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College of Engineering

Department of Civil Engineering



Nonlinear Analysis of Reinforced Concrete Members with Embedded Precast- Prestressed Concrete Units

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By

Ali Hadi Adheem

Supervised By

Assist. Prof. Dr. Mustafa B. Dawood

Prof. Dr. Nameer A. Alwash

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جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة بابل
كلية الهندسة
قسم الهندسة المدنية

التحليل اللاخطي للأعتاب الخرسانية المسلحة مع وحدات خرسانية مطبوعة مسبقة الصب و الجهد

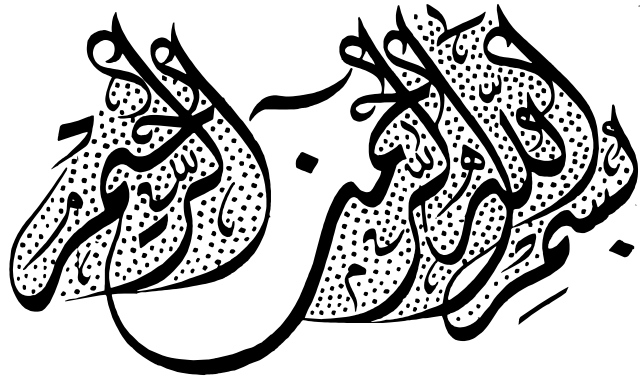
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من قبل

علي هادي عظيم

بإشراف

أ.م.د. مصطفى بلاسم داوود
أ.د. نمير عبد الأمير علوش



وَيَسْأَلُونَكَ عَنِ الرُّوحِ قُلِ الرُّوحُ مِنْ أَمْرِ
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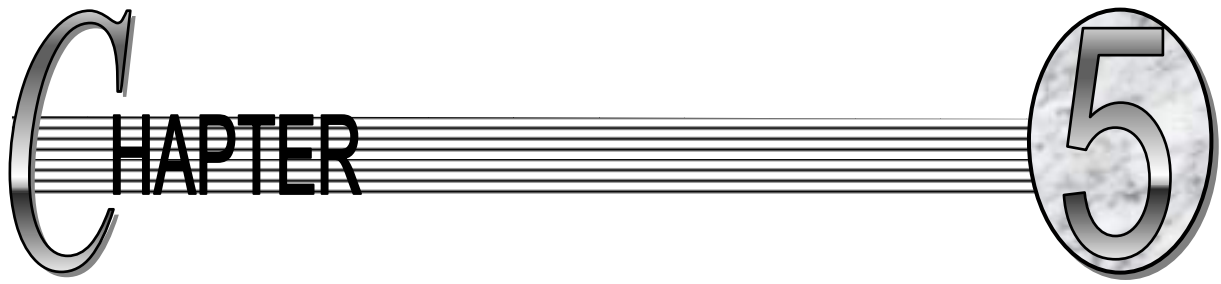
الخلاصة

يقدم هذا البحث نموذجاً لاختبار ثلاثي الأبعاد للعناصر المحددة ملائماً لتحليل العتبات الخرسانية المركبة مسبقة الجهد تحت تأثير عزوم الانحناء وذلك باستخدام برنامج الحاسبة التحليلي (ANSYS V. 9.0).

تم تمثيل الخرسانة العادية والخرسانة المسبقة الجهد باستخدام العناصر الطابوقية ثمانية العقدة. إما قضبان التسليح الاعتيادية والمسبقة الجهد فقد تم تمثيلها كعناصر محورية تربط نقاط متقابلة في العنصر الطابوقي مع فرض وجود تلاصق تام وتم تمثيل السطح البيني بعنصر التصاق لمقاومة الانزلاق والانفصال بين الخرسانة العادية والخرسانة المسبقة الجهد. تم اخذ التصرف اللاخطي للمواد والذي يشمل التشقق والتهشم للخرسانة والخضوع لحديد التسليح المسبق الجهد وقضبان التسليح الاعتيادية بنظر الاعتبار في التحليل. تم تمثيل الخرسانة المتشققة في الشد باستخدام نموذج التشقق المنتشر الثابت مع الأخذ بنظر الاعتبار صلادة الشد وانتقال قوى القص بعد التشقق.

تم حل معادلات التوازن اللاخطية بطريقة تزايدية – تكرارية وذلك بالاعتماد على طريقة (Full Newton-Raphson) لحساب الحل اللاخطي وكذلك الأخذ بمعيار القوة لتدقيق مقدار التقارب. أجريت التكاملات العددية باستخدام قاعدة كاوس (Gauss) في التكامل المختزل ذي الثمان نقاط للعناصر الطابوقية. ولغرض التأكد من كفاءة ودقة النماذج المستخدمة تم حل بعض الأمثلة العملية، كذلك تمت مقارنة الحلول مع النتائج العملية في الأمثلة. لوحظ الاختلاف في قيمتي الحمل الأقصى والهطول الأقصى مع النتائج المختبرية ضمن مدى (1-2.1) % و (1.22-7.7) % على الترتيب.

تم إجراء دراسات لبحث تأثير معاملات مختلفة. تُوضَّح النتائج التي تَرى يُدْكمية نسبة حديد التسليح مسبق الجهد تَرى يُدْصلابة الانثناء التي أفتُ إلى زيَّادة الحمل الأقصى ، الزيَّادة في الحمل الأقصى كَان (42.58 %) للعتب بسيط الإسناد و (29.52 %) للعتب المستمر.



CHAPTER 5

APPLICATIONS AND DISCUSSION

5-1: Introduction

The aim of this chapter is to verify the present nonlinear finite element model developed to investigate the behavior and ultimate load capacity of composite reinforced concrete members with embedded precast-prestressed concrete units with interface element. The numerical treatment has been carried out on four different composite concrete beams chosen from two available experimental studies in order to compare their results with the experimental results and use the same kind of models to solve the basic problem of the study. The results of the analysis of these examples are discussed and the load-deflection and moment-curvature behavior obtained by the finite element analysis are compared with those obtained from the available experimental investigation.

5-2: Verification Study on Bishara and Almeida Simply Supported Tested Beam [10].

Three simply supported composite beams tested by **Bishara** and **Almeida** (1971) [10], namely beams B2, B3 and B4, were selected in the present study for the analysis by the finite element method using the computer program (ANSYS 9.0). The following sections present a brief description of these beams and their numerical analysis under the effect of transverse loading.

5-2-1: Description of The Beam B2

The geometry, loading conditions and the reinforcement details for **Bishara** and **Almeida** composite beam are shown in figures (5-1). The simply supported beam of (2438.4 mm) span, (266.7 mm) width, (381 mm) depth and (338.84 mm) effective depth, was reinforced with two (15.88 mm) diameter mild steel bar at bottom, and top. The beam was provided with stirrups of (9.53 mm) diameter bars whose centers were spaced (76.2 mm) apart, also included one precast-prestressed concrete unit of sectional dimensions (38 mm) width and (68 mm) depth with total area of prestressing straight wire of (51.4 mm²).

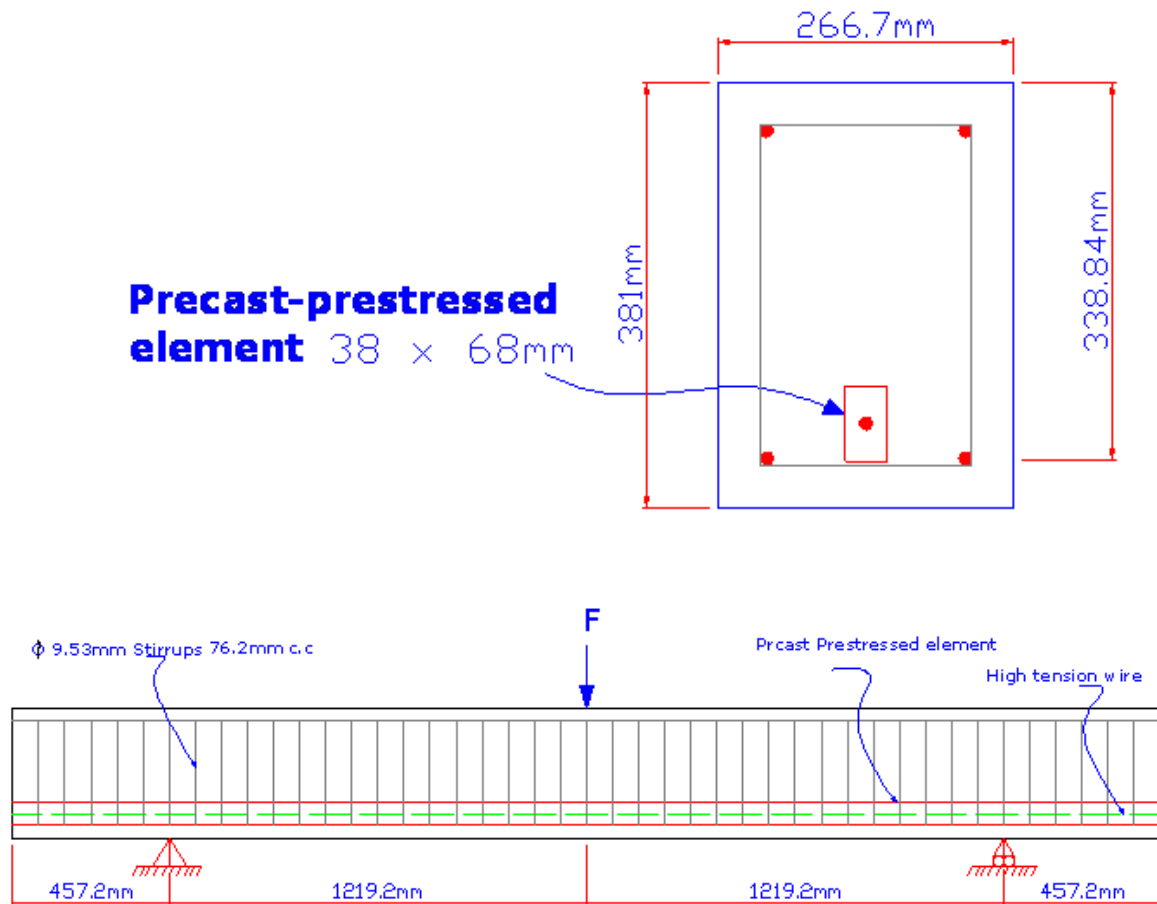


Figure (5-1): Dimensions and reinforcement of beam B2 tested by Bishara and Almeida [10].

5-2-1-1: Number of Elements

In this study, three different finite element meshes have been considered for the selected beam B2. These meshes contain (9948 elements), (6116 elements) and (4578 elements) as shown in figures (5-2), (5-3) and (5-4) respectively.

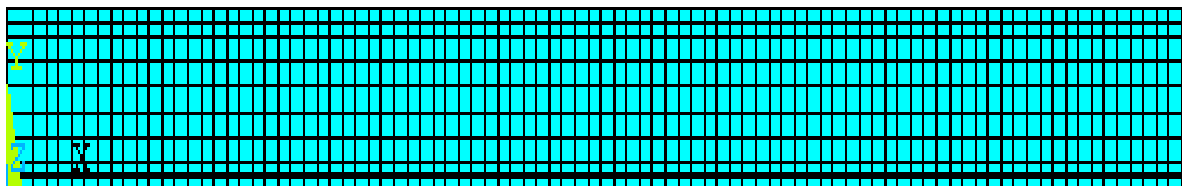


Figure (5-2): Finite element mesh (consists of 9948 –elements)

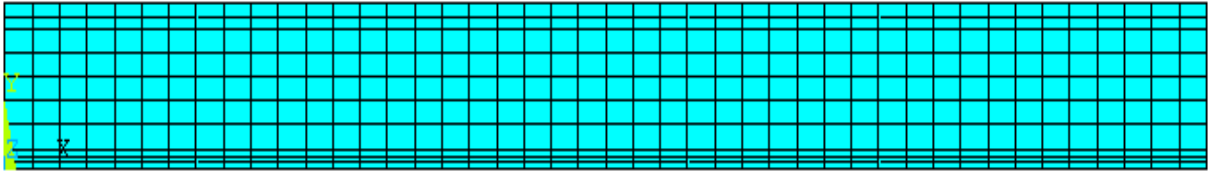


Figure (5-3): Finite element mesh (consists of 6116 –elements)

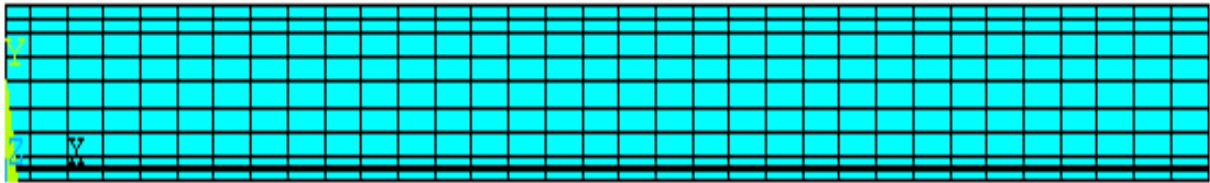


Figure (5-4): Finite element mesh (consists of 4578–elements)

Figure (5-5) shows the effect of number of elements on the load-deflection behavior of composite prestressed concrete beam. The increase in number of element beyond (6116) is insignificant because the response obtained for values of (6116) and (9948) are almost the same. So number of elements equal to (6116) is adopted here.

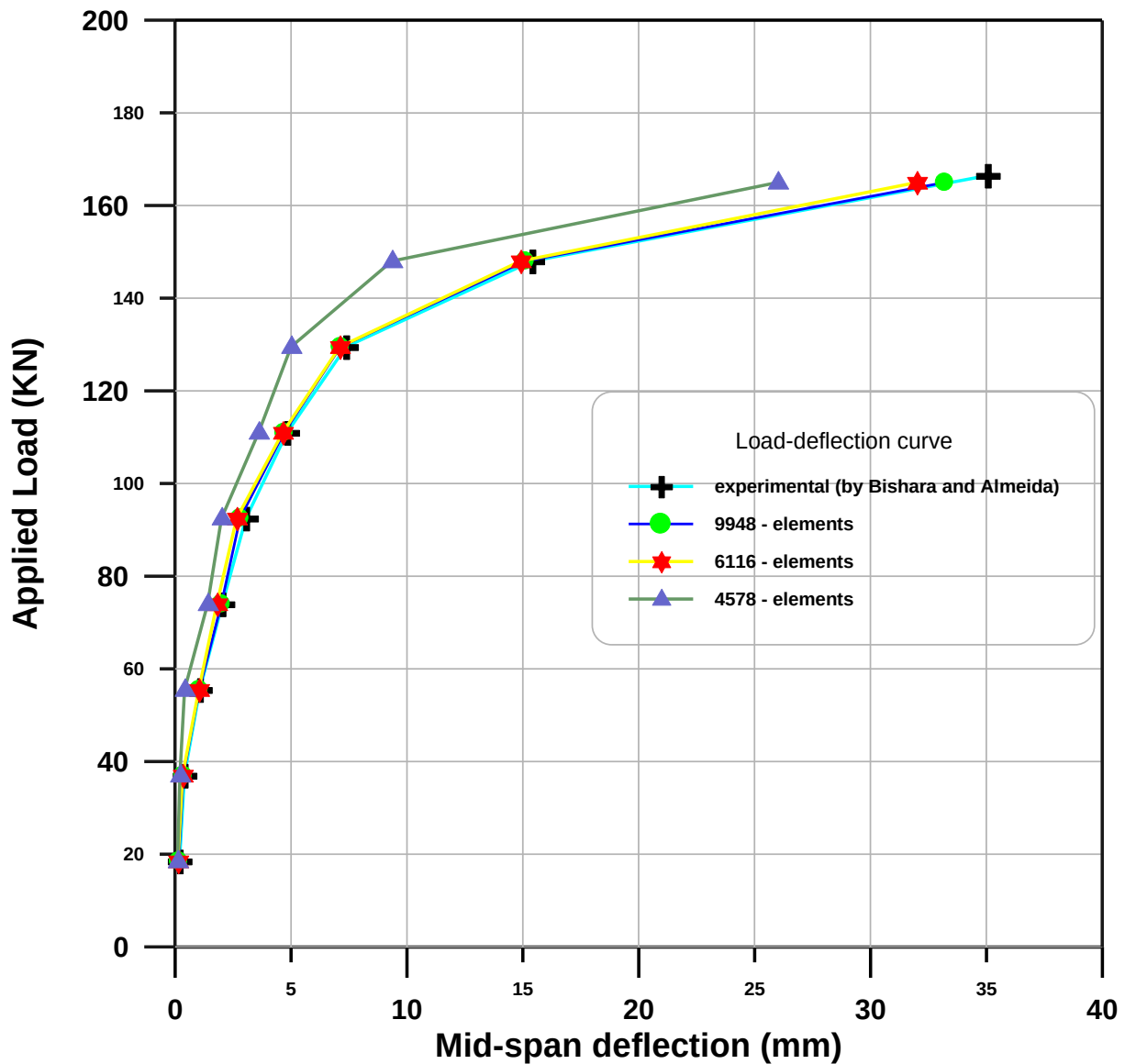


Figure (5-5): Effect of number of elements on load-deflection curve of composite prestressed concrete beam by Bishara and Almeida [10].

5-2-1-2: Finite Element Idealization

The three-dimensional finite element mesh of the Beam B2 (in the present study) has been analyzed by using (ANSYS 9.0) soft ware.

Concrete is idealized by using (3872) eight-nodded brick elements (Solid 65), the reinforcement is idealized by using (1540) link elements (Link 8), and the interface between the ordinary concrete members and prestressed concrete units is idealized by (704) two-nodded contact elements (Contact 52 point-to-point).

The total number of nodes resulting from the above idealization is (5265) nodes, and the total of elements is (6116) elements.

To avoid the crushing of the concrete under the point load, the mid span load was represented in the finite element model by 45 equivalent nodal forces in the middle of the beam, shown in figure (5-6).

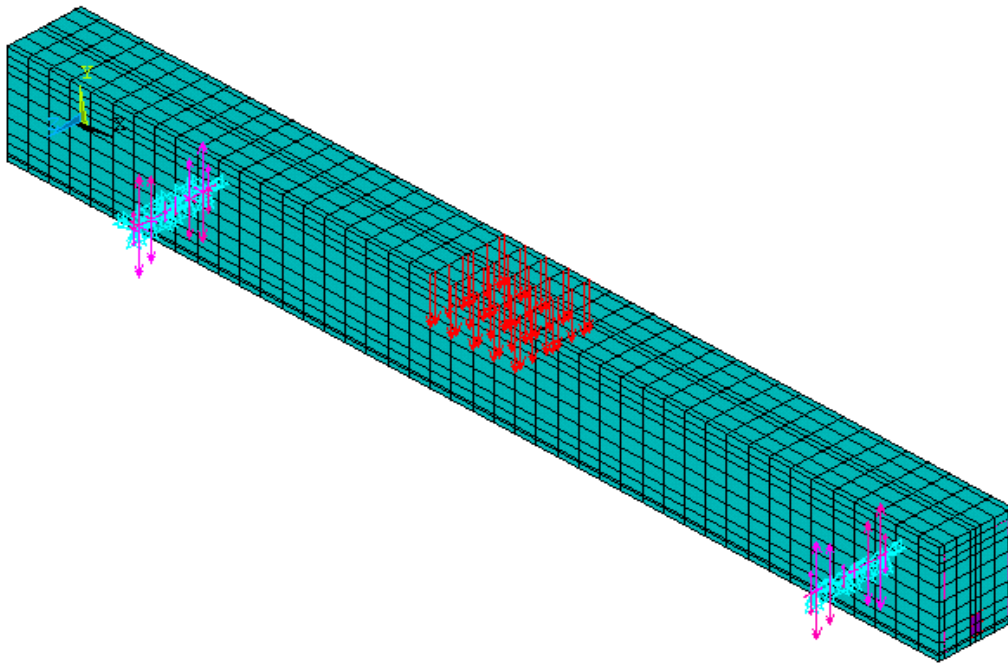


Figure (5-6): Finite elements mesh and load simulation for beam B2 used in ANSYS program

Material properties and the nonlinear solution parameters adopted in the analysis are given in table (5-1).

The mid span load of (166500 N) over 45 nodes is applying after the application of prestressing strain.

Table (5-1): Material properties and parameters used for Bishara and Almeida composite beam B2 [10].

Cast-in-place Concrete	E_c^{**}	Young's modulus	30419 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	41.887 MPa
	f_t^{***}	Tensile strength	4.1887 MPa
	β_o^*	Shear Transfer parameters	.1
	β_c^*		.99
Prestressed concrete	E_{PC}^{**}	Young's modulus	35982 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	58.61 MPa
	f_t^{***}	Tensile strength	5.861 MPa
	β_o^*	Shear Transfer parameters	0.1
	β_c^*		0.99
Reinforcing steel	E_s^*	Young's modulus	200,000 MPa
	f_y	Yield stress	310.275 (ϕ 16) MPa
			376.19 (ϕ 10) MPa
	ν_s^*	Poisson's ratio	0.3
High tension wire	E_s^{****}	Young's modulus	194439 MPa
	f_y	Yield stress	1592.745 MPa
	ν_s^*	Poisson's ratio	0.3

(*) Assumed value, (**) $E_c = 4700\sqrt{f'_c}$, (***) $f_t = 0.1 f'_c$

(**) ACI-Building Code 318M-05 [61].

(***) Chen (1981) [57].

(****) Bishara and Almeida [10].

5-2-1-3: Results of Analysis

This section presents the obtained results of the finite element analysis carried out for the chosen simply supported composite prestressed concrete beam. The experimental is load-deflection curves of **Bishara** and **Almeida** composite beam (B2) and numerical load-deflection curves shown in figure (5-7). Good agreement is obtained between the present finite element and the experimental load-deflection curves throughout the entire range of behavior. The analytical ultimate deflection (32.614 mm) is detected quite well compared with that experimentally observed (35 mm), with an error of only (6.8%). The difference between the experimental ultimate load and that obtained by finite element analysis is not more than (2.1%).

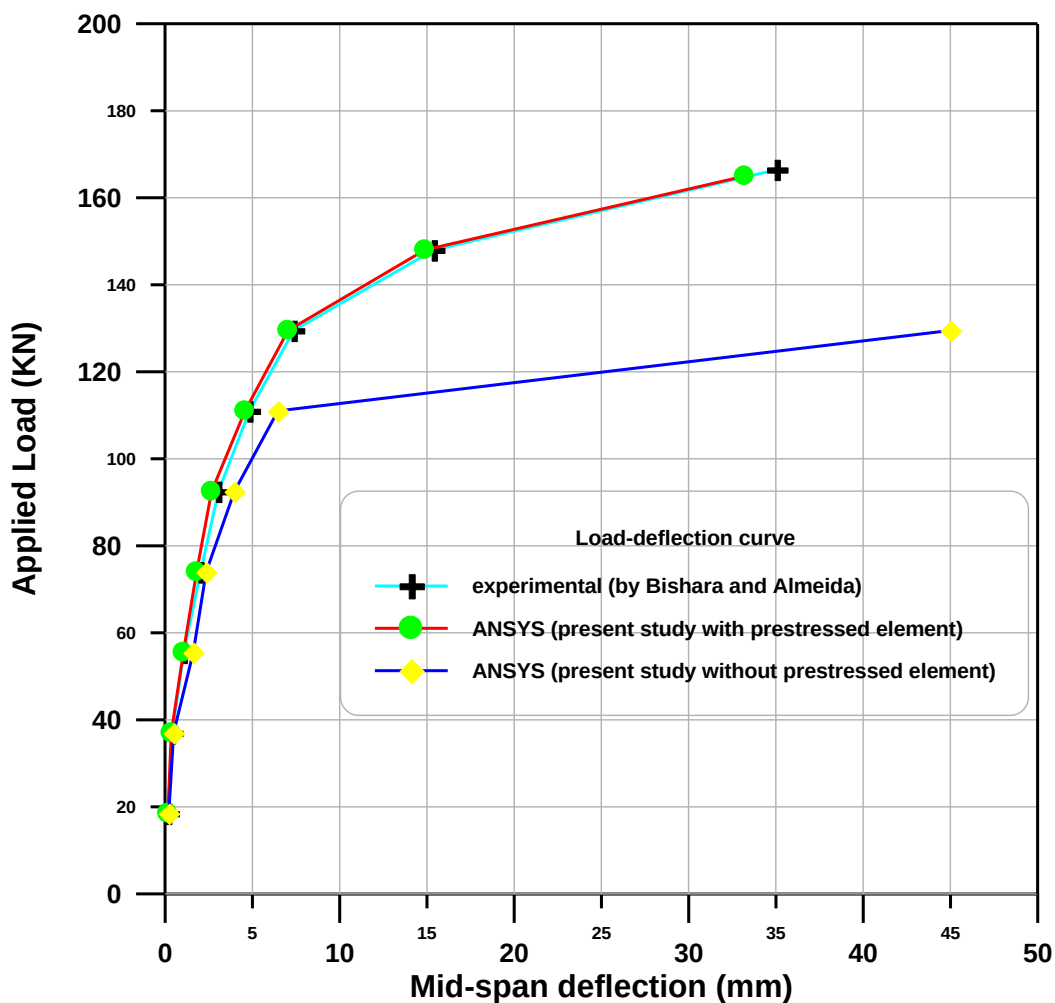


Figure (5-7): Load-deflection curve for beam B2

Figure (5-8) shown the cracks first close to the mid-span of the beam (B2) at

applied level ($F = 51231 \text{ N}$). The cracking and crushing shown in figure (5-9) at the ultimate load. It can be seen the cracks extended towards the mid-depth of the beam as the applied load is increased.

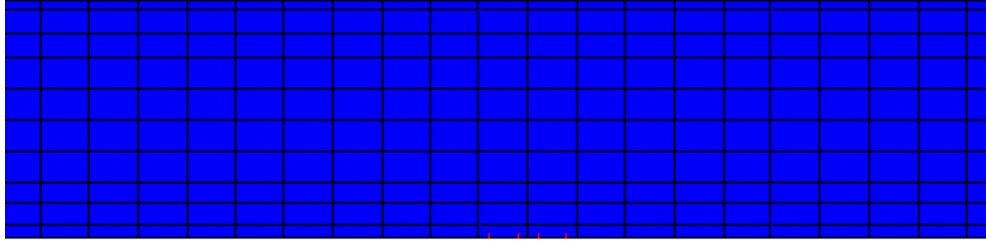


Figure (5-8): First crack for beam B2

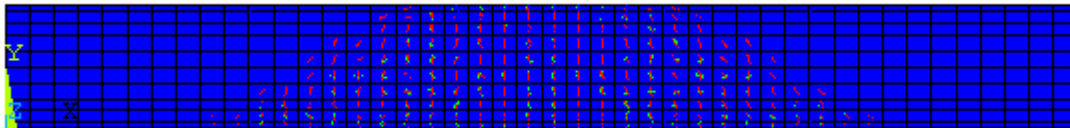


Figure (5-9): Cracking and crushing for beam B2

5-2-2: Description of The Beam B3

A simply supported composite beam under one concentrated load has been analyzed to investigate the applicability and accuracy of the computer program results. Pertinent details for **Bishara** and **Almeida** composite beam are shown in figure (5-10). The rectangular cross-section of the beam was (266.7 mm) wide and (381 mm) deep. The effective depth for the ordinary steel was (338.84 mm). figure (5-10) shows the internal reinforcement, two (15.88 mm) diameter bars were included in the compression zone, one (15.88 mm) diameter bars was included in the tension zone, and (9.53 mm) mild steel bars were provided at (76.2 mm) for shear reinforcement. Also, including two prestressed reinforcing element consisted of a concrete prism (38 mm) wide and (68 mm) depth axially prestressed by (9.52 mm) diameter 7-wire strand.

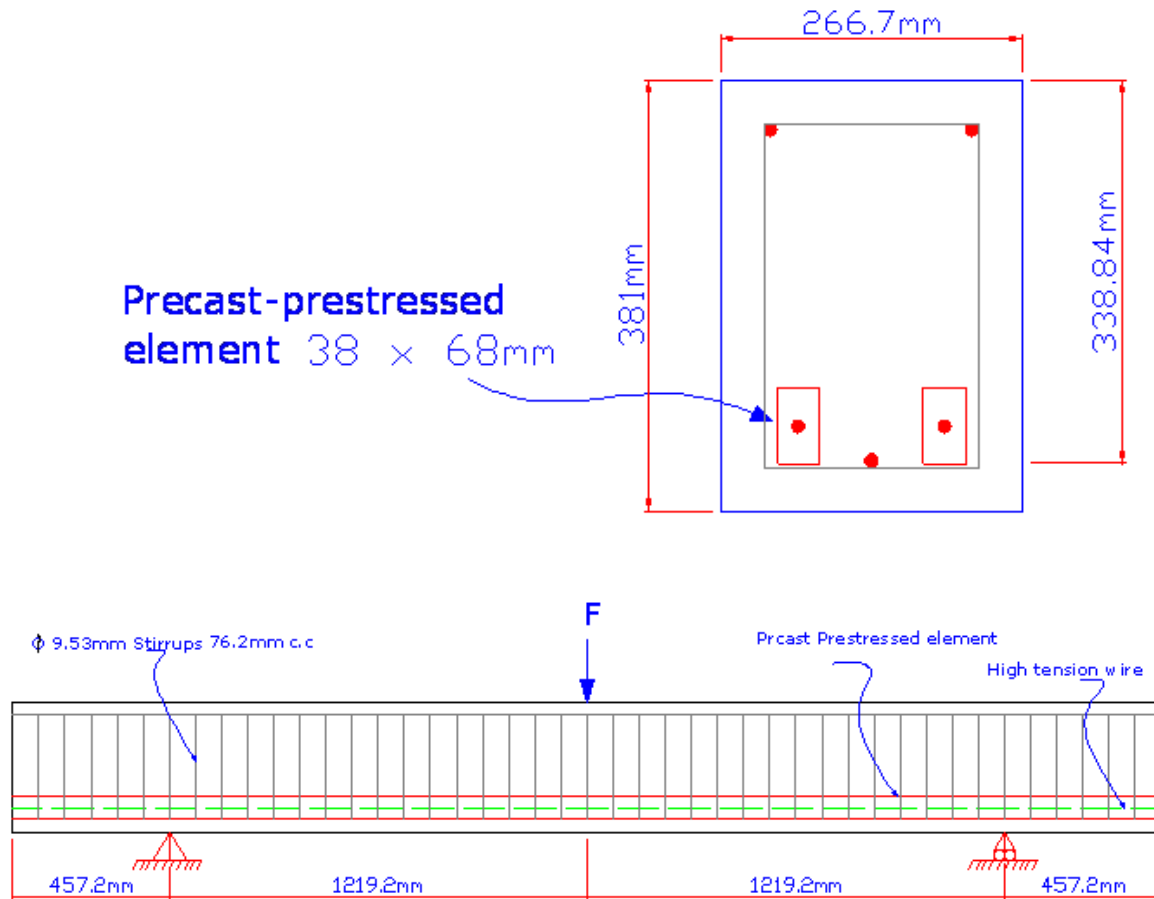


Figure (5-10): Dimensions and reinforcement of beam B3 tested by Bishara and Almeida [10].

5-2-2-1: Finite Element Idealization

The requirement to simulate the nonlinear response of ordinary reinforced concrete beams and precast-prestressed units demands the use of the dedicated (SOLID 65) element in (ANSYS 9.0). Both concrete types are idealized by using (3872) eight-node brick elements.

The reinforcement for the beam B3 is idealized by using spar elements, (Link 8), and the interface between the ordinary concrete beams and prestressed concrete units is idealized by contact elements, (Contact 52 point-to-point). The total number of nodes resulting from the above idealization is (5760) nodes, and the total number of elements is (8008) elements.

The external load (154530 N) was modeled by 45 equivalent nodal forces these were applied at the mid span, shown in figure (5-11) to avoid the crushing of the concrete.

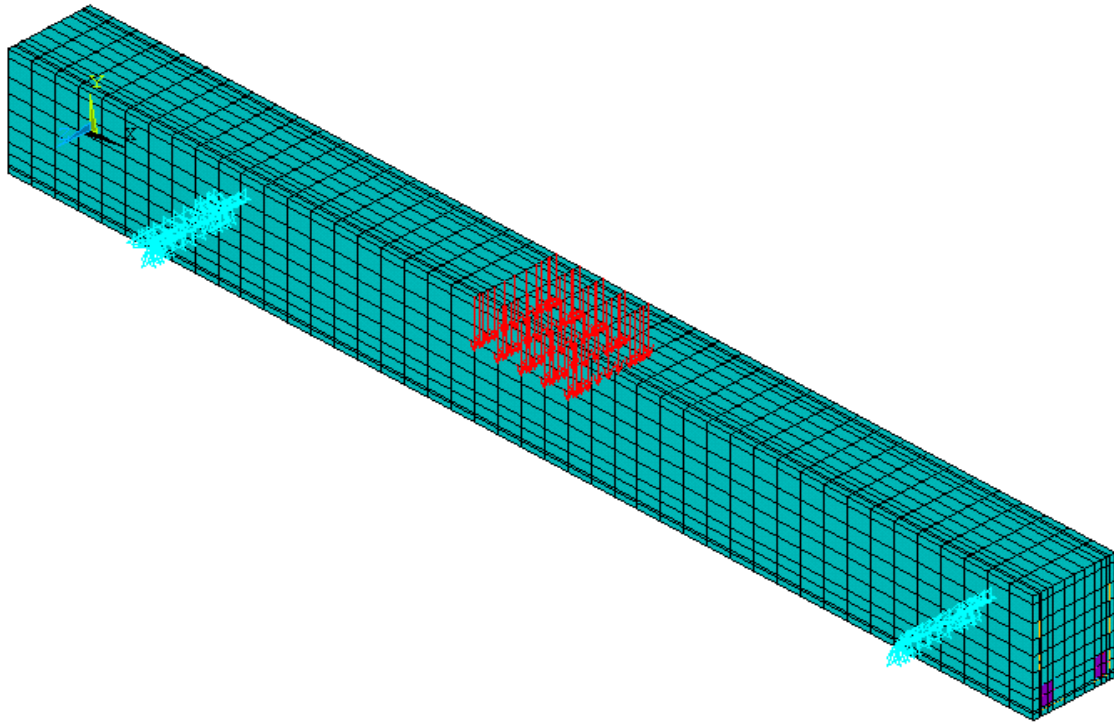


Figure (5-11): Finite elements mesh and load simulation for beam B3 used in ANSYS program

Material properties and the nonlinear solution parameters adopted in the analysis are given in table (5-2).

Table (5-2): Material properties and parameters used for Bishara and Almeida composite beam B3 [10].

Cast-in-place Concrete	E_c^{**}	Young's modulus	31342 MPa
	ν_c^*	Poisson's ratio	0.2
	f_c'	Compressive strength	44.4689 MPa
	f_t^{***}	Tensile strength	4.4.689MPa
	β_o^*	Shear Transfer	.1

	β_c^*	parameters	.99
Prestressed concrete	E_{PC}^{**}	Young's modulus	35982 MPa
	ν_c^*	Poisson's ratio	0.2
	f_c'	Compressive strength	58.61 MPa
	f_t^{***}	Tensile strength	5.861 MPa
	β_o^*	Shear Transfer parameters	0.1
	β_c^*		0.99
Reinforcing steel	E_s^*	Young's modulus	200,000 MPa
	f_y	Yield stress	310.275 (ϕ 16) MPa
			376.19 (ϕ 10) MPa
	ν_s^*	Poisson's ratio	0.3
High tension wire	E_s^{****}	Young's modulus	194439 MPa
	f_y	Yield stress	1592.745 MPa
	ν_s^*	Poisson's ratio	0.3

(*) Assumed value, (**) $E_c = 4700\sqrt{f_c'}$, (***) $f_t = 0.1 f_c'$

(**) ACI-Building Code 318M-05 [61].

(***) Chen (1981) [57].

(****) Bishara and Almeida [10].

5-2-2-2: Results of Analysis

Figure (5-12) shows that the load-deflection plotted from the present finite element analysis agree well with the experimental results by **Bishara** and **Almeida**. The experimental ultimate deflection is obtained by **Bishara** and **Almeida** is equal to (17.78 mm) and the numerical ultimate deflection is equal to (18 mm) with a difference (1.22%). The ultimate load is (157 KN) by finite elements while the experimental load is (154.53 KN) by **Bishara** and **Almeida** tested. The ratio of the predicted ultimate load to the corresponding experimental load is about (1.016). These results indicate very well agreement.

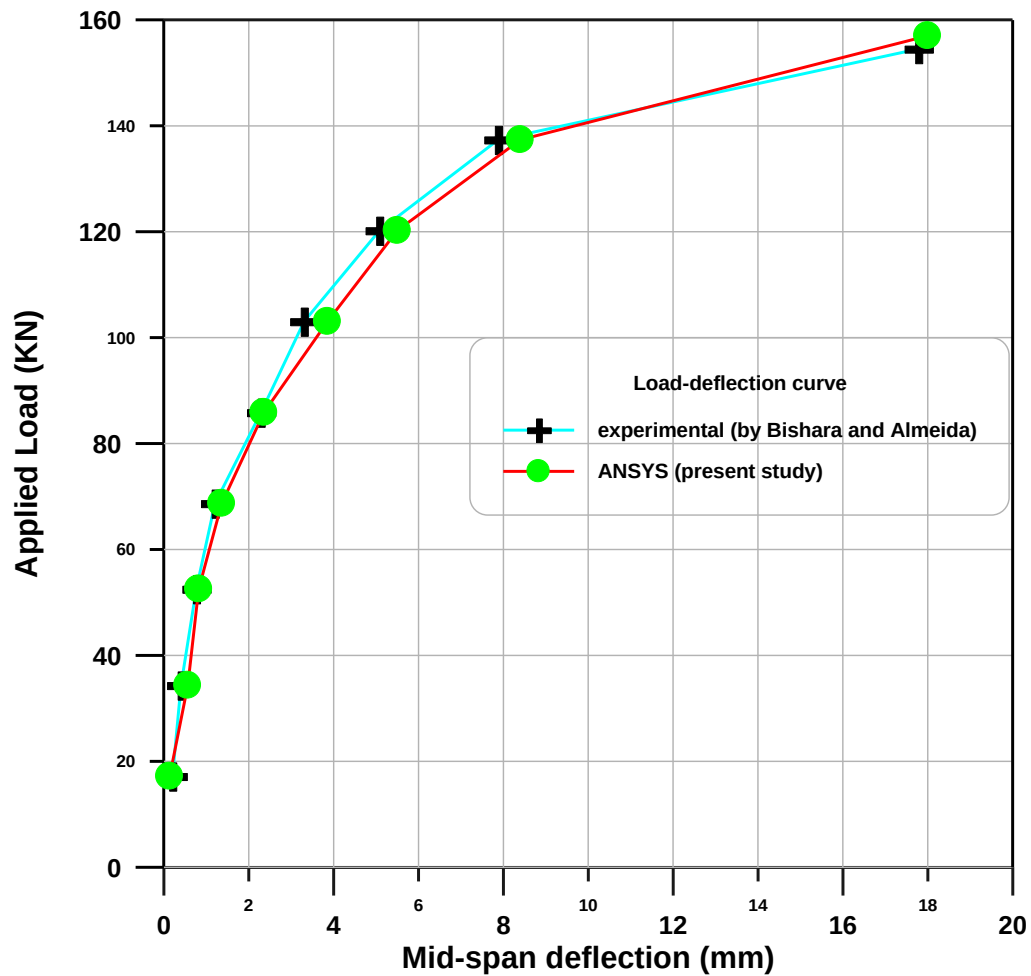


Figure (5-12): Load-deflection curve for beam B3

The cracks first appear at the mid-span of the beam (B3) at applied load (71323 N), shown in figure (5-13). Both concrete types are cracking and crushing at the ultimate load, shown in figure (5-14). The cracks extended towards the mid-depth of the beam as the applied load is increased.

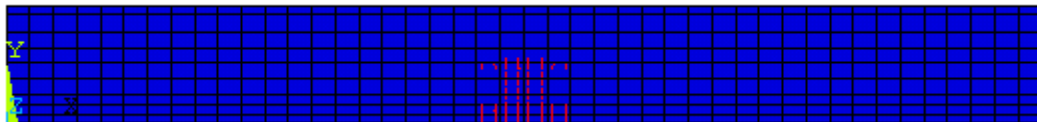


Figure (5-13): First crack for beam B3

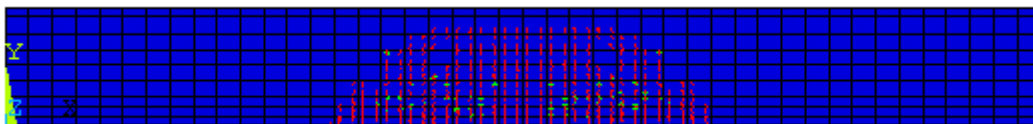


Figure (5-14): Cracking and crushing for beam B3

5-2-3: Description of The Beam B4

A cross section through the long beam, figure (5-15), illustrates the internal reinforcement. Two (15.88 mm) diameter bars were included in the compression zone. Fourteen-three shear links, formed from (9.53 mm) mild steel bars, were provided at (76.2 mm) centers for shear reinforcement in the shear span. Also, include three precast-prestressed units of (38 mm) wide and (68 mm) deep with total area of prestressing straight wire was (154.4 mm²).

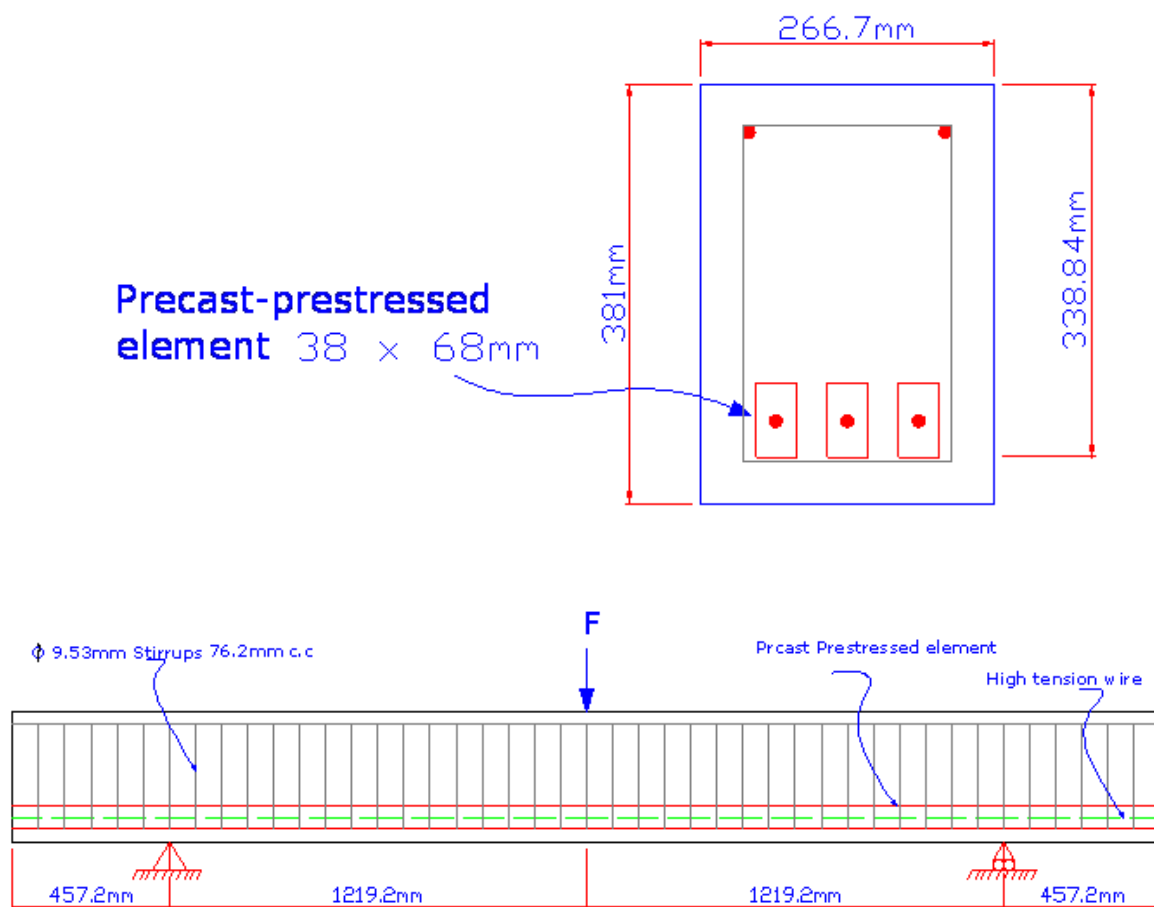


Figure (5-15): Dimensions and reinforcement of beam B4 tested by Bishara and Almeida [10].

5-2-3-1: Finite Element Idealization

The finite element idealization of prestressed concrete composite beam is shown in figure (5-16). This beam is subjected to central point loading. Ordinary reinforced concrete and prestressed concrete were simulated by (5310) isoperimetric eight-nodded brick elements (SOLID 65) connected by (1848) two node interface elements (Contact 52 point-to-point), while the reinforcement was modeled by (184) link elements (Link 8).

The finite element meshes, boundary conditions and loading arrangement are show in figure (5-12).

The mid span load of (137160 N) over 65 nodes is applying after the prestressing strain application.

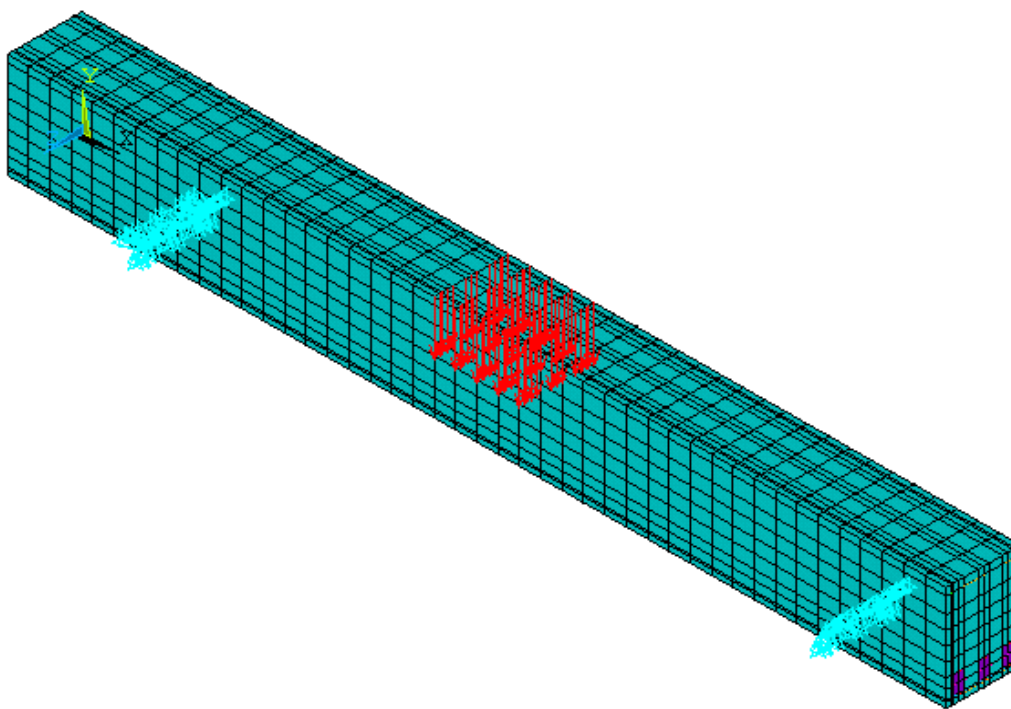


Figure (5-16): Finite elements mesh and load simulation for beam B4 used in ANSYS program

Material properties and the nonlinear solution parameters adopted in the analysis are given in table (5-3).

Table (5-3): Material properties and parameters used for Bishara and Almeida composite beam B4 [10].

Cast-in-place Concrete	E_c^{**}	Young's modulus	31198 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	44.06 MPa
	f_t^{***}	Tensile strength	4.406MPa
	β_o^*	Shear Transfer parameters	.1
	β_c^*		.99
Prestressed concrete	E_{PC}^{**}	Young's modulus	35982 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	58.61 MPa
	f_t^{***}	Tensile strength	5.861 MPa
	β_o^*	Shear Transfer parameters	0.1
	β_c^*		0.99
Reinforcing steel	E_s^*	Young's modulus	200,000 MPa
	f_y	Yield stress	310.275 (ϕ 16) MPa
			376.19 (ϕ 10) MPa
	ν_s^*	Poisson's ratio	0.3
High tension wire	E_s^{****}	Young's modulus	194439 MPa
	f_y	Yield stress	1592.745 MPa
	ν_s^*	Poisson's ratio	0.3

$$(*) \text{ Assumed value }, (**) E_c = 4700\sqrt{f'_c}, (***) f_t = 0.1 f'_c$$

(**) ACI-Building Code 318M-05 [61].

(***) Chen (1981) [57].

(****) Bishara and Almeida [10].

5-2-3-2: Results of Analysis

Again, the results of the beam (B4) show a good agreement between the

experimental results by **Bishara** and **Almeida** and the present finite element analysis. The experimental ultimate deflection was (8.3518 mm) while the present numerical deflection was (8.9 mm) with a difference (6.16%) between experimental and numerical deflection.

A comparison between the experimental load-deflection curve by (**Bishara** and **Almeida**) and the present load-deflection curve were shown in figure (5-17). The difference between the experimental ultimate load and that obtained by finite element analysis is not more than (1%).

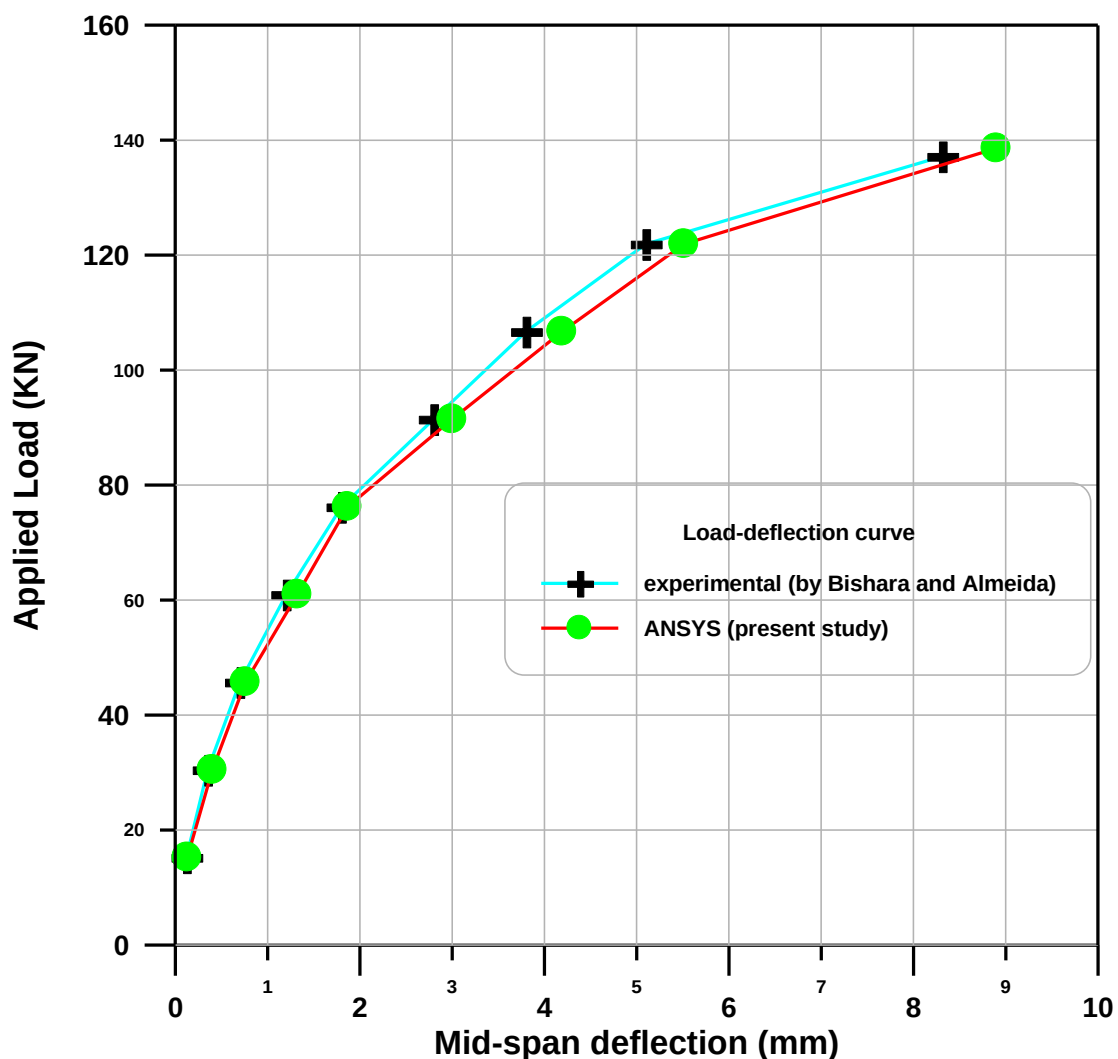


Figure (5-17): Load-deflection curve for beam B4

Figure (5-18) shows distribution of cracks first in the beam at load level ($F=81323$ N). At the ultimate load the cracking and crushing occurred, shown in

figure (5-19). At the applied load is increased the cracks extended towards the mid-depth of beam.

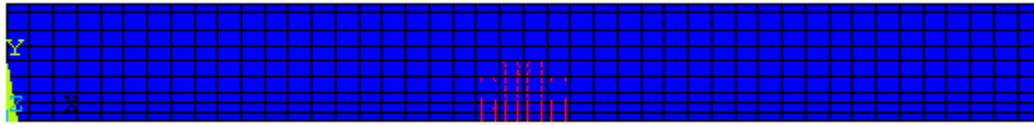


Figure (5-18): First crack for beam B4

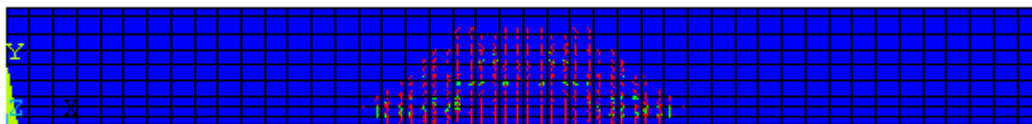


Figure (5-19): Crack and crushing for beam B4

5-3: Verification Study on Bishara et al. [11] Continuous Tested Beam.

The continuous composite beam of **Bishara et al.** was selected in the present study for the analysis by the finite element method using the computer program (ANSYS 9.0). Because that the selected two-span continuous beam is symmetric about its center support only one half of the beam was analyzed.

5-3-1: Description of the Beam BC2

The geometry, loading conditions and the reinforcement details for **Bishara et al.** composite beam are shown in figure (5-20). The continuous composite beam of (4699 mm) span, (266.7 mm) width, (381 mm) depth and the effective depth for the ordinary steel at support was (342.47 mm) and at center was (316.92 mm). The ordinary steel bars were six (15.88mm) mild steel located close to the bottom face and two (9.53) mild steel located close to the top face at load exterior system support. The central support section including two (15.88 mm) mild steel bars located close to the bottom face and two (15.88) mild steel

bars located close to top face. Also, including one precast-prestressed concrete units of (38 mm) width and (68 mm) depth cross-section with total area of prestressing straight wire was (51.4 mm^2) . the load was applied at (2065.8 mm) from central support.

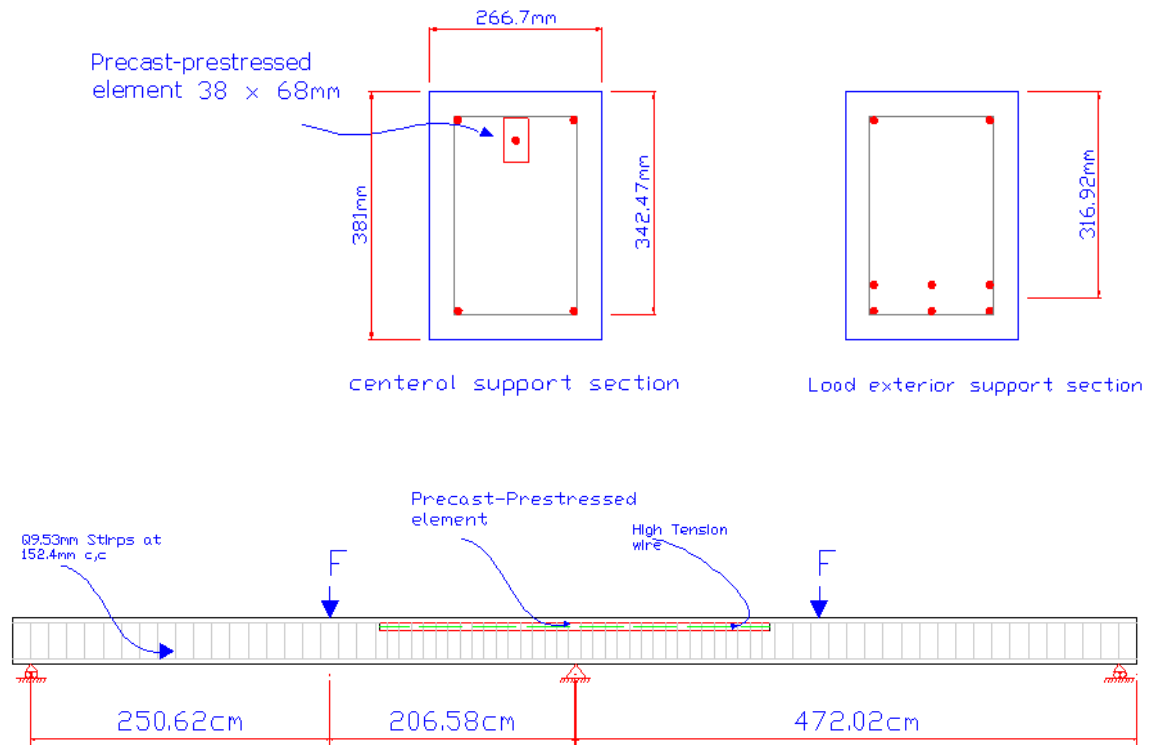


Figure (5-20): Dimensions and reinforcement of beam BC2 tested by Bishara et al. [11].

5-3-1-1: Finite Element Idealization

The three-dimensional finite element mesh of the half of the Beam B2 (in the present study) has been analyzed by using (ANSYS 9.0) soft ware.

The concrete in these beam is represented by (3640) eight-node isoperimetric brick elements (SOLID65) , while the reinforcement is represented by (2015) links element (Link 8) and the two noded interface element (Contact 52 point-to-point) is suitable to represent the bond between the ordinary concrete beam and prestressed concrete units. The external load was applied at (2065.8 mm) from central support. In the finite element model the point load

was modeled by 27 equivalent nodal forces, shown in figure (5-21).

The total number of nodes resulting from the above idealization is (4725) nodes, and the total of elements is (6055) elements.

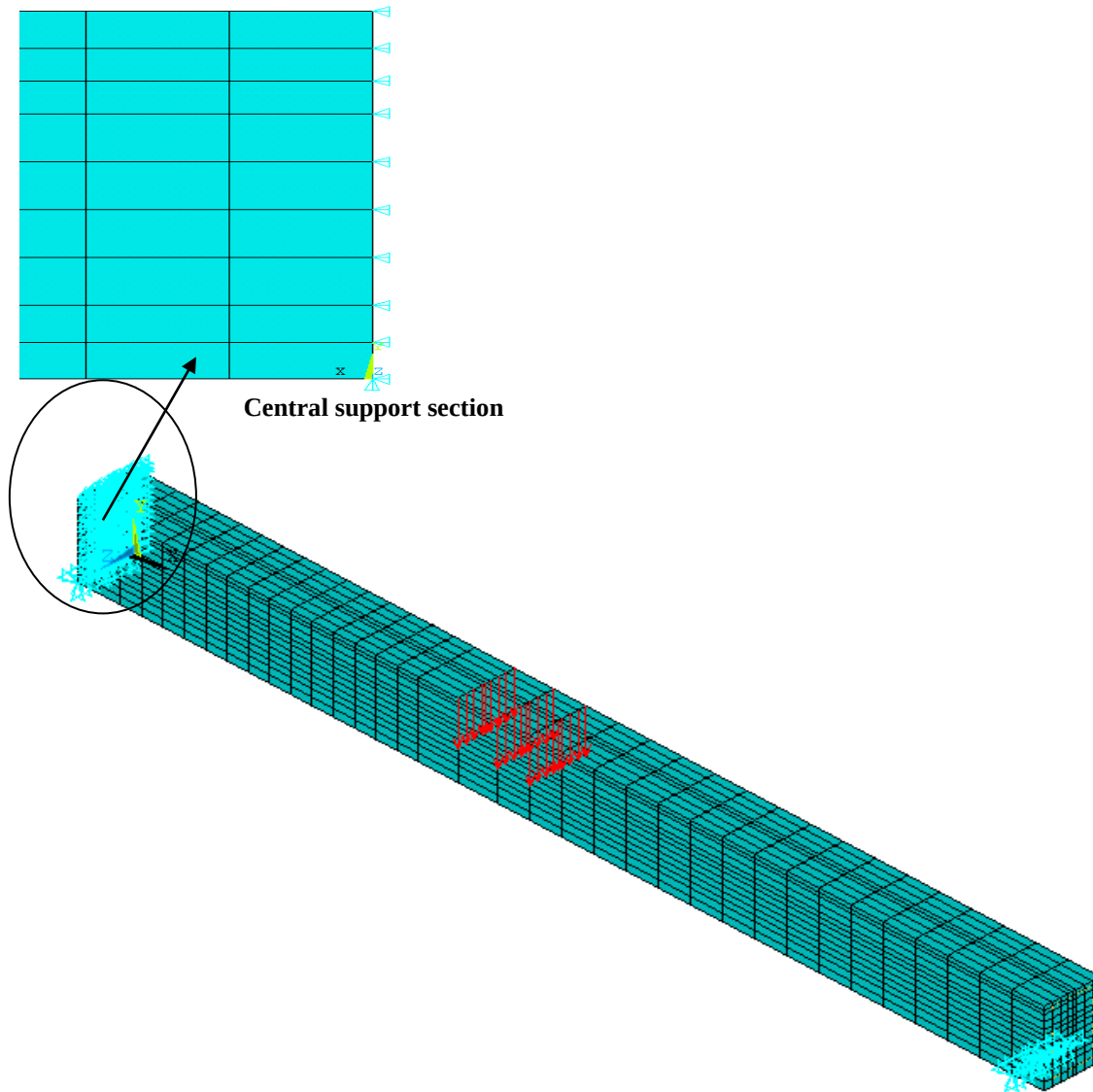


Figure (5-21) Finite elements mesh and load simulation for beam BC2 used in ANSYS program

Material properties and the nonlinear solution parameters adopted in the analysis are given in table (5-4).

Table (5-4): Material properties and parameters used for Bishara et al. composite beam BC2 [10].

Cast-in-place Concrete	E_c^{**}	Young's modulus	29310 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	38.888 MPa
	f_t^{***}	Tensile strength	3.8888 MPa
	β_o^*	Shear Transfer parameters	.1
	β_c^*		.99
Prestressed concrete	E_{PC}^{**}	Young's modulus	35982 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	58.61 MPa
	f_t^{***}	Tensile strength	5.861 MPa
	β_o^*	Shear Transfer parameters	0.1
	β_c^*		0.99
Reinforcing steel	E_s^*	Young's modulus	200,000 MPa
	f_y	Yield stress	310.275 (ϕ 16) MPa
			376.19 (ϕ 10) MPa
	ν_s^*	Poisson's ratio	0.3
High tension wire	E_s^{****}	Young's modulus	194439 MPa
	f_y	Yield stress	1592.745 MPa
	ν_s^*	Poisson's ratio	0.3

(*) Assumed value, (**) $E_c = 4700\sqrt{f'_c}$, (***) $f_t = 0.1 f'_c$

(**) ACI-Building Code 318M-05 [61].

(***) Chen (1981) [57].

(****) Bishara et al. [11].

5-3-1-2: Results of Analysis

The experimental and analytical moment-curvature curve obtained for continuous composite prestressed concrete beam are shown in figure (5-22). Good agreement is obtained between the predicted finite element and the experimental moment-curvature curve throughout the entire range of behavior. The analytical ultimate curvature (6×10^{-5} 1/mm) as detected by finite element is quite well compared with that experimentally observed (6.5×10^{-5} 1/mm), with a difference of only (7.7%).

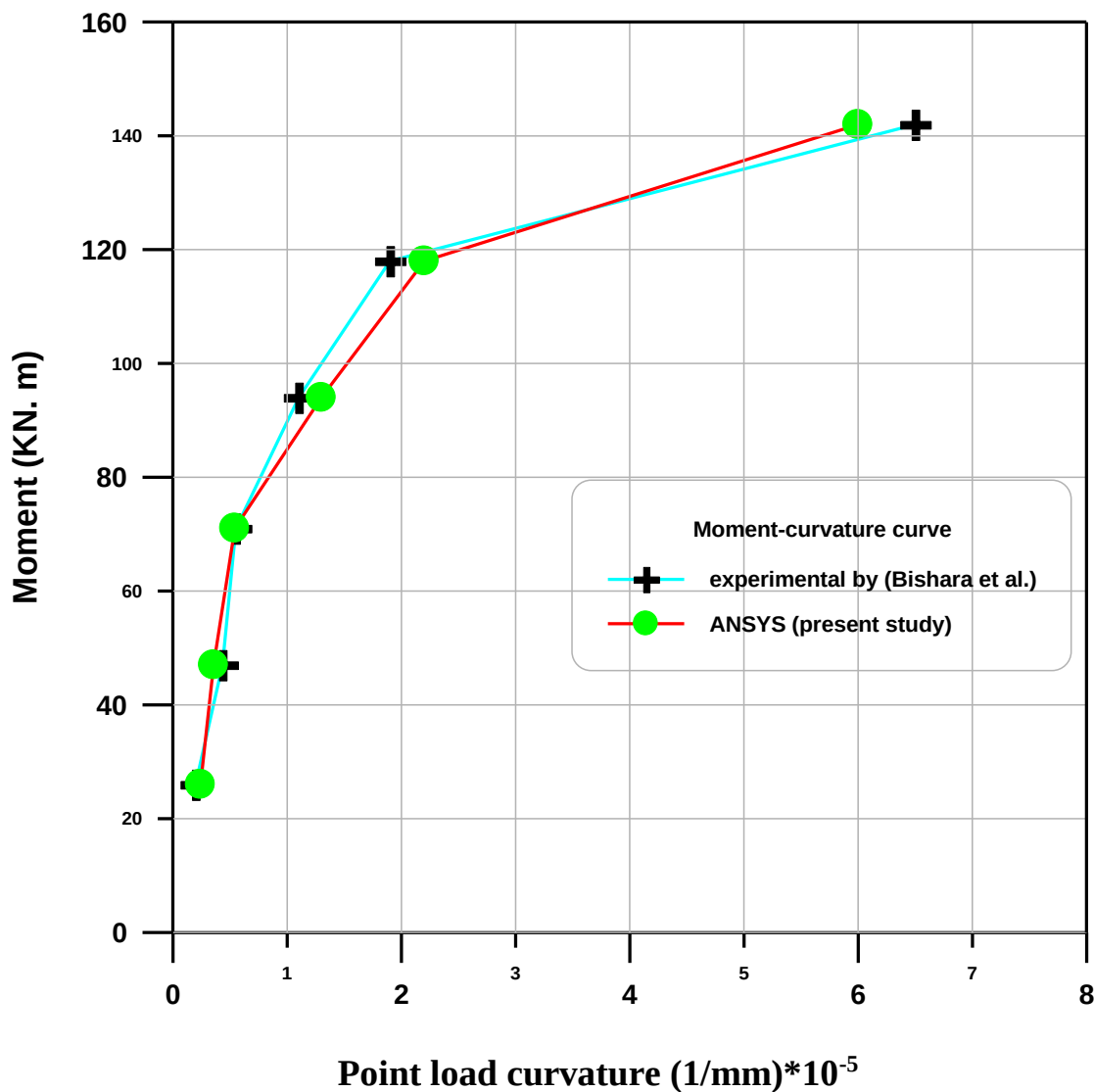


Figure (5-22): Moment-curvature curve for continuous beam BC2

The first cracks appear close to the central support of the beam (BC2) at load level ($F = 42145$ N), shown in figure (5-23). The distribution of cracks at the

mid-span at applied load ($F = 56193 \text{ N}$), shown in figure (5-24). All cracks shown in figure (5-25) at the ultimate load.

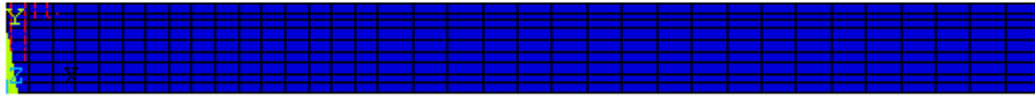


Figure (5-23): First crack for beam BC2

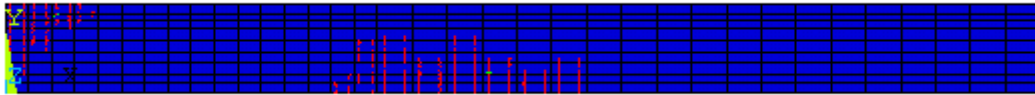


Figure (5-24): Crack at mid-span for beam BC2

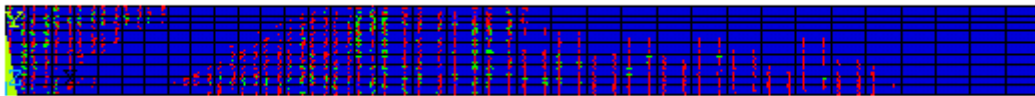
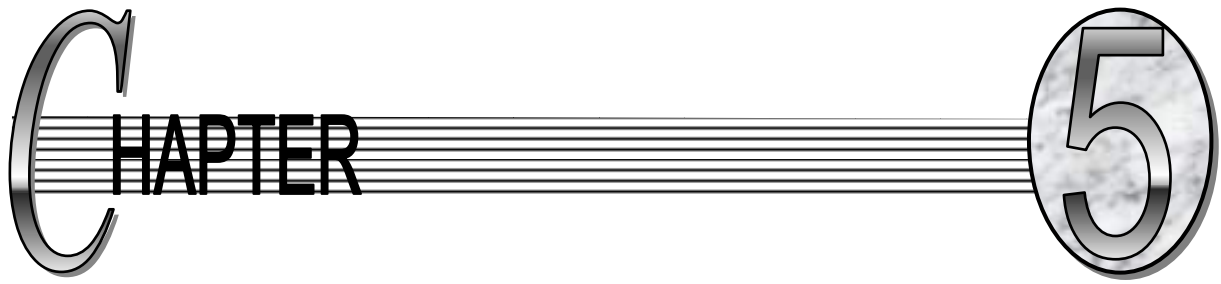


Figure (5-25): Cracking and crushing for beam BC2



CHAPTER 5

APPLICATIONS AND DISCUSSION

5-1: Introduction

The aim of this chapter is to verify the present nonlinear finite element model developed to investigate the behavior and ultimate load capacity of composite reinforced concrete members with embedded precast-prestressed concrete units with interface element. The numerical treatment has been carried out on four different composite concrete beams chosen from two available experimental studies in order to compare their results with the experimental results and use the same kind of models to solve the basic problem of the study. The results of the analysis of these examples are discussed and the load-deflection and moment-curvature behavior obtained by the finite element analysis are compared with those obtained from the available experimental investigation.

5-2: Verification Study on Bishara and Almeida Simply Supported Tested Beam [10].

Three simply supported composite beams tested by **Bishara** and **Almeida** (1971) [13], namely beams B2, B3 and B4, were selected in the present study for the analysis by the finite element method using the computer program (**ANSYS 9.0**). The following sections present a brief description of these beams and their numerical analysis under the effect of transverse loading. The only variable in the beams B2 to B4 was the number of prestressed units and (15.88 mm) deformed bars acting as tension reinforcement. Their ratio was varied to give approximately the same ultimate moment capacity computed on the basis of yield strength of steel.

5-2-1: Description of The Beam B2

The geometry, loading conditions and the reinforcement details for **Bishara** and **Almeida** composite beam are shown in figure (5-1). The simply supported beam of (2438.4 mm) span, (266.7 mm) width, (381 mm) depth and (338.84 mm) effective depth, is reinforced with two (15.88 mm) diameter mild steel bar at bottom, and top. The beam is provided with stirrups of (9.53 mm) diameter bars whose centers were spaced (76.2 mm) apart, also included one precast-prestressed concrete unit of sectional dimensions (38 mm) width and (68 mm) depth with total area of prestressing straight wire of (51.4 mm²).

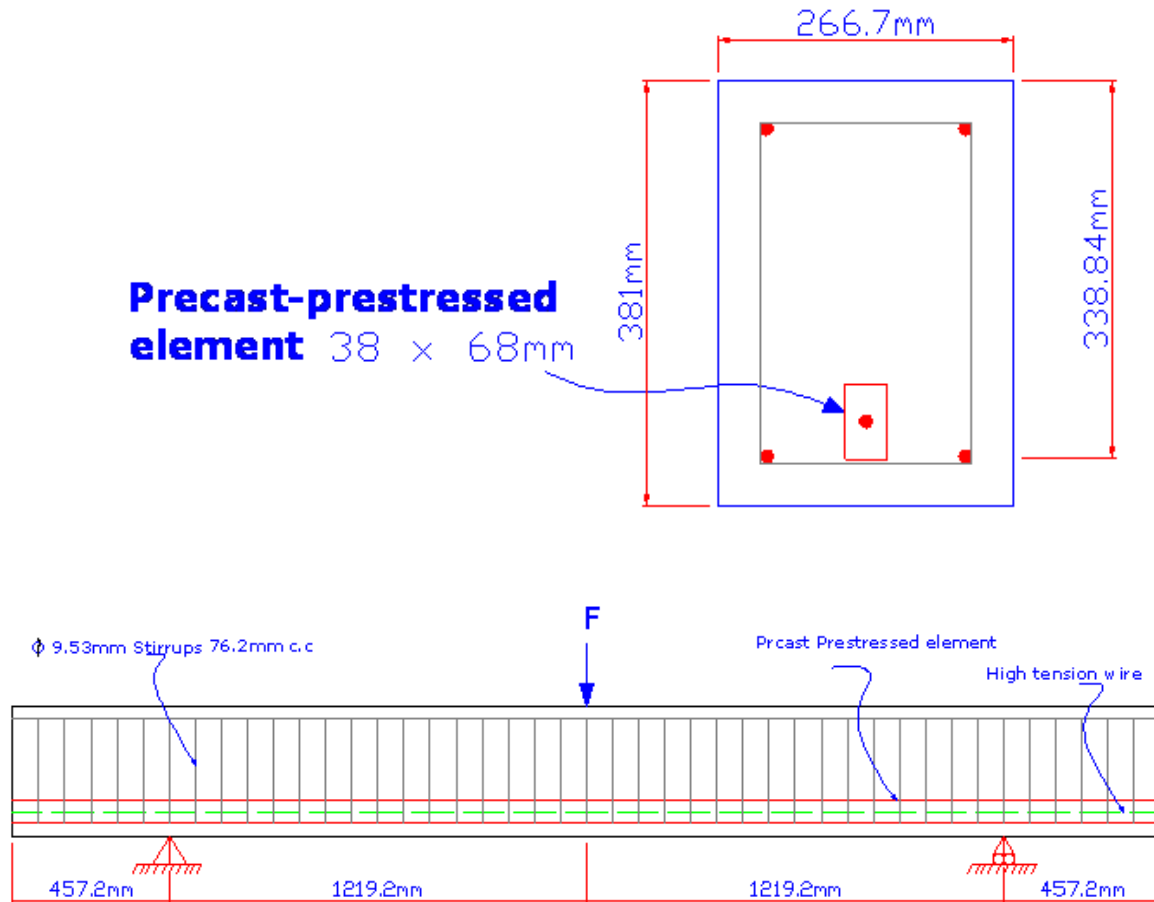


Figure (5-1): Dimensions and reinforcement of beam B2 tested by Bishara and Almeida [13].

5-2-1-1: Number of Elements

In this study, three different finite element meshes have been considered for the selected beam B2. These meshes contain (9948 elements), (6116 elements) and (4578 elements).

Figure (5-2) shows the effect of number of elements on the load-deflection behavior of composite prestressed concrete beam. The increase in number of element beyond (6116) is insignificant because the response obtained for values of (6116) and (9948) are almost the same. So number of elements equal to (6116) is adopted here.

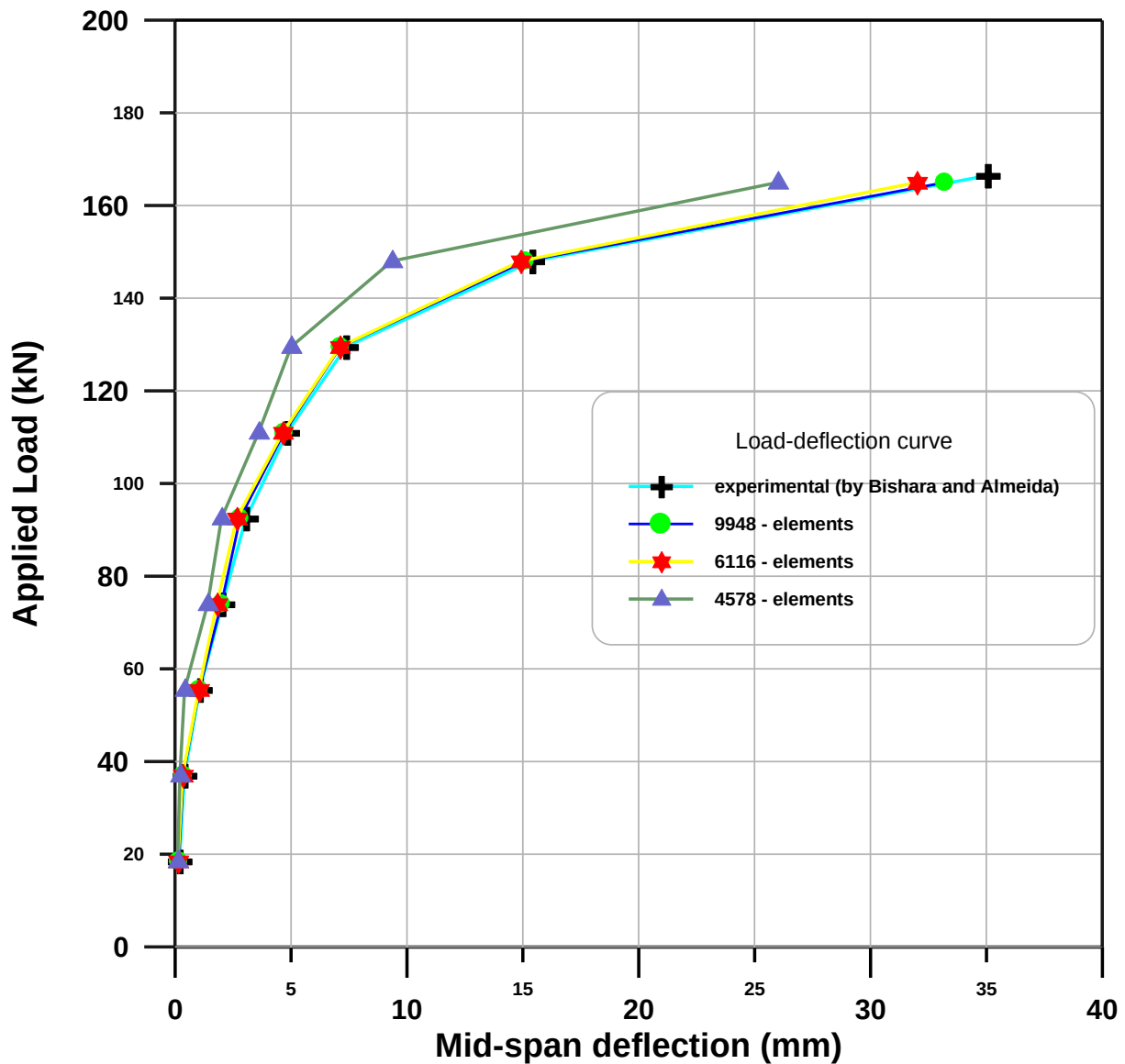


Figure (5-2): Effect of number of elements on load-deflection curve of composite prestressed concrete beam by Bishara and Almeida [13].

5-2-1-2: Finite Element Idealization

The three-dimensional finite element mesh of the Beam B2 (in the present study) has been analyzed by using (ANSYS 9.0) soft ware.

Concrete is idealized by using (3872) eight-node brick elements (Solid 65), the reinforcement is idealized by using (1540) link elements (Link 8), and the interface between the ordinary concrete members and prestressed concrete units is idealized by (704) two-node contact elements (Contact 52 point-to-point).

The total number of nodes resulting from the above idealization is (5265)

nodes, and the total of elements is (6116) elements.

To avoid the crushing of the concrete under the point load, the mid span load was represented in the finite element model by 45 equivalent nodal forces in the middle of the beam, as shown in figure (5-3).

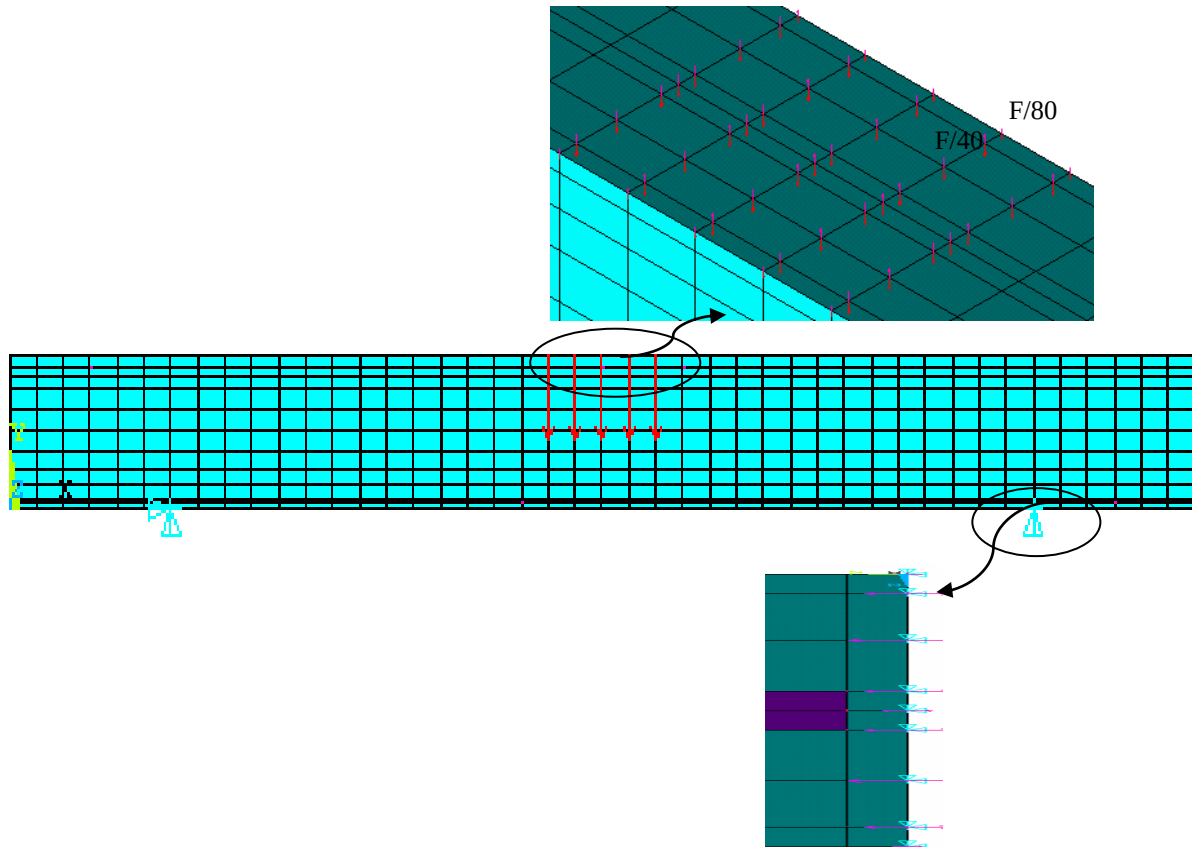


Figure (5-3): Finite elements mesh and load simulation for beam B2 used in ANSYS program

Material properties and the nonlinear solution parameters adopted in the analysis are given in Table (5-1).

The mid span load of (166500 N) over 45 nodes is applied after the application of prestressing strain.

Table (5-1): Material properties and parameters used for Bishara and Almeida composite beam B2 [13].

Cast-in-place Concrete	E_c^{**}	Young's modulus	30419 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	41.887 MPa
	f_t^{***}	Tensile strength	4.1887 MPa
	β_o^*	Shear Transfer parameters	.1
	β_c^*		.99
Prestressed concrete	E_{PC}^{**}	Young's modulus	35982 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	58.61 MPa
	f_t^{***}	Tensile strength	5.861 MPa
	β_o^*	Shear Transfer parameters	0.1
	β_c^*		0.99
Reinforcing steel	E_s^*	Young's modulus	200,000 MPa
	f_y	Yield stress	310.275 (ϕ 16) MPa
			376.19 (ϕ 10) MPa
	ν_s^*	Poisson's ratio	0.3
High tension wire	E_s^{****}	Young's modulus	194439 MPa
	f_y	Yield stress	1592.745 MPa
	ν_s^*	Poisson's ratio	0.3

(*) Assumed value, (**) $E_c = 4700 \sqrt{f'_c}$, (***) $f_t = 0.1 f'_c$

(**) ACI-Building Code 318M-05 [41].

(***) Chen (1981) [37].

(****) Bishara and Almeida [13].

5-2-1-3: Results of Analysis.

The experimental load-deflection curves of **Bishara** and **Almeida** composite beam (B2) and numerical load-deflection curves are shown in figure (5-4). Good agreement is obtained between the present finite element and the experimental load-deflection curves throughout the entire range of behavior. The analytical ultimate deflection (32.614 mm) is detected quite well compared with that experimentally observed (35 mm), with an error of only (6.8%). The difference between the experimental ultimate load and that obtained by finite element analysis is not more than (2.1%). From figure (5-4) find the used precast-prestressed unit increased flexural rigidity and cracking load. That resulted in the increasing of the ultimate load and the decreasing of the ultimate deflection.

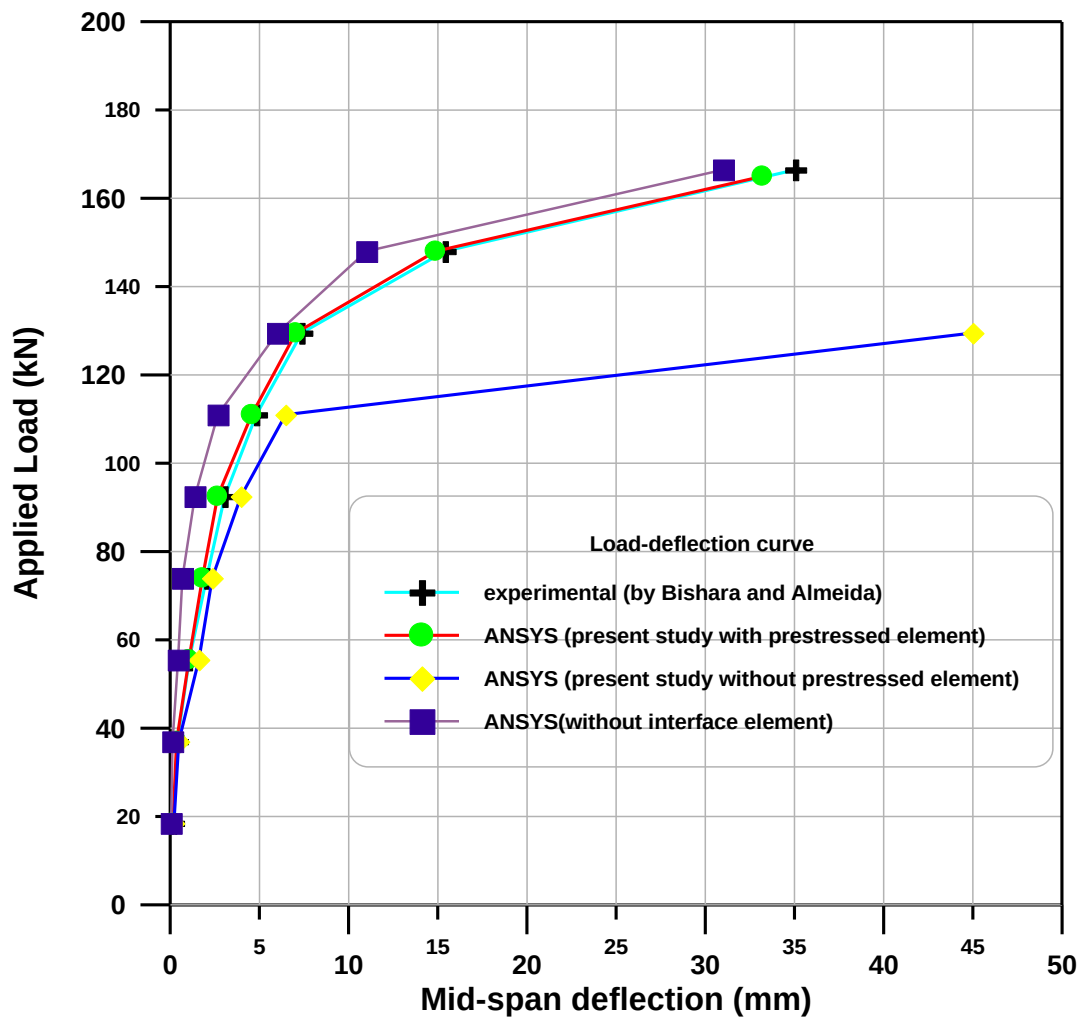


Figure (5-4): Load-deflection curve for beam B2

Figure (5-5) shows the cracks close to the mid-span of the beam (B2) at applied level ($F = 51231 \text{ N}$). Cracks are shown in figure (5-6) at the ultimate load. It can be seen the cracks extended towards the mid-depth of the beam as the applied load is increased.

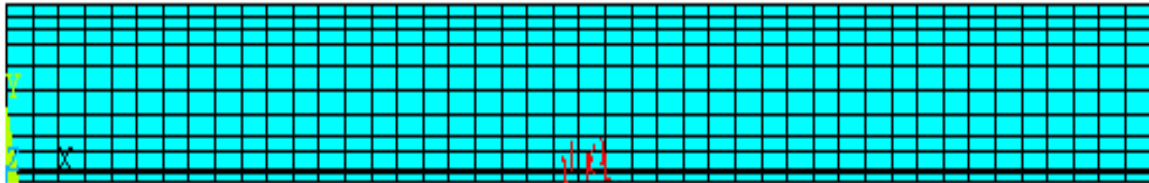


Figure (5-5): Crack for beam B2 at load level ($F = 51231 \text{ N}$)

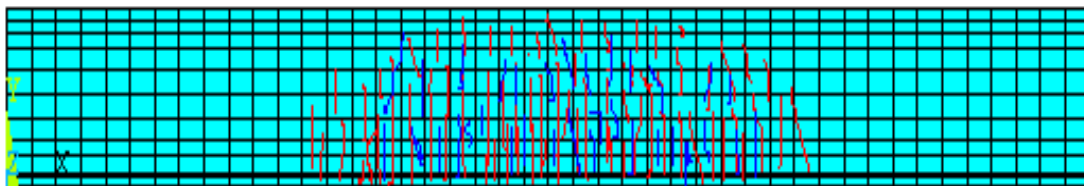
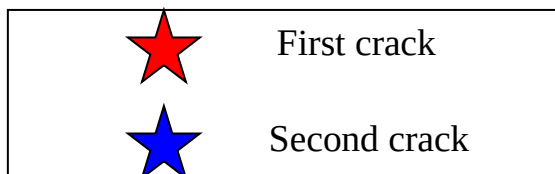


Figure (5-6): Crack for beam B2 at load level ($F = 166500 \text{ N}$)



5-2-2: Description of The Beam B3

A simply supported composite beam under one concentrated load has been analyzed to investigate the applicability and accuracy of the computer program results. Pertinent details for **Bishara** and **Almeida** composite beam are shown in figure (5-7). The rectangular cross-section of the beam was (266.7 mm) wide and (381 mm) deep. The effective depth for the ordinary steel was (338.84 mm). figure (5-7) shows the internal reinforcement, two (15.88 mm) diameter bars were included in the compression zone, one (15.88 mm) diameter bars was included in the tension zone, and (9.53 mm) mild steel bars were provided at (76.2 mm) for shear reinforcement. Also, including two prestressed reinforcing element consisted of a concrete prism (38 mm) wide and (68 mm) depth axially prestressed by (9.52 mm) diameter 7-wire strand.

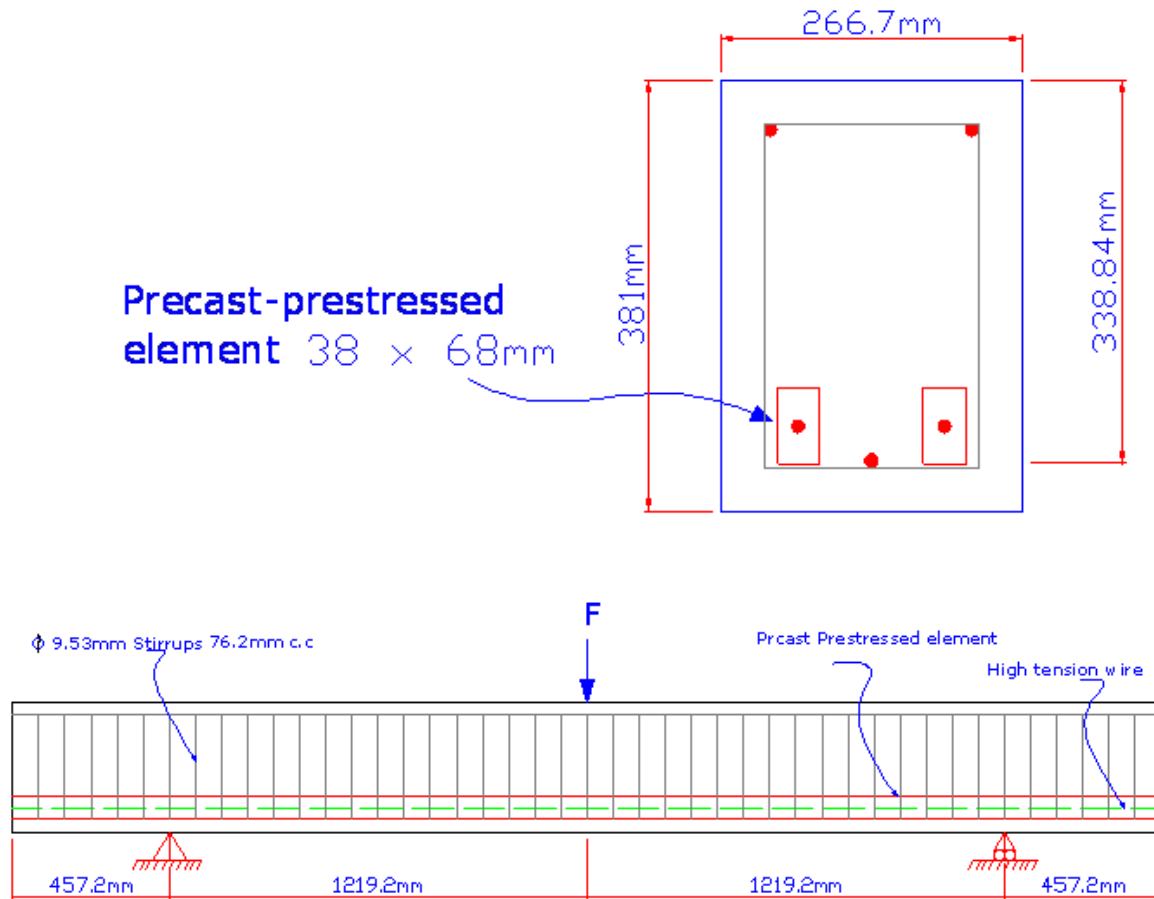


Figure (5-7): Dimensions and reinforcement of beam B3 tested by Bishara and Almeida [13].

5-2-2-1: Finite Element Idealization

The requirement to simulate the nonlinear response of ordinary reinforced concrete beams and precast-prestressed units demands the use of the dedicated (SOLID 65) element in (ANSYS 9.0). Both concrete types are idealized by using (5280) eight-node brick elements.

The reinforcement for the beam B3 is idealized by using spar elements, (Link 8), and the interface between the ordinary concrete beams and prestressed concrete units is idealized by contact elements, (Contact 52 point-to-point). The total number of nodes resulting from the above idealization is (5760) nodes, and the total number of elements is (8008) elements.

The external load (154530 N) was modeled by 45 equivalent nodal forces these were applied at the mid span, shown in figure (5-8) to avoid the crushing of the concrete.

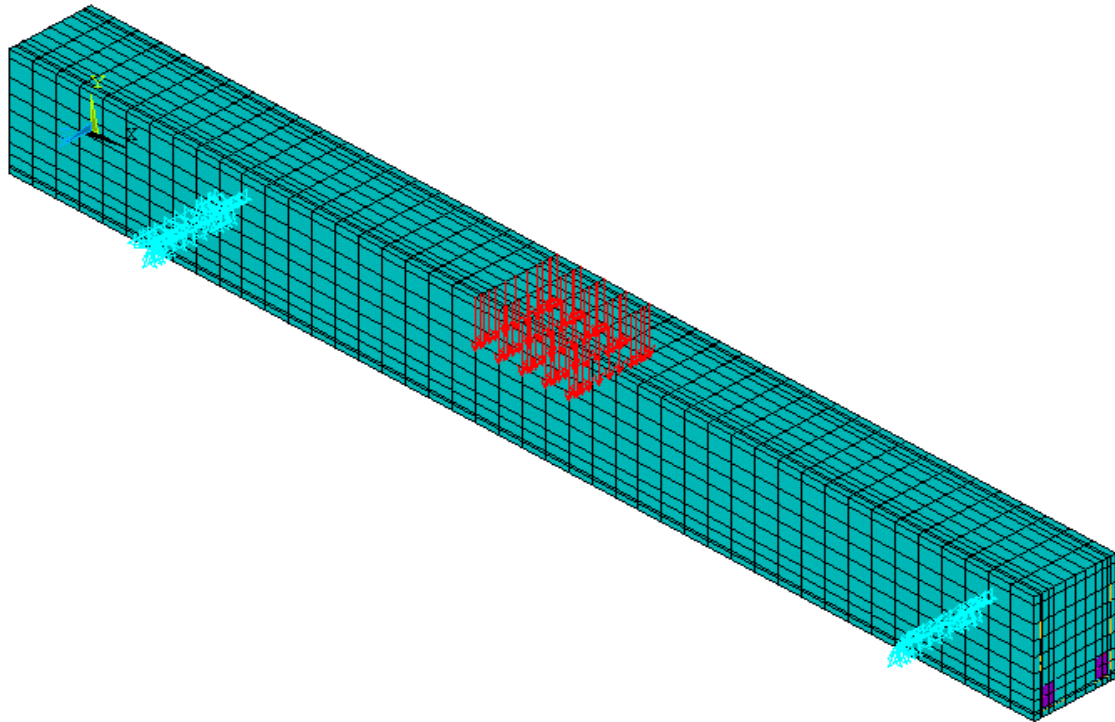


Figure (5-8): Finite elements mesh and load simulation for beam B3 used in ANSYS program

Material properties and the nonlinear solution parameters adopted in the analysis are given in table (5-2).

Table (5-2): Material properties and parameters used for Bishara and Almeida composite beam B3 [13].

Cast-in-place Concrete	E_c^{**}	Young's modulus	31342 MPa
	ν_c^*	Poisson's ratio	0.2
	f_c'	Compressive strength	44.4689 MPa
	f_t^{***}	Tensile strength	4.4.689MPa
	β_o^*	Shear Transfer parameters	.1
	β_c^*		.99
	E_{PC}^{**}	Young's modulus	35982 MPa

Prestressed concrete	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	58.61 MPa
	f_t^{***}	Tensile strength	5.861 MPa
	β_o^*	Shear Transfer parameters	0.1
	β_c^*		0.99
Reinforcing steel	E_s^*	Young's modulus	200,000 MPa
	f_y	Yield stress	310.275 (ϕ 16) MPa
			376.19 (ϕ 10) MPa
	ν_s^*	Poisson's ratio	0.3
High tension wire	E_s^{****}	Young's modulus	194439 MPa
	f_y	Yield stress	1592.745 MPa
	ν_s^*	Poisson's ratio	0.3

(*) Assumed value, (**) $E_c = 4700\sqrt{f'_c}$, (***) $f_t = 0.1 f'_c$

(**) ACI-Building Code 318M-05 [41].

(***) Chen (1981) [37].

(****) Bishara and Almeida [13].

5-2-2-2: Results of Analysis

The figure (5-9) shows the implementation of the interface element to the composite beam of this study. We conclude that the results are closer to the experimental tests than to that without the interface element. The experimental ultimate deflection obtained by **Bishara** and **Almeida** is equal to (17.78 mm) and the numerical ultimate deflection is equal to (18 mm) with a difference (1.22%). The ultimate load is (157 KN) by finite elements while the experimental load is (154.53 KN) by **Bishara** and **Almeida** tested. The ratio of the predicted ultimate load to the corresponding experimental load is about (1.016). These results indicate very well agreement.

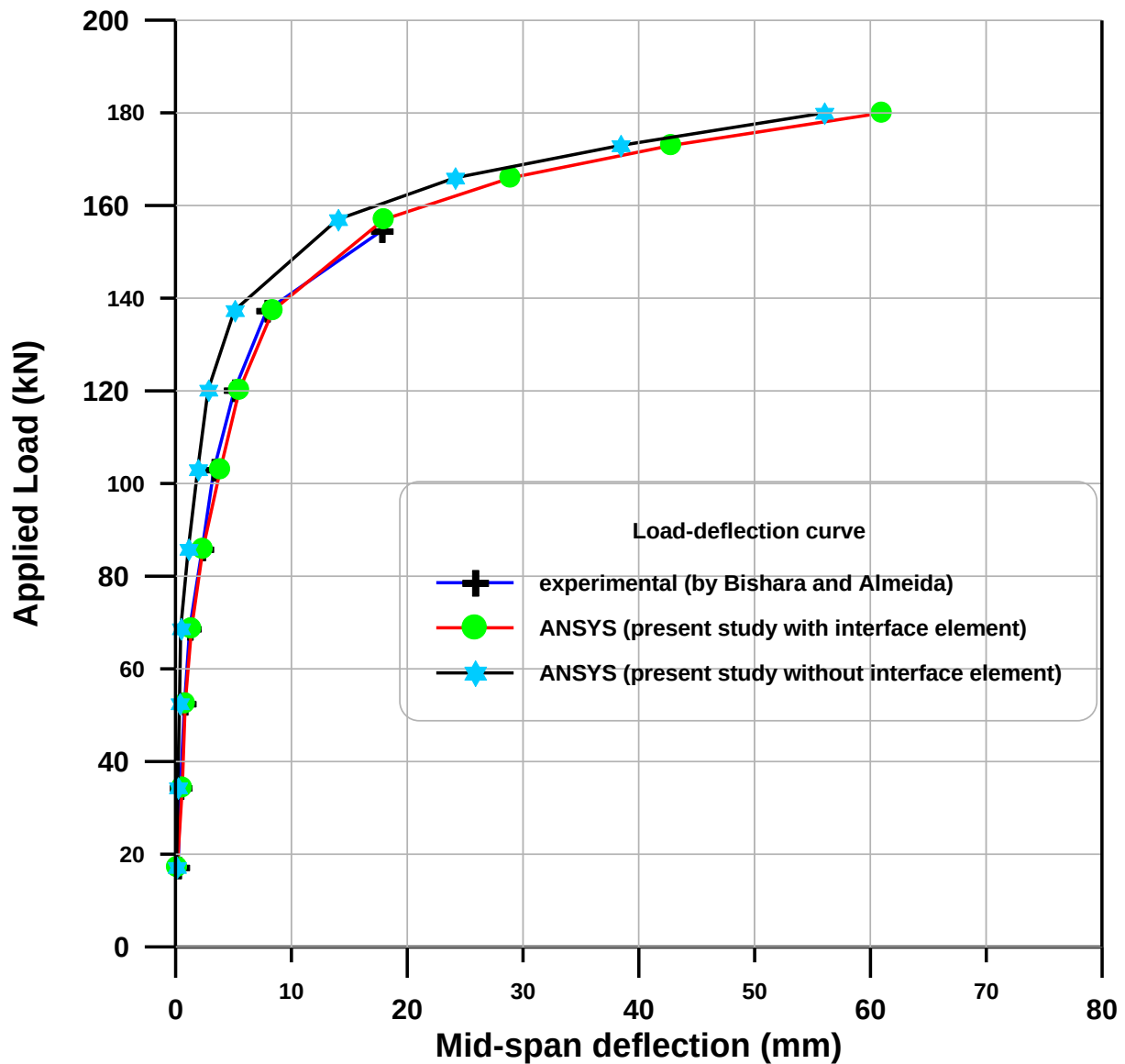


Figure (5-9): Load-deflection curve for beam B3

The cracks appear at the mid-span of the beam (B3) at applied load (71323 N), shown in figure (5-10). Both concrete types are cracking at the ultimate load, shown in figure (5-11). The cracks extended towards the mid-depth of the beam as the applied load is increased.

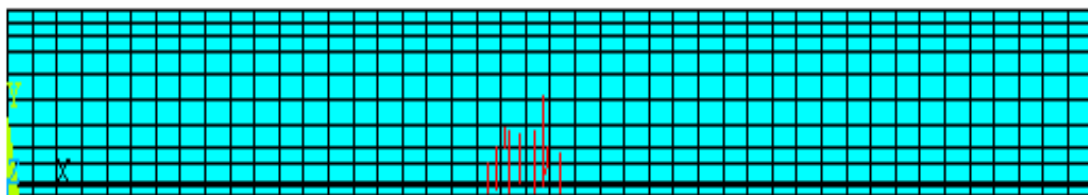


Figure (5-10): Crack for beam B3 at load level ($F=71323$ N)

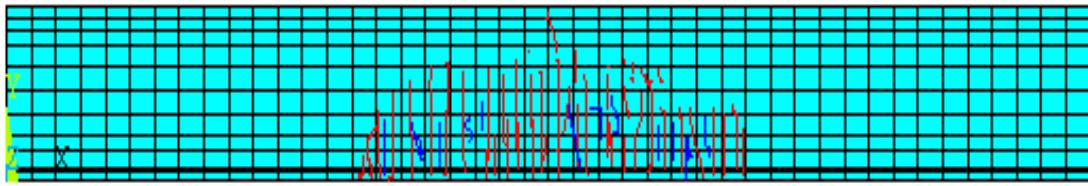


Figure (5-11): Cracks for beam B3 at load level ($F=154530$ N)

5-2-3: Description of The Beam B4

A cross section through the long beam, shown in figure (5-12), illustrates the internal reinforcement. Two (15.88 mm) diameter bars were included in the compression zone. Fourteen-three shear links, formed from (9.53 mm) mild steel bars, were provided at (76.2 mm) on centers for shear reinforcement in the shear span. Also, include three precast-prestressed units of (38 mm) wide and (68 mm) deep with total area of prestressing straight wires of (154.4 mm^2).

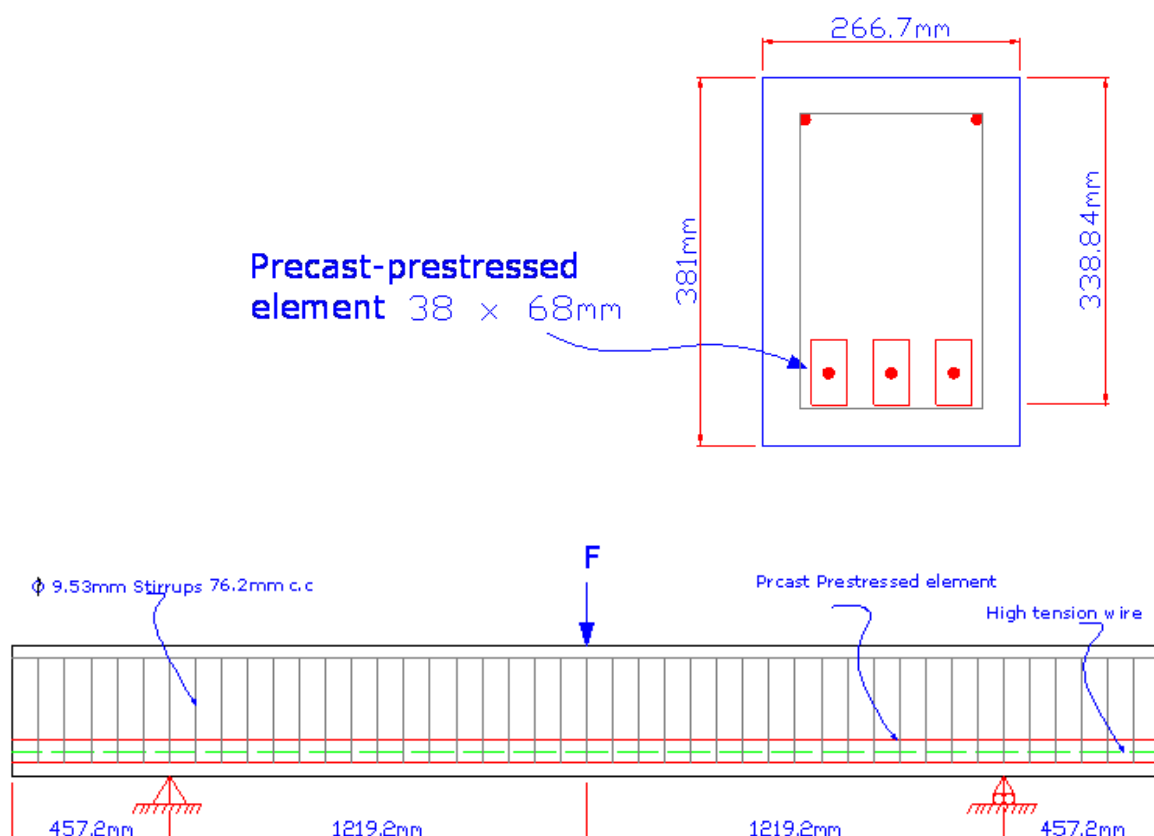


Figure (5-12): Dimensions and reinforcement of beam B4 tested by Bishara and Almeida [13].

5-2-3-1: Finite Element Idealization

The finite element idealization of prestressed concrete composite beam is shown in figure (5-13). This beam is subjected to central point loading. Ordinary reinforced concrete and prestressed concrete were simulated by (5380) isoparametric eight-node brick elements (SOLID 65) connected by (2112) two node interface elements (Contact 52 point-to-point), while the reinforcement was modeled by (1768) link elements (Link 8).

The finite element meshes, boundary conditions and loading arrangement are shown in figure (5-13).

The mid span load of (137160 N) over 65 nodes is applied after the prestressing strain application.

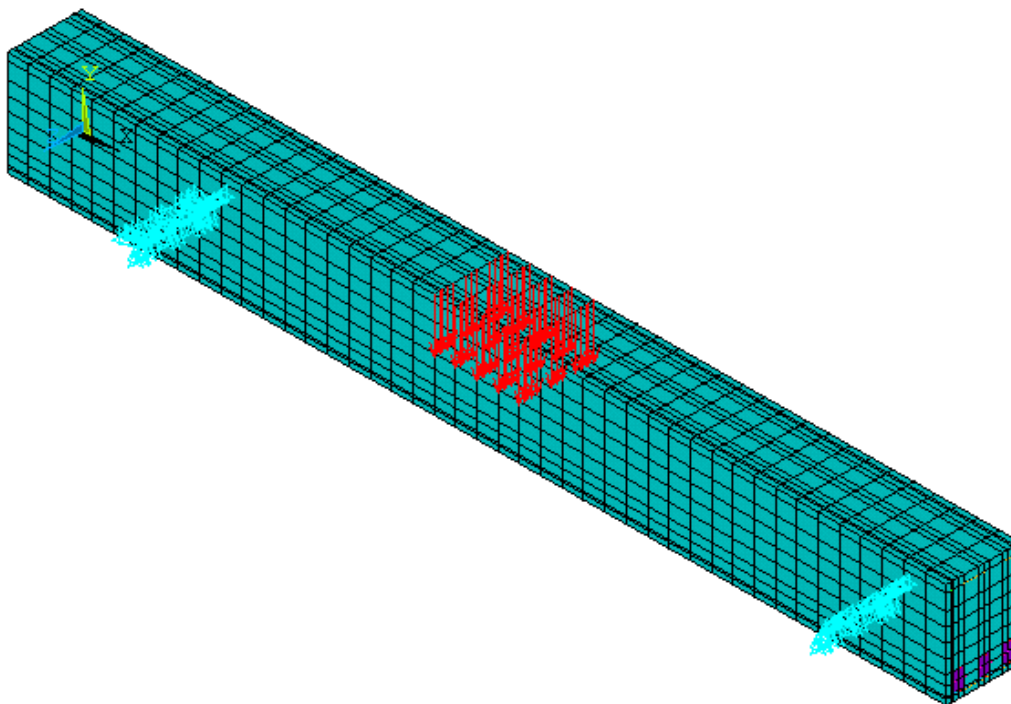


Figure (5-13): Finite elements mesh and load simulation for beam B4 used in ANSYS program

Material properties and the nonlinear solution parameters adopted in the analysis are given in table (5-3).

Table (5-3): Material properties and parameters used for Bishara and Almeida composite beam B4 [13].

Cast-in-place Concrete	E_c^{**}	Young's modulus	31198 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	44.06 MPa
	f_t^{***}	Tensile strength	4.406 MPa
	β_o^*	Shear Transfer parameters	.1
	β_c^*		.99
Prestressed concrete	E_{PC}^{**}	Young's modulus	35982 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	58.61 MPa
	f_t^{***}	Tensile strength	5.861 MPa
	β_o^*	Shear Transfer parameters	0.1
	β_c^*		0.99
Reinforcing steel	E_s^*	Young's modulus	200,000 MPa
	f_y	Yield stress	310.275 (ϕ 16) MPa
			376.19 (ϕ 10) MPa
	ν_s^*	Poisson's ratio	0.3
High tension wire	E_s^{****}	Young's modulus	194439 MPa
	f_y	Yield stress	1592.745 MPa
	ν_s^*	Poisson's ratio	0.3

(*) Assumed value, (**) $E_c = 4700\sqrt{f'_c}$, (***) $f_t = 0.1 f'_c$

(**) ACI-Building Code 318M-05 [41].

(***) Chen (1981) [37].

(****) Bishara and Almeida [13].

5-2-3-2: Results of Analysis

Comparison the deflection for different beams (B2, B3 and B4), shown in figure(5-14), indicates the beneficial effect on flexural rigidity caused by incorporating prestressed elements as a substitute for conventional reinforcement. This was primarily due to an increase in the effective moment of inertia that was clearly manifested by a reduction in the extent of cracking.

A comparison between the experimental load-deflection curve by (**Bishara and Almeida**) and the present load-deflection curve were shown in figure (5-15).The difference between the experimental ultimate load and that obtained by finite element analysis is not more than (1%).

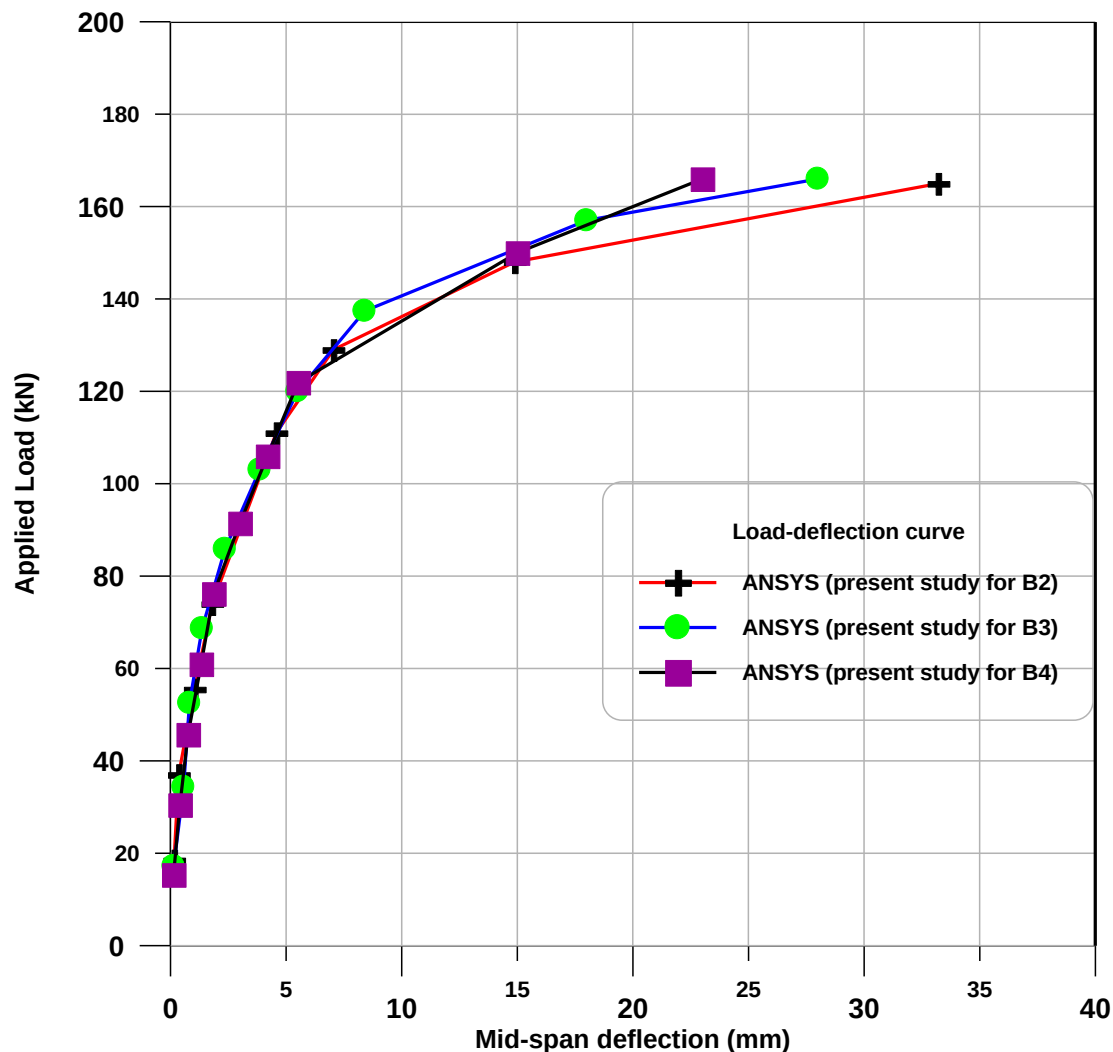


Figure (5-14): Load-deflection curve for beam B2,B3 and B4

Table (5-4): load, cracking load and deflection for beam B2, B3 and B4.

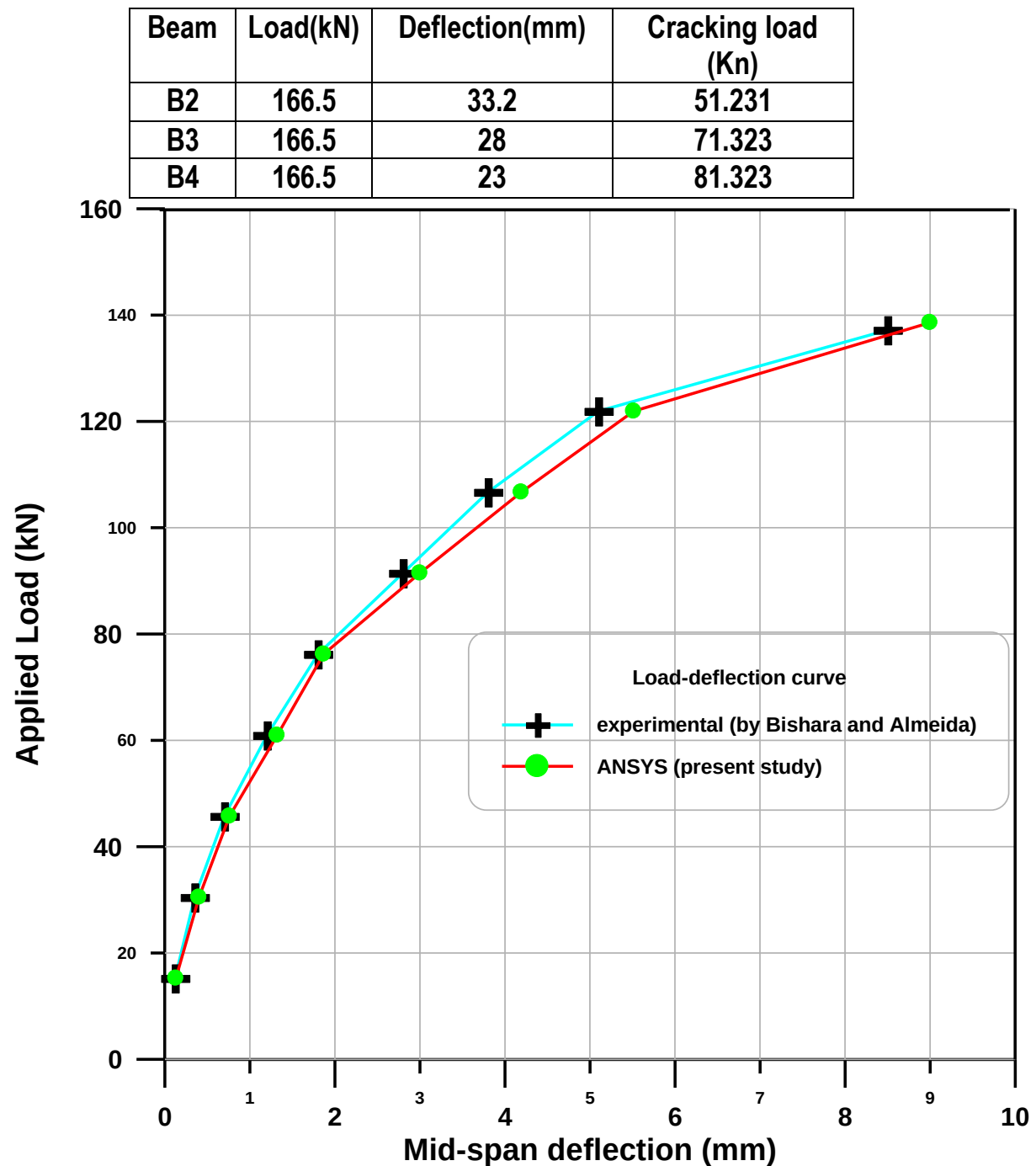


Figure (5-15): Load-deflection curve for beam B4

Figure (5-16) shows distribution of cracks in the beam at load level ($F = 81323$ N). At the ultimate load the first and second cracks occurred, shown in figure (5-17). At the applied load is increased the cracks extended towards the mid-depth of beam.

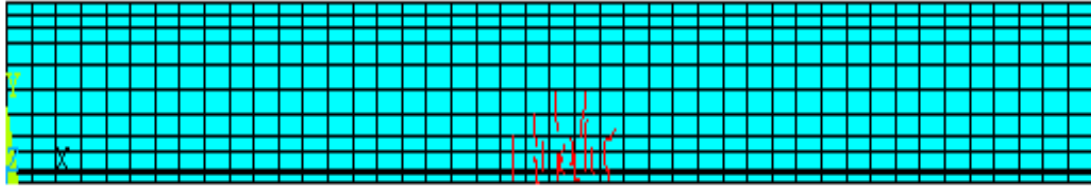


Figure (5-16): Crack for beam B4 at load level ($F = 81323 \text{ N}$)

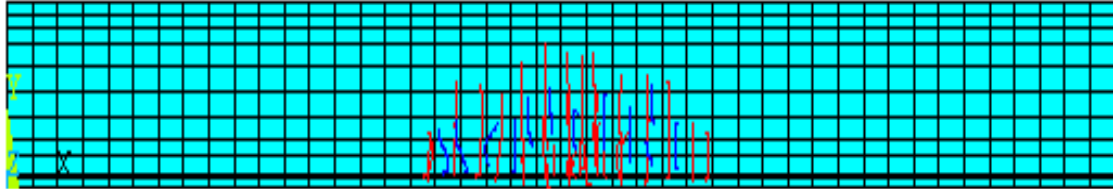


Figure (5-17): Cracks for beam B4 at load level ($F = 137160 \text{ N}$)

5-3: Verification Study on Bishara et al. [14] Continuous Tested Beam.

The continuous composite beam of **Bishara et al.** was selected in the present study for the analysis by the finite element method using the computer program (ANSYS 9.0). Because that the selected two-span continuous beam is symmetric about its center support only one half of the beam was analyzed.

5-3-1: Description of the Beam BC2

The geometry, loading conditions and the reinforcement details for **Bishara et al.** composite beam are shown in figure (5-18). The continuous composite beam of (4699 mm) span, (266.7 mm) width, (381 mm) depth and the effective depth for the ordinary steel at support was (342.47 mm) and at center was (316.92 mm). The ordinary steel bars were six (15.88mm) mild steel located close to the bottom face and two (9.53) mild steel located close to the top face at load exterior system support. The central support section including two (15.88 mm) mild steel bars located close to the bottom face and two (15.88) mild steel bars located close to top face. Also, including one precast-prestressed concrete

units of (38 mm) width and (68 mm) depth cross-section with total area of prestressing straight wire was (51.4 mm^2) . the load was applied at (2065.8 mm) from central support.

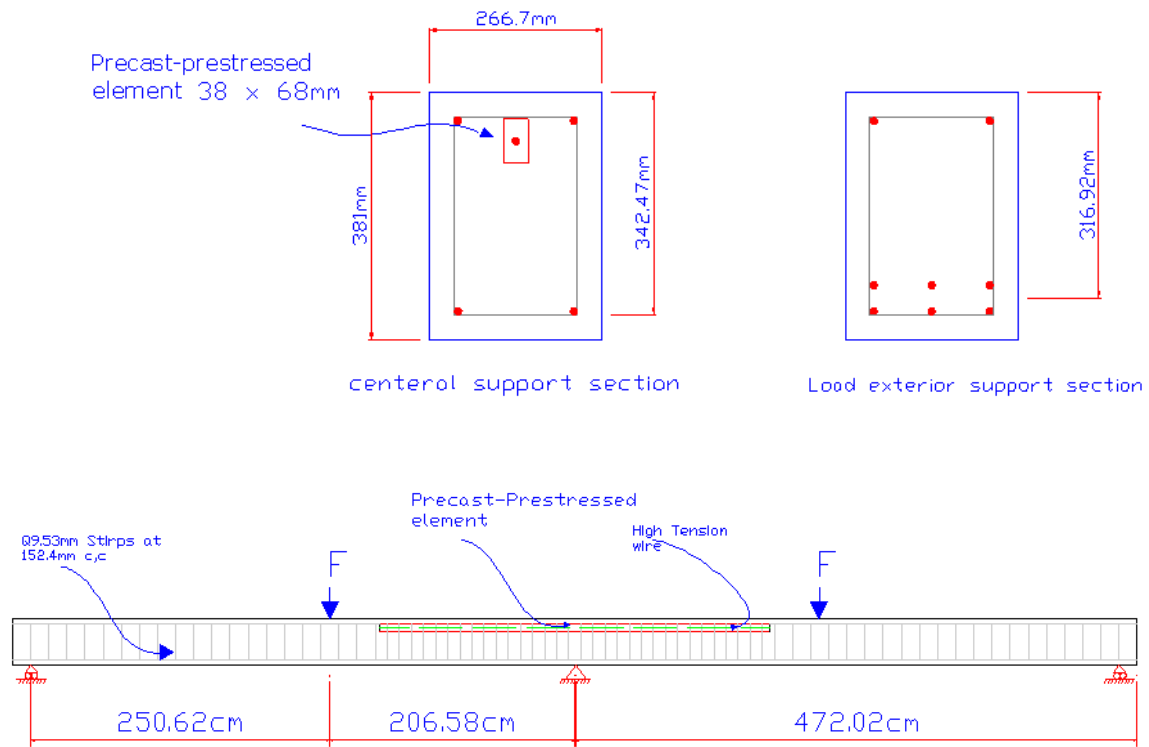


Figure (5-18): Dimensions and reinforcement of beam BC2 tested by Bishara et al. [14].

5-3-1-1: Finite Element Idealization

The three-dimensional finite element mesh of the half of the Beam B2 (in the present study) has been analyzed by using (ANSYS 9.0) soft ware.

The concrete in these beam is represented by (3640) eight-node isoperimetric brick elements (SOLID65) , while the reinforcement is represented by (2015) links element (Link 8) and the two noded interface element (Contact 52 point-to-point) is suitable to represent the bond between the ordinary concrete beam and prestressed concrete units. The external load was applied at (2065.8 mm) from central support. In the finite element model the point load was modeled by 27 equivalent nodal forces, shown in figure (5-19).

The total number of nodes resulting from the above idealization is (4725) nodes, and the total of elements is (6055) elements.

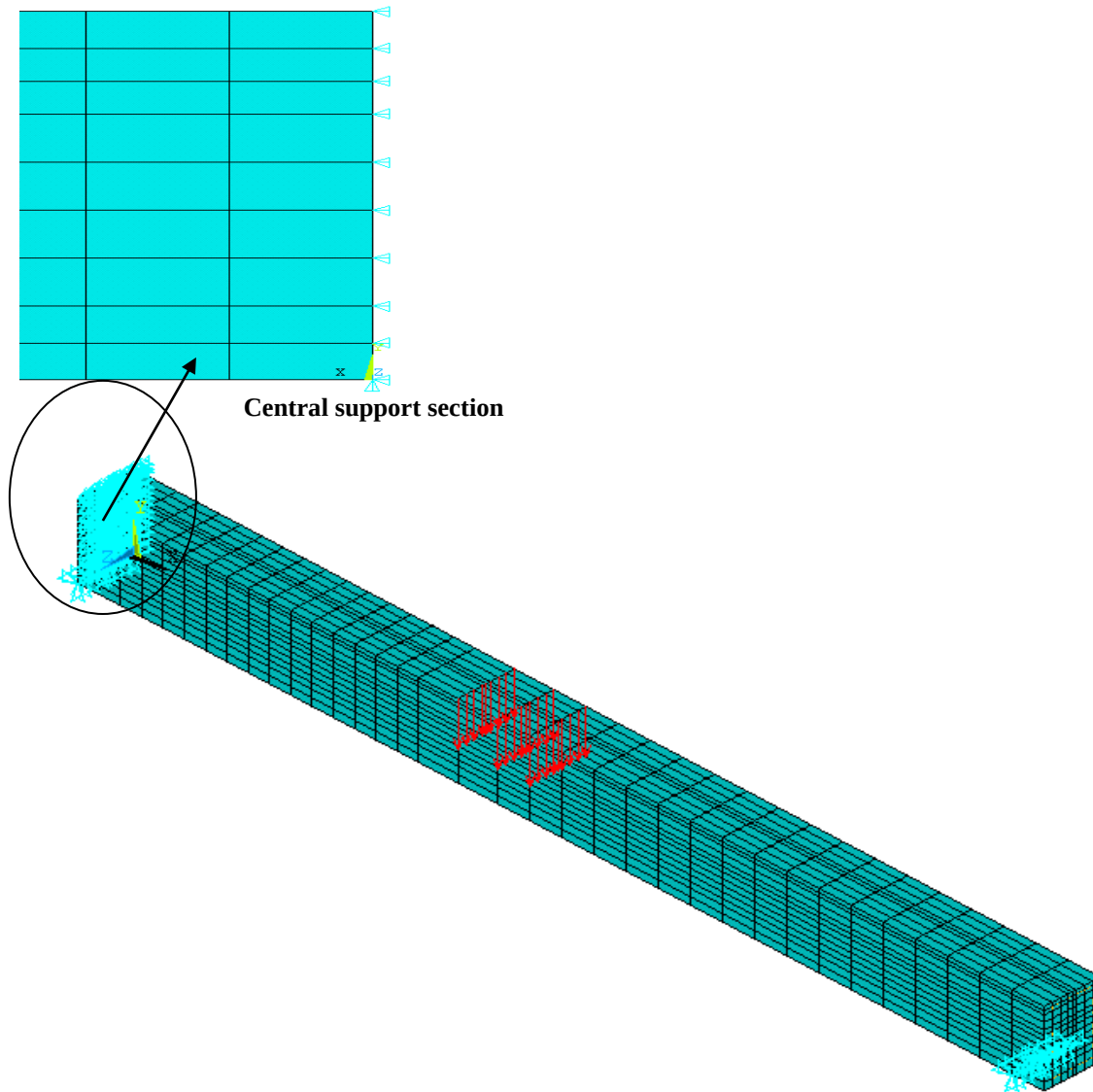


Figure (5-19) Finite elements mesh and load simulation for beam BC2 used in ANSYS program

Material properties and the nonlinear solution parameters adopted in the analysis are given in table (5-5).

Table (5-5): Material properties and parameters used for Bishara et al. composite beam BC2 [14].

	E_c^{**}	Young's modulus	29310 MPa
	ν_c^*	Poisson's ratio	0.2

Cast-in-place	f'_c	Compressive strength	38.888 MPa
	f_t^{***}	Tensile strength	3.8888 MPa
	β_o^*	Shear Transfer parameters	.1
	β_c^*		.99
Prestressed concrete	E_{PC}^{**}	Young's modulus	35982 MPa
	ν_c^*	Poisson's ratio	0.2
	f'_c	Compressive strength	58.61 MPa
	f_t^{***}	Tensile strength	5.861 MPa
	β_o^*	Shear Transfer parameters	0.1
	β_c^*		0.99
Reinforcing steel	E_s^*	Young's modulus	200,000 MPa
	f_y	Yield stress	310.275 (ϕ 16) MPa
			376.19 (ϕ 10) MPa
	ν_s^*	Poisson's ratio	0.3
High tension wire	E_s^{****}	Young's modulus	194439 MPa
	f_y	Yield stress	1592.745 MPa
	ν_s^*	Poisson's ratio	0.3

(*) Assumed value, (**) $E_c = 4700\sqrt{f'_c}$, (***) $f_t = 0.1 f'_c$

(**) ACI-Building Code 318M-05 [41].

(***) Chen (1981) [37].

(****) Bishara et al. [14].

5-3-1-2: Results of Analysis

The experimental and analytical moment-curvature curve obtained for continuous composite prestressed concrete beam are shown in figure (5-20). Good agreement is obtained between the predicted finite element and the experimental moment-curvature curve throughout the entire range of behavior. The analytical ultimate curvature (6×10^{-5} 1/mm) as detected by finite element is quite well compared with that experimentally observed (6.5×10^{-5} 1/mm), with a difference of only (7.7%).

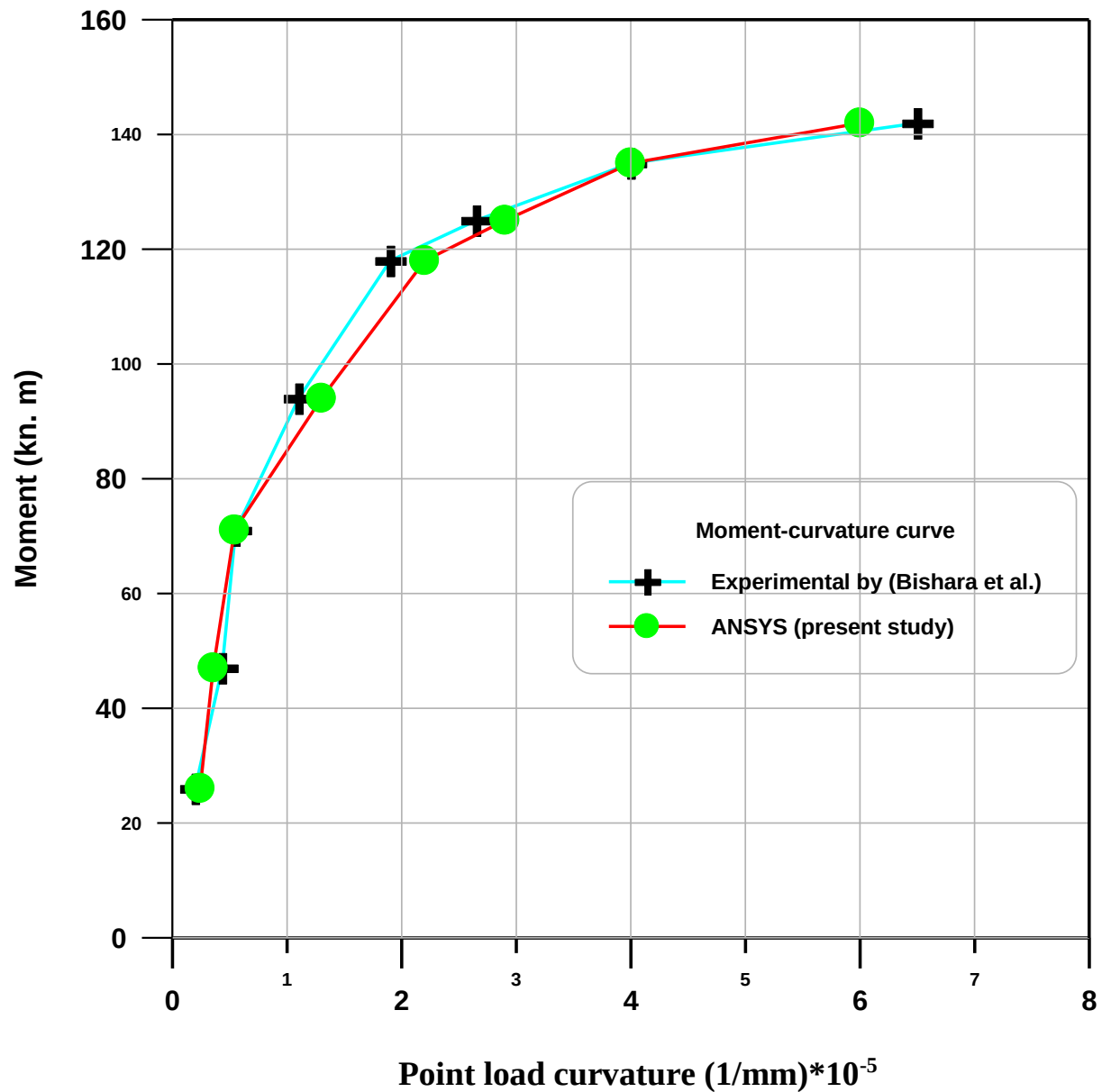


Figure (5-20): Moment-curvature curve for continuous beam BC2

The cracks appear close to the central support of the beam (BC2) at load level ($F = 42145 \text{ N}$), shown in figure (5-21). The distribution of cracks at the mid-span at applied load ($F = 56193 \text{ N}$), shown in figure (5-22). All cracks shown in figure (5-23) at the ultimate load.

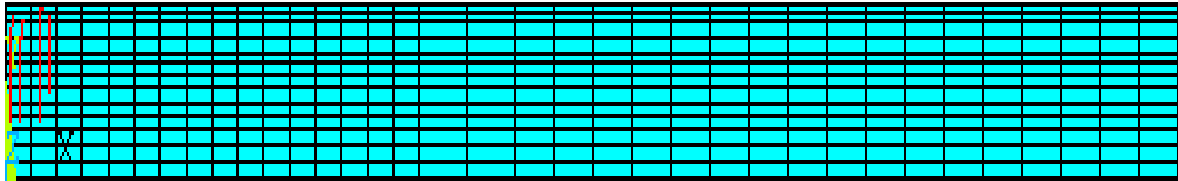


Figure (5-21): Crack for beam BC2 at load level ($F= 42145$ N)

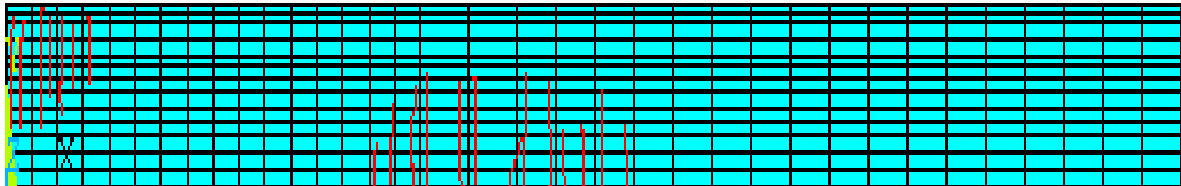


Figure (5-22): Crack at mid-span for beam BC2 at load level ($F= 56193$ N)

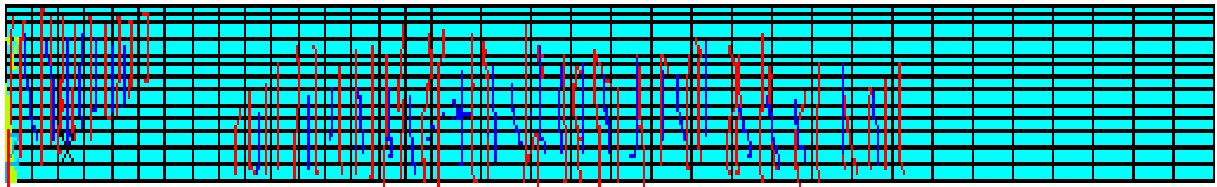


Figure (5-23): Cracks for beam BC2 at the ultimate load

ABSTRACT

This thesis presents a three-dimensional finite element model suitable for the analysis of prestressed concrete composite beams under flexure. For this analysis, a system of computer program (ANSYS V.9.0) is used for this study.

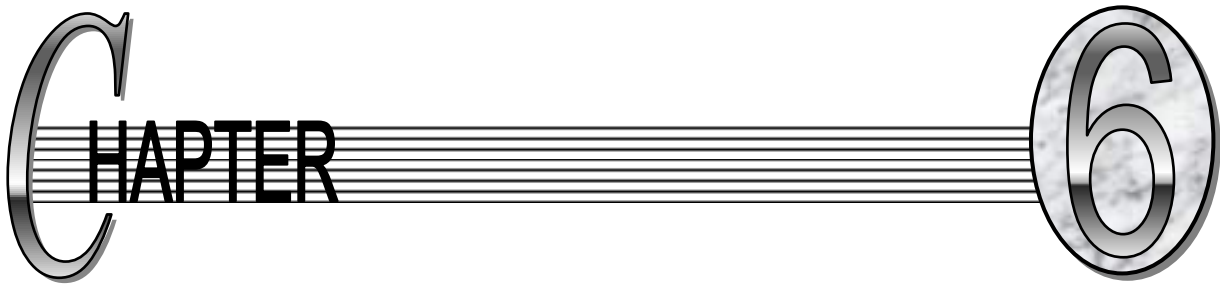
The ordinary reinforced concrete and the precast-prestressed concrete units were modeled by 8-noded isoparametric brick elements, while the steel reinforcing bars and prestressing wires were modeled as axial members (bar elements) connecting opposite nodes in the brick elements with full interaction assumption. The interface elements were modeled by contact element to represent slip and uplift separation between the ordinary concrete beams and precast-prestressed units.

Materials nonlinearities due to the cracking, crushing of the concrete, yielding of prestressing wires and reinforcing bars are taken into consideration during the analysis. In tension, the crack is represented by a fixed smeared crack with post-cracking tension stiffening.

The nonlinear equations of equilibrium have been solved by using an incremental-iterative technique. The full Newton-Raphson method is used as a nonlinear solution algorithm and the force criterion is used as a convergence criterion. The numerical integration has been carried out using eight-point Gaussian rule for brick elements.

In order to insure the validity and accuracy of the adopted models, several experimental examples have been solved and compared. In the considered examples, it is noted that the difference between finite element analysis and experimental test in the range (1 - 2.1)% and (1.22 - 7.7)% for ultimate load and ultimate deflection respectively.

Parametric studies are implemented to study the effect of some important parameters. Among the obtained conclusions, the amount of prestressing ratio. The results explain that increasing the amount of prestressing steel ratio increases the flexural rigidity that resulted in the increasing of ultimate load, the increasing in ultimate load was (42.58%) for simply support and (29.52%) for continuous beam.



CHAPTER 6

PARAMETRIC STUDY

6-1: Introduction

In the present chapter, a parametric study was performed to investigate the influence of several important parameters on the behavior of prestressed composite beam. In the previous chapter, four numerical examples have been analyzed using the computer program ANSYS. These examples demonstrate the validity of the approach.

In this chapter, two examples were chosen to study the effect of several important parameters on the behavior of prestressed composite beam. The first example was a simply supported pretressed composite beam. The other examples were continuous pretressed composite beam.

6-2: Simply Supported and Continuous Prestressed Composite Concrete Beams.

The simply supported and continuous prestressed composite concrete beams with prestressed units tested by **Bishara** and **Almeida** [13] and **Bishara et al.**[14] respectively have been selected to carry out the parametric study. The composite beams were considered to verify the validity of the analytical results with respect to the experimental results. The parameters considered are effect of modular ratio of compressive strength of the cast-in-place concrete to prestressed units, effect of prestressing steel ratio, effect of position of the high tension wires within prestressed units to depth of prestressed units ratio and effect shapes and positions of prestressed units inside reinforced concrete.

6-2-1: Effect of Ratio of Compressive Strength of Cast –in-place Concrete to Precast Units.

To study the effect of modular ratio (n) (ratio of compressive strength of cast-in-place concrete to precast concrete) on the behavior of prestressed concrete composite beam, different values of (n) were considered. The composite beams have been analyzed with six different ratios for beam (B2) and beam (BC2). These values were (0.9, 0.8, 0.7, 0.6, 0.5, 0.4). With reference to figure (6-1) and figure (6-2), the behavior of composite beam with high modular ratio exhibits lower stiffness than those of composite beam having lower modular ratio. The predicted ultimate deflection of this structure decreases (41.67%) for simply support and (31%) for continuous beam. On the other hand, the decreasing of modular ratio lower than (0.6) is insignificant because the failure will happen in the ordinary reinforced concrete. The compressive strength of cast-in-place concrete is constant. From figure(6-1) and figure(6-2) it is shown that the effect of (n) ratio on simply supported and continuous are the same.

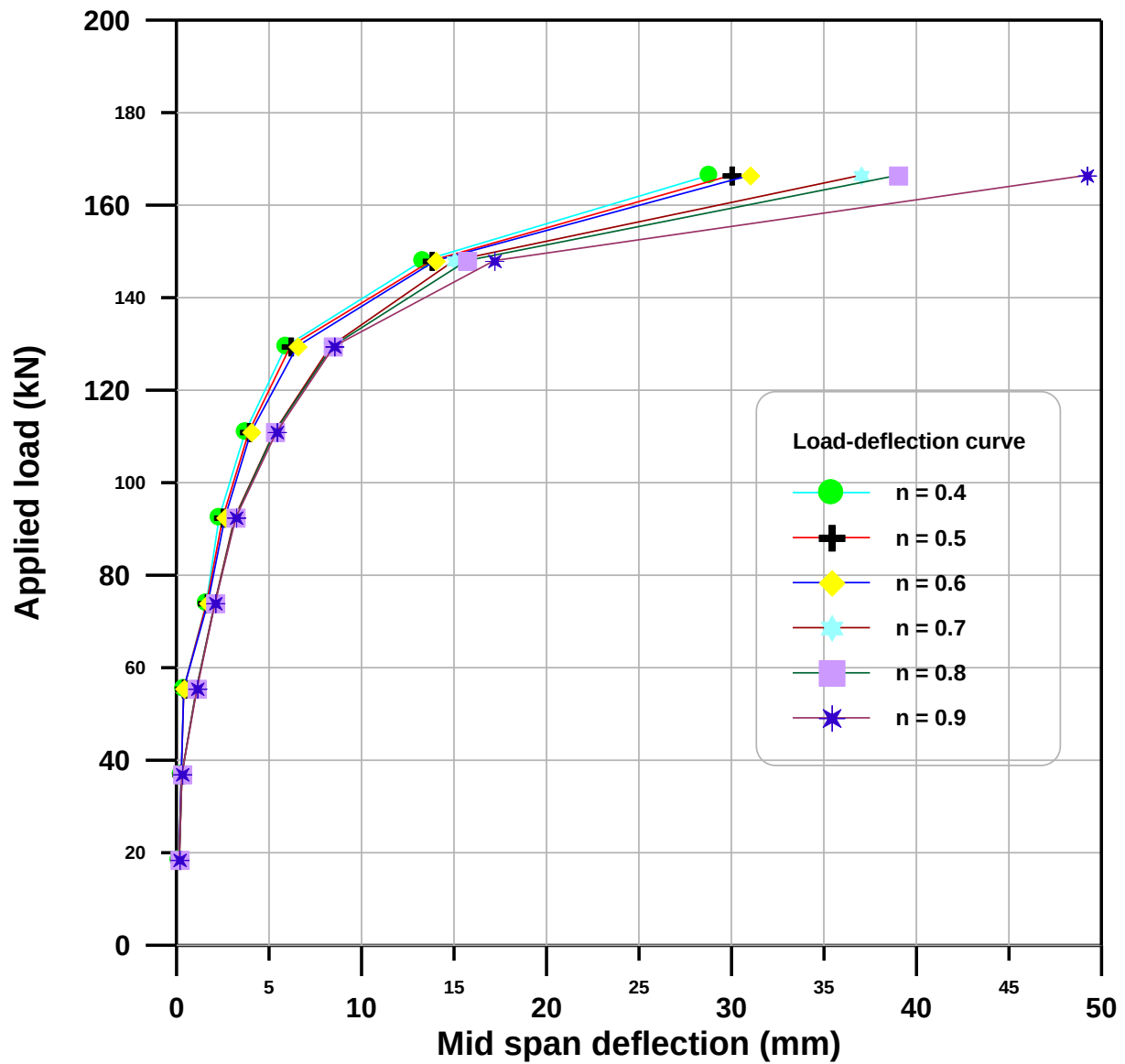


Figure (6-1): Effect of modular ratio (n) on load-deflection curve for simply supported composite prestressed concrete beam (B2) of Bishara and Almeida [13].

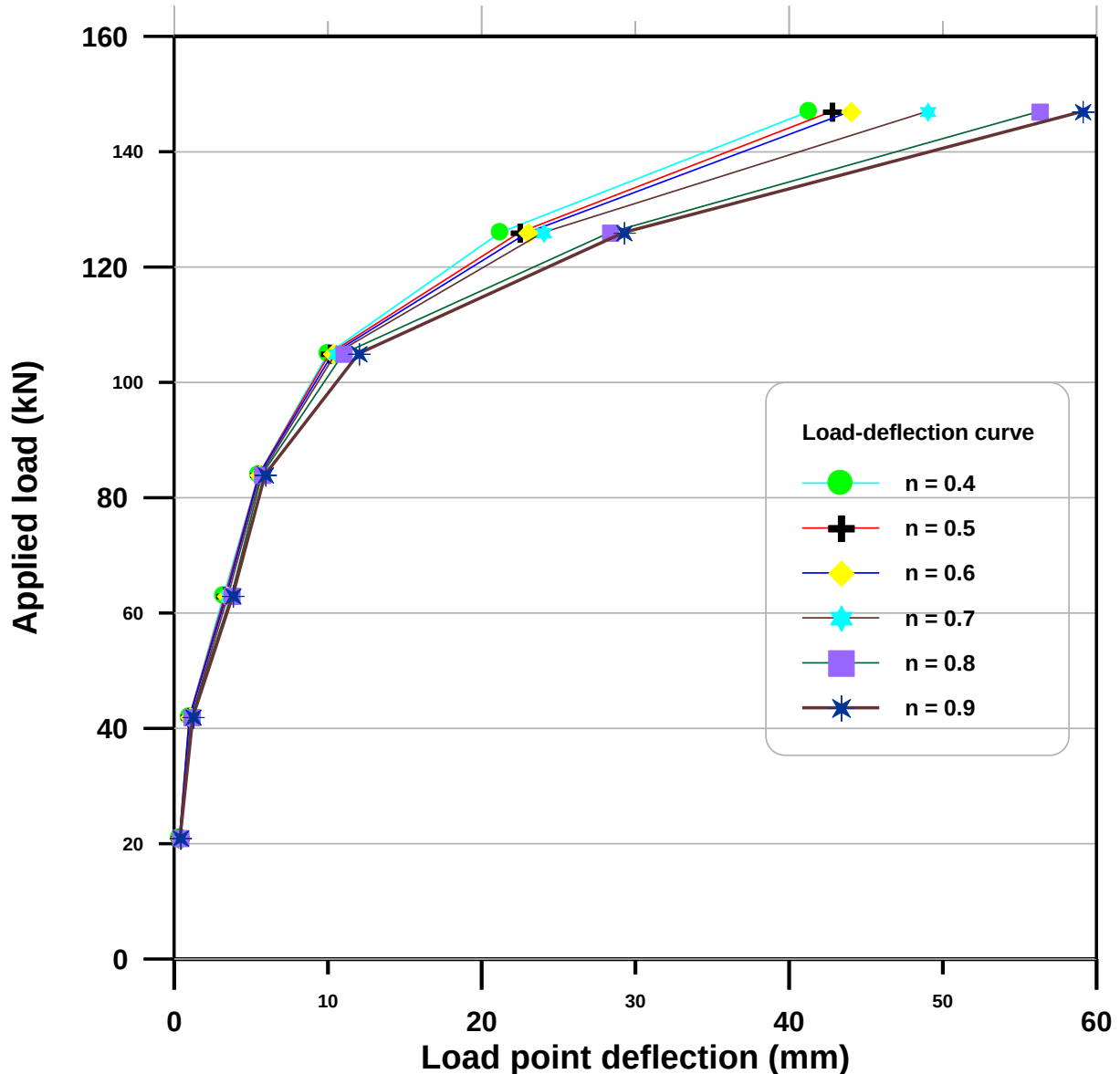


Figure (6-2): Effect of modular ratio (n) on load-deflection curve for continuous composite prestressed concrete beam (BC2) of Bishara et al. [14].

6-2-2: Effect of Eccentricity to Depth Ratio (e_2/h_2).

To study the effect of (e_2/h_2) ratio (e_2 :distance from center of prestressing steel to center of prestressed units, h_2 :depth of prestressed units) as shown in figures (6-3) and (6-4)), on the behavior of prestressed concrete composite beams, different values of (e_2/h_2) were considered. The composite beams have been analyzed with four different ratios for beam (B2) and beam (BC2). These values were (0, 0.1, 0.2, 0.3). Figures (6-5) to (6-8) show the effect of (e_2/h_2)

ratio of the selected beams. The load-deflection curve indicates that the stiffness of the beam increases with increase of (e_2/h_2) up to (0.2). The increase in (e_2/h_2) ratio beyond (0.2) is insignificant since the response obtained for values of (e_2/h_2) equal to (0.2) and (0.3) are almost the same. The total moment and camber increase due to the increasing in eccentricity ratio. As a result, the ultimate load increases (2.63- 2.25)% for simply support and (1.33-1)% for continuous beam when (e_1/h_1) varied from (0.3 to 0.2).

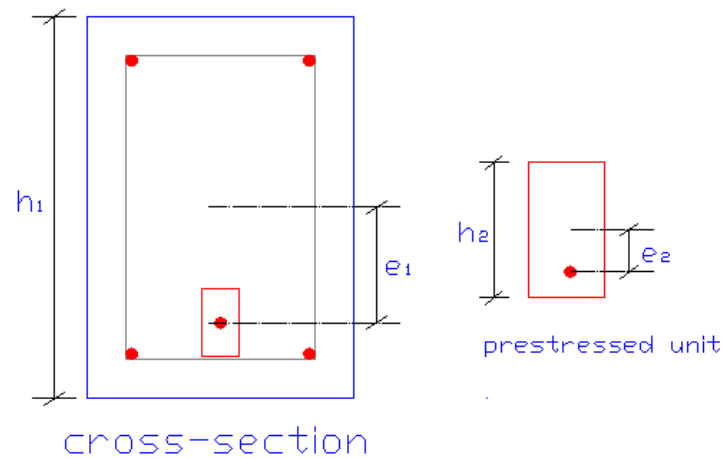


Figure (6-3): Location of the prestressed unit in the cross-section of the simply supported composite beam (B2) and location of the prestressing steel in the prestressed unit.

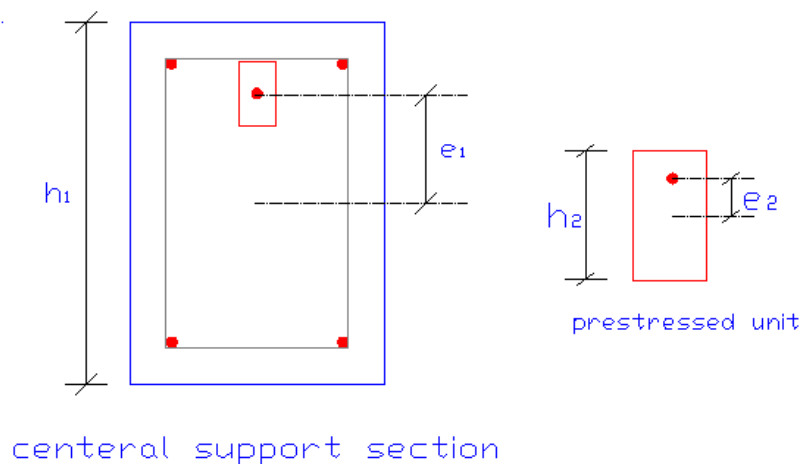


Figure (6-4): Location of the prestressed unit in the cross-section of the continuous composite beam (BC2) and location of the prestressing steel in the prestressed unit.

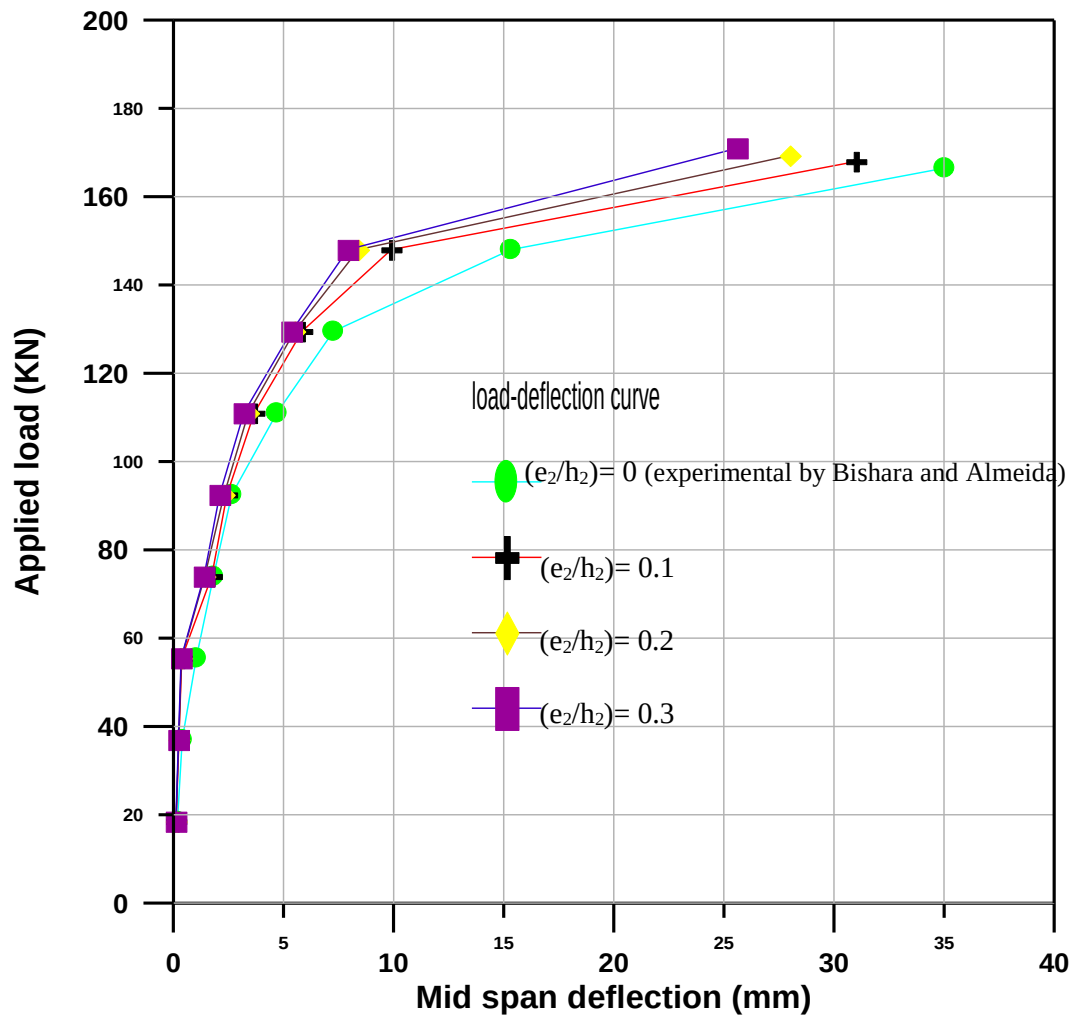


Figure (6-5): Effect of (e_2/h_2) ratio on load-deflection curve for simply supported composite prestressed concrete beam (B2) of Bishara and Almeida [13].
 $(e_1/h_1)=0.3$

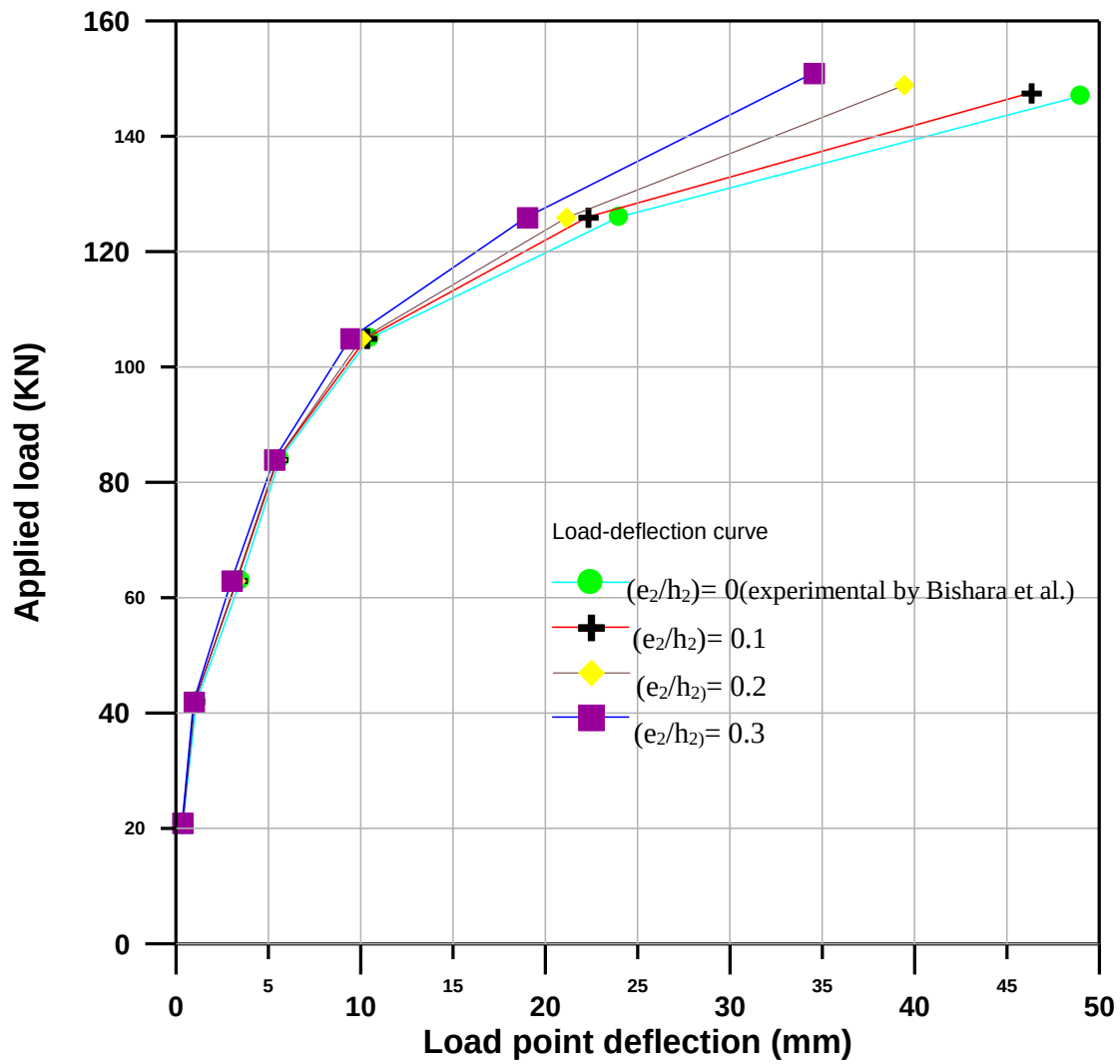


Figure (6-6): Effect of (e_2/h_2) ratio on load-deflection curve for continuous composite prestressed concrete beam (BC2) of Bishara et al. [14].
 $(e_1/h_1) = 0.3$

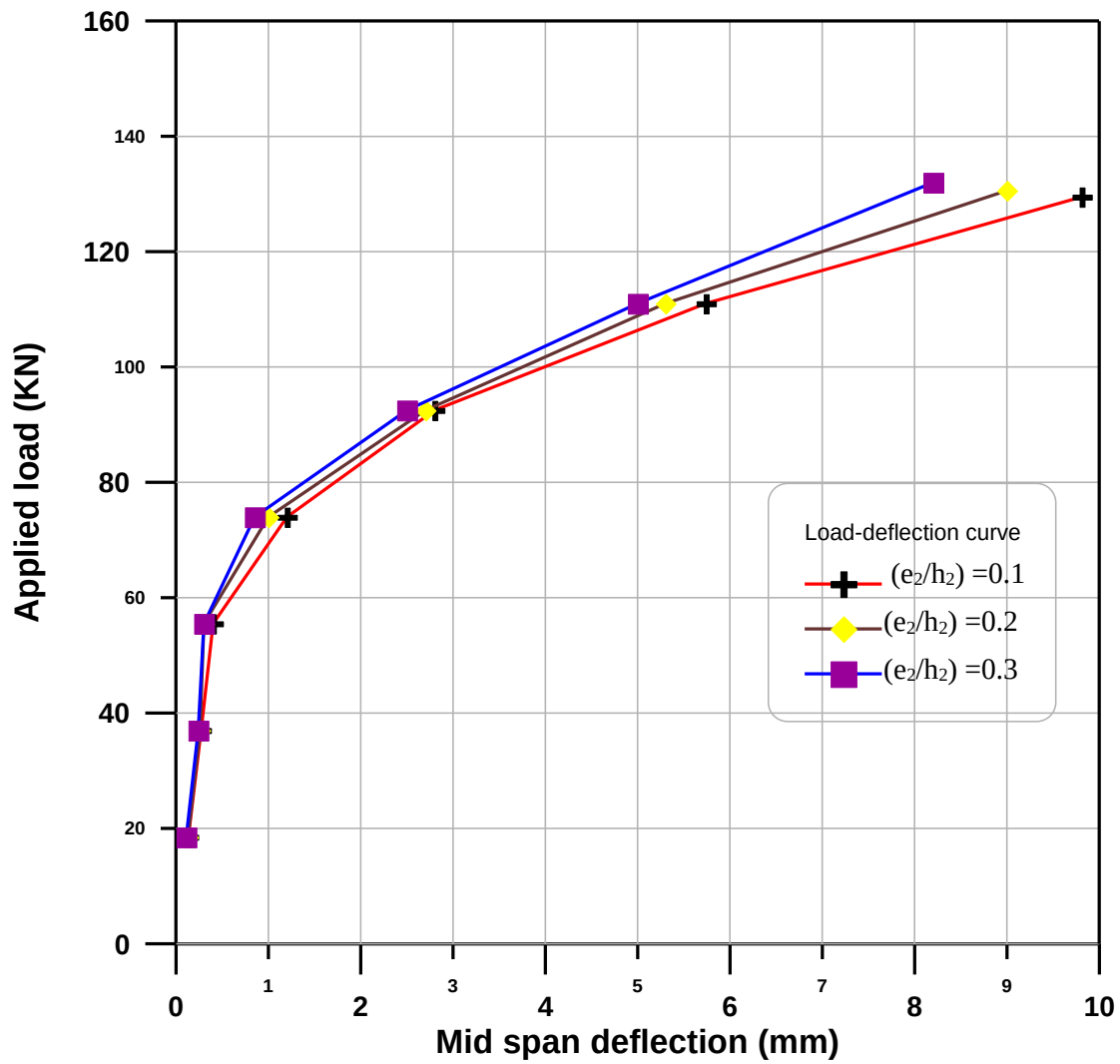


Figure (6-7): Effect of (e_2/h_2) ratio on load-deflection curve for simply supported composite prestressed concrete beam (B2) of Bishara and Almeiad [13].
 $(e_1/h_1) = 0.2$

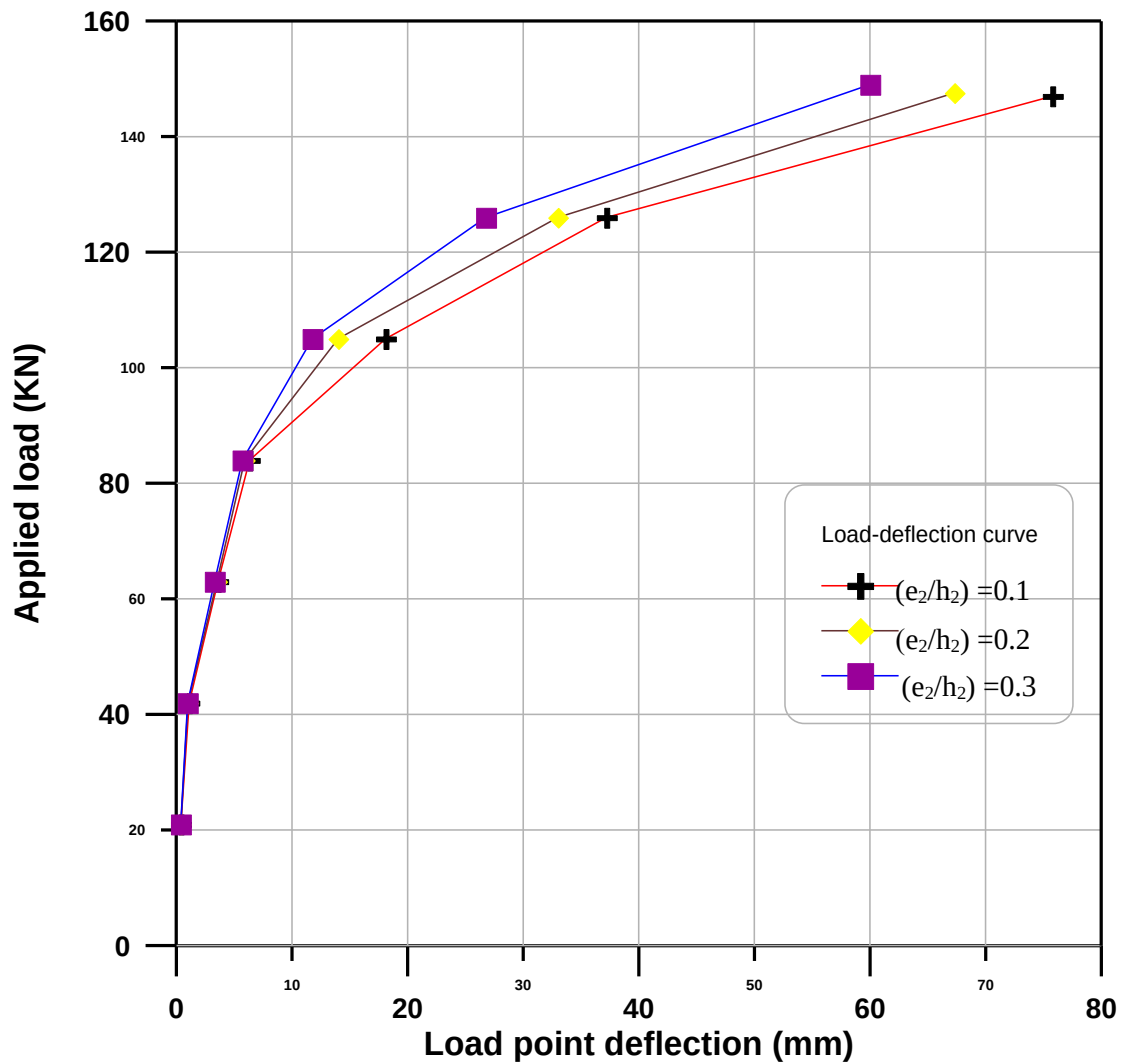


Figure (6-8): Effect of (e_2/h_2) ratio on load-deflection curve for continuous composite prestressed concrete beam (BC2) of Bishara et al. [14].
 $(e_1/h_1)=0.2$

6-2-3 : Effect of Variation of Prestressing Steel Ratio.

Four different ratios of prestressed steel are used in Bishara and Almeida's composite beam (B2) and Bishara et al. composite beam (BC2) to know their effect on the load-deflection curve. These values are $(0.000568(A_{ps}=51.4 \text{ mm}^2)$, $0.00082(A_{ps}=74.19 \text{ mm}^2)$, $0.00111(A_{ps}=98.7 \text{ mm}^2)$, and $0.00155(A_{ps}=140 \text{ mm}^2)$).

Figure (6-9) and figure (6-10), show the behavior (load-deflection curves), from which it can be seen that as the prestressing steel ratio increased, the ultimate load increased since the stiffness of prestressed bars contribute in the total stiffness of the concrete. From figure(6-9) and figure(6-10) it is shown that the effect of prestressing steel ratio is more effective in simply supported than continuous beams. The existence of cracks is delayed by the increasing of the section area for the prestressed reinforcing steel because this increasing in section area will result in increasing of the initial compressive stresses also increasing the flexural rigidity and that leads to the increasing of the ability to sustain loads, the increasing in ultimate load was (42.58%) for simply support and (29.52%) for continuous beam.

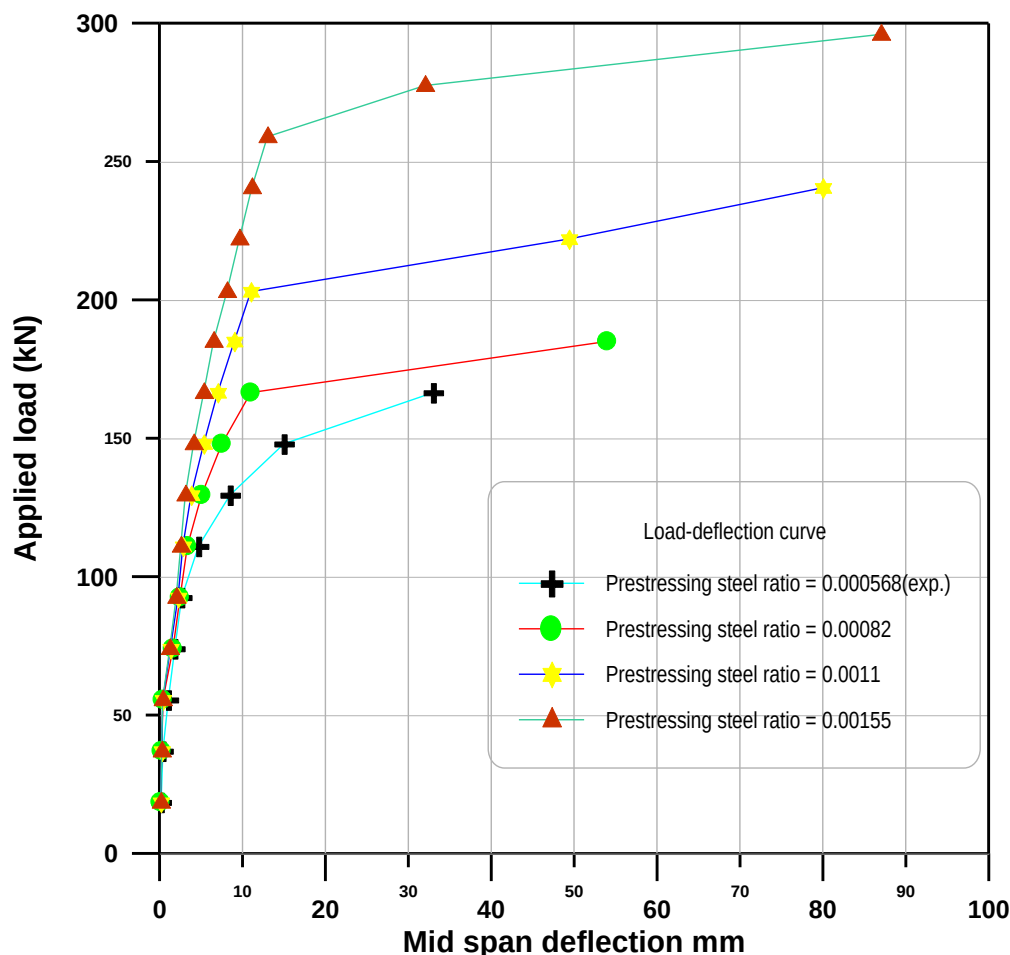


Figure (6-9): Effect of increasing prestressing steel ratio on load-deflection curve of simply supported composite prestressed concrete beam (B2) of Bishara and Almeiad [13].

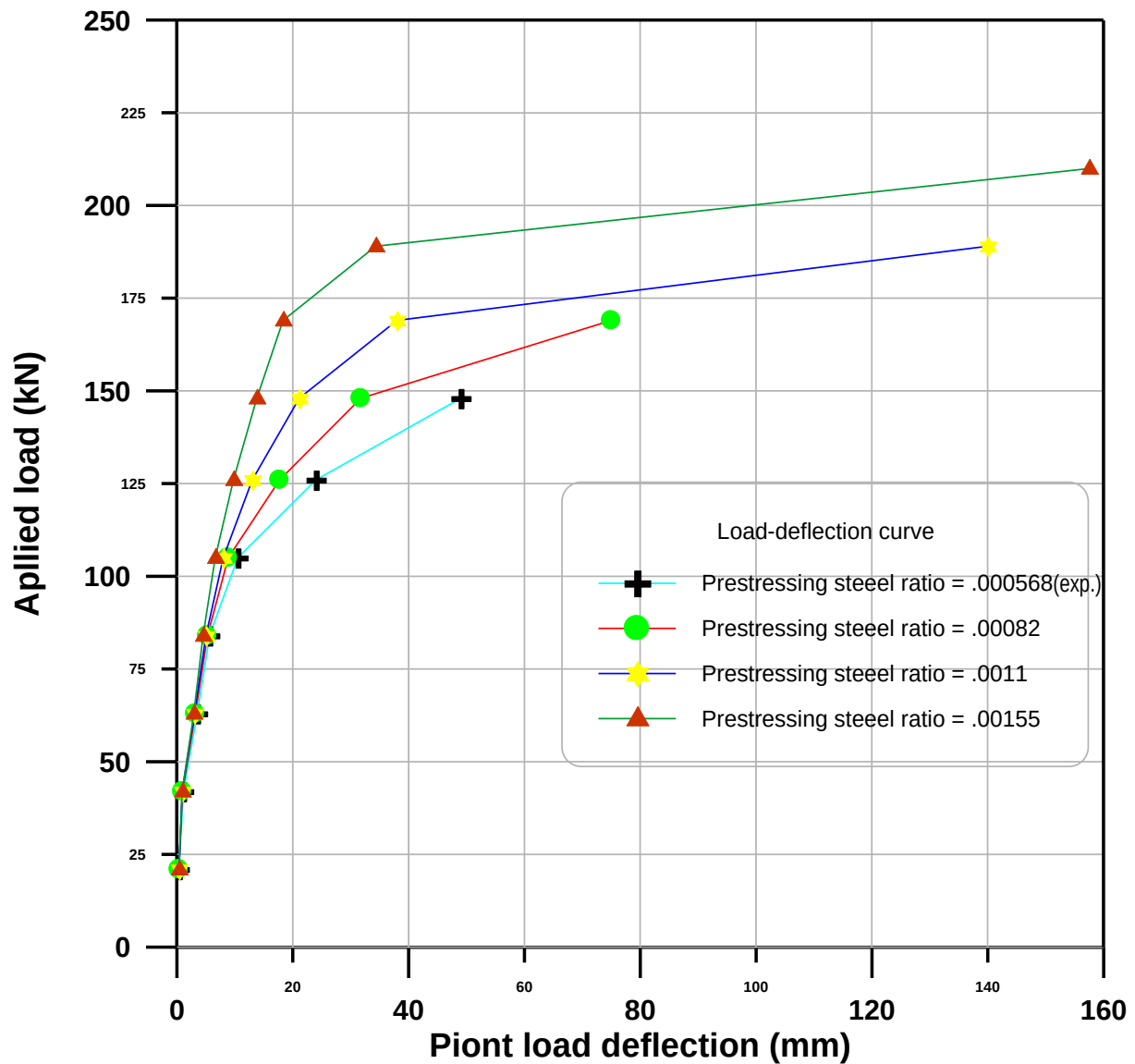


Figure (6-10): Effect of increasing prestressing steel ratio on load-deflection curve of continuous composite prestressed concrete beam (BC2) of Bishara et al. [14].

6-2-4 : Effect of The Shape of The Prestressed Concrete Units.

To study the effect of the shape of the prestressed concrete units on the behavior of the prestressed composite beam shown in figure (6-12). The following cases are considered:

1. The original case has two precast-prestressed units of (38 mm) width and (68 mm) depth shown in figure (6-11a).

2. The second case shown in figure (6-11b) has one precast-prestressed units of (76 mm) width and (68 mm) depth.
3. The third case shown in figure (6-11c) has one precast-prestressed units of (72 mm) width and (72 mm) depth.

In all prescribed three cases, the area of precast concrete unit is kept the same.

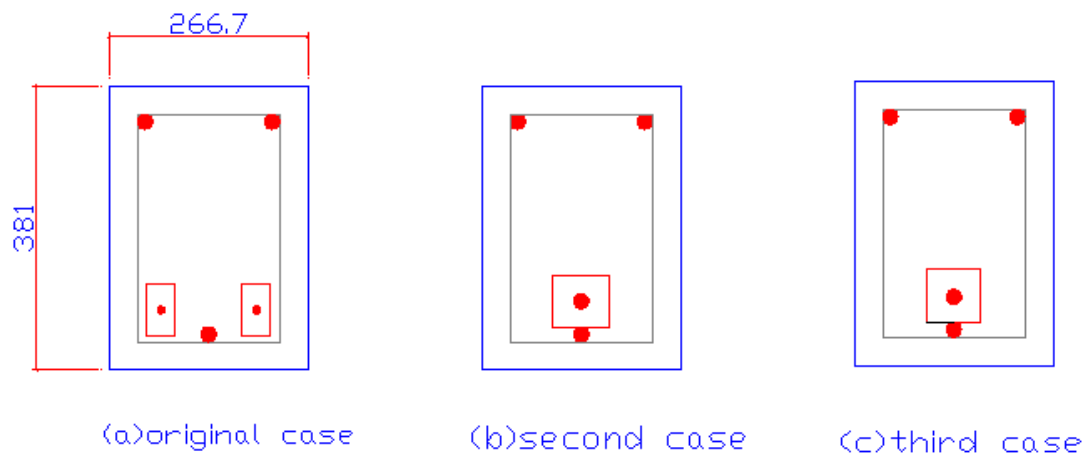


Figure (6-11): Cross-section of the composite prestressed concrete beam (B3) with three different shapes of the precast -prestressed units Bishara and Almeida [13].

Figure (6-12) shows the effect of the shape of the precast-prestressed units on load-deflection behavior of composite beam. It can be noted that one unit in the middle beam is more efficient and results stiffer behavior than the two units in corners. The rectangular shape of the precast-prestressed unit in the middle beam section has surface area less than the two units in the corners so that the bond failure is smaller in the one unit in the middle beam and number of cracks are smaller leading to the stiffness of the beam increases and the ultimate load is increased.

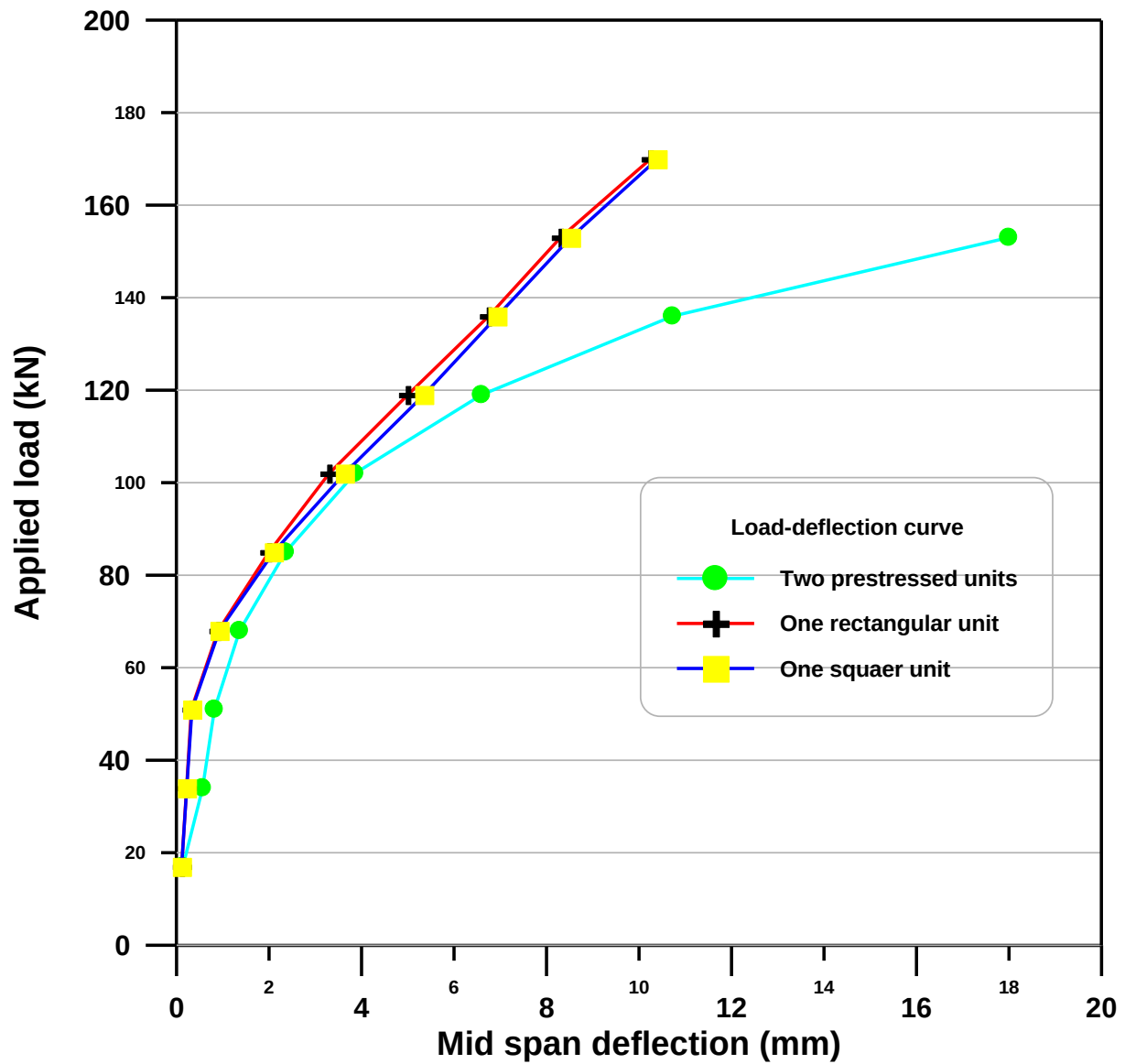


Figure (6-12): Effect shape of the precast- prestressed units on load-deflection curve of simply supported composite prestressed concrete beam (B3) of Bishara and Almeida[13].

Certificate

We certify that this thesis titled “*Nonlinear Analysis of Reinforced Members with Embedded Precast-Prestressed Concrete Units*”, was prepared by *Ali Hadi Adheem* under our supervision at University of Babylon as a fulfillment of the partial requirements for the Degree of Master of Science in Civil Engineering.

Supervisor

Signature :

Name : **Ass. Prof. Dr. Mustafa B. Dawood**

Date : / /2009

Supervisor

Signature :

Name: **Prof. Dr. Nameer A.Alwash**

Date : / /2009

Certificate of The Examining Committee

We certify as an Examining Committee that we have read this thesis, titled “*Nonlinear Analysis of Reinforced Concrete Members With Embedded Precast-Prestressed Concrete Units*”, and examined the student “*Ali Hadi Adheem*” in its content and what related to it, and found it meets the standard of thesis for the degree of Master of science in Civil Engineering (Structure Engineering).

Signature :

Name : **Ass. Prof. Dr. Mustafa B. Dawood**

Date : / / 2009

(Supervisor)

Signature :

Name : **Prof. Dr. Nameer A. Alwash**

Date : / / 2009

(Supervisor)

Signature :

Name : **Prof. Dr. Bayan S. OBaid**

Date : / / 2009

(Member)

Signature :

Name : **Ass. Prof. Dr. Raad K. Shuhur**

Date : / / 2009

(Member)

Signature :

Name : **Prof. Dr. Ammar Y. Ali**

Date : / / 2009

(Chairman)

Approved by the Head of the Civil Engineering Department

Signature :

Name : **Prof. Dr. Ammar Y. Ali**

Date : / / 2009

Approved by the Dean of the College of Engineering

Signature :

Name : **Ass. Prof. Dr. Salah Tawfeek AL-Bazzaz**

(The Dean of the College of Engineering)

Date : / / 2009

CHAPTER 1

INTRODUCTION

1-1: General

Concrete is essentially a compression material. Its strength in tension is much lower than that in compression. The original concept of prestressing concrete is to introduce sufficient axial pre-compression in member so that all tension in concrete is eliminated in member at service load. Therefore, prestressing can be defined in general terms as the pre-loading of a structure before application of service loads, so as to improve its performance in specific ways, **Nilson (1987) [1]**.

Prestressing actually provides tensile strength to the concrete member but does not increase the tensile strength of the concrete material itself. Plain concrete has a low tensile strength and limited ductility due to its brittle property which leads to a less energy absorption and low resistance to crack control.

Composite reinforced concrete beams with precast-prestressed concrete units have been used in the field of several civil engineering applications such as building and for decks of bridges for their advantages, especially in hinging regions of statically indeterminate structures and delays cracking of the in-situ concrete.

Composite construction implies the use, in a single structure acting as a unit, of different structural elements made with similar or different structural materials. Common examples include steel-concrete beams or columns, orthotropic steel decks, and sandwich panel construction. The two structural materials are utilized in the best possible manner. In a steel-concrete composite beam, the steel is designed to carry tension and the concrete to carry compression. Because prestressed concrete is less prone to cracking than ordinary reinforced concrete a combination of the two seems to offer obvious advantages, especially in hinging regions of statically indeterminate structures and where cracking is to be controlled. This combination could be realized in different ways, one is to reinforce the tension side of concrete flexural members by precast-prestressed concrete prisms.

On the other hand there are some places where the direct prestressing is inconvenient, or flexural cracks must be controlled without affecting the moment redistribution capacity of the structure. Then, precast-prestressed elements are used in order to solve this kind of problems.

Most investigators seem to agree that such composite systems do not exhibit sudden changes in deflection after cracking, **Mikhailov (1958) [2]**, **Burns (1966) [3]** and **Hanson (1969) [4]**. Some investigators proclaim that the incorporation of prestressed reinforcement delays cracking of the in-situ concrete, **Hanson (1969) [4]**, **Burns (1966) [3]** and **Evans (1955) [5]**. Others substituted small-sized elements are for conventional reinforcement in pavements, buildings and bridges slabs, rectangular concrete beam, column, continuity connections and other structures. Precast-prestressed elements can be used along the member or in critical sections.

Although composite members may improve construction efficiency, they also demand a more complex analysis. Composite construction usually involves different concrete properties in a single cross-section, changes in the constructed

system and stressing by tendons along the construction system. Consideration of these situations in an ultimate strength by hand calculation constitutes a very cumbersome task. Serviceability check is further complicated because it involves cracking and deflection computations, which are strongly dependent on time-effects acting on components of different ages and material properties.

The composite system consists of two components; the Precast-prestressed units, and in-situ concrete beams. The success of composite action depends on connection between prestressed concrete elements and ordinary concrete beam, each component will freely slide over the other and each carries separately a part of load. However, if adequate connection is provided, the slip between the two elements can be counted on figure (1-1). Thus a good connection between the two elements of the composite system is essential. This can be done artificially by roughening the interface surface, providing a bonding agent, and / or using shear connectors mostly in the form of extended stirrups. The problems of shear transfer in composite beams had generated substantial research work.

At present, with the development of digital computers and numerical techniques, the finite element method has emerged as a powerful analytical tool for analysis of structure. These many aspects of the phenomenological behaviors are encountered in prestressed concrete. These aspects include the nonlinear multiaxial material properties, modeling of cracking and crushing, the changes in material properties after cracking, yielding of prestressed steel and many other properties.

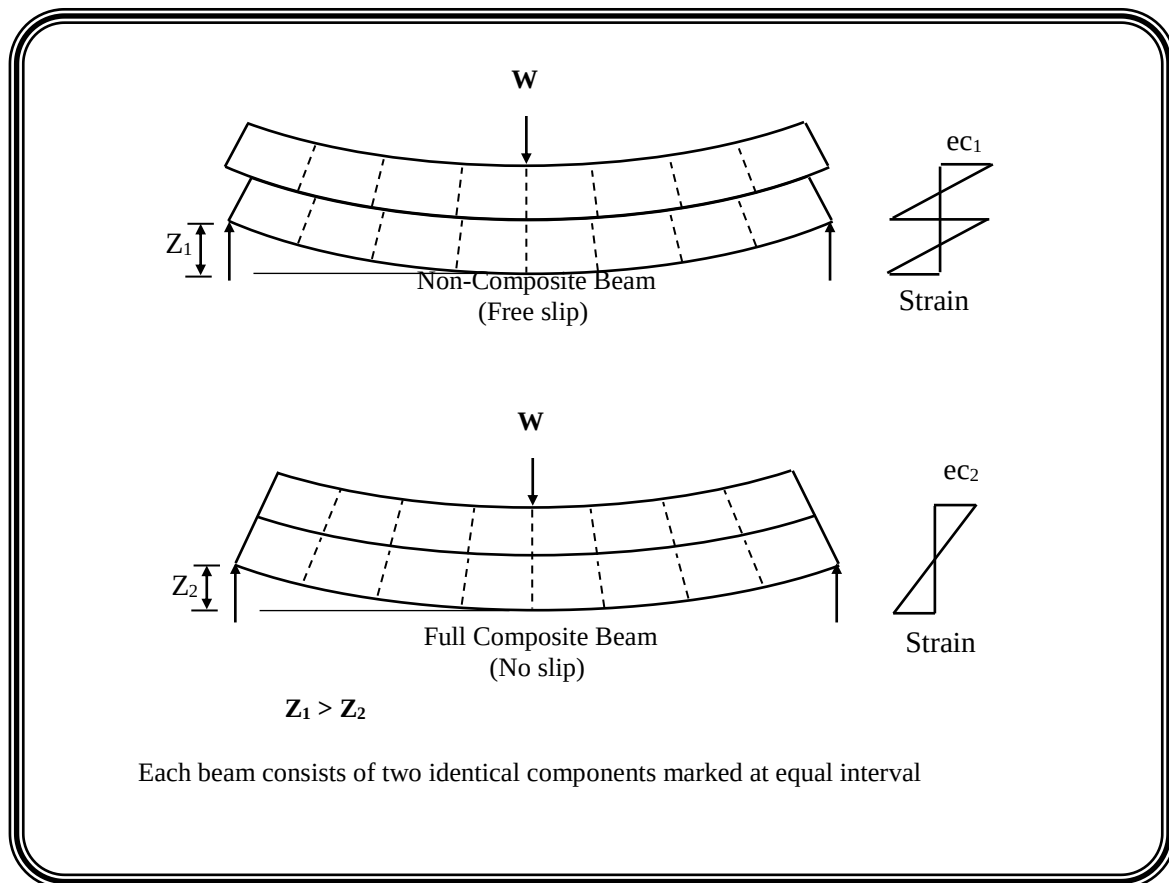


Figure (1-1): Composite and non-composite beams under same load(Al-Thebhawi) [6].

1-2: Scope and objective

This study is devoted to investigate the behavior of composite beams (prestressed concrete units embedded in ordinary reinforced concrete beam subjected to transverse loading). The advantages of using precast-prestressed elements in construction have become apparent through research and testing done in various countries, notably United States, United Kingdom, Germany and Russia. Some investigators substituted small-sized elements for conventional reinforcement in pavement, building and bridge slabs, rectangular concrete beams, columns, continuity connections and another structures, **Mawal 1979 [7]**, Figure (1-2). The main advantage of this oriented- production and economic material is that it can be used in places where direct prestressing is inconvenient

or where flexural cracks must be controlled without affecting the moment redistribution capacity of the structure.

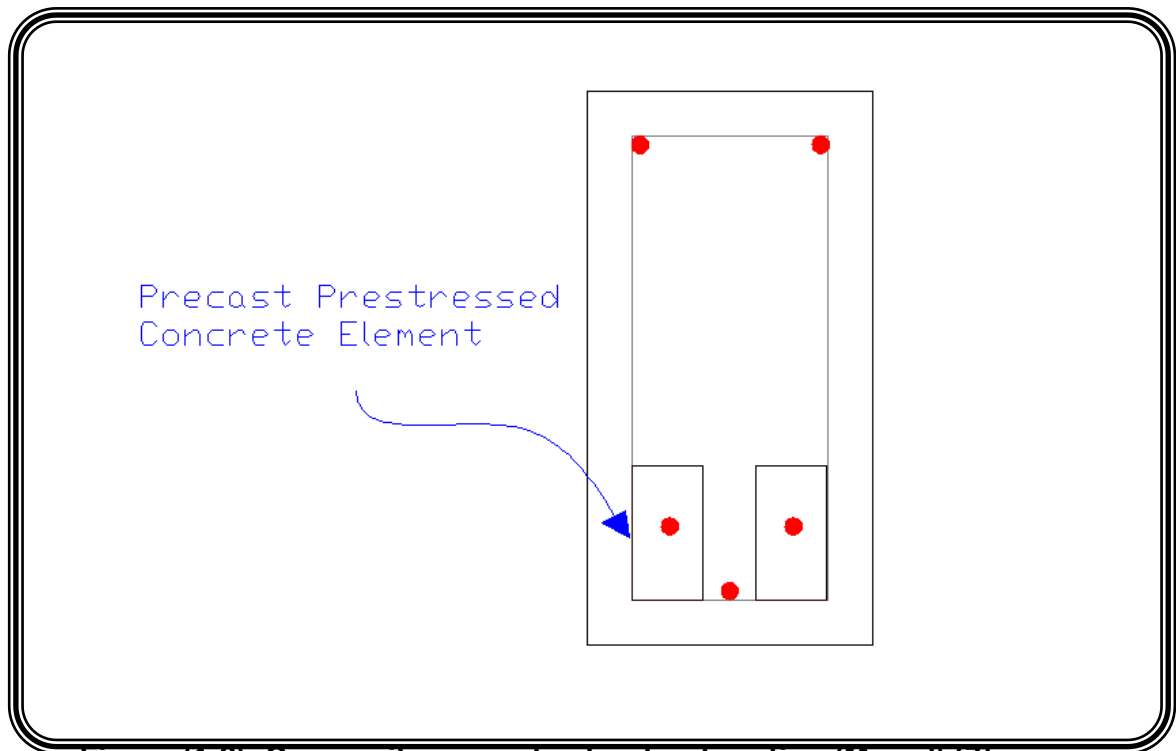


Figure (1-2): Composite concrete structural section (Mawal) [7]

Many testing can give an idea about the structural behavior, but it is not economical and time consuming procedure. Therefore, this study is carried out to develop an analytical procedure based on the finite element method to analyze the composite beams where a three-dimensional nonlinear finite element is utilized.

Material nonlinearities due to cracking, crushing of concrete as well as yielding of steel, are taken into account. Some composite beams have been considered to demonstrate the applicability of the adopted models by comparing the predicted results with available experimental results. This research work also aims to investigate the effect of some important material properties.

1-3: Layout of Thesis

The thesis is presented in seven chapters. A general introduction describing the significance of prestressed concrete elements in composite construction, composite action and description of the objectives of the investigation are presented in the current **Chapter One**.

Chapter Two reviews the work done on composite action of concrete beams. It reviews also the research work on the nonlinear finite element analysis of reinforced and prestressed concrete members.

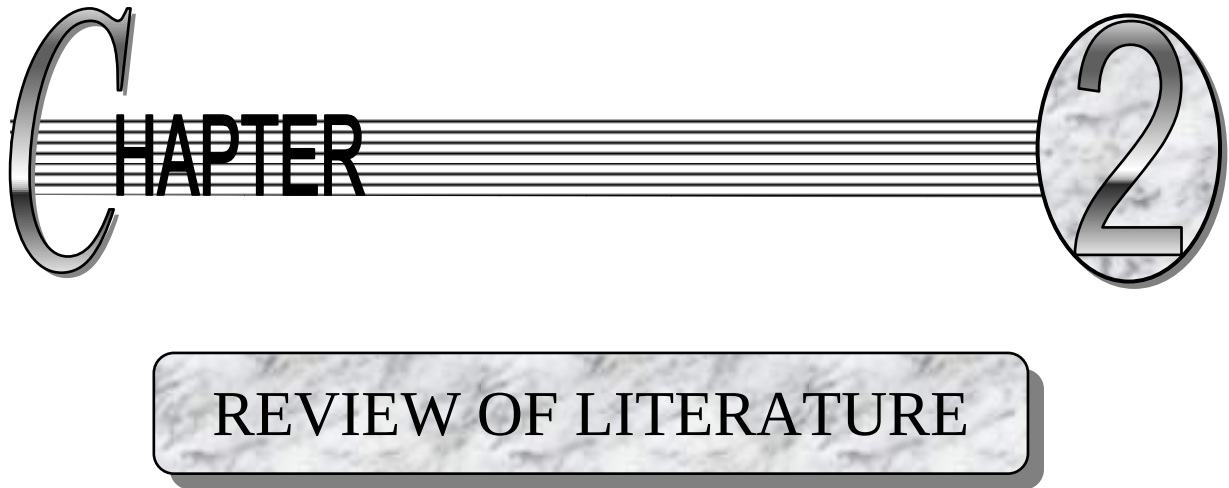
Chapter Three presents the basic concepts of the finite element formulation and the derivation of the governing equilibrium equations using the principle of virtual displacements. Concrete-steel members and the interface representations are described. The nonlinear solution techniques that are used to solve the nonlinear equations are also presented. The numerical integration rules used to determine the element stiffness matrix are given.

Chapter Four describes the behavior of concrete under both compression and tension, the steel reinforcement and the interface between the two bodies of ordinary reinforced concrete beams and prestressed concrete elements.

chapter Five gives the results of the nonlinear analysis of selected beams and the comparison with available experimental results.

A parametric study to investigate the effect of some parameters are presented in **Chapter six**

Chapter seven summarizes the conclusions obtained from the present research work and gives suggestions for future work.

The graphic features the word 'CHAPTER' in a bold, black, serif font, positioned on the left side of a horizontal staff with five lines. To the right of the staff is a large, stylized number '2' enclosed in an oval frame. Below this graphic is a rectangular box with a textured, light gray background and a dark gray border, containing the text 'REVIEW OF LITERATURE' in a bold, black, serif font.

CHAPTER 2

REVIEW OF LITERATURE

2-1: Introduction

The concept of non-linear analysis of reinforced concrete structures has increased steadily because of the wide use of both prestressed and ordinary reinforced concrete. In the mid sixties, the last century, the finite element method had started to be used, extensively, in the non-linear analysis of prestressed, as well as, reinforced concrete members such as beams, slabs and columns due to development of electronic digital computers.

This chapter reviews the theories of flexural behavior which were used to predict the flexural strength of concrete members and previous works that studies the behavior of composite section.

2-2: Studying The Behavior Of Composite Section

Evans and Parker (1955) [4], presented an experimental investigation on the behavior of composite section consisting of in-situ and prestressed concrete acts monolithically. Good bond between prestressed and in-situ concretes can be obtained if the jointing surface is sufficiently roughened. Cracks in the in-situ concrete of a composite beam open more slowly than those in either reinforced concrete or normal prestressed beams. The differential shrinkage between the two elements may affect cracking loads.

Hanson (1960) [5], made a series of tests on the problems of shear connections between precast concrete beams and cast-in-place concrete slabs indicated that the ultimate horizontal shear strength of a smooth bonded joint was about (2.1 MPa) and that of a rough bonded joint was about (3.5 MPa). In addition, it was found that the shear strength of a joint could be increased (1.23 MPa) for each percent of reinforcing steel crossing the joint.

Tests were carried out by **Seaman and Washa (1964) [6]**, on the problem of horizontal shear connections between precast concrete beams and cast-in-place concrete slabs. They tested 42 beams in an attempt to provide information on the degree of roughness of contact surface, length of shear span, percentage of steel across the joint, effect of shear keys, position of the joint with respect to the neutral axis and the effect of concrete compressive strength. The results obtained indicate complex relations between roughness of surface joint, percent steel across joint, and shear span. The ultimate shear strength was computed on the basis of the results from the following formula :

$$V = (2700 / (M + 5) + 300\beta((33 - M) / (M^2 + 6M + 5))) \dots\dots\dots(2-1)$$

Where :

V=ultimate shear strength of joint in psi.

β =percent of steel across joint, M=ratio of shear span to effective depth.

Birkeland and Birkeland (1966) [7], developed a philosophy of connection design in which cracks are assumed to have occurred at disadvantageous locations within the region of the connection. Reinforcement is then designed to transfer shear, normal force and moment across these cracks when the connection is loaded. They proposed that the reinforcement needed to transfer shear across the cracks be designed by using the "Shear-Friction" method of design. Provisions for the shear transfer reinforcement design by using the shear- friction method were subsequently included in the American concrete institute, ACI, Building Code 318M-95.

Badoux and Hulsbos (1967) [8], studied the strength of the joint between a pre-cast concrete beam and cast-in-place slab, when the composite beam was subjected to repeated loading. The test program included 29 beams and the principal variables were the amount of joint reinforcement, the roughness of the joint, and the ratio of the shear span to the effective depth of the beam. Equation have been presented which yield a conservative allowable stress for the horizontal shear in composite members under repeated loads.

Johnson et al. (1969) [9], presented and discussed the results of push-out tests on 8-mm diameter studs set in reinforced concrete slabs in tension. Examination of the applicability of the conclusions to larger studs and in beams was allowed by design recommendations for connectors in hogging moment regions of continuous composite beams. Two groups of push-out tests had been carried out on 8-mm studs connectors set in cracked concrete. The tests showed that the ultimate push-out strength of the connectors were laying between (0.64 – 1.06) from the strength found in a conventional push-out test and that the slip at 0.8 of the ultimate push-out strength was about double that at the same load in a conventional test, however the load /slip curves where widely different. The effect of tensile force was further explored in the second group of tests in which

the forces tending to prevent uplift of the slab were more closely controlled, and transverse reinforcement was introduced. They were expected to influence the state of the concrete surrounding the stud.

Bishara and Almeida (1970) [10], reported tests on six simply supported ordinary reinforced concrete beams with precast-prestressed concrete elements. They studied the effects of precast-prestressed concrete elements on crack width reduction, flexural rigidity and on rotational capacity of hinging regions. For the same load level, maximum cracks width were smaller in composite beams compared with ordinary reinforced concrete beams without precast-prestressed element, and the higher load level, the lower is the percentage reduction in maximum crack width. For the same load level, the deflection are smaller in beams with prestressed element, and the load causing first visible cracking of the in-situ concrete of the beams reported did not seem to be affected by the presence of the prestressed reinforcement.

Bishara et al. (1971) [11], reported tests on four continuous rectangular beams simply supported over two 15-ft spans with precast-prestressed concrete elements subjected to flexural loads. A study of the tests indicated that use of prestressed prisms as a part of tensile reinforcement in the intermediate support regions of continuous beams is quite beneficial for design purposes. It considerably improves the serviceability criteria related to cracking and flexural rigidity. For the same load level maximum crack widths and crack heights are smaller in the beam when prestressed reinforcement was used as tension reinforcement over the intermediate support.

Mallick and Chattopadhyay (1975) [12], investigated the general behavior of sixteen continuous composite beams tested to failure under different arrangements of spans and concentrated loads. Discussion of the results was made with reference to shear connection, slip, uplift, flexural mode of action,

stiffness and contribution of longitudinal reinforcement in the zones of negative bending moment. A very high degree of adaptation has been noticed in the test and the maximum theoretical loads were observed to have been carried within error of 9 percent. In these tests an exploration was also made for the possibility of ultimate strength design of continuous beams in which the first hinges would be formed at the loaded sections with the neutral axes located within the depth of slab at failure.

Frodin et al. (1978) [13], investigated the deflections of simply supported composite beams having full and partial shear connections and compared their behavior. They concluded that, when composite steel-concrete beams with partial shear connection are subjected to their service loads, their central deflections would not be greater than those of the corresponding beam with full shear connection carrying their own service loads. It is, therefore, suggested that the allowable span to depth ratios for beams with partial shear connection when due allowance is made for the reduced moment capacity and reduced service load of the latter. A linear analysis was used in the comparison but some allowance for nonlinear effects was also made. The study was based on a computer program, which analyzed a range of composite beams considered to have typical sections used in practice.

The effect of effective prestressing force and the area of the prestressed element on the behavior of composite prestressed concrete beams was studied by **Manak B. Mawal (1979) [3]**. He tested 16 simply supported and 6 continuous beams. He concluded that the range of linearity of deformation with respect to load increases with the increase in the effective prestressing force and the area of prestressed elements, and it is possible to predict the plastic hinge rotation and the degree of redistribution. In all tested continuous beams, failure occurred after full redistribution of moments had taken place and the structure

had been reduced to a mechanism of collapse. The central support section, where the first hinge was formed, was capable of absorbing large hinge rotation after the prestressed elements embedded at this section had cracked.

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Eighteen beams with a rectangular-shaped section and six beams with a T-shaped section with smooth interface tested by **Patnaik (2001)** []. Horizontal shear strength of composite concrete beams mainly depends on the roughness of the top surface of the precast girder over which the in-situ concrete is placed. A linear equation that represents well the behavior of composite concrete beams with a smooth interface.

AL-Aquly (2003) [18], studied a non-linear response of continuous composite concrete-steel girders with partial interaction by using a finite element analysis. Based on the partial interaction theory, one-dimensional composite beam element was developed. This study adopted a composite beam element with the layered system to incorporate the properties of the composite beam cross-section. In this study, including the constitutive models of non-linear material behavior for concrete, steel, shear connector and reinforcing steel, the proposed method of analysis was capable of predicting the response of composite beams throughout the elastic, inelastic and ultimate load ranges in one complete analysis. The difference between the results of this study and the experimental studies was no more than 5%.

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AL-Saidi (2004) [20], used a three-dimensional non-linear finite element analysis to predict the behavior of beams composed of concrete encased steel section. Eight-node isoparametric brick element has been used to model the reinforced concrete, while the steel I-section are modeled using four-node isoparametric shell elements, the bond between the concrete and steel I-section was represented by using five-node isoparametric interface elements and two nodes combine elements. Material nonlinearity due to cracking and crushing of the concrete and yielding of the steel I-section and reinforcing bars are taken into consideration during the analysis. The difference between the results of this study and the experimental studies was not more than 5%.

Abdul-kareem (2004) [21], used a three dimensional non-linear finite element model to analyze composite beams under concentrated load up to failure. The load-deflection behavior was predicted for composite beams consisting of a precast-prestressed I-section encased in ordinary reinforced concrete with full bond assumption under static loads using the analysis program "ANSYS V.5.4". Eight node isoparametric brick element has been used to model the ordinary reinforced concrete and precast-prestressed units. Material nonlinearity due to cracking and crushing of the concrete and yielding of

reinforcing bars and prestressing tendons are taken into consideration during the analysis. The difference between the results of this study and the experimental studies was not more than 10%.

Mohammed (2006) [22], used a three-dimensional non-linear finite element model to analyze composite prestressed concrete girders. A 20-node isoparametric brick element was used to represent the concrete in the slab and the prestressing concrete girder while the interface was considered as a thin 20-node isoparametric brick element. The prestressing and ordinary reinforced steel bars were idealized as axial members embedded within the concrete element, perfect bond was assumed between the concrete and steel. The concrete in compression was simulated by an elastic-plastic response which is terminated at the onset of crushing. Concrete behavior in tension is modeled as linear elastic prior to cracking. The prestressing effect was introduced as an external prestressing forces at the ends of beams. The analysis shows a difference in results not greater than 11% in comparison with experimental results.

Bachachi (2007) [23], used a three-dimensional non-linear finite element model to analyze composite concrete-steel beams under bending loads up to failure. The load-deflection behavior was predicted for composite beams consisting of a reinforced concrete slab and steel beams with shear connector under static loads using the analysis program "ANSYS". For the concrete slab, 8-node isoparametric brick element has been used. The steel reinforcing bars are modeled as axial members connecting opposite nodes between brick elements with full bond assumption. Four-node isoparametric shell elements have been used to model steel beam while the shear connectors were simulated by truss (bar) element. A three-dimensional point-to-point contact element was used to model the non-linear physical contact behavior of the surface between concrete slab and steel beam. The non-linear spring element and bar element were used to

model the shear connector between two different materials. The maximum percentage difference in ultimate load-carrying capacity is 5.5%.

2-3: Non-linear Finite Element Analysis of Reinforced Concrete Members.

The earliest publication on the application of the finite element method to analyze reinforced concrete structure was presented by **Ngo and Scordelis (1967) [24]**. In their study, simple beam was analyzed with a method in which concrete and reinforcing steel were represented by constant strain triangular elements, and a special bond link element was used to connect the steel to concrete and describe the bond-slip effect. A linear elastic analysis was performed on beam with predefined crack patterns to determine principal stress in concrete, stress in steel reinforcement and bond stresses.

Nilson (1968) [25], extended the work of Ngo and Scordelis to include cracking (discrete crack), bond slips and the non-linear material behavior . The concrete and steel reinforcement were represented by two dimensional triangular elements. The uncracked member was loaded incrementally until the principal tensile stress exceeds the modulus of rupture of concrete at one or more locations. In this analysis , the cracks were allowed to form without bias to location or extent. Thus, it was possible to trace the progressive crack formation due to incremental load process.

A non –linear stress analysis including the elasto-plastic behavior of concrete and steel in addition to limiting tension of concrete is presented by, **Vallioppa and Doolan (1972) [26]**. The triangular elements have been used for

the concrete and, bar elements have been used for reinforcement. Because the ends of the bar elements have been connected rigidly to the adjacent triangular elements of concrete, perfect bond has been assumed. The results of the numerical examples indicate that this type of analysis will be valuable to the design engineer as it offers a complete picture of the stress distribution both in concrete and steel.

Mubbada and William (1973) [27], presented a non-linear finite element model to analyze reinforced concrete beams. This paper used isoperimetric elements to represent the concrete. A linear bar element was introduced to represent the reinforcing bars. This analysis incorporates an elasto-plastic behavior of concrete and reinforcing steel and cracking of concrete stressed in tension if any principal stress exceeded the tensile strength of concrete. Comparison with experimental results were found favorable with difference not more than 7%.

Buyukozturk (1977) [28], presented a non-linear finite element model to analyze reinforced concrete deep beams. Deep beams were simulated by using a plane stress finite element model consisting of isoperimetric triangular elements for concrete with reinforcement being represented by equivalent isotropic steel layers within the concrete elements. The concrete failure law was based on a generalized Mohr-Coulomb behavior developed from biaxial experimental data, while the constitutive relationships were based on plasticity concepts for two dimensional stress state.

By including cracking and post cracking behavior of reinforced concrete members, **Damijanac and Owen (1985) [29]**, used eight-nodded isoperimetric quadrilateral elements to represent the concrete. Steel bars were represented by embedded elements into the isoperimetric elements. Tensile cracking was governed by a maximum tensile stress criterion. The tensile stiffness were

reduced to zero according to the linear descending stress-strain relations. The cracking model was adjusted by tension-stiffening and shear retention parameters.

AL-Ta'an and Mohamed (1988) [30], used the finite element method for the analysis of reinforced concrete deep beams. Concrete was represented by a quadrilateral isoperimetric four nodal element with or without incompatible modes. The main reinforcement was represented by either using an axial single bar element or a constant strain triangle element having an equivalent cross sectional area of the bar as well as equivalent moment of inertia. The linkage elements were introduced to simulate the bond-slip phenomenon, as well as, dowel action. The web reinforcement was also idealized by bar elements with perfect bond the with concrete.

Frantzeskakis and Theillout (1989) [31], developed a computer program for finite element non-linear analysis of two dimensional reinforced concrete structures under monotonic loading. The smeared crack approach is used and the reinforcement is considered uniformly distributed with the element. An elastic work-hardening plastic stress-strain relationship with an associated flow rule are assumed for the concrete in compression and elastic brittle fracture behavior in tension. The steel is considered as an elastic-perfectly plastic material. Non-linear reversible formulation are used for the tension stiffening and the aggregate interlock, while a linear model is adopted for the dowel action of the reinforcement.

AL-Shaarbaf (1990) [32], presented a three-dimensional nonlinear finite element method suitable for the analysis of reinforced concrete or steel members under general states of loading. The 20-noded isoperimetric brick element was used to model the concrete while the reinforcing bars were idealized as axial members embedded within the concrete elements. The behavior of concrete in

compression was simulated by an elasto-plastic work hardening model followed by a perfectly plastic response, which is terminated at the onset of crushing. In tension, a smeared crack model with fixed orthogonal cracks has been used with the inclusion of models for retained post-cracking stress and the reduced shear modulus. Also, a model simulate the reduction in the compressive strength of concrete due to orthogonal cracking was used.

Abbasi et al (1992) [33], applied the finite element method for the nonlinear analysis of reinforced concrete slabs subjected to central patch loads. Using the layered approach, a degenerated quadratic plate element with five degrees of freedom has been employed to model the reinforced concrete slabs. Smeared crack model was used with fixed orthogonal cracking. Several comparative studies have been considered in this work. These studies were conserved with perfectly plastic models versus hardening model, variation of tension- stiffening parameter, degraded shear modulus, and the role of yielding criterion.

Allose (1996) [34], used a three-dimensional nonlinear finite element model to study the behavior of steel fiber reinforced concrete rectangular beams subjected to torsion. The 8 and 20-noded isoperimetric brick element were used to model the concrete. Reinforcing bars were idealized as axial members embedded within the concrete elements. The behavior of concrete in compression was simulated by an elasto-plastic work hardening model followed by a perfectly plastic response, which is terminated at the onset of crushing. In tension, a smeared crack model with fixed orthogonal cracks has been used with the inclusion of models for retained post-cracking stress and the reduced shear modulus.

Antonio and Gabrial (1998) [35], presented a nonlinear finite element model to analyze simply supported reinforced concrete beams subjected to uniformly distributed loading. This paper used a three-dimensional solid element

to represent the concrete. The material is capable of cracking in tension and crushing in compression. A smeared model was used to represent the reinforcing bars. The smeared crack approach was used to model the cracking of concrete and the softening of concrete was considered. The difference between the results of this study and the experimental studies was no more than 15%.

Diab (1998) [36], investigated the behavior of high strength reinforced concrete members that fail in shear and torsion modes using a non-linear finite element model. In this investigation, a verification of proposed softening models was carried out using a numerical nonlinear finite element analysis on high strength reinforced concrete beams in torsions and panels subjected to in-plane loading. A three-dimensional 20-noded isoperimetric brick element was used to model the concrete, and the steel have been modeled as axial members embedded within the concrete brick element. He concluded that the effect of hollowing out the cross section by a certain amount leads to the acceptance of ACI 318M-95 assumption that deals with the solid section loaded in torsion.

Al-Rmahee (2002) [37], used a three-dimensional finite element model for the analysis of high strength reinforced concrete slabs under different types of loading. The 20-noded isoperimetric brick element was used to model the concrete while the reinforcing bars were idealized as axial members embedded within the brick elements. Perfect bond was assumed to occur between the concrete and the reinforcing bars. The behavior of concrete in compression was simulated by an elasto-plastic work hardening model followed by a perfectly plastic response, which is terminated at the onset of crushing. In tension, a smeared crack model with fixed orthogonal cracks has been used with the inclusion of models for retained post-cracking stress and the reduced shear modulus. Also a model simulate the reduction in the compressive strength of concrete due to orthogonal cracking was used. Comparison with experimental results were found favorable with difference not more than 1%.

2-4: Non-linear Finite Element Analysis of Prestressed Concrete Members.

Kang and Scordelis (1980) [38], investigated the behavior of plane prestressed concrete frames based on the finite element method. The material and geometric non-linearities were considered including the time dependent effect due to creep, shrinkage, and relaxation of prestressing force. Multilinear stress-strain curves were assumed for reinforcing and prestressing steel. In order to account for the varied material properties within a frame element the cross section is divided into a discrete number of concrete and reinforcing steel layers each of which is assumed to be in a state of uniaxial stress. The models were capable of predicting the behavior of these structures throughout their service load history as well as through elastic, inelastic, and ultimate load ranges in one complete analysis. The prestressing steel is considered as an integral part of the structure. Thus, the variation of prestress throughout various stages of prestressing and loading is traced automatically in a unified manner.

Abdul- Rahman and Hinton (1986) [39], analyzed reinforced and prestressed concrete cellular slabs based on slab-beam model by using linear and nonlinear two dimensional finite element models. The basic idea of this work was the separation of the cellular plate into main components, a hollow plate, representing the upper and lower flanges and stiffening beams representing the vertical webs between the voids. The voided slab cross sections were represented as different layered elements assuming that the plane sections before deformation remain plane after deformation. The pretensioned prestressing

strands are replaced by ordinary reinforcement and the prestressing force was represented by axial forces acting at the center of the reinforcement. The difference between the results of this study and the experimental studies was no more than 10%.

Ghalib (1990) [40], presented the behavior of reinforced and prestressed three dimensional multi-planer system. The concrete was represented by assemblage of a set of isoperimetric layered 8-noded flat shell elements. The behavior of concrete in compression was modeled as an elastic-plastic material with hardening and as elastic-brittle fracture in tension. A generally curved bonded/unbonded prestressing strand element embedded in the general 8-noded shell element has been developed. An iterative incremental technique was adopted in the study to suit the nonlinear path-dependent response.

Campbell and Kouder (1990) [41], used a finite element model based on a curvature incremental technique to apply the nonlinear analysis of prestressed concrete continuous beams. In this paper, prestressed concrete continuous beams were idealized by dividing it into segments. Each segment was defined as the length between two consecutive joints, with each joint having two degrees of freedom. Nonlinearity was included in the model by means of stepwise linear analysis technique, which made use of the relevant second stiffness obtained from the moment-curvature relationship for segments along the length of beam. For concrete in tension, the stress-strain relationship is assumed to be linear with a slop equal to that of the curve for compression at zero stress. An elastic plastic stress-strain relationship was used for nonprestressed reinforcement, both in compression and tension. A multilinear stress-strain relationship was used for prestressed reinforcement in tension. The prestressing effect was introduced as initial strain in prestressing wire. The analysis shows a difference in results not greater than 2%.

AL-Thebhawe (2001) [42], used a three-dimensional nonlinear finite element model to analyze prestressed concrete member under flexural. The 20-noded isoperimetric brick element was used to model the concrete while the prestressing steel bars were idealized as axial members embedded within the brick elements. The behavior of concrete in compression was simulated by an elasto-plastic work hardening model followed by a perfectly plastic response, which is terminated at the onset of crushing. In tension, a smeared crack model with fixed orthogonal cracks has been used with the inclusion of models for retained post-cracking stress and the reduced shear modulus. The prestressing effect was introduced as an external prestressing forces at the ends of beam. The analysis shows a difference in results not greater than 1.05%.

Fanning and Znidaric (2001) [43], discussed the possibilities of practical application of a nonlinear model in the analysis of prestressed concrete structures. The results of some analyses performed by using the reinforced concrete model in the general-purpose finite element code “ANSYS” were presented and discussed. Differences were observed in the response of the same prestressed concrete beam when some variations were made in the material model. A simply supported prestressed concrete beam subjected to concentrated loading has been analyzed with two kinds of meshes: (1) adopting truss bars as discrete reinforcement connecting the solid elements nodes of the concrete model, (2) adopting smeared reinforcement within the solid elements. The post-tensioning was modeled via a initial strain in the tendon elements, corresponding to tendon forces. The analysis shows a difference in results not greater than 4% between the two models.

Mustafa (2002) [44], used a three-dimensional nonlinear finite element model to analyze prestressed concrete member under torsion. The 20-noded isoperimetric brick element was used to model the concrete. The reinforcing

bars were idealized as axial members embedded within the brick element and perfect bond between the concrete and the reinforcement was assumed to occur. The behavior of concrete in compression was simulated by an elasto-plastic work hardening model followed by a perfectly plastic response, which is terminated at the onset of crushing. In tension, a smeared crack model in connection with using a tension stiffening model was adopted and this account for the retained post-cracking stresses, and a shear retention model that modifies the shear modulus of rigidity as the crack widens. The prestressing effect was introduced as an external prestressing forces at the ends of beam.

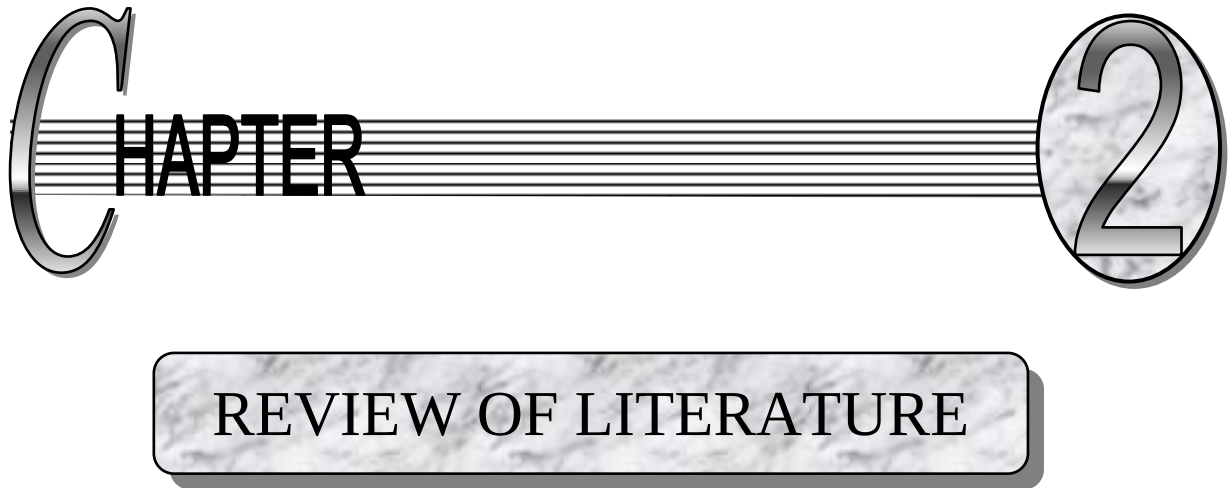
Antho J. Wokanski (2004) [45], presented finite element analysis of prestressed concrete beam including material non-linearity and linear bond-slip relation between the steel and concrete as a relaxation of prestressing steel. Bilinear stress-strain curves were assumed for prestressing steel. These models were capable of predicting the response of the structure over the entire load range up to failure. The prestressing effect was introduced as initial strain in prestressing wire. The analysis shows a difference in results not greater than 2% with previous experimental works.

Amir (2005) [46], investigated the behavior of prestressed concrete members under combined shear and torsion using a non-linear three dimensional finite elements model. The 20-noded isoperimetric brick element was used to model the concrete while the prestressing steel bars were idealized as axial members embedded within the brick elements. The behavior of concrete in compression was simulated by an elasto-plastic work hardening model followed by a perfectly plastic response, which is terminated at the onset of crushing. The behavior in tension was simulated by linear elastic prior to cracking. The prestressing effect was introduced as an external prestressing forces at the ends

of beam. Comparison with experimental results were found favorable with difference not more than 4.82%.

2-5: Summary

In the last decades, few numbers of researches have dealt with precast-prestressed concrete units embedded in reinforced concrete members. Limited numerical study dealt with the behavior of composite reinforced concrete members with embedded precast-prestressed concrete members with interface element as reviewed in the present work. In the present study, three dimensional non-linear finite element analysis using 8-noded brick element with three degree of freedom at each node (u, v, w) are used to model the ordinary concrete and prestressing concrete while the interface is considered as a 3-D (2-nodded) zero thickness interface element. The prestressing and ordinary reinforced steel bars are idealized as discrete elements connecting opposite nodes between brick elements.



CHAPTER 2

REVIEW OF LITERATURE

2-1: Introduction

The concept of non-linear analysis of reinforced concrete structures has increased steadily because of the wide use of both prestressed and ordinary reinforcement concrete. In the mid sixties, the last century, the finite element method had started to be used, extensively, in the non-linear analysis of prestressed, as well as, reinforced concrete members such as beams, slabs and columns due to development of electronic digital computers.

This chapter reviews the theories of flexural behavior which were used to predict the flexural strength of concrete members. The previous works that studies the behavior of composite section. A review is also presented on the research work related to the non-linear finite element analysis of reinforced concrete members, prestressed concrete members and interface model.

2-2: Studying The Behavior Of Composite Section

Evans and Parker (1955) [5], presented an experimental investigation on the behavior of composite section consisting of in-situ and prestressed concrete acts monolithically. Good bond between prestressed and in-situ concretes can be obtained if the jointing surface is sufficiently roughened. Cracks in the in-situ concrete of a composite beam open more slowly than those in either reinforced concrete or normal prestressed beams. The differential shrinkage between the two elements may affect cracking loads.

Hanson (1960) [8], made a series of tests on the problems of shear connections between precast concrete beams and cast-in-place concrete slabs indicated that the ultimate horizontal shear strength of a smooth bonded joint was about (2.1 MPa) and that of a rough bonded joint was about (3.5 MPa). In addition, it was found that the shear strength of a joint could be increased (1.23 MPa) for each percent of reinforcing steel crossing the joint.

Tests were carried out by **Seaman and Washa (1964) [9]**, on the problem of horizontal shear connections between precast concrete beams and cast-in-place concrete slabs. They tested 42 beams in an attempt to provide information on the degree of roughness of contact surface, length of shear span, percentage of steel across the joint, effect of shear keys, position of the joint with respect to the neutral axis and the effect of concrete compressive strength. The results obtained indicate complex relations between roughness of surface joint, percent steel across joint, and shear span. The ultimate shear strength was computed on the basis of the results from the following formula :

$$V = (2700 / (M + 5) + 300\beta((33 - M) / (M^2 + 6M + 5))) \dots\dots\dots(2-1)$$

Where :

V=ultimate shear strength of joint in psi.

β =percent of steel across joint, M=ratio of shear span to effective depth.

Birkeland and Birkeland (1966) [10], developed a philosophy of connection design in which cracks are assumed to have occurred at disadvantageous locations within the region of the connection. Reinforcement is then designed to transfer shear, normal force and moment across these cracks when the connection is loaded. They proposed that the reinforcement needed to transfer shear across the cracks be designed by using the "Shear-Friction" method of design. Provisions for the shear transfer reinforcement design by using the shear- friction method were subsequently included in the American concrete institute, ACI, Building Code 318M-95.

Badoux and Hulsbos (1967) [11], studied the strength of the joint between a pre-cast concrete beam and cast-in-place slab, when the composite beam was subjected to repeated loading. The test program included 29 beams and the principal variables were the amount of joint reinforcement, the roughness of the joint, and the ratio of the shear span to the effective depth of the beam. Equation have been presented which yield a conservative allowable stress for the horizontal shear in composite members under repeated loads.

Johnson et al. (1969) [12], presented and discussed the results of push-out tests on 8-mm diameter studs set in reinforced concrete slabs in tension. Examination of the applicability of the conclusions to larger studs and in beams was allowed by design recommendations for connectors in hogging moment regions of continuous composite beams. Two groups of push-out tests had been carried out on 8-mm studs connectors set in cracked concrete. The tests showed that the ultimate push-out strength of the connectors were laying between (0.64 – 1.06) from the strength found in a conventional push-out test and that the slip at 0.8 of the ultimate push-out strength was about double that at the same load in a conventional test, however the load /slip curves where widely different. The effect of tensile force was further explored in the second group of tests in which

the forces tending to prevent uplift of the slab were more closely controlled, and transverse reinforcement was introduced. They were expected to influence the state of the concrete surrounding the stud.

Bishara and Almeida (1970) [13], reported tests on six simply supported ordinary reinforced concrete beams with precast-prestressed concrete elements. They studied the effects of precast-prestressed concrete elements on crack width reduction, flexural rigidity and on rotational capacity of hinging regions. For the same load level, maximum cracks width were smaller in composite beams compared with ordinary reinforced concrete beams without precast-prestressed element, and the higher load level, the lower is the percentage reduction in maximum crack width. For the same load level, the deflection are smaller in beams with prestressed element, and the load causing first visible cracking of the in-situ concrete of the beams reported did not seem to be affected by the presence of the prestressed reinforcement.

Bishara et al. (1971) [14], reported tests on four continuous rectangular beams simply supported over two 15-ft spans with precast-prestressed concrete elements subjected to flexural loads. A study of the tests that use of prestressed prisms as apart of tensile reinforcement in the intermediate support regions of continuous beams is quite beneficial for design purposes. It considerably improves the serviceability criteria related to cracking and flexural rigidity. For the same load level maximum crack widths and crack heights are smaller in the beam when prestressed reinforcement was used as tension reinforcement over the intermediate support.

Mallick and Chattopadhyay (1975) [15], investigated the general behavior of sixteen continuous composite beams tested to failure under different arrangements of spans and concentrated loads. Discussion of the results was made with reference to shear connection, slip, uplift, flexural mode of action,

stiffness and contribution of longitudinal reinforcement in the zones of negative bending moment. A very high degree of adaptation has been noticed in the test and the maximum theoretical loads were observed to have been carried within error of 9 percent. In these tests an exploration was also made for the possibility of ultimate strength design of continuous beams in which the first hinges would be formed at the loaded sections with the neutral axes located within the depth of slab at failure.

Frodin et al. (1978) [16], investigated the deflections of simply supported composite beams having full and partial shear connections and compared their behavior. They concluded that, when composite steel-concrete beams with partial shear connection are subjected to their service loads, their central deflections would not be greater than those of the corresponding beam with full shear connection carrying their own service loads. It is, therefore, suggested that the allowable span to depth ratios for beams with partial shear connection when due allowance is made for the reduced moment capacity and reduced service load of the latter. A linear analysis was used in the comparison but some allowance for nonlinear effects was also made. The study was based on a computer program, which analyzed a range of composite beams considered to have typical sections used in practice.

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cracked thin-layer 20-node isoperimetric interface element. The behavior of the concrete in compression is simulated by an elasto-plastic work hardening model followed by a perfect plastic response, which is terminated at the onset of the crushing. In tension, a smeared cracked model with fixed orthogonal cracks has been used with the inclusion of models for the retained post-cracking tensile stress and reduced shear modulus. The non-linear equation of equilibrium were solved by using an incremental-iterative technique operated under load control. The difference between the results of this study and the experimental studies was no more than 4%.

AL-Saidi (2004) [22], used a three-dimensional non-linear finite element analysis to predict the behavior of beams composed of concrete encased steel section. Eight-node isoperimetric brick element has been used to model the reinforced concrete, while the steel I-section are modeled using four-node isoperimetric shell elements, the bond between the concrete and steel I-section was represented by using five-node isoperimetric interface elements and two nodes combine elements. Material nonlinearity due to cracking and crushing of the concrete and yielding of the steel I-section and reinforcing bars are taken into consideration during the analysis. The difference between the results of this study and the experimental studies was not more than 5%.

Abdul-kareem (2004) [23], used a three dimensional non-linear finite element model to analyze composite beams under concentrated load up to failure. The load-deflection behavior was predicted for composite beams consisting of a precast-prestressed I-section encased in ordinary reinforced concrete with full bond assumption under static loads using the analysis program "ANSYS V.5.4". Eight node isoperimetric brick element has been used to model the ordinary reinforced concrete and precast-prestressed units. Material nonlinearity due to cracking and crushing of the concrete and yielding of

reinforcing bars and prestressing tendons are taken into consideration during the analysis. The difference between the results of this study and the experimental studies was not more than 10%.

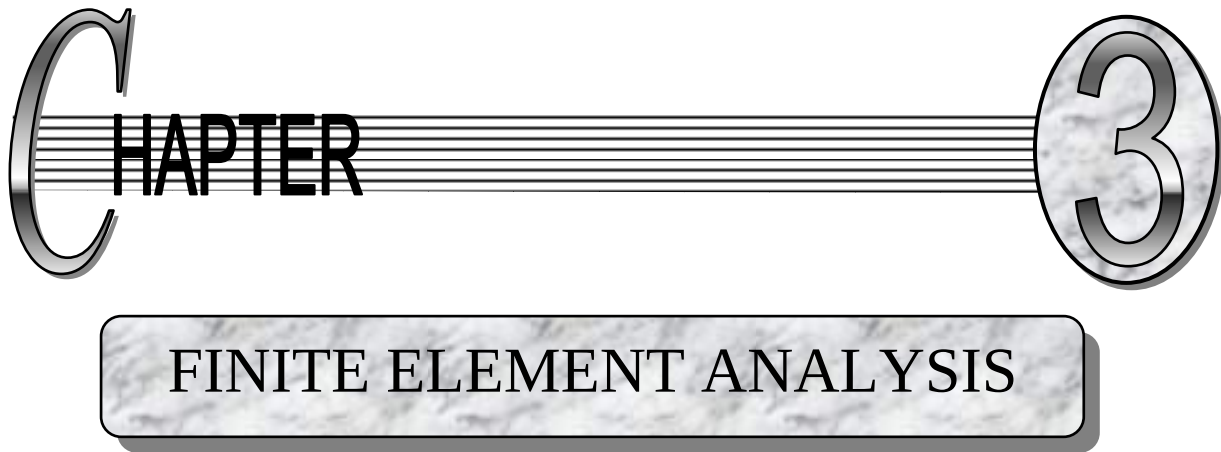
Mohammed (2006) [24], used a three-dimensional non-linear finite element model to analyze composite prestressed concrete girders. A 20-node isoperimetric brick element was used to represent the concrete in the slab and the prestressing concrete girder while the interface was considered as a thin 20-node isoperimetric brick element. The prestressing and ordinary reinforced steel bars were idealized as axial members embedded within the concrete element, perfect bond was assumed between the concrete and steel. The concrete mode, in compression was simulated by an elastic- plastic response which is terminated at the onset of crushing. Concrete behavior in tension is modeled as linear elastic prior to cracking. The prestressing effect was introduces as an external prestressing forces at the ends of beams. The analysis shows a difference in results not greater than 11% in comparison with experimental results.

Bachachi (2007) [25], used a three-dimensional non-linear finite element model to analyze composite concrete-steel beams under bending loads up to failure. The load-deflection behavior was predicted for composite beams consisting of a reinforced concrete slab and a steel beams with shear connector under static loads using the analysis program "ANSYS". For the concrete slab, 8-node isoperimetric brick element has been used. The steel reinforcing bars are modeled as axial members connecting opposite nodes between brick elements with full bond assumption. Four-node isoperimetric shell elements have been used to model steel beam while the shear connectors were simulated by truss (bar) element. A three- dimensional point-to-point contact element was used to model the non-linear physical contact behavior of the surface between concrete

slab and steel beam. The non-linear spring element and bar element were used to model the shear connector between two different materials. The maximum percentage difference in ultimate load-carrying capacity is 5.5%.

2-5: Summary

In the last decades, few numbers of researches have dealt with precast-prestressed concrete units embedded in reinforced concrete members. Limited numerical study dealt with the behavior of composite reinforced concrete members with embedded precast-prestressed concrete members with interface element as reviewed in the present work. In the present study, three dimensional non-linear finite element analysis using 8-noded brick element with three degree of freedom at each node (u, v, w) are used to model the ordinary concrete and prestressing concrete while the interface is considered as a 3-D (2-nodded) zero thickness interface element. The prestressing and ordinary reinforced steel bars are idealized as discrete elements connecting opposite nodes between brick elements.

A decorative graphic for the chapter header. It features a large, stylized letter 'C' on the left, which is part of a horizontal bar. The word 'CHAPTER' is written in a bold, sans-serif font across the middle of this bar. On the right end of the bar is a large, stylized number '3'. Below this graphic is a rectangular box with a textured, light gray background and a dark border. Inside this box, the words 'FINITE ELEMENT ANALYSIS' are written in a bold, serif font.

CHAPTER 3

FINITE ELEMENT ANALYSIS

3-1: Introduction

The nonlinear finite element approach can be used to predict the behavior of composite reinforced concrete members with embedded precast-prestressed concrete units at different stages of loading. The approach can be considered as a theoretical or more accurately a numerical method to test the composite prestressed concrete members, **Amir (2005) [26]**. The efficiency and accuracy of this test depends mainly on suitable modeling of material properties of reinforced concrete members, the precast-prestressed concrete units, and the interface between the reinforced concrete member and the precast-prestressed concrete units. Although this approach is widely used to analyze reinforced concrete members, but very little work has been devoted to study the behavior of composite prestressed concrete members.

In the present study, the 8-node isoperimetric brick element is used for the idealization of concrete, while the steel reinforcement is formulated by using link elements with the assumption of perfect bond between steel and concrete. The interface between reinforced concrete member and embedded precast-prestressed concrete units is formulated by node to node contact element.

3-2: Finite Element formulation

For any homogenous continuum subjected to a set of external forces, consisting of surface tractions and body forces, the element stiffness matrix for the individual finite elements can be determined by using an energy principle, such as the principle of virtual work. The following basic steps are involved no matter what type or shape of element is involved, **AL-Sa'aty (2004)[21]**.

The displacement vector at any point within the element {a} can be expressed as:

$$\{a\}=[N] \{U\} \quad \text{.....(3-1)}$$

Where [N], is a matrix containing the interpolation functions which relate the displacement {a}, to nodal displacement, {U}.

The corresponding strains {ε}, are obtained through the differentiation of the displacements :

$$\{\varepsilon\}=[B] \{U\} \quad \text{..... (3-2)}$$

Where [B], is a matrix that contains the differential operators and it is called the strain-displacement matrix.

Where, [B]=[N']

The stress-strain relationship is usually given by :

$$\{\sigma\}=[D] \{\varepsilon\} \quad \text{.....(3-3)}$$

Where [D], is the material constitutive matrix.

If equation (3-2) is substituted in equation (3-3), the governed equation is the stress-displacement relationship which is :

$$\{\sigma\}=[D] [B] \{U\} \quad \text{.....(3-4)}$$

The principle for virtual displacement of a deformable body is used to establish the governing equations of static equilibrium. It states that " If a

general structure in equilibrium is subjected to a system of small virtual displacements with a compatible state of deformation, the virtual work due to the external forces, W_{ext} is equal to virtual strain energy due to the internal stress, W_{int} , **Dawe (1984) [27]**, the principal can be written as :

$$\delta W_{\text{int}} - \delta W_{\text{ext}} = 0 \quad \dots\dots\dots (3-5)$$

Where δW_{int} , is the internal work, and δW_{ext} , is the external work.

The virtual strain energy is:

$$\delta W_{\text{int}} = \int_v \{\partial \epsilon\}^T \{\sigma\} dv \quad \dots\dots\dots (3-6)$$

where:

$$\{\partial \epsilon\} = \text{virtual strain vector} = [\partial \epsilon_x \quad \partial \epsilon_y \quad \partial \epsilon_z \quad \partial \epsilon_{xy} \quad \partial \epsilon_{yz} \quad \partial \epsilon_{xz}]^T$$

$$\{\sigma\} = \text{actual stress vector} = [\sigma_x \quad \sigma_y \quad \sigma_z \quad \sigma_{xy} \quad \sigma_{yz} \quad \sigma_{xz}]^T$$

v = volume of element.

After substituting Equation (3-3) in (3-6), the virtual internal work can be written as :

$$\delta W_{\text{int}} = \int_v \{\partial \epsilon\}^T [D] \{\epsilon\} dv \quad \dots\dots\dots (3-7)$$

It is assumed that all effects are in the global Cartesian system. Combining Equation (3-2) with Equation (3-7) yields :

$$\delta W_{\text{int}} = \{\delta U\}^T \left[\int_v [B]^T [D] [B] dv \right] \{U\} \quad \dots\dots\dots (3-8)$$

The pressure force vector formation is given by:

$$\delta W_{\text{ext}1} = \int_{A_p} \{\partial W_n\}^T \{P\} dA_p \quad \dots\dots\dots(3-9)$$

Where:

$\{W_n\}$ = motion normal to surface.

$\{P\}$ = The applied pressure vector.

A_p = area over which pressure acts.

The point-wise normal displacement is related to the nodal displacements by:

$$\{W_n\} = [N_n] \{U\} \quad \dots\dots\dots(3-10)$$

Where:

$[N_n]$ = matrix of shape functions for normal motions at the surface.

Combining equation (3-9) and (3-10),

$$\delta W_{\text{ext}1} = \{\partial U\}^T \int_{A_p} [N_n]^T \{P\} dA_p \quad \dots\dots\dots(3-11)$$

Pressure are applied to the out side surface of each element.

Node forces applied to the element can be accounted for by:

$$\delta W_{\text{ext}2} = \{\partial U\}^T \{F_e^{\text{nd}}\} \quad \dots\dots\dots(3-12)$$

Where, $\{F_e^{\text{nd}}\}$ = External nodal forces.

Finally, equation (3-1), (3-4), (3-8) and (3-9) may be combined to give:

$$\{U\} \{\partial U\}^T \int_V [B]^T [D] [B] dV = \{\partial U\}^T \int_{A_p} [N_n]^T \{P\} dA_p + \{\partial U\}^T \{F_e^{\text{nd}}\} \quad \dots(3-13)$$

Noting that the $\{\partial U\}^T$ vector is a set of arbitrary virtual displacement terms, the condition required satisfying equation (3-13) is reduced to:

$$[K^e]\{U\} = \{F_e^{pr}\} + \{F_e^{nd}\} \quad \text{.....(3-14)}$$

Where:

$$[K^e] = \int_V [B]^T [D] [B] dV = \text{Element stiffness matrix.}$$

$$\{F_e^{pr}\} = \int_{A_p} [Nn]^T \{P\} dA_p = \text{Surface load vector.}$$

Equation (3-14) represents the equilibrium equation on one-element. For all elements this equation can be written as:

$$[K]\{U\} = \{F^a\} \quad \text{.....(3-15)}$$

Where:

$$[K] = \sum_n [K^e] = \text{Structural stiffness matrix.}$$

$$\{F^a\} = \{F^{pr}\} + \{F^{nd}\} = \text{Vector of applied loads.}$$

$$\{F^{pr}\} = \text{Structural pressure vector.}$$

$$\{F^{nd}\} = \text{Structural nodal-load vector.}$$

3-3:Finite Element Representation of Reinforced Concrete.

In the field of solid mechanics, the finite element method is usually used to find approximate solutions for structures having complicated shapes and loading arrangement. In this study a hexahedron brick element having 8 nodes will represent the reinforced concrete, where the reinforcing bars are modeled by link elements connecting opposite nodes of the brick elements.

3-3-1: Concrete Brick Element

In this study ordinary concrete and embedded precast-prestressed concrete units are modeled by using 8-noded isoperimetric brick element (SOLID 65) in **ANSYS 9.0** program [28] as shown in figure (3-1). The element has its local coordinate system, ξ , η , ζ , as shown in figure (3-1), with the origin at the center of the element such that each local coordinate range from (-1) to (+1).

Each nodal point has three degrees of freedom of movement (u, v and w) along the Cartesian coordinates (x, y and z), respectively. The geometry of the element can be defined in terms of the shape functions and can be expressed by :

$$\left. \begin{aligned} x(\xi, \eta, \zeta) &= \sum_{i=1}^8 N_i(\xi, \eta, \zeta) x_i \\ y(\xi, \eta, \zeta) &= \sum_{i=1}^8 N_i(\xi, \eta, \zeta) y_i \\ z(\xi, \eta, \zeta) &= \sum_{i=1}^8 N_i(\xi, \eta, \zeta) z_i \end{aligned} \right\} \dots\dots\dots(3-16)$$

Where :

$N_i(\xi, \eta, \zeta)$ = the shape function at the i-th node.

x_i , y_i and z_i = the global coordinates of the node (i).

The shape function for the 8-node brick elements has the form:

$$N_i(\xi, \eta, \zeta) = \frac{1}{8} (1 + \xi \xi_i) (1 + \eta \eta_i) (1 + \zeta \zeta_i) \quad \dots\dots\dots(3-17)$$

where: $\xi_i = \pm 1$, $\eta_i = \pm 1$, $\zeta_i = \pm 1$.

The starting point for the stiffness matrix derivation is the element displacement field. The isoperimetric definition of displacements is :

$$\left. \begin{aligned} u(\xi, \eta, \zeta) &= \sum_{i=1}^8 N_i(\xi, \eta, \zeta) u_i \\ v(\xi, \eta, \zeta) &= \sum_{i=1}^8 N_i(\xi, \eta, \zeta) v_i \\ w(\xi, \eta, \zeta) &= \sum_{i=1}^8 N_i(\xi, \eta, \zeta) w_i \end{aligned} \right\} \quad \dots\dots\dots(3-18)$$

where: u_i , v_i and w_i are the nodal displacements of node i

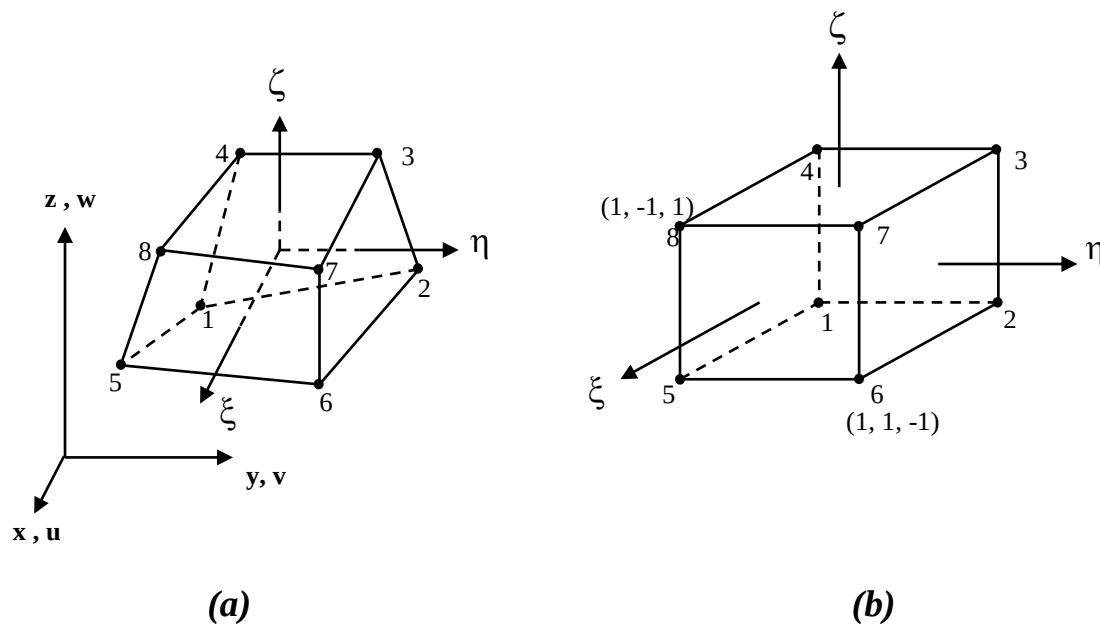


Figure (3-1): Brick element with 8 nodes [Solid 65 in ANSYS 9.0] [28]:

(a) Global coordinate system, (b) Local coordinate system

3-3-1-1: Stress and Strain Fields

In the present study, the geometric nonlinearities are not considered, displacement gradients remain small throughout the loading process and hence the engineering components of strain can be expressed in terms of the first partial derivatives of the displacement components. Therefore, the strain displacement relations can take the form :

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \epsilon_{xy} \\ \epsilon_{yz} \\ \epsilon_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix} = [B]\{U\} \quad \text{..... (3-19)}$$

Where, $[B] = [[B_1][B_2].....[B_8]]$, and

$$[B_i] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix}, i=1,2,\dots,8 \quad \text{.....(3-20)}$$

The shape functions, (N_i) can be expressed in either local or global coordinate system. By using the chain rule, the partial differential relation can be expressed in a matrix form as:

$$\dots\dots\dots(3-21) \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = [J] \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix}$$

Where [J], is known as the Jacobean matrix, can be expressed using equation (3-16) as :

$$[J] = \sum_{i=1}^8 \begin{bmatrix} \frac{\partial N_i}{\partial \xi} \cdot x_i & \frac{\partial N_i}{\partial \xi} \cdot y_i & \frac{\partial N_i}{\partial \xi} \cdot z_i \\ \frac{\partial N_i}{\partial \eta} \cdot x_i & \frac{\partial N_i}{\partial \eta} \cdot y_i & \frac{\partial N_i}{\partial \eta} \cdot z_i \\ \frac{\partial N_i}{\partial \zeta} \cdot x_i & \frac{\partial N_i}{\partial \zeta} \cdot y_i & \frac{\partial N_i}{\partial \zeta} \cdot z_i \end{bmatrix} \dots\dots\dots (3-22)$$

Then, the shape functions derivatives with respect to x, y and z can be obtained as :

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} \end{Bmatrix} \dots\dots\dots(3-23)$$

Where, $[J]^{-1}$ = the inverse of the Jacobean matrix [J].
From which the matrix $[B_i]$, can be evaluated.

The element stiffness matrix presented in equation (3-15) is given by :

$$[K^e] = \int_V [B]^T [D] [B] dv$$

For three-dimensional elements, the differential volume (dv) may be written as :

$$.....(3-24) \quad dv = dx \cdot dy \cdot dz$$

Equation (3-24) can be transformed into the natural coordinates :

$$dv = |J| \cdot d\xi \cdot d\eta \cdot d\zeta \quad(3-25)$$

where, $|J|$ is the determinant of the Jacobean matrix.

The limits of integration in the local coordinate ranges from (-1) to (+1) and the element stiffness matrix can therefore be written as :

$$[K^e] = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D] \cdot [B] \cdot |J| \cdot d\xi \cdot d\eta \cdot d\zeta \quad(3-26)$$

3-3-2: Representation of Reinforcement.

Tavares (2001) [29], discussed three techniques that exist to model steel reinforcement in the finite element model for reinforced concrete, the discrete model, the embedded model and the smeared model.

3-3-2-1: Discrete Representation

In discrete representation the steel bars are replaced by a truss element that are connected to the concrete elements at the nodes, shown in figure (3-2). A drawback to this model is that the concrete mesh is restricted by the locations of the reinforcement and the volume of mid-steel reinforcement is not deducted from the concrete volume.

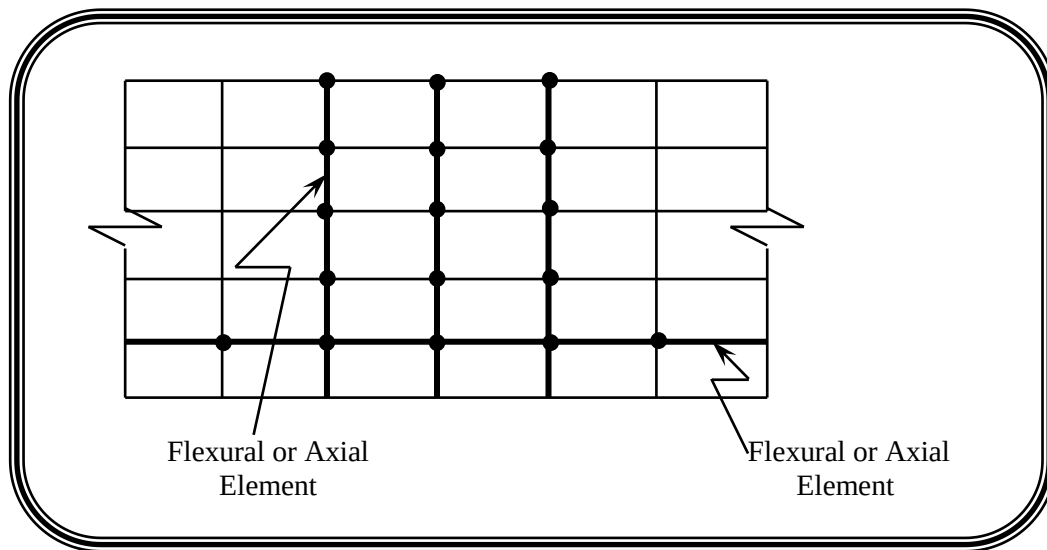


Figure (3-2): Discrete reinforcement representation Tavares (2001) [29]

3-3-2-2: Embedded Representation

The embedded model figure (3-3), overcomes the concrete mesh restriction because the stiffness of the reinforcing steel is evaluated separately from the concrete elements. The model is built in a way that keeps reinforcing steel displacements compatible with the surrounding concrete elements. When reinforcement is complex, this model is very advantageous.

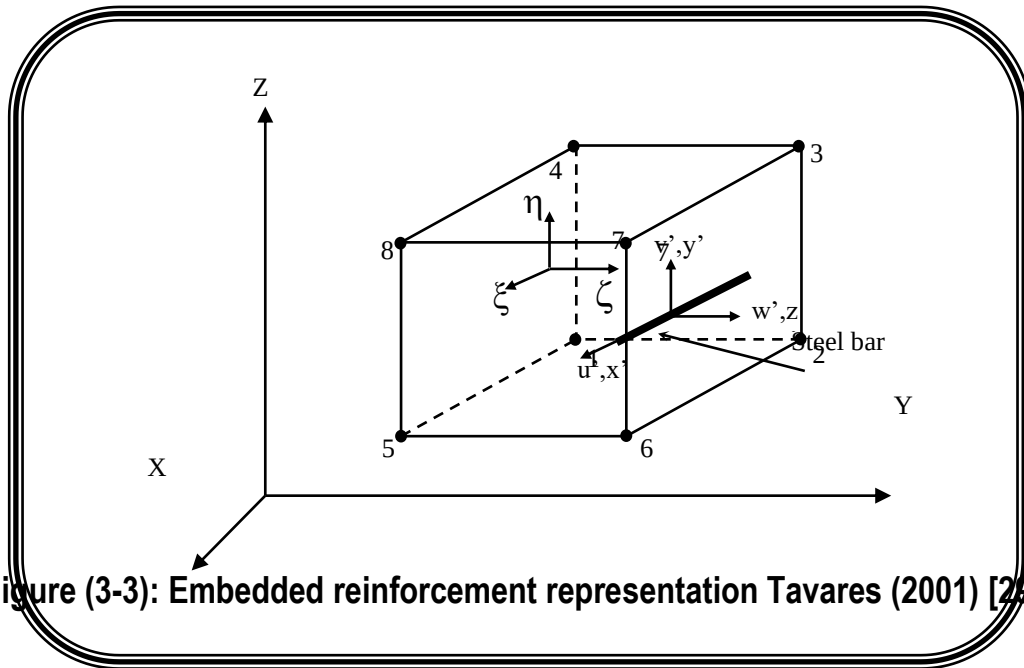


Figure (3-3): Embedded reinforcement representation Tavares (2001) [29]

3-3-2-3: Smeared Representation

The smeared model figure (3-4), assumes that reinforcement is uniformly spread throughout the concrete elements in a defined region of the finite element mesh. This approach is used for large-scale models where the reinforcement does not significantly contribute to the overall response of the structure.

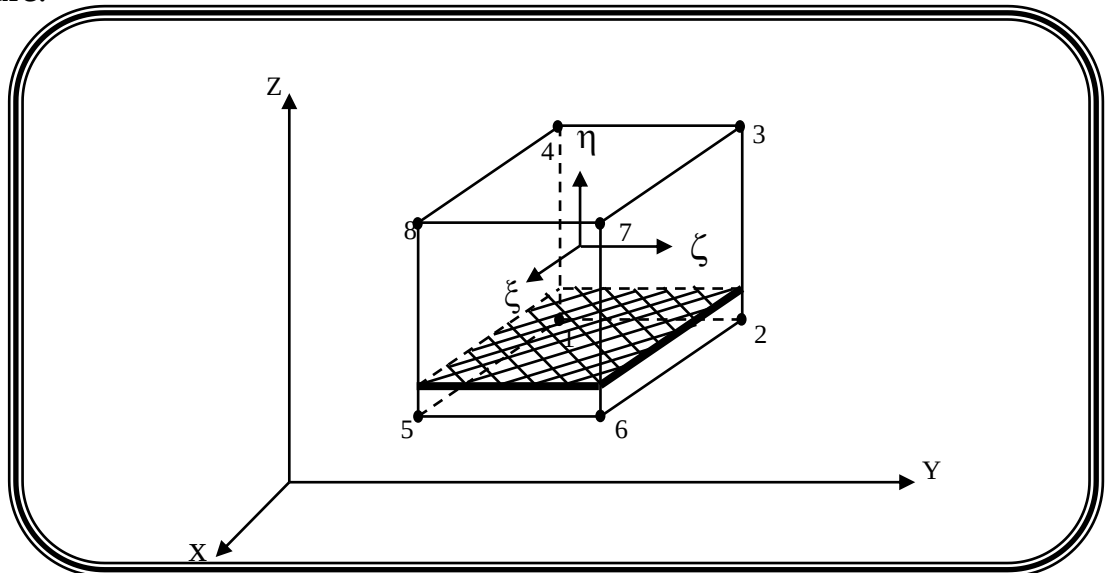


Figure (3-4): Distributed reinforcement representation Tavares (2001) [29]

When **Fanning (2001) [30]**, modeled the response of the reinforcement using the discrete model and the distribution model for reinforcement concrete beam find the discrete model is the best modeling of reinforcement.

In this study the discrete model is used to represent steel reinforcement. The three-dimensional two-node bar element (link8) is a uniaxial tension-compression element with three degrees of freedom at each node (nodal translation in x, y and z) directions. The axial normal stress is assumed to be uniform over the entire element. The element x-axis is oriented along the length of the element from node (1) towards node (2), figure (3-5).

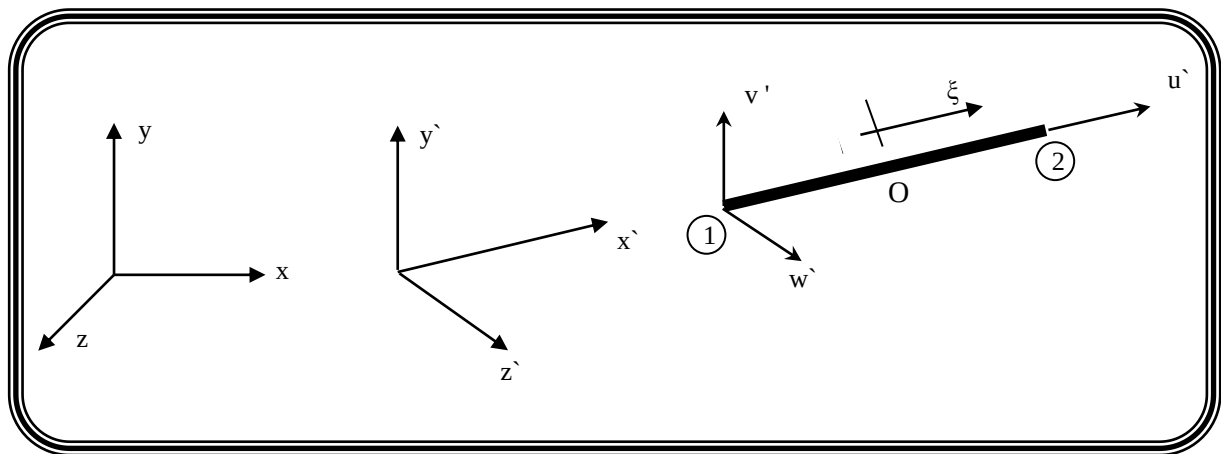


Figure (3-5): Bar element (LINK8) in ANSYS 9.0 [28]

The displacement components within the element (in local coordinates x' , y' and z') are as follows :

$$\begin{Bmatrix} u' \\ v' \\ w' \end{Bmatrix} = \sum_{i=1}^2 N_i \begin{Bmatrix} u'_i \\ v'_i \\ w'_i \end{Bmatrix} \quad \dots\dots\dots(3-27)$$

The shape functions are as follows :

$$\left. \begin{aligned} N_1 &= \frac{1}{2}(1 - \xi) \\ N_2 &= \frac{1}{2}(1 + \xi) \end{aligned} \right\} \dots\dots\dots(3-28)$$

The local coordinates of any point with the element, in terms of the natural (local) coordinates are :

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \sum_{i=1}^2 N_i \begin{Bmatrix} x'_i \\ y'_i \\ z'_i \end{Bmatrix} \dots\dots\dots(3-29)$$

where x'_i, y'_i, z'_i are local coordinates of node i.

As the element is capable of carrying axial loads only, then the strain-displacement relationship is as follows :

$$\varepsilon = \varepsilon_{x'} = \frac{\partial u'}{\partial x'} \dots\dots\dots(3-30)$$

The stiffness matrix of this element is found in the local coordinates (ξ, η, ζ) or (x'_i, y'_i, z'_i) as:

$$[K_\ell] = \frac{AE}{L} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots\dots\dots(3-31)$$

Where :

L = element length

A = element cross-sectional area

E = Young's modulus of the element

All element matrices are generated in the element coordinate system and then converted to the global coordinate system in order to be added to the global stiffness matrix of the entire structure. Thus, the stiffness matrix in the global coordinates system is :

$$[K_G] = \frac{AE}{L} \begin{bmatrix} \ell^2 & \ell m & \ell n & -\ell^2 & -\ell m & -\ell n \\ & m^2 & mn & -\ell m & -m^2 & -mn \\ & & n^2 & -\ell n & -mn & -n^2 \\ & & & \ell^2 & -\ell m & \ell n \\ & & & & m^2 & mn \\ & & & & & n^2 \end{bmatrix} \quad \text{.....(3-32)}$$

where(ℓ , m and n) are the direction cosines of the bar axis (ξ) or (x') with respect to global axes (x , y and z) respectively.

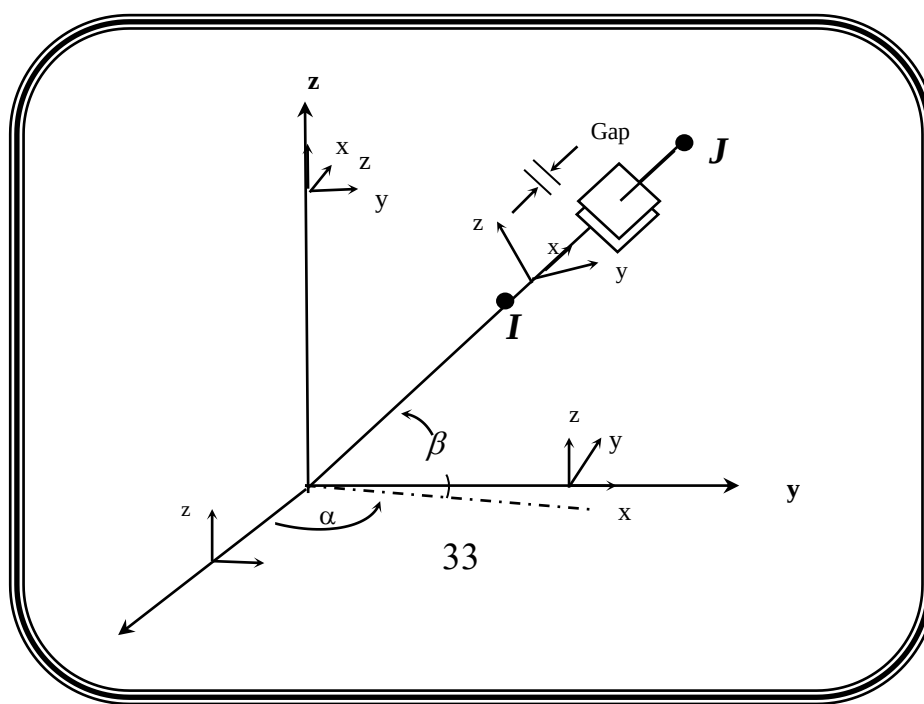
3-4: Interface Finite Element Idealization

Interface problems are highly nonlinear and require significant computer resources to solve. To model the interface behavior between the precast-prestressed concrete units and ordinary concrete beam (CONTAC52) finite element of ANSYS has been used to represent sticking and sliding between two nodes in three dimensions.

3-4-1: Performance a Node to Node Contact Analysis

A three-dimensional point-to-point contact element (Contact 52) **ANSYS 9.0 [28]**, is used to model the nonlinear physical contact behavior of the surface between the precast-prestressed concrete units and the ordinary concrete beam in a composite prestressed concrete beam. This model also includes the definition of the stress transfer. The element joins two surfaces that may maintain or break the physical contact and may slide relative to each other. Also, the element is capable of supporting only compression in the direction normal to the interface between the two surfaces .

The 3-D point-to-point contact element (Contact 52) has three degrees of freedom at each node (u , v and w) in the element coordinate system. The orientation of the interface is defined by the node locations, as shown in Figure (3-6).



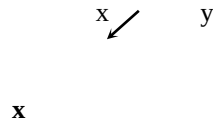


**Figure (3-6): 3-D Point-to-point contact element
ANSYS 9.0 [28]**

The element coordinate system has its origin at node I and the x-axis is directed towards node J, and the element may have one of the three conditions:

1. Closed and sticking.
 2. Closed and sliding.
 3. Open.
- If the element is closed and sticking, its stiffness matrix in the element coordinate system is:

$$[K_L] = \begin{bmatrix} k_n & 0 & 0 & -k_n & 0 & 0 \\ 0 & k_s & 0 & 0 & -k_s & 0 \\ 0 & 0 & k_s & 0 & 0 & -k_s \\ -k_n & 0 & 0 & k_n & 0 & 0 \\ 0 & -k_s & 0 & 0 & k_s & 0 \\ 0 & 0 & -k_s & 0 & 0 & k_s \end{bmatrix} \quad \dots\dots\dots(3-33)$$

where:

K_n = normal stiffness.

K_s = sticking stiffness.

The load vector in local coordinates is:

$$\{F_L\}^T = [F_{n,\xi i} \quad F_{s,\eta i} \quad F_{s,\zeta i} \quad F_{n,\xi j} \quad F_{s,\eta j} \quad F_{s,\zeta j}] \quad \dots\dots\dots(3-34)$$

where:

F_n = normal force across the gap.

F_s = sticking force across the gap.

- If the element is closed and slides, its stiffness matrix in the element coordinate system is:

$$[K_L] = \begin{bmatrix} k_n & 0 & 0 & -k_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -k_n & 0 & 0 & k_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \dots\dots\dots(3-35)$$

The load vector is the same as in equation (3-34)

- If the element is open, there is no stiffness matrix and no load vector

By applying the equilibrium equations in local coordinates, then:

$$\{F_L\} = [K_L] \{\Delta_L\} \quad \dots\dots\dots(3-36)$$

where:

$$\{\Delta_L\}^T = [u_i \quad v_i \quad w_i \quad u_j \quad v_j \quad w_j] \quad \dots\dots\dots(3-37)$$

In the finite element solution procedure, matrices are usually assembled in terms of global coordinates, and these can be obtained by multiplying the respective vector with a transformation matrix $[T]$ **Mazurkiewics and Ostachowics, (1983)[31]**.

$$\{\Delta_L\} = [T] \{\Delta_G\} \quad \dots\dots\dots(3-38)$$

where:

$$\{\Delta_G\}^T = [U_{x,i} \quad U_{y,i} \quad U_{z,i} \quad U_{x,j} \quad U_{y,j} \quad U_{z,j}] \quad \dots\dots\dots(3-39)$$

Also:

$$\{F_G\} = [T]^T \{F_L\} \quad \dots\dots\dots(3-40)$$

where:

$$\{F_G\}^T = [F_{x,i} \quad F_{y,i} \quad F_{z,i} \quad F_{x,j} \quad F_{y,j} \quad F_{z,j}] \quad \dots\dots\dots(3-41)$$

Premultiplying both sides of Equation (3-38) by the matrix $[T]^T$ yields:

$$[T]^T \{F_L\} = [T]^T [K_L] \{\Delta_L\}$$

$$\text{or} \quad \{F_G\} = [T]^T [K_L] \{\Delta_L\} \quad \dots\dots\dots(3-42)$$

Substituting equation (3-37) into equation (3-38) gives:

$$\{F_G\} = [T]^T [K_L] [T] \{\Delta_G\}$$

$$\text{or} \quad \{F_G\} = [K_G] \{\Delta_G\} \quad \dots\dots\dots(3-43)$$

where:

$$[K_G] = [T]^T [K_L] [T] \quad \dots\dots\dots(3-44)$$

3-5: Numerical Integration

The evaluation of the element stiffness matrix involves some difficult

integration. Explicit integration might be difficult or even impossible, for such functions. Usually the Gauss-legender scheme is used to perform the integration required to setup the element stiffness matrix. The method has been found to be both accurate and convenient for finite element work **Zeinkiewicz, (1977)[32]**. In this technique the element stiffness matrix for the brick element may be written in the form.

$$I = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} F(\xi, \eta, \zeta) \cdot d\xi \cdot d\eta \cdot d\zeta \approx \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^l W_i W_j W_k F(\xi_i, \eta_j, \zeta_k) \quad \dots(3-45)$$

where: n , m , and l are the number of Gaussian points in the ξ , η and ζ - directions respectively, and W_i , W_j and W_k are the weight factor of i th, j th and k th integration point. The function $F(\xi, \eta, \zeta)$ represents the matrix multiplication $[B]^T [D] [B] |J|$ in equation (3-26). Generally the number of integration points are taken equal in three directions.

The integration rule, which has been used in this work, $2 \times 2 \times 2 = 8$ -point Gauss quadrature for brick element, are shown in figure (3-7). The distribution of the sampling point over the volume of the 8-node and their weighting factors are given in table (3-1) :

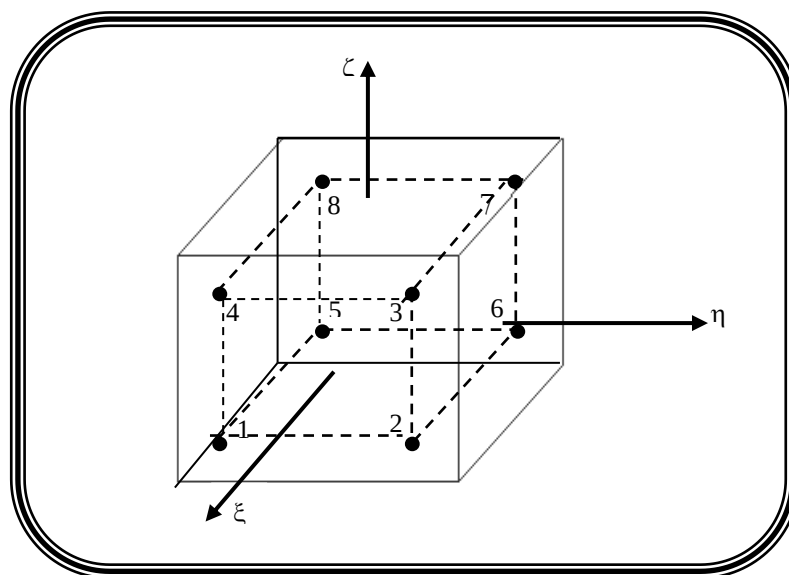


Figure (3-7): Distribution of integration points ANSYS 9.0 [28], Mottarn and Shaw (1996) [33]

Table (3-1) sampling points and associated weights for 2*2*2 integration rules by [28],[33].

Number of Sampling Points	Local Coordinate			Weight
	ξ	η	ζ	
1-8	$0.57734 \pm$	$0.57734 \pm$	$0.57734 \pm$	1

3-6: Nonlinear Solution Techniques

The nonlinear behavior of any reinforced concrete structure may be generally attributed to either material or geometric nonlinearity or a combination of both types. In reinforced concrete members, the geometric nonlinearity, with large deflections occurring only close to structural collapse. So, only material nonlinearity is considered in the present study.

For linear elastic problems, equation (3-15) is used to find the solution of the unknown displacements $\{U\}$. In nonlinear case system, the stiffness matrix $[K]$ is a function of the unknown displacements (or their derivatives). Then equation (3-15) can not be exactly solved before the determination of the unknown displacements $\{U\}$.

The solution of nonlinear problems is usually performed by three basic techniques, these techniques are the incremental technique, iterative technique, and the incremental- iterative technique.

3-6-1: Incremental Techniques

In this approach, figure (3-8a), the load is applied in several small increments, because the purely incremental technique does not account for the redistribution of forces during the application of loading increments, this technique suffers from continuous drift from the true equilibrium

3-6-2: Iterative Techniques

In the iterative approach, figure (3-8b), the total load is applied in one increment at the first iteration, the out of balance forces are then computed and used in the next iteration to obtain progressively improved solution. The iterative process is continued until the final converged solution would be in equilibrium, such that the internal load vector would be equal to the applied load vector or with some tolerance. This process can be written as **Nyssen (1981) [34]** :

$$[K_i] \{\Delta U_i\} = \{F^a\} - \{F^{in}\} \quad \dots\dots\dots(3-46)$$

$$\{U_{i+1}\} = \{U_i\} + \{\Delta U_i\} \quad \dots\dots\dots(3-47)$$

where:

$[K_i]$ = stiffness matrix.

i = subscript representing the current equilibrium iteration.

$\{F^{in}\}$ = internal load vector.

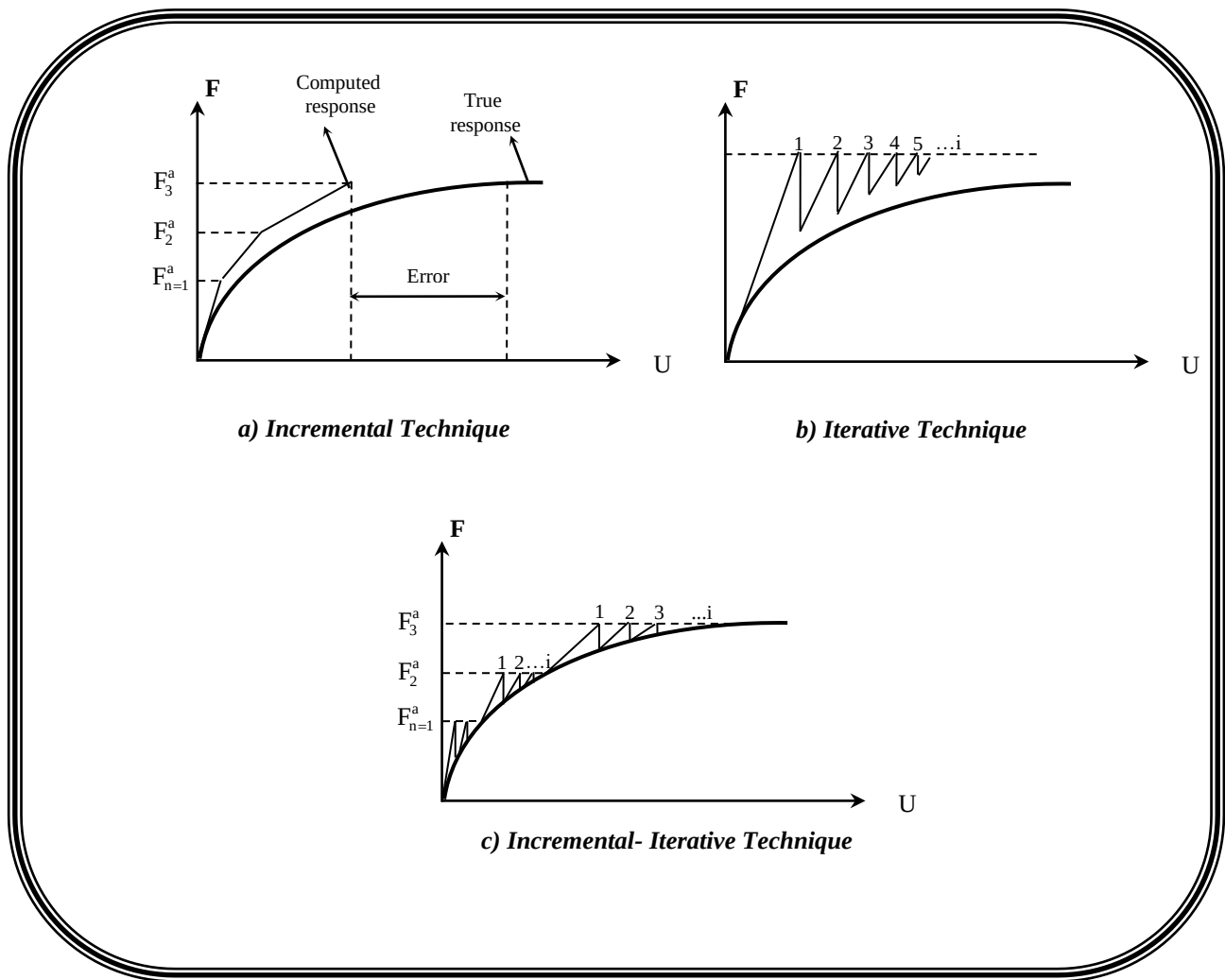


Figure (3-8): Basic techniques for solving the nonlinear equation [34]

3-6-3: Incremental-Iterative Techniques

In the incremental-iterative technique figure (3-8c), the load is applied as a series of increments, and at each increment, iterative solution is carried out to find the truest response of the structure. The conventional, modified and combined Newton-Raphson methods may also be used in the iteration process within each step.

The incremental-iterative method of solution which is used in the current study, uses the following procedures:

3-6-3-1: Initial-stiffness procedure

In this procedure, the stiffness matrix is formed and solved only once at the beginning of the analysis, and the program uses this initial stiffness matrix at every equilibrium iteration within each load increment. For this procedure, the computation cost per iteration is significantly reduced, but in case of strong nonlinearities (such as large deformation analyses), the method often fails to converge [28], Figure (3-9a).

3-6-3-2: Full Newton-Raphson procedure

The stiffness matrix is formed at every iteration. The advantage of this procedure may give more accurate result. The disadvantage of this procedure is that a large amount of computational effort may be required to form and decompose the stiffness matrix, figure (3-9b).

3-6-3-3: Modified Newton-Raphson procedure

In this method, figure (3-9c), the stiffness matrix is updated only once for each increment of loading. As compared with the full Newton-Raphson method, the modified Newton-Raphson method is more economical. However, this method requires more steps for convergence, but each step is done quickly by avoiding time consuming repetitions of forming the tangent stiffness matrix.

The initial-stiffness and the modified Newton-Raphson procedures converge more slowly than the full Newton-Raphson procedure, as they require fewer stiffness matrix reformulation[28]. The full Newton-Raphson is adopted in the current study.

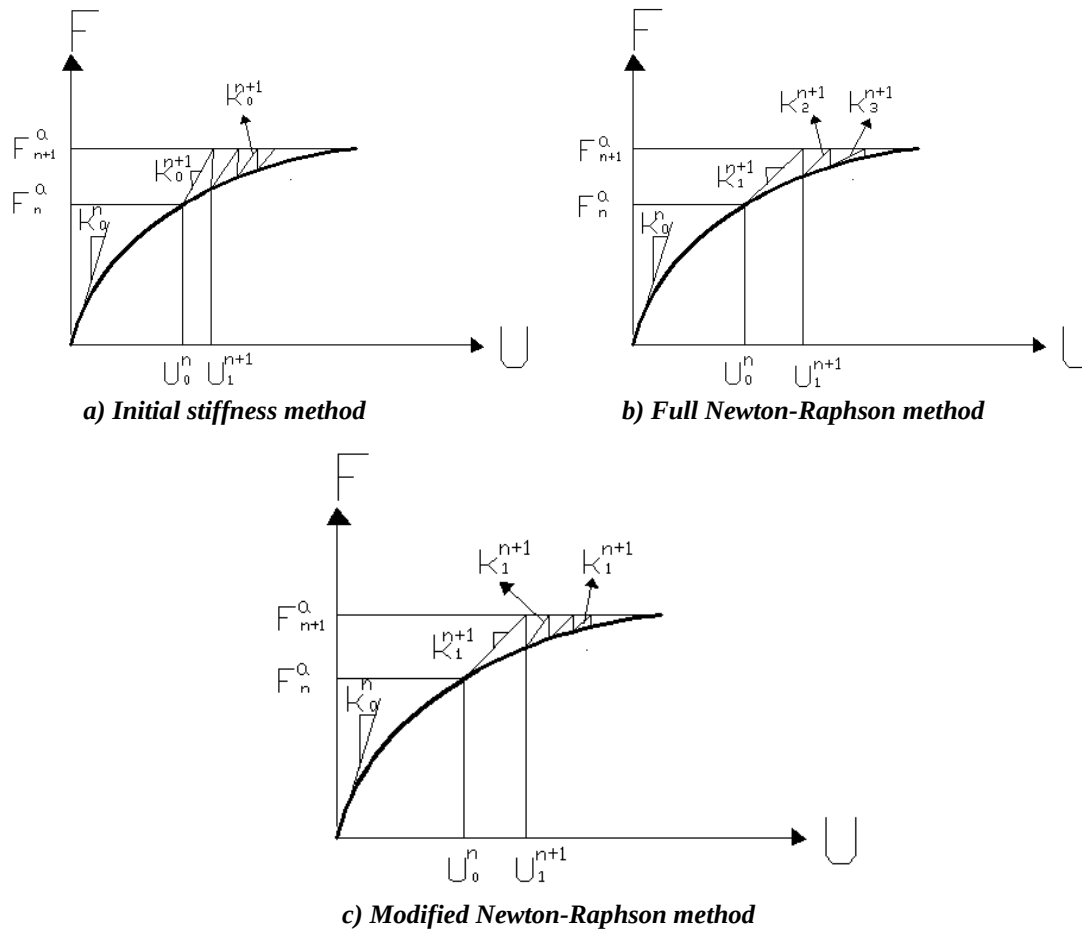


Figure (3-9): Incremental-iterative procedure[28]

3-7: Convergence Criterion

A termination criterion for the iterative process should be used to stop iteration when the difference between the external and the internal forces becomes negligibly small, at which the convergence is assumed to occur. The convergence criteria for non-linear structural problems can usually be classified as :

1. force criterion.
2. displacement criterion.
3. work done criterion.

In the displacement convergence, the incremental displacements and the total displacements at iteration (i) are determined. The solution is

considered to be converged when the norm of the incremental displacements is within a given tolerance of the norm of the total displacements; L norm (Euclidean norm) is used and the form.

$$\|\{\Delta U_i\}\| = (\sum \Delta U_i^2)^{0.5} \leq T_n (\sum U_i^2)^{0.5} \dots\dots\dots(3-48)$$

Where U may equal u, v, w.

For forces convergence the norm of the residual forces at the end of each iteration is checked against the norm of the current applied forces as:

$$\|\{R\}\| = (\sum R_i^2)^{0.5} \leq T_n (\sum F_i^2)^{0.5} \dots\dots\dots(3-49)$$

Where $\{R\}$ is the residual vector:

$$\{R\} = \{F^a\} - \{F^{nr}\} \dots\dots\dots(3-50)$$

In this study, the tolerance T_n is taken equal to 0.1% up to the ultimate load for load control.

3-8: Analysis Termination Criterion

The nonlinear finite element analysis must be provided with a criterion to detect failure of the solution. In the physical test under load control, collapse of a structure takes place when no further loading can be sustained. This is usually indicated in the numerical tests by successively increasing iterative displacements and a continuous growth in the dissipated energy. Hence, the convergence of the iterative process cannot be achieved. A maximum number of iterations for each increment are specified to stop the nonlinear solution if the convergence limit has not been achieved. It has been observed that a minimum number about 15 iterations is generally sufficient to predict the solution divergence or failure. This maximum number of iterations

depends on the type of the problem, extent of nonlinearities, and on the specified tolerance, **Hinton and Owen (1984) [35]**.

3-9: Outline of the ANSYS Computer Program

The computer program ANSYS (Analysis System Version 9.0) is a powerful and impressive engineering tool that may be used to solve a variety of problems. ANSYS is a comprehensive general-purpose finite element computer program that contains about 165 different elements implemented in the program. ANSYS has the capacity of solving linear and nonlinear problems including the effect of cracking, crushing, yielding of reinforcement, creep and bond slip.

In order to use ANSYS or any other finite element analysis computer program intelligently, it is imperative that one first fully understands the underlying basic concepts and limitation of the finite element method.

At this point it is worth noting that:

1. There are three processors that have been frequently used:
 - (a) the preprocessor (PREP 7).
 - (b) the processor (SOLUTION).
 - (c) the general postprocessor (POST 1).
2. The commands of the preprocessor you need to use to built a model:
 - (a) define element type and options.
 - (b) define element real constants.
 - (c) define material properties.
 - (d) create model geometry.
 - (e) define meshing controls.
 - (f) mesh the object created.
3. The command of the solution processor (SOLUTION) that allow to apply boundary condition and loads. The solution processor also solves for

the nodal solutions and calculates other element information.

4. The command for the general postprocessor (POST 1) allows to list and display result of an analysis:

- (a) read result data from result file.
- (b) read element result data.
- (c) plot results.
- (d) list results.

5. ANSYS writes and reads many files during a typical analysis.

6. ANSYS program also offers the capability to select information about a portion of the model, such as certain nodes, elements for future processing.

7. ANSYS program provides numerous features that enhance the visual presented information. Some examples of graphics capabilities of ANSYS are deformed shapes, result contours, sectional views, and animation.

The analysis in this study has been carried out on Lap Top 1600 MHz computer with 1GA RAM.

CHAPTER 4

MODELING OF MATERIAL PROPERTIES

4-1: Introduction

Reinforced concrete structures are made up of two materials with different characteristic, namely, concrete and steel. Steel can be considered a homogenous material and its material properties are generally well defined. Concrete is, on the other hand, a heterogeneous material made up of cement, mortar and aggregates. Its mechanical properties scatter more widely and can not be defined easily. For the convenience of analysis and design, however, concrete is often considered a homogeneous material in the macroscopic sense, **Fillip and Kawk (1990) [36]**.

The information required in any three-dimensional nonlinear finite element analysis of reinforced concrete structures are the material constitutive relationships, which describe the multi-dimensional stress-strain relations that govern the behavior of the structures. The composite prestressed concrete is a complicated composite material therefore its behavior can not obtained without considering the constitutive relations of its constituents (ordinary concrete, precast-prestressed units, prestressing wire, and ordinary steel bars)

independently. Perfect bond between concrete and steel bars, and interaction between ordinary concrete and precast-prestressed units has been considered throughout the present work.

4-2: Sources of Nonlinearities in Reinforced Concrete Members.

The nonlinear behavior of reinforced concrete members may be due to the material or geometrical nonlinearities. The geometric nonlinearities is related to the large deformations and, in many cases this nonlinearity is neglected. The material nonlinear behavior is caused by two major material effects, cracking of the concrete and plasticity of the reinforcement and of the compression concrete. Other time-independent nonlinearities arise from the nonlinear action between concrete and reinforcement such as bond slip between steel and concrete, aggregate interlock of a cracked concrete and dowel action of reinforcement steel, as shown in figures (4-1) and (4-2). The time-dependent effects such as creep, shrinkage and temperature change and also contribute to the nonlinear response behavior, **Chen (1981) [37]**.

In this study only time-independent material nonlinearities (cracking and plasticity) are included.

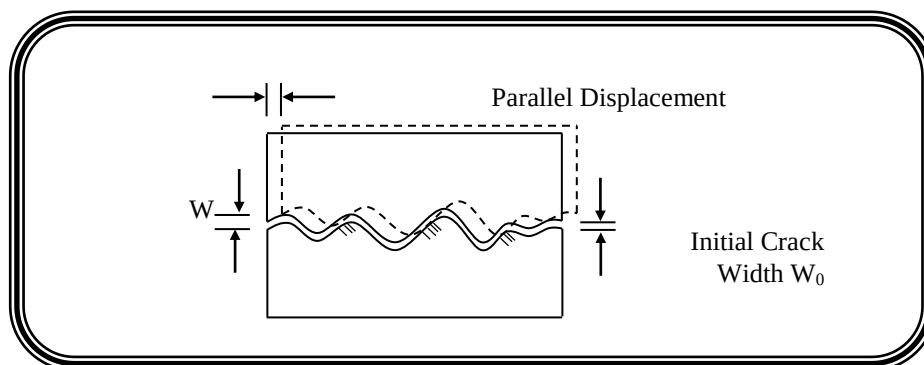
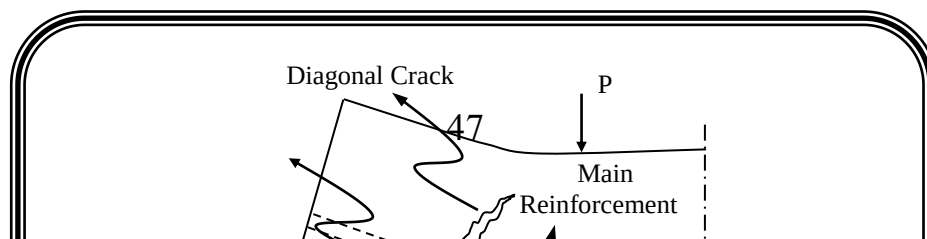


Figure (4-1): Aggregate interlock Chen (1981) [37]



Splitting
Along Bar

Dowel Displacement

Figure(4-2): Dowel action of shear transfer across crack Chen (1981) [37]

4-3: The Observed Behavior of Concrete.

Concrete contains a large number of microcracks, especially at interface between coarse aggregate and mortar, even before any load has been applied. The propagation of these microcracks during loading contributes to the nonlinear behavior of concrete. Many of these microcracks are caused by segregation, shrinkage, or thermal expansion in the mortar. Some microcracks may be developed during loading because of the differences in stiffness between aggregates and mortar, **Chen (1981) [37]**.

4-3-1: Uniaxial Behavior of Concrete.

4-3-1-1: Uniaxial Behavior of Concrete in Compression.

A typical stress-strain relationship for concrete subjected to uniaxial compression is shown in figure (4-3). The stress-strain curve has a nearly linear-elastic behavior up to about (30) percent of its maximum compressive strength, (f'_c). The stress level of about (30) percent of (f'_c) has been termed onset of localized cracking and has been proposed as a limit of elasticity. For stress levels ranging between ($0.3f'_c$ and $0.5f'_c$) the stress-strain curve exhibits a slight nonlinearity due to the extension of stress concentrations at the crack tips. When the stress level increases form ($0.5 f'_c$ and $0.75 f'_c$), mortar cracks and other

cracks continue and grow slowly with a gradual increase in curvature of the curve, **Chen (1981) [37]**.

Beyond this level of stress the rate of crack propagation increases rapidly and the stress-strain curve bends sharply until the peak stress level is reached. Beyond the peak stress level, concrete shows a softening response, presented by the descending portion of the stress-strain curve, **Gerstle (1980) [38]**.

The shape of stress-strain curve is similar for concrete of low, normal and high strength. A high-strength concrete behaves in a linear fashion to a relatively higher stress level than the low-strength concrete, **Chen (1981) [37]**.

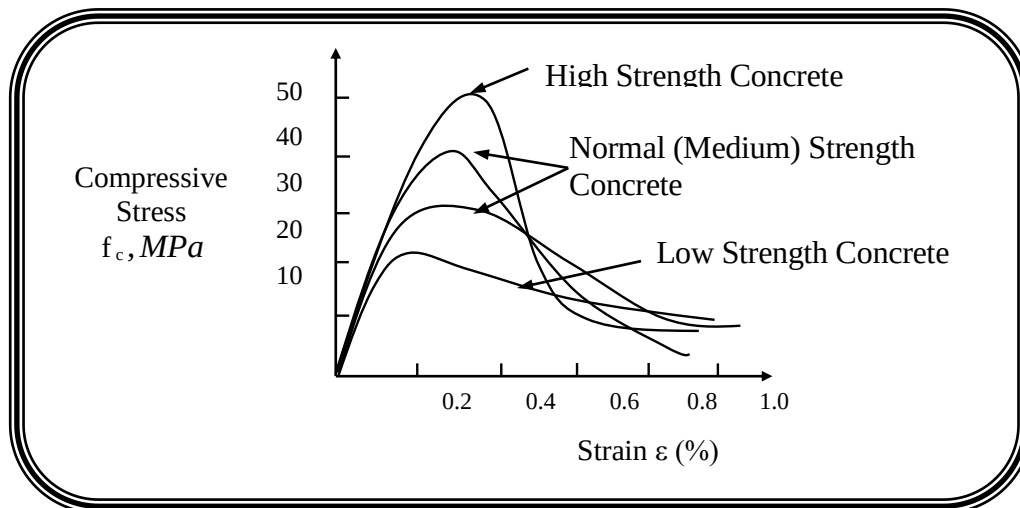


Figure (4-3): Stress-strain curve under uniaxial compression for concrete Chen (1981) [37].

4-3-1-2: Uniaxial Behavior of Concrete in Tension

Under uniaxial tensile stress, the stress-strain response of concrete is nearly linear up to a relatively high stress level. Figure (4-4), show many similarities to

the uniaxial compression curves. For stress less than about (60) percent of the uniaxial tensile strength (f_t), the creation of new microcracks is negligible. Thus, this stress level will correspond to limit elasticity. Above this level, the bond microcracks start to grow and onset of unstable crack propagation which will therefore be about (75) percent of (f_t), **Chen (1981) [37]**.

As a result of the difficulties in implementing the direct testing of concrete in pure axial tension, indirect tests are alternatively used to determine the concrete cracking strength. These tests are the modulus of rupture (f_r), which is determined by splitting a concrete cylinder with a line load. Also, the double punching test is another indirect test method for determining concrete, **Chen (1981) [37]**.

$$f_t = 0.1 f'_c \quad \text{in (MPa)} \quad \text{.....(4-1)}$$

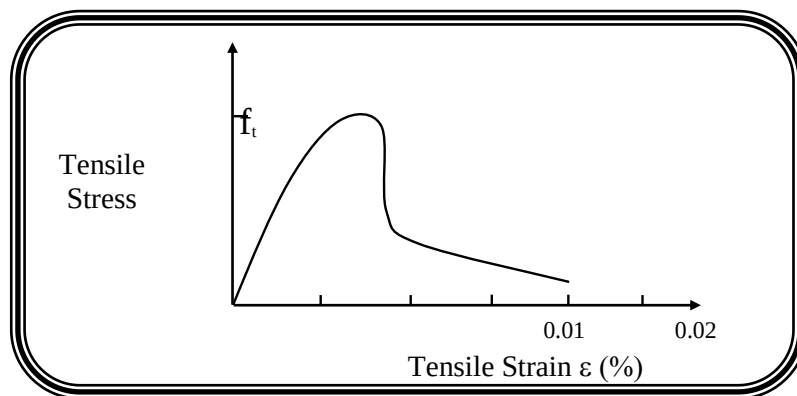


Figure (4-4): Stress-strain curve under uniaxial tension for concrete Chen (1981) [37]

4-3-1-3: Multiaxial Behavior of Concrete

The governed strength of concrete under a combination of many types of loading is shown in the multiaxial behavior of concrete under different combinations of biaxial and triaxial loading. Concrete exhibits strength and stress-strain behavior some different from that under uniaxial condition. The maximum compressive strength increases for the biaxial compression state, and

this increase is approximately (25) percent for principal stress ratio of (0.5) and about (16) percent for principal stress ratio (1.0). Under biaxial compression-tension, the compressive strength decreases almost linearly as the applied tensile stress is increased and under biaxial tension. The strength is the same as that of uniaxial tensile strength, **Tasuji et al. (1978) [39]**. Figure (4-5) shows a typical biaxial failure envelope of concrete.

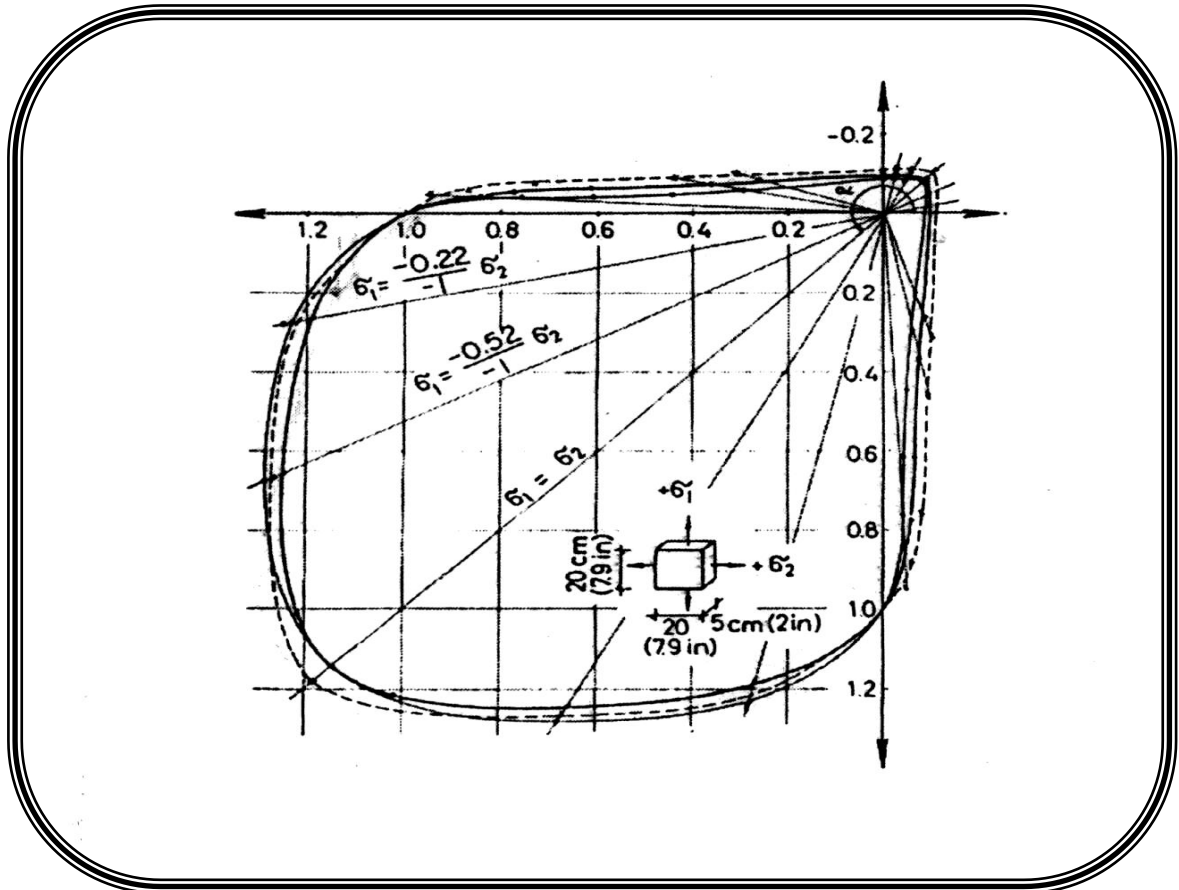


Figure (4-5): Biaxial strength envelope for concrete Chen (1981) [37].

Under triaxial loading, concrete appears to generate a fairly consistent failure surface, which is a function of three principal stresses, **Chen (1981) [37]**, figure (4-6). It exhibits an increased strength as a result of the confinement of concrete. Experimental studies indicate that the three-dimensional failure surface can be expressed in terms of three stress invariants. These invariants are the first invariant of the stress tensor, I_1 , the second and third invariant of the stress deviatoric tensor J_2 and J_3 , **Chen (1981) [37]**.

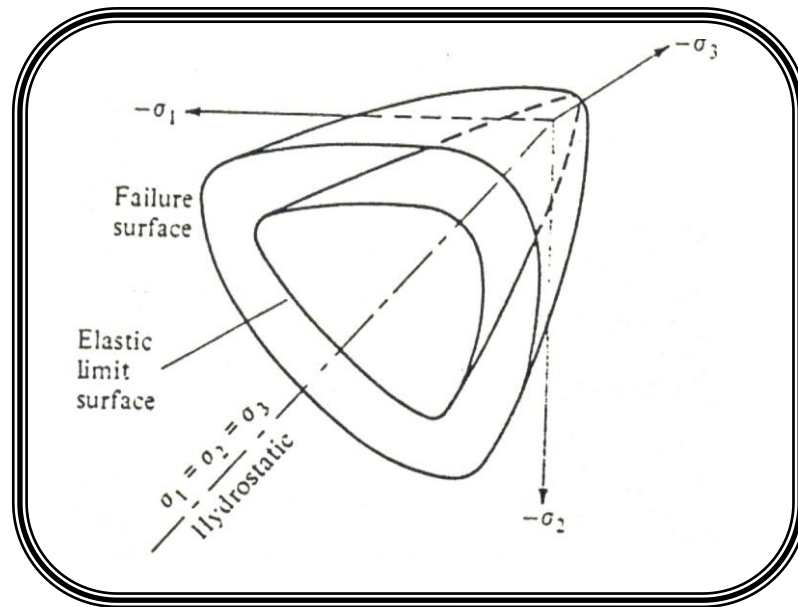


Figure (4-6): Typical triaxial failure envelop of concrete Chen (1981) [37].

4-3-1-4: The Modulus of Elasticity and Poisson's Ratio of Concrete.

The modulus of elasticity of concrete depends on its compressive strength. The initial modulus of elasticity for high strength concrete can be calculated with reasonable accuracy from the following formula recommended by Carrasaquillo et al. (1981) [40].

$$E_c = 3320 \sqrt{f'_c} + 6900 \quad \text{in (MPa)} \quad \dots\dots\dots(4-2)$$

Where :

E_c = Modulus of elasticity in (MPa).

f'_c = Cylindrical compressive strength of concrete in (MPa).

Also modulus of elasticity of concrete may be estimated according to the **ACI-Building Code 318-05 [41]** as:

$$E_c = 4700 \sqrt{f'_c} \quad \text{in (MPa)} \quad \dots\dots\dots(4-3)$$

Poisson's ratio, (ν), for concrete under compressive loading equals (0.2) for concrete with strength up to (73) (MPa) as proposed by, **Shideler (1957) [42]**. However, **Fillip and Kawk (1990) [36]**, reported values of Poisson's ratio between (0.15 and 0.22). the ratio, (ν), remain constant until approximately (80) percent of (f'_c), and then Poisson's ratio begins to increase.

4-4: Numerical Modeling of Concrete

The performance of any structure under load depends to a large degree on the stress-strain relationship of the material from which it is made. Composite concrete beams are made of different materials (concrete and reinforcing steel) which are brought together to work as a composite.

In this finite element analysis, the numerical models of concrete should cover the overall behavior of composite prestressed concrete members with sufficient degree of accuracy and at the same time models must kept as simple as possible for easy implementation in the finite element program.

4-4-1: Concrete Material Modeling

Many mathematical methods of the mechanical behavior of concrete are currently used in the analysis of concrete structures. These can be divided into two main groups, elasticity models and plasticity models. **Chen (1981) [37]**, and **ASCE (1982) [43]**.

4-4-1-1: Elasticity Based Model

In recent years, there have been a large number of nonlinear elastic stress-strain relationships proposed for concrete materials. They can be generally classified into two categories, **Yashiro and Chen (1987) [44]**.

- The total stress-strain formulation in the form of secant stiffness relation.

- The increment stress-strain formulation in the form of tangential stiffness formulation.

The total relationships between the stresses and the strains are reversible and load path independents. Since concrete is an irreversible and load-path dependent material, it is not appropriate to use this type of formulation for a general structural analysis involving severe nonproportional and reversed loading situation. The incremental stress-strain formulation, known as the hypo-elastic model, can incorporate the loading history effect, which leads to the load-path dependency behavior including the unloading behavior. The incremental formulation in the form of tangential stress-strain relationship is more general and more flexible to describe the inelastic irreversible behavior of concrete materials. These models are capable of reflecting the irreversible characteristic of concrete material under general loading condition, **Willam and Warnke (1975) [45]**.

4-4-1-2: Plasticity Based Model

The plasticity-based model has been used to describe the nonlinear behavior of concrete in compression. In this approach, the nonlinear deformation occurs when concrete is stressed beyond its limit of elasticity. The total strain may be separated into recoverable and irrecoverable parts. The recoverable part represents the framework of elasticity, while the treatment of the irrecoverable part is based on the theory of plasticity. Under triaxial compression, concrete can flow like a ductile material on the yield or failure surface before reaching its crushing strains, **Chen (1981) [37]**.

Arnesen et al. (1980) [46], stated that classical elasticity and plasticity theories can be sufficiently accurate for many applications. This is particularly true when the concrete structure fails in cracking of concrete and yielding of reinforcement bars. Plasticity theory for concrete is sufficiently accurate for

uniaxial stress state and two-dimensional state of stress with proportional loading. But for complex loading history and three-dimensional state of stress, it has been shown that these theories are inadequate.

4-5: Concrete Models Adopted in the Present Study

In the present study, concrete behavior is considered to be a non-linear elastic-perfectly plastic-brittle fracture material. This model should include the following.

- Stress-strain relationship.
- Failure criterion to simulate cracking and crushing types of fractures.
- Cracking modeling.
- Crushing modeling.

These models are implemented in **ANSYS** program and used in the present theoretical investigation.

4-5-1: Stress-Strain Relationship

A parabolic stress-strain curve is used for the equivalent uniaxial stress-strain relationship beyond the limit of elasticity, $c_p f'_c$, Figure (4-7). The parabolic curve represents the work-hardening stage of behavior. When the peak compressive stress is reached, a perfectly plastic response is assumed to occur.

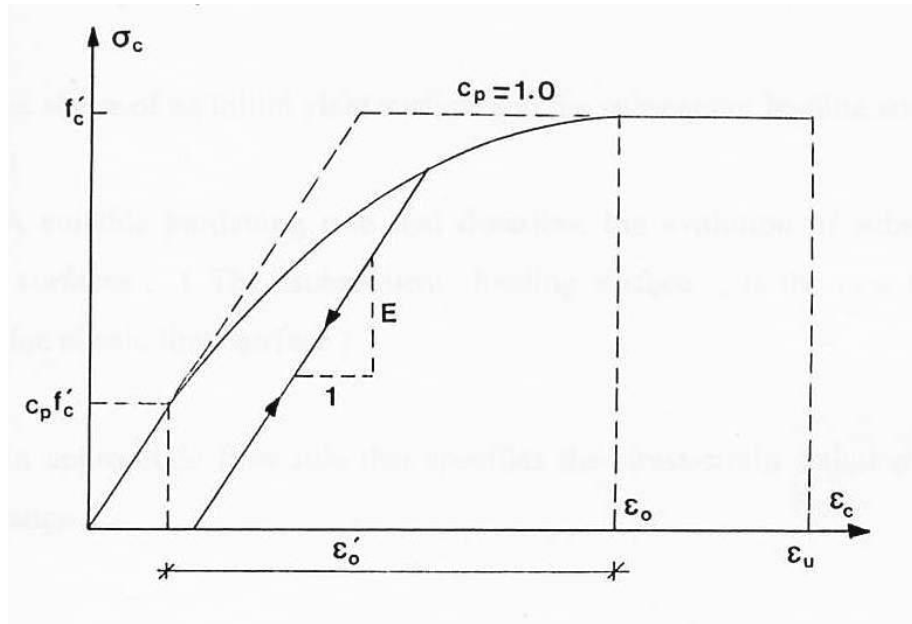


Figure (4-7): Uniaxial stress-strain curve for concrete in Compression AL-Shaarabaf (1990)[47].

The equivalent uniaxial stress, $\bar{\sigma}$, at the various stages of behavior, is given by:

a) for $\bar{\sigma} \leq c_p f'_c$

$$\bar{\sigma} = E \varepsilon_c \quad \dots(4-4)$$

b) for $c_p f'_c \leq \bar{\sigma} \leq f'_c$

$$\bar{\sigma} = c_p f'_c + E \left[\varepsilon_c - \frac{c_p f'_c}{E} \right] - \frac{E}{2\varepsilon'_o} \left[\varepsilon_c - \frac{c_p f'_c}{E} \right]^2 \quad \dots(4-5)$$

where, ε'_o is the total strain corresponding to the parabolic part of the curve, Figure(4-7), and given by:

$$\varepsilon'_o = 2(1 - c_p) f'_c / E \quad \dots(4-6)$$

If $c_p = 1.0$ then elastic perfectly plastic behavior is specified. When $c_p = 0$, Eqs. (4-5) and (4-6) reduce to the equations that are commonly used in

the parabolic uniaxial stress-strain curve given by the following Eqs. respectively:

$$\varepsilon_0 = 2 \frac{f'_c}{E} \quad \dots(4-7)$$

$$\sigma_c = E\varepsilon_c - \frac{E}{2\varepsilon_0} \varepsilon_c^2 \quad \dots(4-8)$$

where σ_c and ε_c are the uniaxial compressive stress and strain respectively and ε_0 is the strain at peak stress f'_c .

$$\begin{aligned} \text{c) For } \varepsilon_c &\geq \frac{(2 - c_p)f'_c}{E} \\ \bar{\sigma} &= f'_c \end{aligned} \quad \dots(4-9)$$

4-5-2: Failure Criteria For Concrete

The model to be used is capable of predicting failure for concrete materials. Both cracking and crushing failure modes are accounted for. The two input strength parameters (i.e., ultimate uniaxial tensile and compressive strengths) are needed to define a failure surface for the concrete. Consequently, a criterion for failure of the concrete due to a multiaxial stress state can be calculated, **Willam and Warnke (1975) [45]**.

A three-dimensional failure surface for concrete is shown in Figure (4-8). The most significant nonzero principal stresses are in the x and y -directions, represented by x_p and y_p , respectively. Three failure surfaces are shown as projections on the $x_p y_p$ plane. The mode of failure is a function of the sign of z_p (principal stress in the z direction). For example, if x_p and y_p are both negative (compressive) and z_p is slightly positive (tensile), cracking would be predicted in a direction perpendicular to z_p . However, if z_p is zero or slightly negative, the material is assumed to crush, Figure (4-8) demonstrate the 3D-failure surface for concrete, **ANSYS 9.0 [28]**.

In a concrete element, cracking occurs when the principal tensile stress in any direction lies outside the failure surface. After cracking, the elastic modulus of the concrete element is set to zero in the direction parallel to the principal tensile stress direction. Crushing occurs when all principal stresses are compressive and lie outside the failure surface; subsequently, the elastic modulus is set to zero in all directions, **ANSYS 9.0** [28], and the element effectively disappears.

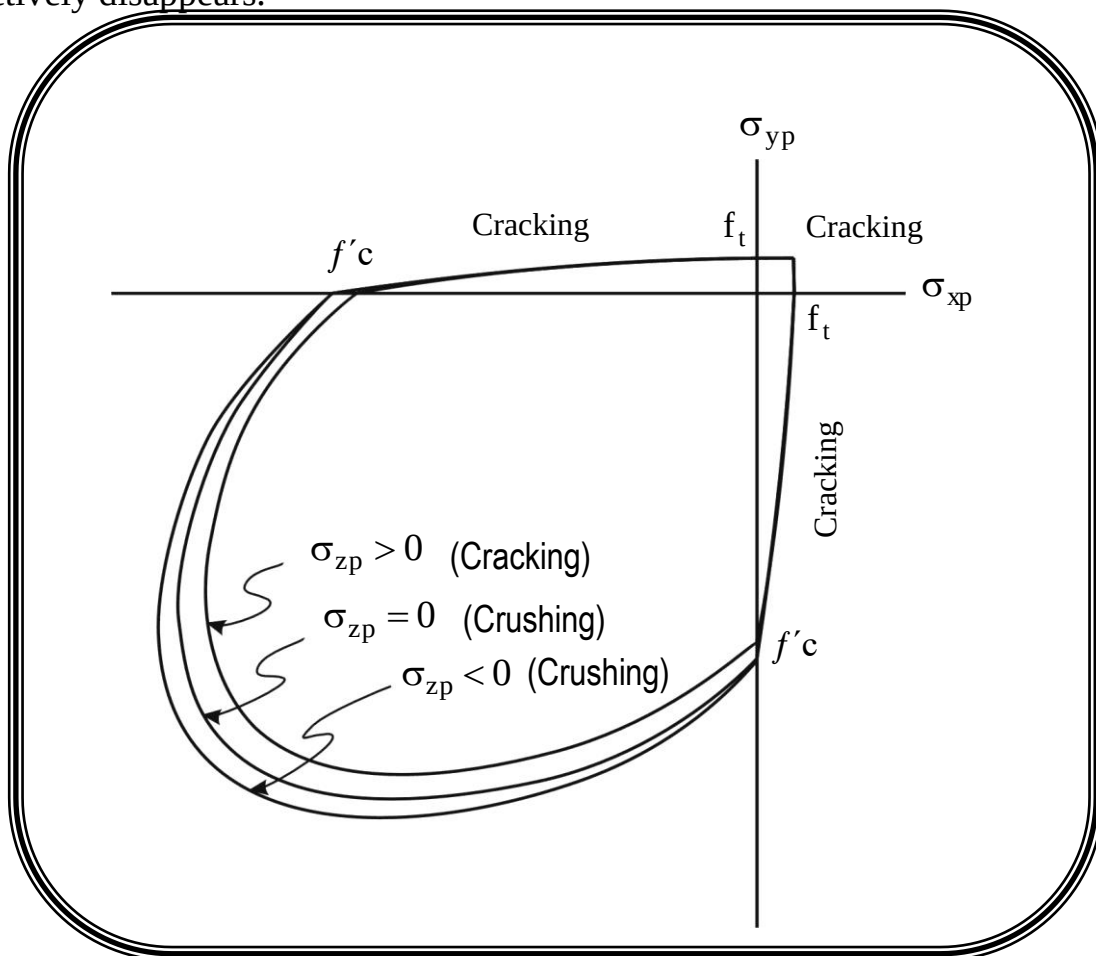


Figure (4-8):3D-Failure Surface for Concrete ANSYS 9.0 [28].

4-5-3: Cracking Modeling

In general, cracks in concrete members may be caused by direct tension, flexural, combined shear and flexure in beams webs, torsion, and other actions. The behavior of members often changes abruptly when tensile cracks form, **ANSYS 9.0** [37].

In the finite element analysis of concrete structures, three different approaches have been employed for cracking modeling, **ANSYS 9.0 [37]**.

- 1) Smeared- cracking model.
- 2) Discrete-cracking model.
- 3) Fracture-mechanism model.

The particular cracking model to be selected from the three alternatives depends upon the purpose of the analysis. If overall load-deflection behavior is desired, without regard to completely realistic crack patterns and local stress, the smeared-crack model is probably the best choice. In this approach, the crack concrete is assumed remain a continuum. It is assumed that the concrete becomes orthotropic or transversely isotropic after the first cracking has occurred, one of the material axes being oriented along the direction of cracking as shown in figure (4-8). For most structural engineering application, the smeared-cracking modeling is generally used, **ANSYS 9.0 [37]**.

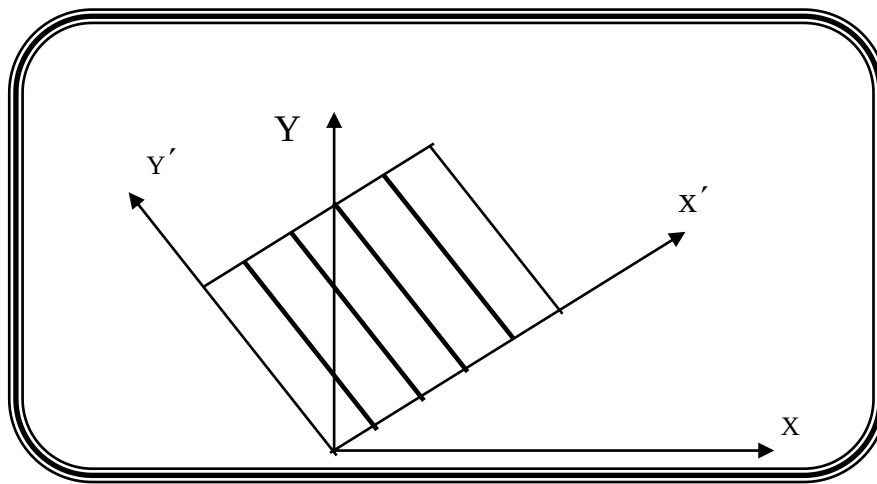


Figure (4-9): Smeared crack modeling [37]

In the discrete-cracking model, the disconnected or separating the concrete elements on either side of the crack. This is done by assigning a different nodal point number on each side of the crack as shown in figure (4-9), **Ngo (1967) [48]**.

One obvious difficulty in such an approach is that the location and orientation of the cracks are not known. While the success fracture-mechanics

theory had been used in solving various types of cracking problems in metals, ceramics and rocks, its use in the finite-element analysis of reinforced concrete structures, is still questionable, **ANSYS 9.0 [37]**.

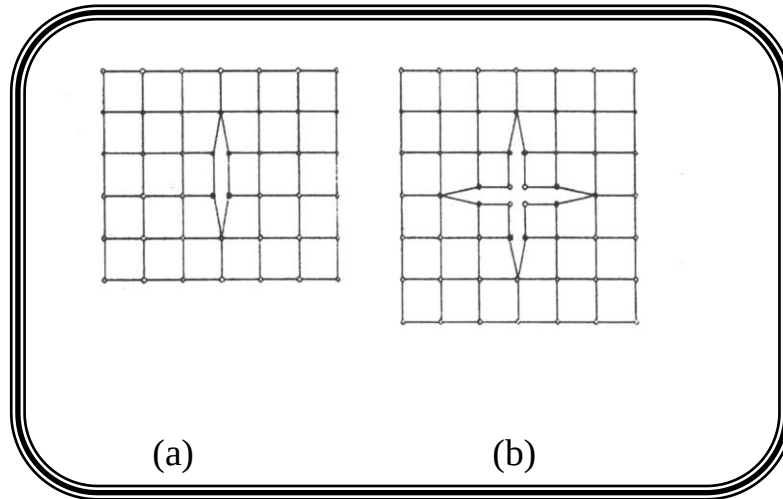


Figure (4-10): Discrete crack model Anthon (2004)[49]: (a) One-dimension crack, (b) Two-dimensional crack ANSYS 9.0 [37].

In the rotating crack model, the crack direction is always aligned normal to the direction of maximum principal strain, and the principal stress and strain directions are assumed to coincide. Since the overall load-deflection behavior is desired, the cracking of concrete in the present study is considered through an adjustment of material properties which effectively treats the cracking as a smeared band of cracks with fixed crack orientation. This model is described in terms of shear model and closing and reopening of cracks, **ANSYS 9.0 [28]**.

4-5-3-1: Shear Transfer Model

After cracking has taken place, cracked reinforced concrete can still transfer shear forces through aggregate interlock or shear friction and dowel action. However, to take the shear stiffness of cracked concrete into account in the smeared crack model, shear modulus may be reduced gradually. Using a reduced shear modulus not only improves the realism of the cracking

representation during the finite element analysis but also removes most of the numerical difficulties caused by the singularity of the composite materials constitutive matrix, **William et al. (1990) [50]**.

The concrete is assumed to behave linearly in tension up to the onset of cracking. When concrete cracks, its shear stiffness is reduced. Although a crack may be open in normal direction, but it is still possible that opposite faces will interlock when they are subjected to parallel differential movement. This depends on the texture of the cracked surface as well as the constrained force that can keep the cracked surfaces from moving apart **Chen and Saleeb (1982) [51]**. This constrained force may be provided by surface roughness, thus, the rough surface is capable of transferring some shear force by “aggregate interlock”.

However, as stated before, the cracked concrete can partially transmit shear across the crack due to aggregate interlock and due to dowel action of the reinforcement as the reinforcing bars can provide some constraint somewhere along the cracked surface.

To estimate such an effect, a shear transfer coefficient (β) is introduced which represents a shear strength reduction factor for concrete across the cracked face.

In the present study, two coefficients of shear strength reduction are used, thus (β_o) is introduced for the case of opened crack and (β_c) is introduced for the case of closed crack. The values of β_o and β_c are always in the range of $1 > \beta_c > \beta_o > 0$, **ANSYS 9.0 [28]**. These values depend on the texture of the cracked surface.

4-5-3-2: Closing and Reopening of Cracks

The reopening of a cracked integration point may occur as a result of stress redistribution at neighboring points. Cracks are assumed to open perpendicular

to the highest principal tensile stress direction, when the failure surface has been reached.

4-5-3-3: Tension Stiffening Model

The tension stiffening may be defined as the ability of the cracked reinforced concrete to decrease the strain in reinforcement due to tensile stresses in the concrete between the cracks, the shear stresses at the reinforcing steel-concrete interface feed tension into concrete between cracks. This phenomenon can be explained more clearly as, after cracking the concrete can not carry force across the cracks, **Sebastian and Cannel (2000) [52]**.

The tensile stress normal to the cracked plane is gradually released, and represented by an average stress-strain curve, **ANSYS 9.0 [28]**, as shown in figure (4-10).

a) For $\varepsilon_c^{cr} \leq \varepsilon_n \leq \alpha_1 \varepsilon_c^{cr}$

$$\sigma_n = \alpha_2 \cdot \sigma_{cr} \frac{\left[\alpha_1 - \frac{\varepsilon_n}{\varepsilon_c^{cr}} \right]}{[\alpha_1 - 1.0]} \quad \text{.....(4-10)}$$

b) For $\varepsilon_n > \alpha_1 \varepsilon_c^{cr}$

$$\sigma_n = 0 \quad \text{.....(4-11)}$$

where (σ_n) and (ε_n) are the stress and strain normal to the cracked plane, (ε_c^{cr}) is the cracking strain associated with cracking stress (σ_{cr}) which is equal to (f_t) and (α_1) and (α_2) are the tension stiffening parameters. (α_1) , represents the rate of stress release as the crack widens, while (α_2) represents the sudden loss of stress at instant of cracking.

Unloading and reloading are assumed to follow a secant path “ E_1 ”, which can be evaluated from previously stored maximum strain developed normal to the cracked plane.

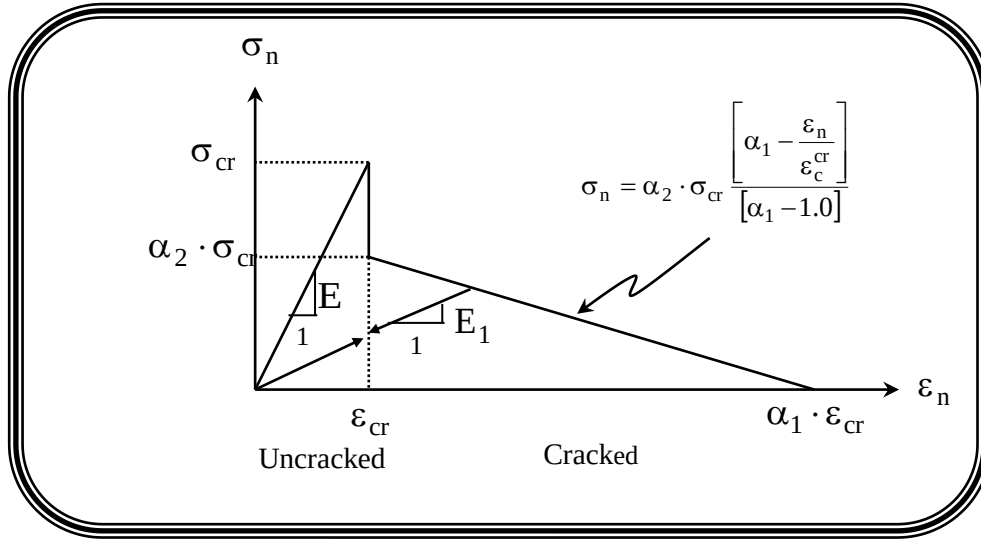


Figure (4-11): Post-cracking model for concrete [28]

4-5-4: Modeling of Crushing

If the material at an integration point fails in uniaxial, biaxial, or triaxial compression, the material is assumed to crush at that point. Under this condition, material strength is assumed to have degraded to an extent such that the contribution to the stiffness of an element at the integration point in equation can be ignored, ANSYS 9.0 [28].

4-6: Modeling of Reinforcement

The properties of reinforcing steel, unlike concrete, are generally not dependent on environmental conditions or time. Thus, the specification of a single stress-strain relation is sufficient to define the material properties needed in the analysis of reinforced structures.

Typical stress-strain curves for reinforcing steel bars used in concrete construction are obtained from coupon tests of bars loaded monotonically in

tension. For all practical purposes steel exhibits the same stress-strain curve in compression as in tension. The steel stress-strain relation exhibits an initial linear elastic portion, a yield plateau, a strain-hardening range in which stress again increases with strain and finally a range in which the stress drops off until fracture occurs. The extent of the yield plateau is a function of the tensile strength of steel, **Cervenka et al. (1990) [53]**.

Where, $E_w = 0.1E_s$

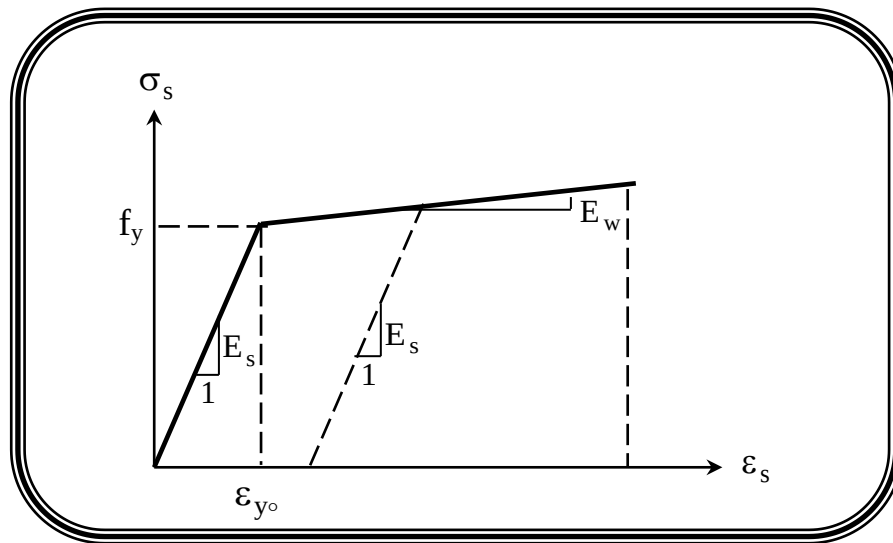


Figure (4-12): Typical stress-strain curve for steel bar Mattock (1981)[54]

4-7: Interface Model

Under static or dynamic loading, an interface can experience various modes of deformation. Shear may be transferred by means of friction when there is a normal compressive stress acting on the interface. This normal stress may be due to an externally imposed load, **ANSYS 9.0 [28]**.

The shear capacity of the interface could be influenced by the type of surface penetration used for the contact element and condition of the interface in composite specimens (the interaction, or stress transfer between prestressed concrete element and ordinary concrete beam) **Mattock (1981) [54]**.

The normal stiffness (k_n) is used in the gap resistance and it should be large enough that it reasonably restrains the model from over-penetration, yet it should not be so large that it causes ill – conditioning in the global stiffness matrix and convergence difficulties.

$$k_n = (E_c \times A) / L \quad (\text{N/mm}) \quad \dots\dots\dots(4-12)$$

Where;

E_c = concrete Young's modulus (MPa).

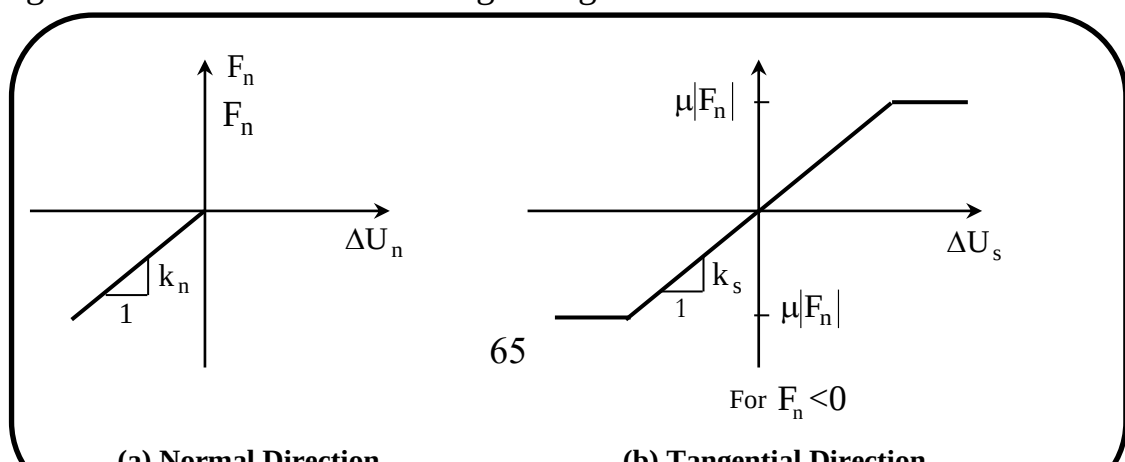
A = area of surrounding or contributed area surrounding (mm^2).

LP = thickness of brick element in direction perpendicular to sliding surface (mm).

The tangential (sticking) stiffness (k_s) is found from special laboratory tests. As a guideline, the sticking stiffness should be 1, 2 or 3 orders of magnitude less than the normal stiffness **ANSYS V.9.0 program [28]**.

k_s from load-slip curve, **Patnaik (2001) [19]**, shown in figure (4-13).

The **ACI-code 318 (2005)[41]**, stated that the coefficient of friction (μ) is assumed to be dependent on the concrete placement. Thus, ($\mu = 1\lambda$) is specified for concrete placed against hardened concrete with surface intentionally roughened, ($\mu = 0.6\lambda$) for concrete placed against hardened concrete not intentionally roughened, and ($\mu = 1.4\lambda$) for concrete placed monolithically, while ($\mu = 0.7\lambda$) for concrete anchored to structural steel by headed studs or by reinforcing bars where, $\lambda = 1.0$ for normal weight concrete, 0.85 for sand-lightweight concrete and 0.75 for all lightweight concrete.



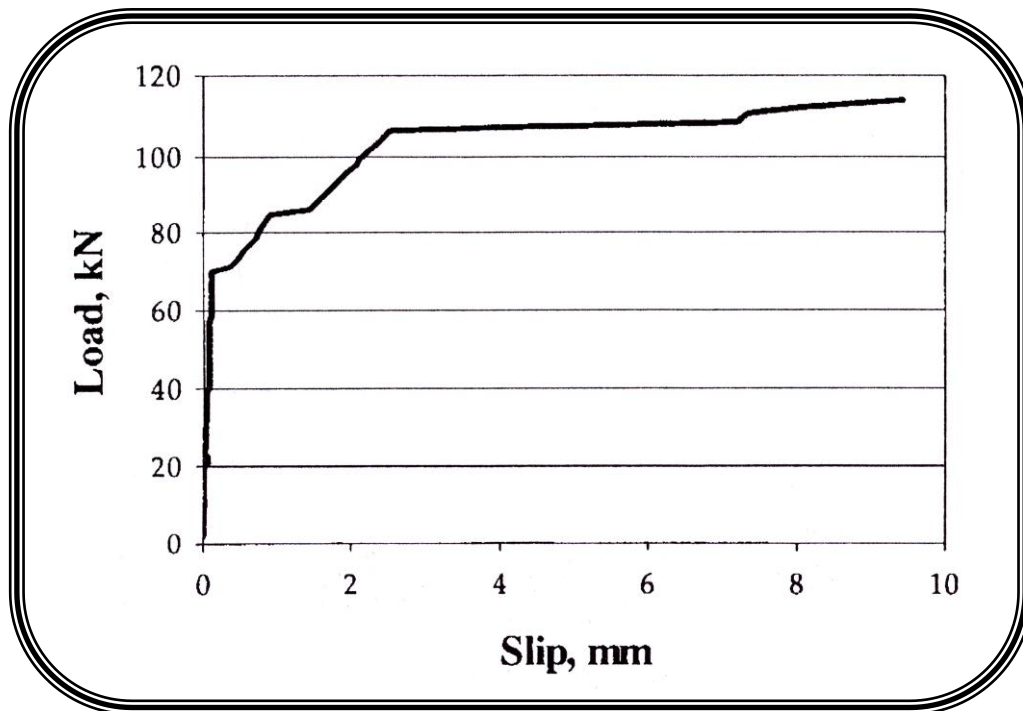


Figure (4-14): Experimental slip curve Patnaik [19].

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7-1 General

In this chapter, based on the numerical finite element analysis described in this work, the following conclusions can be drawn with regard to the results obtained for reinforced concrete members with precast-prestressed units subjected to flexural loading. The suggested recommendations for further work are also presented.

7-2 Conclusions

1. The three-dimensional nonlinear finite element model presented in this study by using the computer program (ANSYS V.9.0) is able to simulate the behavior of reinforced concrete member with precast-prestressed units. The numerical results were in good agreement with experimental load-deflection curves and moment-curvature curves throughout the entire range of behavior.
2. According to the results obtained from the numerical analyses of composite beams find the used precast-prestressed units as tension reinforcements increased flexural rigidity, that resulted in the increasing

of ultimate load, cracking load and the decreasing of the ultimate deflection.

3. Concerning the effect of the modular ratio on the load-deflection curve, it can be shown that the decrease of modular ratio from 0.9 to 0.6 has a clear effect on stiffness. On the other hand, decreasing of modular ratio beyond 0.6 is insignificant.
4. The ultimate load and the behavior of composite prestressed beam are affected by increasing of prestressing steel ratio. It can be concluded that one can obtain an increase that may reach to (43.75)% for simply supported beam and to (29.52)% for continuous beam in ultimate load when the amount of prestressing steel ratio increased from (0.00082 to 0.00155) (increase 63.35%) for the considered experimental studies without exceeding the allowable stresses calculated by using **ACI-Code 318M-05** equation.
5. According to the results obtained from the numerical analysis of composite prestressed concrete members, it is found that the ultimate load is increased with increasing the eccentricity to depth ratio. The amount of increase in the ultimate load may reach (2.63-2.55)% for simply support and (1.33-1)% for continuous beam when the eccentricity to depth ratio in the prestressed unit(e_2/h_2) increase from (0.1) to (0.3) and (e_1/h_1) varied from (0.3 to 0.2). This is done without exceeding the allowable stresses calculated by using **ACI-Code 318M-05** equation.
6. It was found that the variation of precast-prestressed unit shape has a real effect on the predicted and ultimate load of composite prestressed concrete members. It can be noted that one unit in the middle beam section is better than two in the corners.

7. Coming from extensive analysis that carried out in this study, it is concluded that implementing of the precast-prestressed units in reinforced concrete beams has expanded the values of the cracking loads and that resulted in increasing the ultimate loads.

7-3 Recommendations and Suggestions for Further Work

1. Analysis of composite prestressed concrete beams subjected to dynamic and impact loading.
2. Analysis of composite prestressed concrete beams subjected to bending and torsional load
3. Analysis of continuous composite prestressed beams with precast-prestressed units along the length of beam.

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NOTATIONS

The major symbols used in the text are listed below; others are defined, as they first appear.

General Symbols

A, a	Scalars
$[\]$	Matrix
$\{ \}$	Vector
$[\]^T$	Transpose of a matrix
$\{ \}^T$	Transpose of a vector
$[\]^{-1}$	Inverse of matrix
d, ∂	Differential symbol
Δ	Denoting incremental quantity
δ	Denoting iterative quantity
$ $	Determinant of a matrix, or absolute value
$\ \ $	Norm symbol

Scalars

E_c	Modulus of Elasticity of Concrete
E_s	Modulus of Elasticity of Steel
E_{pc}	Modulus of Elasticity of Prestressed Concrete
f	Function
f'_c	Uniaxial Compressive Strength of Concrete (Cylinder Test)
f_t	Uniaxial Tensile Strength of Concrete
J_2	Second Deviatoric Stress Invariant
F	Applied Concentrated Load
T_n	Convergence Tolerance

V	Volume
u, v, w	Displacement Components in x, y and z Directions, Respectively
X, Y, Z	Global Coordinate System
β_o, β_c	Shear Transfer Coefficients
ε	Strain
ν	Poisson's Ratio
σ	Stress
τ	Shear Stress
ξ, η, ς	Local Coordinates
C_p	Plasticity Coefficient
W	Weight Vector
S	Failure Surface
α_1, α_2	Tension-Stiffening Parameters

Matrices

$[B]$	Strain-Displacement Matrix
$[D]$	Material Constitutive Relations Matrix
$[J]$	Jacobian Matrix
$[K]$	Stiffness Matrix
$[N]$	Shape Function
$[T]$	Transformation Matrix

Vectors

$\{F^a\}$	External Load Vector
$\{F^{nr}\}$	Internal Load Vector
$\{R\}$	Residual Load Vector
$\{U\}$	Displacement Vector

$\{\varepsilon\}$	Strain Vector
$\{\sigma\}$	Stress Vector

Subscripts and Superscripts

c	Concrete
c^r	Cracking
e	Elastic Component, or Element Quantity
s	Steel
t	Tension
x,y,z	Denoting Cartesian Coordinates
1,2,3	Denoting Principal Directions

DEDICATION

To My Family...

The reason of my existence and diligence.

To My Teachers...

The reason of my educational ambition.

To My Friends...

*Who gave me unconditional support and
encouragement*