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CERTAIN MATHEMATICAL MODELS FOR RELIABILITY ALLOCATION

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بسم الله الرحمن الرحيم

((وما اوتيتم من العلم الا قليلاً))

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Dedication

Thanks go to Allah and at all the whole grace prayer, and peace upon the best creation Mohammad (peace be upon him and his family) and his divine good…

To my father who has given me love and care, and taught me how to face life.

To the spirit of my mother and brother martyr (Falah) are the source of light in my life.

To my wife and children, the source of love and affection.

To my teachers the lighted candles who are showing us the road of life.

To my close friends

To My country with honour and dignityand Everyone who helped me.

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R_s	**Reliability of the system**
R_i	**Reliability of the component i**
$R_s(0_i, R_i)$	**Reliability of the system when R_i=0**
$R_s(1_i, R_i)$	**Reliability of the system when R_i=1**
MT T F	**Mean time to failure**
R_{sub}	**Reliability of subsystem**
λ_s	**System failure rate.**
m_i	**The number of modules in subsystem i.**
w_i	**The importance of subsystem i.**
t_i	**The operating time of subsystem i.**
m	**The total number of modules in the system=** $\sum_{i=1}^{n} m_i$
T	**Mission duration.**
c_k	**Complexity of subsystem (k)**

λ_k^*	**Failure rate allocated to each subsystem.**
W	**Sum of the rated product's**
W_k	**Rating for subsystem (k).**
ARINC	**Aeronautical research Inc**
AGREE	**Advisory Group of Reliability of Electronic Equipment**

Abstract

This thesis aims to using some applications to improve the reliability of the systems that we adopted in this thesis. These are series, parallel and complex systems. The complex system is focused of attention in this thesis. Then, two methods were used to calculate the reliability function for the complex system. Applying two techniques (redundancy and allocation) to increase value the reliability of the three systems (series, parallel and complex). The redundancy technique was being in two methods, namely redundancy of elements and redundancy of the system, the two methods are applied to the three systems. Then ,applying the allocation technique to increase the reliability value of the three systems. Study the reliability importance of in simple and complex systems in multiple and different situations. then a new measure was be studied to evaluate the reliability importance of the minimal cuts for more than one component.

Introduction

The reliability allocation process, an important element of reliability engineering, helps system, designers select components, and apply appropriate design strategies to meet system reliability requirements. The allocation process involves generating a reliability block diagram and specifying system reliability requirements **[1].** Reliability requirements are then assigned to subsystems and related components. The allocation process is dynamic and is typically considered throughout the design phases (Rau, 1984). At the conceptual phase when little or nothing is known about the system, simple allocation methods such as equal allocation, can be applied to assign equal reliability to subsystems. As the system design develops and more information about components and the operating environment becomes available, different allocation methods and reliability improvement techniques may also be considered. The allocation of system reliability involves solving the following inequality:

$$f(R_1\ (t),R_2(t),...,R_n\ (t)) \geq R_s\ (t)$$

where f is a function that relates components reliabilities to system reliability, R_1 is the reliability at time t of the i^{th} component, $R_s(t)$ is reliability goal at time t.

In this work, we discussed some basic definitions and concepts related to reliability, as well as some systems related to reliability. We also discussed in some methods of allocating reliability **[2]**.

Many researchers focus their work on the study of various reliability systems such as series, parallel, series - parallel, parallel - series and complex systems that enter all areas of life.

CHAPTER ONE

MATHEMATICAL CONCEPTS AND BASIC DEFINITIONS

1.1 Introduction

In this chapter, relevant definitions, basic concepts and the background required for this study are presented. Two sections in this chapter, the first section contains some of the basic definitions and graph theory that we need at our study. The second section provides the basic definition of the reliability function and reliability network and some important definitions that we need in this work.

1.2 Basic Definitions and Concepts

In this section, several definitions, Probability, System, Reliability Mean Time to Failure (MTTF)[, etc.[31, 36, 42], and also Relations between F(t) , f(t) , R(t) and h(t) have all been discussed.

Definition (1.2.1) Probability p(t) [31]

Probability is a numerical measure of the likelihood of an event in comparison to a set of possible outcomes. When flipping a coin, for example, there's a 50% chance of seeing heads vs tails (assuming a fair or unbiased coin) Probability Properties P(B) is the probability of an occurrence B, and it has the following properties:

1) $0 \leq P(B) \leq 1$

2) $P(B) = 1 - P(B^c)$

3) $P(\emptyset) = 0$

4) $P(S) = 1$

In other words, when an event is certain to occur it has a probability equal to(1) and when it is impossible to occur it has a probability equal to (0) it can be shown that the probability of the union of two events A and B is

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Similarly the probability of the union of three events A , B and C is given by :

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Definition (1.2.2) System [16,32]

A collection of compounds in a specific order that communicate in order to perform with each other and with external components or other structures given purpose.

Definition (1.2.3) Reliability [22,30]

The probability that for a certain period of time the system will survive is established This can be expressed as the time to system failure in terms of random variable T. The probability of survival or reliability R(t) at time t, has the following properties:

1) $0 \le R(t) \le 1$

2) $R(\infty) = 0$, no device can work forever

3) R(0) =1 the device is assumed to be working properly at time t=0

4) $R(t)$ in general is a decreasing function of time t.

$$R(t) = p\{T > t\} \tag{1.1}$$

$$R(t)=1-\int_0^t f(t)dt \tag{1.2}$$

Or

$$R(t)=\int_t^{\infty} f(t)dt \quad (1.3)$$

Definition (1.2.4) Mean Time to Failure (MTTF)[27]

This is a simple metric of reliability for non-repairable systems. Under specified conditions, the average time between failures during a measurement cycle.

$$\text{MTTF}=\int_0^{t} tf(t)dt \quad (1.4)$$

$$\text{MTTF}=\int_0^{\infty} R(t)dt \quad (1.5)$$

Definition (1.2.5) Hazard Rete h(t)[28]

It is the rate of immediate failure and the maximum rate of failure, both indicated by the symbol h(t), where the rate of change is the chance that the remaining product will fail in the next several time periods the equation can be written as follows:

$$h(t) = lim_{\Delta t\to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t R(t)} \quad (1.6)$$

The Hazard rate unit is the failure cases per unit time t

Definitions (1.2.6) Probability Density Function(p. d .f)[15]

If x is a continuous random variable , then the probability density function ,p.d.f, of X is a function , $\mathcal{F}(\chi)$, such that for two numbers , a and b with a ≤b

P(a$\leq x \leq b) = \int_a^b f(x)dx \; and \; f(x) \geq 0, for \; all \; x$ (1.7)

Definitions (1.2.7) Cumulative Distribution Function (c.d.f)[24]

The cumulative distribution function (c.d.f) is a random variable x function F(x) that is defined for a number x by:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(s)ds \tag{1.8}$$

$$F(x) = d(F(x))/dx$$

The value of c.d.f at x is the area under the probability density function up to x if so chosen. It should also be pointed out that the total area under the p.d.f is always equal to or mathematically.

Definitions(1.2.9) Conditional Probability[17]

The probability of one of two events A and B happening knowing that the other event has already occurred is known as conditional probability. Provided that B has already occurred, the expression below denotes the likelihood of an occurring.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \tag{1.9}$$

Definitions (1.2.10) Conditional Reliability Function[19]

Conditional reliability is defined as the probability that a component or system will operate without failure for a mission time t given that it has already survived to a given time t mathematically this is expressed as [0,1].

$$R(t / T) = R(t+T))/(R(t)) \tag{1.10}$$

Definitions (1.2.11) Reliability Allocation[18]

The process of assigning reliability requlrements to individual reliability is called reliability allocation.

1.3 Relations between F(t) , f(t) , R(t) and h(t)

All of the functions F(t), f(t), R(t), and h(t) can be transformed into each other consider the example below:

$$F(t) = \int_{0}^{t} f(x)dx \tag{1.11}$$

This leads

$$\text{F(t)}= \frac{df(t)}{dt} = -\frac{dR(t)}{dt} \tag{1.12}$$

And R(t)= 1-F(t)

According to equation(1.12) , h(t)=$\frac{f(t)}{R(t)}$

Compensate equation(1.12) in equation (1.11) to get

$$\text{h(t)}=-\frac{dR(t)}{dt} \times \frac{1}{R(t)} \tag{1.13}$$

and from them we get:

$$\frac{dR(t)}{d(t)} = -h(t)dt \tag{1.14}$$

Integrity of the parties ,

$$\text{LnR(t)}=-\int_0^t h(x)dx \tag{1.15}$$

And taking the exponential of both sides equation we get :

$$\text{R(t)}= e^{-\int_0^t h(x)dx} \tag{1.16}$$

Also we can obtian R(t) from its equation (1.16)

$$R(t) = \frac{f(t)}{h(t)}$$

Then :-

$$f(t) = R(t)\, h(t)$$

And

$$f(t) = h(t)\, e^{-\int_0^t h(x)dx}$$

CHAPTER TWO

SYSTEM RELIABILITY

2.1 Introduction

The system may varies from simple to complex. The system can be analyzed by decomposing it into smaller subsystems and estimating reliability of each subsystem to assess the total system reliability. There are several types of configurations (systems) available, such as series, parallel, series-parallel, parallel-series, and Complex Reliability Systems. We mentioned some of these systems for their importance in the study among reliability analysts.

2.2 Simple Reliability Systems

Assume that we need to ascertain the Reliability of a framework made up of a few parts. The complete dependability can be determined by ascertaining the unwavering quality of every individual unit, and consolidating these individual reliabilities. The ways by which the framework components can be consolidated are the accompanying: [14]

2.2 .1. Series System

In a series system, the failure of any component causes the entire system to fail. Also, the success of all components leads to a successful system [7,29]. structural function is determined by**:**

$$\boldsymbol{\theta}(\boldsymbol{x}) = min\{x_1, \dots, x_n\} = \prod_{i=1}^{n} x_i \qquad (2.1)$$

In other word if R_1, R_2, , R_n are the reliabilities of the individual components , then the reliability of the system is given by :

$$R_s = R_1 . R_2 \dots\dots R_n = \prod_{i=1}^{n} Ri \qquad (2.2)$$

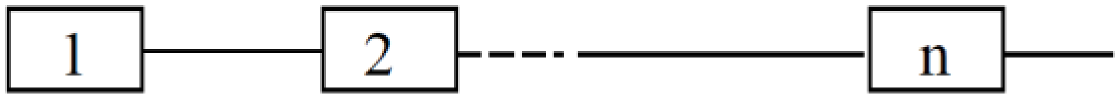

Figure 2.1: Series system.

2.2 .2. Parallel System

In a parallel system, at least one of the components must succeed for the system to succeed. The probability of failure, or unreliability, for a system with n statistically independent parallel components is the probability that component 1 fails and component 2 fails and all of the other components in the system are failed. Putting another way, if component 1 succeeds or component 2 succeeds or any of the n components succeeds [8, 33], the system succeeds. A system's structural function is determined by :

$$\boldsymbol{\theta}(\boldsymbol{x}) = max\{x_1, \ldots\ldots x_n\} = 1 - \prod_{i=1}^{n}(1 - x_i) \tag{2.3}$$

Thus, the reliability of the system (again assuming independent failures) is :

$R_{system} = 1 - p\ (all\ fail) = 1 - [p(A_1\ fail) \times p\ (A_2\ fail) \times \ldots. \times p\ (An\ fail)]$

$= 1 - (1 - R_1)(1 - R_2)\ldots(1 - R_n)$

$$R_s = 1 - \prod_{i=1}^{n}(1 - \boldsymbol{R_i}) \tag{2.4}$$

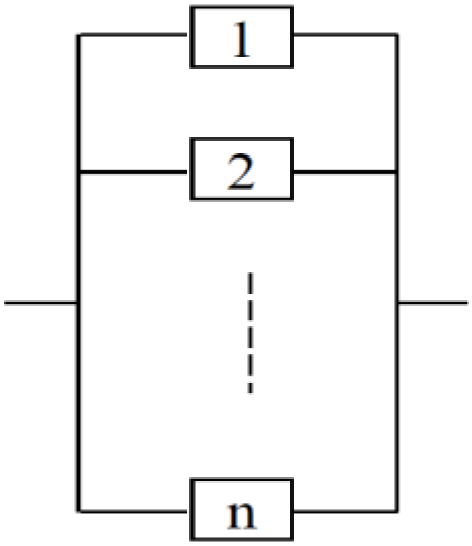

Figure 2.2: Parallel system.

The part with the greatest quality has the greatest impact on the efficiency of the system in a parallel system, since the most secure component is the one most probable to last fail. The reliability of the system improves for a parallel setup as the amount of parts rises.

2.2 .3. Series-Parallel System

The reliability of this system that consist m series redundancy connected components,is (2.4)

where R is the reliability of an individual component[1, 20]. If we place in sets in parallel ,where each has m component in series. Thus:

$$R_s = \prod_{i=1}^{m} \langle 1 - \prod_{i=1}^{n} (1 - R) \rangle \tag{2.5}$$

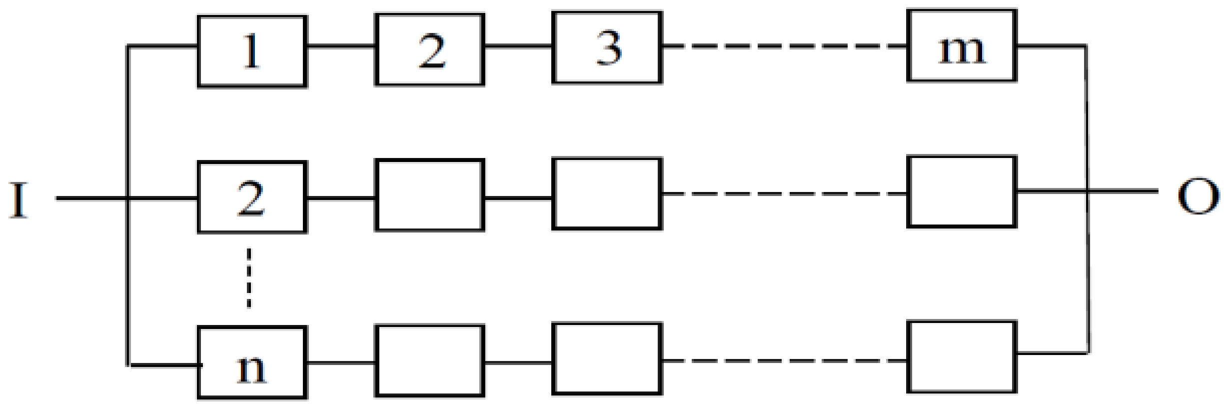

Figure 2.3: The Series - parallel system

2.2 .4. Parallel-Series System

The reliability of the parallel linked n element is determined by :

$$R_s = 1 - (1 - R)^n \tag{2.6}$$

Where R is a single component's reliability[13, 23]. If m such sets are connected in series, where each set consists of n parallel components, then the system's reliability is determined :

$$R_s = 1 - \prod_{i=1}^{m} (1 - \prod_{i=1}^{n} Ri). \tag{2.7}$$

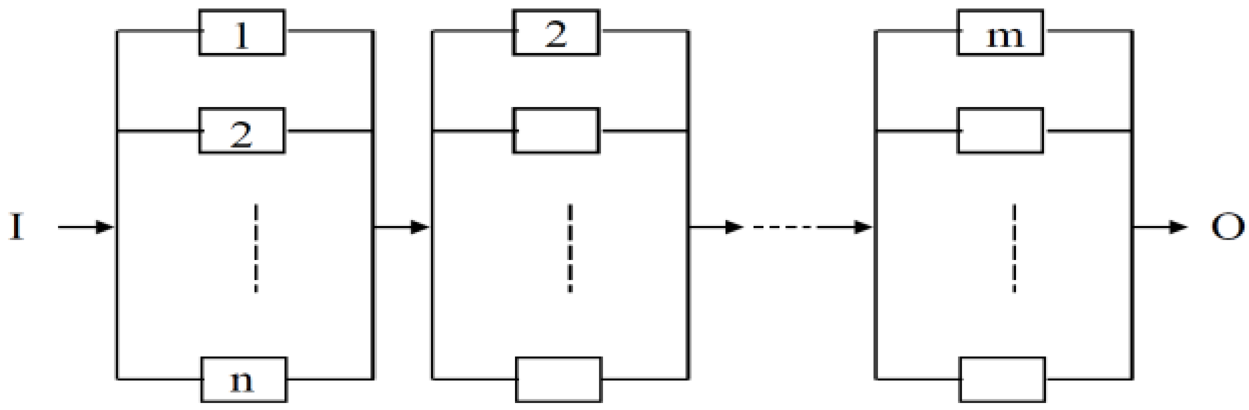

Figure 2.4: The Parallel series system

2.3 Complex Reliability Systems

A system that can not be categorized straight into the previous basic structure instances is called a complex structure system[9, 35]. The ideas of minimum paths and cuts (in the event of monotonous structure) must be implemented for the research of structures with complicated structure. The system in fig.1.18, below is a complicated structure scheme. This system can not be categorized straight into modules that have traditional structures. If this system's input / output is situated at component 3 extremities, it will then involve a system with elementary structure. There are several methods exist for obtaining the reliability of a complex system :

1. Path Tracing Method(PTM).
2. Minimal Cut Method(MCM).
3. Reduction to Series Element Method.
4. The Delta-Star technique for simplified equivalent reliability polynomial.

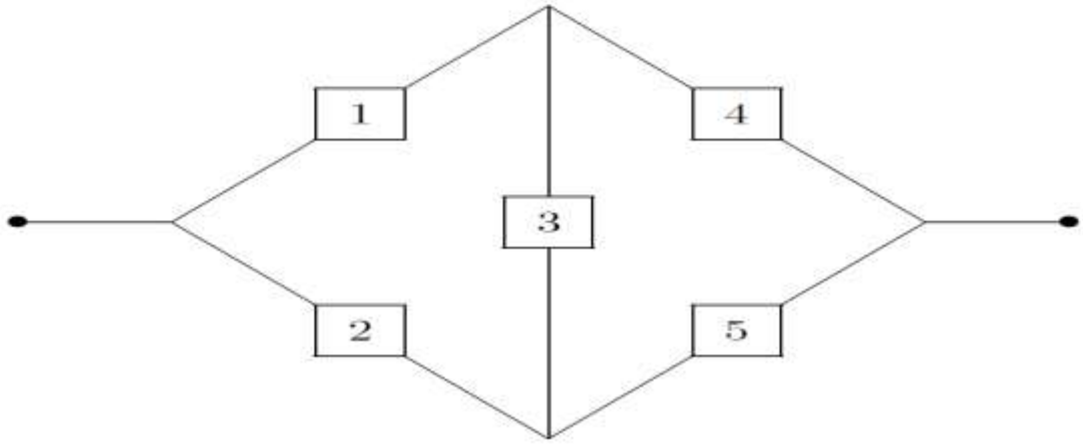

Figure 2.5: Example of complex reliability system

CHAPTER THREE

RELIABILITY ALLOCATION METHODS

&Techniques to Improve Reliability of Systems

3.1Introduction

Many researches worked to compute the reliability allocation in series, and complex systems. The reliability factor of a system is known or specified on the basis of the overall mission requirements. If the system comprises many elements and units, we must have method to determine the reliability factor for each of them. So that when they are assembled to form a system. The system reliability will have the desired value. The purpose of reliability allocation is to establish a goal or objective for the reliability of each component so that the manufactures can have an idea of the performance required of this product. This chapter consists of three section where the third section deals with some methods of reliability allocating.

3.2 Reliability allocation

The process by which the failure of a system is determined by using a logical method through the systems of the sub-system and its components. We define the system reliability objective of the individual components within the system that ensure access to the overall goal of system reliability. For each formatting we use the component to refer to a typical unit or sub- system, which can be formulated in the allocation of reliability.

$$h(R_1^* , R_2^* \,\ldots\ldots\ldots R_n^*) \geq R^*. \tag{3.1}$$

where R^* is the system reliability goal and R_1^* , R_2^*, ……, R_n^* is the components reliability, and h has a relationship between system reliability and component reliability and obtained from the reliability analysis of the system to be allocated.

The allocation of reliability is important in the overall reliability program especially when components or vehicles are under development and complex.

Among the advantages of reliability allocation it is:

1) The complex product contains a number of components planned, designed and tested for manufacture by external suppliers. It is important that all partners involved in the objective of the product reach customers with the required reliability.

2) The parties are responsible for improving their reliability through the use of reliability techniques, better engineering designs, high quality manufacturing processes and rigorous and accurate testing methods.

3) The allocation reliability aims at a board and clear understanding of the hierarchical product structure which is the relationship between components in terms of function, subsystem, the final product. This leads to week design knowledge and subsequent improvement.

4) The requirements for reliability depend on the compulsory reliability functions to be considered on an equal footing with engineering activities are designed to meet other customers' ideas such as cost, performance and weight in the process of product realization.

5) It is possible that output reliability allocation as input to other reliability tasks.

3.3 Reliability allocation methods of series system

There are many method's to reliability allocation and these methods are vary in complexity depending on how much the definition of the subsystem is type and the degree of accuracy required. In this part of the chapter we will look at the most common and used methods, such as:

1. Equal allocation technique method.
2. Agree allocation method.
3. Feasibility - of - objectives technique method.
4. Wang and Others Method

3.3.1 Equal allocation technique method

The Equal Allocation Technique use all standards for all components within the system and assigns a common reliability target for all components to achieve the overall objective of system reliability(7,19). This method is simple and useful especially at the beginning of the design when no detailed information is available for the series system. Reliability of the system is the reliability results of the individual components.

Thus, can be written as:

$$1 - \prod_{i=1}^{n} (1 - \mathrm{R}_\mathrm{i}^{*}) \geq \mathrm{R}^{*}. \qquad (3.2)$$

The minimum reliability requirement of a component is given by:

$$\mathrm{R}_\mathrm{i}^{*} = (1 - (1 - R^{*}))^{\frac{1}{n}} \qquad \text{i=1,2,….n} \qquad (3.3)$$

If all components are exponential,(3.1) becomes

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} \lambda_\mathrm{i}^{*} \leq \lambda^{*} \qquad (3.4)$$

where λ_i^{*} and λ^{*} are the maximum allowable failure rates of component i and system, respectively. Then the maximum allowable failure rate of a component is

$$\lambda_\mathrm{i}^{*} = \frac{\lambda^{*}}{\mathrm{n}}, \mathrm{i} = 1,2, \ldots \mathrm{n}. \qquad (3.5)$$

$$\mathrm{F(t)} = 1 - \mathrm{e}^{-\lambda \mathrm{t}}$$

$$\mathrm{R(t)} = 1 - \mathrm{F(t)}$$

$$\mathrm{R(t)} = \mathrm{e}^{-\lambda \mathrm{t}} \Rightarrow \mathrm{Ln(Rt))} = -\lambda \mathrm{t}$$

$$\lambda_i = -\frac{\mathrm{Ln}\big(R(t)\big)}{n}, i = 1,2, \dots n$$

Example 3.1 The parallel electromagnetic subsystem consists of two components ,The lifetime of all subsystems of allocation is distributed evenly over components. If the above system reliability target at 42 months in service is 0:96, determine the reliability requirement at this time and the maximum allowable failure rate of each subsystem.

Solution

From eq.(3.3), the reliability of each individual subsystem is

R_i^*=(42) =(1-(1-0.96))$^{1/2} = 0.979$, $i = 1,2.$

The maximum allowable failure rate of the electromagnetic subsystem in accordance with the overall reliability target is

$\lambda^* = -\frac{\mathrm{In}[R^*(42)]}{42} = \frac{\mathrm{In}(0.96)}{42} = 9.72 \times 10^{-4}$ failures per month.

From eq.(3.4), the maximum allowable failure rate of each individual subsystem is

$$\lambda_i^* = \frac{9.72 \times 10^{-4}}{2} = 4.86 \times 10^{-4} \text{ failure per month}, i = 1, \mathbf{2}, \dots, \boldsymbol{n}$$

3.3.2 AGREE Method

The AGREE established and developed by Advisory Group on Reliability of Electronic Equipment, determines the minimum allowable mean failure time for each subsystem to meet the system reliability goal [24, 33]. This strategy to distribution explicitly requires into account subsystem complexity. Complexity is described in terms of modules and their related circuitry, where an equal failure rate is assumed for each module. When defining the limit of modules, this hypothesis should be held in mind. In particular, module counts should be decreased for extremely secure subsystems like pcs because the failure rates are much smaller than those of less secure subsystems like actuators. The AGREE allocation technique also takes into account the significance of individual subsystems, where significance is described as the likelihood of system failure if a subsystem fails. The significance represents the subsystem's essential to system achievement. The significance of 1 implies that for the system to operate the subsystem must work effectively. The importance of 0 indicates that the failure of the subsystem has no effect on system operation. Assume the subsystems are distributed, separately, exponentially, and function in sequence as to their impact on system achievement. You can then rewrite eq.(3.1)as

$$\prod_{i=1}^{n}\{1 - w_i[1 - R_i^*(t_i)]\} = R^*(T), \tag{3.6}$$

where $R^*(t)$ is the system reliability target at time t_i; R_i^* (t_i) the reliability target allocated for subsystem i at time t_i $(t_i \leq t)$, w_i the importance of subsystem i, and n the number of subsystems. It can be seen that the method of allocation allows the subsystem's mission time to be less than the system. Since the times to failure of subsystems are distributed exponentially and we have the approximation exp(-x) ≈ 1 - x for a very small x, can be written as

$$\sum_{i=1}^{n}\lambda_i^* w_i t_i = -Ln[R^*(t)], \tag{3.7}$$

where $\lambda_i^*$is the failure rate allocated to subsystem i. Taking the complexity into account, λ_i^*can be written as:

$$\lambda_i^* = \frac{m_i[InR^*(t)]}{mw_i t_i}, \qquad i= 1,2,\ldots,n, \tag{3.8}$$

$$R_i^*(t_i) = 1 - \frac{1 - (R^*(t))^{\frac{m_i}{m}}}{w_i}, \qquad (3.9)$$

If w_i is equal or close to 1, simplifies to

$$R_i^*(t_i) = (R^*(t))^{\frac{m_i}{m w_i}} \qquad (3.10)$$

It can be seen that eq.(3.6) and eq.(3.7) for a small subsystem would result in a very low reliability goal. A very tiny w_i value distortedly outweighs the complexity impact and results in an unfair distribution. The technique only operates well when each subsystem's significance is close to 1.

Example 3.2 The electromagnetic subsystem consists of two sub-components. If the failure of the two components leads to a system failure, the reliability of the subsystems is required to achieve the system reliability target of 0:95 at a time of 12 hours.

Subsystem	Number of Modules (m_i)	Importance (w_i)	Operating Time (t_i)
component 1	11	1	12
component 2	11	1	12

Table 3.1: Data for AGREE reliability allocation

From Table 3.1, the total number of modules in the system is m = 11 + 11 = 22. Substituting the given data into eq.(3.8) yields the maximum allowable failure rates (in failures per hour) of the two subsystems as

$$\lambda_1^* = -\frac{11[\mathrm{Ln}(0.95)]}{22 \times 1 \times 12} = 0.0021$$

$$\lambda_2^* = -\frac{11[\mathrm{Ln}(0.95)]}{22 \times 1 \times 12} = 0.0021$$

From eq.(3.9), the corresponding reliability targets are

$$R_1^*(12) = 1 - \frac{1 - (0.95)^{\frac{11}{22}}}{w_i} = 0.975$$

$$R_2^*(12) = 1 - \frac{1 - (0.95)^{\frac{11}{22}}}{w_i} = 0.975$$

Now we substituted the allocated reliabilities into eq.(3.6) to check the system reliability,which is

$R^*(t) = [1 - 1(1 – 0.975)] \times [1 - 1(1 – 0.975)] = 0.95$

This approximately equals the system reliability target of 0.95.

3.3.3 Feasibility -of- objectives technique

This method was developed as a means of allocating reliability without repair of mechanical and electrical systems [22, 12, 19]. In this method, subsystem allocation factors, numerical classifications of complexity system, performance time, state of the art and environmental conditions are calculated. The classification of this method is estimated on a scale from 1 to 10, with values determined as discussed:

(1) The complexity of the system. The complexity is assessed by looking at the components that make up the system. The least complex system is rated at 1, and the highly complex system is rated at 10.

(2) Performance time. The component that runs for the duration of task 10 is rated, the component that runs the least time during the task is at 1.

(3) State-of-the-art. The lowest design or method is 10, and the most sophisticated is to assign a value of 1.

(4) Environment. Components are also classified according to environmental conditions from 10 to 1. Components facing extremely harsh environments during their operation are expected to be classified as 10, which are expected to experience the least harsh environments classified as 1. Each subsystem classification will be between 1 and 10 normalized so that a total is 1.

The basic equations in this method are:

$$\lambda_s = \sum_{i=1}^{n} \lambda_{\mathcal{K}}^{*} \quad (3.11)$$

As that

$$\mathcal{C}_{\mathcal{K}} = \frac{W_{\mathcal{K}}}{W} \quad (3.12)$$

$$\lambda_{\mathcal{K}}^{*} = \mathcal{C}_{\mathcal{K}} . \lambda_s$$

$$W_{\mathcal{K}} = r_{1k} . r_{2k} . r_{3k} . r_{4k} .$$

And

$$\mathrm{W} = \sum_{k=1}^{n} W_k \quad (3.13)$$

Where λ_s is system failure rate

$\mathcal{C}_{\mathcal{K}}$ Complexity of subsystem (k)

$\lambda_{\mathcal{K}}^{*}$ is failure rate allocated to each subsystem

$W_k$is rating for subsystem (k)

W is sum of the rated product's

Example 3.3 The electromagnetic subsystem consists of two subsystems with reliability (0:96) in 120 hours as shown in fig.2.6. Engineering estimates of complexity, state of art, performance time and environment are given in Table (3.2) Calculate the failure rate for each subsystem.

Table 3.2: Value of subsystem allocation factors

Subsystem	Complexity	State of the art	Time of performance	Environment
component1	7	6	8	3
component2	6	5	6	2

solution *by using equation (3.13) we get:*

$$W_{\mathcal{K}} = r_{1k}.r_{2k}.r_{3k}.r_{4k}.$$

$W1 = (7)(6)(8)(3) = 1008$

$W2 = (6)(5)(6)(2) = 360$

by equation (3.23) where

$$W = \sum_{i=1}^{n} W_{\mathcal{K}}$$

SO , W=1008+360

W= 1368

Now, we find $C_{\mathcal{K}}$ *by using equation* (3.12)

$$\mathcal{C}_1 = \frac{w_1}{W} = \frac{1008}{1368} = 0.737$$

$$\mathcal{C}_2 = \frac{w_2}{W} = \frac{360}{1368} = 0.263$$

$$\lambda_s = \frac{-\mathcal{J}n[R^*(t)]}{t} = \frac{-\mathcal{J}n(0.96)}{120} = 0.00034$$

Since :

$$\lambda_{\mathcal{K}}^* = \mathcal{C}_{\mathcal{K}} . \lambda_s$$

$$\lambda_1^* = (0.737)(0.00034) = 0.00025$$

$$\lambda_2^* = (0.263)(0.00034) = 0.000089$$

By equation (3.11) where

$$\lambda_s = \sum_{k=1}^{n} \lambda_{\mathcal{K}}^* = 0.000339$$

$$And \quad \sum_{k=1}^{2} C_{\mathcal{K}} = 1$$

$Then$:

$$\sum_{k=1}^{2} C_{\mathcal{K}} = 0.737 + 0.263 = 1$$

$This\ is\ the\ desired\ goal.$

3.3.4 Wang Method

Wang (2001) developed a method to allocate reliability requirements to a CNC system composed of units connected in a series configuration. They defined seven factors that are considered important for CNC lathe system performance: frequency of failure, criticality of failure, maintainability, complexity, manufacturing technology, working condition, and cost. In essence the method consists of estimating the weighting factors using objective and subjective assessment, adjusting each factor using important weighting factors and calculating the failure rate of each subsystem.

Estimating the weighting factors involves calculating the relative ratio for failure rate and the average relative ratio for failure rate. The estimation of the relative ratio factor for the frequency of failure, criticality of failure, maintainability, and complexity require historical failure data and criticality analysis data. The mathematical expression for calculating these factors is included in Appendix C.Once the relative ratio is estimated, the average relative ratio for failure rate is calculated as follows:

$$\gamma_{\kappa\iota} = \frac{1}{2}\sum_{j=1}^{n} \beta_{ij}^{k}, \qquad k = 1,2,\ldots\ldots,m; \qquad i = 1,2,\ldots.,n \tag{3.14}$$

where

γ^{ki} average relative ratio for failure rate of k^{th} criterion and i^{th} subsystem

n number of subsystems

m number of allocation factors

β^{k}_{ij} relative ratio for ratio rate allocation between the i^{th} subsystem and the j_{th} subsystem

The failure rate allocation factors are calculated as follows:

$$\alpha_i = \sum_{k=1}^{m} \gamma_{\kappa\iota} \,.\, w_k, \qquad i = 1,2,\ldots,n \tag{3.15}$$

where

a_i comprehensive allocation factor for the i^{th} subsystem

wk weight or the importance of the k^{th} criterion

γki average relative ratio for failure rate of k^{th} criterion and i^{th} subsystem

n number of subsystems

m number of allocation criteria

Once the failure rate allocation factor *ki* is calculated, the failure rate for the *i*th subsystem is estimated as follows:

$$\lambda_i = \frac{a_i}{\sum_{i=1}^{n} a_i} \lambda_s \tag{3.16}$$

Where

λ_i failure rate of i^{th}subsystem

a_i failure rate allocation factor of the i^{th}subsystem

λ_s system failure rate

n number of subsystems

This method applies only to series configurations and requires the availability of equipment historical data in order to reduce subjectivity and produce credible and reasonable allocation estimates. Although the Wang and others method was developed for CNC lathe systems, it can also be adapted for other systems by adjusting the weighting factors. As stated earlier, this method utilizes objective and subjective assessments to estimate allocation factors and requires calculation of importance weight factors which involves expert judgment.

Example (3.4) : A computer consists of two subsystems, let the system failure rate be 0.30 and relative ratio for ratio rate allocation β :0.25 and the importance of the k^{th} criterion is 0.15, Find the failure rate of the subsystems.

Soloution:

By equation(3.14) we get:

$$\gamma_{\kappa\iota} = \frac{1}{2}\sum_{j=1}^{n} \beta_{ij}^{k}, \qquad k = 1,2,\ldots\ldots,m; \qquad i = 1,2,\ldots.,n$$

$$\gamma_{k1} = \frac{1}{2}[(0.25) + (0.25)^2] = 0.375$$

$$\gamma_{k2} = \frac{1}{2}[(0.25).2 + (0.25)^2.2] = 0.312$$

By equation (3.15) we get:

$$\alpha_i = \sum_{k=1}^{m} \gamma_{\kappa\iota} . w_k, \qquad i = 1,2,\ldots,n$$

$$\alpha_1 = (0.375)(0.15) = 0.05625$$

$$\alpha_2 = (0.312)(0.15) = 0.0468$$

$$\lambda_i = \frac{a_i}{\sum_{i=1}^{n} a_i}\lambda_s \qquad \text{i=1,2,3}$$

$$\lambda_1 = \frac{0.05625}{0.05625 + 0.0468} \times 0.30 = 0.163$$

$$\lambda_2 = \frac{0.0468}{0.05625 + 0.0468} \times 0.30 = 0.136$$

This is the desired goal.

3.4 Techniques to Improve Reliability of Systems

There are many different methods to improve the reliability of systems and devices. In this chapter, we discuss two techniques to improve the reliability of three types of systems: the series system, the parallel system, and the complex system [26, 29]. Techniques are: redundancy, and be in two cases, namely redundancy for the element and redundancy for the system.

3.4.1 Redundancy Technique to Improve Reliability Systems

Used a techniques to improve the systems reliability, which is the redundancy technique when one piece of equipment or component may replace another piece of equipment or component that has failed, the general term ”Redundancy” is used [5, 21]. More specifically,redundancy is a technique for increasing reliability that includes putting two or more components, substructures, or complex devices in parallel. Were two methods for this technique.

3.4.1.1 Element redundancy method

Each component of the system has its own path. The series components C1 and C_2, and also the other components of C_1 and C_2 that constitute the system's individual components, are as a results shown in figure (3.1) the system as a whole has improved, as has the reliability of each component [5]. The reliability of this system is $R_s = R_{C1}R_{C2}$, the formula for redundancy the element be:

$R^*_{C1} = 1 - (1 - R_1)^2$, $\quad R^*_{C2} = 1 - (1 - R_2)^2$,

we get, $R^*_{Es} = R^*_1 * R^*_2$,

$$R^*_i = 1 - (1 - R_i)^2 , \; i = 1, \ldots\ldots, n \qquad (3.17)$$

Then,

$$R^*_{Es} = R^*_1 * R^*_2 \ldots R^*_i \qquad ,i=3,4\ldots\ldots n \qquad (3.18)$$

WhereR^*_{Es} represent reliability of system is after adding backup components for each component in the system.

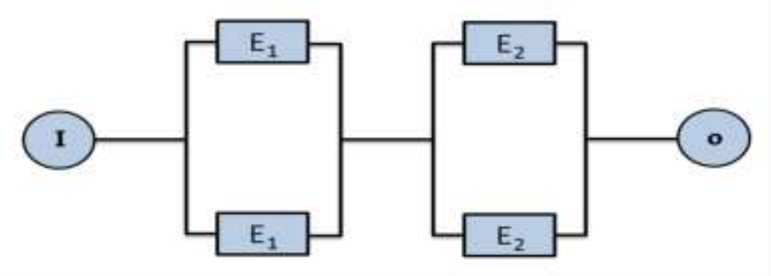

Figure 3.1: Element redundancy

3.4.1.2 Unit Redundancy Method

The entire system has an additional path. C1 and C2 are system components in fig. (3.2), and by providing two of them simultaneously, the reliability of the overall system is increased [30].

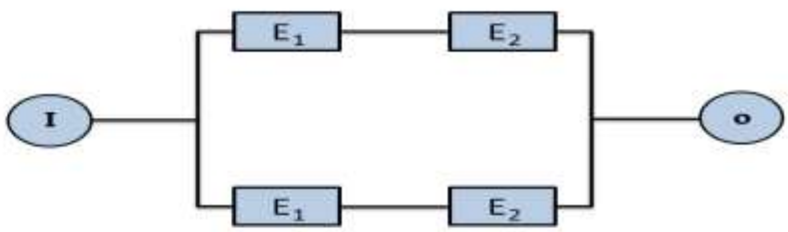

Figure 3.2: Unit redundancy.

Unit redundancy can be computed using the equation :

$$R^*_{Us} = 1 - (1 - R_S)^2 \qquad (3.19)$$

R^*_{Us} represent the unit redundancy and *RS* represent system reliability.

3.5 Evaluating the Reliability Importance of Each Units in Some Types of Systems

The reliability importance of the components must be studied. Regarding the systems design or suggestions for maintaining and operating it optimally [13]. Identificationdentifying weaknesses critical system components, determining the effects of failure atthese components, are the main objectives of a reliability study. Lambert (1975) accordingto a components importance should be determined by two reasons 37:

- The components location in the system.
- The components reliability is in explanation

The reliability of a component can be increased by choosing higher quality components ,adding redundant components, reducing the operational and environmental conditions imposed on the component, or improving the components main portability [41]. In this part, we discuss the reliability importance of each component in relation to the systems, and find the reliability importance of the components depending on the first method of Birnbaum measure(1969)[5],according to equation :

$$I(i) = \frac{\partial R_s}{\partial R_i} \qquad (3.20)$$

Which depends on the derivative of the systems for each component present in the system.

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