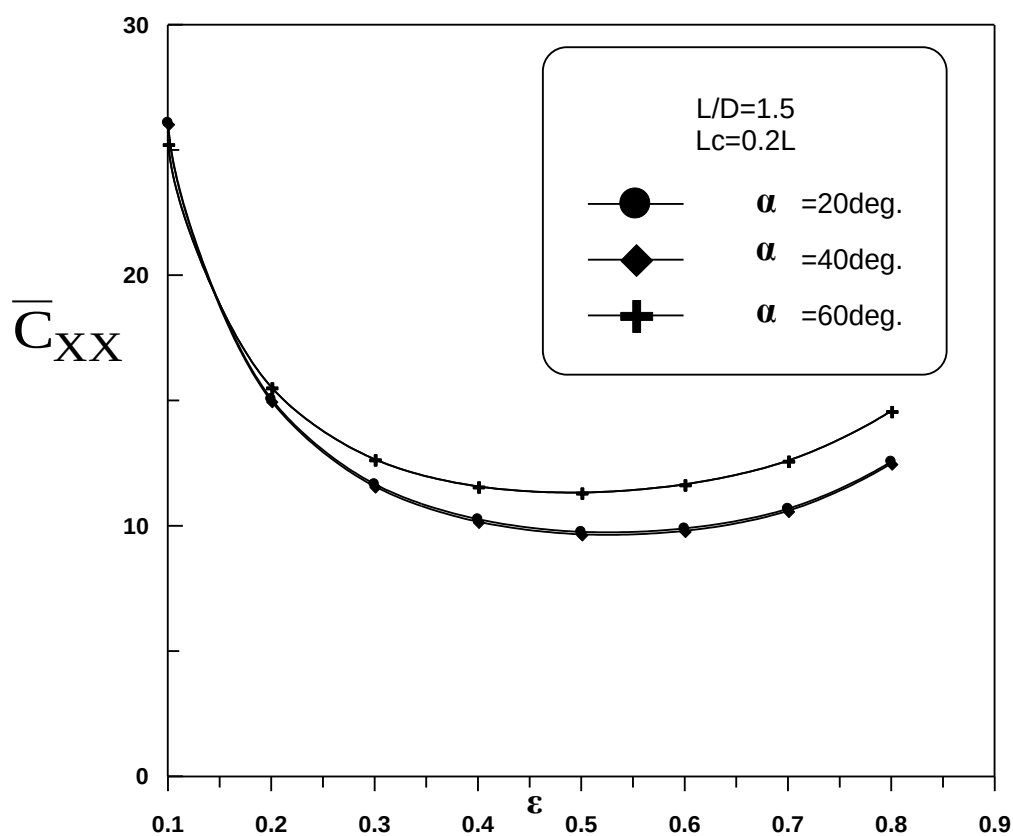


Figure (5-31), Effect of Chamfer Angle on Direct Damping Coefficient for Different (L/D).

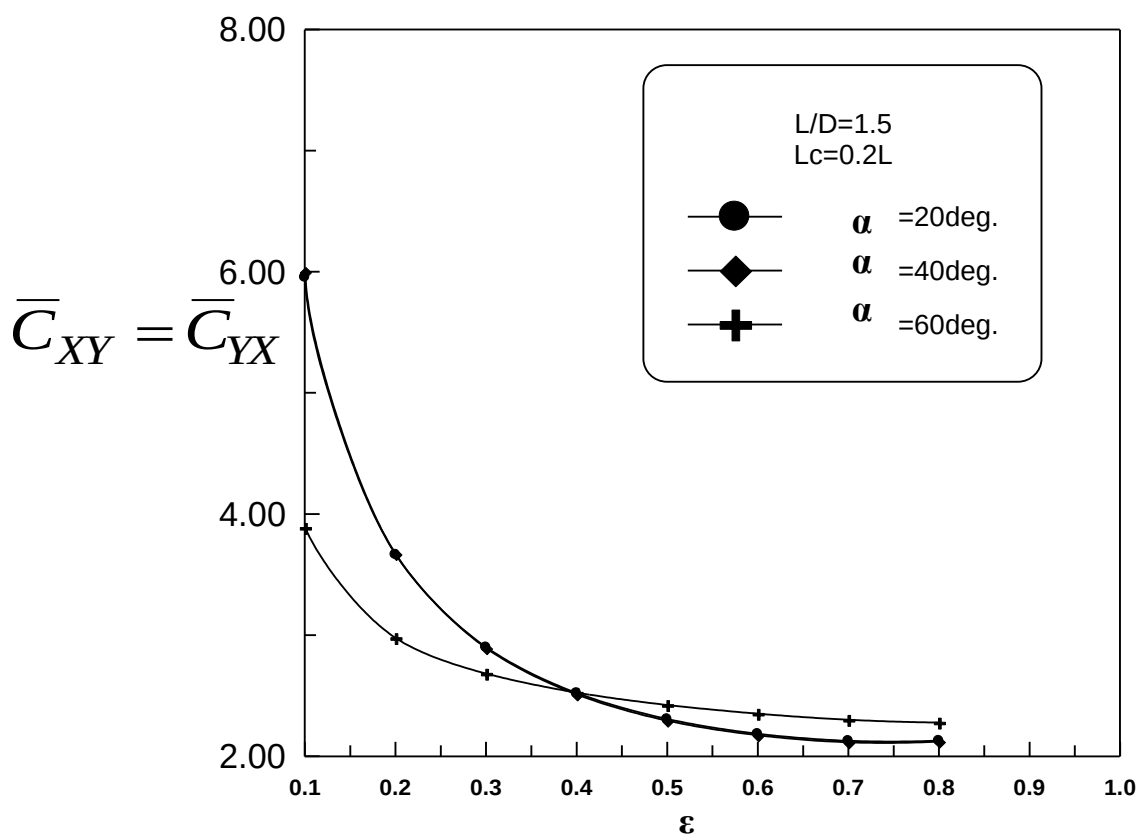


Figure (5-32), Effect of Chamfer Angle on Cross- Coupled Damping Coefficient for Different (L/D).

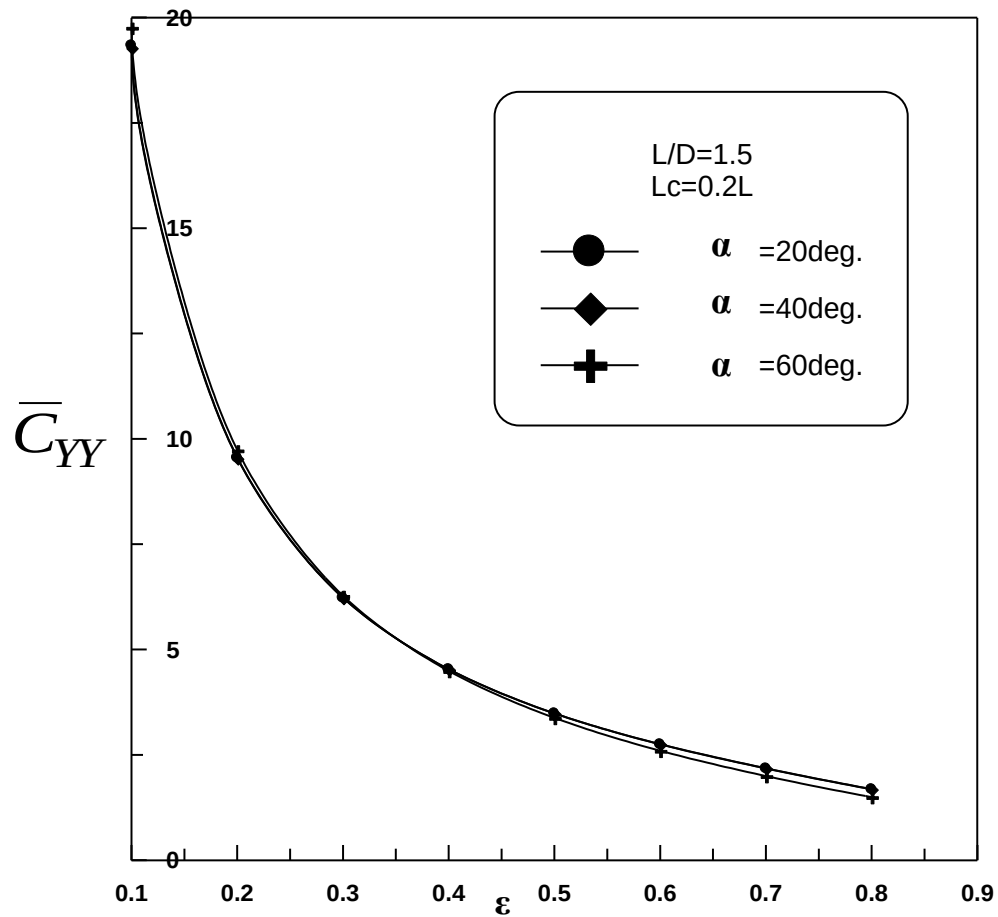


Figure (5-33), Effect of Chamfer Angle on direct damping coefficient for Different (L/D).

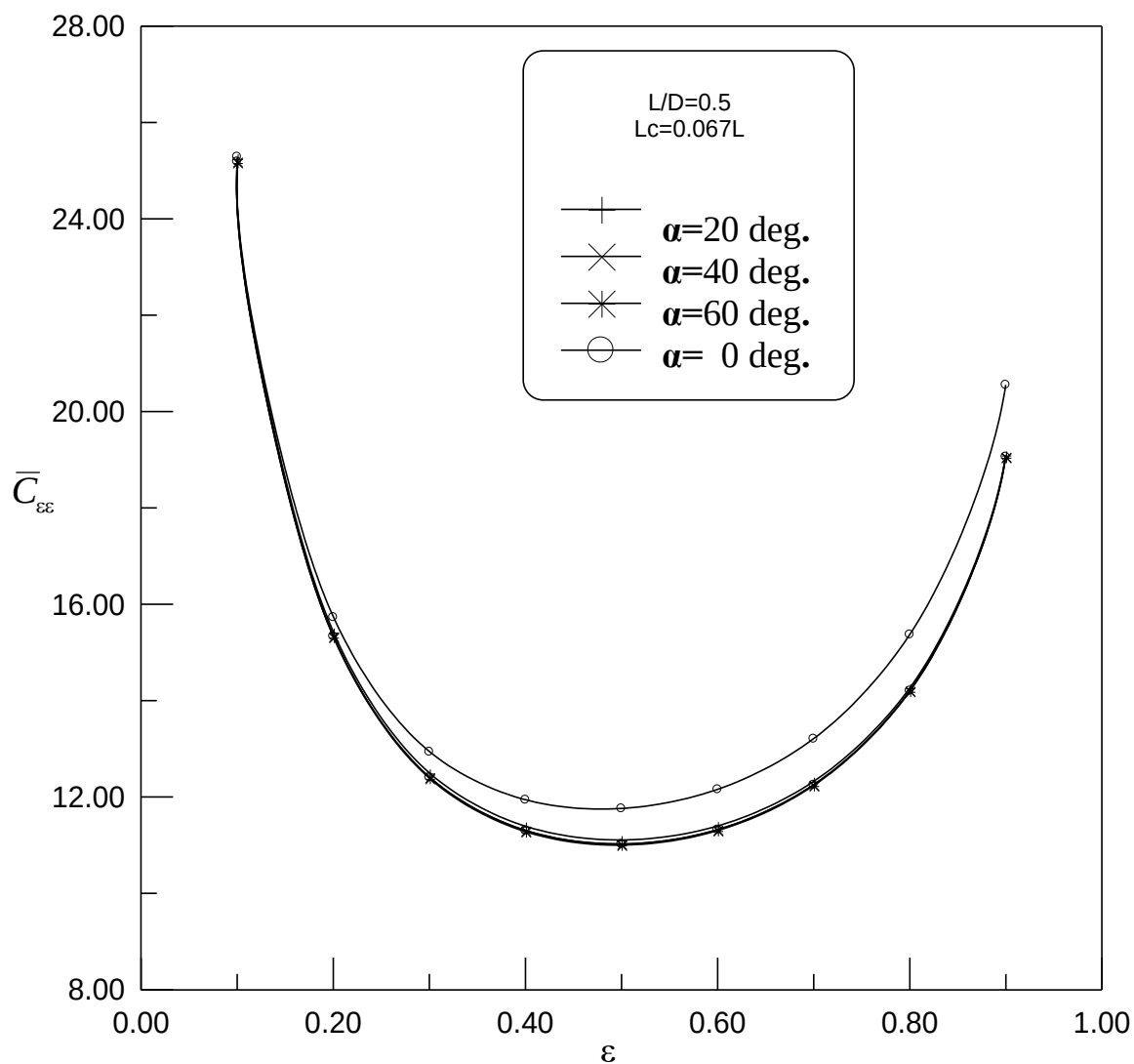


Figure (5-34),Computed Results in Case of Chamfer Addition for Dimensionless Direct Damping Coefficient Versus Eccentricity Ratio .

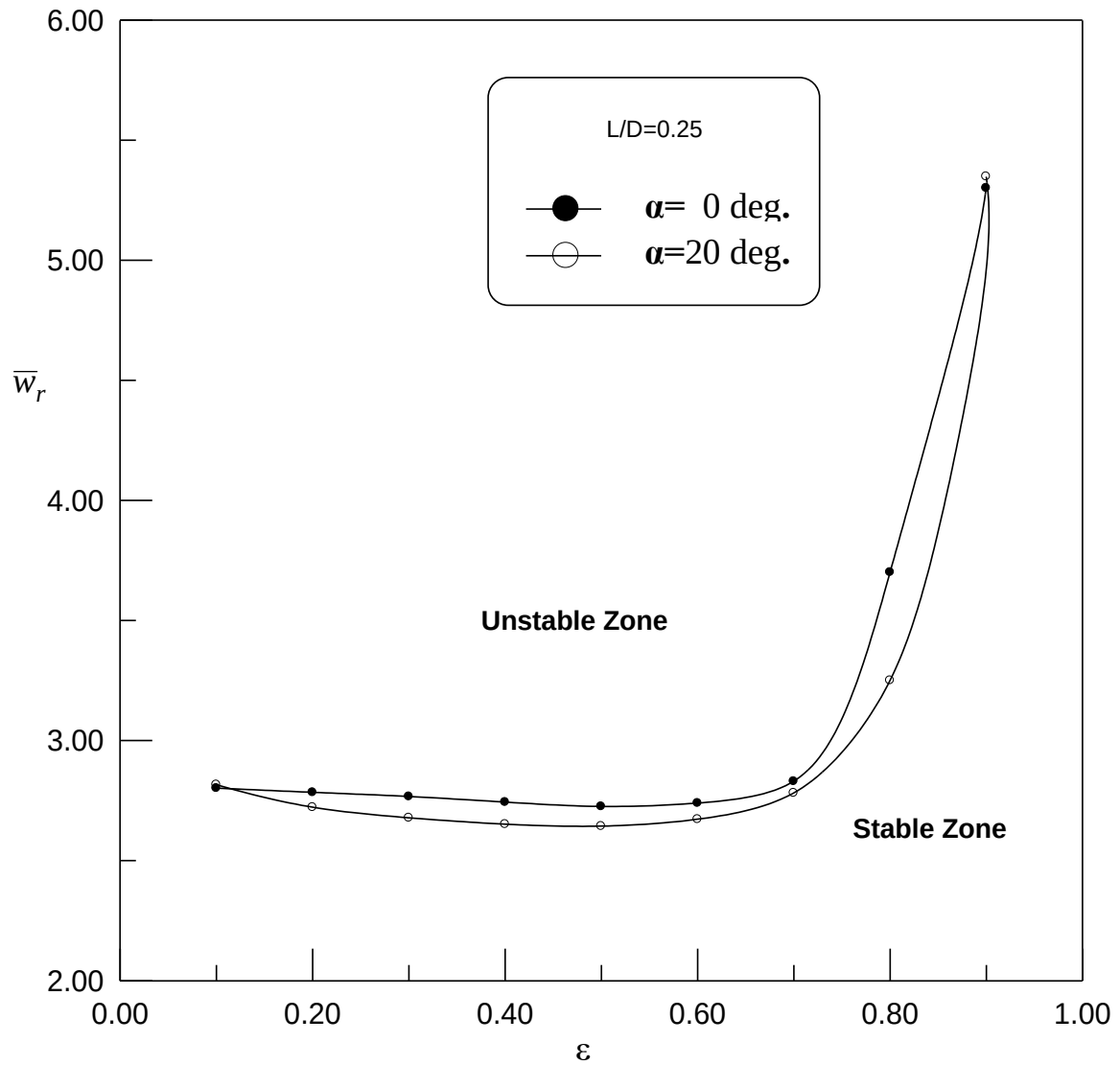


Figure (5-35),Effect of Chamfer Addition of Journal Bearing on Critical Speed versus Eccentricity ratio .

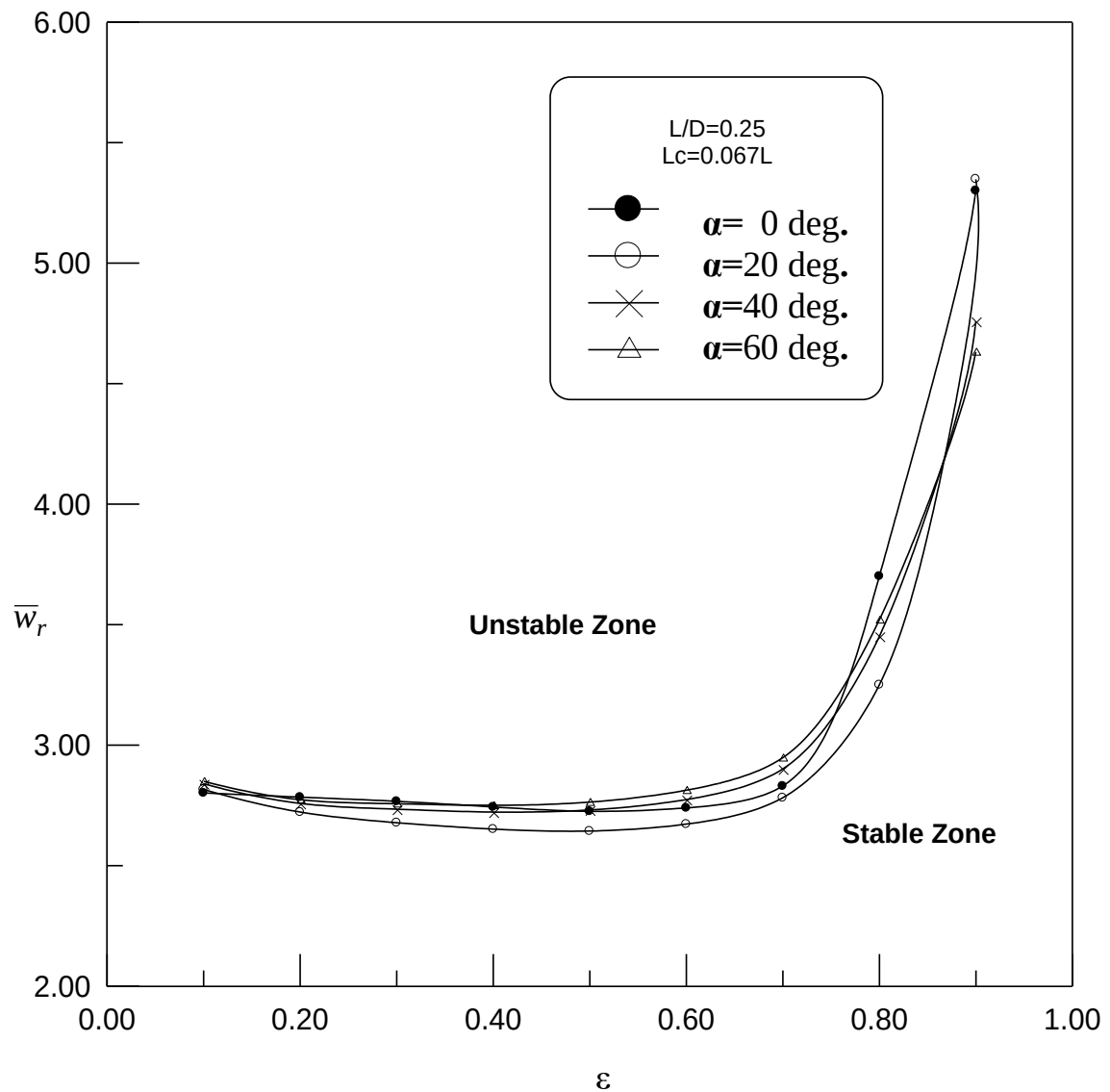


Figure (5-36), Effect of Chamfer angle on the Critical Speed (Threshold speed) at the bearing.

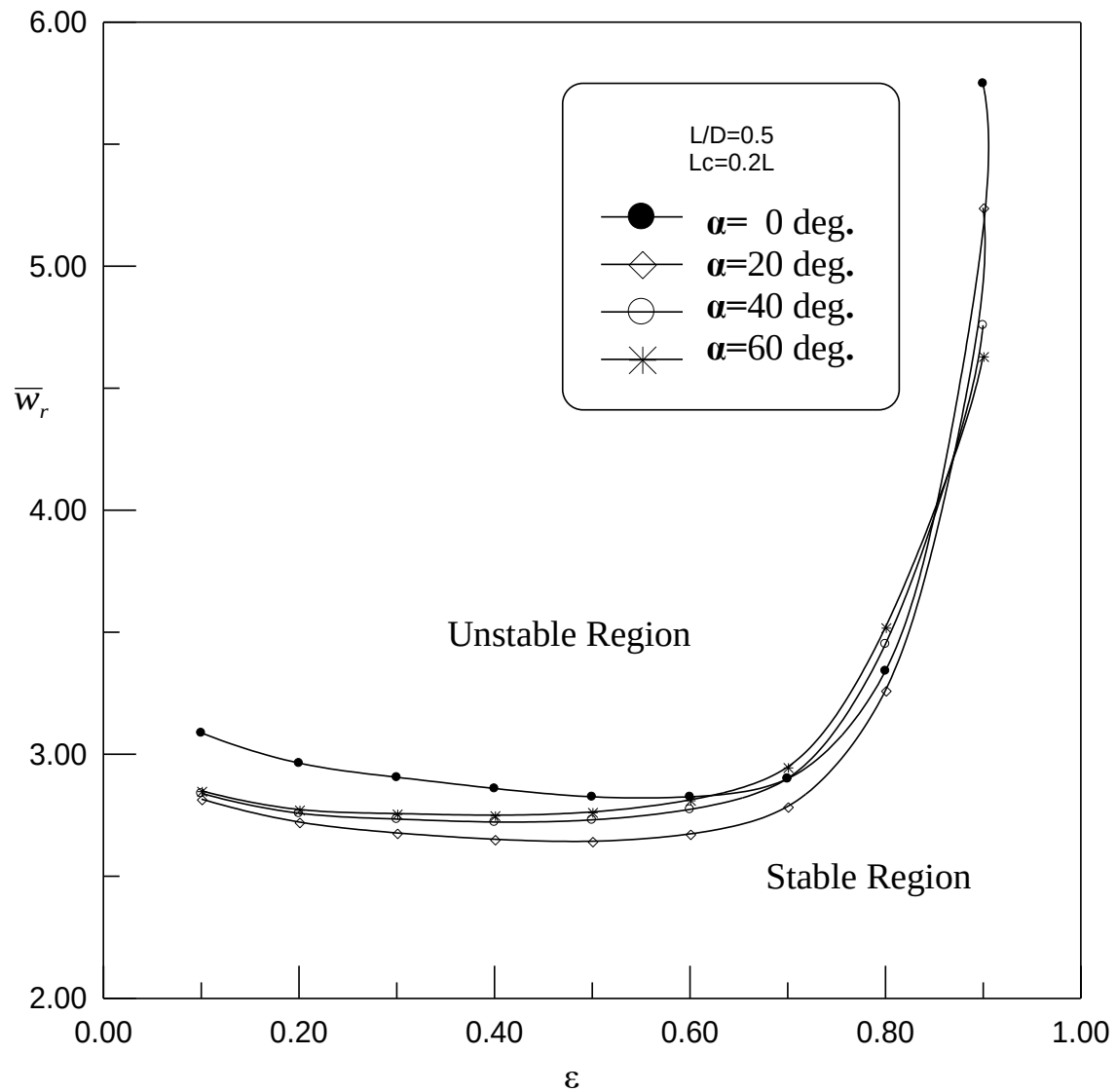
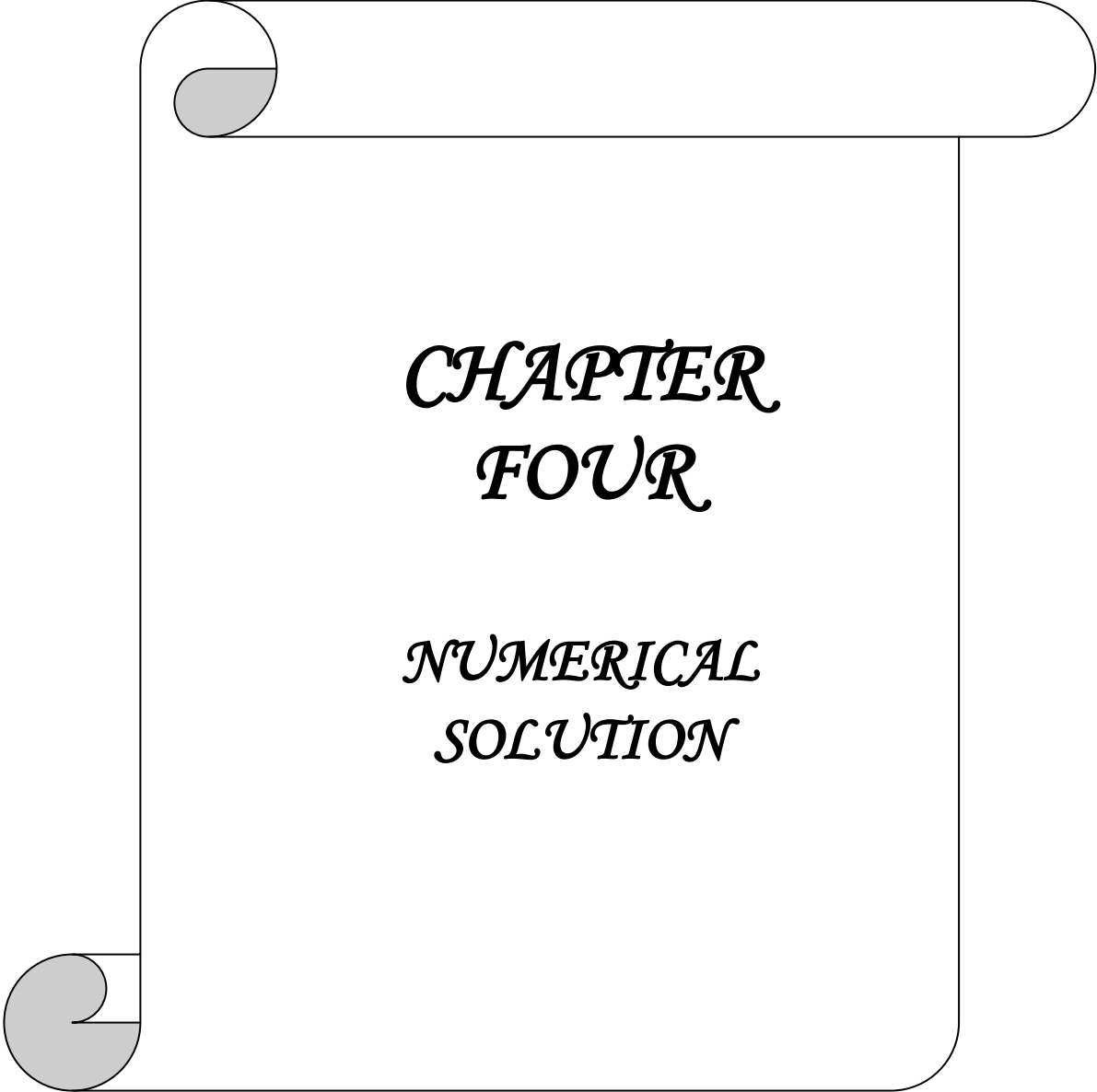


Figure (5-37), Effect of Chamfer angle and length on the Critical Speed (Threshold speed) at the bearing.



CHAPTER FOUR

NUMERICAL SOLUTION

Chapter Four

“Numerical Solution”

4.1.Introduction

The governing equations mentioned previously in last chapter are solved numerically using the conventional finite difference method. This technique was presented for transient state and isothermal conditions.

This chapter consists of constructing the grid generation and numerical solution of the governing partial differential equations. The algorithm of the solution and the computer program which prepared to solve the problem are also described later :

4.2.1.Pertubation Symbols

Solution of the differential equations are greatly simplified by small constructed grid . The perturbed oil film pressures distribution (P_0) (zeroth-order or equilibrium pressures distribution) with (e_{x0}, e_{y0}) at steady operating conditions and P_1 the first-order or perturbed dynamic pressure field can be obtained by simultaneously solving equations (3.1).

4.2.2.Pertubation Technique

The dynamic characteristics of the journal bearings can be described by eight coefficients ,four stiffness and four damping coefficients. In order to compute these coefficients the Reynolds equation (3.1) was perturbed as shown in appendix D.

4.3. Mesh Generation

Reynolds equation was discretized respectively with the mesh size of (N_1) in the circumferential direction, (N_2) in the axial direction and (N_c) in the chamfer zone.

The division are as follow:

The interval $0 < \theta < 2\pi$, is divided into circumferentially (100) equal intervals, (N_1).

The interval $-(L-L_c)/2 < Z < (L-L_c)/2$, is divided axially into (20) equal intervals, (N_2).

The final interval $0 < Z < L_c/2.0$ at both sides of the bearing is divided axially into (6) equal intervals, (N_c). It is well known that the number of nodes is equal to the division plus one.

$$\text{nodes} = \text{divisions} + 1$$

The circumferential mesh size can be defined as :-

$$\Delta\theta = (2\pi / N_1) \quad (4.1)$$

The axial mesh size for no chamfer journal bearing zone is defined as :-

$$\Delta Z = L / N_2 \quad (4.2)$$

Which can be normalized as follows:-

$$\bar{\Delta Z} = \Delta Z / L \quad (4.3)$$

This mesh size is considered in the computer program to solve the Reynolds equation by using the common finite difference method to get the rotordynamic coefficients for the different bearing length to diameters ($L/D=0.25, 0.5, 1.5$).

Mesh size in the axial direction for chamfered journal bearing is calculated as:-

$$\Delta Z_c = L_c / N_c \quad (4.4)$$

The axial mesh size for the rest length is taken as :-

$$\Delta Z = (L - L_c) / N_L \quad (4.5)$$

Where; $N_L = N_2 - N_c$

Which can be normalized as follows:

$$\bar{\Delta Z} = \Delta Z / (L - L_c) \quad (4.6)$$

Figure(4-1) show the finite differences grids schematically.

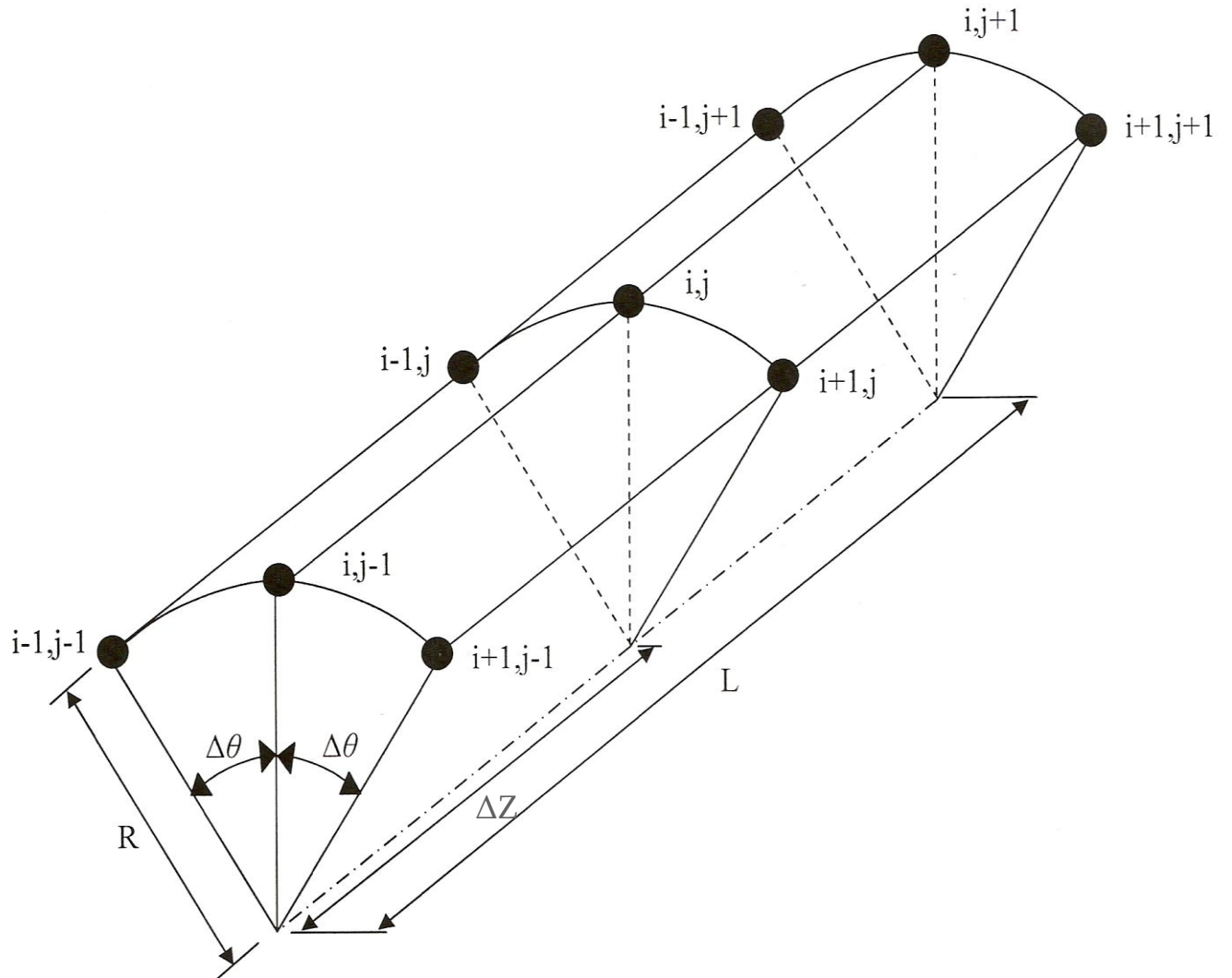


Figure (4-1), Mesh Generation and Finite Differences Grid of the Oil Film.

4.4. Discretized Form of the Governing Equation

The numerical technique used for solving the differential equations is based on replacing these by algebraic equations.

In the case of popular finite difference method, this is done by replacing the derivatives by differences so that the governing equation may be written in discrete form.

The oil film pressure of the journal bearing can be evaluated from the Reynolds equation which can be rewritten as follows;

$$\frac{\partial}{\partial \theta} \left[\frac{h^3}{12\mu R^2} \frac{\partial P}{\partial \theta} \right] + \frac{\partial}{\partial z} \left[\frac{h^3}{12\mu} \frac{\partial P}{\partial z} \right] = \frac{\omega}{2} \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t} \quad (4.7)$$

Where;

(t) is time.

(μ) is lubricant viscosity.

(ω) is the journal angular speed of rotation.

(h) is the oil film thickness.

The oil film thickness can be evaluated as;

$$h = C + X \cos \Theta + Y \sin \Theta \quad (4.8)$$

Where;

(C) is the radial clearness.

(X,Y) are the coordinates of the journal center.

The finite difference analog to Reynolds' equation can be written as ,[30,36,37] :-

$$\begin{aligned}
 & \frac{1}{12R^2} \frac{1}{\mu\Delta\theta^2} [h_{i+0.5,j}^3 P_{i+1,j} + h_{i-0.5,j}^3 P_{i-1,j}] + \frac{1}{12\mu\Delta Z^2} [h_{i,j+0.5}^3 P_{i,j+1} + h_{i,j-0.5}^3 P_{i,j-1}] \\
 & - \left[\frac{1}{12R^2} \frac{1}{\mu\Delta\theta^2} [h_{i+0.5,j}^3 + h_{i-0.5,j}^3] + \frac{1}{12\mu\Delta Z^2} [h_{i,j+0.5}^3 + h_{i,j-0.5}^3] \right] P_{i,j} \\
 & = \frac{\omega}{2} \left[\frac{h_{i+0.5,j} - h_{i-0.5,j}}{\Delta\theta} \right]
 \end{aligned} \tag{4.9}$$

Simplify equation (4.9) to get:-

$$P_{i,j} = \left[\frac{A_{1i,j} P_{i+1,j} + A_{2i,j} P_{i-1,j} + A_{3i,j} P_{i,j+1} + A_{4i,j} P_{i,j-1} - A_{5i,j}}{A_{6i,j}} \right] \tag{4.10}$$

Where;

$$A_{1i,j} = \left(\frac{h_{i+0.5,j}^3}{12R^2 \mu \Delta\theta^2} \right) \tag{4.11}$$

$$A_{2i,j} = \left(\frac{h_{i-0.5,j}^3}{12R^2 \mu \Delta\theta^2} \right) \tag{4.12}$$

$$A_{3i,j} = \left(\frac{h_{i,j+0.5}^3}{12R^2 \mu \Delta Z^2} \right) \tag{4.13}$$

$$A_{4i,j} = \left(\frac{h_{i,j-0.5}^3}{12R^2 \mu \Delta Z^2} \right) \tag{4.14}$$

$$A_{5i,j} = \frac{\omega}{2} \left(\frac{h_{i+0.5,j} - h_{i-0.5,j}}{\Delta\theta} \right) \tag{4.15}$$

$$A_{6i,j} = A_{1i,j} + A_{2i,j} + A_{3i,j} + A_{4i,j} \tag{4.16}$$

The film thickness with chamfer end can be represented by[1] as;

$$h_{i,j} = c(1 + \varepsilon \cos \theta_i) + Zc_j \tan \alpha \quad (4.17)$$

$$h_{i+0.5,j} = c(1 + \varepsilon \cos \theta_{i+0.5}) + Zc_j \tan \alpha \quad (4.18)$$

$$h_{i-0.5,j} = c(1 + \varepsilon \cos \theta_{i-0.5}) + Zc_j \tan \alpha \quad (4.19)$$

$$h_{i,j+0.5} = c(1 + \varepsilon \cos \theta_i) + Zc_{j+0.5} \tan \alpha \quad (4.20)$$

$$h_{i,j-0.5} = c(1 + \varepsilon \cos \theta_i) + Zc_{j-0.5} \tan \alpha \quad (4.21)$$

The angular position of the bearing can be represented as;

$$\theta_i = (i-1)\Delta\theta \quad (4.22)$$

$$\theta_{i+0.5} = (i-0.5)\Delta\theta \quad (4.23)$$

$$\theta_{i-0.5} = (i-1.5)\Delta\theta \quad (4.24)$$

The coordinates of the chamfer end in the both sides of the bearing can be declared as;

-In the left end (its length $Lc/2$);

$$Zc_j = \frac{Lc}{2} - (j-1)\Delta Zc \quad (4.25)$$

$$Zc_{j+0.5} = \frac{Lc}{2} - (j-0.5)\Delta Zc \quad (4.26)$$

$$Zc_{j-0.5} = \frac{Lc}{2} - (j-1.5)\Delta Zc \quad (4.27)$$

-In the right end and its length ($Lc/2$);

$$Zc_j = (L - \frac{Lc}{2}) - (j-1)\Delta Zc \quad (4.28)$$

$$Zc_{j+0.5} = (L - \frac{Lc}{2}) - (j-0.5)\Delta Zc \quad (4.29)$$

$$Zc_{j-0.5} = (L - \frac{Lc}{2}) - (j-1.5)\Delta Zc \quad (4.30)$$

After all the parameter are evaluated then the Reynolds equation (4.10) can be solved iteratively for the pressure distribution at each node ($P_{i,j}$) using the Guass –Siedel iterative method with successive over relaxation scheme. Over relaxation factor of (1.98) was used to accelerate the rate of convergence through the iterative process .It is the most widely used iterative method for solving the equation like the Reynolds equation for incompressible fluid . The procedure of solution has been shown in appendix C.

The load carrying capacity can be calculated from the pressure distribution over the bearing surface. Using equation (3.9) and (3.10),[47,36]as follows;

$$F_{\varepsilon} = \sum_{j=1}^{N2-1} \sum_{i=1}^{N1-1} \left[\frac{P_{i-1,j-1} + P_{i,j-1} + P_{i-1,j} + P_{i,j}}{4} \right] \cos \theta_{i-0.5} R \Delta \theta \Delta Z \quad (4.31)$$

$$F_{\phi} = \sum_{j=1}^{N2-1} \sum_{i=1}^{N1-1} \left[\frac{P_{i-1,j-1} + P_{i,j-1} + P_{i-1,j} + P_{i,j}}{4} \right] \sin \theta_{i-0.5} R \Delta \theta \Delta Z \quad (4.32)$$

Hence the total load carrying capacity can calculated from equation (3.11) as;

$$F = \sqrt{(F_{\varepsilon})^2 + (F_{\phi})^2} \quad (4.33)$$

Consequently ,the attitude can be evaluated from equation (3.12) as;

$$\phi = \tan^{-1} \left[\pm \frac{F_{\phi}}{F_{\varepsilon}} \right] \quad (4.34)$$

The bearing side-leakage flow rate ,given by equation (3.14)can be written as[32,30] :

$$q = \frac{C \omega L R}{24} \left(\frac{D}{L} \right)^2 \sum_{i=1}^{N1} h_{i,j}^3 \frac{(P_{i,j} - P_{i,j+1})}{\Delta Z} \Delta \theta \quad (4.35)$$

From this equation it is clear that the side-leakage flow rate is dependent on the pressure gradient in axial direction at the outermost boundaries of the bearing ($Z=\pm L/2$).

Chamfering effect of the bearing on the dynamic behavior of fluid film bearing can be studied numerically through evaluating eight displacement and velocity coefficients.

As seen in equations (3.18), and (3.19), K_{xx}, K_{xy}, K_{yx} and K_{yy} are the stiffness (displacement) coefficients, whilst C_{xx}, C_{xy}, C_{yx} and C_{yy} are damping (velocity) coefficients respectively.

To employ these coefficients the perturbation analysis must be used as follows;

4.5. Perturbation Analysis

Consider small amplitude of the journal center motion about a static equilibrium position (e_{x_0}, e_{y_0}) , i.e.

$$e_x = e_{x_0} + \Delta x(t) \quad , \quad e_y = e_{y_0} + \Delta y(t) \quad (4.36)$$

Where $\Delta x, \Delta y$ as shown in figure (4-2), in reference [2].

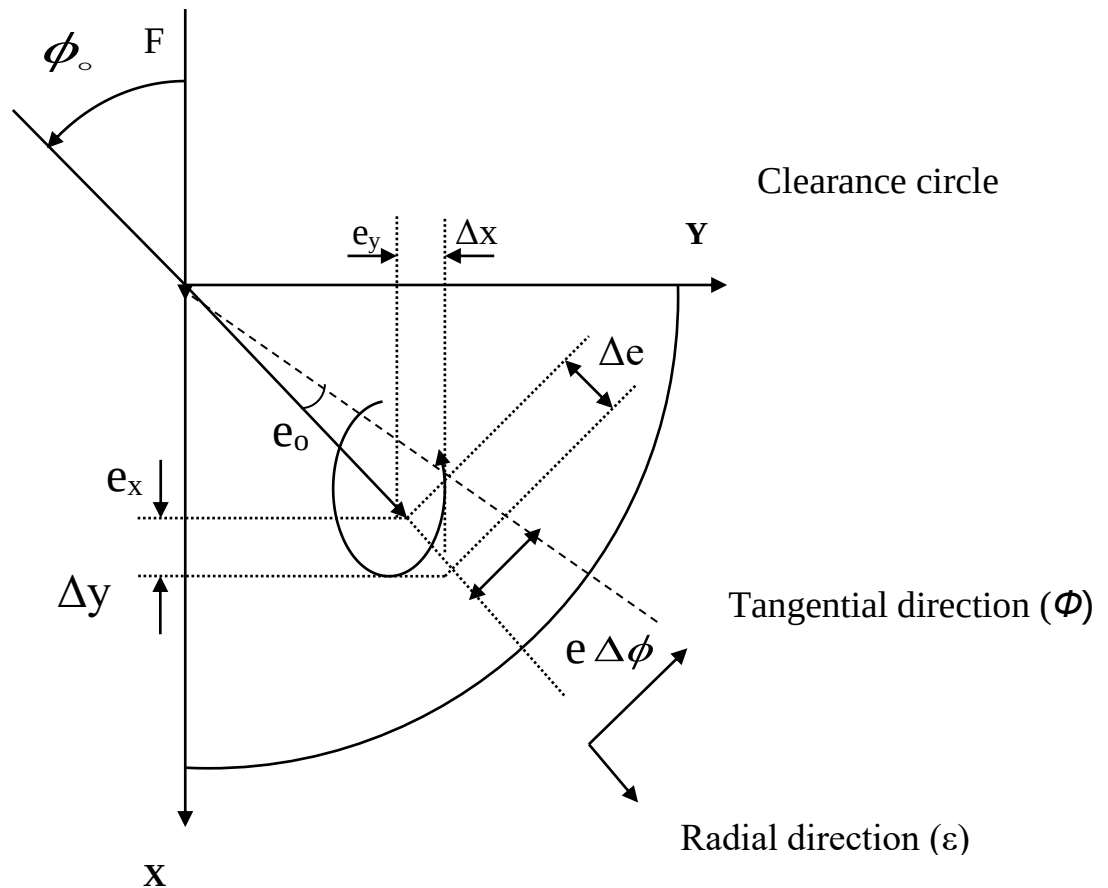


Figure (4-2) Small amplitude journal motions about an equilibrium position

The film thickness is described by the superposition of an equilibrium (zeroth-order) h_0 film thickness and a perturbed (first-order) Δh_1 film thickness,

$$h = h_0 + \Delta h(t) \quad (4.37)$$

Where h_0 in no chamfer (region II) can be written as ;

$$h_0 = C + ex_0 \cos \Theta + ey_0 \sin \Theta \quad (4.38)$$

While h_0 in chamfer (region I) can be written as ;

$$h_0 = C + ex_0 \cos \Theta + ey_0 \sin \Theta + Zc \tan \alpha \quad (4.39)$$

and

$$\Delta h = \Delta x(t) \cos \Theta + \Delta y(t) \sin \Theta \quad (4.40)$$

The fluid film forces are functions of the journal center displacements and velocities;

$$F_\zeta = F_\zeta [e_{x_0}, e_{y_0}, \Delta x, \Delta y, \Delta \dot{x}, \Delta \dot{y}], \zeta = x, y \quad (4.41)$$

A Taylor series expansion of these forces around the static journal position (e_{x_0}, e_{y_0}) leads to (disregarding second order and higher terms):-

$$\begin{aligned} F_x &= F_{x_0} + \frac{\partial F_x}{\partial x} \Delta x + \frac{\partial F_x}{\partial y} \Delta y + \frac{\partial F_x}{\partial \dot{x}} \Delta \dot{x} + \frac{\partial F_x}{\partial \dot{y}} \Delta \dot{y} \\ F_y &= F_{y_0} + \frac{\partial F_y}{\partial x} \Delta x + \frac{\partial F_y}{\partial y} \Delta y + \frac{\partial F_y}{\partial \dot{x}} \Delta \dot{x} + \frac{\partial F_y}{\partial \dot{y}} \Delta \dot{y} \end{aligned} \quad (4.42)$$

The bearing stiffness $\{K_{i,j}\}$ and damping $\{C_{i,j}\}$ coefficients are defined as;

$$K_{i,j} = -\frac{\partial F_i}{\partial x_j}; C_{i,j} = -\frac{\partial F_i}{\partial \dot{x}_j} \quad (4.43)$$

Hence, the hydrodynamic forces in the oil shown in equations (4.42) can be written in matrix form as;

$$\begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix} = \begin{bmatrix} F_{x_0} \\ F_{y_0} \end{bmatrix} - \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} - \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{pmatrix} \Delta \dot{x} \\ \Delta \dot{y} \end{pmatrix} \quad (4.44)$$

Note that at the equilibrium position (e_{xo}, e_{yo}) , the fluid film reaction forces balances the applied static load (W), i.e.;

$$Fx_o = -Wx, Fy_o = -Wy.$$

The perturbation of the film thickness leads naturally to perturbation of the film pressure (P), and defined as in [4];

$$P(\Theta, z, t) = P_o(\Theta, z) + \Delta P_1(\Theta, z, t) \quad (4.45)$$

Where P_o is the zeroth-order or equilibrium pressure field defined by (e_{xo}, e_{yo}) at steady operating conditions, and ΔP_1 is first-order or perturbed dynamic pressure field given by;

$$\Delta P_1 = P_x \Delta x + P_y \Delta y + P_{\dot{x}} \Delta \dot{x} + P_{\dot{y}} \Delta \dot{y} \quad (4.46)$$

Denote the linear differential operator;

$$L[] = \frac{\partial}{R \partial \Theta} \left[\frac{h^3}{12\mu} \frac{\partial}{R \partial \Theta} \right] + \frac{\partial}{\partial z} \left[\frac{h^3}{12\mu} \frac{\partial}{\partial z} \right] [] \quad (4.47)$$

and write the Reynolds lubrication equation as;

$$L[P] = \frac{\partial h}{\partial t} + \frac{\omega}{2} \frac{\partial h}{\partial \Theta} \quad (4.48)$$

Note that:-

$$h^3 \partial P = (h_o + \Delta h_1)^3 \partial (P_o + \Delta P_1) = h_o^3 \partial P_o + h_o^3 \partial \Delta P_1 + 3h_o^2 \Delta h_1 \partial P_o$$

+Second order terms

Thus, Reynolds equation is decomposed as;

$$L[P] = L[P_o + \Delta P_1] = L[P_o] + L[\Delta P_1] \quad (4.49)$$

Where;

$$L[P_o] = \frac{\omega}{2} \frac{\partial h_o}{\partial \Theta} = \frac{\omega}{2} \{ -ex_o \sin \Theta + ey_o \cos \Theta \} \quad (4.50)$$

And

$$L[\Delta P_1] = \frac{\omega}{2} \frac{\partial \Delta h_1}{\partial \Theta} + \frac{\partial \Delta h_1}{\partial t} - \frac{\partial}{R \partial \Theta} \left(\frac{3h_o^3}{12\mu} \frac{\Delta h_1}{h_o} \frac{\partial p_o}{R \partial \Theta} \right) - \frac{\partial}{\partial Z} \left(\frac{3h_o^3}{12\mu} \frac{\Delta h_1}{h_o} \frac{\partial p_o}{\partial Z} \right) \quad (4.51)$$

$$\Delta h_1 = \Delta X \cos \Theta + \Delta Y \sin \Theta; \quad \Delta P_1 = P_x \Delta x + P_y \Delta y + P_x^* \Delta \dot{x} + P_y^* \Delta \dot{y}; \quad (4.52)$$

$$\frac{\partial \Delta h_1}{\partial t} = \Delta \dot{X} \cos \Theta + \Delta \dot{Y} \sin \Theta; \quad \frac{\partial \Delta h_1}{\partial \Theta} = -\Delta X \sin \Theta + \Delta Y \cos \Theta$$

Substitution of equation (4.52) into equation (4.51) renders a set of linear partial differential equations for the first-order pressures due to each perturbation in displacement and velocity and for two region , as shown in [2],i.e.;

$$\begin{aligned} L[P_x] &= \frac{\omega}{2} \sin \Theta - \frac{\partial}{R \partial \Theta} \left\{ \frac{3h_o^3}{12\mu} \frac{\cos \Theta}{h_o} \frac{\partial p_o}{R \partial \Theta} \right\} - \frac{\partial}{\partial Z} \left\{ \frac{3h_o^3}{12\mu} \frac{\cos \Theta}{h_o} \frac{\partial p_o}{\partial Z} \right\} \\ L[P_y] &= \frac{\omega}{2} \cos \Theta - \frac{\partial}{R \partial \Theta} \left\{ \frac{3h_o^3}{12\mu} \frac{\sin \Theta}{h_o} \frac{\partial p_o}{R \partial \Theta} \right\} - \frac{\partial}{\partial Z} \left\{ \frac{3h_o^3}{12\mu} \frac{\sin \Theta}{h_o} \frac{\partial p_o}{\partial Z} \right\} \end{aligned} \quad (4.53)$$

The first two equations can be simplified using the zeroth order Reynolds equation (4.50) to:-

-No chamfer region II :-

$$L[P_x] = \frac{\omega}{2} \left(-\sin \Theta - 3 \frac{\cos \Theta}{h_o} \frac{\partial h_o}{\partial \Theta} \right) - \frac{h_o^3}{4\mu} \left\{ \frac{\partial p_o}{R \partial \Theta} \frac{\partial}{R \partial \Theta} \left(\frac{\cos \Theta}{h_o} \right) \right\} \quad (4.54)$$

$$L[P_y] = \frac{\omega}{2} \left(+ \cos \Theta - 3 \frac{\sin \Theta}{h_o} \frac{\partial h_o}{\partial \Theta} \right) - \frac{h_o^3}{4\mu} \left\{ \frac{\partial p_o}{R \partial \Theta} \frac{\partial}{R \partial \Theta} \left(\frac{\sin \Theta}{h_o} \right) \right\} \quad (4.55)$$

$$L[P_x] = \cos \Theta \quad (4.56)$$

$$L[P_y] = \sin \Theta \quad (4.57)$$

-chamfer region II :-

$$L[P_x] = \frac{\omega}{2} \left(- \sin \Theta - 3 \frac{\cos \Theta}{h_o} \frac{\partial h_o}{\partial \Theta} \right) - \frac{h_o^3}{4\mu} \left\{ \frac{\partial p_o}{R \partial \Theta} \frac{\partial}{R \partial \Theta} \left(\frac{\cos \Theta}{h_o} \right) \right\} - \frac{h_o^3}{4\mu} \cos \Theta \left\{ \frac{\partial p_o}{\partial Z} \frac{\partial}{\partial \Theta} \left(\frac{1}{h_o} \right) \right\} \quad (4.58)$$

$$L[P_y] = \frac{\omega}{2} \left(+ \cos \Theta - 3 \frac{\sin \Theta}{h_o} \frac{\partial h_o}{\partial \Theta} \right) - \frac{h_o^3}{4\mu} \left\{ \frac{\partial p_o}{R \partial \Theta} \frac{\partial}{R \partial \Theta} \left(\frac{\sin \Theta}{h_o} \right) \right\} - \frac{h_o^3}{4\mu} \sin \Theta \left\{ \frac{\partial p_o}{\partial Z} \frac{\partial}{\partial \Theta} \left(\frac{1}{h_o} \right) \right\} \quad (4.59)$$

$$L[P_x] = \cos \Theta \quad (4.60)$$

$$L[P_y] = \sin \Theta \quad (4.61)$$

Where h_o is a function of the circumferential coordinate only, i.e. aligned journal within its bearing.

The appropriate boundary conditions for the solution of the zeroth and first-order pressure fields are obtained from a perturbation of the boundary pressures given in equations (3.4) and (3.7). Note that in those boundaries where the pressure is not ambient value, the perturbed pressures must vanish, i.e. homogeneous boundary conditions.

However, recommends a different set of more complex boundary conditions for the perturbed pressure fields by [2,25,16].

Once the zeroth and first-order pressure fields are calculated numerically, the bearing reaction forces and dynamic force coefficients can be evaluated.

Substitution of the pressure field equation (4.46) into the force expressions (3.9), and (3.10) then write it in matrix forms gives;

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \int_0^L \int_0^{\Theta_c} \left\{ P_o + P_x \Delta x + P_y \Delta y + P_{\dot{x}} \Delta \dot{x} + P_{\dot{y}} \Delta \dot{y} \right\} \begin{bmatrix} \cos \Theta \\ \sin \Theta \end{bmatrix} R d\Theta dz \quad (4.62)$$

The bearing reaction equilibrium forces (F_{x0}, F_{y0}) are easily obtained from the next equation;

$$\begin{bmatrix} F_{x0} \\ F_{y0} \end{bmatrix} = - \begin{bmatrix} W_x \\ W_y \end{bmatrix} = \int_0^L \int_0^{\Theta_c} \{ P_o \} \begin{bmatrix} \cos \Theta \\ \sin \Theta \end{bmatrix} R d\Theta dz \quad (4.63)$$

and the stiffness dynamic force coefficients are determined from the integration of perturbed dynamic pressures field;

$$\begin{bmatrix} K_{xx} \\ K_{yx} \end{bmatrix} = - \int_0^L \int_0^{\Theta_c} \{ P_x \} \begin{bmatrix} \cos \Theta \\ \sin \Theta \end{bmatrix} R d\Theta dz \quad (4.64)$$

$$\begin{bmatrix} K_{xy} \\ K_{yy} \end{bmatrix} = - \int_0^L \int_0^{\Theta_c} \{ P_y \} \begin{bmatrix} \cos \Theta \\ \sin \Theta \end{bmatrix} R d\Theta dz \quad (4.65)$$

and the damping dynamic force coefficients are also determined from the integration of perturbed dynamic pressures field;

$$\begin{bmatrix} C_{xx} \\ C_{yx} \end{bmatrix} = - \int_0^L \int_0^{\Theta_c} \{ P_{\dot{x}} \} \begin{bmatrix} \cos \Theta \\ \sin \Theta \end{bmatrix} R d\Theta dz \quad (4.66)$$

$$\begin{bmatrix} C_{xy} \\ C_{yy} \end{bmatrix} = - \int_0^L \int_0^{\Theta_c} \{ P_{\dot{y}} \} \begin{bmatrix} \cos \Theta \\ \sin \Theta \end{bmatrix} R d\Theta dz \quad (4.67)$$

4.6. Computation Algorithm

The prior numerical calculation described can be transformed into a computational algorithm, as follows;

4.6.1. Without Chamfer (Region II):-

In this case the chamfer length and angle will be zero;

1. Choose material, geometry, dimensions.
2. Perform the mesh generation for the bearing oil films in (θ, Z) directions.
3. Choose a value for the journal-bearing eccentricity ratio (ϵ) .
4. Calculate a value of perturbed oil film thickness $(h_{i,j})$ using equation (4.38).
5. Calculate the pressure distribution through the oil film $(P_{i,j})$ by solving zeroth-order Reynolds equations (4.10), using Gauss – Seidel iterative method with over relaxation factor, then test the convergence using the convergence criterion (C.4).
6. Calculate the load carrying capacity (F) of the journal-bearing oil film using equation (4.33).
7. Calculate the value of reference Sommerfeld number (S) , attitude angle (ϕ) using equation (3.13) and (3.12) respectively.
8. Calculate the perturbed dynamic pressure distribution (P_x) by solving first-order Reynolds equation (4.54) using the same mentioned iterative method which solved the zeroth-order pressure distribution.
9. Calculate the stiffness dynamic force coefficients (K_{xx}, K_{yx}) by using equation (4.64).
10. Calculate the perturbed dynamic pressure distribution (P_y) by solving first-order Reynolds equation (4.55) using the same mentioned iterative method.
11. Calculate the stiffness dynamic force coefficients (K_{yx}, K_{yy}) by using equation (4.65).
12. Calculate the perturbed dynamic pressure distribution (P_x^*) by solving first-order Reynolds equation (4.56) using the same mentioned iterative method.

13. Calculate the damping dynamic force coefficients (C_{XX}, C_{YX}) by using equation (4.66).
14. Calculate the perturbed dynamic pressure distribution (P_y^*) by solving first-order Reynolds equation (4.57) using the same mentioned iterative method.
15. Calculate the damping dynamic force coefficients (C_{YX}, C_{YY}) by using equation (4.67).
16. Calculate the threshold speed instability by using equation (3.20).
17. Calculate the static radial and tangential forces ($F_{\epsilon 0}, F_{\phi 0}$) using equation (B.4) and (B.5), then calculate the modified Sommerfeld number (σ) and the attitude angle using equations (B.3) and (B.6) as these equations amidst in Lund ref.[4]
18. Calculate the force coefficients in the (X,Y) coordinate using equations from (B.14) to (B.20) in succession.
19. Print the output results.

4.6.1. With Chamfer (Region I):-

In this case the chamfer length and angle will be known ;

1. Choose material, geometry, dimensions.
2. Perform the mesh generation for the bearing oil films in (θ, Z) directions.
3. Choose a value for the journal-bearing eccentricity ratio (ϵ).
4. Calculate a value of perturbed oil film thickness ($h_{ci,j}$) using equation (4.39).
5. Calculate the pressure distribution through the oil film ($P_{i,j}$) by solving zeroth-order Reynolds equations (4.10), using Gauss – Seidel iterative method with over relaxation factor, then test the convergence using the convergence criterion (C.4).
6. Calculate the load carrying capacity (F) of the journal-bearing oil film using equation (4.33).
7. Calculate the value of reference Sommerfeld number (S), attitude angle (ϕ) using equation (3.13) and (3.12) respectively.

8. Calculate the perturbed dynamic pressure distribution (P_x) by solving first-order Reynolds equation (4.58) using the same mentioned iterative method which solved the zeroth-order pressure distribution.
9. Calculate the stiffness dynamic force coefficients (K_{xx}, K_{yx}) by using equation (4.64).
10. Calculate the perturbed dynamic pressure distribution (P_y) by solving first-order Reynolds equation (4.59) using the same mentioned iterative method.
11. Calculate the stiffness dynamic force coefficients (K_{yx}, K_{yy}) by using equation (4.65).
12. Calculate the perturbed dynamic pressure distribution (P_x^*) by solving first-order Reynolds equation (4.60) using the same mentioned iterative method.
13. Calculate the damping dynamic force coefficients (C_{xx}, C_{yx}) by using equation (4.66).
14. Calculate the perturbed dynamic pressure distribution (P_y^*) by solving first-order Reynolds equation (4.61) using the same mentioned iterative method.
15. Calculate the damping dynamic force coefficients (C_{yx}, C_{yy}) by using equation (4.67).
16. Calculate the threshold speed instability by using equation (3.20).
17. Print the output results.

4.7. Computer Program

Computer Program were implement to predict theoretical results by solving the governing equations of the problem. The programs is designed in a suitable programming language (FORTRAN-90).

1- Prog.1: A Computer Program utilized to predict the steady state case without chamfer adding to predict 8th rotor dynamic coefficients shown in figure (4-4).

To verify it then with that published in References (Lund [4], Sumit [3])

2- Prog.2: A Computer Program utilized to predict the characteristics of the chamfered journal bearing shown in figure (4-5). According to this Program the results is the final resultant for showing the effect of chamfering.

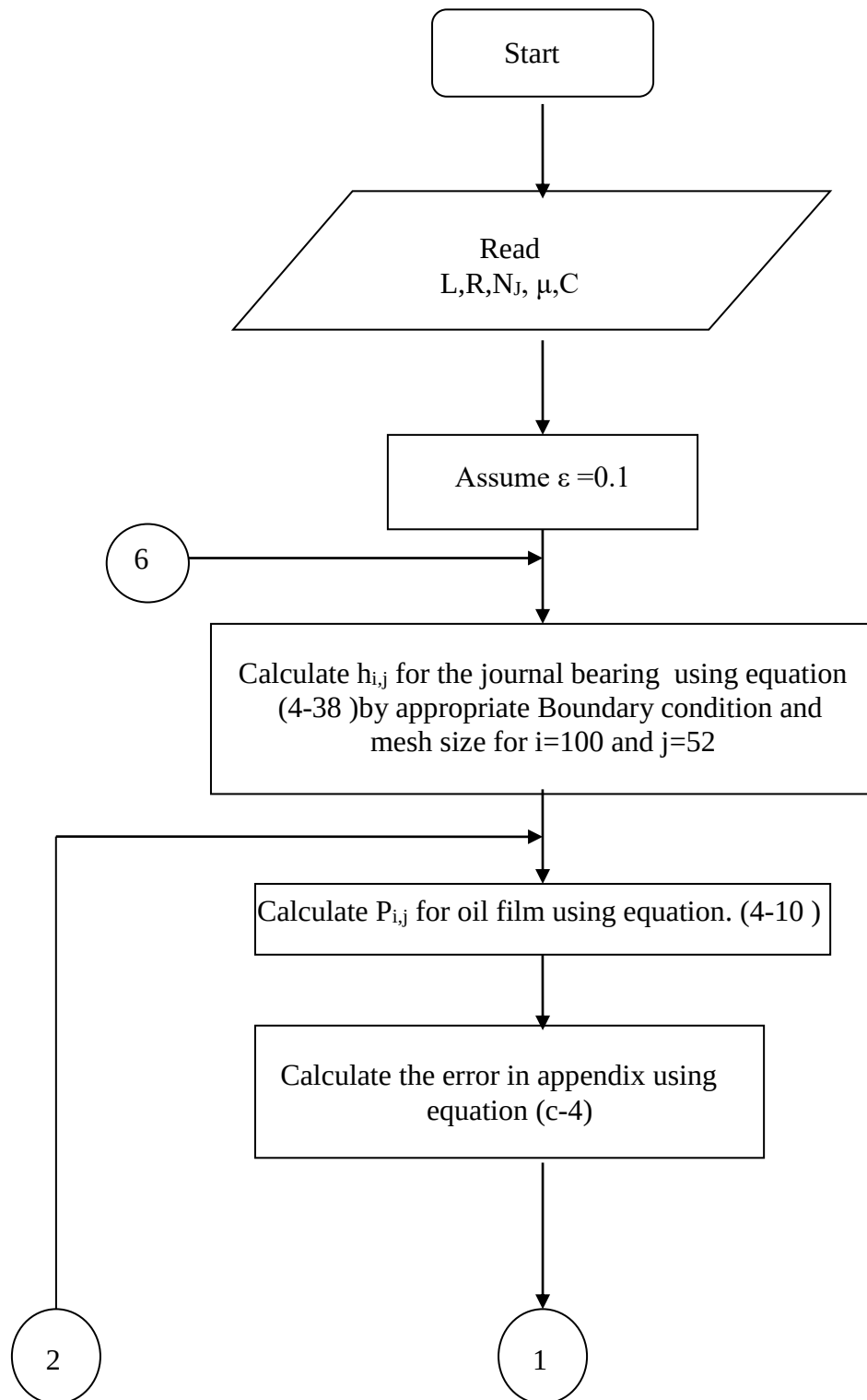
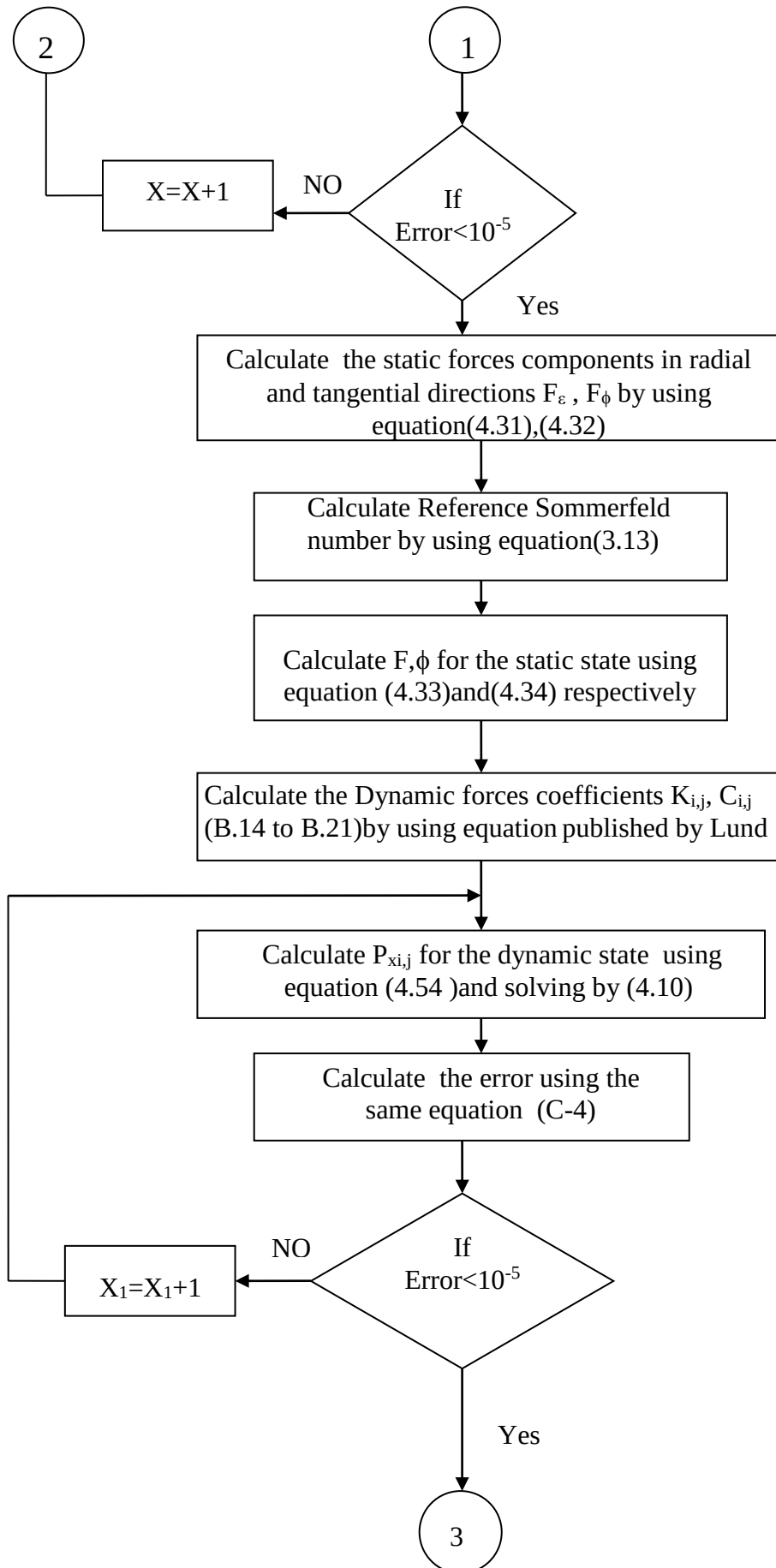
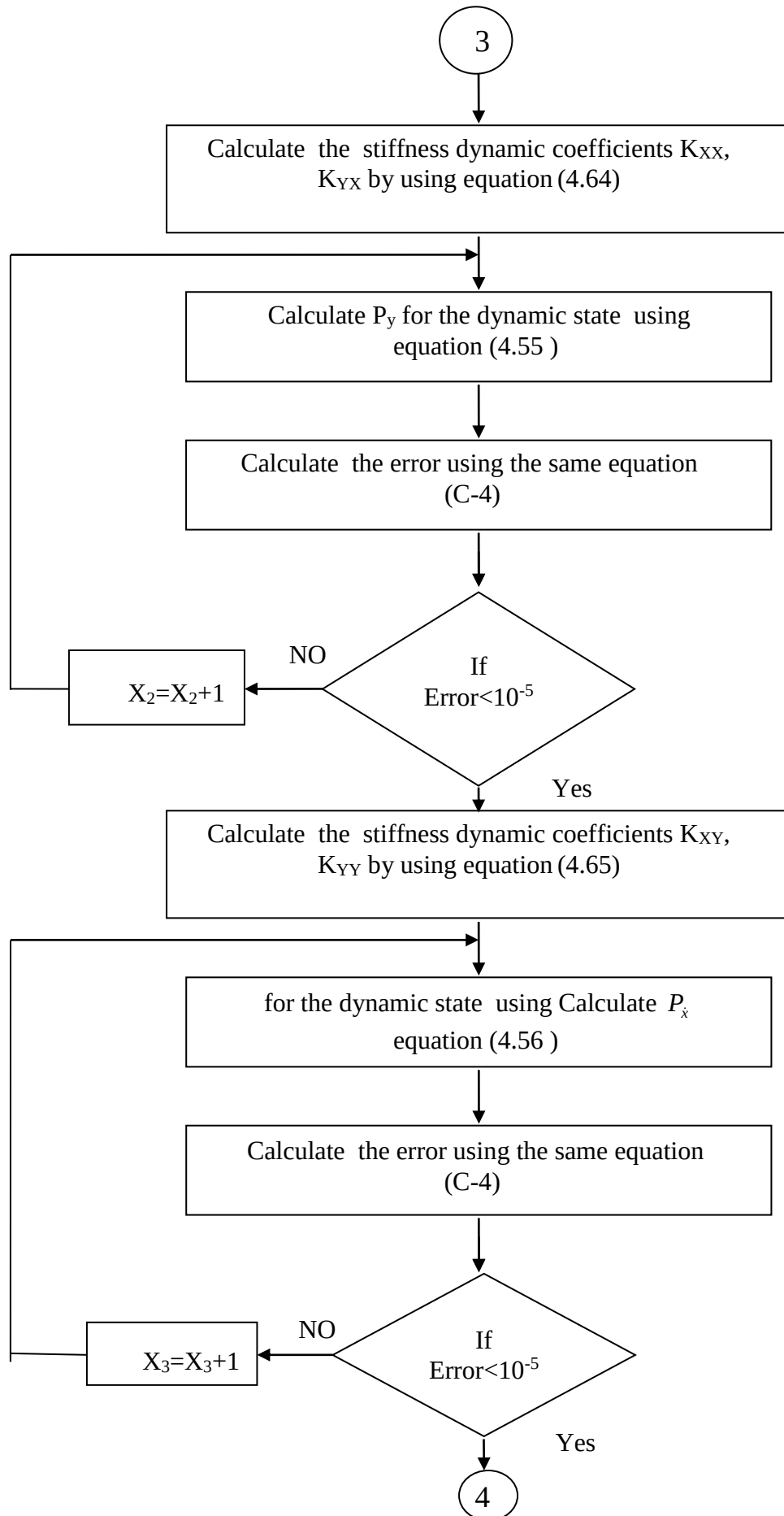


Figure (4-4), Flow Chart of Computer Program 1.





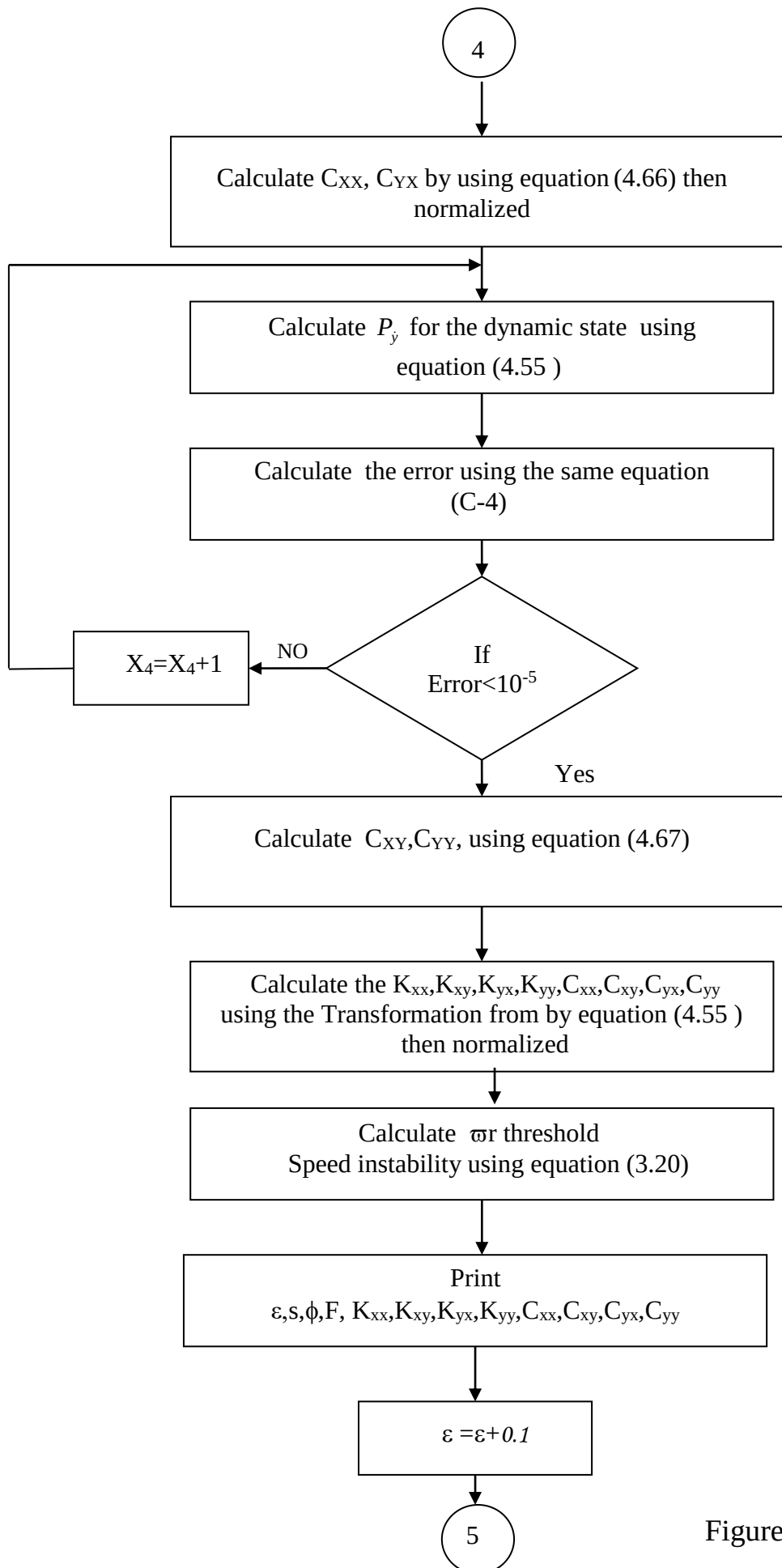


Figure (4-4), Continued.

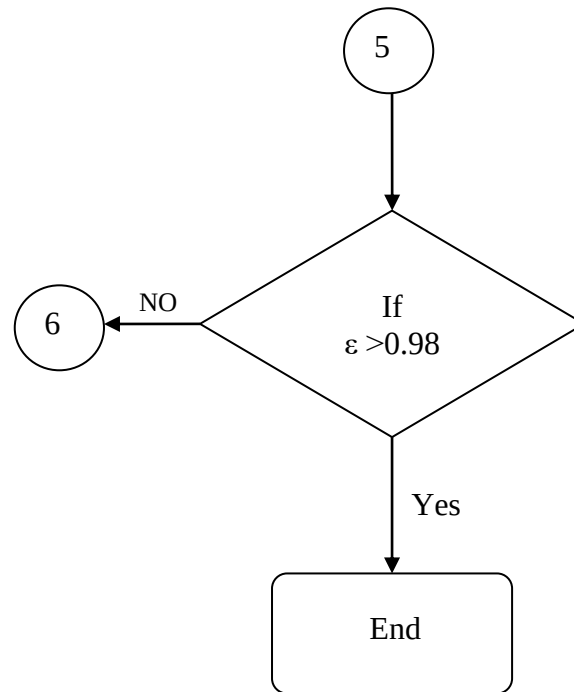


Figure (4-4), Flow Chart of Computer Program 1.

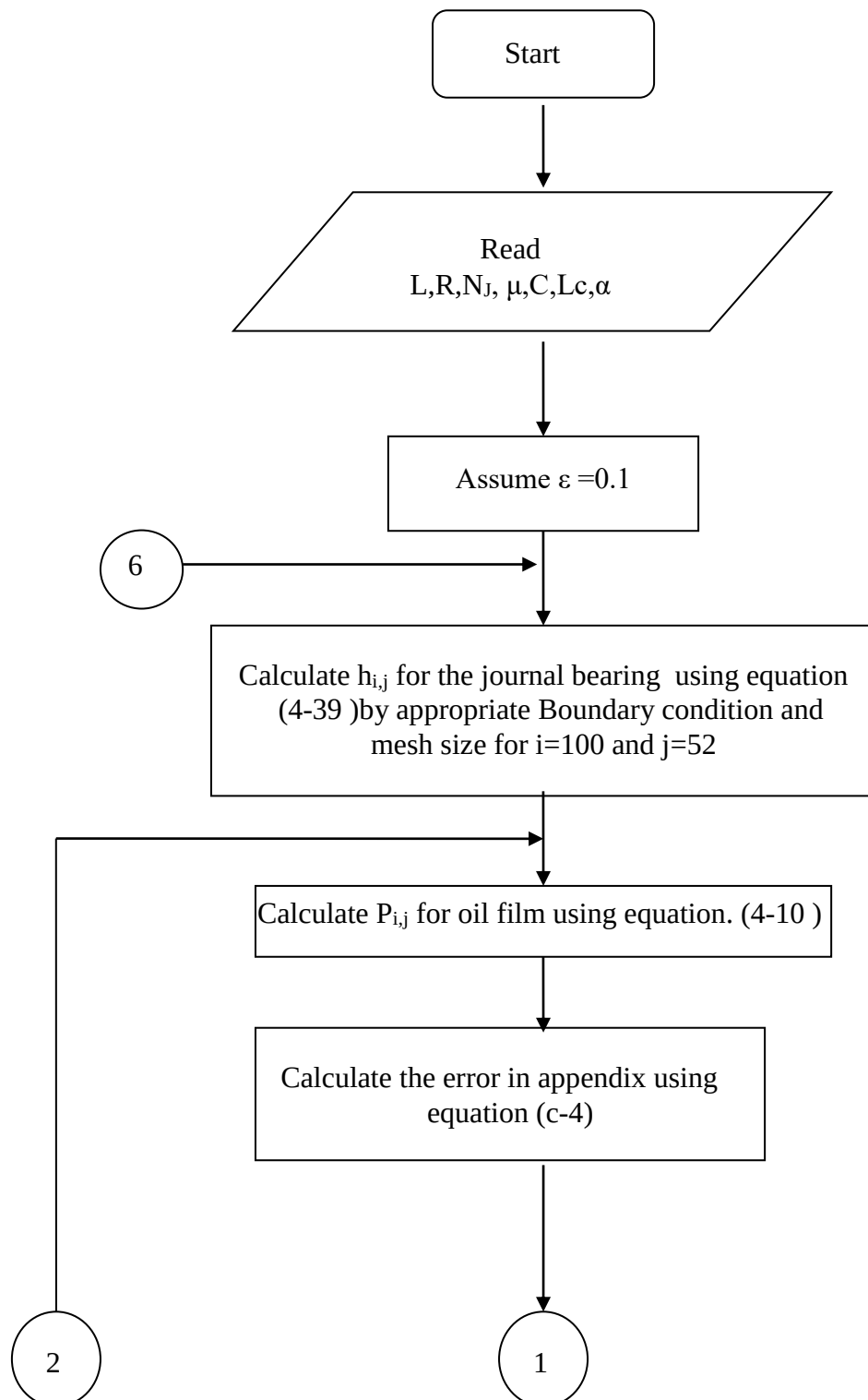


Figure (4-5), Flow Chart of Computer Program 2.

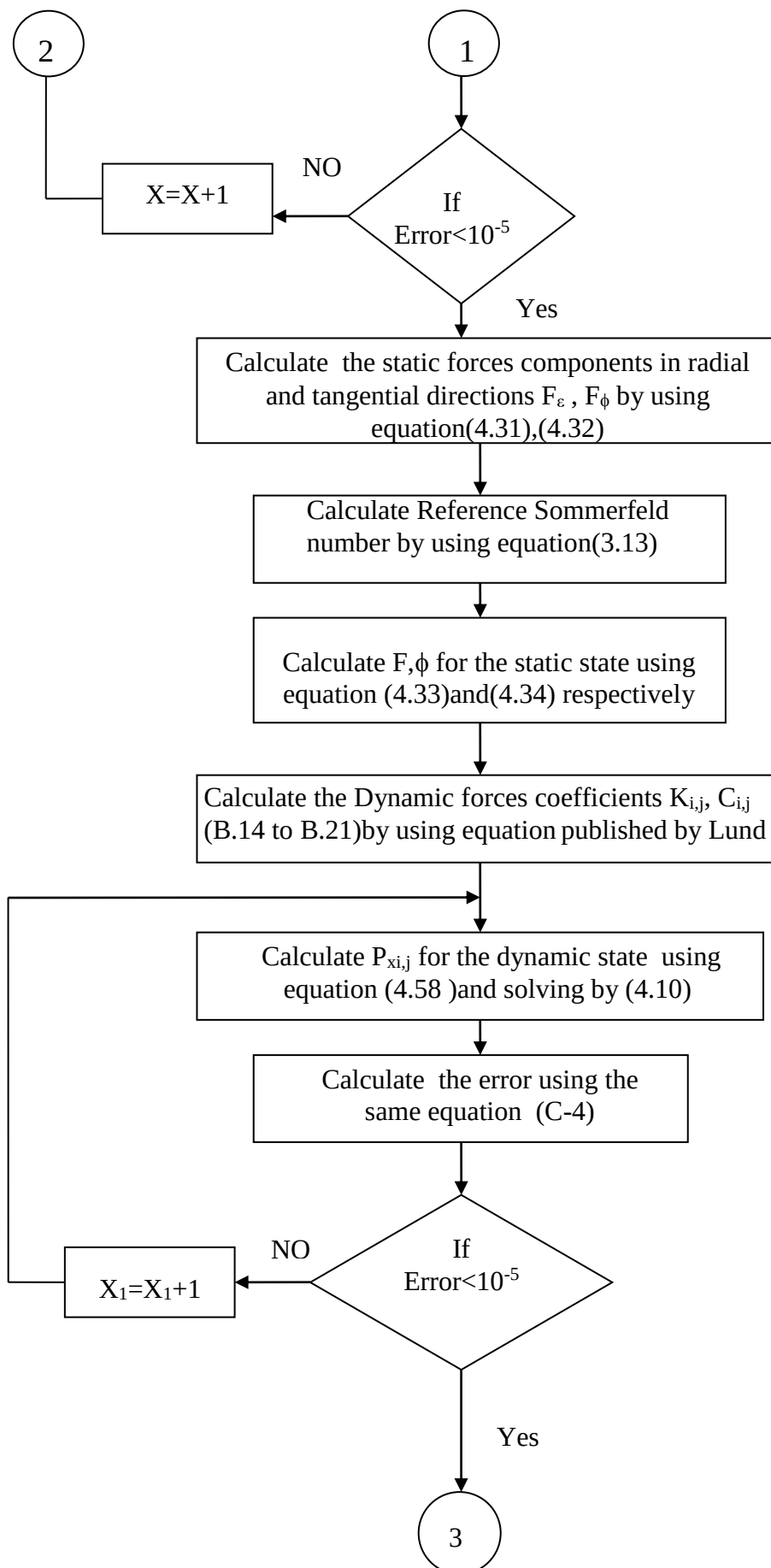


Figure (4-5), Continued.

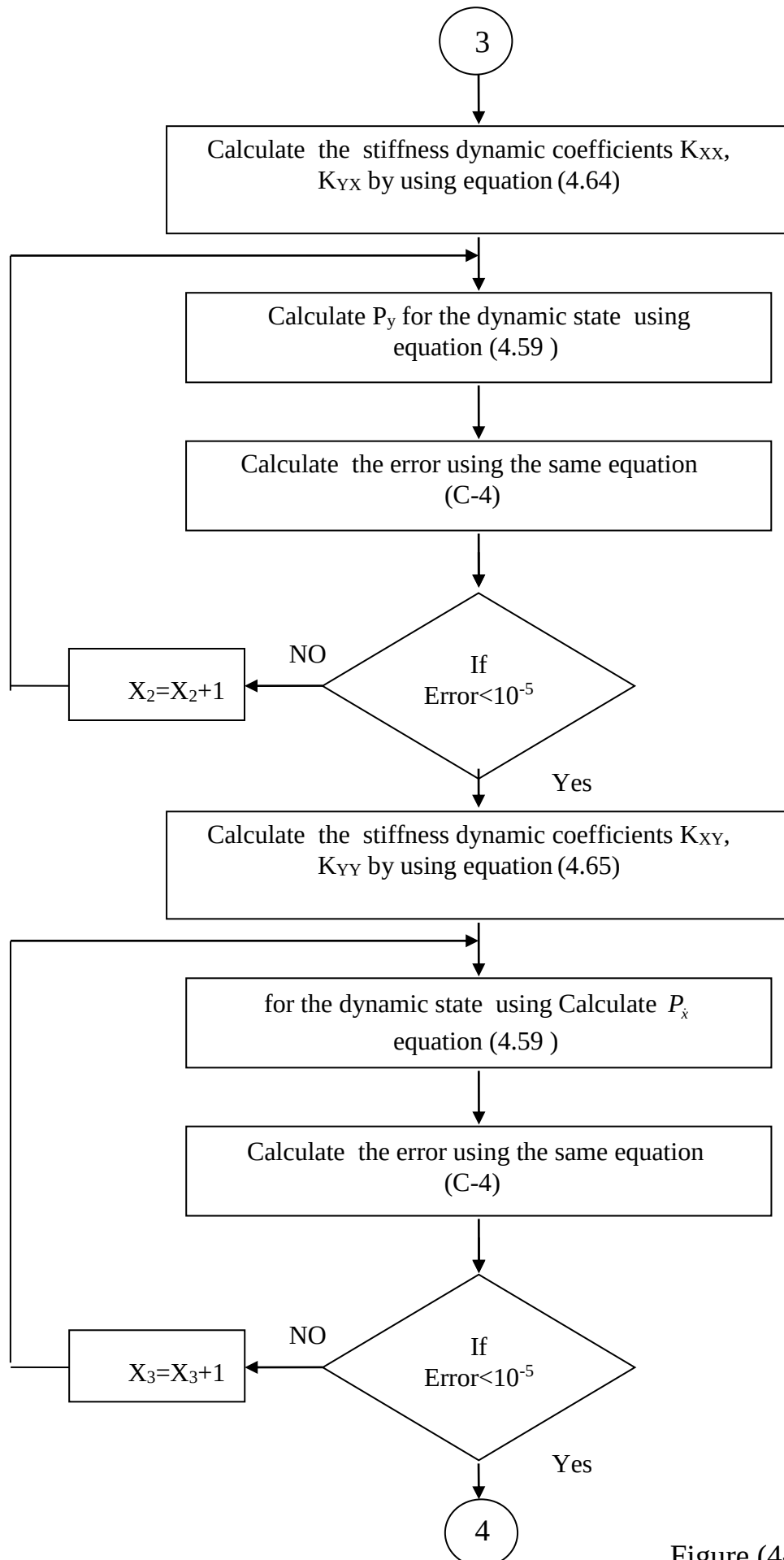


Figure (4-5), Continued.

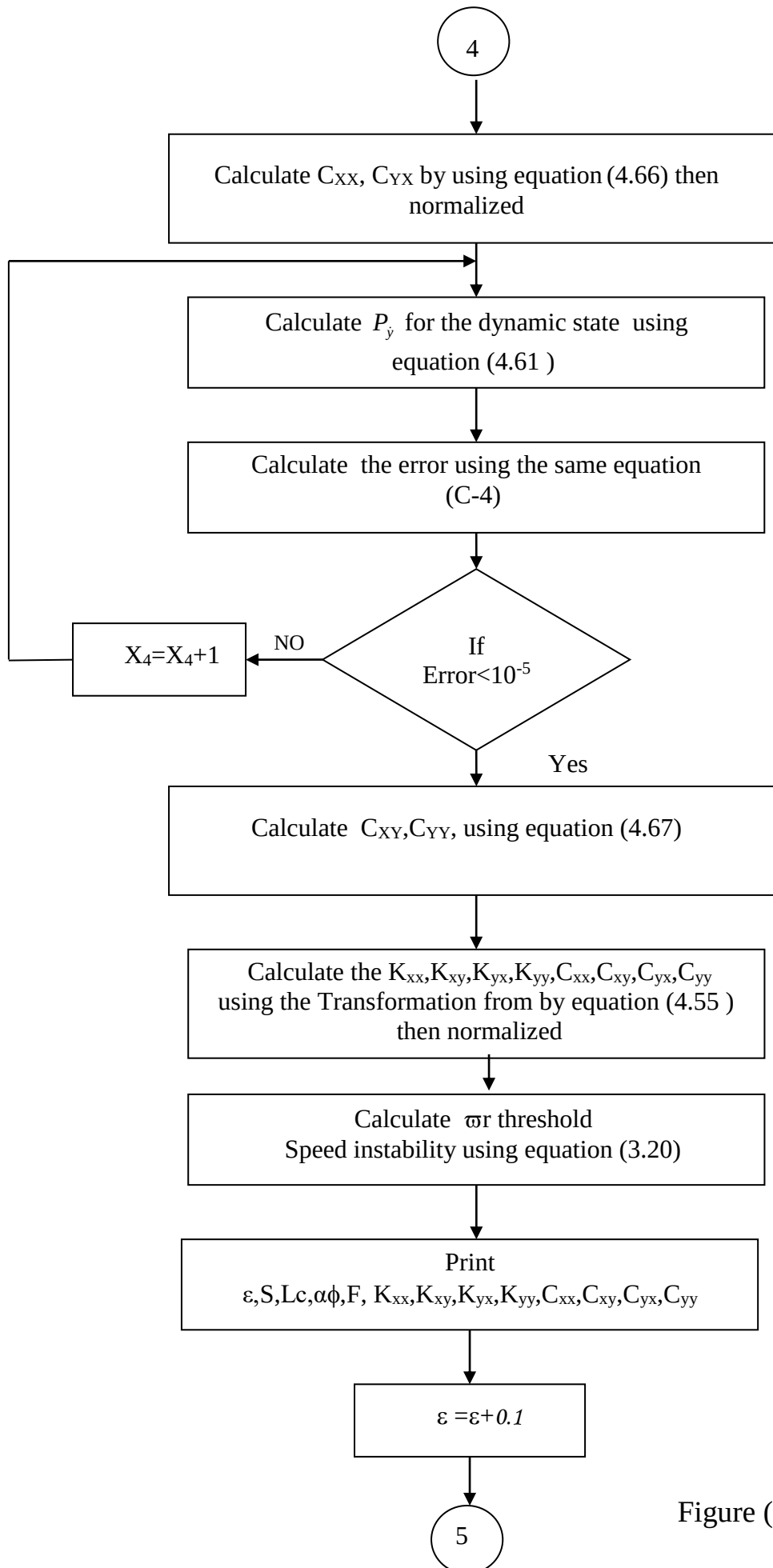


Figure (4-5), Continued.

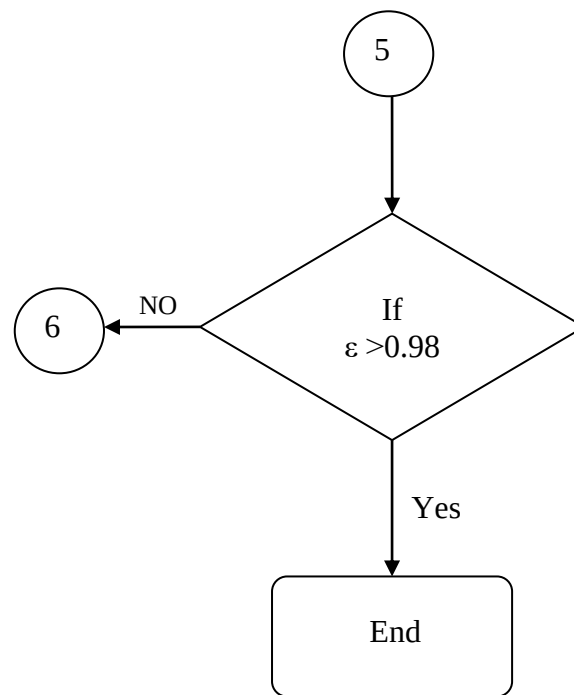


Figure (4-5), Flow Chart of Computer Program 2.

*CHAPTER
THERE*

*THEORETICAL
CONSIDERATION*

Chapter (3)

“ Theoretical Consideration ”

3.1.Introduction

The mathematical model developed to treat the problem under consideration was explained her in . The main governing equations with appropriate assumptions and boundary conditions used to investigate the performance of hydrodynamically lubricated chamfered journal bearing and also to solve the problem related to the stability characteristics of the journal bearing with end chamfer were highly lighted in the following sections.

3.2. Bearing Geometry

The geometry of the journal bearing with end chamfer and the suitable coordinates used in this work are shown schematically in figure (3.1).

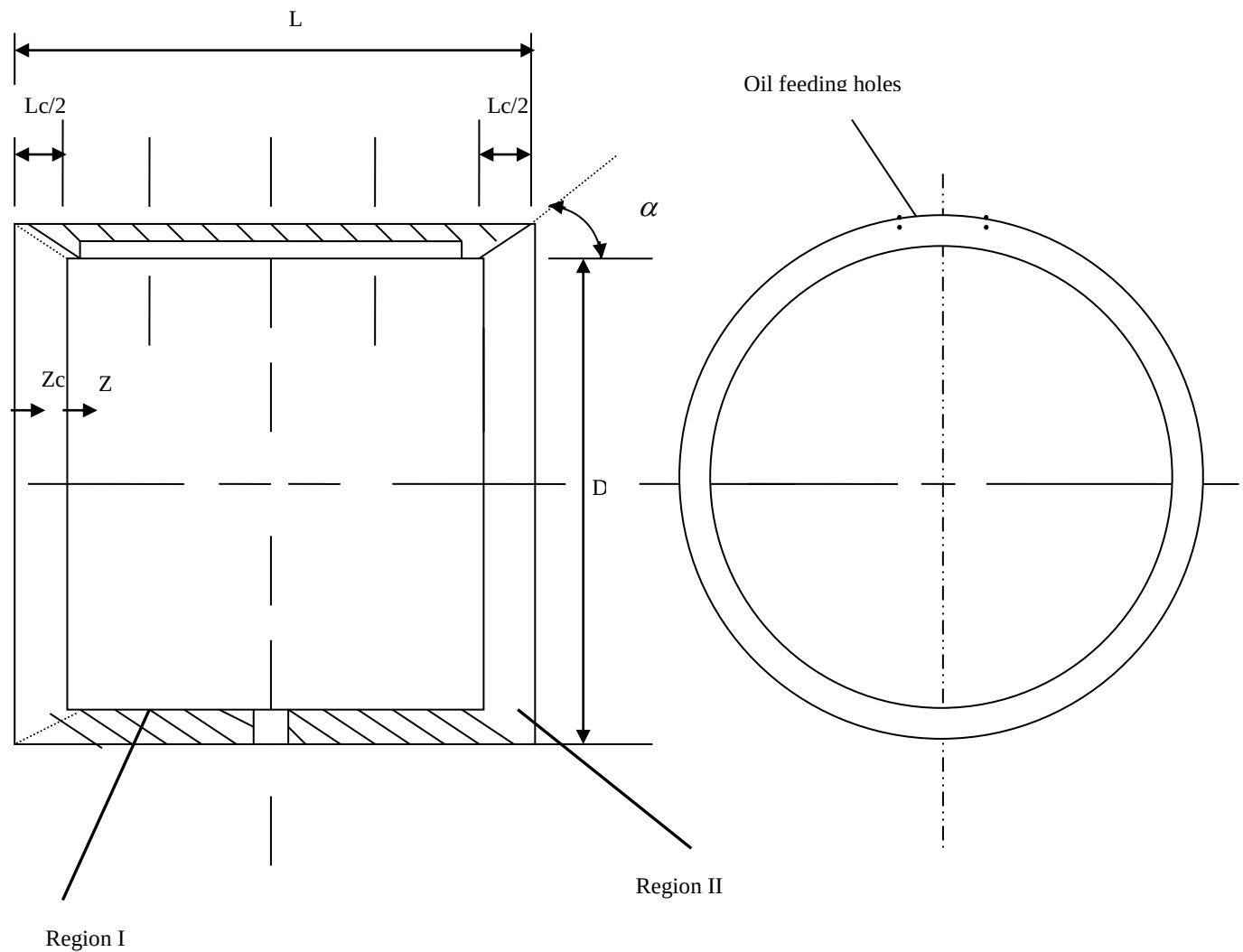
The journal spins with angular speed (ω) about its center denoted by (O_j) and the bearing center (O_b) may also describe translational or housing which is stationary (not moving) in most application as shown schematically in figure (3.2) as referenced by [3].

A chamfer is machined circumferentially at both sides of the bearing inner surface, to minimize or almost prevent the side leakage flow rate in lightly loaded bearings as claimed by [1].

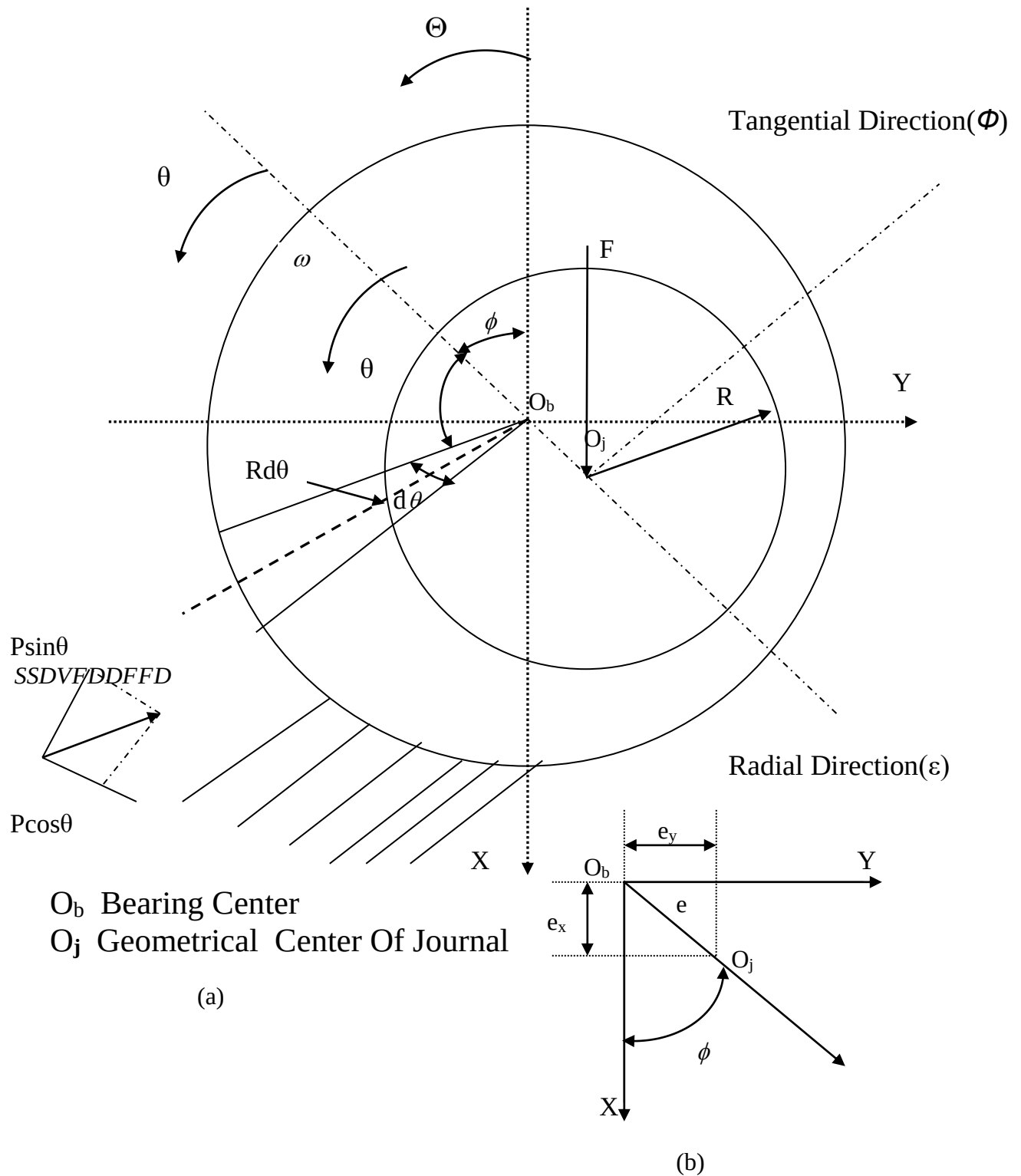
3.3. Coordinates System

The smallness of the oil film dimensions relative to the geometry of the bearing allows for cartesian coordinate to be located on the bearing surfaces and for cylindrical coordinates(θ, Z) in the circumferential and axial directions. Two dimensional Cartesian coordinates (X, Y) were developed to describe the behavior

of the oil film , while the coordinates (ε) and (Φ) are used to describe the dynamic coefficients of the bearing .



Figure(3-1) Chamfered Bearing Terminology



Figure(3-2) (a)Radial and Tangential Force Components.

(b) Schematic diagram of the bearing geometry and the suitable coordinate used.

3.4. Assumptions

The following assumptions are made to drive the governing equations used to solve the problem :-

1. All the assumptions used in the derivation of Reynolds equation are valid .
2. No slip at the boundaries (always valid ,since the velocity of the oil layer adjacent to the boundary is the same of the boundary).
3. Lubricant behaves as a Newtonian fluid .
4. Surface roughness effect is neglected (value of wavelength and amplitude).
5. Heat transfer through the solid boundaries is neglected .
6. Shaft is rigid and there is no misalignment .

3.5. Governing Equations

The following governing equations is adopted in this work :-

3.5.1. Reynolds Equations

The fluid film pressure generated in the oil film can obtained by solving the lubricant equation (Reynolds equation) which can be expressed in cylindrical coordinates as follows,[3] :-

$$\frac{1}{R^2} \frac{\partial}{\partial \theta} \left\{ \frac{h^3}{12\mu} \frac{\partial p}{\partial \theta} \right\} + \frac{\partial}{\partial z} \left\{ \frac{h^3}{12\mu} \frac{\partial p}{\partial z} \right\} = \frac{\omega}{2} \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t} \quad (3.1)$$

3.5.2. Oil Film Thickness

The oil film thickness for the bearing with and without chamfer at both end as shown in figure (3.1) can be obtained [1];

$$h = C(1 + \varepsilon \cos \theta) \quad (3.2)$$

$$h_c = C(1 + \varepsilon \cos \theta) + Zc \tan \alpha \quad (3.3)$$

3.5.3. Boundary Conditions

The Reynolds equation was used with an appropriate boundary condition to define the pressure through the oil film as used by [1].

In cylindrical bearing the pressure field must be continuous in the circumferential direction , hence :-

$$P=0 \quad \text{at } \theta=0 \quad (\text{at position of maximum film thickness}). \quad (3.4)$$

The classical Reynolds boundary condition was used in the present work .

$$P_{\text{cav}} = P (\pi + \alpha_c , Z) = 0 \quad (3.5)$$

α_c = the angle at which cavitations occurs.

$$P = \frac{\partial P}{\partial \theta} = 0 \quad \text{at } \theta = \theta_c \quad (\text{at film rupture boundaries}). \quad (3.6)$$

$$P=P_c \text{ at } Z=0 \text{ and } Z=L \quad (\text{at axial ends}). \quad (3.7)$$

Where; $P_c = \sigma/h_c$ and

$$h_c = C(1 + \varepsilon \cos \theta) + (Lc/2) \tan \alpha \quad (3.8)$$

σ = surface tension for the lubricating oil used to be (0.0366 N m⁻¹) as measured in reference[1].

h_c = oil film thickness at the bearing chamfer.

3.6. Bearing Parameters

The solution of the Reynolds equation with the appropriate boundary conditions yields the bearing pressure distribution .

Integrate the pressure in radial and tangential direction give the fluid film forces which can be obtained as ,[2,3];

$$F_{\varepsilon} = \int_0^L \int_0^{\theta_c} RP \cos \theta d\theta dz \quad (3.9)$$

$$F_{\phi} = \int_0^L \int_0^{\theta_c} RP \sin \theta d\theta dz \quad (3.10)$$

The total load carried by the bearing can be evaluated as:-

$$F = \sqrt{(F_{\varepsilon})^2 + (F_{\phi})^2} \quad (3.11)$$

The attitude angle can be evaluated as :-

$$\phi = \tan^{-1} \left(\pm \frac{F_{\phi}}{F_{\varepsilon}} \right) \quad (3.12)$$

The reference Sommerfeld number can be expressed from the following expression,[2]:-

$$S = 1/F = (R\omega\mu) * (R/C)^2 / (F/L) \quad (3.13)$$

The side-leakage flow rate can be determined as ,[30,32]:-

$$q = \left(\frac{CR\omega L}{24} \right) \left(\frac{D}{L} \right)^2 \int_0^{\theta_c} h^3 \frac{\partial p}{\partial Z} \Big|_{Z=\pm L} d\theta \quad (3.14)$$

which as appear it's dependent upon the pressure gradient in the axial direction at the outermost boundaries of the bearing ,the non-dimension form can be presented as:-

$$Q = \left(\frac{1}{24} \right) \left(\frac{D}{L} \right)^2 \int_0^{\theta_c} \bar{h}^3 \frac{\partial \bar{P}}{\partial \bar{Z}} \Big|_{\bar{Z}=\pm 1} d\theta \quad (3.15)$$

Where, the dimensionless parameters as follows;

$$\bar{h} = h / C$$

$$\bar{Z} = Z / L$$

$$\bar{P} = PC^2 / (\mu \omega R^2)$$

3.7. Theoretical Prediction Of Dynamic Characteristics

The source of oil whirl instability is the hydrodynamic forces generated in the lubricant film .These forces are strong function of the journal eccentricity (ε) and attitude ,angle(ϕ)if the squeeze film terms in the Reynolds linear equation are considered , then these forces will also be function of velocity components ($d\varepsilon/dt$) and ($d\phi/dt$).Squeeze film terms makes the Reynolds linear equation transient [3].

The dynamic characteristics of the journal bearing can be defined by means of eight coefficient, four stiffness factors and four damping factor as shown in fig(3-3).

Perturbation technique was used to identify these coefficients . the fluid film forces may be expanded in Taylor series about static equilibrium position . For sufficiently small motions , using Taylor series expansion , a small perturbation of the displacement and velocity about steady state position (ε_0, ϕ_0) in the radial and tangential directions . Fluid film forces can be expressed as [2,3]:-

$$F_\varepsilon = F_{\varepsilon_0} + \frac{\partial F_\varepsilon}{\partial \varepsilon} \Delta \varepsilon + \frac{\partial F_\varepsilon}{\varepsilon_0 \partial \phi} \varepsilon_0 \Delta \phi + \frac{\partial F_\varepsilon}{\partial \dot{\varepsilon}} \Delta \dot{\varepsilon} + \frac{\partial F_\varepsilon}{\varepsilon_0 \partial \dot{\phi}} \varepsilon_0 \Delta \dot{\phi} \quad (3.16)$$

$$F_\phi = F_{\phi_0} + \frac{\partial F_\phi}{\partial \varepsilon} \Delta \varepsilon + \frac{\partial F_\phi}{\varepsilon_0 \partial \phi} \varepsilon_0 \Delta \phi + \frac{\partial F_\phi}{\partial \dot{\varepsilon}} \Delta \dot{\varepsilon} + \frac{\partial F_\phi}{\varepsilon_0 \partial \dot{\phi}} \varepsilon_0 \Delta \dot{\phi} \quad (3.17)$$

Where :

$$K_{\varepsilon\varepsilon} = \frac{\Delta F_\varepsilon}{\Delta \varepsilon}; K_{\phi\phi} = \frac{\Delta F_\phi}{\varepsilon_0 \Delta \phi}; K_{\varepsilon\phi} = \frac{\Delta F_\varepsilon}{\varepsilon_0 \Delta \phi}; K_{\phi\varepsilon} = \frac{\Delta F_\phi}{\Delta \varepsilon} \quad (3.18)$$

$$\begin{aligned}
C_{\varepsilon\varepsilon} &= \frac{\Delta F_{\varepsilon}}{\Delta \dot{\varepsilon}}; C_{\phi\phi} = \frac{\Delta F_{\phi}}{\varepsilon_0 \Delta \dot{\phi}} \\
C_{\varepsilon\phi} &= \frac{\Delta F_{\varepsilon}}{\varepsilon_0 \Delta \dot{\phi}}; C_{\phi\varepsilon} = \frac{\Delta F_{\phi}}{\Delta \dot{\varepsilon}}
\end{aligned} \tag{3.19}$$

Where $(\Delta \dot{\varepsilon})$ and $(\varepsilon_0, \Delta \dot{\phi})$ are the journal radial and tangential velocities in (ε, ϕ) coordinates system .

3.8. Whirl Speed

The whirl threshold for rigid shaft as a function of dynamic stiffness and damping coefficients which are functions of eccentricity ratio is given by (3.20) [4,31,32]:-

$$\omega_r = \sqrt{\frac{g_2 g_{32} g_4}{g_4^2 - g_2 g_{31} g_4 + g_2^2 g_5}} \tag{3.20}$$

Where,

$$g_2 = \bar{C}_{xx} + \bar{C}_{yy}$$

$$g_{31} = \bar{K}_{xx} + \bar{K}_{yy}$$

$$g_{32} = \bar{C}_{xx} + \bar{C}_{yy} - \bar{C}_{xy} \bar{C}_{yx}$$

$$g_4 = \bar{K}_{xx} + \bar{C}_{yy} + \bar{K}_{yy} \bar{C}_{xx} - \bar{K}_{xy} \bar{C}_{yx} - \bar{K}_{yx} \bar{C}_{xy}$$

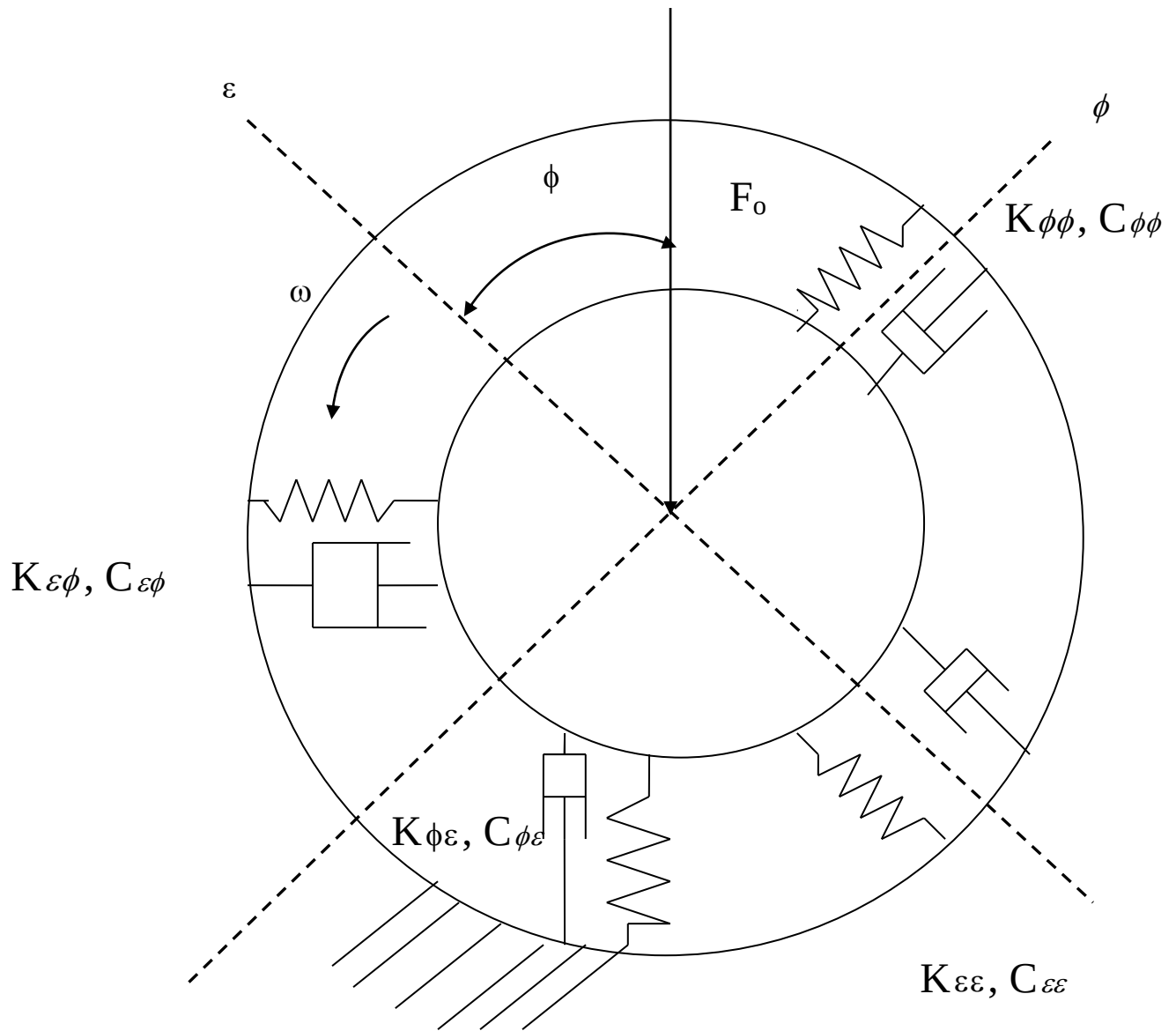
$$g_5 = \bar{K}_{xx} + \bar{K}_{yy} - \bar{K}_{xy} \bar{K}_{yx}$$

Since the eccentricity is a function of the Sommerfeld number (or the bearing parameters) as the equation (B-3) given later shown, a relation between the whirl threshold and the bearing parameters could be formulated. For a particular system under given conditions, an evaluation of the eccentricity ratio and the dimensionless speed would produce a point on this graph. If that point lies above the whirl threshold line, the journal bearing system is whirling.

Conversely, if the point lies below the line, the bearing system is stable with respect to oil whirl. And if the point were to lie on the whirl threshold line, the bearing system would be considered marginally stable (on the brink of whirling),[31].

The stability of a system is defined as the reaction to any external perturbation. In other words, consider a rotor that is rotating in journal bearings. Imagine tapping the rotor with a hammer (perturbation). If the rotor comes back to its original position in a finite amount of time, the rotor is said to be stable. If the rotor amplitude increases with time (eventually distressing the machine), the rotor is said to be unstable,[44].

If the shaft receives a disturbing force such as a sudden surge or external shock, it can momentarily increase the eccentricity from its equilibrium position. When this occurs, additional oil is immediately pumped into the space vacated by the shaft. This results in an increased pressure of the load-carrying film, creating additional force between the oil film and shaft. In this case, the oil film can actually drive the shaft ahead of it in a forward circular motion and into a whirling path around the bearing within the bearing clearance. If there is sufficient damping within the system, the shaft can be returned to its normal position and stability. Otherwise, the shaft will continue in its whirling motion, which may become violent depending on several parameters,[47].



Figure(3-3),Dynamic Coefficients of a Journal Bearing System