Republic of Iraq

Ministry of Higher Education

and Scientific Research

University of Babylon

College of Education for Pure Sciences

Department of Mathematics

Study of Some Methods to Solve the Reliability of Complex Systems

A Research

Submitted to the College

of Education for Pure Sciences

of the University of Babylon

as Partial Fulfillment of the Requirements for the Requirement for the Degree of Higher Diploma Education/ Mathematics

by

Zytoon Muhsin Abdullah Abid

Supervised by

Dr. Ghazi Abdullah Madlool

2024 – A.D 1444 A-H

بسم الله الرحمن الرحيم

(ويسألونك عن الروح قل الروح من امر ربي وما اوتيتم من العلم الا قليلا)

صدق الله العلي العظيم

الاسراء 85

Supervisor Certification

I certify that this research entitled **" Study of Some Methods to Solve the Reliability of Complex Systems "** was prepared by the student **" Zytoon Muhsion Abdullah Abid "** under my supervision at University of Babylon, College of Education for Pure Sciences, Department of Mathematics as a partial fulfillment of the requirement the Diploma Education / Mathematics

. **Signature:**

Name: Dr.

Scientific grade:

Date: / / 2023

In view of available recommendation, I forward this research for debate by the examining committee

Signature:

Name: Dr. Ali Hussein Al-Obaidi

Address: Head of Department of Mathematics

Scientific grade: Assistant Professor

Date: / / 2023

Scientific Certification

This is to certify that I have read this research which is entitled **" Study of Some Methods to Solve the Reliability of Complex Systems "** by the student **" Zytoon Muhsion Abdullah Abid "** and I found that this research is qualified for debate.

Signature :

Name : Dr. Ali Hussein Al-Obaidi

Title : Assistant Professor

Address : Head of Department of Mathematics

Date: / /2023

Linguistic Certification

This is to certify that I have read this research which is entitled **" Study of Some Methods to Solve the Reliability of Complex Systems "** by the student **" Zytoon Muhsion Abdullah Abid "** and I found that this research is qualified for debate.

Signature :

Name : Amera Al-Funjan

Title : Assistant Professor

Address : Lecturer

Date: / /2023

Examing Committee Certification

We certify that we have read this research entitled " **Study of Some Methods to Solve the Reliability of Complex Systems** " and as board, tested the student " **Zytoon Muhsion Abdullah Abid** " in its contents and that in our opinion it is adequate for the partial fulfillment of the requirement for the Degree of Higher Diploma Education\ Mathema

Signature:

Name:

Title:

Date: / / 2023

Signature:

Name:

Title:

Date: / / 2023

Signature:

Name: Dr.

Title:

Date: / / 2023

I hereby certify the decision of college of the examining committee.

Signature:

Name: Dr. Bahaa Hussien Salih Rabee

Title: Professor

Address: Dean of College of Education for Pure Sciences

Date: / / 2023

Dedication

Under her feet lies paradise. To my loving mother. To the one who made my scientific journey possible. To my merciful father. To whom I walked the path. To my husband. To my dear brother and sister and all of them I dedicate this humble effort he prayed. To god to prolong their lives so that they may see the fruit of their labors.

Acknowledgement

It gives me great pleasure to thank my supervising Dr. Ghazi Abdullah for peeling excellent supervisor during the preparation of the research that this earful. Guidance and encouragement have on me. Would not for many think the present hard of the mathematics Department , and the staff of the department for their cooperation.

Zytoon muhsin…….

Contents

Table of Figures

Abstract

In this thesis, we calculated the reliability of simple systems, which dealt with calculating series systems, parallel systems, series-parallel systems, and parallel - series systems. We also calculated mixed systems. As for complex systems, we calculated them in three different ways,(Path Tracing Method ,Minimal Cut Method andReduction to Series Elements) and it turned out to be the same reliability with these three methods in terms of accurac

INTRODECTION

In everyday language, we use the word reliable to mean that something is dependable, and that it will give behave , predictably every time . we might talk of a football player as reliable meaning that he gives a good performance playing in the game

In science, the idea is similar, but the definition is much narrower. reliability is a property of any measure tool test The aim of this is research is to prove that solving any way will get the same result .

This research consists of two chapters , in the first chapter, we studied some basic definition of reliability then we studied the definition of reliability in both types , the simple system with branches with examples . The second chapter which complex system with three methods to solve it . The we studied some complex system and treat them in the three ways .

1.1- INTRODECTION

In this chapter we studed some definitions of reliability as well as studided the simple system and its types with examples for each one

1.2- Some basic definitions in reliability

In the current report, we will present some concepts in structure function, network topology and graph theory which concern our work for the calculation the network reliability

1- Success[12]

The system performs its function satisfactorily for a given period of time, where the criterion for success is clearly defined.

2- failure[12]

Failure rate can be defined as the anticipated number of times that an item fails in a specified period of time. It is a calculated value that provides a measure of reliability for a product.

3- success indicator[12]

The success indicator for component i is the binary random variable X_i that indicates the status of component i.

$X_i = 1$ implies component i is working
$X_i = 0$ implies component i is failed

4- status vector[12]

The status vector is the vector of component status indicators.
X = (X_1;X_2,…, X_n):
There are 2^n possible realizations of this vector

5- Relevant component[12]

A component is irrelevant if it has no effect on the functioning of the system, i.e., the i^{th}component is irrelevant
$\Phi(x_1,\ldots, x_{i-1}, 0, x_{i+1}, \ldots,x_n) = \Phi(x_1,\ldots, x_{i-1}, 1, x_{i+1}, \ldots,x_n)$ for all $x_1,\ldots, x_{i-1}, x_{i+1}\ldots,x_n$.
Otherwise it is called relevant.

6- coherent systems (Monotone Systems)[12]

A system is coherent if all of its components are relevant and its structure function is non-decreasing.
In other words, a coherent system has the property that when the system is successful for some status vector X, it remains successful if some components of X change from 0 to 1. Alternatively, if the system is failed for some status vector X, it remains failed if some components of X are changed from 1 to 0 . More formally , a coherent system has the property that when X and Y are two status vectors such that Y > X, $\Phi(Y) \geq \Phi(X)$:

7- graph[12]

A network can be represented in the form of a graph G = (V,E), where V is the set of vertices (or nodes) and E the set of edges (or arcs).

8- path[24]

A path for the network is a set of components, such that if all the components in the set are successful, the system will be successful.

For example, the set of all components is a path. For example, in Figure (1-1) the set [1, 3, 5] [1, 4] is a path.

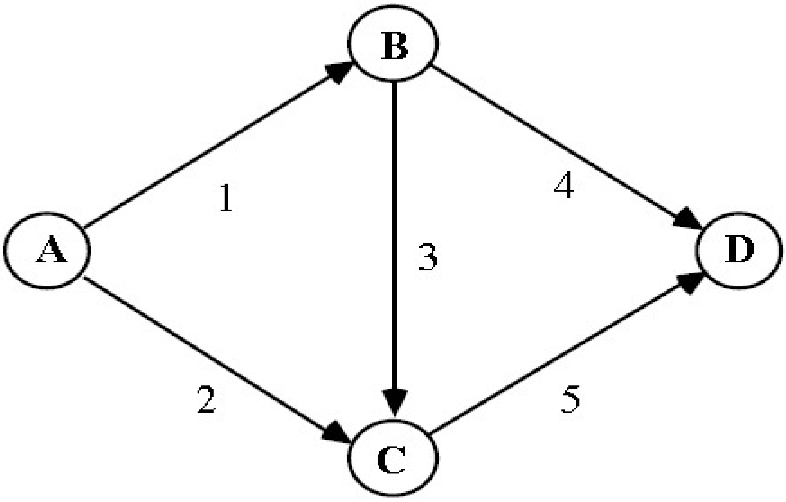


Figure (1-1) Bridge Network

9- minimal path[12]

A minimal path is a set of components that comprise a path, but the removal of any one component will cause the resulting set to not be a path.

In other words, if all the components in a minimal path are successful while all other components have failed, the system will be successful. If any one of the components in the minimal path subsequently fails, the system will fail. In terms of the network model, the minimal path corresponds to a simple path from the source to the sink in the network. For example in fig (1-1) the sets [1, 4], [1, 3, 5], [2, 5] are minimal paths.

10- cut set[15]

A cut is a set of components such that if all the components in the cut fail, while all other components are successful, the system will fail. Again, the set of all components is a cut.

11- minimal cut[5]

The minimal cut is a set of components that comprise a cut, but the removal of any one component from the set causes the resulting set to not be a cut. In the network a minimal cut breaks all simple paths from the source
to the sink. From Fig. 1 we observe that the minimal cuts are: [1, 2], [1,5],[2, 3] ,[2 ,3 ,4] and [4,5]

1.3- Network Model [2]

We describe the system as a directed network consisting of nodes and arcs ,as illustrated in Fig. (1.1) One node is defined as the source (node A in the figure), and a second node is defined as a sink (node D). Each component of the network is identified as an arc passing from one node to another. The arcs are numbered for identification. A failure of a component is equivalent to an arc being removed or cut from the network. The system is successful if there exists a successful path from the source to the sink. The system is failed if no such path exists. The reliability of the system is the probability .

1.4 - Structure Function [11]

The structure function is a binary function that indicates the status of the system (success or failure) given the status of each component. Φ ($X_1;X_2;…;X_n$) or Φ(X):

We can rewrite the structural function in another way, as follows

X_i =1; if component i performs satisfactorily during time [0, t]

X_i =0; if component i fails during time [0, t].

The performance of the system is measured by the binary random variable Φ ($X_1;X_2;…;X_n$) where Φ ($X_1;X_2;…;X_n$) =1; if system performs satisfactorily during time [0, t]

Φ ($X_1;X_2;…;X_n$) =0; if system fails during time [0, t].

The structure function is a complete model of the failure and success characteristics of the system. There are several important structure functions to consider, depending upon how the components are assembled. Given the structure function of the system, one can compute its reliability. The component reliability, pi, is the probability that component i is operating Correctly.

The component failure probability, qi, is the probability that a component has failed. In terms of the success indicators,

Pi = P[X_i = 1] and qi = P[X_i = 0] = 1 - pi

When the probability of success or failure of a component does not depend on the status of some other component, the components are said to be independent. The assumption of this report is that all components are independent.

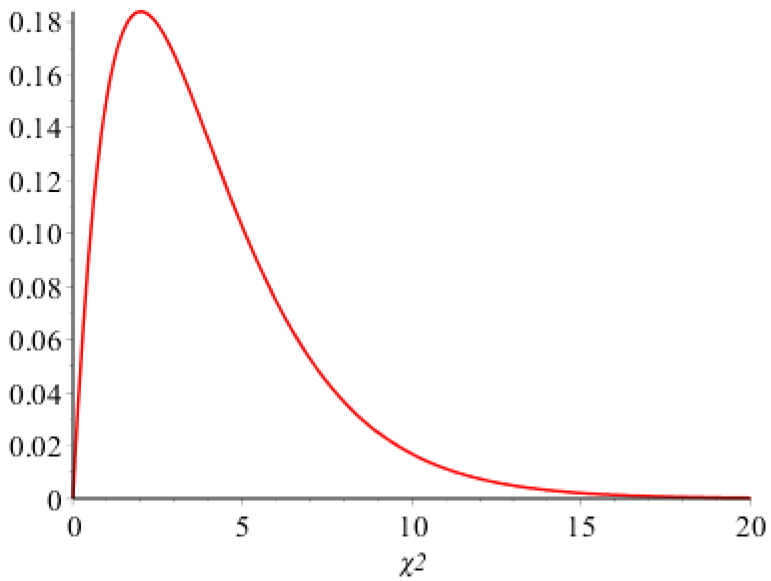
The probability that the system is operating correctly is the system reliabilityIt is the probability that the structure function is 1.

R = P[(X) = 1] = E[Φ(X)]………………. (1)

1.5 - probability density function p.d.f [6]

If X is continuous random variable , then the probability density function p.d.f of X is function f(x) such that for two numbers a and b with a ≤ b

p (a ≤ X ≤ b) = $\int_a^b f(x)dx = 1$ **and f(x) ≥ 0 for all x ….. (2)**



Figure

(1- 2) probability density function

1.6 -probability mass function[7]

The probability mass function of a discrete random variable X is f (x) = P{X = x}. The mass function has two basic properties:

f (x) ≥ 0 for all x in the state space.

F(x) = $\sum$ f(x)=1 ....(3)

1.7 – Cumulative distribution function [8]

The cumulative distribution function CDF is a function , F(x) , of a random variable X , and is defined for a number x , by F(x) = p(X ≤ x) = $\int_{-\infty}^{x} f(s)ds$

The c.d.f is used to measure the probability that the item in question will before the associated time value . and it also called unreliability .

1.8- bathtub curve [8]

is a particular shape of a failure rate graph . This graph is used in reliability engineering and deterioration modeling. The 'bathtub' refers to the shape of a line that curves up at both ends, similar in shape to a bathtub. The bathtub curve has 3 regions:

1. The first region has a decreasing failure rate due to early failures.
2. The middle region is a constant failure rate due to random failures.
3. The last region is an increasing failure rate due to wear-out failures.

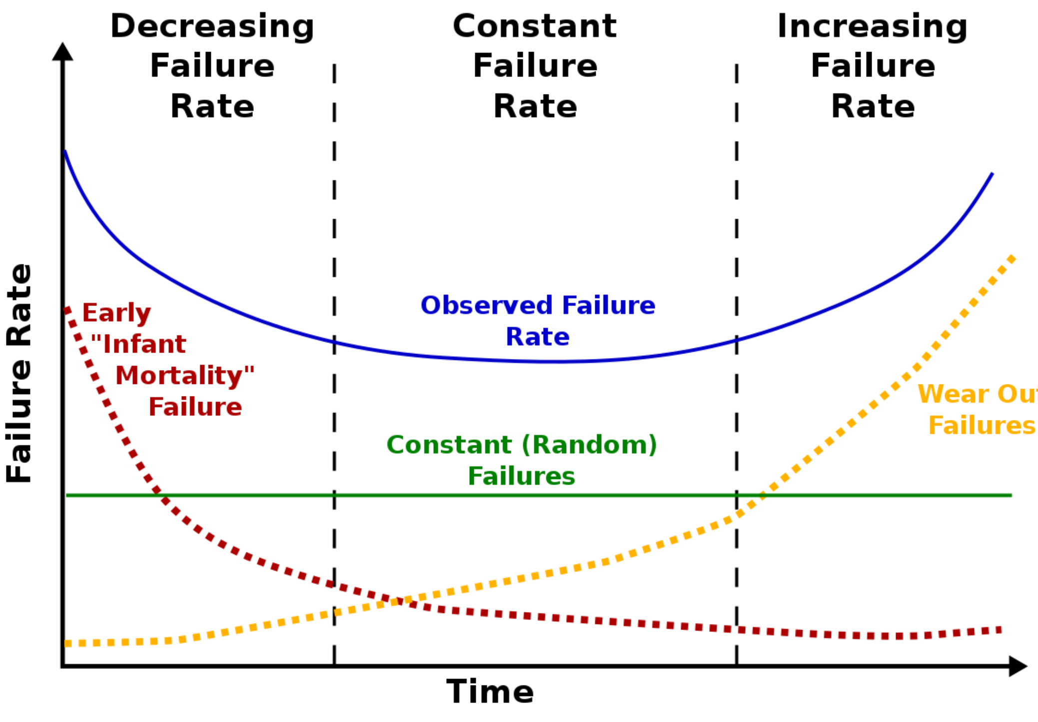


Figure (1- 3) bathtub curve

1.9- Reliability [2]

The probability of a system or component to perform its intended function under specified conditions for an extended period without failure . A reliable product has a high probability of performing as expected over time . the reliability of a product can be measured by how long it will last before failing .

1.10- Reliability Simple Systems [2]

Suppose that we have to calculate the Reliability of a system made up of several components. The total reliability can be calculated by calculating the reliability of each individual component, and combining these individual reliabilities. The way in which they are combined depends on the way in which the components are connected. That is, whether they are connected:

1.10.1- Series System[8]

A two-state system is called series if its lifetime T given by
$T = \max\{T_i\}$ s.t $1 \leq i \leq n$
The above definition means that the series system is not failed if and only if all its components are not failed, and therefore its reliability function is given by

$R_{System} = R_1 R_2 \ldots. R_n$ ....(4)

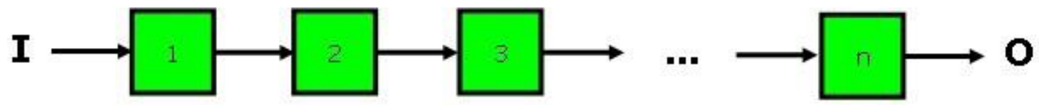


Figure (1-4) series system

Example 1[9]

Consider a system of 3 components connected in series, each component having a reliability 0.8 , 0.7 , 0.9 respectivliely
$R_s = R_1 \times R_2 \times R_3 = 0.8 \times 0.7 \times 0.9 = 0.504$

1.10.2- parallel system[1]

A two-state system is called parallel if its lifetime T is given by
$T = \min\{T_i\}$ s.t $1 \leq i \leq n$
The above definition means that the parallel system is failed if and only if all its components are failed and therefore its reliability function is given by

$$R_{system} = 1 - (1 - R_1)(1 - R_2)....(1 - R_n) \qquad ...(5)$$

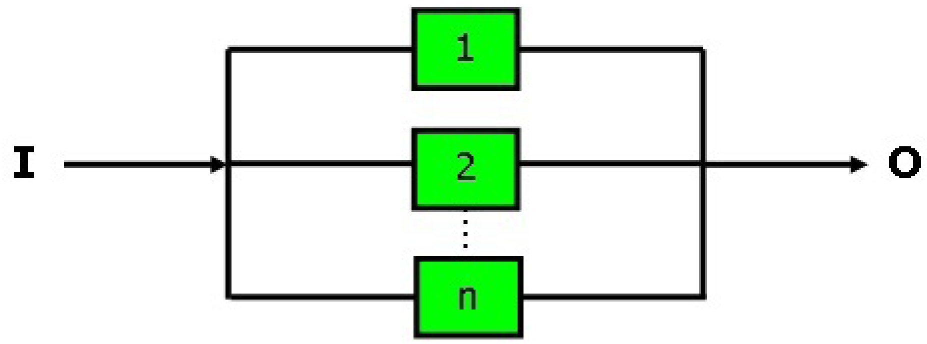


Figure (1-5) parallel system

Example 2[5]

Consider a system of 3 components connected in parallel, each component having a reliability 0.8 , 0.7 , 0.9

$R_{system} = 1 - (1 - R_1)(1 - R_2)(1 - R_3)$

$R_{system} = 1 - (1 - 0.8)(1 - 0.7)(1 - 0.9) = 1 - 0.2 \times 0.3 \times 0.1$

$= 1 - 0.006 = 0.994$

1.10.3- series_parallel system[12]

A two-state system is called series_parallel if its lifetime T is given by T = max{min (Ti) } s.t 1≤ i ≤ n

therefore its reliability function is given by

$R_{system} = 1 - (\prod_{i=1}^{m} R\ i)^{n}$ ...(6)

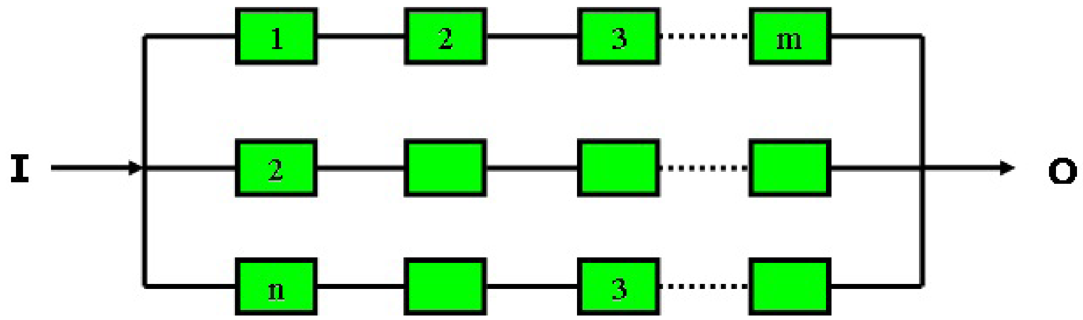


Figure (1-6) series parallel system

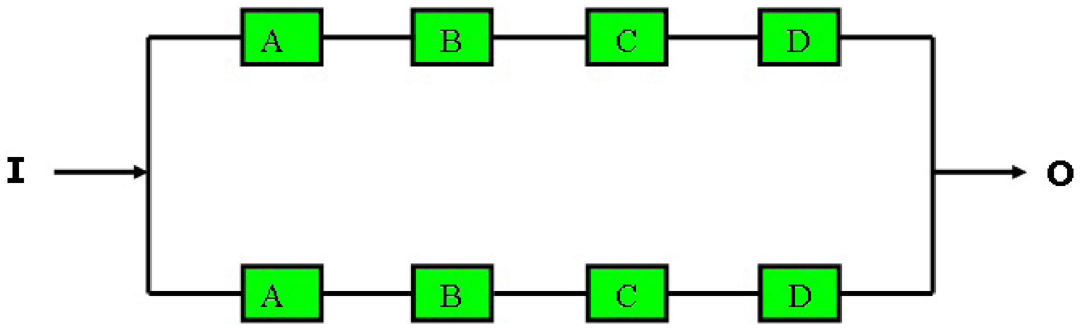
Example 3[2]

Consider a system of components connected in parallel-series, each component having a reliability A= 0.8 ,B= 0.7 ,C= 0.9 , D= 0.6

$R_{system} = 1 - (\prod_{i=1}^{m} R\ i)^{n}$

$R_{system} = 1 - (0.8 \times\ 0.7 \times\ 0.9 \times .06)^{2}$ = 1-0.09144576

R_{system} = 0.9088424



Figure(1-7) series parallel system

1.10.4 - parallel_series system[12]

A two-state system is called parallel_series if its lifetime T is given by T = min{max (Ti) } s.t 1≤ i ≤ n

therefore its reliability function is given by

$$R_{system} = (1-(1-R)^n)^m \quad ....(7)$$

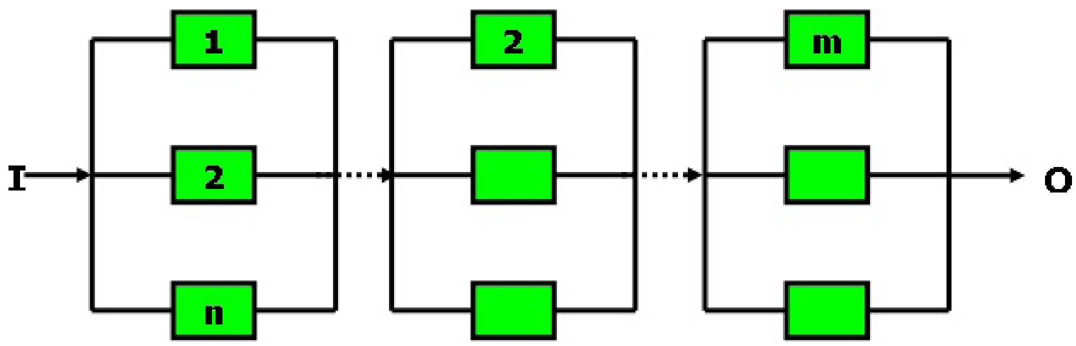


Figure (1-8) parallel-series system

Example 4[2]

Compute the reliability of the system for the connections given fig . Assuming the reliability of each component is 0.8

R$_{system}$ **=**$(\mathbf{1}-(\mathbf{1}-\boldsymbol{R})^{n})^{m}$

=$(\mathbf{1}-(\mathbf{1}-\mathbf{0.8})^{2})^{4}$**= 0.84934656**

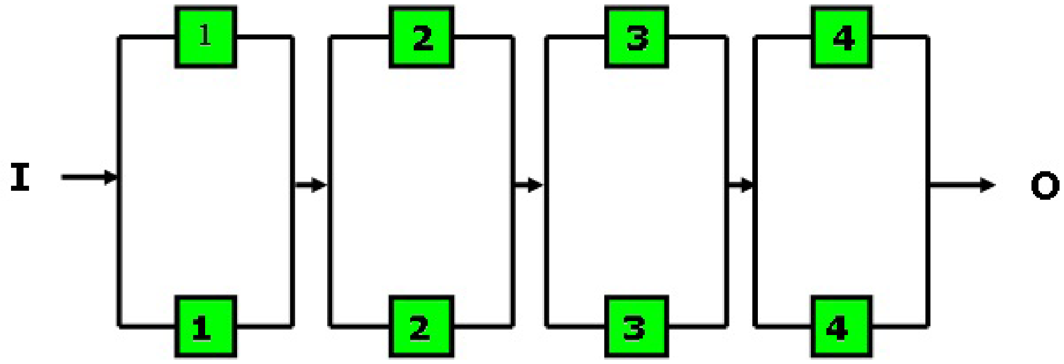


Figure (1-9) parallel-series system

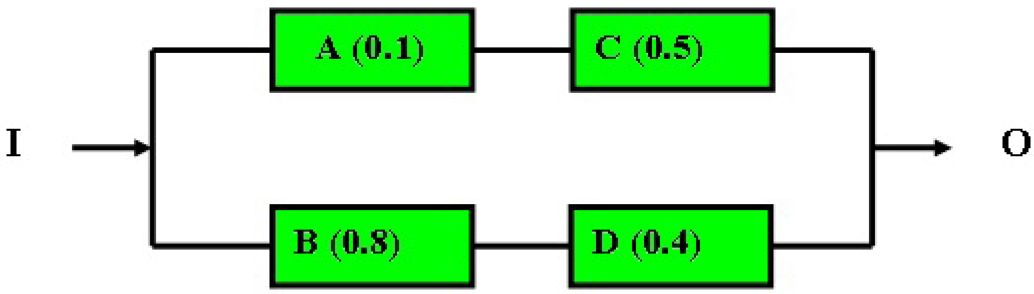
Assuming the reliability of each component is 0.80. Here, we are given n=2, m= 4,

1.10.5 - Mixed of Series and Parallel Systems[12]

To evaluate reliability of a system comprising both parallel and series sect the reliability of each subsystem as above then combine in a suitable manner. Note that this system is contained in some sources, under the name of mixed systems.

Example 5[2]

Find the reliability of the system shown where the number in brackets indicates the reliability for each of the components , divide the system into series only and parallel only subsystems.



Figure(1-10) Mixed system

Solution

Consider the two branches (AC) and (BD) . These are both series 'subsystems'. Then Reliability of top branch =

$R_{AC}(t) = R_A(t) \times R_C(t) = 0.1 \times 0.5 = 0.05$

Reliability of bottom branch =

$R_{BD}(t) = R_B(t)$ _ $R_D(t) = 0.8 \times 0.4 = 0.32$

We can then consider these two subsystems (AC) and (BD) as part of a larger system , namely

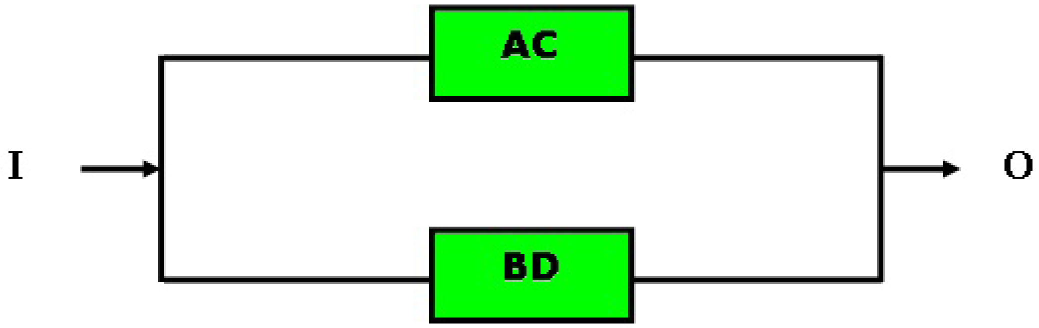


Figure (1-11) Mixed system

And for this parallel system

R_{system}=1-{ (1- R_{AC}) × (1- R_{BD})} = 1- { (1- 0.05) × (1-0.32)}
R_{system}= 1- { 0.95× 0.68 }= 1- 0.646 = 0. 354

1.10.6 - k Out of n System[12]

Some systems are assembled such that the system operates if k out of n components function properly. Note that the series system is a k out of n system, with k = n, and the parallel system is a k out of n system, with k = 1.
To compute the reliability, assume that all components have the same reliability, pi = p for all i. Then the reliability of the system is the probability that k or more components are successful. because of the independence assumption, we can the Binomial distribution to compute the reliability.

$$R = \sum_{i=k}^{n} \binom{n}{i} p^{i} (1-p)^{n-i} \quad \text{....(8)}$$

Example 6[12]

A space vehicle has three identical computers operating simultaneously and solving the same problems. The outputs of the three computers are compared, and if two or three of them are identical, that result is used. This is called a majority vote system, and in this mode one of the three computers can fail without causing the system to fail. This is a two out of three system. Identifying the success or failure of each of the computers with the variables X_1 , X_2 , and X_3,
Any combination of two or more Xi set equal to 1 will cause this function to assume the value 1, while fewer than two will result in a 0 value. With the reliability of each computer equal to 0.9, we use the Binomial distribution to compute the system reliability.

R= $\sum_{i=k}^{n}\binom{n}{i}p^{i}(1-p)^{n-i}$

R= $\sum_{i=2}^{n}\binom{3}{i}(0.9)^{i}(1-0.9)^{3-i}$

R= $3\times(0.9)^{2}(0.1)+\ (0.9)^{3}$**= 0.972**

The reliability of the combination of three computers is much greater than that of an individual. This is an example of the use of redundancy to increase reliability. Since only one computer is required to perform the function, the other two are redundant from a functional point of view. They do play an important role, however, in increasing the reliability of the system. The independence of failures is important here. If some failure mechanism causes all three computers to fail simultaneously, the reliability improvement will not be realized. For example, if all three computers used the same program and the program had an error, the combination of results will certainly be no better than any one of the individual results. The use of redundancy is common in systems for which failure has particularly severe consequences, such as in the space program or for very complex systems with many components.

2.1- INTRODECTION

In this chapter, we studied the complex system and some methods of solving five complex systems and finding their reliability in three methods

2.2 Complex systems [2]

Generally, a complex system in reliability is defined as a system having a combination of series, parallel, *R* out of *N*, and standby components. Each of these models has corresponding mathematical formulations for reliability computations leading to decomposition of the original system (or subsystem) into an equivalent one with a known cumulative distribution function (CDF) or reliability function. Continuing the decomposition procedure enables the decision maker to reduce the whole system to a unique component with a known CDF.

2.2.1 Path Tracing Method [1, 9,12]

In this method, we convert the complex system into a series-parallel system to facilitate the calculation of its reliability, by using the following steps.

1:- we extract the minimal path sets, as each of these sets causes the system to success.

2:- we link the elements of each of the groups of the minimal path with a series connection, because the failure of any component causes the failure of the path.

3:- we link parallel between these sets. Fourthly, we will have a series parallel system.

5:- we calculate its reliability according to its law .

2.2.2- Minimal Cut Method [1, 9,12]

In this method, we convert the complex system into a parallel series system to facilitate the calculation of its reliability, using the following steps.

1:- we extract all the minimal cat sets. Each of these sets causes the system to failure.

2:- we linked the elements of the cat sets to the parallel link, so the success of any component causes the success of the set.

3:- we linked the minimal cat groups with a series connection, because the failure of any group causes the failure of the system.

4:- we had a parallel series system.

5:- we calculated its reliability according to its own law.

2.2.3- Reduction to Series Elements [1, 9,12]

Reduction to series elements is simple and useful method for determining the reliability of systems consisting of independent series and parallel subsystems . The method sequentially reduces the series and parallel configurations to equivalent units until the whole system becomes a single hypothetical unit . The primary advantage of this method is easy to understand and use .

2.3 Some examples for compute the reliability

Example 1

Compute the reliability of the system by using three methods

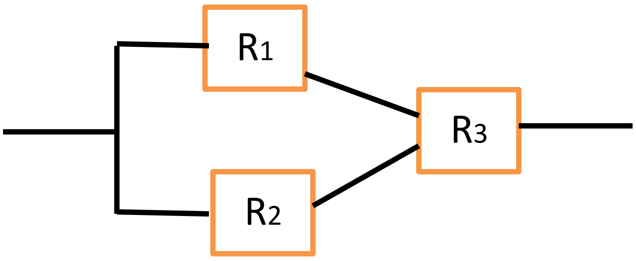


Figure (2-1) complex system

1- Path Tracing Method

Minimal path set [(R_1 R_3) , (R_2 R_3)]

We used the minimal cut method to obtain the structure function of the system illustrated by Fig (2-1). The minimal path sets of the system are

[(R_1 R_3) , (R_2 R_3)]

Thus our system can be replaced by the series-parallel system illustrated by Figure (2-2)

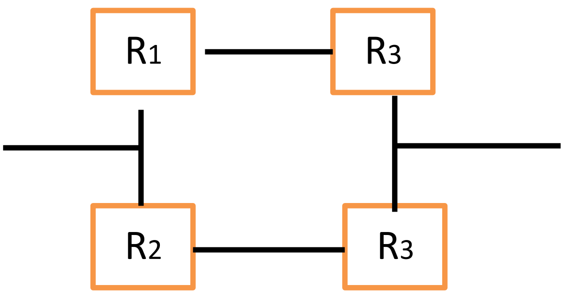


Figure (2-2) Tie-sets system

$\Phi 1 = (x_1, x_3) = x_1 x_3$

$\Phi 2 = (x_2, x_3) = x_2 x_3$

The structure function of the entire system is then

$\Phi : \{0 , 1\}^{3} = \{0 , 1\}$

$\Phi(x_1\ x_2\ x_3) = 1 - [\ (1 - \Phi_1\ (\ x_1\ x_3\ ))\ (1 - \Phi_2\ (\ x_2\ x_3\ ))]$

Note that Ri^2= Ri. Therefore, we obtain the canonical expression of the structure function

$\Phi(x_1\ x_2\ x_3) = x_2\ x_3 + x_1\ x_3 - x_1\ x_2\ x_3$

So we find the reliability R of the system

$R_s = R_1\ R_3 + R_2\ R_3 - R_1\ R_2\ R_3$

2- minimal cut sets method

The minimal cut sets of the system are $[R_1\ R_2]$, $[R_3]$

Thus our system can be replaced by the parallel-series system illustrated

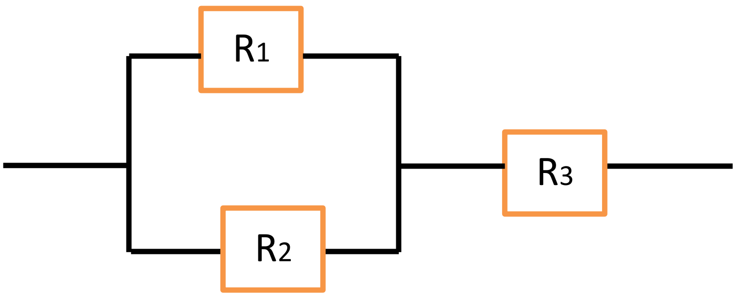


Fig (2-3) minimal cut sets system

Denote $R_i \in [0, 1]$ the state of the arc (component) R_i (i.e. $R_i = 1$ if Ri is functioning, and $R_i = 0$ if Ri is failed), for

i ϵ [1, 2 , 3].

The three parallel subsystems have the following structure functions

$\Phi_1(x_1\ x_2) = [1- (1-x_1)(1-x_2)] = x_1+x_2 - x_1\ x_2$

$\Phi_2(x_3) = [1-(1-x_3)] = x_3$

The structure function of the entire system is then

$\Phi : \{0 , 1\}^3 = \{0 , 1\}$

$\Phi(x_1\ x_2\ x_3) = \Phi_1(x_1\ x_2)\ \Phi_2(x_3)$

Note that Ri^2= R_i. Therefore, we obtain the canonical expression of the structure function

$\Phi(x_1\ x_2\ x_3) = (x_1+x_2 - x_1\ x_2)(x_3)$

$\Phi(x_1\ x_2\ x_3) = x_2\ x_3 + x_1\ x_3 - x_1\ x_2\ x_3$

This leads to system reliability

$R_s = R_2\ R_3 + R_1\ R_3 - R_1\ R_2\ R_3$

3- Reduction to series Elements

The system in Figure (2-1) has been modified to another system

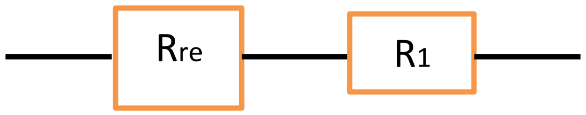


Fig (2-4) reduction system

$R_{re} = 1- (1 - R_1) (1 - R_2) = 1- (1- R_2 – R_1 + R_1 R_2)$

$R_{re} = R_1 + R_2 – R_1 R_2$

**The system has now been reduced to a system contains series-
elements**

now to solve the system
$R_s = [1-(1- R_{re})] [1- (1-R_3)]$

$R_s = R_{re} R_3 = (R_2 + R_1 – R_1 R_2) R_3$

$R_s = R_1 R_3 + R_2 R_3 – R_1 R_2 R_3$

Note that, we got in that three methods, the same results

Example 2

Compute the reliability of the system by using three methods

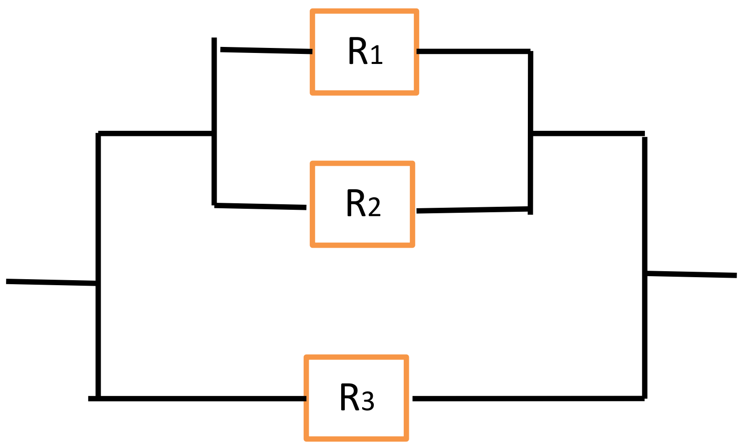


Figure (2-5) complex system

1- Path Tracing Method

Minimal path set [(R_1) ,(R_2) , (R_3)]

We shall use the minimal cut method to obtain the structure function of the system illustrated by Figure (2-5). The minimal path sets of the system are

[(R_1) ,(R_2) , (R_3)]

Thus our system can be replaced by the series-parallel system illustrated by Figure (2-5).

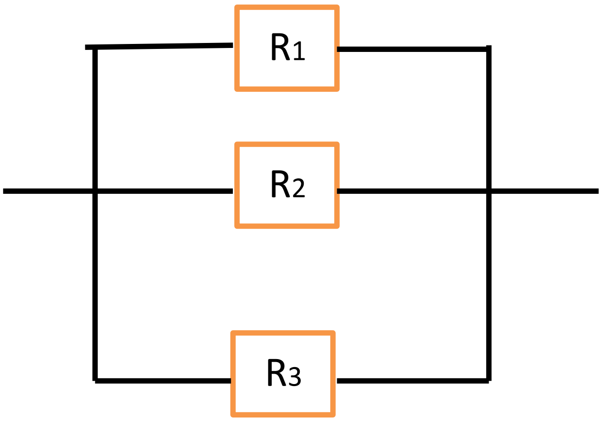


Figure (2-6) Tie-sets system

$\Phi_1 = (x_1) = x_1$
$\Phi_2 = (x_2) = x_2$
$\Phi_3 = (x_3) = x_3$

The structure function of the entire system is then

$\Phi : \{0\,,1\}^{3} = \{0\,,1\}$

$\Phi(x_1\ x_2\ x_3) = 1 - [\ (1 - \Phi_1\ (\ x_1\ ))\ (1 - \Phi_2\ (\ x_2\ ))\ (1 - \Phi_3\ (\ x_3\ ))\ ]$

Note that, $Ri^2 = R_i$. Therefore, we obtain the canonical expression of the structure function

$$\Phi(x_1\ x_2\ x_3) = 1 - [\ (\ 1 - x_1\ )\ (1 - x_2\ )\ (1 - x_3\ )]$$
$$= 1 - [\ (1 - x_1 - x_2 + x_1\ x_2\ )\ (1 - x_3\ )]$$
$$= 1 - [\ 1 - x_3 - x_2 + x_1\ _3 - x_1 + x_1\ x_3 + x_2 x_3 + x_1\ x_2 - x_1 x_{23}]$$
$$= x_1 + x_2 + x_3 - x_1\ x_2 - x_2\ x_3 - x_1\ x_3 + x_1\ x_2\ x_3$$

So we find the reliability R of the system

$R_s = R_1 + R_2 + R_3 - R_1\ R_2 - R_2\ R_3 - R_1\ R_3 + R_1\ R_2\ R_3$

2- minimal cut sets method

The minimal cut sets of the system are [R_1 R_2] , [R_3]

Thus our system can be replaced by the parallel-series system illustrated

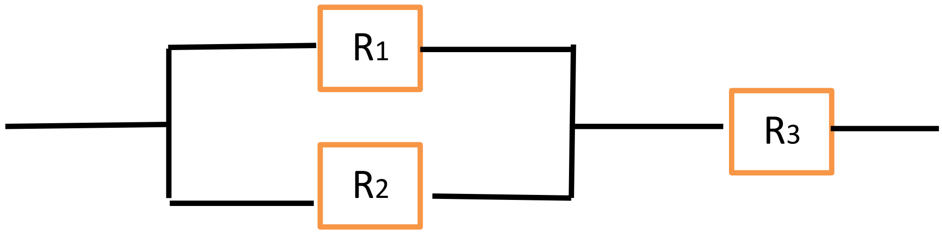


Figure (2-7) minimal cut sets system

Denote R_i ϵ [0, 1] the state of the arc (component) R_i (i.e. R_i = 1 if Ri is functioning, and R_i = 0 if Ri is failed), for i ϵ [1, 2 , 3].
The three parallel subsystems have the following structure functions

$$\Phi_1(x_1\, x_2\, x_3) = [1-(1-x_1)(1-x_2)\,(1-x_3)]$$
$$= 1-(1 - x_1 - x_2 - x_3 + x_1x_2 + x_2\, x_3 + x_1\, x_3 - x_1\, x_2\, x_3)$$
$$= x_1 + x_2 + x_3 - x_1\, x_2 - x_2\, x_3 - x_1\, x_3 + x_1\, x_2\, x_3$$

The structure function of the entire system is then

$$\Phi : \{0 , 1\}^{3} = \{0 , 1\}$$

$\Phi(x_1\ x_2\ x_3) = x_1 + x_2 + x_3 - x_1 x_2 - x_2 x_3 - x_1 x_3 + x_1 x_2 x_3$

Note that, Ri^2= Ri. Therefore, we obtain the canonical expression of the structure function

$\Phi(x_1\ x_2\ x_3) = x_1 + x_2 + x_3 - x_1 x_2 - x_2 x_3 - x_1 x_3 + x_1 x_2 x_3$

This leads to system reliability

$R_s = R_1 + R_2 + R_3 - R_1 R_2 - R_2 R_3 - R_1 R_3 + R_1 R_2 R_3$

3- Reduction to series Elements

The system in Figure (2-5) has been modified to another system

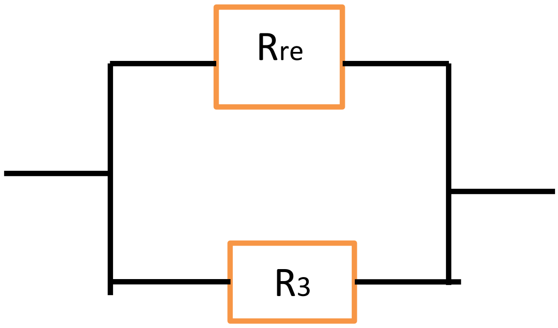


figure (2-8) reduced system

$R_{re} = 1- (1 - R_1) (1 - R_2) = 1- (1- R_2 – R_1 + R1\ R_2)$

$R_{re} = R_1 + R_2 – R_1 R_2$

The system has now been reduced to a system contains series- elements

now to solve the system
$R_s = [1-(1- R_{re}) (1-R_3)]$

$R_s = 1- (1- R_3 – R_{re} + R_{re} R_3)= R_3 + R_{re} – R_{re}R_3$

$R_s = R_3 + R_1 + R_2 – R_1 R_2 - R_1 R_3 - R_2 R_3 + R_1 R_2 R_3$

Note that, we got in that three methods, the same results

Example 3

Compute the reliability of the system by using three method

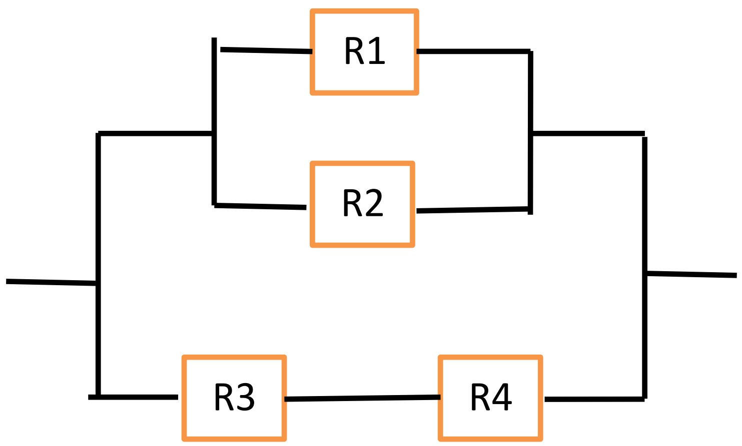


figure (2-9) complex system

1- Path Tracing Method

Minimal path sets [(R1) ,(R2) , (R3 R4)]

We used the minimal cut method to obtain the structure function of the system. The minimal path sets of the system are

[(R1) ,(R2) , (R3 R4)]

Thus, our system can be replaced by the series-parallel system

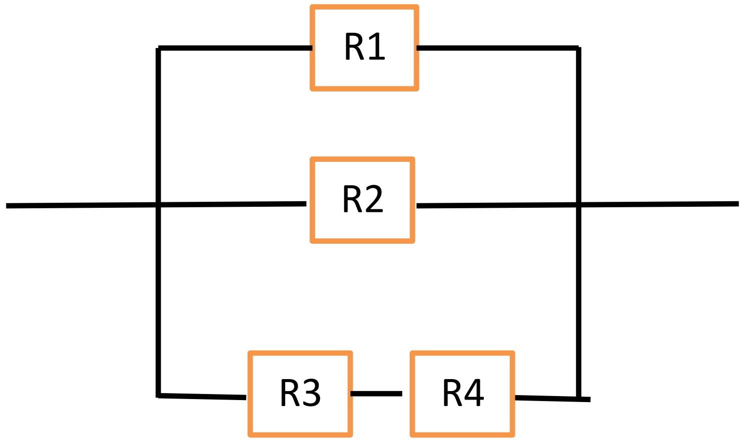


Figure (2-10) Tie-sets system

Φ1= (x1) = x1
Φ2= (x2) = x2
Φ3= (x3,x4) = x3 x4
The structure function of the entire system is then
Φ : $\{0\ ,1\}^{\ 4} = \{0\ ,1\}$
Φ(x1 x2 x3 x4) = 1- [(1- Φ1 (x1)) (1- Φ2 (x2)) (1- Φ3 (x3 x4))]

Note that, Ri^2**= Ri. Therefore, we obtain the canonical expression of the structure function**

Φ(x1 x2 x3 x4) = 1 - [(1- x1) (1- x2) (1- x3 x4)]
= 1-[1- x2 – x1 + x1 x2 – x3 x4 + x2 x3 x4 + x1x3x4 – x1x2x3x4]
So we find the reliability R of the system

Rs = R2 + R1 - R1 R2 + R3 R4 - R2 R3 R4 - R1R3R4 + R1R2R3R4

2- minimal cut sets method

The minimal cut sets of the system are [R1 R2 R3] , [R1 R2 R4]

Thus our system can be replaced by the parallel-series system illustrated

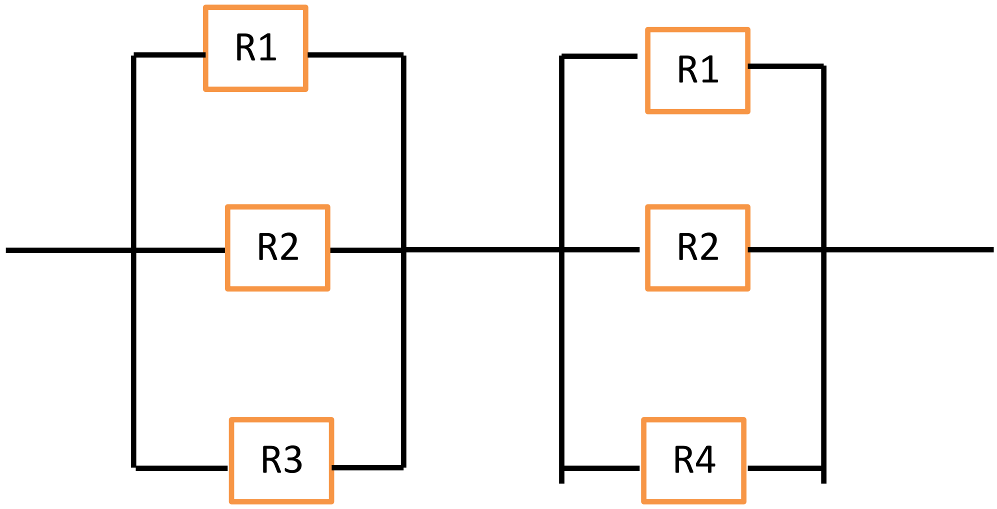


Figure (2-12) minimal cut sets system

Denote Ri ϵ [0, 1] the state of the arc (component) Ri
(i.e. Ri = 1 if Ri is functioning, and Ri = 0 if Ri is failed), for i ϵ [1, 2 , 3 , 4].
The three parallel subsystems have the following structure functions
Φ1(x1 x2 x3)= [1- (1-x1)(1-x2)(1- 3)]
Φ2(x1 x2 x4) = [1-(1-x3) (1-x2)(1- x4)]

The structure function of the entire system is then
Φ $: \{0\,,1\}^{4} = \{0\,,1\}$
Φ(x1 x2 x3 x4) = Φ1(x1 x2x3) Φ2(x1 x2 x4)

Note that Ri^2**= Ri. Therefore, we obtain the canonical expression of the structure function**

$\Phi(x1\ x2\ x3\ x4) = [1-(1-x1)(1-x2)(1-x3)]\ [1-(1-x3)(1-x2)(1-x4)]$
$\Phi(x1\ x2\ x3) = [x1 + x2\ x4 + x1 - x1\ x4 - x1\ x3 + x1\ x2\ x4]$

$[x3 + x2 - x2\ x3 + x1 - x1x3 - x1x2 + x1\ x2\ x4]$

This leads to system reliability

$Rs = R2 + R1 - R1\ R2 + R3\ R4 - R2\ R3\ R4 - R1R3R4 + R1R2R3R4$

3- Reduction to series Elements

The system in figure (2-9) has been modified to another system

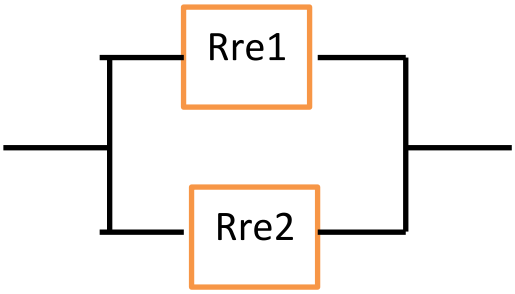


Fig (2-13) reduced system

Rre1 = 1- (1 - R1) (1 - R2) = 1- (1- R2 – R1 + R1 R2)

Rre1 = R1 + R2 – R1 R2

Rre2 = R3 R4

The system has now been reduced to a system contains series Element

now to solve the system

Rs = [1-(1- Rre1)] [1- (1-Rre2)]

Rs = Rre1 Rre2 = (R2 + R1 – R1 R2)(R3 R4)

Rs = R2 + R1 - R1 R2 + R3 R4 - R2 R3 R4 - R1R3R4 + R1R2R3R4

Note that, we got in that three methods, the same results

.

Example 4 [12]

Compute the reliability of the system by using three method

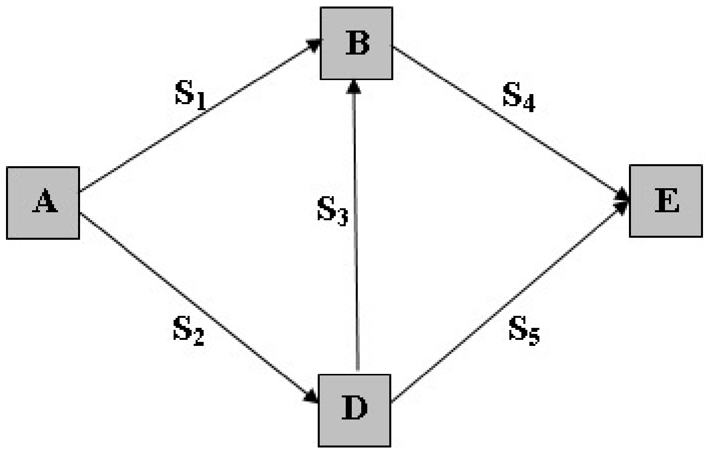


fig (2-14) complex system

1- Minimal path sets

Minimal path set = [(S1, S4), (S2, S5) , (S2, S3, S4)]
We shall use the minimal cut method to obtain the structure function of the system. The minimal path sets of the system are
[S1, S4] , [S2, S5] , [S2, S3, S4]

Thus our system can be replaced by the series-parallel system illustrated by

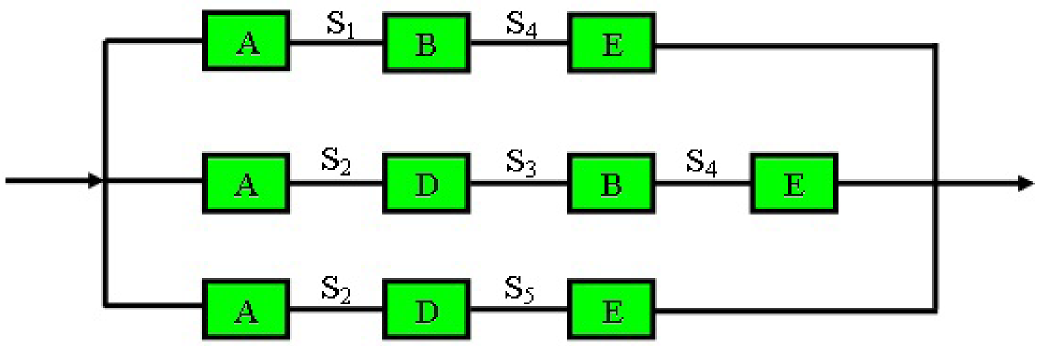


Fig. (2-15) series-parallel system

Denote xi є [0,1] the state of the arc (component) Si (i.e. xi = 1 if Si is functioning, and xi = 0 if Si is failed), for i 2 [1, _ _ _, 5]. The three parallel subsystems have the following structure functions
Φ1 (x1, x4) = x1x4
Φ2 (x2, x3, x4) = x2 x3 x4
Φ3 (x2 x5) = x2 x5
The structure function of the entire system is then
Φ: $[0;\ 1]^{5}$= [0, 1],
Φ(x1; x2; x3; x4; x5) = 1-[(1-[1-Φ1 (x1, x4))(1-Φ2 (x2, x5))(1-Φ3 (x2, x3,x4))]:

Note that xi^2= xi. Therefore, we obtain the canonical expression of the structure function
Φ (x1, _ _ _ , x5) = x1x4 +x2x5 +x2x3x4- x1x2x3x4- x1x2x4x5 - x2x3x4x5+x1x2x3x4x5:

So we and the reliability R of the system given by (20) as
R = Φ (R1, _ _ _ ,R5).
R(t) = R1R4 + R2R5 + R2R3R4 - R1R2R3R4 - R1R2R4R5 - R2R3R4R5 + R1R2R3R4R5

2- Minimal Cut Method

We used the minimal cut method to obtain the structure function of the system see Minimal cut The minimal cut sets of the system are

[S1, S2] , [S4, S5] , [S2, S4] , [S1,S3, S5]

Thus ,our system can be replaced by the parallel-series system

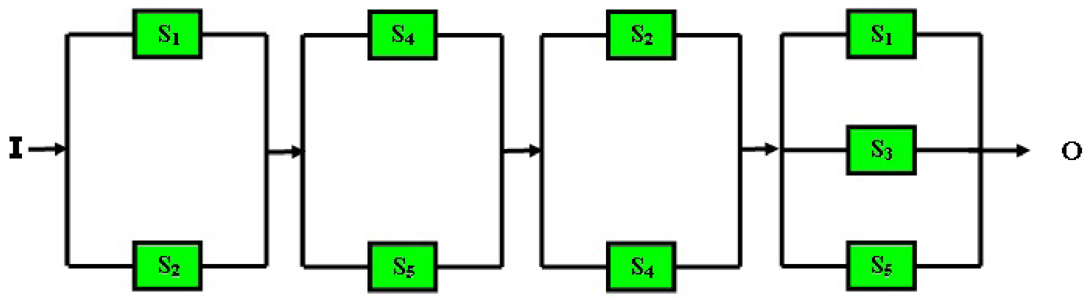


Fig (2-16) parallel-series system

Denote $x_i \in [0,1]$the state of the arc (component) S_i (i.e. $x_i = 1$ if S_i is functioning, and $x_i = 0$ if S_i is failed), for $i \in [1; \_\_\_ ; 5]$.

The four parallel subsystems have the following structure functions

$\Phi 1 (x1; x2) = 1 - (1 - x1) (1 - x2) = x1 + x2 - x1x2$

$\Phi 2 (x4; x5) = 1 - (1 - x4) (1 - x5) = x4 + x5 - x4x5$

$\Phi 3 (x2; x4) = 1 - (1 - x2) (1 - x4) = x2 + x4 - x2x4$

$\Phi 4 (x1; x3; x5) = 1-(1 - x1) (1 - x3) (1 - x5) = x1+x3+x5 - x1x3 - x3x5 - x5x1 + x1x3x5$

The structure function of the entire system is then Φ : $[0; 1]^5 = [0, 1]$

$\Phi (x1; x2; x3; x4; x5) = \Phi 1 (x1; x2) \Phi 2 (x4; x5) \Phi 3 (x2; x4) \Phi 4 (x1; x3; x5)$:

Note that, x^2i = xi. Therefore, we obtain the canonical expression of the structure function

$\Phi(x_1, \_\_\_, x_5) = x_1x_4 + x_2x_5 + x_2x_3x_4 - x_1x_2x_3x_4 - x_1x_2x_4x_5 - x_2x_3x_4x_5 + x_1x_2x_3x_4x_5$:

So the reliability R of the system given

$R = \Phi(R_1, \_\_\_, R_5)$

$R(t) = R_1R_4 + R_2R_5 + R_2R_3R_4 - R_1R_2R_3R_4 - R_1R_2R_4R_5 - R_2R_3R_4R_5 + R_1R_2R_3R_4R_5$

Note that, we got in that three methods, the same results

Example 5

Compute the reliability of the system by using three method

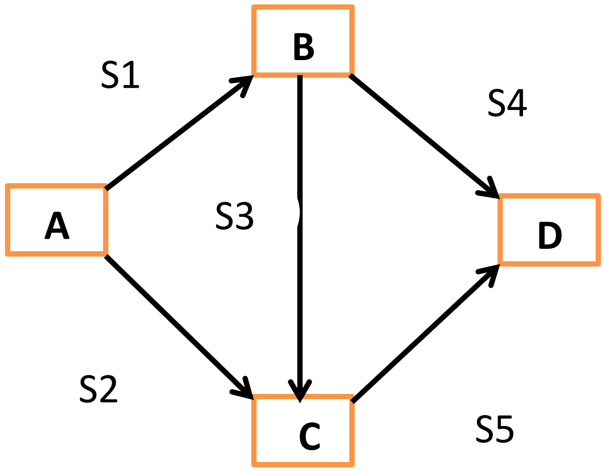


fig (2-17) complex system

1- Minimal path sets

Minimal path set = [(S_1, S_4), (S_2, S_5) , (S_1, S_3, S_5)]
We used the minimal cut method to obtain the structure function of the system. The minimal path sets of the system are [S_1, S_4] , [S_2, S_5] , [S_1, S_3, S_5]

Thus , our system can be replaced by the series-parallel system
Denote xi ϵ [0,1] the state of the arc (component) Si (i.e. xi = 1 if Si is functioning, and xi = 0 if Si is failed), for i 2 [1, _ _ _, 5].
The three parallel subsystems have the following structure functions
Φ1 (x1, x4) = x1x4
Φ2 (x1, x3, x5) = x2 x3 x5
Φ3 (x2 x5) = x2 x5
The structure function of the entire system is then
Φ: $[0;\ 1]^{5}$**= [0, 1],**

$\Phi(x1; x2; x3; x4; x5) = 1-[(1-[1-\Phi1 (x1, x4))(1-\Phi2 (x2, x5))(1-\Phi3 (x1, x3,x5))]:$

Note that, $xi^2 = xi$. Therefore, we obtain the canonical expression of the structure function

Φ (x1, _ _ _ , x5) = 1- [(1- x1x4)(1-x2x5)(1-x1x3x5)]
= 1- [(1- x2x5 – x1x4 + x1x4x2x5)(1-x1x3x5)]
= 1-[1- x1x3x5 – x2x5 – x1x4 + x1x4x2x5 + x1x4x3x5 + x1x3x4x5 – x1x2x3x4x5]= x1x3x5 + x2x5 +x1x4 - x1x4x2x5 - x1x4x3x5 - x1x3x4x5 + x1x2x3x4x5

So the reliability R of the system given by
R = Φ (R1, _ _ _ ,R5).
R(t) = R1R3R5 + R2R5 +R1R4 - R1R4R2R5 - R1R4R3R5 - R1R3R4R5 + R1R2R3R4R5

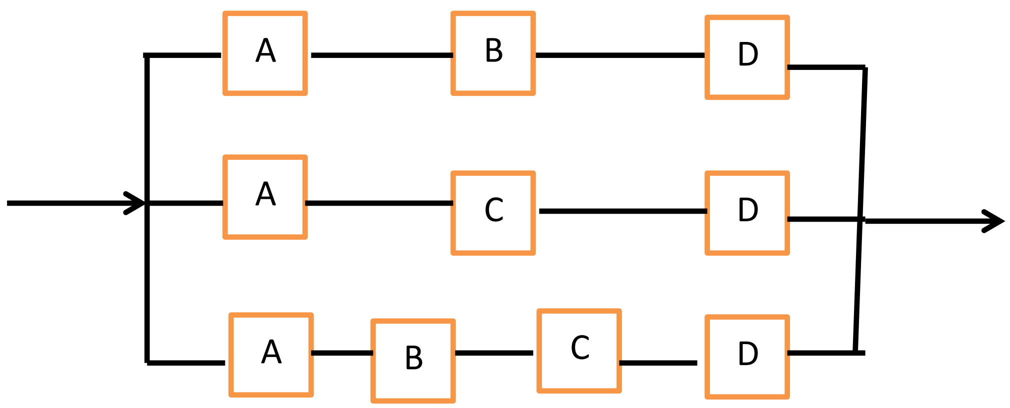


Fig. (2-18) Minimal path sets

2- Minimal Cut sets Method

We used the minimal cut method to obtain the structure function of the system . see Minimal cut The minimal cut sets of the system are

[S1, S2] , [S4, S5] , [S1, S5] , [S2,S3, S4]

Thus, our system can be replaced by the parallel-series system.

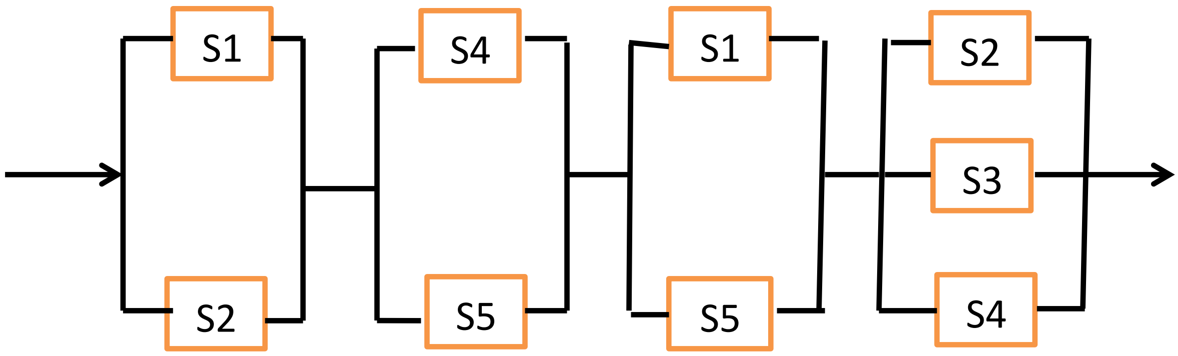


fig (2-19) Minimal Cut sets

Denote xi ϵ [0,1]the state of the arc (component) Si (i.e. xi = 1 if Si is functioning, and xi = 0 if Si is failed), for i ϵ [1; _ _ _ ; 5].
The four parallel subsystems have the following structure functions

Φ1 (x1; x2) = 1 - (1 - x1) (1 - x2) = x1 + x2 - x1x2
Φ2 (x4; x5) = 1 - (1 - x4) (1 - x5) = x4 + x5 - x4x5
Φ3 (x1; x5) = 1 - (1 – x1) (1 – x5) = x1 + x5 – x1x5
Φ4 (x2; x3; x5) = 1-(1 – x2) (1 - x3) (1 - x5) = x2+x3+x5 - x2x3 - x3x5 - x5x2 + x2x3x5

The structure function of the entire system is then

Φ : $[0;\ 1]^5$ **= [0, 1]**

Φ (x1; x2; x3; x4; x5) = Φ1 (x1; x2) Φ2 (x4; x5) Φ3 (x1; x5) Φ4 (x2; x3; x5) :

Note that x^2i = xi. Therefore, we obtain the canonical expression of the structure function
Φ (x1, _ _ _ , x5) = x1x3x5 + x2x5 +x1x4 - x1x4x2x5 - x1x4x3x5 - x1x3x4x5 + x1x2x3x4x5
So we and the reliability R of the system given

R = Φ (R1, _ _ _ ,R5)

R(t) = R1R3R5 + R2R5 +R1R4 - R1R4R2R5 - R1R4R3R5 - R1R3R4R5 + R1R2R3R4R5

Note that, we got in that three methods, the same results

Conclusio

In this work , we have dealt with some methods of solving complex system , and we found that when solving the system in any that method , the same output will be appeared .

References

1) Advisory Group on Reliability of Electronic Equipment (AGREE), "Reliability of Military Electronic Equipment, "Office of the Assistant Secretary of Defense, 1957.
2) Al-Ali A.A. and Hassan Z.A., "Reliability methods for solving complex system", Journal of university Babylon, (to appear), 2005.
3) Al-Ali A.A., "Reliability of complex system", Basra J. science, vol.16, No.1, pp.115-122. Basra, 1998.
4) Al-Ali A.A., "Two unit reliability systems subject to effective random shocks", journal of Indian society of statistics and operation research, Vol. IX, NO. 1-4, PP. 1-8, 1988, India.
5) Al-Ali A.A., and Murari K. "One unit reliability systems subject to random shocks and preventive maintenance", microelectronic and reliability, Vol. 28, NO. 31, PP. 373-377, 1988, England.
6) Al-Ali A.A., and Subri U.A., "Evaluation of reliability and mean time to failure of power plant", Journal of Babylon university, Vol. 7, NO. 3, 2002, Iraq.
7) aan informa business / © 2006 by Taylor & Francis Group, LLC
8) Blanchard, B.S. and W.J. Fabricky, System Engineering Analysis, Prentice-Hall, 1990.
9) Dummer, G.W., and N.B. Griffin, Electronic Reliability: Calculation and Design. Elmsford,NY: Pergamon, 1966.
10) Fuqua, N., Reliability Engineering for Electronic Design. New York, NY: Marcel Dekker,1987.
11) Furqa N.B. "The applicability of Markov analysis methods to reliability, maintainability and safety", Vol.10, No.2, 2003.
12) Reliability of Complex networks/By Prof. Dr. Zahir Abdul Haddi Hassan/This course i s dedicated to Postgraduate students 2022

الخلاصة

في هذه الأطروحة قمنا بحساب موثوقية الأنظمة البسيطة، والتي تناولت حساب الأنظمة المتسلسلة، والأنظمة المتوازية، والأنظمة المتوازية متسلسلة ، والأنظمة المتسلسلة المتوازية. قمنا أيضًا بحساب الأنظمة المختلطة. أما بالنسبة للأنظمة المعقدة فقد قمنا بحسابها بثلاث طرق مختلفة (طريقة تتبع المسار، طريقة القطع الأدنى، والاختزال إلى عناصر السلسلة) وتبين أنها نفس الموثوقية مع هذه الطرق الثلاثة من حيث الدقة.

جمهورية العراق

وزارة التعليم العالي والبحث العلمي

جامعة بابل

كلية التربية للعلوم الصرفة

قسم الرياضيات

دراسة بعض الطرق لحل موثوقية الأنظمة المعقدة

بحث مقدم الى

مجلس كلية التربية للعلوم الصرفة – جامعة بابل

كجزء من متطلبات نيل شهادة الدبلوم العالي تربية / الرياضيات

من قبل

زيتون محسن عبدالله

بأشراف

د. غازي عبدالله مدلول

2024 م **1444ه**