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Computation of Cohomology Operations of Groups of Order 128 and Stiefel-Whitney Classes of Real Representation for Non-Prime Power Groups

A Dissertation
Submitted to the council of the College of Education for Pure Sciences /University of Babylon in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Education / Mathematics.

By

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Supervised by

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بسم الله الرحمن الرحيم

۞ هُوَ الَّذِي خَلَقَ السَّمَاوَاتِ وَالْأَرْضَ فِي سِتَّةِ أَيَّامٍ ثُمَّ اسْتَوَىٰ عَلَى الْعَرْشِ ۚ يَعْلَمُ مَا يَلِجُ فِي الْأَرْضِ وَمَا يَخْرُجُ مِنْهَا وَمَا يَنْزِلُ مِنَ السَّمَاءِ وَمَا يَعْرُجُ فِيهَا ۖ وَهُوَ مَعَكُمْ أَيْنَ مَا كُنْتُمْ ۚ وَاللَّهُ بِمَا تَعْمَلُونَ بَصِيرٌ ۞

صدق الله العلي العظيم

سورة الحديد (4)

DEDICATION

To who are absent from my sight and present in my heart

My father

To the source of kindness

My mother

To my second half

My husband and children

To my support in life

My brothers and sisters

I dedicate the fruit of my humble effort

Marwah Yasir

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Praise be to God, Lord of the worlds, and prayers and peace be upon the most honorable of the prophets and messengers, our Master Muhammad, his family, his companions, and those who followed them with kindness until the Day of Judgment, and after...

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Finally, my great thanks, love and respect to all who help me.

Marwah Yasir

Approval of Scientific Supervisor

I certify that this dissertation **"Computation of Cohomology operations of groups of order 128 and Stiefel-Whitney classes of real representation for non-prime power groups"** by student **"Marwah yasir Mohsin naji"** was prepared under my supervision at the University of Babylon, Faculty of Education for Pure Sciences, in a partial fulfillment of the requirements for the degree of Doctor of Philosophy in Education / Mathematics.

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Abstract

The aim of this dissertation is the computation of Steenrod squares on the Mod-2 cohomology of all groups of order 128. By using the package Homology Algebra programming (HAP), which is part the Group algorithm programming (GAP). Steenrod squares were computed for more than 200 groups of order 128 and Give a group cohomology construction and implementation of the cup-i product with implements an algorithm the cup-i product used to compute the Steenrod square on the classifying space of groups of order 128. Three methods were utilized to compute the Steenrod square of groups of order 128 two of them by using HAP and the third by hand. we devised and implemented an algorithm to compute all groups that can be represented as a direct product of two groups of order less than 128. The study also demonstrates how the underlying techniques can be used to compute the Stiefel-Whitney classes of certain real representations of non-prime power groups. Other contributions of the study are introducing the definition of fuzzy cofibration and fuzzy Serre cofibration with mixing case of fuzzy (Serre) cofibration. The study concluded properties and theorems.

CONTENTS

List of Symbols

Symbols	Description
$\mathbb{Z}$	Integer set
$\mathbb{Z}G$	Integral group ring
H_n	Homology module
H^n	Cohomology module
R_*^G, $\mathcal{M}_*$	Free resolution for group G
$\oplus$	Direct sum
$\otimes$	Tensor product
$\mathbb{Z}_2$	Field of integers $\{\bar{0}, \bar{1}\}$
β	Bockstein homomorphism
Sq^i	Steenrod square
P^i	Steenrod power
S^∞	CW-complex universal cover of the projective plane
$\smile_i$	The cup-i product
$GL_n(\mathbb{R})$	The general liner group
$O_n(\mathbb{R})$	The orthogonal group
τ	Thom isomorphism
$R_j = \prec j \succ$	Ring generated by j
ω_i	The i^{th} Stiefel-Whitney classes

Publication

Four papers from this dissertation were accepted and published

1. "Fuzzy cofibration and fuzzy serre cofibration" accepted for publication in the Journal of Interdisciplinary Mathematics.Taylor&Francis, ISSN: 0972-0502, Cite Score (2.2), No. JIM-1740(SICC_2024_Batch 8_6).

2. "Computation of cohomology operation of groups of order 128", accepted in the first international conference for physics and mathematics (1^{st} ICPM 2022) held for the period 19-20/October/2022. Will be published in the IEEE Publisher conference.
3. "Computations Stiefel-Whitney classes of real rerepresentation for non-prime power groups", Wasit Journal for Pure Science Vol. (2) ,Issue.(3), No. (104). This volume will begin publication in september, 2023.
4. "Mixing fuzzy cofibration and mixing fuzzy serre cofibration", accepted in 5^{th}. International Conference in Physical Science and Advanced Materials (PAM 2023) held for the period sep. 24-27 /2023 KSA. Will be published in AIP conference Proceedings.

Introduction

Introduction

The cohomology operations were introduced in 1940, given as lectures by N. E. Steenrod. Later, they were Written and revised by D. B. A. Epstein in 1962[23]. Cohomology operation has many important applications in algebraic and differential topology. Our study is about one type of these operations is called steenrod operation $(\mathbb{Z}_2, n, \mathbb{Z}_2, n+1)$. (For more information of the homomorphism Sq^i we follow [7], [24] and [22], [28] and [5]). The first pillar of the study presented in this dissertation is computation of Steenrod squares on the mod 2 cohomology of all groups of order 128 we used the (HAP)[4] Homological Algebra Package in the GAP programming[8] software. This type computation was identified at the beginning of by Rusin(1989) [15], such that he compute All 2-groups of order 32 except two groups of number 8 and 44 in the GAP library of small groups and most of the computations were done by hand. Guillot(2008)[11] computed The 5 groups of order 16, 28 groups of order 32, and 61 groups of order 64. and Thanh Tung Vo(2011)[26] computed All 2-groups of order less than or equal 32, except group number 8 by using detection method.(He mentions to compute 210 groups of order 64 but did not explain any details). The summary of their studies used by Daher(28)[2018] who managed to compute all Steenrod squares of finite groups of order less than 64 by using cup i- product and detection method . Results of all groups of order 2, 4, 8, 16, 32 were obtained, but only two of order 64. The second and fundamental pillar of our work is also demonstrates how the underlying techniques can be used to compute the Stiefel-Whitny classes of certain real representations of non-prime power groups.

Based on the space that we worked on when we compute the Stiefel-Whitney classes (classifying space "fiber space") we touched on the topic of fuzzy cofibration and fuzzy serre cofibration. Then this subject was examined by taking the mixing cofibration and the mixing serre cofibration with some important concepts, we benefited from Zadeh classical paper such that he introduced the fundamental conscepts of fuzzy sets [19]. Fuzzy sets have applications in many fields of engineering, social science, economics, etc. Later in mathgematics, the theory of fuzzy topology was developed by many researchers starting with chang[2]. Since then various notions in classical topology have been extended to fuzzy topological space [18],[9] . The topological algebra at first studied by Allen Hatcher[13] developed some of its concepts. The fuzzy homotopy theory was studied and submitted by Cuvalcioglu and citil [10]. In this dissertation the concepts of fuzzy homotopy, the fuzzy lowering homotopy property, fuzzy cofiberation, fuzzy serre cofiberation were examined. Later Z .Y Habeeb, D . Al Baydli(2022) [29] study the concept of mixed serre fibration and some of its properties which enabled us to study the cofibration and mixing cofibration concepts in fuzzy case with their properties and theorems.

This dissertation consists of five chapters:

Chapter One, consist of two sections; the first one contains of the most important basics and definition of cohomological algebra and classifying space and fiber space. The second one contains the data type of all functions and relationships we used for computing by using GAP program.

Chapter Two consists of five sections. In section one, we introduced the concept of fuzzy cofibration and fuzzy serre cofibration and studied their definition with the most important properties and relationships. In section two, we introduced the concept of mixing fuzzy cofibration and studied the most important properties of this concept. The third section, introduced mixing fuzzy serre cofibration, and studied its definition, the most important properties and relationships. In the last section, we studied the Fuzzy homotopy Extention property. For the fifth section we have introduced the concept of a mixing criterion for a fuzzy map to be a mixing fuzzy Serre Cofibration.

Chapter Three consists of three sections. Section one introduced the definition of cohomology operation and Steenrod squares and studied its properties. Section two explicated the structure of cup-i product and their relationships in this section also, we developed algorithm. In section three, we investigated the relation between cup-i product and steenrod square, the computation of steenrod squares on the cohomology ring of 2-groups of order 128 and the methods for computing. In our computation, we followed the way of Rusin [15], Guillot [11], Thanh Tung Vo [26] and Daher W. Al Baydli [28]. Farthermore, we use the direct product method which depends on the results of previous groups of order less than 128 and we get the results for more than 200 groups of order 128.

Chapter four, contains of two sections. In both sections we explained how Steenrod squares with cupi- product, orbit polytopes and the thom isomorphism are used to compute the Stiefel-Whitney classes (pithiness SW-classes) comparable to a real representation $\rho : G \rightarrow O_n(\mathbb{R})$ of a non-prime power groups. The main obstacle was how to calculate the steenrod squares on cohomology rings of non-prime power groups, which were computed by Mahmood [21]. However, if the cohomology ring of the sylow subgroup of any non-prime power groups is homeomorphic to finite groups then they are less than or equal to 128 groups. The SW-classes comparable to a real representation $\rho : G \rightarrow O_n(\mathbb{R})$ of a finite groups studies were taken up Ellis(1989and 2006) [5,6], Guillot [26] and Daher [28]. In our computation we used three programs GAP, polymaker and texmaker.

Chapter Five is composed of two sections. In section one; we introduced all results of compute the Steenrod squares of groupes of order 128. In section two we introduced all results of compute stiefel-whitney classes of real represntation of non-prime power groups.

CHAPTER ONE

Review of necessary background material

Chapter One:

In this chapter, we deal with the most important special mathematical tools in the process of constructing a dissertation, as they are the mainstay of our work, This chapter contains two parts: The first topic deals with the concept modules, topological algebra, the concept of fiber space and homological algebra with there's most important characteristics, theories . The second topic dealt with the data type.

1.1 Review of cohomological algebra.

In this part, we review the concept of homological and cohomological algebra and its basic properties and theories that we need in our work. In other side we review the concept of fiber space, fibration and cofibration with there's theories and properties.

Definition 1.1.1[25]

Let R be a ring and M an abelian group (M,+). We say M is a left R-module if there is function $\psi : R \times M \to M$ such that $\psi(r, m) \to rm$, called a scalar multiplication and satisfying:

- $(r+s)m = rm + sm$,
- $r(m+n)=rm+rn$,
- $(rs)m = r(sm)$, for all $r,s \in R$, $m,n \in M$.

Definition 1.1.2[25]

Let R be a ring and M an abelian group (M,+). We say M is a right R-module if there is function $\psi : M \times R \to M$ such that $\psi(m, r) \to mr$, called a scalar multiplication and satisfying:

- $m(r+s)=mr + ms$,
- $(m+n)r=mr + nr$,
- $m(rs) = (mr)s$, for all $r,s \in R$, $m,n \in M$.

If R is a commutative ring, then we say M is an R-module.

Definition 1.1.3.[25]

Let M and N be two R-modules over arbitrary ring R. Their tensor product $M \otimes_R N$ is the R-module generated by the formal symbols $m \otimes n$ for some $m \in M$, $n \in N$ subject to the following relations.

- $(m_1 + m_2) \otimes n = (m_1 \otimes n) + (m_2 \otimes n)$,
- $m \otimes (n_1 + n_2) = (m \otimes n_1) + (m \otimes n_2)$
- $r.m \otimes n = m \otimes r.n$, for all $r \in R$, $m_1, m_2 \in M$ and $n_1, n_2 \in N$.

Definition 1.1.4.[25]

Let A and B be submodules of a module M over arbitrary ring R. we say that M is the direct sum of A and B and denoted by $A \oplus B = M$,if $A + B = M$ and $A \cap B = 0$. And we also say $A \oplus B$ is a decomposition of M.

Definition 1.1.5. [25]

An R-module M is a free R-module if and only if M is isomorphic to a direct sum of copies of R, that is $M \cong \bigoplus_{j \in J} R_j$, where $R_j = \prec j \succ \cong R$ for all $j \in J$. We call J a basis of M. By Definition of direct sum each $m \in M$ has a unique expression of the form $m = \sum_{j \in J} r_j j$, $r_j \in R$ and almost all $r_j = 0$.

Definition 1.1.6.[25]

Let R be a ring. A chain complex $C = (C_n, d_n)_{n \in Z}$ of R-modules

is a sequence of homomorphisms of R-modules

$$\cdots \rightarrow C_{n+1} \xrightarrow{d_{n+1}} C_n \xrightarrow{d_n} C_{n-1} \xrightarrow{d_{n-1}} \cdots \qquad (1.1)$$

such that $d_n d_{n+1} = 0$ for all n. The chain complex C is called an exact sequence

if $\mathrm{Im} d_{n+1} = \mathrm{Ker} d_n$ for all n. The operation d_n are boundary maps. An element of the kernel of d_n is called a cycle and an element of the image of d_{n+1} is called a boundary.

We dualize the chain complex (1.1), that is, we apply Hom(−, R) to it, to get the cochain complex.

$$\cdots \xleftarrow{\delta n+1} C^{n+1} \xleftarrow{\delta n} C^n \xleftarrow{\delta n-1} C^{n-1} \leftarrow \cdots \qquad (1.2)$$

with $C^n := \mathrm{Hom}(C_n, R)$, and the coboundary map $\delta^n : C^n \to C^{n+1}$ is defined by

$$(\delta^n \psi)(\alpha) = \psi(d_{n+1}\alpha), \text{ for } \psi \in C^n \text{ and } \alpha \in C_{n+1} \qquad (1.3)$$

with the property that $\delta^n \circ \delta^{n+1} = 0$. An element of the kernel of δ^n is called a cocycle and an element of the image of δ^{n-1} is called a coboundary.

Definition 1.1.7.[25]

Let C = $(C_n, d_n)_{n\in\mathbb{Z}}$ be a chain complex of R-modules and let $C = (C^n, \delta^n)_{n\in\mathbb{Z}}$ be a cochain complex of R-modules . For each n $\in\mathbb{Z}$, the nth (co)homology module of C is defined to be the quotient module

$$H_n(C) = \frac{\mathrm{Ker}d_n}{\mathrm{Im}d_{n+1}}$$

$$H^n(C) = \frac{Ker\delta^n}{Im\delta^{n-1}}$$

Definition 1.1.8.[25] Let $C = (C_*, d_*)$ and $\acute{C} = (\acute{C}_*, \acute{d}_*)$ be two *chain complexes* of R-modules. A *chain map* $f : C \to \acute{C}$ is a sequence of homomorphisms of R-modules $\{f_n\}_{\{n\in\mathbb{Z}\}} : C_n \to \acute{C}_n$ such that the following diagram commutes

$$\begin{array}{cccccc} \cdots \longrightarrow C_{n+1} & \xrightarrow{d_{n+1}} & C_n & \xrightarrow{d_n} & C_{n-1} & \longrightarrow \cdots \\ f_{n+1}\downarrow & & f_n \downarrow & & \downarrow f_{n-1} & \\ \cdots \longrightarrow C'_{n+1} & \xrightarrow{d'_{n+1}} & C'_n & \xrightarrow{d'_n} & C'_{n-1} & \longrightarrow \cdots \end{array}$$

Figure 1.1.1

A *chain map* $f : C \to \acute{C}$ induces R-module homomorphisms

$$f_* = H_n(f) : H_n(C) \to H_n(\acute{C}) \textit{ for all } n.$$

That means a chain map f takes cycles to cycles and boundaries to boundaries. The map $z_n + B_n(C) \to f_n(z_n) + B_n(\acute{C})$ is a well-defined R-module homomorphism because if $z \in Z_n(C)$, then $d_n(z) = 0$, since f is chain map, $\acute{d}_n f_n(z) = f_{n-1} d_n(z) = f_{n-1}(0) = 0$, therefore $f_n(z) \in Z_n(\acute{C})$

However if $b \in B_n(C)$, then $d_{n+1}(c) = b$, for some $c \in C_{n+1}$, then $f_n(b) = f_n(d_{n+1}(c)) = \acute{d}_{n+1} f_{n+1}(c)$, so $f_n(b) \in B_n(\acute{C})$.

Definition 1.1.9.[13]

Let X and Y be two topological spaces ,and let $f, g: X \to Y$ be two maps. We say that f is homotopic to g(by symbols$f \cong g$) if and only if there exist a continuous function $H: X \times I \to Y$, I=[0, 1] such that
$H(x, 0) = f(x), H(x, 1) = g(x), \forall x \in X$. H is called a homotopy.

Definition 1.1.10.[25]

Let $f, g : C \to \acute{C}$ be *chain maps*. A *chain homotopy h* between f and g, denoted by $h : f \cong g$, is a sequence of homomorphisms $h_n : C_n \to \acute{C}_{n+1}$ such that
$f_n - g_n = d'_{n+1} h_n + h_{n-1} d_n$; see the diagram below.

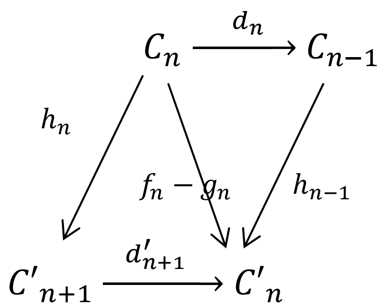

Figure 1.1.2

If there exists a *chain homotopy* between f and g, then f and g are said to be *chain homotopic*.

Lemma 1.1.11.[25]

If f; g : $C \to \acute{C}$ *are* chain homotopic *then they induce the same homomorphisms* $H_n(C) \to H_n(C')$ *for all n.*

Note: Let G denote an arbitary discrete group. Let $\mathbb{Z}$ denote the additive group of integers considered as a $\mathbb{Z}G$-module in which the group G acts trivially by $gn=n$ for $g \in G$, $n \in \mathbb{Z}$.

Definition 1.2.12.[25]

Let G be a group and $\mathbb{Z}$ be the group of integers considered as a trivial $\mathbb{Z}G$-module. The map ϵ: $\mathbb{Z}\,G \to \mathbb{Z}$ from the integral group ring to $\mathbb{Z}$, given by $\Sigma m_g g \mapsto \Sigma m_g$, is called the *augmentation*.
It easy to see that $\in$ is a $\mathbb{Z}G$-module homomorphism.

Definition 1.1.13.[25]

Let G be a group. A *free $\mathbb{Z}G$-resolution* of $\mathbb{Z}$ is an exact sequence of $\mathbb{Z}G$-modules

$$R_*^G: \ldots \to R_{n+1}^G \xrightarrow{\vartheta_{n+1}} R_*^G \xrightarrow{\vartheta_n} R_{n-1}^G \to \cdots \to R_1^G \xrightarrow{\vartheta_1} R_0^G \xrightarrow{\epsilon} R_{-1}^G = \mathbb{Z} \to 0$$

with each R_i^G a free $\mathbb{Z}G$-module for all $i \geq 0$. Given R_*^G a $\mathbb{Z}G$-resolution of $\mathbb{Z}$ and any $\mathbb{Z}G$ module A we can use the tensor product of Definition (1.1.3) to construct an induced chain complex $R_*^G \otimes_{\mathbb{Z}G} A$ of abelian groups:

$$\ldots \to R_{n+1}^G \otimes_{\mathbb{Z}G} A \xrightarrow{d_{n+1}} R_*^G \otimes_{\mathbb{Z}G} A \xrightarrow{d_n} R_{n-1}^G \otimes_{\mathbb{Z}G} A \to \cdots$$

In addition, we can also construct an induced cochain complex

$Hom_{\mathbb{Z}G}(R_*^G, A)$ of abelian groups:

$$\ldots \to Hom_{\mathbb{Z}G}(R_{n-1}^G, A) \xrightarrow{\delta^{n-1}} Hom_{\mathbb{Z}G}(R_n^G, A) \xrightarrow{\delta^n} Hom_{\mathbb{Z}G}(R_{n+1}^G, A) \ldots$$

Definition 1.1.14. [25]

Let R_*^G be a free $\mathbb{Z}G$-resolution of $\mathbb{Z}$. A *contracting homotopy h* of R_*^G is a sequence of homomorphisms of $\mathbb{Z}$ -modules $h_n : R_n^G \to R_{n+1}^G (n \geq -1)$ such that

$$\epsilon h_{-1} = 1_{\mathbb{Z}},$$

$$h_{-1}\epsilon + \vartheta_1 h_0 = 1_{R_0^G}$$

$$h_{i-1}\vartheta_i + \vartheta_{i+1} h_i = 1_{R_i^G} \; for \; i > 0$$

Definition 1.1.15.[25]

Let R_*^G be a free $\mathbb{Z}G$-resolution of $\mathbb{Z}$ and A be a $\mathbb{Z}G$-module. The *homology* and *cohomology* of a group G with coefficients in a $\mathbb{Z}G$-module A are defined to be the $\mathbb{Z}G$-modules

$$H_n(G,A) = H_n(R_*^G \otimes_{\mathbb{Z}G} A)$$

$$H^n(G,A) = H^n\big(Hom_{\mathbb{Z}G}(R_*^G, A)\big), for\ all\ n \geq 0$$

Definition 1.1.16. [3, 5]

Suppose that (C_*,ϑ) and (D_*,ϑ) are two chain complexes of free R-module, let

$$(C_* \otimes_R D_*)_n = \bigoplus_{p+q=n} C_p \otimes D_q$$

where the tensor product $C_p \otimes_R D_q$ is the module of Definition(1.1.3.) one can check that $(C_* \otimes_R D_*)_n$ is the free R-module with basis $a_\curlywedge^p \otimes b_\mu^q$ where $a_\curlywedge^p \in Cp$ and $b_\mu^q \in Dq$, p, $q \geq 0$, $p + q = n$: The boundary operator

$$d_n\colon (C_* \otimes_R D_*)_n \to (C_* \otimes_R D_*)_{n-1}$$

is the homomorphism defined on the free generators by

$$d_n(a_\curlywedge^p \otimes b_\mu^q) = \vartheta a_\curlywedge^p \otimes b_\mu^q + (-1) a_\curlywedge^p \otimes \vartheta b_\mu^q$$

Definition 1.1.17. [5]

Let A be a $\mathbb{Z}G$-module and let B be a $\mathbb{Z}H$-module for two groups G, H. By regarding both A and B as abelian groups, we can form their tensor product $A \otimes_{\mathbb{Z}} B$. There is an action of the direct product of groups $G \times H$ on the tensor product $A \otimes_{\mathbb{Z}} B$ define by

$$(g, h) \cdot (a \otimes b) = (ga) \otimes (hb),\ (g, h) \in G \times H,\ (a \otimes b) \in A \otimes_{\mathbb{Z}} B \qquad (1.4)$$

If A is a free $\mathbb{Z}G$-module and B is a free $\mathbb{Z}H$-module then $A \otimes_{\mathbb{Z}} B$ is a free $\mathbb{Z}(G \times H)$- module under this action. Let R_*^G be a chain complex of $\mathbb{Z}G$-modules and let S_*^H be a chain complex of $\mathbb{Z}H$-modules. By regarding these chain complexes as complexes of abelian groups we can form their tensor product $R_*^G \otimes_{\mathbb{Z}} S_*^H$ as defined in (1.1.15.) Under the action (1.4), the chain complex $R_*^G \otimes_{\mathbb{Z}} S_*^H$ is a chain complex of $\mathbb{Z}(G \times H)$- modules.

Let R_*^G be a free $\mathbb{Z}G$-resolution of $\mathbb{Z}$ with contracting homotopy

$h_n : R_n^G \to R_{n+1}^G$,

let S_*^H be a free $\mathbb{Z}H$-resolution of $\mathbb{Z}$ with contracting homotopy $h_n^{'}: S_n^G \to S_{n+1}^G$ Then the tensor product $R_*^G \otimes_{\mathbb{Z}} S_*^H$ is a free $\mathbb{Z}(G \times H)$-resolution of $\mathbb{Z}$ with contracting homotopy $h_n^{''}$ given by

$$h_n^{''}(u \otimes v) = (h_p u) \otimes v + (-1)^p u \otimes (h_q^{'} v),$$

for $u \in R_p^G$, $v \in S_q^G$, $p, q \geq 0$

Definition 1.1.18. [25]

Let $\psi : G \to H$ be a group homomorphism, let A be a $\mathbb{Z}G$-module and let B be a $\mathbb{Z}H$-module. A function $f : A \to B$ is a ψ *-equivariant homomorphism* if $f(u_1+u_2) = f(u_1)+f(u_2)$ and $f(g.u) = \psi(g) . f(a)$ for all $u, u_1, u_2 \in A,\ g \in G.$ Also, let R_*^G be a $\mathbb{Z}G$-resolution and let S_*^H be a $\mathbb{Z}H$-resolution. A ψ *-equivariant chain map* f_* : $R_*^G \to S_*^H$ consists of sequence of ψ–equivariant homomorphism $f_n : R_n^G \to S_n^H$ $n \geq 0$, satisfying the usual chain map condition

$$f_{n-1}\vartheta_n = \vartheta_n f_n \qquad for\ n \geq 1$$

Definition 1.1.19. [13]

Let $X \xrightarrow{\Delta} X \times X$, such that $x \to (x, x)$ be the diagonal map,

$$C_{p+q}(X) \xrightarrow{d\Delta} C_{p+q}(X \times X) \xrightarrow{\varphi} C_p(X) \otimes C_q(X) \xrightarrow{f \otimes g} Z \otimes Z \longrightarrow Z$$

$$(f \smile g)(c) = (f \otimes g)\varphi \circ d_\Delta(c) = f \otimes g\ (\varphi\ (d_\Delta(c)))\ ,\ c \in C_{p+q}(X)$$

then the cup product is the homomorphism

$$\smile:\ H^p(X) \otimes H^q(X) \to H^{p+q}(X)$$

defined by $\quad f \smile g = d^*(f \times g).$

(Coefficients are in any commutative ring with unity) An immediate consequence of the rules for the cross product is that f ⌣ g = $(-1)^{pq}$g ⌣ f Where p and q are the degrees of f and g respectively.

Definition 1.1.20 [13] Let X be a topological space and $\phi \in C^k$ (X, R) Let $\psi \in C^l$ (X, R) where R is a ring from which wepick the coefficients ($\mathbb{Z}$, $\mathbb{Z}_n$) , Define the Cup Product $\phi \smile \psi \in C^{k+l}$ (X, R) is given by $\phi \smile \psi$ ($\sigma : \Delta_{k+1} \to X$) = ϕ ($\sigma|_{[v_{0,\dots} v_k]}$) .$\psi$ ($\sigma|_{[v_{k,\dots} v_{k+1}]}$). where is the multiplication in ring R. To see that this cup product induces a cup product of cohomology classes, we will use the lemma below which relates the cup product to coboundary maps.

Lemma 1.1.21 [13] if $\phi \in C^n$ (X) and $\psi \in C^n$ (X) then

$$\delta (\phi \smile \psi) = \delta\phi \smile \psi + (-1)^n \phi \smile \delta\psi$$

for more explicit description of cup product follow [[3], [13]] and[[22]]

Definition 1.1.22.[25]

Let G be a group, A a $\mathbb{Z}G$-module and R_*^G a free $\mathbb{Z}G$-resolution of $\mathbb{Z}$. An element $\alpha \in Hom_{\mathbb{Z}G}$ (R_n^G, A) is called a n-cochain with coefficients in A.For the trivial $\mathbb{Z}G$-module A = $\mathbb{Z}$ and any group G there is a bilinear mapping:

$$H^p(G,\mathbb{Z}) \otimes H^q(G,\mathbb{Z}) \to H^{p+q}(G,\mathbb{Z}), ([\alpha],[\beta]) \mapsto [\alpha] \smile [\beta]$$

called the cup product. The cup product is associative and graded commutative,
i.e. $[\alpha] \smile [\beta] = (-1)^{pq}[\beta][\alpha]$ for cohomology classes $[\alpha]$, $[\beta]$ of degree p, q.

The cup product gives a multiplication on the direct sum of the cohomology groups
$H^*(G, \mathbb{Z}) = \bigoplus_{n\in N} H^n(G, \mathbb{Z})$. This multiplication turns $H^*(G, \mathbb{Z})$ into a ring which is called the integral cohomology ring of G. For more details the construction of the cup product $\alpha \smile \beta \in Hom_{\mathbb{Z}G}(R_{p+q}^G, A)$, where $\alpha \in Hom_{\mathbb{Z}G}$ (R_p^G, A) and $\beta \in Hom_{\mathbb{Z}G}$ (R_p^G, A) see **[5]**. There are several ways to

construct the cup product. One way is to lift α to family of $\mathbb{Z}G$-equivariant homomorphisms $\alpha_n : R^G_{p+n} \to R^G_n, n \geq 0$, statisfying

$$\alpha_{n-1}\vartheta_{p+n} = \vartheta_n\alpha_n \quad for\ n \geq 1\ and\ \alpha = \epsilon\alpha_0 \quad with\ \epsilon\colon R^G_0 \twoheadrightarrow H_0(R^G_*) \cong \mathbb{Z}$$

the canonical quotient map.

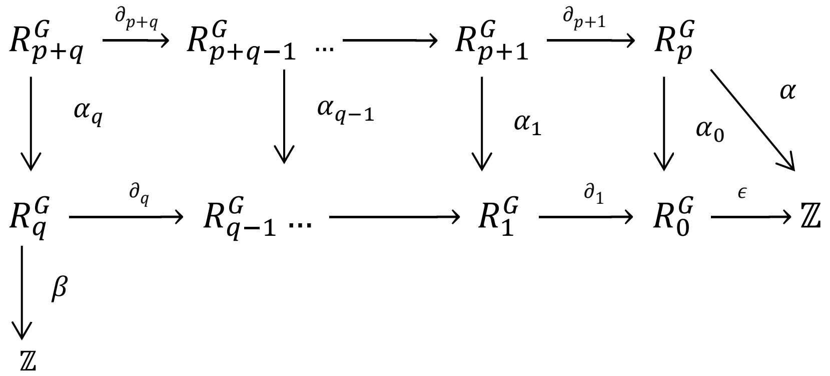

Figure 1.1.3

The composite homomorphism $\beta\alpha_q : R^G_{p+q} \to \mathbb{Z}$ is then a $(p + q)$-cocycle. Any contracting homotopy on R^G_* can be used to compute the homomorphisms α_n (for more details see [5],[25],[20])

Definition 1.1.23:[5, 27]

A segment connecting any two points $a, b \in X$, a subset $X\ of\ \mathbb{R}^n$ is called Convex, and it implies that for every pair and $\lambda \in [0,1]$, which is real number, one has $\lambda a + (1 - \lambda)b \in X$. The finite set $X = \{a_1, a_2, \dots, a_d\}$ of points in $\mathbb{R}^n$ called *convex hull* and denoted *conv*(X),

$$conv(X) = \{\textstyle\sum_{i=1}^{d} \lambda_i a_i : a_i \epsilon X, \lambda_i \geq 0, \sum_{i=1}^{d} \lambda_i = 1\}$$

Definition 1.1.24 :[13]

CW-complex is an abbreviation of what means "closure-finite weak topology" which is obtained by taking the union of a sequence of topological spaces

$$X_0 \subset X_1 \subset X_2 \subset \cdots$$

Each space X_n is formed from X_{n-1} such that $X_n = X_{n-1} \coprod_{\propto} e^n_{\propto}$ where each $e^n_{\propto}$ is an open n-disk. X is called CW-complex if $X = \cup_n X_n$

Definition 1.1.25:[17]

To construct a contractible CW-space EG, there are various ways for any group G, where G acts freely; the action changes the cell when referring to EG as a total space for G. The base space $BG = EG/G$ is a quotient space. A classification space for G is produced by omitting the action. We have a free $\mathbb{Z}$G $-$ resolution which is the cellular chain complex $C_*(EG)$ and $C_*(BG) = C_*(EG) \otimes_{\mathbb{Z}G} \mathbb{Z}$. Thus, $H^*(BG, \mathbb{Z}_p) = H^*(G, \mathbb{Z}_p)$.

Definition 1.1.26:[14]

(Fibre Bundle). A fibre bundle(locally trivial fibration) is a collection (E, B, F, ϑ) where E, B , F are topological spaces and $\vartheta: E \to B$ is acontinuous surjection.

- E is the total space.
- B is the Bace space.
- F is the fibre.
- $\vartheta: E \to B$ is the (bundle) projection.

For each $b \in B$ there exist an open neighborhood $U \subseteq B$and homeomorphism $\varphi: \vartheta^{-1}(U) \to U \times F$ such that the following diagram is commutes

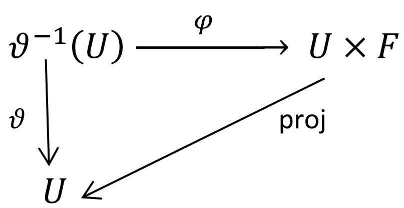

Figure 1.1.4

It follows that for every $b \in B, \vartheta^{-1}(b) \cong F$

Noting that $F_b \coloneqq \vartheta^{-1}(b) \xrightarrow{\varphi} \{b\} \times F$.

Definition 1.1.27:[14]

let E, B , F are topological spaces an n-dimensional real(resp. complex)vector bundle over B is a fibre bundle $\vartheta: E \to B$ along with a real vector space structure on each fiber F_b such that for every $b \in$ B there exist a neiborhood U and

$\varphi: \vartheta^{-1}(U) \to U \times \mathbb{R}^n$ (resp.$\mathbb{C}^n$) where $\varphi|_{F_b}: F_b \xrightarrow{\cong} \mathbb{R}^n$(resp.$\mathbb{C}^n$) is an isomorphism of vector spaces.

Definition 1.1.28:[29]

Let E, B, X are topological spaces a map $\vartheta: E \to B$ is said to have the covering homotopy property (CHP) with respect to X, iff given $f: X \to E$ and homotopy $h_t: X \to B$ satisfying $\vartheta f = h_0$.

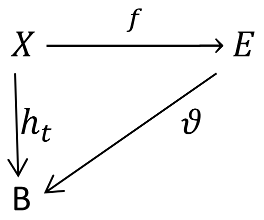

Figure 1.1.5

Then there exists a homotopy $f_t: X \to E$ such that $f_0 = f$ and $\vartheta f_t = h_t$ for all $t \in I = [0, 1]$. A map ϑ ia a fibration with respect any class of spaces (Q) if it has the (CHP) for every $X \in Q$.

Definition 1.1.29:[27]

Let E, B, X are topological spaces a map $\vartheta: E \to B$ is said to have the lowering homotopy property (LHP) with respect to X, iff given $h: B \to X$ and homotopy $f_t: X \to B$ satisfying $h\vartheta = f_0$,

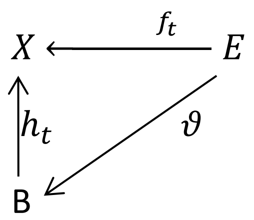

Figure 1.1.6

Then there exists a homotopy $h_t: B \to X$ such that $h_0 = h$ and $h_t\vartheta = f_t$. for all $t \in I = [0, 1]$. A map ϑ ia a cofibration with respect any class of spaces (Q) if it has the (LHP) for every$X \in Q$.

1.2 Data type

In this chapter we give an explain of the data types which we used in our work especially in chapter four and five with some examples.

1.2.1 HAP free $\mathbb{Z}\boldsymbol{G}-\boldsymbol{resolution}$.[4,5]

A free $\mathbb{Z}G-resolution$ is an exact sequence of $\mathbb{Z}G-modules$

$$\mathcal{M}_* = \cdots \to \mathcal{M}_n \to \mathcal{M}_{n-1} \to \cdots \to \mathcal{M}_1 \to \mathcal{M}_0 \to \mathbb{Z}$$

$such\ that\ \mathcal{M}_n$ is a free $\mathbb{Z}G-modules$

<u>Definitions 1.2.1</u>: A free $\mathbb{Z}G-resolution$ is represented as a component object $\mathcal{M}$ with the following components:

- $\mathcal{M}$!. Dimension (k) is a function which return the $\mathbb{Z}\boldsymbol{G}-\boldsymbol{r}ank$ of the module $\mathcal{M}_k$.
- $\mathcal{M}$!.elts is a (partial) list of (possibly duplicate) elements in G
- $\mathcal{M}$!. Boundary(k,j) is a function which returns the image in $\mathcal{M}_{k-1}$ of the j-th free generator of the $\mathbb{Z}\boldsymbol{G}-modules\ \mathcal{M}_k$. Let us denote by e_{ij}^k the free $\mathbb{Z}\boldsymbol{G}-generators$ of $\mathcal{M}_k$. To represent a word $\omega = \eta_{i1}e_{i1}^{k-1} + \eta_{i2}e_{i2}^{k-1} + \cdots + \eta_{ij}e_{ij}^{k-1}$, where $\eta_{ij} \in \mathbb{Z}\boldsymbol{G}$, with $\eta_{ij} = \sum_{i=1}^{j} a_i g_i$, $(g_i \in G\ and\ a_i \neq \pm 1)$

We use a list of integer pairs $\omega = [[a_1 i_1, i_1'], [a_2 i_2, i_2'], \dots, [a_k i_k, i_k']]$

With $\mathcal{M}$!.elts $[i_k'] = g_k$ for k=1, 2, 3, …, h

- $\mathcal{M}$!. Homotopy(k,[I,g]) is a function which returns the image in $\mathcal{M}_{k+1}$,of the element ge_i^k in $\mathcal{M}_k$.
- $\mathcal{M}$!. Group is the group G
- $\mathcal{M}$!. Properties is a list of pairs ["name" ,value] where "name" is a string and value is a numerical or Boolean value.

Examples 1.2.2:

1) let we take the resolution for the eight- elements quaternion group Q_8 such that

$Q_8 \coloneqq \{\pm 1, \pm i, \pm j, \pm k\}$, such that, $ij = k \; and \; jk = i \; and \; ki = j$

Another representation of Q_8 is

$$Q_8 \coloneqq \langle \mathrm{i}, \mathrm{j} \;/\; \mathrm{i}^2 = \mathrm{j}^2 = (\mathrm{ij})^2 = -1, \mathrm{ji} = -\mathrm{ij}, (-1)^2 = 1 \rangle$$

$$= \{1, a, b, b^2, \; ab, ab^2, a^2b, abb^2\}; \; a, b \text{ are ageneratores } of \, Q_8$$

$$\dots \xrightarrow{d_4} (\mathbb{Z}[Q_8])^3 \xrightarrow{d_3} (\mathbb{Z}[Q_8])^3 \xrightarrow{d_2} (\mathbb{Z}[Q_8])^2 \xrightarrow{d_1} \mathbb{Z}[\,Q_8] \xrightarrow{\varepsilon} 0$$

To compute $d_1$we have

$d_1(1, a) = a - 1$

$d_1(1, b) = b - 1$, we get a (1×2) matrix rerepresentation for d_1 such that $d_1 = (a - 1 \quad b - 1)$

To compute d_2 by the same way, we get a (2×3) matrix rerepresentation for d_2 such that $d_2 = \begin{pmatrix} ab + 1 & ab^2 + b^2 + a + 1 & -a - 1 \\ a - 1 & 0 & 1 + b \end{pmatrix}$

And $d_3 = \begin{pmatrix} 0 & ab - 1 & -a - 1 \\ a - 1 & 0 & b \\ 0 & a - 1 & b - 1 \end{pmatrix}$.

We tensor with $\mathbb{Z}$ over the group ring, i.e $(- \otimes_{\mathbb{Z}[Q_8]} \mathbb{Z})$ to get,

$$\dots \longrightarrow \mathbb{Z}^3 \xrightarrow{\begin{pmatrix} 0 & 0 & -2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}} \mathbb{Z}^3 \xrightarrow{\begin{pmatrix} 2 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}} \mathbb{Z}^2 \xrightarrow{(0 \quad 0)} \mathbb{Z} \longrightarrow 0$$

GAP session
gap> G:=QuaternionGroup(8); <pc group of size 8 with 3 generators>

```
gap> A:=Mod2SteenrodAlgebra(G,8);
<algebra of dimension 15 over GF(2)>
gap> gens:=ModPRingGenerators(A);
[ v.1, v.2, v.3, v.7 ]
gap> List(gens,A!.degree);
[ 0, 1, 1, 4 ]
gap> R:=ResolutionFiniteGroup(G,8);
Resolution of length 8 in characteristic 0 for <pc group of size 8 with
3 generators> .
gap> Elements(R!.elts);
[ <identity> of ..., x, y, y2, x*y, x*y2, y*y2, x*y*y2 ]
gap> R!.elts[5];
x*y
gap> R!.elts[6];
x*y2
gap> R!.elts[4];
y2
gap> List([0..8], R!.dimension);
[ 1, 2, 3, 3, 2, 2, 3, 3, 2 ]
gap> R!.boundary(1,1);
[ [ 1, 2 ], [ -1, 1 ] ]
gap> R!.boundary(1,2);
[ [ 1, 3 ], [ -1, 1 ] ]
gap> R!.boundary(2,1);
[ [ 1, 5 ], [ 2, 2 ], [ 1, 1 ], [ -2, 1 ] ]
gap> R!.boundary(2,2);
[ [ 1, 6 ], [ 1, 4 ], [ 1, 2 ], [ 1, 1 ] ]
gap> R!.boundary(2,3);
[ [ 2, 3 ], [ 2, 1 ], [ -1, 2 ], [ -1, 1 ] ]
gap> R!.boundary(3,1);
[ [ 2, 2 ], [ -2, 1 ] ]
gap> R!.boundary(3,2);
[ [ 1, 5 ], [ 3, 2 ], [ -3, 1 ], [ -1, 1 ] ]
gap> R!.boundary(3,3);
[ [ 3, 3 ], [ 2, 3 ], [ -3, 1 ], [ -1, 1 ], [ -1, 2 ] ]
gap> R!.boundary(4,1);
[ [ 1, 6 ], [ 1, 4 ], [ 1, 2 ], [ 1, 1 ] ]
gap> R!.boundary(4,2);
[ [ 2, 5 ], [ 3, 2 ], [ 1, 5 ], [ 2, 1 ], [ -3, 1 ] ]
gap> T:=TensorWithIntegers(R);
Chain complex of length 9 in characteristic 0 .
```

```
gap> BoundaryMatrix(T,1);
[ [ 0, 0 ] ]
gap> BoundaryMatrix(T,2);
[ [ 2, 4, -2 ], [ 0, 0, 2 ] ]
gap> BoundaryMatrix(T,3);
[ [ 0, 0, -2 ], [ 0, 0, 1 ], [ 0, 0, 0 ] ]
gap> BoundaryMatrix(T,4);
[ [ 4, 1 ], [ 0, 2 ], [ 0, 0 ] ]
```

2) For another example on computing the boundarys let we take G:=SymmetricGroup(3); we have S_3 ={ e , (2,3), (1,2), (1,2,3), (1,3,2), (1,3) }

$$\dots \xrightarrow{d_4} (\mathbb{Z}[S_3])^4 \xrightarrow{d_3} (\mathbb{Z}[S_3])^3 \xrightarrow{d_2} (\mathbb{Z}[S_3])^2 \xrightarrow{d_1} \mathbb{Z}[\, S_3] \xrightarrow{\varepsilon} 0$$

$$d_1 = (g_2 - g_1 \quad g_3 - g_1) \Rightarrow d_1 = ((2,3) - 1 \quad (1,2) - 1)$$

$$d_2 = \begin{pmatrix} g_2 + g_1 & 0 & g_3 - g_4 - g_1 \\ 0 & g_3 + g_1 & g_5 + g_4 + g_1 \end{pmatrix} \Rightarrow$$

$$d_2 = \begin{pmatrix} (2,3) + 1 & 0 & (1,2) - (1,2,3) - 1 \\ 0 & (1,2) + 1 & (1,3,2) + (1,2,3) + 1 \end{pmatrix}$$

by the same way we can found d_3 such that

$$d_3 = \begin{pmatrix} g_2 - g_1 & 0 & g_3 - g_1 & -g_4 \\ 0 & g_2 - g_4 & 0 & g_5 + g_4 + g_1 \\ 0 & 0 & g_4 - g_1 & -g_3 - g_1 \end{pmatrix}$$

We tensor with $\mathbb{Z}$ over the group ring, i.e $(- \otimes_{\mathbb{Z}[S_3]} \mathbb{Z})$ to get:

$$\dots \longrightarrow \mathbb{Z}^4 \xrightarrow{\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -2 \end{pmatrix}} \mathbb{Z}^3 \xrightarrow{\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 3 \end{pmatrix}} \mathbb{Z}^2 \xrightarrow{(0 \quad 0)} \mathbb{Z} \longrightarrow 0$$

GAP session

```
gap> G:=SymmetricGroup(3);;
gap> R:=ResolutionFiniteGroup(G,6);
Resolution of length 6 in characteristic 0 for Group([ (1,2), (1,2,3) ]) .
gap> List([0..6],R!.dimension);
[ 1, 2, 3, 4, 5, 6, 7 ]
gap> R!.boundary(1,1);
[ [ 1, 2 ], [ -1, 1 ] ]
gap> R!.boundary(1,2);
[ [ 1, 3 ], [ -1, 1 ] ]
gap> R!.boundary(2,1);
[ [ 1, 2 ], [ 1, 1 ] ]
gap> R!.boundary(2,2);
[ [ 2, 3 ], [ 2, 1 ] ]
gap> R!.boundary(2,3);
[ [ 2, 5 ], [ 2, 4 ], [ 1, 3 ], [ 2, 1 ], [ -1, 4 ], [ -1, 1 ] ]
gap> R!.boundary(3,1);
[ [ 1, 2 ], [ -1, 1 ] ]
gap> R!.boundary(3,2);
[ [ 2, 2 ], [ -2, 4 ] ]
gap> R!.boundary(3,3);
[ [ 3, 4 ], [ 1, 3 ], [ -3, 1 ], [ -1, 1 ] ]
gap> R!.boundary(3,4);
[ [ 2, 5 ], [ 2, 4 ], [ 2, 1 ], [ -3, 3 ], [ -1, 4 ], [ -3, 1 ] ]
gap> R!.elts;
[ (), (2,3), (1,2), (1,2,3), (1,3,2), (1,3) ]
gap> Elements(R!.elts);
[ (), (2,3), (1,2), (1,2,3), (1,3,2), (1,3) ]
gap> R!.elts[2];
```

```
(2,3)
gap> R!.elts[3];
(1,2)   ...ets
gap> R!.elts[4];
(1,2,3)
gap> R!.elts[5];
(1,3,2)

gap> T:=TensorWithIntegers(R);
Chain complex of length 6 in characteristic 0 .

gap> BoundaryMatrix(T,1);
[ [ 0, 0 ] ]
gap> BoundaryMatrix(T,2);
[ [ 2, 0, -1 ], [ 0, 2, 3 ] ]
gap> BoundaryMatrix(T,3);
[ [ 0, 0, 0, -1 ], [ 0, 0, 0, 3 ], [ 0, 0, 0, -2 ] ]
gap> BoundaryMatrix(T,4);
[ [ 2, 0, -1, 1, 0 ], [ 0, 2, 0, 1, 0 ], [ 0, 0, 3, 0, 0 ], [ 0, 0, 0, 0, 0 ] ]
```

1.2.2 HAP chain complex and cochain complex

We have a chain complex of free $\mathbb{Z}-modules$.

$$\cdots \to C_n \longrightarrow C_{n-1} \longrightarrow \cdots \longrightarrow C_1 \longrightarrow C_0$$

We can compute the co-chain complex and chain complex by recording C with the following components.

- C!.dimension(h) is a function which returns the $\mathbb{Z}-rank$ of the module $C_{h.}$
- C!.boundary(h,i) is a function which returns the image in C_{h-1} of the i-th free generator of $C_{h.}$(Elements in C_{h-1} are represented in the obvious way as free vectors of length to the rank of C_{h-1}).
- C!.properties is a list of pairs["name",value] where "name" is a string and value is numerical or Boolean value. Example pairs are:["length", n] which would record that there are n terms in the chain complex. A

cochain complex

$$\cdots \to C_0 \longrightarrow C_1 \longrightarrow \cdots \longrightarrow C_n \longrightarrow C_{n+1} \longrightarrow \ldots$$

Is represented by a similar record C. the difference is that C!.boundary(h,i) returns an element in C_{h+1}).

By taking the same example we can get the chain complex and the cochain complex.

GAP session

```
gap> T:=TensorWithIntegers(R);
Chain complex of length 8 in characteristic 0 .

gap> List([0..8],T!.dimension);
[ 1, 2, 3, 3, 2, 2, 3, 3, 2 ]
gap> T!.boundary(1,1);
[ 0 ]
gap> T!.boundary(1,2);
[ 0 ]
gap> T!.boundary(2,1);
[ 2, 0 ]
gap> T!.boundary(2,2);
[ 4, 0 ]
gap> T!.boundary(2,3);
[ -2, 2 ]
gap> T!.boundary(3,1);
[ 0, 0, 0 ]
gap> T!.boundary(3,2);
[ 0, 0, 0 ]
gap> T!.boundary(3,3);
[ -2, 1, 0 ]
gap> F:=HomToIntegers(R);
Cochain complex of length 8 in characteristic 0 .

gap> List([0..8],F!.dimension);
[ 1, 2, 3, 3, 2, 2, 3, 3, 2 ]
gap> F!.boundary(0,1);
[ 0, 0 ]
gap> F!.boundary(1,1);
[ 2, 4, -2 ]
```

```
gap> F!.boundary(2,1);
[ 0, 0, -2 ]
gap> F!.boundary(1,2);
[ 0, 0, 2 ]
```

CHAPTER TWO

Fuzzy (Serre)Cofibration

Chapter Two:

This chapter consists of five sections. In section one, are clarified the basic concepts of fuzzy cofibration and the concept of the fuzzy serre cofibration, with an explanation of the most important fuzzy topological algebraic properties is presented. In section two, we introduce the concept of mixing fuzzy cofibration and in the last sections in this chapter we study the concept of mixing fuzzy serre cofibration, Fuzzy homotopy Extention property, and mixing criterion for a fuzzy map to be a mixing fuzzy Serre Cofibration.

2.1 Fuzzy cofibration and a fuzzy serre cofibration

In this section, we study and introduce the concept of the fuzzy cofibration, the fuzzy serre cofibration and presented some characteristics and theories of these concepts with examples.

Definition 2.1.1[19]

A fuzzy set γ is a function $\gamma: X \to I$ such that $X \neq \emptyset$ and $I = [0,1]$ the family of all fuzzy sets is denoted by I^X.

Definition 2.1.2 [2]

Let X be a set and τ be a family of fuzzy subsets of X, then τ is called fuzzy topology on X if satisfies the following condition

- $0_X \; and \; 1_X \; \in \tau.$
- If $\gamma, \mu \in \tau, then \;\; \gamma \wedge \mu \in \tau.$
- If $\gamma_i \in \tau \; for \; each \; i \in I, then \; \vee \gamma_i \in \tau.$

The order (X, τ) is said to be fuzzy topological space (F.T.S) moreover the members of τ are said to be the fuzzy open sets and their complements are said to be the fuzzy closed sets.

Definition 2.1.3 [10]

Let $(X, \tau_1) and (Y, \tau_2)$ be fuzzy topological spaces,and $u, v: X \to Y$are fuzzy continuous functions, then . u is a fuzzy homotopy to v if there exist a family of fuzzy continuous functions $F_t: X \to Y$ such that for all $t \in I$

- $F_t(x) = u_t(x)$
- $F_0(x) = u(x)$
- $F_1(x) = v(x)$

if u and v are fuzzy homotopic functions, then we write "$u \sim v$"

For example every fuzzy continuous function is a fuzzy homotopic to itself.

Definition 2.1.4: (fuzzy cofiberation and fuzzy serre cofibration)

a map $\rho: A \to X$ is said to have the fuzzy lowering homotopy property (FLHP) with respect to Y if and only if given a fuzzy map $f: X \to Y$ and a fuzzy homotopy $\bar{F}_t: A \to Y$ satisfying $f\rho = \bar{F}_0$; then there exist another fuzzy homotopy $F_t: X \to Y, with\ F_0 = f\ and\ F_t\rho = \bar{F}_t\ for\ all\ t \in I.$

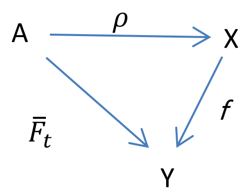

Figure 2.1.1

- Let Q be a class of fuzzy topological spaces a fuzzy map $\rho: A \to X$ is a fuzzy cofibration with respect to Q if and only if ρ has the (FLHP) with respect to each X ∈ Q.
- If Q is the class of all fuzzy topological spaces ,then $\rho: A \to X$ is called fuzzy Hurewicz cofiberation.
- If Q is the class of all fuzzy CW-complex spaces ,then $\rho: A \to X$ is called fuzzy serre cofibration.
- If $A \subset X$, the inclusion map is a fuzzy cofiberation.
- The pair (X,A) is called fuzzy cofibred pair.
- A necessary condition for (X, A) to be a cofibred pair is the existence of a retraction

$$r: X \times I \to (X \times 0) \cup (A \times I).$$

If A is closed, this condition is also sufficient.

Example 2.1.5:

The fuzzy inclusion map $i: \mu_{S^{n-1}} \to \mu_{D^n}$ of the boundary sphere of solid disk is a fuzzy cofibration. In addition, if the space CW-complex then, the fuzzy inclusion map $i: \mu_{S^{n-1}} \to \mu_{D^n}$ is a fuzzy serre cofibration.

Definition 2.1.6:

$A \subset X$ is said to have the fuzzy homotopy extension property (FHEP) with respect to Y, if and only if any fuzzy partial homotopy: $\bar{H}: X \times \{0\} \cup A \times I \to Y$ can be extended to afuzzy homotopy $H: X \times I \to Y$.

If $A \subset X$ has the (FHEP) with respect to every Y,then $A \subset X$is said to have the fuzzy absolute homotopy extension property (FAHEP)

Theorem 2.1.7:

The cofibred pair (X, A) has (FHEP) with respect to Y, if and only if given a fuzzy homotopy $f_t: A \to Y$, and a fuzzy map $h_0: X \to Y$ such that $f_0 = h_0|A$, there exists a fuzzy homotopy $h_t: X \to Y$ that starts from the given fuzzy map h_0 and extends the fuzzy homotopy f_t, in the sense that $f_t = h_t|A$.

Proof: Suppose $\gamma: X \times I \to (X \times 0) \cup (A \times I)$ is a retraction. A fuzzy homotopy f_t and a fuzzy map h_0 as above combine to give a fuzzy map $g: X \times \{0\} \cup A \times I \to Y$. The desired fuzzy homotopy is $h_t(x) = g\big(\gamma(x,t)\big)$. Conversely, take Y=$(X \times 0) \cup (A \times I)$, $f_t(a) = (a,t)$, and $h_0(x) = (x,0)$.if (X, A) has the FHEP, the resulting homotopy $h_t: X \to Y$ can be rewritten as a retraction $X \times I \to Y$.

Example2.1.8:

The same pair $(\mu_{D^n}, \mu_{S^{n-1}})$has the FHEP. If given a fuzzy homotopy $f_t: \mu_{S^{n-1}} \to Y$ and a map $h_0: I^{D^n} \to Y$ such that $h_0|\mu_{S^{n-1}} = f_0$. We assemble these to form a fuzzy map $g: \mu_{D^n} \to Y$ from $\mu_{D^n} = \{\mu_x: D^2 \to I: \|x\| \leq 2\}$ by setting

$$g(\mu_x) = \begin{cases} h_0(\mu_x) & \text{for} \|x\| \leq 1 \\ f_{\|\mu_x\| \leq 1}\left(\frac{\mu_x}{\|\mu_x\|}\right) & \text{for } 1 \leq \|x\| \leq 2 \end{cases}.$$

The desired fuzzy homotopy $h_t: \mu_{D^n} \to Y$ is then just $h_t = g((1+t)x)$.

Theorem 2.1.9:

If $\rho: A \rightarrow X$ is a fuzzy serre cofibering of a fuzzy compact Hausdorff A into a fuzzy Hausdorff space X, then ρ must be an imbedding.

Proof:

First let us prove that ρ must be injective. This will be proved by contradiction. Assume that ρ were not one-one then there exist two distinct points u and v in A with $\rho(u) = \rho(v)$. Since A is fuzzy normal, and {u}, {v} are closed subsete of A, there exists a fuzzy continuous function $\gamma: A \rightarrow I$, with $\gamma(u) = 0 \; and \; \gamma(v) = 1$.

Define a fuzzy homotopy $\bar{F}_t: A \rightarrow I$ by $\bar{F}_t(a) = t\gamma(a), \; a \in A, t \in I = [0,1]$ and define$f: X \rightarrow I, with \; f(x) = 0 \;\; for \; all \; x \in X$. Then we have $f\rho = \bar{F}_0$.

But $\rho: A \rightarrow X$ is a fuzzy serre cofiberation, hence there exists a fuzzy homotopy $F_t: X \rightarrow Y, with \; F_0 = f \; and \; F_t\rho = \bar{F}_t \; for \; all \; t \in I.$

Now $\gamma(u) = \bar{F}_1(u) = F_1(\rho(u))$ and $\gamma(v) = \bar{F}_1(v) = F_1(\rho(v))$

But $\rho(u) = \rho(v) \Longrightarrow \gamma(u) = \gamma(v)$

But $\gamma(u) = 0 \neq 1 = \gamma(v) \Rightarrow \rho$ must be injective.

But A is a fuzzy compact Hausdorff and X is a fuzzy Hausdorff space, hence ρ must be an imbedding.

Proposition 2.1.10:

(X, A) is fuzzy closed cofibration iff there are a fuzzy maps $D: X \times I \rightarrow X$, $\varphi: X \rightarrow I$ such that:

1) $D(x, 0) = x \quad for \; all \;\; x \in X$
2) $D(a, t) = a \quad for \; all \;\; a \in A, t \in I.$
3) $A = \varphi^{-1}(1) \quad and \;\; D(\varphi^{-1}(0,1] \times 1) \subset A.$

Proof: $A \subset X$ is a fuzzy cofibration iff there is a retraction

$$r: X \times I \rightarrow (X \times 0) \cup (A \times I).$$

If $A \subset X$ is a fuzzy cofibration, define $D: X \times I \rightarrow X$ by $D(x, t) = \pi_1 r(x, t)$ where π_1 is the projection on the first coordinate. Then D satisfies "1" and "2" since r is a retraction. Define $\varphi_1: X \rightarrow I$, by

$\varphi_1(x) = \max_{t\in I}|\pi_2 r(x,t) - t|$. Then $\varphi_1(x) = 0 \ \ iff \ \ x \in A$ since $\varphi_1(x) = 0 \ \ \Longrightarrow \pi_2 r(x,t) = t$ for all $t \in I$, for $t \neq 0 \ \ r(x,t) \in A \times I$. However $r(x,0) = (x,0) \ and \ thus \ \ x \in A$. Define $\varphi_2 : X \to I$,by $\varphi_2(x) = \pi_2 r(x,1)$. Then $\varphi_2(a)$=1 if $a \in A$ and if $\varphi_2(x) \neq 0$, $D(x,1) \in A$. Thus if we let $\varphi(x) = \big(1 - \varphi_1(x)\big) \cdot \varphi_2(x)$, φ satisfied condition "3".

Conversely, given D and φ, define a retraction

$$r: X \times I \to (X \times 0) \cup (A \times I).$$

As follows:

$$r(x,t) = \begin{cases} (x,0) & if\ \varphi(x) = 0. \\ (D(x, 2\varphi(x)\cdot t), 0) & if \ \ 0 < \varphi(x) \leq \frac{1}{2} \\ \left(D\left(x, \frac{t}{2\big(1-\varphi(x)\big)}\right), 0\right) & if\ \frac{1}{2} \leq \varphi(x) < 1, \ \ t \leq 2\big(1-\varphi(x)\big) \\ \Big(D(x,1), t - 2\big(1-\varphi(x)\big)\Big), & if \frac{1}{2} \leq \varphi(x) < 1 \ and \ \ 2\big(1-\varphi(x)\big) \leq t \leq 1 \\ (x,t) & if\ \varphi(x) = 1 \end{cases}$$

Definition 2.1.11:

let $\rho: A \to X$ be a fuzzy cofiberation and let Y be any fuzzy topological space and $f: X \to Y$ be any fuzzy map and $Af \subset A \times Y$ be any fuzzy subspace such that $Af = \{(a,y) \in A \times Y;\ \rho(a) = f^{-1}(y)\}$ of the Cartesian product.

Let $\rho^*: Af \to Y$ be the projection $\rho^*(a,y) = y$ then ρ^* is called the fuzzy pullback of ρ by f. Define a fuzzy map $\pi: A \to Af$ such that $\pi(a) = (a, \rho(a))$, it is immediate from the definitions that the diagram is commutative such that $\rho^* \circ \pi = f \circ \rho$

$$\begin{array}{ccc} Af & \xleftarrow{\ \pi\ } & A \\ \rho^* \downarrow & \circlearrowleft & \downarrow \rho \\ Y & \xleftarrow[\ f\ ]{} & X \end{array}$$

Figure 2.1.2

Example 2.1.12:

Let we take the fuzzy cofibration $i: \mu_{S^{n-1}} \hookrightarrow \mu_{D^n}$ and let Y=μ_{S^n} , we have a fuzzy inclusion map $f: \mu_{D^n} \hookrightarrow \mu_{S^n}$ and $\mu_{S^{n-1}}f \subset \mu_{S^{n-1}} \times \mu_{S^n}$, the projection $\rho^*: \mu_{S^{n-1}}f \to \mu_{S^n}$ such that $\rho^*(\mu_x, \mu_y) = \mu_y$ is a fuzzy pullback of i by f .

Theorem 2.1.3: The fuzzy pullback of a fuzzy cofibration is also fuzzy cofibration**.**

Proof:

Let Z be fuzzy topological space we will show that ρ^* has the (FLHP) with respect to Z.

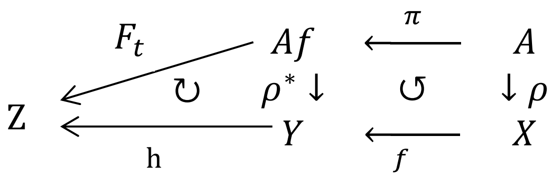

Figure 2.1.3

Let $h: Y \to Z$ be a fuzzy map and $F_t: Af \to Z$ be a fuzzy homotopy such that $h\rho^* = F_0$

Consider $F_t \circ \pi : A \to Z$ and $h \circ f : X \to Z$ are a fuzzy homotopic

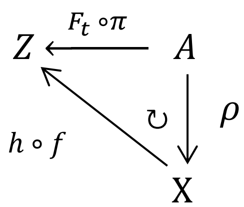

Figure 2.1.4

$h \circ f \circ \rho = F_t \circ \pi$ Since ($h\rho^* = F_0$ *and* ρ^* *is a fuzzy pullback* $\Rightarrow f \circ \rho = \rho^* \circ \pi$)

$h \circ \rho^* \circ \pi = F_t \circ \pi \Rightarrow F_0 \circ \pi = F_t \circ \pi \Rightarrow F_0 = F_t \ \forall\, t \in I$

$h \circ f \circ \rho = h \circ \rho^* \circ \pi$

But ρ is a fuzzy cofibration then there exist a fuzzy homotopy $H_t: X \to Z$; such that $H_0 = h \circ f$ and $H_t \circ \rho = F_t \circ \pi$

Define a fuzzy homotopy $\overline{F_t}: Y \to Z$ such that $\overline{F_t}(y) = h(y)$

$\Longrightarrow \overline{F_0}(y) = h(y)$ and $\overline{F_t} \circ \rho^*(y) = h \circ \rho^*(y) = F_t$

$\Longrightarrow (X, Af)$ is a cofiberd pair

2.2 Mixing fuzzy Cofibration

We will study the new concept of Mixing fuzzy (Hurewicz) cofibration (MF(H) Cofibration)

Definition 2.2.1:

Let , X_1,X_2 be three fuzzy topological spaces and let $f_1: X_1 \to Y$, $f_2: X_2 \to Y$ are two fuzzy fiber space and $\alpha: X_2 \to X_1$ such that $f_1 \circ \alpha = f_2$, let $X_i = \{X_1, X_2\}$, $f_i = \{f_1, f_2\}$, then$\{X_i,\ f_i,\ Y,\ \alpha\}$ where i=1,2, has Mixing fuzzy lowering homotopy property (M-FLHP) with respect to a space Z if and only if given a fuzzy map $h: Y \to Z$ and a fuzzy homotopy $g_t: X_1 \to Z$ satisfying $h \circ f_2 = g_0 \circ \alpha$ then there exist a fuzzy homotopy $h_t: Y \to Z$, with $h_0 = h$ and $h_t \circ f_1 = g_t$; $\forall\, t \in I = [0, 1]$, M-fuzzy fiber space is called M-fuzzy cofibration for class of all fuzzy topological spaces if f has (M-FLHP) for each fuzzy topological spaces Z.

Proposition 2.2.2: Every fuzzy Cofibration is mixing fuzzy Cofibration.

Proof:

Let $\{X_i,\ f_i,\ Y,\ \alpha\}$ be a M-fuzzy fiber space such that $X_2 = X_1 = X$, α is the fuzzy identity map $f_i = f_1 = f_2$,let $h: Y \to Z$ and a fuzzy homoyopy $g_t: X_1 \to Z$ such that $h \circ f_2 = g_0 \circ \alpha$, then there exist a fuzzy homotopy $h_t: Y \to Z$, with $h_0 = h$ and $h_t \circ f_1 = g_t$; $\forall\, t \in I$ then f has (M-FLHP) with respect to Z therefore (Y,X) is a mixed fuzzy cofibred pair.

Proposition 2.2.3:

Let $f_i: X_i \to Y$, $f_i{}': X_i{}' \to Y'$ be two M-fuzzy cofibration then $f_i \times f'{}_i: X_i \times X'{}_i \to Y \times Y'$, is also M-fuzzy cofibration.

Proof:

let Z be a fuzzy topological space and let $h^*: Y \times Y' \to Z$ be a fuzzy map where $h: Y \to Z$ and $h': Y' \to Z$ and we define a fuzzy homotopy $g_t{}^*: X \times X' \to Z$; as $h^* \circ (f_2 \times f_2{}') = g_0{}^* \circ (\alpha \times \alpha')$, such that $g_t{}': X_1{}' \to Z$ and $g_t: X_1 \to Z$,

Since f and f' are M- fuzzy cofibration then there exist a fuzzy homotopy $h_t: Y \to Z$, with $h_0 = h$ and $h_t \circ f_1 = g_t$; $\forall\, t \in I$ and a homotopy $h_t{}': Y' \to Z$ with $h_0{}' = h'$ and $h_t{}' \circ f_1{}' = g_t$; now for the fuzzy homotopy $h_t{}^*: Y \times Y' \to Z$,define as $h_t{}^* \circ (f_1 \times f_1{}') = g_t{}^*$, and $h_0{}^* = h^*$, therefore $f_i \times f'{}_i: X_i \times X'{}_i \to Y \times Y'$, is also M-fuzzy cofibration.

Definition 2.2.4:

Let $\{X_i,\ f_i,\ Y,\ \alpha\}$ be a M-fuzzy fiber space such that X is any fuzzy topological space and $h: Y' \to Y$ any continuous fuzzy map into base Y let $X_1{}' = \{(x_1, y) \in X_1 \times Y: f_1(x_1) = h(y')\}$ and $X_2{}' = \{(x_2, y) \in X_2 \times Y: f_2(x_2) = h(y')\}$,then $X_i{}' = \{ X_1{}', X_2{}'\}$ is called M-fuzzy pullback of f by h and $f_i{}' = \{f_1{}', f_2{}'\}: X' \to Y$ is called induced M-fuzzy function of f_i by h. Define $\alpha': X_2{}' \to X_1$ by $\alpha'(x_2, y') = (\alpha(x_2), y')$ to show that α' is continuous since $\alpha' = \alpha \times I_{y'}$, α is continuous and $I_{y'}$ is continuous then α' is continuous to show that α' is a commutative $f_1{}' \circ \alpha'(x_2, y') = f_1{}'(\alpha(x_2), y') = y'$, $f_2{}'(x_2, y') = y'$ therefore $f_1{}' \circ \alpha' = f_2{}'$.

Proposition 2.2.5:

The Mixing-fuzzy pullback of Mixing-fuzzy cofibration is also Mixing-fuzzy cofibration.

Proof:

let $h: Y \to Z$ and $h': Y' \to Z$ be two fuzzy map . define a fuzzy homotopy $g_t: X_1 \to Z$ such that $h \circ f_2 = g_0 \circ \alpha$. Since f has M-fuzzy cofibration then there exist afuzzy homotopy $h_t: Y \to Z$, with $h_0 = h$ and $h_t \circ f_1 = g_t$; $\forall\, t \in I$ define a fuzzy homotopy $g_t{}': X_1{}' \to Z$,such that $h' \circ f_2{}' = g_0{}' \circ \alpha$ and $g_t{}' = g_t \circ L$ then there exist a fuzzy homotopy $h_t{}': Y' \to Z$ with $h_0{}' = h'$ and $h_t{}' \circ f_1{}' = g_t$ therefore $f': X' \to Y'$ has a M-fuzzy cofibration.

2.3 Mixing fuzzy Serre Cofibration (M-FS Cofibration).

Definition 2.3.1:

Let $\mathbb{Z}$ be a fuzzy CW-complex $f_1: X_1 \to \mathbb{Z}$, $f_2: X_2 \to \mathbb{Z}$ are two fuzzy fiber space and let $\alpha: X_2 \to X_1$ such that $f_1 \circ \alpha = f_2$, let $X_i = \{X_1, X_2\}$, $f_i = \{f_1, f_2\}$, the $\{X_i, f_i, \mathbb{Z}, \alpha\}$, has Mixing fuzzy lowering homotopy property (M-FLHP) with respace to a fuzzy CW-complex Y if and only if $K: \mathbb{Z} \to Y$ and a fuzzy homotopy $g_t: X_1 \to Y$ satisfying $K \circ f_2 = g_0 \circ \alpha$ then there exist a fuzzy homotopy $K_t: Y \to \mathbb{Z}$, with $K_0 = K$ and $K_t \circ f_1 = g_t$; $\forall\, t \in I = [0,1]$, M-fuzzy fiber space is called M-fuzzy cofibration for class of all fuzzy topological spaces if f has (M-FLHP) .

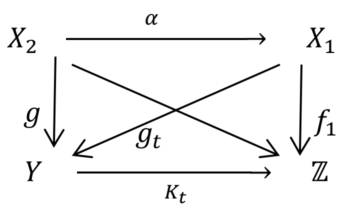

Figure 2.3.1

Proposition 2.3.2:

Every fuzzy Serre Cofibration is mixing fuzzy Serre Cofibration.

Proof:

let $\{X_i,\ f_i,\ Y,\ \alpha\}$ be a M-fuzzy fiber space such that $X_2 = X_1 = X$, α is the fuzzy identity map $f = f_1 = f_2$,let $K\colon Y \to Z$ and a fuzzy homoyopy $g_t\colon X_1 \to Z$ such that $K \circ f_2 = g_0 \circ \alpha$, then there exist a fuzzy homotopy $K_t\colon Y \to Z$, with $K_0 = K$ and $K_t \circ f_1 = g_t$; $\forall\ t \in I$ then f_i has (M-FLHP) with respect to Z, therefore $(X_i,\ Y)$ is a mixing fuzzy cofibred pair.

Corollary 2.3.3:

Let $f_i\colon X_i \to Y$, $f_i{}'\colon X_i{}' \to Y'$ be two M-fuzzy cofibration then $f_i \times f_i{}'\colon X_i \times X_i{}' \to Y \times Y'$, is also M-fuzzy cofibration.

Proof:

let Z be a fuzzy topological space and let $K^*\colon Y \times Y' \to Z$ be a fuzzy map where $K\colon Y \to Z$ and $K'\colon Y' \to Z$ and we define a fuzzy homotopy $g_t{}^*\colon X_1 \times X_1{}' \to Z$;

as $K^* \circ (f_2 \times f_2{}') = g_0{}^* \circ (\alpha \times \alpha')$, such that $g_t{}'\colon X_1{}' \to Z$ and $g_t\colon X_1 \to Z$,

Since f_i *and* $f_i{}'$ are M- fuzzy cofibration. then there exist a fuzzy homotopy $K_t\colon Y \to Z$, with $K_0 = K$ and $K_t \circ f_1 = g_t$; $\forall\ t \in I$ and a homotopy $K_t{}'\colon Y' \to Z$ with $K_0{}' = K'$ and $K_t{}' \circ f_1{}' = g_t$; now for the fuzzy homotopy $K_t{}^*\colon Y \times Y' \to Z$,define as $K_t{}^* \circ (f_1 \times f_1{}') = g_t{}^*$, and $K_0{}^* = K^*$, therefore $f_i \times f_i{}'\colon X_i \times X_i{}' \to Y \times Y'$, is also M-fuzzy cofibration.

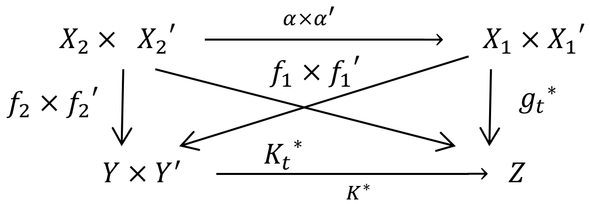

Figure 2.3.2

Proposition 2.3.4:

The Mixing fuzzy pullback of Mixing fuzzy Serre cofibration is also Mixing fuzzy Serre cofibration.

Proof:

let $h: Y \to Z$ and $h': Y' \to Z$ be two fuzzy map . define a fuzzy homotopy $g_t: X_1 \to Z$ such that $h \circ f_2 = g_0 \circ \alpha$. Since f has M-fuzzy cofibration then there exist afuzzy homotopy $h_t: Y \to Z,$ with $h_0 = h$ and $h_t \circ f_1 = g_t$; $\forall\, t \in I$ define a fuzzy homotopy $g_t{}': X_1{}' \to Z$,such that $h' \circ f_2{}' = g_0{}' \circ \alpha$ and $g_t{}' = g_t \circ L$ then there exist a fuzzy homotopy $h_t{}': Y' \to Z$ with $h_0{}' = h'$ and $h_t{}' \circ f_1{}' = g_t$ therefore $f': X' \to Y'$ has a M-fuzzy cofibration.

2.4 **Fuzzy homotopy Extention property:**

A fuzzy subset $A \subset Y$ is said to have the fuzzy homotopy extension property (FHEP) with respect to Y , iff any fuzzy partial homotopy*:*

$$\overline{H}: Y \times \{0\} \cup A \times I \to Z$$

can be extended to a fuzzy homotopy $H: Y \times I \to Z$ If A has the (FHEP) with respect to every Z, then A is said to have the absolute fuzzy homotopy extension property (AFHEP).

Definition 2.4.1:

A fuzzy subset $\mathrm{A} \subset \mathrm{Y}$ has the Fiber fuzzy Homotopy Extension Property (FFHEP) iff for any fuzzy Serre fibration $\mathrm{q}: \mathrm{E} \to \mathrm{B}$ and any fuzzy map $G: Y \times \{0\} \cup A \times I \to E$ such that $qG(y,t) = qG(y,0)$ where $y \in Y, 0 \le t \le 1$ there is an extension $H: Y \times I \to E$ of G such that $qG(y,t) = qG(y,0)$.

Definition 2.4.2:

A fuzzy subset $A_1, A_2 \subset Y$ has the mixing fiber fuzzy homotopy extension property (M-FFHEP) iff for any M- fuzzy Serre fibration $q_i : E_i \rightarrow B$ where $i = 1,2$, and map $G_1: Y \times \{0\} \cup A_1 \times I \rightarrow E_1$, $G_2: Y \times \{0\} \cup A_2 \times I \rightarrow E_2$ such that $p_1 G_1(y_1, t) = p_1 G_1(y_1, 0)$, $p_2 G_2(y_2, t) = p_2 G_2(y_2, 0)$, where $y_1, y_2 \in Y, 0 \leq t \leq 1$, there is an extention $H_1: Y \times I \rightarrow E_1$, $H_2: Y \times I \rightarrow E_2$ of G_1, G_2 such that

$q_1 H_1(y_1, t) = q_1 H_1(y_1, 0)$, $q_2 H_2(y_2, t) = q_2 H_2(y_2, 0)$.

Theorem 2.4.3:

If $q_i: E_i \rightarrow B$ is M-Fuzzy Serre cofibration of a fuzzy compact Hausdorff E_i into a fuzzy Hausdorff space B, then q_i must be an imbedding.

Proof:

First let us prove that q_i must be injective. This will be proved by contradiction.

Assume that q_i where not one-one. Then there exists two distinct points u_i and v_i in E_i with $p(u_i) = p(v_i)$. Since E_i is normal, and $\{u_i\}, \{v_i\}$ are closed subsets of E_i, there exists a fuzzy continuous function

$$\lambda_1: E_1 \rightarrow I$$

and

$$\lambda_2: E_2 \rightarrow I$$

where $I = [0,1]$, with $\lambda_1(u_1) = 0, \lambda_1(v_1) = 1$, and $\lambda_2(u_2) = 0, \lambda_2(v_2) = 1$.

Define a fuzzy homotopy $f_t: E_1 \rightarrow I$ and $g_t: E_2 \rightarrow I$ by $f_t(e_1) = t\lambda_1(e_1)$ and $g_t(e_2) = t\lambda_2(e_2)$, $e_1 \in E_1, e_2 \in E_2, t \in I$ and define $h: B \rightarrow I$ by $h(b) = 0$ for all $b \in B$

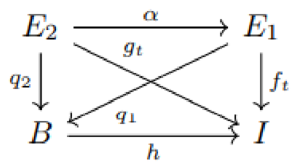

Figure 2.4.1

Then we have $hq_1 = f_0$ and $hq_2 = g_0$.

But $q_1: E_1 \to B$ and $q_2: E_2 \to B$ are a M-Fuzzy Serre cofibration, hence there exists a fuzzy homotopy $h_t: B \to I$ such that $h_0 = h$, and $h_t q_1 f_t$,

$h_t q_2 = g_t$ For all $t \in I$. Now

$$\lambda_1(u_1) = f_1(u_1) = h_1(q_1(u_1))$$
$$\lambda_1(v_1) = f_1(v_1) = h_1(q_1(v_1))$$

and

$$\lambda_2(u_2) = g_1(u_2) = h_1\big(q_2(u_2)\big)$$
$$\lambda_2(v_2) = g_1(v_2) = h_1\big(q_2(v_2)\big)$$

Since $q_1(u_1) = q_1(v_1)$, therefore $\lambda_1(u_1) = \lambda_1(v_1)$ and $q_2(u_2) = q_2(v_2)$ therefore $\lambda_2(u_2) = \lambda_2(v_2)$ But

$$\lambda_1(u_1) = 0 \neq 1 = \lambda_1(v_1)$$

and

$$\lambda_2(u_2) = 0 \neq 1 = \lambda_2(v_2)$$

Therefore q_i must be injective. Since E_i is fuzzy compact Hausdorff and B is fuzzy Hausdorff, hence q_i must be an imbedding.

Hence $q_1: E_1 \to B$ and $q_2: E_2 \to B$ are a M-fuzzy Serre cofibration, when $E_i \subset B$ has the FHEP with respect to all fuzzy CW-complex spaces.

Theorem 2.4.4:

Let A_i be a fuzzy closed subspace of a fuzzy topological space Y_i. Where $i = 1,2$. Then (Y_i, A_i) is a M- fuzzy Serre cofibered pair if and only if there exist:

1) A fuzzy neighborhood U_i of A_i which is deformable in Y_i to A_i $\mathrm{rel}A_i$ (there exists a fuzzy homotopy $H_1: U_1 \times I \to Y_1$ such that $H_1(y_1, 0) = y_1$, $H_1(a_1, t) = a_1$ and $H_2: U_2 \times I \to Y_2$ such that $H_2(y_2, 0) = y_2, H_2(a_2, t) = a_2$ and $H_1(y_1, 0) \in A_1, H_2(y_2, 0) \in A_2$ for all $y_1 \in U_1 \wedge y_2 \in U_2, a_1 \in A_1 \wedge a_2 \in A_2, t \in I$.
2) A fuzzy continuous function $\varphi_i: Y_i \to I$ such that $A_1 = \varphi_1^{-1}(0)$, $\varphi_1(y_1) = 1$ and $A_2 = \varphi_2^{-1}(0)$, $\varphi_2(y_2) = 1$ for all $y_1 \in Y_1 - U_1$, $y_2 \in Y_2 - U_2$.

Proof: Suppose that (Y_i, A_i) is a M- fuzzy Serre cofibered pair. Then there exists a retraction

$$r_i: Y_i \times I \to (Y_i \times 0) \cup (A_i \times I)$$

Where $i = 1,2$, and U_i, H_i and φ_i may be chosen as follows:

$$U_i = \{y_i \in Y_i | pr_i r_i(y_i, 1) \in A_i\}$$

$$H_i = pr_i r_i | U_i \times I$$

$$\varphi_i(y_i) = \sup_{t \in I} |t - pr_2 r_i(y_i, t)|$$

pr_1 and pr_2 denoting projections on Y_i and I, respectively.

Conversely, suppose that U_i, H_i and φ_i are given and satisfy the conditions of the theorem. Since A_i is fuzzy closed it suffices to prove the existence of a retraction.

$$r_i: Y_i \times I \to (Y_i \times 0) \cup (A_i \times I)$$

The required retraction may be constructed as follows:

- If $\varphi_i(y_i) = 1$, let $r_i(y_i, t) = (y_i, 0)$.
- If $1/2 \leq \varphi_i(y_i) < 1$,

  let $r_i(y_i, t) = \begin{cases} \big(H_1(y_1), 2\big(1 - \varphi_1(y_1)\big)t\big), 0). \\ \big(H_2(y_2), 2\big(1 - \varphi_2(y_2)\big)t\big), 0). \end{cases}$

- If $0 < \varphi_i(y_i) \leq 1/2$ and $0 \leq t \leq 2\,\varphi_i(y_i)$,

  let $r_i(y_i, t) = \begin{cases} (H_1(y_1, t/(2\varphi_1(y_1))),0). \\ (H_2(y_2, t/(2\varphi_2(y_2))),0). \end{cases}$

- If $0 < \varphi_i(y_i) \leq 1/2$ and $2\varphi_i(y_i) \leq t \leq 1$,

  Let $r_i(y_i, t) = \begin{cases} \big(H_1(y_1, 1), t - 2\varphi_1(y_1)\big). \\ \big(H_2(y_2, 1), t - 2\varphi_2(y_2)\big). \end{cases}$

- If $\varphi_i(y_i) = 0$, let $r_i(y_i, t) = (y_i, t)$.

Lemma 2.4.5:

If (Y_i, A_i) is Mixing fuzzy Serre cofibered pair, where $i = 1,2$, then $(Y_i \times 0) \cup (A_i \times I)$ is a strong deformation retract of $Y_i \times I$

Proof:

Let $\eta_i: (Y_i \times 0) \cup (A_i \times I) \subset Y_i \times I$ be the inclusion map, and let

$$r_i : Y_i \times I \to (Y_i \times 0) \cup (A_i \times I)$$

be a retraction. A fuzzy homotopy

$$D_1: \eta_1 r_1 \simeq 1_{Y_1 \times I} \qquad releated\ to\ (Y_1 \times 0) \cup (A_1 \times I)$$

and

$$D_2: \eta_2 r_2 \simeq 1_{Y_2 \times I} \qquad releated\ to\ (Y_2 \times 0) \cup (A_2 \times I)$$

Is given by

$$D_i(y_i,t,\acute{t}) = (pr_j r_i(y_i,(1-\acute{t})t),(1-\acute{t})pr_i r_j(y_2,t)+\acute{t}t)$$

Where j = 1,2 , i = 3 .

$$D_1(y_1,t,\acute{t}) = (pr_1 r_1(y_1,(1-\acute{t})t),(1-\acute{t})pr_3 r_1(y_1,t)+\acute{t}t).$$

$$D_2(y_2,t,\acute{t}) = (pr_2 r_2(y_2,(1-\acute{t})t),(1-\acute{t})pr_3 r_2(y_2,t)+\acute{t}t).$$

Theorem 2.4.6:

Suppose that $p_i: E_i \to B$ is M-fuzzy fibration, that A_i is a strong deformation retract of Y_i , and that there exists a map $\varphi_i: Y_i \to I$ such that $A_1 = \varphi_1^{-1}(0)$and $A_2 = \varphi_2^{-1}(0$.Then any commutative diagram

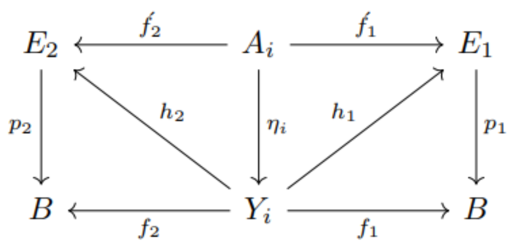

Figure 2.4.2

may be filled in with a fuzzy map $h_i: Y_i \to E_i$ such that $p_1 h_1 = f_1$, $p_2 h_2 = f_2$ and $h_1\eta_1 = \acute{f}_1$, $h_2\eta_2 = \acute{f}_2$,. h_i is unique up to fuzzy homotopy relA_i.

Proof: By hypodissertation there exists a retraction $r_1: Y_1 \to A_1$, $r_2: Y_2 \to A_2$and a fuzzy homotopy

$D_1: \eta_1 r_1 \simeq 1_{Y_1}$ relA_1 ,

$D_2: \eta_2 r_2 \simeq 1_{Y_2}$ relA_2.

If $h_1: Y_1 \to E_1$ and $h_2: Y_2 \to E_2$ such that $h_i\eta_i = \acute{f}_i$, then $h_i \simeq h_i\eta_i r_i = \acute{f}_i$ relA which proves the last assertion of the theorem. Define $\overline{D_i}: Y_i \times I \to Y_i$ by

$$\overline{D_1}(y_1, t) = \begin{cases} D_1(y_1, t/(\varphi_1(y_1))) & t < \varphi_1(y_1) \\ D_1(y_1, 1) & t \geq \varphi_1(y_1) \end{cases}$$

$$\overline{D_2}(y_2, t) = \begin{cases} D_2(y_2, t/(\varphi_2(y_2))) & t < \varphi_2(y_2) \\ D_2(y_2, 1) & t \geq \varphi_2(y_2) \end{cases}$$

D_i is easily shown to be fuzzy continuous. Because p_i is a fuzzy fibration there exists a fuzzy homotopy $\overline{F_1}: Y_1 \times I \to E_1$, $\overline{F_2}: Y_2 \times I \to E_2$ such that $p_1\overline{F_1} = f_1\overline{D_1}$, $p_1\overline{F_1} = f_1\overline{D_1}$ and $\overline{F_1}(y_1, 0) = \acute{f}_1 r_1(y_1)$, $\overline{F_2}(y_2, 0) = \acute{f}_2 r_2(y_2)$ for each $y_1 \in Y_1, y_2 \in Y_2$. h_i is given by $h_i(y_i) = \overline{F_i}(y_i, \varphi_i(y_i))$, where $i = 1,2$.

Theorem 2.4.7:

Suppose that $p_i: E_i \to B$ is a M-fuzzy fibration, that (Y_i, A_i) is a M-fuzzy Serre cofibred pair, and that A_i is fuzzy closed. Then any commutative diagram

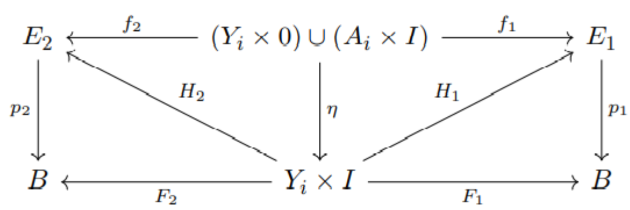

Figure 2.4.3

may be filled in with a fuzzy homotopy $\overline{F}: Y_i \times I \to E_i$ such that $p_1\overline{F_1} = F_1$, $p_2\overline{F_2} = F_2$ and $\overline{F_i}|(Y_i \times 0) \cup (A_i \times I) = f_i$.

Proof:

According to the lemma (2.4.5), and by theorem (2.4.4) there exists a fuzzy Function $\psi_1: Y_1 \to I$, $\psi_2: Y_2 \to I$ such that $A_1 = \psi_1^{-1}(0), A_2 = \psi_2^{-1}(0)$.

Define $\varphi_1: Y_1 \times I \to I$ and $\varphi_2: Y_2 \times I \to I$ by $\varphi_1(y_1, t) = t\psi_1(y_1)$ and $\varphi_2(y_2, t) = t\psi_2(y_2)$. Then $(Y_1 \times 0) \cup (A_1 \times I) = \varphi_1^{-1}(0)$ and $(Y_2 \times 0) \cup (A_2 \times I) = \varphi_2^{-1}(0)$ and the theorem follows from theorem (5.7) The condition that A_i be fuzzy closed is not very restrictive. For instance, A will always be fuzzy closed if Y_i is fuzzy Hausdorff. Not all M- fuzzy Serre cofibration are fuzzy closed, however. The most trivial example of a non-fuzzy closed

M-fuzzy Serre cofibration is the pair $(Y_1, a_1), (Y_2, a_2)$ where Y_i is the two-point spacea_i, b_i with the trivial fuzzy topology

2.5 A mixing criterion for a fuzzy map to be a mixing fuzzy Serre Cofibration.

In this section the M-criterion that allows us to recognize M-fuzzy Serre Cofibration when we see them. We shall often consider pair (Y_i, A_i) consisting of a fuzzy space Y_i and a fuzzy subspace A_i. M-fuzzy Serre Cofibration pairs will be those pairs that "behave fuzzy homologically" just like the associated fuzzy quotient spaces Y_i/A_i.

Definition 2.5.1:

A pair (Y_i, A_i) is an Mixing fuzzy Neighborhood Deformation Retract pair (MFNDR-pair) if there is a fuzzy map $u: Y_1 \to I$, $v: Y_2 \to I$ such that $u^{-1}(0) = A_1$, $v^{-1}(0) = A_2$ and a fuzzy homotopy $h: Y_1 \times I \to$

Y_1, $k: Y_2 \times I \to Y_2$ such that $h_0 = id$, $h(a_1, t) = a_1$ and $k_0 = id$, $k(a_2, t) = a_2$ for $a_1 \in A_1, a_2 \in A_2$ and $t \in I$, and $h(y_1, 1) \in A_1$ if $u(y_1) < 1$, and $k(y_2, 1) \in A_2$ if $v(y_2) < 1$. (Y_i, A_i) is a MFDR-pair if $u(y_1) < 1$, $v(y_2) < 1$ for all $y_1 \in Y_1, y_2 \in Y_2$, in which case A_i is a deformation retract of Y_i where $i = 1,2$.

Lemma 2.5.2:

If (h_i, u_i) and (k_i, v_i) represent (Y_i, A_i) and (Z_i, B_i) as (MFNDR-pairs), then (l_i, w_i) represents the (product pair) $(Y_i \times Z_i, Y_i \times B_i \cup A_i \times Z_i)$ as an MFNDR-pair, where
$w_i(y_i, z_i) = \min(u_i(y_i), v_i(z_i))$ and

$$l_i(y_i, z_i, t) = \begin{cases} \left(h_i(y_i, t), k_i(z_i, t u_i(y_i)/v_i(z_i))\right) \\ \quad \text{if } v_i(z_i) \geq u_i(y_i) \\ \left(h_i\left(y_i, t v_i(z_i)/u_i(y_i)\right), k_i(z_i, t)\right) \\ \quad \text{if } u_i(y_i) \geq v_i(z_i) \end{cases}$$

If (Y_i, A_i) or (Z_i, B_i) is a DR-pair, then so is $(Y_i \times Z_i, Y_i \times B_i \cup A_i \times Z_i)$.

Proof:

If $v_i(z_i) = 0$ and $v_i(z_i) \geq u_i(y_i)$, then $u_i(y_i) = 0$ and both (Z_i, B_i) and (Y_i, A_i) , therefore we can and must understand $l_i(y_i, z_i, t)$ to be (y_i, z_i). It is easy to check from this and the symmetric observation that l_i is a well-defined continuous fuzzy homotopy as desired.

Theorem 2.5.3:

Let A_i be a closed subspace of Y_i, where $i = 1,2$. Then the following are equivalent:

- (Y_i, A_i) is an MFNDR-pair.

- $(Y_i \times I, Y_i \times \{0\} \cup A_i \times I)$is a MFDR-pair.
- $Y_i \times \{0\} \cup A_i \times I$ is a M-fuzzy retract of $Y_i \times I$.
- The Fuzzy inclusion $\eta_i: A_i \to Y_i$ is a M-Fuzzy Serre cofibration.

Proof: The lemma gives that (a) implies (b), (b) trivially implies (c), and we have already seen that(c) and (d) are equivalent. Assume given a retraction $r_i : Y_i \times I \to Y_i \times \{0\} \cup A_i \times I$.

Let $pr_i: Y_i \times I \to Y_i$ and $pr_2: Y_i \times I \to I$ be the projections and define $u: Y_1 \to I$ by

$$u(y_1) = \sup\{t - pr_2\, r_1(y_1, t) | t \in I\},$$

and $v: Y_2 \to I$ by

$$v(y_2) = \sup\{t - pr_2\, r_2(y_2, t) | t \in I\}$$

$h: Y_1 \times I \to Y_1$ by

$$h(y_1, t) = pr_1 r_1(y_1, t)$$

and $k: Y_2 \times I \to Y_2$ by

$$k(y_2, t) = pr_2 r_2(y_2, t)$$

Then,(h, u), (k, v) represents (Y_i, A_i) as an MFNDR-pair. Here $u^{-1}(0) = A_1$ since $u(y_1) = 0$ and $v^{-1}(0) = A_2$ since $v(y_2) = 0$ implies that $r_i(y_i, t) \in A_i \times I$ for $t > 0$ and thus also for $t = 0$ since $A_i \times I$ is closed in $Y_i \times I$, where $i = 1,2$

Example 2.5.4:

Let $\eta_1: A_1 \to Y_1$and $\eta_2: A_2 \to Y_2$ be a M-Fuzzy Serre cofibration, where $i = 1,2$. We then have the commutative diagram

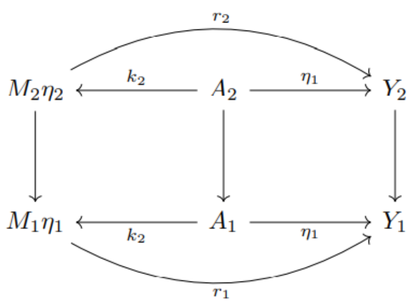

Figure 2.5.1

Where $k_1(a_1) = (a_1, 1)$ and $k_2(a_2) = (a_2, 1$ where $M_i\eta_i \equiv Y_i \cup_\eta (A_i \times I)$. The obviou fuzzy homotopy inverse $l_i: Y_i \rightarrow M_i\eta_i$ has $l_i(y_i) = (y_i, 0)$ and is thus very far from being a map A_i. The proposition ensures that l_i is fuzzy homotopic to a fuzzy map under A_i that is fuzzy homotopy inverse to r_i under A_i.

CHAPTER THREE

Computation of Cup-i product and Steenrod operations on classifying space for groups of order 128

Chapter Three:

In this chapter, we give a computational treatment to compute the cup i-product and Steenrod operations on cohomology rings of as many groups as possible. There are some available approaches for computing Steenrod operations on these cohomology rings. We compute all Steenrod squares on the Mod 2 cohomology ring of groups mod 2 of order 128. Spacial to compute Steenrod operation for all groups of order 128 as possible and we construct some algorithm and technique for computing by using HAP programming.

3.1 Steenrod operations

In this section, we introduce our treatment to compute Steenrod operations on cohomology rings of as many groups of order 128 as possible.

<u>Definition 3.1.1:</u> [25]

The type (A, n, B, m) is a natural transformation, $\emptyset : H^n(-,A) \rightarrow H^m(-,B)$, which called cohomology operation that for any spaces, X, Y and for any map

$f : X \rightarrow Y$ there are functions $\emptyset_X, \emptyset_Y$ satisfying the naturality condition

$f^*\emptyset_Y = \emptyset_X f^*$ (i.e., the following diagram commutes).

$$\begin{array}{ccc} H^n(X,A) & \xrightarrow{\emptyset_X} & H^m(X,B) \\ f^* \uparrow & & \uparrow f^* \\ H^n(Y,A) & \xrightarrow{\emptyset_Y} & H^m(Y,B) \end{array}$$

Figure 3.1.1

We know the Bockstein homomorphism as an example of a cohomology operation of type ($\mathbb{Z}_p$, n, $\mathbb{Z}_p$, n + 1).

Definition 3.1.6. [28]

Consider the cohomology of a space B with co-efficients in the field of p elements, p a prime number. The Steenrod squares are cohomology operations of type ($\mathbb{Z}\, p, n, \mathbb{Z}\, p, n + i$) for $p = 2$,

$$Sq^i : H^n(B, \mathbb{Z}_2) \to H^{n+i}(B, \mathbb{Z}_2),\ i \geq 0 \qquad (3.1)$$

and the Steenrod powers are cohomology operations of type ($\mathbb{Z}_p, n, \mathbb{Z}_p, n + i(p - 1)$) for $p > 2$,

$$P^i : H^n(B, \mathbb{Z}_p) \to H^{n+i(p-1)}(B, \mathbb{Z}_p),\ i \geq 0. \quad (3.2)$$

The Steenrod squares $Sq^i$of 1, defined for $i \geq 0$ satisfy the following properties:

1. Sq^1 is the Bockstein homomorphism (denoted β) [26].
2. Sq^0 is the identity homomorphism.
3. $Sq^i(x) = x^2$ when $degree(x) = i$.
4. $Sq^i(x) = 0$ if $degree(x) < i$.
5. "Cartan formula" $Sq^n(xy) = \sum_{i+j=n} Sq^i(x) \smile Sq^i(y)$.
6. $Sq^i(x + y) = Sq^i(x) + Sq^i(y)$.
7. Naturality: means that for any map $f : B \to B'$,
   $Sq^i(f^*) = f^*(Sq^i)$ for the cohomology homomorphism
   f^* induced by the map f.
8. $Sq^a Sq^b = \sum_{c=0}^{a/2} Sq^{a+b-c} Sq^c, \quad for\ a < 2b,$

Where $Sq^a Sq^b$ denotes the composition of the Steenrod squares and the binomial coefficient is taken modulo 2 is called "Adem relations".

3.2 $cup - i$ product :

The cup-i product is introduced by [[23], [24]] and [[28]] such that for each $i \geq 0$ and cochains $u, v \in C^p\ (R_*^G)$ of dimension p, q respectively, where the cochain complex $C^p\ (R_*^G) = Hom_{\mathbb{Z}G}(R_*^G\ , \mathbb{Z})$ such that R_*^G is free $\mathbb{Z}G$-resolution of $\mathbb{Z}$ a product $u \smile_i v \in C^{p+q+i}\ (R_*^G)$ the general case of cup product is the cup-i product such that they are equal if $i = 0$, [$i.e$: = $u \smile_0 v = u \smile v$].

3.2.1. Structure of $cup - i$ product

Let S^∞ represent the infinite-dimensional sphere such that $S^0 \subset S^1 \subset \ldots \subset S^\infty = \bigcup_{n=0}^{\infty} S^n$

There exists a cell structure on S^∞ as a CW-complex, with two cells in each dimension. the group of $\mathbb{Z}_n$ acts on S^∞ by the antipodal action induces an action in S^∞ , whose orbit space denoted by RP^∞ . We have some steps to follow.

1) It has one cell in each dimension. Let $R_*^{\mathbb{Z}_2}$ be the free $\mathbb{Z}\mathbb{Z}_2$-resolution of $\mathbb{Z}$ obtained from the cellular chain complex with one free $\mathbb{Z}\mathbb{Z}_2$-generator k_n in each degree n.

$$R_*^{\mathbb{Z}_2} : \ldots\ \mathbb{Z}[\mathbb{Z}_2] \xrightarrow{t-1} \mathbb{Z}[\mathbb{Z}_2] \xrightarrow{t+1} \mathbb{Z}[\mathbb{Z}_2] \xrightarrow{t-1} \mathbb{Z}[\mathbb{Z}_2]$$

2) Let $B = B_G$ for some group G and set $R_*^G = C*(\bar{B})$. Let e_1^n , e_2^n, ... denote free generators for the free $\mathbb{Z}$ G-module R_n^G .
3) The group $\mathbb{Z}_2$ acts on $R_p^G \otimes \mathbb{Z}\ R_p^G = R_n^{GxG}$ by the interchange map

$$\tau : R_*^G \otimes R_*^G \rightarrow R_*^G \otimes R_*^G$$

$$t \cdot g' e_i^p \otimes g'' e_j^q = (-1)^{pq} g'' e_j^q \otimes g' e_i^p$$

4)The tensor product $R_*^{G\times G} = R_*^G \otimes \mathbb{Z}\, R_*^G$ by(The Eilenberg-Zilber theorem)is a free $\mathbb{Z}$ [G × G]-resolution of $\mathbb{Z}$ with free $\mathbb{Z}$ [G × G]-generators $e_i^p \otimes e_j^q$ in degree n = p + q. With a free a belian group $R_n^{G\times G}$ is freely generated via $g' e_i^p \otimes g'' e_j^q$, *such that* $(g', g'') \in G\text{x}G$.

5) The action extends to an action of $\mathbb{Z}_2\times G$ via the formula:

$$(t,g)\cdot(g' e_i^p \otimes g'' e_j^q) = (-1)^{pq} g g'' e_j^q \otimes g g' e_i^p.$$

We view $R_*^{G\times G}$ as an exact chain complex of $\mathbb{Z}[G\times G]$ modules.

6) The tensor product $R_*^{\mathbb{Z}_2\times G} = R_*^{\mathbb{Z}_2} \otimes_{\mathbb{Z}} R_*^G$ is a free $\mathbb{Z}[\mathbb{Z}_2\times G]$-resolution of $\mathbb{Z}$.

7) We will consider the $\mathbb{Z}\,[\mathbb{Z}_2\times G]$-equivariant homomorphism $\phi_0 : R_0^{\mathbb{Z}_2} \otimes R_0^G \to R_0^G \otimes R_0^G$,

defined by $\phi_0\,(k^0 \otimes e_i^0) = e_i^0 \otimes e_i^0$.

8) The map ϕ_0 extends, using the freeness of $R_0^{\mathbb{Z}_2} \otimes R_*^G$ and 1 the exactness of $R_*^G \otimes R_*^G$to a $\mathbb{Z}\,[\mathbb{Z}_2\times G]$ equivariant chain map

$$\phi_* : R_*^{\mathbb{Z}_2} \otimes R_*^G \to R_*^G \otimes R_*^G \qquad (3.3)$$

and the diagram below describes the ϕ_n,

$$\begin{array}{ccccccc}
R_n^{\mathbb{Z}_2\times G} & \xrightarrow{\partial_n} & R_{n-1}^{\mathbb{Z}_2\times G} & \ldots \to & R_1^{\mathbb{Z}_2\times G} & \xrightarrow{\partial_1} & R_0^{\mathbb{Z}_2\times G} \\
\downarrow \phi_n & & \downarrow \phi_{n-1} & & \downarrow \phi_1 & & \downarrow \phi_0 \\
R_n^{G\times G} & \xrightarrow{\partial_n} & R_{n-1}^{G\times G} & \ldots \to & R_1^{G\times G} & \xrightarrow{\partial_1} & R_0^{G\times G}
\end{array}$$

Figure 3.2.1

and ϕ_* is unique up to chain homotopy. The chain map ϕ_* is computed from a contracting homotopy (or discrete vector field) on $R_*^{G\times G}$. The diagonal map $\Delta : G \to G \times G$ define by $\Delta(x) = (x, x)$, the first projection $\pi_1 : \mathbb{Z}_2 \times G \to \mathbb{Z}_2$ given by $\pi_1(t, x) = t$, and $\pi_2 : C_2 \times G \to G$ be the second projection given by $\pi_2(t, x) = x$. Let $i_1 : G \to G \times G$, $i_1(x) = (x, 1)$ and $i_2 : G \to G \times G$, $i_2(x) = (1, x)$ the specified embedding to $G\times G$. We now consider the cochain complex $C^*(R_*^G) = Hom_{\mathbb{Z}G}(R_*^G, \mathbb{Z})$. The group $C^n(R_*^G)$ is a free abelian group with free abelian generators e_i^n corresponding to the free $\mathbb{Z}G$-generators $e_i^n$of R_n^G. More precisely, e_i^n: $R_n^G \to \mathbb{Z}$ is the $\mathbb{Z}$ G-equivariant homomorphism sending $e_i^n \mapsto 1$, $e_j^n \mapsto 0$ for $j \neq i$. This notation describes a homomorphism

$$R_*^G \to C^n (R_*^G), u \mapsto \bar{u}.$$

For each integer i ≥ 0 define a $\mathbb{Z}$*-linear cup-i product*

$$C^p(R_*^G)\otimes_{\mathbb{Z}} C^q(R_*^G) \to C^{p+q-1} (R_*^G), \bar{u}\otimes\bar{v} \mapsto \bar{u} \smile_i \bar{v} \quad (3.4)$$

by the formula

$$(\bar{u} \smile_i \bar{v})(c) = (\bar{u}\otimes\bar{v})\, \phi_{p+q} (k^i \otimes c) \qquad (3.5)$$

for $c \in R_{p+q-i}^G$

Algorithm 3.2.2 The function HAP-PHI

Input:

• G finite group and

• an integer n ≥ 0.

Output: A list $[\phi_*, R_*^{\mathbb{Z}_2\times G}, R_*^{G\times G}]$.

Procedure:

1: Construct diagonal function $G \to G \times G$.

2: Construct interchange map $\tau : R_*^G \otimes R_*^G \to R_*^G \otimes R_*^G$,

$t \cdot (g'e_i^p \otimes g''e_j^q) = (-1)^{pq} gg''e_j^q \otimes g'e_i^p$.

3: Construct $\phi_0 : R^{\mathbb{Z}_2} \otimes R_0^G \to R_0^G \otimes R_0^G$, $\phi_0(k^0 \otimes e_i^0) =$

$e_i^0 \otimes e_i^0$

4: The output $\{\phi_1, \phi_2, \ldots, \phi_n\}$, $R_n^{\mathbb{Z}_2\times G}$ and $R_n^{G\times G}$

5: EndProcedure:

Example: the following command compute the cup-i product for G=(128,988)

GAP Session

```
gap> G:=SmallGroup(128,988);;
gap> P:=HAP_PHI(G,8);
[ function( n, a, ii ) ... end, Resolution of length 9 in characteristic
   2 for <group of size 256 with 8 generators> .
   , Resolution of length 9 in characteristic 2 for <pc group of size 16384
with 14 generators> .
   , Resolution of length 9 in characteristic 2 for <pc group of size 128 with 7
generators> .    ]
```

3.3 The computation of Steenrod squares for groups of order 128.

In this section we will give some methods to compute Steenrod squares on cohomology rings of groups of order 128 and we will study the relationship between the The cup-i product and the computation of Steenrod squares of groups of order 128.

Theorem 3.3.1 [24]

The operation

$C^n(R_*^G) \to C^{2n-i}(R_*^G),\ \bar{u} \mapsto \bar{u} \smile_i \bar{u}$ (3.6)

Induces a homomorphism

$Sq^i: H^n(G, \mathbb{Z}_2) \to H^{2n-i}(G, \mathbb{Z}_2)$ (3.7)

The homomorphism

$Sq^i = sq^{n-i}: H^n(G, \mathbb{Z}_2) \to H^{n+i}(G, \mathbb{Z}_2)$ (3.8)

is independent of the choices in ϕ_* made in (3.3) and satisfies the properties of Definition (3.1.6)

We use the HAP function Mod2Steenrodalgebra(G,n) which is an implementation of Sq^i defined in 6 that inputs a finite 2-group G and a non-negative integer and returns the first nth degree of Steenrod squares.

Example 3.3.2

Let us take the small group of order 128 andnumber 206 To compute the Steenrod square Sq^k for each generator and each positive 2-power k = 2^i < degree(x), x ∈ $H^*(K, \mathbb{Z}_2)$ for $G_{128,206}$ see the following

GAP session.
gap> M:=SmallGroup(128,206);;
gap> P:=Mod2SteenrodAlgebra(M,8);;
gap> gens:=ModPRingGenerators(P);
[v.1, v.2, v.3, v.4, v.5, v.6, v.7]
gap> List(gens,P!.degree);
[0, 1, 1, 1, 2, 2, 2]
gap> List(gens,y->Sq(P,2,y));
[0*v.1, 0*v.1, 0*v.1, 0*v.1, v.31
v.27+v.28+v.35, 0*v.1]

```
gap> PrintAlgebraWordAsPolynomial(P,
List(gens,y->Sq(P,2,y))[5]);
v.5*v.5
gap> PrintAlgebraWordAsPolynomial(P,
List(gens,y->Sq(P,2,y))[6]);
v.6*v.6
```

Also we using the HAP command Cohomological Data(G,n) to determine details of the groups, It prints
correct data for the cohomology ring $H^*(K, \mathbb{Z}_2)$ of a 2-group G on the condition that the integer n is at least the maximal degree of a relater in a minimal set of relaters for the ring. Moreover, n terms of a free $\mathbb{Z}$ G-resolution are sufficient to compute the whole mod-2 cohomology ring by the tables of King and Green [16].
When Steenrod squares are composed, the composetion satisfy certain relations known the Adem relation (3.1.6),

$$Sq^a Sq^b = \sum_{j=1}^{a/2} \binom{b-j-1}{a-2j} Sq^{a+b-j} Sq^j$$

for $a < 2b$, where $Sq^a Sq^b$ denotes the composition of the Steenrod squares and the binomail coefficient is taken modulo 2. A detailed proof the Adem relation can found in [3], [24].
Suppose that $i = a + b$ where $b = 2^k$and $0 < a < 2^k$. Then we can rewritten the Adem relations in the form,

$$\binom{b-1}{a} Sq^i = Sq^a Sq^b + \sum_{j=1}^{a/2} \binom{b-j-1}{a-2j} Sq^{a+b-j} Sq^j$$

if $a \leq b - 1$ that $\binom{b-1}{a} \equiv 1(\text{mod}2)$ which proof in [Proposition 15.6 [3]], we can used recursively to express Sq^i in terms of Sq^{2^k}. For instance,there are relations

$Sq^1Sq^1 = 0$, $Sq^1\ Sq^3 = 0$,...; $Sq^1Sq^{2n+1} = 0$ and $Sq^3 = Sq^1Sq^2$, $Sq^5 = Sq^1Sq^4$,... $Sq^{2n+1} = Sq^1Sq^{2n}$. The expression of Sq^6 in terms of squares of the form Sq^{2^k}as $Sq^6 = Sq^2Sq^4 + Sq^5Sq^1, \dots, Sq^{4n+2} = Sq^2Sq^{4n}+Sq^{4n+1}Sq^1$. Also, $Sq^3Sq^{4n+2} = 0, Sq^{2n-1}Sq^n = 0$, and more details see [15].

Group order: 128

Group number: 206

Group description: C2 x ((C4 : C8) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Cohomology relations

1: f^2

2: a*f

3: a*b

4: a^2

Steenrod squares

$Sq^1(d) = 0$

$Sq^1(e) = e * a + e * b$

$Sq^1(f) = f * b$

3.3.3 Method of computation

In this part we will give some methods to compute Steenrod squares on cohomology rings of groups of order 128

1) The first method we use the HAP command **CohomologicalData(*G, n*)** [[5], [28]]to compute and print details of the group order,group number, cohomology ring generators with degree and relations and the Steenrod square $Sq^k(x)$ for each generator x and each positive 2-power $k =$

$2^i degree(x)$ .this method is used with groups that have generators with degree one or two. If we want the cohomology ring details printed to a file then this file name is included as an optional third input to the command.

Example 3.3.4

Consider the group G=$G_{128,459}$ namely the small group of order 128 and number 459 in GAP's library(see [1]). The eight generators of $H^*(G, \mathbb{Z}_2)$ can be denoted a1, b_1, c_1, d_2, e_2, f_2, g_2, h_2 such that the index of each generator indicates the degree of the element. we can check that the cohomology of G by using The command CohomologicalData(G,n) taking $n = 8$ the integer n is at least the maximal degree of a relator in a minimal set relators for the ring, moreover n terms of a free ZG-resolution is enough to compute the whole mod-2 cohomology ring by the tables of king and Green [16] the output we get are the group order,group number, cohomology ring generators with degree and relationsand the Steenrod square $Sq^k(x)$ for each generator x and each positive 2-power $k = 2^i < degree(x)$. If we want the cohomology ring details printed to a file then this file name is included as an optional third input to the command, the output will be as follows.

Group order: 128

Group number: 459

Group description: C2 x ((C8 x C2) : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g, h

Cohomology relations

1 : f^2

2 : $e * f + f * g$

3 : $e^2 + g^2$

4 : $b * e + b * g$

5 : b^2

6 : $a * f$

7 : $a * e + a * g + b * f$

8 : $a * b$

9 : a^2

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = e ∗ a + e ∗ b

Sq^1 (f) = e ∗ a + f ∗ b

Sq^1 (g) = e ∗ a + f ∗ b + h ∗ a

Sq^1 (h) = 0

2) DETECTION METHOD

we use the following HAP command ***Cohomological Detected*(G, K, n)** and the command *Cohomological Detected Intersection*(*G, K, n*) instead *Cohomological Data*(*G, n*) because the higher degree of generators, such that there inputs a finit 2-group , K are maximal subgroups and positive integer n.

Definition 3.3.5 [16]

Let K be a a proper subgroup from finite group G such that $K \leq G$ and $i : K \to G$ the inclusion map. The induced cohomology homomorphism

$$i_K^G : H^*(G, \mathbb{Z}_2) \to H^*(K, \mathbb{Z}_2)$$

is called restriction map.

Definition 3.3.6. [17]

Let k a collection of proper subgroups

of a finite group G. If the producct of the restriction maps,

$\prod_{K \in \kappa} i_K^G : (H^*(G, \mathbb{Z}_2)) \to \prod_{K \in \kappa} H^*(K, \mathbb{Z}_2)$

is an injection. We say that k detects the cohomology
$H^*(K, \mathbb{Z}_2)$.
The HAP function Mod2Steenrodalgebra(G,n) and ModPSteenrodAlgebra(G,n) become less practical as the size of the finite 2-group G increases. For large G it can be useful using Definition (3.3.6). We are using the HAP function InducedSteenrodHomomorphism(f,n) which input a homomorphism $f : K \rightarrow G$ of finite 2-groups and a positive integer n.
It returns a triple $[HG, HK, l]$, where $HG = H^{\leq n}(G, \mathbb{Z}_2)$,
$HK = H^{\leq n}(K, \mathbb{Z}_2)$ and l is a list $[l_1, l_2, ..., l_n]$ with
$l_i : H^i(G, \mathbb{Z}_2) \rightarrow H^i(K, \mathbb{Z}_2)$ the linear homomorphism
induced by f. For each element v in the generating set of the cohomology ring
$H^*(G, \mathbb{Z}_2)$ and each Steenrod square Sq^{2^k},
we have

$$\prod_{K\in\kappa} i_K^G\left(Sq^{2^k}(v)\right) = \prod_{K\in\kappa} Sq^{2^k}\left(i_K^G(v)\right)$$

If $i_K^G(Sq^{2^k}(v)) = Sq^{2^k}(i_K^G(v)) = 0$, then the Steenrod square
$Sq^{2^k}(v) \in \mathrm{Kernel}(l_1) \cap \mathrm{Kernel}(l_2) \cap ... \cap \mathrm{Kernel}(l_n)$.

Example 3.3.7.

Consider the group $G=G_{128,181}$ namely the
small group of order 128 and number 181 in GAP's library. The six generators of $H^*(G, \mathbb{Z}_2)$ can be denoted
$x_1, y_1, z_2, w_2, u_3, u_4$, so that each generator index indicates the degree of the item. One can verify the cohomology of G is not detected by any family of proper subgroups as in example (3.3.2). However it is possible to determine some information about Steenrod squares for G using Steenrod square computations in a proper subgroup K. The group $G_{128,181}$ has seven subgroups of order 64; we denote them by $[K_{64,2}, K_{64,85}, K_{64,3}, K_{64,83}, K_{64,2}, K_{64,112}, K_{64,3}]$. Let $K_{64,3} < G$ denote the subgroup of order 64 and number 3. The following GAP

session uses a Steenrod square computation in $H^*(K, \mathbb{Z}_2)$ in order to determine that $Sq^2(f_4) = f * a * b + f * b * b$.

GAP session

```
gap > G:= SmallGroup(128,181);;
gap > K:= MaximalSubgroups(G)[1];
[ Group([ f1, f2, f4, f5, f6, f7 ])]
gap > f:= GroupHomomorphismByFunction(K, G, x-> x)
gap > L:= InducedSteenrodHomomorphisms(f, 8);;
gap > HG:= L[1];;
gap > HK:= L[2];;
gap > iota:= L[3];;
gap > gens:= ModPRingGenerators(HG);
[ v.1, v.2, v.3, v.4, v.7, v.10, v.17 ]
gap > List(gens, HG!.degree);
[ 0, 1, 1, 1, 2, 3, 4 ]
gap > P:= Sq(HK, 2, Image(iota[5], gens[7]));
0 * v.1
gap > ker:= Kernel(iota[7]);
< vector space overGF(2), with 13generators >
gap > P:= Elements(ker);
[ 0 * v.1, v.57, v.54, v.54 + v.57, .... ]
gap > PrintAlgebraWordAsPolynomial(HG, P[3]);
v.7 * v.7 * v.4 * v.4 + v.17 * v.3 * v.4
```

Also, we are using the HAP command **Cohomological Detected(G,K,n)** which inputs a finite 2-group, K are maximal subgroups and positive integer n to determine and print details of the group order, group number, a list of maximal subgroups, cohomology ring generators with their degree, and Steenrod squares Sq^k for each generator. If a file name is included as an optional fourth input to the command then the details are printed to this file.

Example 3.3.8.

From the previous example we can use the command

CohomologicalDetected(G,K,n), which returns the following information for $G_{128,181}$, n = 8 and the seven maximal subgroups of order 64.

Group order: 128

Group number: 181

Group description: C8 x (C8 : C2)

Subgroup List:[[64, 2], [64, 85], [64, 3], [64, 83], [64, 2], [64, 112], [64, 3]]

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

$Sq^1(d) = [0, a * b * c]$

$Sq^1(e) = [0]$

$Sq^2(e) = [d * a * b * c + e * c * c + f * a, e * c * c + f * a]$

$Sq^1(f) = [0, d * a * b * c]$

$Sq^2(f) = [f * a * b + f * b * b]$

In the example above, we found the Steenrod square by taking the intersection of the Steenrod squares over all maximal subgroups. For instance $Sq^1(d) = 0, a * b * c$ by taking the intersection between the list of $Sq^1(d)$ over [$K_{64,2}$, $Sq^1(d)$ over $K_{64,85}$, $Sq^1(d)$ over $K_{64,3}$, Sq^1 (d) over $K_{64,83}$, Sq^1 (d) over $K_{64,2}$ $Sq^i(d)$ over

$K_{64,112}$, $Sq^1(d)$ over $K_{64,3}$] We proceed analogously with the other generators. Occasionally, the HAP command **CohomologicalDetected(G,K,n)** gives us a huge information therefore we use the HAP commend **CohomologicalDetectedIntersection(G,K,n)** which input a finite 2-group, K a maximal subgroups and positive integer *n*, and output the information of Steenrod square.

3) DIRECT PRODUCT METHOD

In special cases, we use direct product as a third method to compute the Steenrod square and another cohomlogy operations relying on the previous information where we can represent the groups which has generators of degree highest than generators in the groups which the previous two methods succeede in computing as a direct product of groups of order less than 128 (order 2, 4, 8, 16,32 and 64) .

we can use the direct products to compute the Steenrod square.by using the proposition below.

Proposition 3.3.9.[3]

Let L and K are groups and G = L×K *be the direct product, for* $l \in H^*(L, \mathbb{Z}_2)$, $k \in H^*(K, \mathbb{Z}_2)$ *and* $(l \times k) \in H^*(L \times K, \mathbb{Z}_2)$ *we have*

$Sq^n(l \times k) = \sum_{i=j=n} Sq^i(l) \times Sq^j(k)$ *The first projection* $p_1 : L\times K \twoheadrightarrow L$ *and the second projection* $p_2 : L \times K \twoheadrightarrow K$ *induce ring inclusions* $p_1^*: H^*(L, \mathbb{Z}_2) \rightarrowtail H^*(L \times K, \mathbb{Z}_2)$ *and* $p_2^* : H^*(K, \mathbb{Z}_2) \rightarrowtail H^*(L \times K, \mathbb{Z}_2)$ *from property 5 we have*

$Sq^n(l\times k) = Sq^n((l\times 1)\cup(1\times k)) = \sum_i Sq^i(l \times 1)\, Sq^{n-i}(1 \times k)$, *then* $Sq^i(l \times 1) = Sq^i(p_1^*(l)) = p_1^* Sq^i(l) = Sq^i(l) \times 1$.

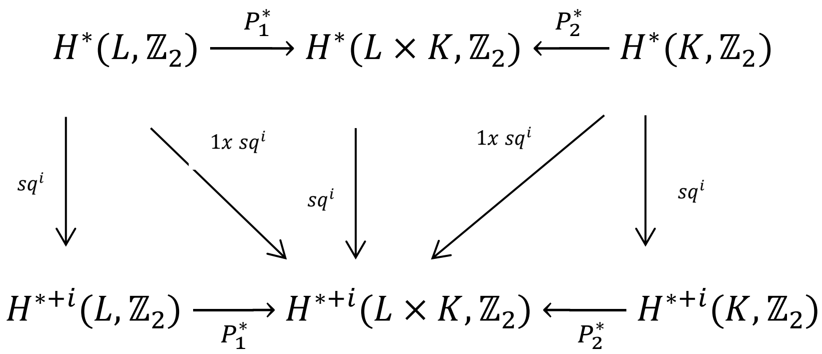

Figure 3.3.1

Accordingly $Sq^n(\mathrm{l} \times \mathrm{k}) = \sum_{i+j=n} Sq^i(l \times 1)Sq^j(1 \times k) = \sum_{i+j=n} (Sq^i(l) \times 1) \cup (1 \times Sq^j(k)) = \sum_{i+j=n} Sq^i(l) \times Sq^j(k)$.

Example 3.3.10.

let we take the small 2-group $G_{128,878}$ of order 128 and number 878 in the small library of the computer algebra system GAP, To compute all Steenrod squares on H^*(G, $\mathbb{Z}_2$), by using the proposition above Let L = C2 and K = ((C16: C2) : C2) are groups and G = L × K =C2 x ((C16 : C2) : C2) be the direct product. Then it is known that the projections G ↠ L and G ↠ K induce ring inclusions H^*(L, $\mathbb{Z}_2$) ↣ H^*(G, $\mathbb{Z}_2$) and H^*(K, $\mathbb{Z}_2$) ↣ H^*(G, $\mathbb{Z}_2$) see Proposesion(3.3.9) , and so we can think of H^*(L, $\mathbb{Z}_2$) and H^*(K, $\mathbb{Z}_2$) as subrings of H^*(G, $\mathbb{Z}_2$). It is known that the ring H^*(G, $\mathbb{Z}_2$) is generated by the generators of two subrings H^*(L, $\mathbb{Z}_2$) and H^*(K, $\mathbb{Z}_2$). In fact we have an isomorphism H^*(G, $\mathbb{Z}_2$) ≅ H^*(L × K, $\mathbb{Z}_2$) ≅ H^*(L, $\mathbb{Z}_2$) $\otimes_{\mathbb{Z}_2}$ H^*(K, $\mathbb{Z}_2$). All of this means that the cohomology ring and the Steenrod squares for H^*(G, $\mathbb{Z}_2$) are completly determined by the ring and operations for H^*(L, $\mathbb{Z}_2$) and H^*(K, $\mathbb{Z}_2$). In other words, if a group G of order 128 is a direct product then we already have the Steenrod operations since we have computed the rings and operations for groups of order less than 128.

By using the Cartan formula, The Steenrod operations on cohomology ring H^*(G, $\mathbb{Z}_2$) are determined by

$$Sq^i(\mathrm{a} \times \mathrm{b}) = \sum_{n+j=i} Sq^n(a) \times Sq^j(b) .$$

Group order: 128

Group number: 878

Group description: C2 x ((C16 : C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f, g

Degree 4: h, p

Steenrod squares

Sq^1 (d) = [d ∗ b]

Sq^1(e) = [0]

Sq^1(f) = [d ∗ d]

Sq^2 (f) = [d ∗ e ∗ b + f ∗ d + p ∗ a]

Sq1(g) = [0]

Sq^2 (g) = [d ∗ e ∗ b + g ∗ b ∗ b + g ∗ d + p ∗ b]

Sq^1 (h) = [d ∗ e ∗ b + g ∗ e]

Sq^2 (h) = [d∗d∗b∗b+g ∗d∗b+g ∗e∗b+f ∗g +p∗d+p∗e]

Sq^1 (p) = [d ∗ e ∗ b]

Sq^2(p) = [d ∗ e ∗ b ∗ b + d ∗ d ∗ d + g ∗ d ∗ b + g ∗ g + p ∗ d]

$\mathbf{G_{128,878} \simeq G_{2,1} \times G_{64,42}}$

to statistic all groups of order 128 that can be represent as direct product of groups of order less than 128 we use the next algorithm

---

Algorithm 3.3.11 direct product decomposition of groups of

---

order ”n”

Input:

• Order of a finite group G. as an integer n ≥ 0

Output:

• A list of all direct product groups of order n;

Procedure:

```
L:=NormalSubgroups(G);
L:=Filtered(L,N-.not Order(N)=Order(G) and not Order(N)=1);
for M in L do
    for N in L do
        if Order(M) ∗ Order(N) = Order(G) then
          if Order(Intersection(M,N))=1thenreturn[M,N]
            then;
          end if
        end if
    end for
end for
return fail;
EndProcedure:
```

This algorithim is new and we get all the direct product for the order 128, and we show some of them.

[128, 42] is a direct product $C_{16} \times C_8$

[128, 128] is a direct product $C_{32}\times C_4$

[128, 159] is a direct product $C_{64} \times C_2$

[128, 164] is a direct product $C_4\times ((C_4 \times C_2) : C_4)$

[128, 179] is a direct product $C_8\times C_8 \times C_2$

[128, 180] is a direct product $C_2 \times (C_8 : C_8)$

[128, 181] is a direct product $C_8 \times (C_8 : C_2)$

. . . .

[128, 2319] is a direct product $C_2 \times C_2 \times C_2 \times C_2 \times C_2 \times C_2$

[128, 2320] is a direct product $C_2 \times C_2 C_2 \times C_2 \times C_2 \times D_8$

[128, 2321] is a direct product $C_2 \times C_2 \times C_2 \times C_2 \times Q_8$

[128, 2322] is a direct product $C_2 \times C_2 \times C_2 \times ((C_4 \times C_2) : C_2)$

[128, 2323] is a direct product $C_2 \times C_2 \times ((C_2 \times C_2 \times$ C2) : $(C_2 \times C_2))$

[128, 2324] is a direct product C2 × C2 × ((C2 × Q8) : C2)

[128, 2325] is a direct product C2×((C2×((C4×C2) : C2)) :C2

CHAPTER FOUR

Stiefel-Whitney classes of a real representations for non-prime power groups

Chapter Four:

In this chapter, the current study, the researchers conduct computation of the Stiefel − Whitney classes of real representations of non − prime power groups such as Mathieu groups, symmetric groups, alternating groups and Janko groups. Stiefel- Whitney classes are conducted by using polytope convex hull with vector in n-dimention our computation using the HAP (Homological Algebra Programming).

4.1 Construct Stiefel- Whitney classes:

In this section, we discuss the method of computation or construction SW-classes. We have some steps to find them.

1) Let $\rho: G \to O_n(\mathbb{R})$ or $GL_n(\mathbb{R})$ be an any (orthogonal) matrix rerepresentation of a finite group G.
2) Take $v \in \mathbb{R}^n$ be any vector.
3) Let $\Omega(g, v) = \{g.v: g \in G\}$ represents convex hull of $\Omega(g, v)$ in $\mathbb{R}^n$
4) The vertices of the orbit polytope $P = P(\rho, v) = \rho(g)v = g.v = \Omega(g, v)$, convex hull of $\Omega(g, v)$ in $\mathbb{R}^n$,

We have the convex polytope by choosing a vector $v \in \mathbb{R}^n$,

$$P = P(\rho, v) = conv\{\rho(g).v: g \in G\}$$

is a CW-space homotopy equivalent to a ball B^n and P^{n-1} is equivalence to a sphere S^{n-1}. $C_*(P)$ is cellular chain complex of the polytope P which is a complex of $\mathbb{Z}$G-modules (may be not free). "A contractible m − dimensional and G acts on it by permuting cells" is the polytope P, and "the action endows P_*^G with structure of a $\mathbb{Z}$G − resolution of $\mathbb{Z}$."

$$C_*(P): 0 \to C_K(P) \to C_{K-1}(P) \to \cdots \to C_0(P)$$

And $C_*(P^{K-1}): 0 \to C_{K-1}(P) \to C_{K-2}(P) \to \cdots \to C_0(P)$

R_*^G *is* $\mathbb{Z}$G − resolution of $\mathbb{Z}$

Then the tensor product $R_*^G \otimes C_*(P)$ if (non- free) $\mathbb{Z}$G –resolution with

$g(a * b) = ga * gb$, where $P = P(\rho, v) = ball$, $P^{n-1} = sphere$.

And we have the equivalence $R_*^G \otimes C_*(P) \cong EG \times B^n$

$$R_*^G \otimes C_*(P^{n-1}) \cong EG \times S^{n-1}$$

In general the module $C_K(P)$ is not always free for $1 \leq m \leq K$, such that K represents the dimension of P. The free abelian group on one generator is the module $C_K(P)$ given a possibly non-trivial G action, we denote $C_K(P)$ by $\mathbb{Z}^{\epsilon}$. For the i^{th} orbit of p-cells, let $G_*^k \subseteq G$ represent "the stabilizer group of some cell in the orbit", and let $R_*^{G_i^k}$ designate some "free $\mathbb{Z}G_i^k$ − resolution of $\mathbb{Z}$ ".

$$R_*^{G_i^k} : \ldots \rightarrow R_2^{G_i^k} \rightarrow R_1^{G_i^k} \rightarrow R_0^{G_i^k}$$

The polytope P has dimension $K = dim(P)$. In addition, $R^{G_i^K} = R_*^G$ denotes a free $\mathbb{Z}G$ –resolution, when the unique K-dimensional cell has stabilizer group $G_1^K = G$. The direct sum of $\mathbb{Z}G$-modules is a module $C_p(P)$.

$$C_p(P) = \oplus_{1 \leq i \leq d_k} \mathbb{Z}G \otimes_{\mathbb{Z}G_i^k} \mathbb{Z}$$

By defining $D_{p,q} := \oplus_{1 \leq i \leq d_k} \mathbb{Z}G \otimes_{\mathbb{Z}G_i^k} R_q^{G_i^k}$ to obtain a free ZG-resolution

$$D_{p,*} : \ldots \rightarrow D_{p,q} \rightarrow \cdots \rightarrow D_{p,1} \rightarrow D_{p,0}$$

of the module $C_p(P)$. The boundary maps in $C_*(P)$ induce chain maps ∂h : $D_{p,q} \rightarrow D_{p-1,q}$ to yield a diagram $D_{*,*}$ of free $\mathbb{Z}G$-module

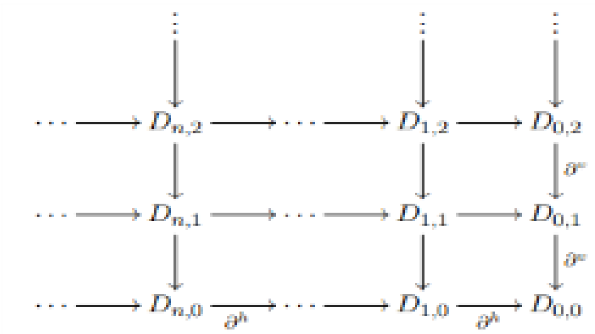

Figure 4.1.1

The vertical maps ∂_v : $D_{p,q} \rightarrow D_{p,q-1}$ in $D_{*,*}$ Satisfy $\partial^v \partial^v = 0$.

$$\ldots \rightarrow R_2^G \otimes \mathbb{Z} \xrightarrow{\alpha} R_1^G \otimes \mathbb{Z} \xrightarrow{\alpha} R_0^G \otimes \mathbb{Z}$$

$$\ldots \rightarrow C_2(P) \xrightarrow{\partial} C_1(P) \xrightarrow{\partial} C_0(P)$$

$$\begin{array}{ccccc}
\ldots \to C_2(P)\otimes R_2^G\otimes\mathbb{Z} & \xrightarrow{\partial_1\otimes I} & C_1(P)\otimes R_2^G\otimes\mathbb{Z} & \xrightarrow{\partial_0\otimes I} & C_0(P)\otimes R_2^G\otimes\mathbb{Z}\\
I\otimes\alpha_1 \downarrow & & I\otimes\alpha_1 \downarrow & & I\otimes\alpha_1 \downarrow\\
\ldots \to C_2(P)\otimes R_1^G\otimes\mathbb{Z} & \xrightarrow{\partial_1\otimes I} & C_1(P)\otimes R_1^G\otimes\mathbb{Z} & \xrightarrow{\partial_0\otimes I} & C_0(P)\otimes R_1^G\otimes\mathbb{Z}\\
I\otimes\alpha_0 \downarrow & & I\otimes\alpha_0 \downarrow & & I\otimes\alpha_0 \downarrow\\
\ldots \to C_2(P)\otimes R_0^G\otimes\mathbb{Z} & \xrightarrow{\partial_1\otimes I} & C_1(P)\otimes R_0^G\otimes\mathbb{Z} & \xrightarrow{\partial_0\otimes I} & C_0(P)\otimes R_0^G\otimes\mathbb{Z}
\end{array}$$

Figure 4.1.2

$$\begin{array}{ccccc}
C_0(P)\otimes R_2^G\otimes\mathbb{Z} & & & & \\
\oplus & & C_0(P)\otimes R_1^G\otimes\mathbb{Z} & & \\
C_1(P)\otimes R_1^G\otimes\mathbb{Z} & \xrightarrow{\partial_0\otimes I\oplus I\otimes\alpha_1\oplus\partial_1\otimes I\oplus I\otimes\alpha_0} & \oplus & \xrightarrow{\partial_0\otimes I\oplus I\otimes\alpha_0} & C_0(P)\otimes R_0^G\otimes\mathbb{Z}\\
\oplus & & C_1(P)\otimes R_0^G\otimes\mathbb{Z} & & \\
C_2(P)\otimes R_0^G\otimes\mathbb{Z} & & & &
\end{array}$$

Figure 4.1.3

Now, Hom($R_*^G \otimes C_*(P)$)=E_*^G, and it is denoted by E_*^G, that can construct a second $\mathbb{Z}G$-resolution. This second resolution has

$$E_n^G = \bigoplus_{\substack{p+q=n\\0\le p\le D}} D_{p,q} = \bigoplus_{\substack{p+q=n\\0\le p\le D}} \bigoplus_{1\le i\le d_p} R_q^{G_i^k} \oplus_{ZG_i^k} \mathbb{Z}G$$

We can define $F_*^G < E_*^G$ as a "ZG − subchain complex "by setting

$$F_n^G = \bigoplus_{\substack{p+q=n\\0\le p\le D-1}} D_{p,q} = \bigoplus_{\substack{p+q=n\\0\le p\le D-1}} \bigoplus_{1\le i\le d_p} R_q^{G_i^k} \oplus_{ZG_i^k} \mathbb{Z}G$$

We let $\bar{R}_* = E_*^G/F_*^G$ Then

$$\overline{R_n} = \begin{cases} 0 & n<K \\ R_{n-K}^G \otimes_{\mathbb{Z}} \mathbb{Z}^{\epsilon} & n\ge K \end{cases} \qquad \ldots(4.1)$$

An Addition, there is a short exact sequene .

$$F_*^G \rightarrowtail E_*^G \twoheadrightarrow \overline{R_*} \qquad \ldots(4.2)$$

Taking $\mathbb{Z}_2$ as a field and applying the contravariant functor $\mathrm{Hom}_{\mathbb{Z}_2 G}(-, \mathbb{Z}_2)$ we acquire "the short exact sequence of cochain complexes"

$$Hom_{\mathbb{Z}_2 G}(\overline{R_*}, \mathbb{Z}_2) \rightarrowtail Hom_{\mathbb{Z}_2 G}(E_*^G, \mathbb{Z}_2) \twoheadrightarrow Hom_{\mathbb{Z}_2 G}(F_*^G, \mathbb{Z}_2) \quad \ldots (4.3)$$

The explanation behind working over $\mathbb{Z}_2$ is that we can ignore the action of G on $\mathbb{Z}^{\epsilon}$. In equation (4.3) the inclusion of cochain complexes induces cohomology homomorphisms

$$\rho^t : H^t(G, \mathbb{Z}_2) \to H^{t+K}(G, \mathbb{Z}_2) \quad \ldots (4.4)$$

The cohomology homomorphism ρ^k is only dependent on "ρ" and to a certain extent on the vector u. The cochain complexes in (4.3) as

$$L^* = Hom_{\mathbb{Z}_2 G}(\bar{R}_*, \mathbb{Z}_2), \; M^* = Hom_{\mathbb{Z}_2 G}(E_*^G, \mathbb{Z}_2), \; M^*/L^* = Hom_{\mathbb{Z}_2 G}(F_*^G, \mathbb{Z}_2)$$

The relative cohomology can be define by setting

$$H^n(M^*, L^*) = H^n\left(\frac{M^*}{L^*}\right) \quad \ldots (4.5)$$

in low dimension $\mathrm{n} < K$

$$H^n(G, \mathbb{Z}_2) = H^n(M^*, L^*) \quad \ldots (4.6)$$

We obtain the isomorphism from (4.1)

$$\tau : H^n(\mathrm{G}, \mathbb{Z}_2) \cong \mathrm{H}^{\mathrm{n+K}}(M^*, L^*), n \geq 0 \quad \ldots (4.7)$$

And $H^n(M^*, L^*) = 0$, where $n < K$. The isomorphism τ is called the Thom isomorphism (for more information see [4, 1]) .

The cohomology cup product is defined as

$$\smile : H^p(M^*) \times H^q(M^*) \to H^{p+q}(M^*) \quad \ldots (4.8)$$

A cup product can extend to

$$H^p(K^*, L^*) \otimes_{\mathbb{Z}_2} H^q(M^*, L^*) \to H^{p+q}(M^*, L^*) \qquad \dots (4.9)$$

Taking ψ to signify the non-zero component of $H^K(M^*, L^*)$, it is possible to demonstrate that the formula (3.5) yields the Thom isomorphism in terms of (4.9).

$$\tau(x) = \psi \smile x \quad \dots (4.10)$$

And by using the properties of a cup-i product

$$\smile_i : M^p \otimes_{\mathbb{Z}_2} M^q \to M^{p+q-i}.$$

We can restrict $L^* < M^*$ be the subcochain complex producing a cup-i product

$$\smile_i : L^p \otimes_{\mathbb{Z}_2} L^q \to L^{p+q-i} \qquad \dots (4.11).$$

We can use properties of Steenrod square and formula (3.8) in order to define Steenrod squares on relative cohomology, with the restricted cup-i product.

$Sq^i : H^n(M^*, L^*) \to H^{n+i}(M^*, L^*)$

Definition 4.1.1:[3]

the i^{th} *SW- class* $\omega_i \in H^i(G, \mathbb{Z}_2)$ is described by the formula

$$\omega_i = \tau^{-1}(Sq^i(\psi))$$

If we have the rerepresentation $\rho : G \to O_n(R)$ of a finite group G, and $v \in R^n$ as a vector, or similarly $\psi \smile \omega_i = Sq^i(\psi)$, *where* $0 \neq 1 \in H^0(M^*, L^*) \cong \mathbb{Z}_2$.

In particular $\omega_0 = 1$ *and* $\psi = \tau(1)$.

The total SW- class can be explained as

$$\omega(\rho, v) = \omega_0 + \omega_1 + \cdots + \omega_K$$

We recommend reading [2, 4] for a theoretical explanation of SW-classes. A direct computer implementation of specification (4.1.1) may not be practical because "the size of the resolution E_*^G underlying the definition". Although, the homomorphism $\rho^0 : H^0(M^*, L^*) \to H^K(G, \mathbb{Z}_2)$ it is feasible to calculate $\rho^0(1) = \psi$. If it occurs that $\rho^0(1)$ is non-zero, then ρ^0 is an isomorphism. The Naturality formula is

$$\begin{array}{ccc} H^0(M^*, L^*) & \xrightarrow{\rho^0} & H^K(G, \mathbb{Z}_2) \\ Sq^i \downarrow & & \downarrow Sq^i \\ H^i(M^*, L^*) & \xrightarrow{\rho^i} & H^{K+i}(G, \mathbb{Z}_2) \end{array}$$

Figure 4.1.4

$$\rho^i(Sq^i(1)) = Sq^i(\rho^0(1))$$

Hence, any solution ω_i to $\psi \cup \omega_i = Sqi(\psi)$ yields $\rho^0(1) \cup \omega_i = Sq^i(\rho^0(1))$. is totally within the ring $H^*(M) = H^*(G, \mathbb{Z}_2)$. This final formula can be used to calculate the multiple $\rho^0(1) \cup \omega_i$. No doubt, this multiple contains helpful information when $\rho^0(1) \neq 0$.

4.2 Computing method SW- classes of real representations of non-prime power groups:

We previously learned the method of computing method SW-classes. As for the non-prime power groups the method revolves around how to compute Steenrod square for this type of groups. To compute Steenrod square on cohomology rings of non – prime power groups are less than or equal to 128 group. However, if the cohomology ring of the sylow subgroup of the given group known, we could calculate the steenrod square on cohomology rings of several non-prime groups whose sylow subgroup is order ≤ 128. For example, the group $G_{8,5}$ of order 8 and number 5 belong to $syl_2(G)$. Structure Description is $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, where G isomorphic to J_1 (janko group) is a realizable group of type J_1. Also, $G_{16,8}$ the group of order 16 and number 8 in GAP library belong to $syl_2(G)$ where Structure Description is QD_{16} ,where $G \cong M_{11}$. Is means that $G_{16,8}$ is a realizable group of type M_{11}.We applied the above for all groups, and it is realizable .

Example4.2.1:

Consider the group G= $Syl_2(M_{11})$ of order 16 and number 5, emerging as the sylow $2-$subgroup of the M_{11} . let ρ signify the representation that transfers each permutation g$\in G$ to 11×11 permutation matrix $\rho(g)$ and let $v = (1,2,3,4,5,6,7,8,9,10,11) \in R^{11}$. The total SW-class will be calculated at the next GAP session.

GAP session
$gap > G := SylowSubgroup(MathieuGroup(11),2);$
Group([(2,4)(5,9)(6,11)(7,8), (3,10)(5,9)(6,8)(7,11), (2,5)(3,10)(4,9)(7,8), (2,6,9,8,4,11,5,7)(3,10)])
$gap > A := Mod2SteenrodAlgebra(G, 12);;$
$gap > gens := ModPRingGenerators(A);$
$[\,v.1, v.2, v.3, v.6, v.8\,]$
$gap > List(gens, A!.degree);$
$[\,0, 1, 1, 3, 4\,]$
$gap > rho := PermToMatrixGroup(G);$
[(2,4)(5,9)(6,11)(7,8), (3,10)(5,9)(6,8)(7,11), (2,5)(3,10)(4,9)(7,8), (2,6,9,8,4,11,5,7)(3,10)] ->
[[[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]……
gap> v:=[1,2,3,4,5,6,7,8,9,10,11];;

```
gap > sw:= FundamentalMultiplesOfStiefelWhitneyClasses(rho, v, A, true);
[ [ 0*v.1, v.1 ], [ 0*v.1, v.3, v.2, v.2+v.3 ], [ 0*v.1, v.5, v.4, v.4+v.5 ], [ 0*v.1,
v. 7, v. 6, v.6+v.7 ],[ 0*v.1, v.10, v.9, v.9+v.10, v.8, v.8+v.10, v.8+v.9,
v.8+v.9+v.10 ],…..
gap > TotalStiefelWhitneyClass:=
sw[1][2]+sw[2][2]+sw[3][2]+sw[4][2]+sw[5][2]+sw[6][2]+sw[7][2]+sw[8][2]+
sw[9][2];
v. 1+v.3+v.5+v.7+v.10+v.14+v.18+v.22+v.27
gap > PrintAlgebraWordAsPolynomial(A, TotalStiefelWhitneyClass);
v. 1 + v.3 + v.2*v.2 + v.2*v.3 + v.2*v.2*v.2 + v. 2 * v. 2 * v. 2 * v. 2 +
v.2*v.2*v.2*v.2*v.2 + v.6*v.2*v.2 +  v.2*v.2*v.2*v.2*v.2*v.2 + v.6*v.2*v.2*v.2
+ v.2*v.2*v.2*\ v.2*v.2*v.2*v.2 + v.2*v.2*v.2*v.2*v.2*v.2*v.2*v.2
```

CHAPTER FIVE

Experimental results

Chapter Five:

5.1 **Experimental results of Steenrod squares on a a finite 2-groups of order 128.**

We present a sample of results from our implementation of Steenrod squares on a a finite 2-groups of order 128. We have completed the computation of more than 200 groups of order 128. The implementation can compute the Steenrod squares.

• all direct product groups of orders 128 by using Direct product method.

• some small groups of order 128 wich have generators of degree 1, 2, 3 or 4 by using The CohomologicalData(G,n) method or Detection Method as well as using the HAP command CohomologicalDetectedIntersection(G,K,n) on the specified maximal subgroups.

Group order: 128

Group number: 1

Group description: C128

Cohomology generators

Degree 1: a

Degree 2: b

Cohomology relations

1 : a^2

Steenrod squares

$Sq^1 (b) = 0$

****

Group order: 128

Group number: 2

Group description: (((C4 x C2) : C8) : C2) : 1

Maximal Subgroup list:[[64, 17], [64, 71], [64, 17]]

Cohomology generators

Degree 1: a, b

Degree 2: c, d, e, f

Degree 3: g

Steenrod squares

Sq^1 (c) = 0

Sq^1 (d) = c ∗ b + d ∗ b + e ∗ a

Sq^1 (e) = c ∗ b + d ∗ a + e ∗ b

Sq^1 (f) = e ∗ a

Sq^1 (g) = c ∗ b ∗ b

Sq^2 (g) = c ∗ c ∗ a + c ∗ c ∗ b + e ∗ e ∗ a + e ∗ e ∗ b + g ∗ b ∗ b

****

Group order: 128

Group number: 3

Group description: (C4 : C4) : C8

Maximal Subgroup list:[[64, 17], [64, 70], [64, 17]]

Cohomology generators

Degree 1: a, b

Degree 2: c, d, e

Degree 3: f, g

Degree 4: h, p

Steenrod squares

Sq^1 (c) = 0

Sq^1 (d) = b ∗ b

Sq^1 (e) = 0

Sq^1 (f) = d ∗ b

Sq^1 (g) = c ∗ a + c ∗ b + d ∗ b + f

Sq^1 (h) = c ∗ b ∗ b

Sq^1 (p) = c ∗ b ∗ b

Sq^2 (f) = c ∗ c ∗ a + c ∗ c ∗ b + d ∗ d ∗ a + f ∗ c + p ∗ a, c ∗ c ∗ a + c ∗ c ∗ b + d ∗ d ∗ a + g ∗ b ∗ b + f ∗ c + p ∗ a

Sq^2 (g) = c ∗ d ∗ a + c ∗ d ∗ b + f ∗ d, c ∗ d ∗ a + c ∗ d ∗ b + g ∗ b ∗ b + f ∗ d

Sq^2 (h) = ?
****

Group order: 128

Group number: 5

Group description: (C8 x C2) : C8

Maximal Subgroup list:[[64, 83], [64, 83], [64, 83]]

Cohomology generators

Degree 1: a, b

Degree 2: c, d, e, f, g

Steenrod squares

Sq^1 (c) = 0, e ∗ b

Sq^1 (d) = d ∗ a + d ∗ b, d ∗ a + d ∗ b + e ∗ b

Sq^1 (e) = d ∗ a, d ∗ a + e ∗ b

Sq^1 (f) = d ∗ a, d ∗ a + e ∗ b

Sq^1 (g) = 0, e ∗ b

****

Group order: 128

Group number: 6

Group description: (C8 x C4) : C4

Maximal Subgroup list:[[64, 112], [64, 57], [64, 112]]

Cohomology generators

Degree 1: a, b

Degree 2: c, d, e, f

Degree 3: g, h

Degree 4: p, q

Steenrod squares

Sq^1 (c) = 0, d ∗ a

Sq^1 (d) = 0, d ∗ a

Sq^1 (e) = 0, d ∗ a

Sq^1 (f) = 0, d ∗ a

Sq^1 (g) = 0

Sq^2 (g) = c ∗ d ∗ a + c ∗ f ∗ b + g ∗ f + q ∗ b, c ∗ f ∗ b + g ∗ f + q ∗ b

Sq^1 (h) = 0

Sq^2 (h) = c ∗ d ∗ a + h ∗ f + q ∗ a, h ∗ f + q ∗ a

Sq^1 (p) = 0, c ∗ d ∗ a

Sq^2 (p) = c ∗ c ∗ d + c ∗ c ∗ f + c ∗ d ∗ f + c ∗ e ∗ f + c ∗ f ∗ f + d ∗ f ∗ f + e ∗ f ∗ f + g ∗ h + q ∗ d + q ∗ e

****

Group order: 128

Group number: 7

Group description: (C8 x C2) : C8

Maximal Subgroup list:[[64, 112], [64, 84], [64, 112]]

Cohomology generators

Degree 1: a, b

Degree 2: c, d, e, f

Degree 3: g, h

Degree 4: p, q

Steenrod squares

Sq^1 (c) = 0, d ∗ a

Sq^1 (d) = d ∗ a + d ∗ b, d ∗ b

Sq^1 (e) = d ∗ a + d ∗ b, d ∗ b

Sq^1 (f) = d ∗ a + d ∗ b, d ∗ b

Sq^1 (g) = 0

Sq^2 (g) = c ∗ d ∗ a + g ∗ d + g ∗ f + q ∗ a, g ∗ d + g ∗ f + q ∗ a

Sq^1 (h) = d ∗ d

Sq^2 (h) = c ∗ d ∗ a + h ∗ d + q ∗ b, h ∗ d + q ∗ b

Sq^1 (p) = c ∗ d ∗ a + d ∗ d ∗ b + g ∗ d + g ∗ f, d ∗ d ∗ b + g ∗ d + g ∗ f

Sq^2 (p) = d ∗ d ∗ f + g ∗ h + q ∗ d + q ∗ e

Sq^1 (q) = c ∗ d ∗ a + d ∗ d ∗ b, d ∗ d ∗ b

****

Group order: 128

Group number: 42

Group description: C16 x C8

Cohomology generators

Degree 1: a, b

Degree 2: c, d

Cohomology relations\

1 : b^2

2 : a^2

Steenrod squares

Sq^1 (c) = 0

Sq^1 (d) = 0

****

Group order: 128

Group number: 128

Group description: C32 x C4

Cohomology generators

Degree 1: a, b

Degree 2: c, d

Cohomology relations

1 : b^2

2 : a^2

Steenrod squares

Sq1 (c) = 0

Sq1 (d) = 0

****

Group order: 128

Group number: 159

Group description: C64 x C2

Cohomology generators

Degree 1: a, b

Degree 2: c

Cohomology relations

1 : a^2

Steenrod squares

Sq^1 (c) = 0

****

Group order: 128

Group number: 164

Group description: C4 x ((C4 x C2) : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g, h, p

Cohomology relations

1: f^2

2: $e * f + f * g$

3: $e^2 + g^2$

4: c^2

5: $b * e + b * g$

6: b^2

7: $a * f$

8: $a * e + a * g + b * f$

9: $a * b$

10: a^2

Steenrod squares

$Sq^1(d) = 0$

$Sq^1(e) = e * a + e * b$

$Sq^1(f) = d * b + e * a + f * b$

$Sq^1(g) = e * a + f * b + h * a$

$Sq^1(h) = 0$

$Sq^1(p) = 0$

****

Group order: 128

Group number: 179

Group description: C8 x C8 x C2

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

1 : b^2

2 : a^2

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = 0

****

Group order: 128

Group number: 180

Group description: C2 x (C8 : C8)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

1 : b^2

2 : a^2

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = 0

****

Group order: 128

Group number: 181

Group description: C8 x (C8 : C2)

Maximal Subgroup list:[[64, 2], [64, 85], [64, 3], [64, 83], [64, 2], [64, 112], [64, 3]]

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1 (d) = 0, a ∗ b ∗ c

Sq^1 (e) = 0

Sq^2 (e) = d ∗ a ∗ b ∗ c + e ∗ c ∗ c + f ∗ a, e ∗ c ∗ c + f ∗ a

Sq^1 (f) = 0, d ∗ a ∗ b ∗ c

Sq^2 (f) = f ∗ a ∗ b + f ∗ b ∗ b

****

Group order: 128

Group number: 188

Group description: C2 x (((C8 x C2) : C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Degree 3: g, h, p

Degree 4: q, r

Degree 5: s

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = d ∗ a + e ∗ b + g

Sq^1 (f) = d ∗ a + g

Sq^1 (g) = 0

Sq^2 (g) = d ∗ d ∗ a + g ∗ e + q ∗ a

Sq^1 (h) = e ∗ e

$Sq^2(h) = d*b*b*b + d*d*a + e*e*b + h*b*b + g*e + h*e*q*b$
$Sq^1(p) = d*b*b$

$Sq^2(p) = p*b*b + g*e$

$Sq^1(q) = q*a$

$Sq^2(q) = d*b*b*b*b + d*d*b*b + d*e*b*b + d*e*e + e*e*e + e*e*f + h*e*b + h*h + q*e$

$Sq^1(r) = d*d*a + g*d$

$Sq^2(r) = d*b*b*b*b + d*d*b*b + d*e*b*b + d*d*e + d*e*e + e*e*e + e*e*f + h*e*b + p*e*b + h*h + q*d + q*f + r*e$

$Sq^1(s) = d*b*b*b*b + d*d*b*b + e*e*f + h*d*b + h*p + q*e + r*e$

$Sq^2(s) = d*b*b*b*b*b + d*d*b*b*b + d*e*e*b + h*d*c*c + g*d*e + h*d*e + p*d*d + h*p*c + q*p + r*h$

$Sq^4(s) = 0$

G128,188 ≅ G2,1 × G64,4

****

Group order: 128

Group number: 189

Group description: C2 x ((C4 x C2) : C8)

Cohomology generators

Degree 1: a, b,c

Degree 2: d, e, f

Degree 3: g, h

Degree 4: p

Steenrod squares

$Sq^1(d) = 0$

$Sq^1(e) = c*c*c + e*c$

$Sq^1(f) = d*b + e*b + g$

$Sq^1 (g) = 0$

$Sq^2 (g) = d * c * c * c + d * d * b + f * c * c + g * e + p * b$

$Sq^1 (h) = e * c * c + e * e$

$Sq^2 (h) = d * c * c * c + d * d * b + d * e * c + h * c * c + h * e + h * c$

$Sq^1 (p) = d * c * c * c + d * d * b + g * d * p * b$

$Sq^2 (p) = d * d * c * c + c * e * c * c + e * e * c * c + d * d * d + d * d * e + d * d * f + d * e * e + e * e * e + e * e * f + p * d + p * e + p * f$

G128,189 ≅ G2,1 × G64,5

****

Group order: 128

Group number: 206

Group description: C2 x ((C4 : C8) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Cohomology relations

1 : f^2

2 : $a * f$

3 : $a * b$

4 : a^2

Steenrod squares

$Sq^1 (d) = 0$

$Sq^1 (e) = e * a + e * b$

$Sq^1 (f) = f * b$

****

Group order: 128

Group number: 207

Group description: C2 x (Q8 : C8)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Steenrod squares

$Sq^1 (d) = 0$

$Sq^1 (e) = e * b$

$Sq^1 (f) = 0$

$Sq^2 (f) = e * b * b * c + g * a, g * a$

$Sq^1 (g) = 0$

$Sq^2 (g) = d * d * c * c + d * e * c * c$

$G128,207 \cong G2,1 \times G64,7$

****

Group order: 128

Group number: 230

Group description: C2 x (((C8 x C2) : C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Degree 3: g, h

Degree 4: p, q

Steenrod squares

$Sq^1 (d) = d * a + d * b$

$Sq^1 (e) = d * a + g + h$

$Sq^1 (f) = 0$

$Sq^1 (g) = 0$

$Sq^2 (g) = g * b * b + q * b$

$Sq^1 (h) = 0$

$Sq^2 (h) = g * b * b + h * f + q * a + q * b$

$Sq^1 (p) = d * b * b * b + d * d * a + q * b$

$Sq^2 (p) = d * b * b * b * b + d * d * b * b + d * d * e + d * e * f + p * f + q * e$

$Sq^1 (q) = d * d * a + d * f * a + g * f + h * f + q * a$

$Sq^2 (q) = d * d * e + d * d * f + g * g + q * e + q * f$

G128,230 ≅ G2,1 × G64,8

****

Group order: 128

Group number: 231

Group description: C2 x ((C4 : C4) : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g, h , p

Degree 4: q

Steenrod squares

$Sq^1 (e) = e * a + e * c$

$Sq^1 (f) = 0$

$Sq^1 (g) = e * a * a + e * a * b + g * c$

$Sq^2 (g) = e * e * a + e * e * c + e * f * c + g * b * c + q * c, e * e * a + e * e * c + e * f * c + q * c$

$Sq^1 (h) = e * b * c + g * c$

$Sq^2 (h) = e * f * a + g * b * c + q * a, e * f * a + q * a$

$Sq^1 (p) = e * a * a + e * a * b + f * a * b + g * c$

$Sq^2 (p) = e * f * b + g * b * c + q * b, e * f * b + q * b$

$Sq^1 (q) = e * f * a + e * f * c, e * f * a + e * f * c + g * b * c$

$Sq^2 (q) = e * e * a * a + e * e * a * b + e * e * b * c + e * e * f + e * f * f + g * g$

G128,231 ≅G2,1 × G64,68

****

Group order: 128

Group number: 254

Group description: C2 x ((C8 : C4) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g, h

Steenrod squares

$Sq^1 (d) = d * b + f$

$Sq^1 (e) = d * b + f$

$Sq^1 (f) = 0$

$Sq^2 (f) = g * a$

$Sq^1 (g) = d * b * b * b$

$Sq^2 (g) = d * d * b * b + g * b * b + g * d + g * e$

$Sq^1 (h) = h * b$

$Sq^2 (h) = d*b*b*b+d*d*b*b+d*d*d+d*d*e+g*b*b+h*b*b+g*d+g*e$
G128,254 ≅ G2,1 × G64,10

****

Group order: 128

Group number: 255

Group description: C2 x ((C2 x C2) . ((C4 x C2) : C2) = (C4 x C2) . (C4 x C2))
Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e, f

Degree 4: g, h, p, q

Degree 5: r

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = 0

Sq^2 (e) = g ∗ a

Sq^1 (f) = 0

Sq^2 (f) = g ∗ a + p ∗ a

Sq^1 (g) = h ∗ b

Sq^2 (g) = e ∗ f + g ∗ b ∗ b + q ∗ b ∗ b

Sq^1 (h) = h ∗ b

Sq^2 (h) = g ∗ b ∗ b + q ∗ b ∗ b + q ∗ c

Sq^1 (p) = 0

Sq^2 (p) = e ∗ f + g ∗ b ∗ b + q ∗ c

Sq^1 (q) = h ∗ b

Sq^2 (q) = e ∗ f + g ∗ b ∗ b + q ∗ b ∗ b

Sq^1 (r) = 0

Sq^2 (r) = g ∗ e + g ∗ f + q ∗ e

Sq^4 (r) = g ∗ g ∗ a + g ∗ q ∗ a

G128,255≅G2,1 × G64,11

****

Group order: 128

Group number: 270

Group description: C2 x((C4 : C8) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Steenrod squares

Sq^1 (d) = d ∗ a + d ∗ b

Sq^1 (e) = e ∗ b + f ∗ b

Sq^1 (f) = 0

G128,270 ≅ G2,1 × G64,12

****

Group order: 128

Group number: 271

Group description: C2 x ((C2 x C2) . ((C4 x C2) : C2) = (C4 x C2) . (C4 x C2))

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(d)=d*b

Sq^1(e)=0

Sq^1(f)=e*b*b

Sq^2(f)=d*d*a+g*a

Sq^1(g)=0

Sq^2(g)=d*d*b*b

G 128,271 ≅ G 2,1 × G 64,13

****

Group order: 128

Group number: 273

Group description: C2 x ((C2 x C2) . ((C4 x C2) : C2) = (C4 x C2) . (C4 x C2))

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(d)=d*b

Sq^1(e)=0

Sq^1(f)=e*b*b

Sq^2(f)=d*d*a+g*a

Sq^1(g)=0

Sq^2(g)=d*d*b*b

G 128,273 ≅ G 2,1 × G 64,13

****

Group order: 128

Group number: 294

Group description: C2 x (C8 : C8)

Maximal Subgroup list:[[64, 15], [64, 103], [64, 15], [64, 83], [64, 15], [64, 103], [64, 15]]

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Steenrod squares

Sq^1(d)=0,a*b*c

Sq^1(e)=a*b*c+d*a+e*a,d*a+e*a

****

Group order: 128

Group number: 295

Group description: C2 x (C8 : C8)

Maximal Subgroup list:[[64, 16], [64, 103], [64, 16], [64, 83], [64, 16], [64, 103], [64, 16]]

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Steenrod squares

Sq^1(d)=0,a*b*c

Sq^1(e)=a*b*c+d*a+e*a,d*a+e*a

****

Group order: 128

Group number: 307

Group description: C8 x D16

Maximal Subgroup list:[[64, 6], [64, 115], [64, 16], [64, 115], [64, 118], [64, 2], [64, 6]]

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Steenrod squares

Sq^1(d)=d*a+d*b

Sq^1(e)=0

****

Group order: 128

Group number: 308

Group description: C8 x QD16

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1(d)=0

Sq^1(e)=0

Sq^2(e)=e*b*b+f*b

Sq^1(f)=0

Sq^2(f)=e*e

G 128,308 ≅ G 8,1 × G 16,8

****

Group order: 128

Group number: 309

Group description: C8 x Q16

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 4: e

Steenrod squares

Sq^1(d)=0

Sq^1(e)=0

Sq^2(e)=0

G 128,309 ≅ G 8,1 × G 16,9

****

Group order: 128

Group number: 456

Group description: C8 x C4 x C4

Maximal Subgroup list:[[64, 83], [64, 83], [64, 83], [64, 55], [64, 83], [64, 83], [64, 83]]

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Steenrod squares

Sq^1(d)=0,a*b*c

Sq^1(e)=0,a*b*c

Sq^1(f)=0,a*b*c

****

Group order: 128

Group number: 457

Group description: C4 x (C8 : C4)

Maximal Subgroup list:[[64, 84], [64, 83], [64, 84], [64, 55], [64, 84], [64, 83], [64, 84]]

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Steenrod squares

Sq^1(d)=a*b*c+e*a,e*a

Sq^1(e)=0,a*b*c

Sq^1(f)=0,a*b*c

****

Group order: 128

Group number: 459

Group description: C2 x ((C8 x C2) : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g, h

Cohomology relations

1: f^2

2: e*f+f*g

3: e^2+g^2

4: b*e+b*g

5: b^2

6: a*f

7: a*e+a*g+b*f

8: a*b

9: a^2

Steenrod squares

Sq^1(d)=0

Sq^1(e)=e*a+e*b

Sq^1(f)=e*a+f*b

Sq^1(g)=e*a+f*b+h*a

Sq^1(h)=0

****

Group order: 128

Group number: 463

Group description: C2 x (C4 . (C4 x C4))

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g

Degree 3: h, p

Degree 5: q, r

Degree 6: s, t

Degree 8: u

Steenrod squares

Sq^1(d)=0, d*a+d*b+e*a

Sq^1(e)=d*a+d*b+e*b, e*a+e*b

Sq^1(f)=d*a+e*a+e*b, d*b+e*b

Sq^1(g)=d*a+e*a+e*b, d*b+e*b

Sq^1(h)=d*d

Sq^2(h)=h*e+h*f+q+r

Sq^1(p)=d*d+e*e

Sq^2(p)=d*d*b+e*e*b+h*g+p*e+r

Sq^1(q)=d*d*f+d*d*g+d*e*f+e*e*e+e*e*f

Sq^2(q)=d*d*d*a+d*d*d*b+q*e

Sq^4(q)=q*d*d+q*d*e+u*a+u*b

Sq^1(r)=d*d*d+d*d*e+d*e*g+e*e*e+e*e*f

Sq^2(r)=d*d*d*a+h*d*d+h*d*e+p*d*d+p*d*e+q*d+q*e

Sq^4(r)=d*d*d*d*a+d*d*d*d*b+h*d*d*d+h*d*d*e+p*d*d*d+p*d*d*e+q*d*d +q*d*e+r*d*d+u*b

Sq^1(s)=d*d*d*a+h*d*d+h*d*e+h*e*g+p*d*d+p*d*e+q*e

Sq^2(s)=d*d*d*e+d+d*d*f+d*d*d*g+d*d*e*e+d*e*e*e+e*e*e*e+e*e*e*f+q*h+q*p

Sq^4(s)=d*d*d*d*e+d*d*d*d*f+d*d*d*e*e+d*d*d*e*f+d*d*e*e*e+d*e*e*e*e*e*e*e*e+e*e*e*e*f+q*h*d+q*p*d+u*d+u*f+u*g

Sq^1(t)=d*d*d*a+d*d*d*b+h*d*e+h*d*g+h*e*e+h*e*g+p*d*d+p*d*e+q*d+q*e+r*d

Sq^2(t)=d*d*d*d+d*d*d*e+d*d*e*f+e*e*e*f+q*p+r*h

Sq^4(t)=d*d*d*d*d+d*d*d*e*f+d*e*e*e*e+e*e*e*e*e+e*e*e*e*g+h*p*d*d+h*p*e*e+q*h*d+q*h*e+r*h*d+r*p*d+u*g

Sq^1(u)=h*d*d*d+h*d*d*e+h*d*d*g+h*e*e*g+p*d*d*e+p*d*e*e+q*d*d+q*e*e+r*d*d

Sq^2(u)=d*d*d*e*e+d*d*d*e*f+d*d*d*e*g+d*d*e*e*e+d*e*e*e*e+e*e*e*e*e+e*e*e*e*f

Sq^4(u)=d*d*d*d*d*d+d*d*d*d*d*f+d*d*d*d*e*e+d*d*d*d*e*g+u*d*d+u*d*f+u*e*e+u*e*g

G 128,463 ≅ G 2,1 × G 64,19

****

Group order: 128

Group number: 464

Group description: C2 x ((C4 : C4) : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g

Degree 3: h, p

Degree 4: q, r

Cohomology relations

1 : $h^2 + p^2$

2 : $g * a + h * p$

3 : f^2

4 : $e * h + e * p + f * p$

5 : $e * f$

6 : e^2

7 : $b * a + e * p$

8 : $b * p + e * g + f * g$

9 : $b * h + e * g$

10 : $b * f$

11 : b^2

12 : $a * a$

13 : $a * p$

14 : $a * h$

15 : $a * g + b * e$

16 : $a * f + b * e$

17 : $a * e + b * e$

18 : $a * b$

19 : a^2

20 : $f * h * p + f * p^2$

21 : $f * g * h + f * g * p$

22 : $d * p^2 + a^2$

23 : $d * g * h + p * a$

24 : $d * g^2 + p^2$

25 : $d * f * g + f * a$

26 : $d * e * g + e * a + f * a$

27 : $b * e * a + d * g * p + h * a$

28 : $b * e * g$

29 : $b * d * g + e * p + f * h$

30 : $b * d * e + f * h + f * p$

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = d * b + e * a

Sq^1 (f) = e * a

Sq^1 (g) = d * b + e * a

Sq^1 (h) =?

****

Group order: 128

Group number: 466

Group description: C2 x ((C4 : C4) : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g, h

Cohomology relations

1 : f^2

2 : $e * f + f * g$

3 : $e^2 + g^2$

4 : $b * e + b * g$

5 : b^2

6 : $a * f$

7 : $a * e + a * g + b * f$

8 : $a * b$

9 : a^2

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = e ∗ a + e ∗ b

Sq^1 (f) = d ∗ b + e ∗ a + f ∗ b

Sq^1 (g) = 0

Sq^1 (h) = h ∗ a

****

Group order: 128

Group number: 469

Group description: C2 x ((C4 x C2) . D8 = C4 . (C4 x C4))

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g

Degree 3: h, p

Degree 4: q, r

Steenrod squares

Sq^1(d)=0

Sq^1(e)=e*a+e*b

Sq^1(f)=d*b+e*a+f*b

Sq^1(g)=0

Sq^1(h)=d*d

Sq^2(h)=e*f*b+h*d+r*a

Sq^1(p)=d*g

Sq^2(p)=d*d*b+e*f*b+h*e+h*g+p*d+r*b

Sq^1(q)=d*d*b+e*f*b+h*g+p*d

Sq^2(q)=d*d*eh*p+r*f

Sq^1(r)=d*d*b+e*f*b+h*g+p*d

Sq^2(r)=d*d*d+d*d*e+d*d*f+r*d+r*e

G 128,469 ≅ G 2,1 × G 64,22

****

Group order: 128

Group number: 473

Group description: C2 x ((C8 : C2) : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g

Degree 3: h, p

Degree 4: q, r

Steenrod squares

Sq^1(d)=0

Sq^1(e)=e*b

Sq^1(f)=d*b+e*b+g*a

Sq^1(g)=e*b

Sq^1(h)=0

Sq^2(h)=h*e+r*a

Sq^1(P)=e*e

Sq^2(p)=e*g*b+h*f+p*e+r*b

Sq^1(q)=d*g*+e*e*b*h*e+h*f

Sq^2(q)=d*e*g+e*e*f+h*p+r*d

Sq^1(r)=d*g*b+e*e*b+e*g*b+h*e+h*f

Sq^2(r)=d*e*e+d*e*g+e*e*e+e*g*g+r*d+r*e

G 128,473 ≅ G 2,1 × G 64,24

****

Group order: 128

Group number: 475

Group description: C2 x ((C8 x C2) : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g

Degree 3: h, p

Degree 5: q, r

Degree 6: s, t

Degree 8: u

Steenrod squares

Sq^1(d)=0

Sq^1(e)=e*a+e*b

Sq^1(f)=e*a+e*b

Sq^1(g)=d*b+e*a+e*b

Sq^1(h)=d*e+d*f

Sq^2(h)=e*e*b+h*d+h*e+r

Sq^1(P)=d*e+d*f+e*e

Sq^2(p)=d*d*a+d*d*b+h*g+p*d+p*e+q*r

Sq^1(q)=d*d*e

Sq^2(q)=d*d*d*a+d*d*d*b+q*d+q*f

Sq^4(q)=d*d*d*d*a+e*e*e*e*b+h*d*d*g+q*d*e+u*a+u*b

Sq^1(r)=d*d*e+d*e*f+e*e*e+e*e*f

Sq^2(r)=d*d*d*a+h*d*e+h*e*g+p*d*e+q*d+q*e+q*f

Sq^4(r)=d*d*d*d*a+d*d*d*d*b+e*e*e*e*b+h*d*d*e+h*d*d*g+p*d*d*e+q*d*d+r*d*d+u*a

Sq^1(s)=h*d*e+h*e*e

Sq^2(s)=d*e*e*e+e*e*e*f+r*h

Sq^4(s)=d*d*d*d*f+d*d*d*e*e+d*d*d*e*g+d*e*e*e*e+e*e*e*e*f+e*e*e*e*g+h*p*e*e+q*h*e+u*e+u*f

Sq^1(t)=d*d*d*a+e*e*e*b+q*d+q*e+r*d

Sq^2(t)=d*d*d*d+d*d*d*e+d*d*d*f+d*d*d*g+d*d*e*f+q*h+q*p

Sq^4(t)=d*d*d*d*d+d*d*d*d*f+d*d*d*e*e+d*d*d*e*f+d*e*e*e*e+e*e*e*e*e+e*e*e*e*f+q*h*d+q*p*d+r*h*d+r*p*d+u*e+u*f+u*g

Sq^1(u)=d*d*d*d*a+e*e*e*e*b+h*d*d*g+h*d*e*g+q*d*d+r*d*d

Sq^2(u)=d*d*d*d*g+d*d*d*e*e+d*d*e*e*e+e*e*e*e*g+h*p*d*d+q*h*d+r*h*d+r*p*d

Sq^4(u)=d*d*d**e*e*e+d*d*e*e*e*e+d*e*e*e*e*e+e*e*e*e*e*e+h*p*d*d*e+q*h*d*e+u*d*d+u*d*f+u*d*g+u*e*e

G 128,475 ≅ G 2,1 × G 64,25

****

Group order: 128

Group number: 480

Group description: C4 x ((C8 x C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g

Cohomology relations

1 : $d^2 + e^2$

2 : $b * d + b * e$

3 : b^2

4 : $a * b$

5 : $a^2 + c^2$

6 : $b * c^2$

Steenrod squares

Sq^1 (d) = d ∗ a + d ∗ b

Sq^1 (e) = e ∗ a

Sq^1 (f) = 0

Sq^1 (g) = a ∗ a ∗ a + a ∗ a ∗ c

****

Group order: 128

Group number: 483

Group description: C8 x ((C4 x C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g

Cohomology relations

1 : $b * d + b * e$

2 : b^2

3 : $a * b$

4 : $a^2 + c^2$

5 : $c^2 * f + d^2 + e^2$

6 : b $* c^2$

Steenrod squares

Sq^1 (d) = d ∗ a + d ∗ b

Sq^1 (e) = e ∗ a + f ∗ a

Sq^1 (f) = 0

Sq^1 (g) = 0

****

Group order: 128

Group number: 487

Group description: C4 x ((C8 : C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Degree 3: g, h

Degree 4: p, q

Steenrod squares

Sq^1(d)=d*a

Sq^1(e)=0

Sq^1(f)=0

Sq^1(g)=0

Sq^2(g)=f*b*b*b+e*f*b+g*f+q*a

Sq^1(h)=e*b*b+f*f

Sq^2(h)=e*f*b+h*b*b+h*f+q*a+q*b

Sq^1(p)=e*b*b*b+e*f*b+f*f*b+g*f+h*e

Sq^2(p)=e*b*b*b*b+h*e*b+h*f*b+g*h+q*e

Sq^1(q)=0

Sq^2(q)=e*b*b*b*b+e*f*b*b+f*f*b*b+h*f*b+h*h+q*f

G 128,487 ≅ G 4,1 × G 32,7

****

Group order: 128

Group number: 488

Group description: C4 x (C2 . ((C4 x C2) : C2) = (C2 x C2) . (C4 x C2))

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Degree 3: g

Degree 5: h, p

Degree 6: q

Degree 8: r

Steenrod squares

Sq^1(d)=0

Sq^1(e)=0

Sq^1(f)=b*b*b+f*b

Sq^1(g)=e*b*b

Sq^2(g)=g*f+h

Sq^1(h)=e*f*b*b+f*f*b*b

Sq^2(h)=p*b*b

Sq^4(h)=r*a

Sq^1(p)=f*f*f+p*b

Sq^2(p)=0

Sq^4(p)=e*f*f*f*b+p*f*b*b+p*f*f+r*a+r*b

Sq^1(q)=e*f*f*b+g*f*f

Sq^2(q)=f*f*f*b*b+e*f*f*f+p*e*b

Sq^4(q)=f*f*f*f*b*b+p*g*f+r*e

Sq^1(r)=e*f*f*f*b

Sq^2(r)=f*f*f*f*b*b+e*f*f*f*f

Sq^4(r)=f*f*f*f*f*b*b+f*f*f*f*f*f+p*f*f*f*b+p*p*f+r*f*f

G 128,488 ≅ G 4,1 × G 32,8

****

Group order: 128

Group number: 490

Group description: C4 x ((C4 x C4) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(d)=d*a

Sq^1(e)=0

Sq^1(f)=0

Sq^1(g)=0

Sq^2(g)=f*f*a+g*f+h*a

Sq^1(h)=0

Sq^2(h)=f*f*f+h*b*b+h*f

G 128,490 ≅ G 4,1 × G 32,11

****

Group order: 128

Group number: 492

Group description: C4 x ((C8 x C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f, g

Cohomology relations

1 : $b * d + b * e$

2 : b^2

3 : $a * b$

4 : $a^2 + c^2$

5 : $c^2 * e + d^2 + e^2$

6 : $b * c^2$

Steenrod squares

$Sq^1 (d) = d * a + d * b$

$Sq^1 (e) = 0$

$Sq^1 (f) = f * a$

$Sq^1 (g) = 0$

****

Group order: 128

Group number: 493

Group description: C4 x (Q8 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(d)=d*a+d*b

Sq^1(e)=0

Sq^1(f)=a*a*c

Sq^1(g)=0

Sq^2(g)=h*a

Sq^1(h)=e*e*b+e*f*b

Sq^2(h)=e*e*f

G 128,493 ≅ G 4,1 × G 32,10

****

Group order: 128

Group number: 498

Group description: C4 x (C4 : C8)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Cohomology relations

1 : b^2

2 : $a * b + c^2$

3 : $a^2 + c^2$

4 : $b * c^2$

5 : $a * c^2$

6 : c^4

Steenrod squares

Sq^1 (d) = d ∗ b

Sq^1 (e) = 0

Sq^1 (f) = 0

****

Group order: 128

Group number: 501

Group description: C8 x (C4 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Cohomology relations

1 : b^2

2 : a ∗ b + c^2

3 : a^2 + c^2

4 : b ∗ c^2

5 : a ∗ c^2

6 : c^4

Steenrod squares

Sq^1 (d) = d ∗ b

Sq^1 (e) = 0

Sq^1 (f) = 0

****

Group order: 128

Group number: 506

Group description: C4 x (C8 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Cohomology relations

1: b^2

2: a*b+c^2

3: a^2+c^2

4: b*c^2

5: a*c^2

6: c^4

Steenrod squares

Sq^1(d)=d*b+e*a

Sq^1(e)=0

Sq^1(f)=0

****

Group order: 128

Group number: 507

Group description: C4 x (C8 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Cohomology relations

1 : b^2

2 : a $*$ b + c^2

3 : a^2 + c^2

4 : b $*$ c^2

5 : a $*$ c^2

6 : c^4

Steenrod squares

Sq^1 (d) = d ∗ b

Sq^1 (e) = 0

Sq^1 (f) = 0
****

Group order: 128

Group number: 509

Group description: C4 x (C4 . D8 = C4 . (C4 x C2))

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(d)=0

Sq^1(e)=0

Sq^1(f)=e*e

Sq^2(f)=f*e+g*a

Sq^1(g)=e*e*b

Sq^2(g)=e*e*e+g*e

G 128,509 ≅ G 4,1 × G 32,15

****

Group order: 128

Group number: 837

Group description: C16 x C4 x C2

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

1: b^2

2 : a^2

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = 0

****

Group order: 128

Group number: 838

Group description: C2 x (C16 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

1: b^2

2 : a^2

Steenrod squares

Sq^1 (d) = e $*$ a

Sq^1 (e) = 0

****

Group order: 128

Group number: 839

Group description: C4 x (C16 : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = 0

Sq^2 (e) = e ∗ b ∗ b + f ∗ b ∗ b

Sq^1 (f) = 0

Sq^2 (f) = f ∗ a ∗ b + f ∗ b ∗ b

G 128,839 ≅G 4,1 × G 32,17

****

Group order: 128

Group number: 841

Group description: C2 x (C16 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1(d)=0

Sq^1(e)=0

Sq^2(e)=e*d+f*a

Sq^1(f)=e*d

Sq^2(f)=f*d

G 128,841 ≅ G 2,1 × G 64,28

****

Group order: 128

Group number: 843

Group description: C2 x ((C16 x C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Cohomology relations

1 : f^2

2 : $a * f$

3 : $a * b$

4 : a^2

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = e ∗ a + e ∗ b

Sq^1 (f) = e ∗ a + f ∗ b

****

Group order: 128

Group number: 846

Group description: C2 x ((C16 : C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Degree 5: h, p

Degree 6: q

Degree 7: r

Degree 8: s, t

Steenrod squares

Sq^1(d)=0, d*b

Sq^1(e)=d*b+e*b, e*b

Sq^1(f)=0

Sq^2(f)=e*b*b*b+f*e+h

Sq^1(g)=e*e*b+f*e+g*b

Sq^2(g)=g*b*b+q

Sq^1(h)=d*b*b

Sq^2(h)=h*e

Sq^4(h)=t*a

Sq^1(p)=e*b*b*b*b+d*e*e+e*e*e+g*b*b+p*b

Sq^2(p)=e*b*b*b*b*b+e*e*b*b*b+g*b*b*b+p*b*b+h*e

Sq^4(p)=e*b*b*b*b*b*b*b+e*e*e*b*b*b+e*e*e*e*b+f*e*e*e+g*b*b*b*b*b+g*e*e*b+p*b*b*b*b+p*e*b*b+h*e*e+p*e*e+p*g+t*b

Sq^1(q)=e*e*e*b+f*e*e

Sq^2(q)=0

Sq^4(q)=p*e*e*b+p*f*e+t*d

Sq^1(r)=0

Sq^2(r)=e*e*e*e*b+f*e*e*e+h*e*e+t*a

Sq^4(r)=e*e*e*e*e*b+f*e*e*e*e+g*e*e*b*b*b+q*p+r*e*e+t*e*b+t*f, g*e*e*b*b*b+h*e*e*e+r*e*e+t*e*b+t*f

Sq^1(s)=e*e*e*e*b+f*e*e*e+g*e*b*b*b+h*e*e+p*g+r*e

Sq^2(s)=g*e*b*b*b*b+g*e*e*b*b+p*e*e*b+p*f*e+t*d

Sq^4(s)=d*e*e*e*e*e+g*e*e*b*b*b*b+g*e*e*e*b*b+p*g*e*b+s*p+t*g

Sq^1(t)=e*b*b*b*b*b*b*b+e*e*b*b*b*b*b+e*e*e*b*b*b+f*e*e*e+g*b*b*b*b*b+p*e*b*b+h*e*e+p*g+r*e

Sq^2(t)=e*e*b*b*b*b*b*b+d*e*e*e*e+g*e*e*b*b+p*b*b*b*b*b+p*f*e+t*d

Sq^4(t)=e*e*e*e*b*b*b*b+*e*e*e*e*e*b*b+e*e*e*e*e*e++g*e*e*e*b*b+p*b*b*b*b*b*b*b+p*e*b*b*b*b*b*b+p*e*e*e*b+p*g*e*b+p*o*e+t*e*e+t*g

G 128,846 ≅ G 2,1 × G 64,30

****

Group order: 128

Group number: 848

Group description: C2 x ((C16 x C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(d)=0

Sq^1(e)=d*a+e*b

Sq^1(f)=d*d

Sq^2(f)=f*d+g*a

Sq^1(g)=0

Sq^2(g)=g*b*b+g*d

G 128,848 ≅ G 2,1 × G 64,31

****

Group order: 128

Group number: 850

Group description: C2 x (((C8 : C2) : C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Degree 3: g, h, p

Degree 4: q, r

Degree 5: s

Steenrod squares

Sq^1(d)=d*b+g+h

Sq^1(e)=0

Sq^1(f)=f*b+g+h

Sq^1(g)=0

Sq^2(g)=g*d+g*e+q*a

Sq^1(h)=h*b

Sq^2(h)=h*f+q*b

Sq^1(p)=d*d+f*f

Sq^2(p)=d*f*b+f*f*b+h*b*b+p*b*b+g*d+g*e+h*d+p*d+p*f+q*a+q*b

Sq^1(q)=q*a+q*b

Sq^2(q)=q*e+q*f

Sq^1(r)=g*d+h*d+h*f+p*d+s

Sq^2(r)=d*d*b*b+d*d*e+d*f*f+h*d*b+p*d*b+h*h+h*p++q*d+q*f+r*f

Sq^1(s)=d*d*d+d*f*f+p*d*b+h*p

Sq^2(s)=0

Sq^4(s)=0

G 128,850 ≅ G 2,1 × G 64,32

****

Group order: 128

Group number: 856

Group description: C2 x (((C2 x C2 x C2) : C4) : C2)

Maximal Subgroup list:[[64, 34], [64, 90], [64, 34], [64, 211], [64, 34], [64, 90], [64, 34]]

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Degree 3: g, h

Degree 4: p

Steenrod squares

Sq^1(d)=0

Sq^1(e)=e*b+h

Sq^1(f)=0

Sq^1(g)=0

Sq^2(g)=g*b*b+g*e+p*b

Sq^1(h)=0

Sq^2(h)=g*d+h*e+h*f+p*a

Sq^1(p)=p*a

Sq^2(p)=d*d*d+d*e*e+g*e*b+g*g+p*d+p*e+p*f

****

Group order: 128

Group number: 868

Group description: C2 x ((C16 x C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e, f

Cohomology relations

1: a*d+a*f

2: a*b

3: a^2

4: b^2*e+d^2+f^2

Steenrod squares

Sq^1(d)=d*a+d*b

Sq^1(e)=0

Sq^1(f)=d*a+e*b+f*b

****

Group order: 128

Group number: 869

Group description: C2 x (Q16 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(d)=d*b+e*b

Sq^1(e)=0

Sq^1(f)=e*b*b

Sq^2(f)=e*e*a+e*e*b+g*a

Sq^1(g)=0

Sq^2(g)=0

G 128,869 ≅ G 2,1 × G 64,39

****

Group order: 128

Group number: 874

Group description: C2 x ((C16 x C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(d)=0

Sq^1(e)=d*a+e*b

Sq^1(f)=d*d

Sq^2(f)=f*d+g*a

Sq^1(g)=0

Sq^2(g)=g*b*b+g*d

G 128,874 ≅ G 2,1 × G 64,31

****

Group order: 128

Group number: 876

Group description: C2 x ((C16 : C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(d)=0

Sq^1(e)=d*a+e*b

Sq^1(f)=d*d

Sq^2(f)=f*d+g*a

Sq^1(g)=0

Sq^2(g)=g*b*b+g*d

G 128,876 ≅ G 2,1 × G 64,31

****

Group order: 128

Group number: 878

Group description: C2 x ((C16 : C2) : C2)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Degree 3: f, g

Degree 4: h, p

Steenrod squares

Sq^1(d)=d*b

Sq^1(e)=0

Sq^1(f)=d*d

Sq^2(f)=d*e*b+f*d+p*a

Sq^1(g)=0

Sq^2(g)=d*e*b+g*b*b+g*d+p*b

Sq^1(h)=d*e*b+g*e

Sq^2(h)=d*d*b*b+g*d*b+g*e*b+f*g+p*d+p*e

Sq^1(p)=d*e*b

Sq^2(p)=d*e*b*b+d*d*d+g*d*b+g*g+p*d

G 128,878 ≅ G 2,1 × G 64,42

****

Group order: 128

Group number: 881

Group description: C2 x (C4 : C16)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

1 : $a * b + b^2$

2 : a^2

3 : b^3

Steenrod squares

Sq^1 (d) = 0

Sq^1 (e) = e ∗ a

****

Group order: 128

Group number: 884

Group description: C2 x (C8 . D8 = C4 . (C8 x C2))

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1(d)=0

Sq^1(e)=0

Sq^2(e)=e*d+f*a

Sq^1(f)=0

Sq^2(f)=d*d*d+f*d

G 128,884 ≅ G 2,1 × G 64,45

****

Group order: 128

Group number: 886

Group description: C2 x (C16 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1(d)=0

Sq^1(e)=0

Sq^2(e)=e*d+f*a+f*b

Sq^1(f)=d*d*a+e*d

Sq^2(f)=e*d*a+f*d

G 128,886 ≅ G 2,1 × G 64,46

****

Group order: 128

Group number: 888

Group description: C2 x (C16 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

1 : $a * b + b^2$

2 : a^2

3 : b^3

Steenrod squares

$Sq^1 (d) = 0$

$Sq^1 (e) = e * a$

****

Group order: 128

Group number: 889

Group description: C2 x (C16 : C4)

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

1 : $a * b + b^2$

2 : a^2

3 : b^3

Steenrod squares

$Sq^1 (d) = 0$

$Sq^1 (e) = d * a + d * b + e * a$

****

Group order: 128

Group number: 892

Group description: C2 x (C4 . D16 = C8 . (C4 x C2))

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1(d)=0

Sq^1(e)=d*d

Sq^2(e)=e*d+f*a

Sq^1(f)=0

Sq^2(f)=f*d

G 128,892 ≅ G 2,1 × G 64,49

****

Group order: 128

Group number: 899

Group description: C16 x D8

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

$1 : a * b$

$2 : a^2 + c^2$

$3 : b * c^2$

Steenrod squares

$Sq^1 (d) = d * a + d * b$

$Sq^1 (e) = 0$

****

Group order: 128

Group number: 904

Group description: C4 x D32

Cohomology generators

Degree 1: a, b, c

Degree 2: d, e

Cohomology relations

1 : $a * b$

2 : $a^2 + c^2$

3 : $b * c^2$

Steenrod squares

$Sq^1 (d) = d * a + d * b$

$Sq^1 (e) = 0$

****

Group order: 128

Group number: 905

Group description: C4 x QD32

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^(d)=0

Sq^1(e)=0

Sq^2(e)=f*b

Sq^1(f)=e*b*b

Sq^2(f)=e*b*b*b

G 128,905 ≅ G 4,1 × G 32,19

****

Group order: 128

Group number: 906

Group description: C4 x Q32

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 4: e

Steenrod squares

Sq^1(d)=0, a*a*a+a*a*c+b*b*c

Sq^1(e)=0

Sq^2(e)=0

G 128,906 ≅ G 4,1 × G 32,20

****

Group order: 128

Group number: 914

Group description: C16 x Q8

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Degree 4: e

Steenrod squares

Sq^1(d)=0

Sq^1(e)=0

Sq^2(e)=0

G 128,914 ≅ G 16,1 × G 8,4

****

Group order: 128

Group number: 988

Group description: C32 x C2 x C2

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Cohomology relations

1: a^2

Steenrod squares

Sq^1(d)=0

****

Group order: 128

Group number: 989

Group description: C2 x (C32 : C2)

Cohomology generators

Degree 1: a, b, c

Degree 3: d

Degree 4: e

Steenrod squares

Sq^1(d)=0

Sq^2(d)=d*b*b+e*a

Sq^1(e)=0

Sq^2(e)=e*a*b+e*b*b

G 128,989 ≅ G 2,1 × G 64,51

****

Group order: 128

Group number: 991

Group description: C2 x D64

Cohomology generators

Degree 1: a, b, c

Degree 2: d

Steenrod squares

Sq^1(d)=d*a+d*b

G 128,991 ≅ G 2,1 × G 64,52

****

Group order: 128

Group number: 992

Group description: C2 x QD64

Cohomology generators

Degree 1: a, b, c

Degree 3: d

Degree 4: e

Steenrod squares

Sq^1(d)=0

Sq^2(d)=d*b*b+e*b

Sq^1(e)=0

Sq^2(e)=d*d

G 128,992 ≅ G 2,1 × G 64,53

****

Group order: 128

Group number: 993

Group description: C2 x Q64

Cohomology generators

Degree 1: a, b, c

Degree 4: d

Steenrod squares

Sq^1(d)=0

Sq^2(d)=0

G 128,993 ≅ G 2,1 × G 64,54

****

Group order: 128

Group number: 997

Group description: C4 x C4 x C4 x C2

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: c^2

2: b^2

3: a^2

Steenrod squares

Sq^1(e)=0

Sq^1(f)=0

Sq^1(g)=0

****

Group order: 128

Group number: 998

Group description: C2 x C2 x ((C4 x C2) : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h, p

Cohomology relations

1: g^2

2: f*g+g*h

3: f^2+h^2

4: b*f+b*h

5: b^2

6: a*g

7: a*f+a*h+b*g

8: a*b

9: a^2

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a+f*b

Sq^1(g)=e*b+f*a+g*b

Sq^1(h)=f*a+g*b+p*a

Sq^1(p)=0

****

Group order: 128

Group number: 999

Group description: C2 x ((C4 x C4) : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: b^2

2: a*b+c^2

3: a^2

4: b*c^2

5: a*c^2

6: c^4

Steenrod squares

Sq^1(e)=0

Sq^1(f)=0

Sq^1(g)=e*b+f*a

****

Group order: 128

Group number: 1000

Group description: C2 x C4 x ((C4 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h

Cohomology relations

1: b*e+b*f

2: b^2

3: a*b

4: a^2+c^2

5: c^2*g+e^2+f^2

6: b*c^2

Steenrod squares

Sq^1(e)=e*a+e*b

Sq^1(f)=f*a+g*a

Sq^1(g)=0

Sq^1(h)=0

****

Group order: 128

Group number: 1001

Group description: C2 x C4 x (C4 : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: b^2

2: a*b+c^2

3: a^2+c^2

4: b*c^2

5: a*c^2

6: c^4

Steenrod squares

Sq^1(e)=e*b

Sq^1(f)=0

Sq^1(g)=0

****

Group order: 128

Group number: 1002

Group description: C4 x ((C4 x C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(e)=0

Sq^1(f)=a*a*b+f*a

Sq^1(g)=a*a*a*b+f*a*b+f*a*c

Sq^2(g)=g*a*a+h*a+h*b+h*c

Sq^1(h)=0

Sq^2(h)=a*a*a*a*a*b+g*g

$G\ 128{,}1002 \cong G\ 4{,}1 \times G\ 32{,}31$

****

Group order: 128

Group number: 1003

Group description: C4 x C4 x D8

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: b^2+c^2

2: a*b

3: a^2+d^2

4: b*d^2

5: a*c^2

6: c^2*d^2

Steenrod squares

Sq^1(e)=e*a+e*b

Sq^1(f)=0

Sq^1(g)=0

****

Group order: 128

Group number: 1004

Group description: C4 x C4 x Q8

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 4: g

Steenrod squares

Sq^1(e)=0

Sq^1(f)=0

Sq^1(g)=0

Sq^2(g)=f*f*a*a

G 128,1004 ≅ G 4,1 × G 32,26

****

Group order: 128

Group number: 1009

Group description: C2 x ((C2 x ((C4 x C2) : C2)) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h, p, q

Degree 3:r

Steenrod squares

Sq^1(e)=0

Sq^1(f)=e*a+e*b+e*c+f*b+f*c+g*a+p*a

Sq^1(g)=g*a+g*c

Sq^1(h)=e*c+g*a+h*b+h*c+p*c

Sq^(p)=e*a+e*b+p*a+p*b

Sq^1(q)=e*c+f*c+g*a+q*b

Sq^1(r)=e*c*c+e*g+e*h+e*q+f*h+g*q+p*q

Sq^2(r)=0

G 128,1009 ≅ G 2,1 × G 64,60

****

Group order: 128

Group number: 1010

Group description: C2 x (((C4 x C2) : C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h

Cohomology relations

1: a*g

2: a*c

3: a*b+b^2

4: a^2

5: c^2*e+g^2

6: b^2*g

7: b^2*c

8: b^3

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a+f*c

Sq^1(g)=e*c+f*a+g*c

Sq^1(h)=h*a

****

Group order: 128

Group number: 1011

Group description: C2 x (((C4 x C2) : C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h

Degree 3: p, q

Degree 4: r, s

steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a+f*c

Sq^1(g)=e*c+f*a+g*c

Sq^1(h)=e*a+e*b+f*c+h*b+p

Sq^1(p)=e*a*b+f*b*c+p*b

Sq^2(p)=f*b*b*c+e*e*a+e*e*b+s*a

Sq^1(q)=e*a*b+e*b*b+f*a*b+f*b*b+p*b

Sq^2(q)=f*b*b*c+q*b*b+s*c

Sq^1(r)=f*b*b*c+e*e*b+e*f*a+e*g*c+e*h*b+f*g*c+f*h*b+p*e+p*f+p*h+q*e

Sq^2(r)=e*f*a*b+e*g*b*c+f*f*a*b, e*e*a*b, e*f*a*b, e*g*b*c, e*e*a*b+e*g*b*c+f*g*b*c, e*b*b*b*b+f*b*b*b*b+p*b*b*b+q*q

Sq^1(s)=0

Sq^2(s)=0, e*f*a*b+e*g*b*c+f*f*a*b, e*e*a*b, e*f*a*b, e*g*b*c, e*e*a*b+e*g*b*c+f*g*b*c, e*b*b*b*b+f*b*b*b*b+p*b*b*b+q*q

G 128,1011 ≅ G 2,1 × G 64,62

****

Group order: 128

Group number: 1013

Group description: C2 x ((C4 x C4) : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: a*c+b^2

2: a*b+c^2

3: a^2

4: b*c^2

5: b^2*c

6: b^3+c^3

7: c^4

Steenrod squares

Sq^1(e)=0

Sq^1(f)=e*a+e*b+e*c+g*a

Sq^1(g)=e*a+e*b+e*c+f*a

****

Group order: 128

Group number: 1014

Group description: C2 x ((C4 x C4) : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: a*c+b^2

2: a*b+b^2+c^2

3: a^2

4: b*c^2+c^3

5: b^2*c

6: b^3+c^3

7: c^4

Steenrod squares

Sq^1(e)=0

Sq^1(f)=a*b*c+e*b+g*a

Sq^1(g)=e*c+f*a+g*a

****

Group order: 128

Group number: 1016

Group description: C2 x ((C4 x C4) : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: a*c+c^2

2: a*b+b^2

3: a^2

4: c^3

5: b^2*c+b*c^2

6: b^3

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a

Sq^1(g)=g*a

****

Group order: 128

Group number: 1018

Group description: C2 x ((C2 x (C4 : C4)) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h

Cohomology relations

1: a*e+a*g

2: a*c

3: a*b

4: a^2+b^2

5: b^2*g+b*c*e+b*c*g+c^2*f+e^2+g^2

6: b^2*e+b*c*e+b*c*g+c^2*f+e^2+g^2

7: b^2*c

8: b^3

9: b*c*e^2+b*c*g^2+c^2*e*f+c^2*f*g+e^3+e^2*g+e*g^2+g^3

10: b*c^2*f+b*e^2+b*g^2

11: b*c^2*e+b*c^2*g+c^3*f+c*e^2+c*g^2

12: c^3*f^2+b*e^3+b*e^2*g+b*e*g^2+b*g^3+c*e^2*f+c*f*g^2

13: c^4*f^2+e^4+g^4

Steenrod squares

Sq^1(e)=e*a+e*c

Sq^1(f)=f*a+f*b

Sq^1(g)=e*a+f*c+g*b+g*c

Sq^1(h)=e*a+f*a+f*b+f*c+g*b+g*c

****

Group order: 128

Group number: 1021

Group description: C2 x (((C4 x C2) : C4) : C2)

Cohomology generators
Degree 1: a, b, c, d

Degree 2: e, f, g

Degree 3: h

Steenrod squares

$Sq^1 (e) = e * a + e * c$

$Sq^1 (f) = 0$

$Sq^1 (g) = f * a + g * a$

$Sq^1 (h) = a * a * a * a + e * a * a + e * a * b + e * b * c + g * a * a + g * a * b$

$Sq^2 (h) = f * a * a * a + e * e * c + g * g * a + h * a * a$

$G128,1021 \cong G2,1 \times G64,69$

****

Group order: 128

Group number: 1023

Group description: C2 x ((C4 : C4) : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: a*c+c^2

2: a*b

3: a^2+b^2

4: b*c^2

5: b^2*c+c^3

6: b^3

7: c^4

Steenrod squares

Sq^1(e)=e*a+e*b

Sq^1(f)=e*a+e*b

Sq^1(g)=g*a

****

Group order: 128

Group number: 1024

Group description: C2 x ((C2 x (C4 : C4)) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h

Cohomology relations

1: a*e+a*f+b*e+b*f

2: a*c+c^2

3: a*b+b^2

4: a^2+b^2

5: b*c*e+b*c*f+c^2*e+c^2*f

6: b*c^2+c^3

7: b^2*e+b^2*f+b^2*g+e^2+f^2

8: b^2*c+c^3

9: c^3*e+c^3*f+c^3*g+c*e^2+c*f^2

Steenrod squares

Sq^1(e)=a*a*a+e*b

Sq^1(f)=a*a*a+e*a+g*a

Sq^1(g)=e*a+e*b+g*a

Sq^1(h)=h*a

****

Group order: 128

Group number: 1026

Group description: C2 x ((C2 x Q8) : C4)

Maximal Subgroup list:[[64, 72], [64, 56], [64, 72], [64, 194], [64, 72], [64, 56], [64, 72], [64, 192], [64, 72], [64, 56], [64, 72], [64, 262], [64, 72], [64, 56], [64, 72]]

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Degree 3: h

Degree 4: p

Steenrod squares

$Sq^1 (e) = e * a + e * c$

$Sq^1 (f) = a * a * b + e * b + f * b + g * a$

$Sq^1 (g) = e * b + g * a + g * b + g * c$

$Sq^1 (h) = e * a * b + e * b * b + f * a * a + f * a * b + g * a * a + g * a * b + h * c$

$Sq^2 (h) = e * e * a * a + e * e * a * b + g * g * a * a + h * h$

G128,1026 ≅ G2,1 × G64,76

****

Group order: 128

Group number: 1031

Group description: C4 x ((C2 x C2 x C2 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h

Steenrod squares

Sq^1(e)=e*a+e*c

Sq^1(f)=f * a+ f * b

Sq^1(g)=e*a+ f*c+ g*b+ g*c

Sq^1(h)=0

G 128,1031 ≅ G 4,1 × G 32,27

****

Group order: 128

Group number: 1032

Group description: C4 x ((C4 x C2 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: a*c

2: a*b+b^2

3: a^2+d^2

4: c*d^2

5: b^2*c

6: b^3+b*d^2

Steenrod squares

Sq^1(e)=e*a+e*c

Sq^1(f)=f*a

Sq^1(g)=0

****

Group order: 128

Group number: 1033

Group description: C4 x ((C4 x C2 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g, h

Degree 4: p

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a+f*b

Sq^1(g)=f*a*b+g*b

Sq^2(g)=f*a*a*a+f*b*b*c+f*f*c+p*a

Sq^1(h)=f*b*b+f*b*c+g*b

Sq^2(h)=f*f*c+f*b*b+p*c

Sq^1(p)=f*a*a*a+f*b*b*c+g*a*a

Sq^2(p)=f*a*a*a*a+f*b*b*b*b+f*f*a*a+f*g*a*b+f*f*b*b+f*g*b*c+g*a*a*a+g*b*b*b+h*h+p*b*b

G 128,1033 ≅ G 4,1 × G 32,30

****

Group order: 128

Group number: 1034

Group description: C4 x ((C2 x Q8) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a+f*c

Sq^1(g)=f*a*a+g*c

Sq^2(g)=f*c*c*c+f*f*a+f*f*c+h*c

Sq^1(h)=f*c*c*c

Sq^2(h)=f*c*c*c*c+f*f*a*a

G 128,1034 ≅ G 4,1 × G 32,29

****

Group order: 128

Group number: 1035

Group description: C4 x ((C4 x C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(e)=0

Sq^1(f)=0

Sq^1(g)=f*a*b+f*b*b

Sq^2(g)=g*b*b+h*a

Sq^1(h)=f*b*b*b+g*b*b

Sq^2(h)=f*b*b*b*b+f*f*a*b+f*f*a*c+f*f*b*b+h*b*b

G 128,1035 ≅ G 4,1 × G 32,24

****

Group order: 128

Group number: 1036

Group description: C4 x ((C4 x C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f, g, h

Degree 4: p, q

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*c

Sq^2(f)=f*b*c+p*c+q*a

Sq^1(g)=f*c

Sq^2(g)=f*a*b+p*b+p*c+q*c

Sq^1(h)=f*c

Sq^2(h)=h*a*a+p*a+p*b+q*b

Sq^1(p)=f*b*c

Sq^2(p)=f*g+g*g+p*a*a

Sq^1(q)=f*a*a

Sq^2(q)=f*a*a*a+g*g+h*h+p*a*a+p*a*c

G 128,1036 ≅ G 4,1 × G 32,33

****

Group order: 128

Group number: 1037

Group description: C4 x ((C2 x C2) . (C2 x C2 x C2))

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f, g

Degree 4: h, p

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*b+f*c

Sq^2(f)=h*a+p*b+p*c

Sq^1(g)=0

Sq^2(g)=f*a*a+h*b+h*c+p*a+p*b+p*c

Sq^1(h)=f*a*a+f*b*c

Sq^2(h)=f*a*a*b+f*f+h*a*b+p*a*b+p*b*c

Sq^1(p)=f*a*a+f*a*b

Sq^2(p)=f*f+h*a*a+h*a*b+h*a*c+h*b*c+p*a*b

G 128,1037 ≅ G 4,1 × G 32,32

****

Group order: 128

Group number: 1038

Group description: C4 x ((C4 x C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: a*c+c^2

2: a*b+b^2

3: a^2+d^2

4: c^3+c*d^2

5: b^2*c+b*c^2

6: b^3+b*d^2

Steenrod squares

Sq^1(e)=e*a

Sq^1(f)=f*a

Sq^1(g)=0

****

Group order: 128

Group number: 1039

Group description: C4 x (C4 : Q8)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 4: g

Cohomology relations

1: a*c+c^2

2: a*b+b^2+d^2

3: a^2+d^2

4: c^3+c*d^2

5: b^2*c+b*c^2+c*d^2

6: b^3

7: a*d^2

8: d^4

9: c^2*d^2

10: b^2*d^2

Steenrod squares

Sq^1(e)=e*a

Sq^1(f)=0

Sq^1(g)=e*a*a*b+e*a*a*c

Sq^2(g)=e*e*a*b+e*e*a*c

****

Group order: 128

Group number: 1070

Group description: D8 x ((C4 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a+f*b

Sq^1(g)=e*b+f*a+g*b

Sq^1(h)=h*a+h*b

$G_{128,1070} \cong G_{8,3} \times G_{16,3}$

****

Group order: 128

Group number: 1072

Group description: Q8 x ((C4 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Degree 4: h

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a+f*b

Sq^1(g)=e*b+f*a+g*b

Sq^1(h)=0

Sq^2(h)=0

G 128,1072 ≅ G 8,4 × G 16,3

****

Group order: 128

Group number: 1080

Group description: D8 x (C4 : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a

Sq^1(g)=g*a+g*b

G 128,1080 ≅ G 8,3 × G 16,4

****

Group order: 128

Group number: 1082

Group description: Q8 x (C4 : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 4: g

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a

Sq^1(g)=0

Sq^2(g)=0

G 128,1082 ≅ G 8,4 × G 16,4

****

Group order: 128

Group number: 1116

Group description: C2 x ((C2 x C2 x D8) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h, p

Steenrod squares

Sq^1(e)=e*a+e*c

Sq^1(f)=f*a+f*b

Sq^1(g)=e*a+e*c+g*b+g*c

Sq^1(h)=e*a+f*b+f*c+g*c+p*a

Sq^1(p)=e*b+f*c+p*a+p*b+p*c

G 128,1116 ≅ G 2,1 × G 64,73

****

Group order: 128

Group number: 1118

Group description: C2 x ((C2 x ((C4 x C2) : C2)) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g, h

Steenrod squares

Sq^1(e)=e*a+e*c

Sq^1(f)=e*a+e*c+f*b+f*a

Sq^1(g)=e*b+g*a+g*b+g*c

Sq^1(h)=h*a+h*b

G 128,1118 ≅ G 2,1 × G 64,75

****

Group order: 128

Group number: 1119

Group description: C2 x ((C4 x C2) : Q8)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Degree 3: h

Degree 4: p

Steenrod squares

Sq^1(e)=e*a+e*c

Sq^1(f)=a*a*b+e*b+f*b+g*a

Sq^1(g)=e*b+g*a+g*b+g*c

Sq^1(h)=e*a*b+e*b*b+f*a*a+f*a*b+g*a*a+g*a*b+h*c

Sq^2(h)=e*e*a*a+e*e*a*b+g*g*a*a+h*h

G 128,1119 ≅ G 2,1 × G 64,76

****

Group order: 128

Group number: 1121

Group description: C2 x ((C2 x (C4 : C4)) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Degree 3: h

Steenrod squares

Sq^1(e)=e*c

Sq^1(f)=e*b+f*b+f*c

Sq^1(g)=e*a+g*a+g*c

Sq^1(h)=e*c*c+h*c

Sq^2(h)=f*f*a+f*f*c+g*g*a+g*g*c+h*c*c

$G_{128,1121} \cong G_{2,1} \times G_{64,77}$

****

Group order: 128

Group number: 1122

Group description: C2 x ((C2 x (C4 : C4)) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Degree 3: h

Steenrod squares

Sq^1(e)=e*c

Sq^1(f)=e*b+f*b+f*c

Sq^1(g)=e*a+e*b+f*a+f*b

Sq^1(h)=e*a*a+e*a*b+e*b*b+f*a*a, e*a*a+e*a*b+e*b*b+f*a*a+f*a*b, e*a*a+e*a*b+e*b*b+f*a*a+f*a*b+g*a*b, e*a*a+e*a*b+e*b*b+f*a*a+g*a*b, e*a*a+e*b*b+f*a*a, e*a*a+e*b*b+f*a*a+f*a*b, e*a*a+e*b*b+f*a*a+f*a*b+g*a*b,e*a*a+e*b*b+f*a*a+g*a*b

Sq^2(h)=e*e*a+e*e*c+f*f*a+h*b*b

G 128,1122 ≅ G 2,1 × G 64,78

****

Group order: 128

Group number: 1123

Group description: C2 x ((C2 x C2 x C2) . (C2 x C2 x C2))

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g, h, p

Degree 4: q

Steenrod squares

Sq^1(e)=e*c

Sq^1(f)=e*b+f*b+f*c

Sq^1(g)=e*a*a+f*a*a+g*a+g*b

Sq^2(g)=e*e*a+e*e*b+e*e*c+g*a*b+q*b, e*e*a+e*e*b+e*e*c+q*b

Sq^1(h)=e*a*a+g*a+g*b+h*c

Sq^2(h)=e*e*a+e*e*b+e*e*c+g*a*b+q*a+q*b+q*c,
e*e*a+e*e*b+e*e*c+q*a+q*b+q*c

Sq^1(p)=e*a*a+e*a*b+f*a*a+f*a*b+g*a+g*b

Sq^2(p)=e*e*b+e*e*c+g*a*b+q*c, e*e*b+e*e*c+q*c

Sq^1(q)=0, g*a*b

Sq^2(q)=e*e*a*a+f*f*a*a+g*e*a+h*e*c+g*h

G 128,1123 ≅ G 2,1 × G 64,79

****

Group order: 128

Group number: 1601

Group description: C8 x C4 x C2 x C2

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Cohomology relations

1: b^2

2: a^2

Steenrod squares

Sq^1(e)=0

Sq^1(f)=0

****

Group order: 128

Group number: 1602

Group description: C2 x C2 x (C8 : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Cohomology relations

1: b^2

2: a^2

Steenrod squares

Sq^1(e)=f*a

Sq^1(f)=0

****

Group order: 128

Group number: 1603

Group description: C2 x C4 x (C8 : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=0

Sq^1(f)=0

Sq^2(f)=e*a*b*c+f*c*c+g*a, f*c*c+g*a

Sq^1(g)=0, e*a*b*c

Sq^2(g)=g*a*c+g*c*c

G 128,1603 ≅ G 2,1 × G 64,85

****

Group order: 128

Group number: 1604

Group description: C2 x ((C8 x C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=a*b*c+a*c*c, a*c*c

Sq^1(f)=a*c*c*c+e*b*c+e*c*c

Sq^2(f)=e*a*b*c+e*c*c+g*b, f*c*c+g*b

Sq^1(g)=e*a*b*c+e*a*c*c+e*c*c*c+f*c*c, e*a*c*c+e*c*c*c++f*c*c

Sq^2(g)=a*c*c*c*c*c+e*c*c*c*c+e*e*c*c+g*a*b+g*c*c

G 128,1604 ≅ G 2,1 × G 64,86

****

Group order: 128

Group number: 1606

Group description: C4 x ((C8 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 4: f

Steenrod squares

Sq^1(e)=0

Sq^1(f)=0

Sq^2(f)=f*a*a+f*b*b+f*c*c

G 128,1606 ≅ G 4,1 × G 32,42

****

Group order: 128

Group number: 1608

Group description: C2 x C2 x ((C8 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: g^2

2: a*g

3: a*b

4: a^2

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a+f*b

Sq^1(g)=f*a+g*b

****

Group order: 128

Group number: 1610

Group description: C2 x ((C8 x C2 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(e)=e*a+e*b

Sq^2(f)=b*b*c+f*b

Sq^1(g)=f*b*c+f*c*c

Sq^2(g)=e*b*b*c+e*c*c*c+f*b*b*c+f*c*c*c+g*c*c+h*a

Sq^1(h)=b*b*b*b*c+e*a*c*c+f*c*c*c+g*c*c

Sq^2(h)=b*b*b*b*b*c+e*a*c*c*c+e*b*b*b*c+e*c*c*c*c+e*e*a*c+e*e*b*c+h*a*c+h*b*b+h*b*c+h*c*c

G 128,1610 ≅ G 2,1 × G 64,89

****

Group order: 128

Group number: 1622

Group description: C2 x C2 x ((C8 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f, g

Cohomology relations

1: a*e+a*f

2: a*b

3: a^2

4: b^2*g+e^2+f^2

Steenrod squares

Sq^1(e)=e*a+e*b

Sq^1(f)=e*a+f*b+g*b

Sq^1(g)=0

****

Group order: 128

Group number: 1623

Group description: C2 x C2 x (Q8 : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(e)=0

Sq^1(f)=e*b+f*b

Sq^1(g)=0

Sq^2(g)=h*a

Sq^1(h)=e*e*b+e*f*b

Sq^2(h)=e*e*f

G 128,1623 ≅ G 2,1 × G 64,96

****

Group order: 128

Group number: 1624

Group description: C2 x (((C4 x C4) : C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(e)=e*b

Sq^1(f)=e*b+e*c+f*a+f*c

Sq^1(g)=0

Sq^2(g)=e*b*c*c+f*c*c*c+e*f*c+g*a*a+g*f+h*a

Sq^1(h)=e*b*b*b

Sq^2(h)=e*f*c*c+f*f*c*c+e*f*f+g*f*a+g*g+h*b*b+h*b*c+h*c*c+h*e+h*f

G 128,1624 ≅ G 2,1 × G 64,134

****

Group order: 128

Group number: 1631

Group description: C2 x C2 x ((C4 x C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Degree 3: g

Degree 4: h

Steenrod squares

Sq^1(e)=e*b

Sq^1(f)=0

Sq^1(g)=0

Sq^2(g)=e*f*c+f*f*a+g*f+h*a

Sq^1(h)=g*c*c

Sq^2(h)=e*f*c*c+f*f*a*c+f*f*f+h*a*v+h*b*b+h*f

$G_{128,1631} \cong G_{2,1} \times G_{64,101}$

****

Group order: 128

Group number: 1634

Group description: C2 x C2 x (C4 : C8)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Cohomology relations

1: a*b+b^2

2: a^2

3: b^3

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a

****

Group order: 128

Group number: 1635

Group description: C2 x ((C4 : C8) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=e*a

Sq^1(f)=0

Sq^2(f)=e*a*b*c+f*c*c+g*a, f*c*c+g*a

Sq^1(g)=0, e*a*b*c

Sq^2(g)=g*a*c+g*c*c

G 128,1635 ≅ G 2,1 × G 64,104

****

Group order: 128

Group number: 1636

Group description: C2 x ((C4 : C8) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=a*c*c+e*a

Sq^1(f)=e*b*c+f*a

Sq^2(f)=a*c*c*c*c+e*a*b*c+e*c*c*c+e*e*c+f*a*c+f*c*c+g*a+g*b, a*c*c*c*c+e*c*c*c+e*e*c+f*a*c+f*c*c+g*a+g*b

Sq^1(g)=a*c*c*c*c+e*a*b*c+f*a*c+f*c*c, a*c*c*c*c+f*a*c+f*c*c

Sq^2(g)=e*e*a*c+e*e*c*c+f*a*c*c+f*f+g*a*c+g*c*c

G 128,1636 ≅ G 2,1 × G 64,105

****

Group order: 128

Group number: 1639

Group description: C2 x C2 x (C8 : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Cohomology relations

1: a*b+b^2

2: a^2

3: b^3

Steenrod squares

Sq^1(e)=0

Sq^1(f)=e*a+e*b+f*a

****

Group order: 128

Group number: 1640

Group description: C2 x C2 x (C8 : C4)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Cohomology relations

1: a*b+b^2

2: a^2

3: b^3

Steenrod squares

Sq^1(e)=0

Sq^1(f)=f*a

****

Group order: 128

Group number: 1641

Group description: C2 x ((C8 : C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=0

Sq^1(f)=e*a*b

Sq^2(f)=e*a*b*c+e*c*c*c+e*e*c+f*c*c+g*a, e*c*c*c+e*e*c+f*c*c+g*a

Sq^1(g)=0

Sq^2(g)=e*b*c*c*c+e*c*c*c*c+e*e*a*c+e*e*b*c+e*e*c*c+g*a*c+g*c*c

G 128,1641 ≅ G 2,1 × G 64,108

****

Group order: 128

Group number: 1642

Group description: C2 x ((C8 : C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=0

Sq^1(f)=e*a*b

Sq^2(f)=e*a*b*c+e*e*a+e*e*b+f*a*c+f*c*c+g*a+g*b,
e*e*a+e*e*b+f*a*c+f*c*c+g*a+g*b

Sq^1(g)=a*c*c*c*c, a*c*c*c*c+e*a*b*c

Sq^2(g)=e*a*c*c*c+e*e*a*b+e*e*b*c+e*e*c*c+f*a*c*c+f*f+g*a*c+g*b*c+g*c*c

G 128,1642 ≅ G 2,1 × G 64,109

****

Group order: 128

Group number: 1646

Group description: C2 x C2 x (C4 . D8 = C4 . (C4 x C2))

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=0

Sq^1(f)=e*e

Sq^2(f)f*e+g*a

Sq^1(g)=0

Sq^2(g)=a*b*c*c*c*c+e*e*e+g*a*b+g*e, e*e*e+g*a*b+g*e

G 128,1646 ≅ G 2,1 × G 64,110

****

Group order: 128

Group number: 1647

Group description: C2 x ((C4 . D8 = C4 . (C4 x C2)) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 4: f

Degree 5: g, h, p

Degree 8: q, r

Steenrod squares

Sq^1(e)=0

Sq^1(f)=e*e*b

Sq^1(g)=e*e*b*c+e*e*c*c+g*a

Sq^1(h)=b*c*c*c*c*c+e*c*c*c*c+e*e*c*c+f*c*c

Sq^1(p)=b*c*c*c*c*c+e*e*e+g*a

Sq^1(q)=?

Sq^1(r)=?

G 128,1647 ≅ G 2,1 × G 64,111

****

Group order: 128

Group number: 1649

Group description: C2 x ((C8 x C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=0

Sq^1(f)=0

Sq^2(f)=e*a*b*c+f*b*b+g*a, f*b*b+g*a

Sq^1(g)=e*a*b*c+f*b*b, f*b*b

Sq^2(f)=e*e*a*b+g*b*b

G 128,1649 ≅ G 2,1 × G 64,112

****

Group order: 128

Group number: 1650

Group description: C2 x ((C8 : C4) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=e*a+e*c

Sq^1(f)=f*c

Sq^2(f)=e*b*c*c+e*e*c+g*c

Sq^1(g)=e*a*a*b

Sq^2(g)=e*b*c*c*c+e*e*a*b+e*e*b*c+e*e*c*c

G 128,1650 ≅ G 2,1 × G 64,155

****

Group order: 128

Group number: 2011

Group description: D8 x D16

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e, f

Cohomology relations

1: a*c

2: a*b+b*d

3: b*c*d

Steenrod squares

Sq^1(e)= e*a+e*b

Sq^1(f)=f*a+f*b

****

Group order: 128

Group number: 2013

Group description: D8 x QD16

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 3: f

Degree 4: g

Steenrod squares

Sq^1(e)=e*a+e*b

Sq^1(f)=0

Sq^2(f)=f*b*b+g*b

Sq^1(g)=0

Sq^2(g)=f*f

G 128,2013 ≅ G 8,3 × G 16,8

****

Group order: 128

Group number: 2018

Group description: D8 x Q16

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 4: f

Steenrod squares

Sq^1(e)=e*a+e*b

Sq^1(f)=0

Sq^2(f)=0

G 128,2018 ≅ G 8,3 × G 16,9

****

Group order: 128

Group number: 2110

Group description: Q8 x D16

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Degree 4: f

Steenrod squares

Sq^1(e)=e*a+e*b

Sq^1(f)=0

Sq^2(f)=0

G 128,2110 ≅ G 8,4 × G 16,7

****

Group order: 128

Group number: 2111

Group description: Q8 x QD16

Cohomology generators

Degree 1: a, b, c, d

Degree 3: e

Degree 4: f, g

Steenrod squares

Sq^1(e)=0

Sq^1(e)=e*b*b+g*b

Sq^1(f)=0

Sq^2(f)=0

Sq^1(g)=0

Sq^2(g)=e*e

G 128,2111 ≅ G 8,4 × G 16,8

****

Group order: 128

Group number: 2114

Group description: Q8 x Q16

Cohomology generators

Degree 1: a, b, c, d

Degree 4: e, f

Steenrod squares

Sq^1(e)=0

Sq^2(e)=0

Sq^1(f)=0

Sq^2(f)=0

G 128,2114 ≅ G 8,4 × G 16,9

****

Group order: 128

Group number: 2136

Group description: C16 x C2 x C2 x C2

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Cohomology relations

1: a^2

Steenrod squares

Sq^1(e)=0

****

Group order: 128

Group number: 2137

Group description: C2 x C2 x (C16 : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1(e)=0

Sq^2(e)=e*b*b+f*a

Sq^1(f)=0

Sq^2(f)=f*a*b+f*b*b

G 128,2137 ≅ G 4,2 × G 32,17

****

Group order: 128

Group number: 2138

Group description: C2 x ((C16 x C2) : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 4: e

Steenrod squares

Sq^1(e)=0

Sq^2(e)=e*a*a+e*b*b+e*c*c

G 128,2138 ≅ G 2,1 × G 64,189

****

Group order: 128

Group number: 2140

Group description: C2 x C2 x D32

Cohomology generators

Degree 1: a, b, c, d

Degree 2: e

Cohomology relations

1: a*b

Steenrod squares

Sq^1(e)=e*a+e*b

****

Group order: 128

Group number: 2141

Group description: C2 x C2 x QD32

Cohomology generators

Degree 1: a, b, c, d

Degree 3: e

Degree 4: f

Steenrod squares

Sq^1(e)=0

Sq^2(e)=f*b

Sq^1(f)=e*b*b

Sq^2(f)=e*b*b*b

G 128,2141 ≅ G 4,2 × G 32,19

****

Group order: 128

Group number: 2142

Group description: C2 x C2 x Q32

Cohomology generators

Degree 1: a, b, c, d

Degree 4: e

Steenrod squares

Sq^1(e)=0

Sq^2(e)=0

G 128,2142 ≅ G 4,2 × G 32,20

****

Group order: 128

Group number: 2145

Group description: C2 x (QD32 : C2)

Cohomology generators

Degree 1: a, b, c, d

Degree 5: e, f

Degree 8: g

Steenrod squares

Sq^1(e)=b*b*c*c*c*c+e*a

Sq^1(f)=f*b

Sq^1(g)=?

G 128,2145 ≅ G 2,1 × G 64,191

****

5.2 Experimental results of Stiefel-Whitney classes of real representation of non-prime power groups.

Group name: SymmetricGroup(6)

Stiefel-Whitney Classes: [[v.1], [v.2, v.2+v.3], [v.8+v.10, v.8+v.9+v.10, v.6+v.8+v.10, v.6+v.8+v.9+v.10], [v.11+v.12+v.17, v.11+v.12+v.17+v.19, v.11+v.12+v.17+v.18, v.11+v.12+v.17+v.18+v.19, v.11+v.12+v.14+v.17, v.11+v.12+v.14+v.17+v.19, v.11+v.12+v.14+v.17+v.18, v.11+v.12+v.14+v.17+v.18+v.19, v.11+v.12+v.13+v.15+v.17, v.11+v.12+v.13+v.15+v.17+v.19, v.11+v.12+v.13+v.15+v.17+v.18, v.11+v.12+v.13+v.15+v.17+v.18+v.19, v.11+v.12+v.13+v.14+v.15+v.17, v.11+v.12+v.13+v.14+v.15+v.17+v.19,v.11+v.12+v.13+v.14+v.15+v.17+v.18 , v.11+v.12+v.13+v.14+v.15+v.17+v.18+v.19], [v.22, v.22+v.34, v.22+v.33, v.22+v.33+v.34, v.22+v.27, v.22+v.27+v.34, v.22+v.27+v.33, v.22+v.27+v.33+v.34, v.22+v.26+v.29, v.22+v.26+v.29+v.34, v.22+v.26+v.29+v.33, v.22+v.26+v.29+v.33+v.34, v.22+v.26+v.27+v.29, v.22+v.26+v.27+v.29+v.34, v.22+v.26+v.27+v.29+v.33, v.22+v.26+v.27+v.29+v.33+v.34, v.22+v.24, v.22+v.24+v.34,v.22+v.24+v.33, v.22+v.24+v.33+v.34, v.22+v.24+v.27, v.22+v.24+v.27+v.34, v.22+v.24+v.27+v.33, v.22+v.24+v.27+v.33+v.34, v.22+v.24+v.26+v.29, v.22+v.24+v.26+v.29+v.34, v.22+v.24+v.26+v.29+v.33, v.22+v.24+v.26+v.29+v.33+v.34, v.22+v.24+v.26+v.27+v.29, v.22+v.24+v.26+v.27+v.29+v.34, v.22+v.24+v.26+v.27+v.29+v.33,v.22 +v.24+v.26+v.27+v.29+v.33+v.34, v.22+v.23+v.28, v.22+v.23+v.28+v.34, v.22+v.23+v.28+v.33, v.22+v.23+v.28+v.33+v.34, v.22+v.23+v.27+v.28,v.22+v.23+v.27+v.28+v.34, v.22+v.23+v.27+v.28+v.33, v.22+v.23+v.27+v.28+v.33+v.34, v.22+v.23+v.26+v.28+v.29, v.22+v.23+v.26+v.28+v.29+v.34, v.22+v.23+v.26+v.28+v.29+v.33, v.22+v.23+v.26+v.28+v.29+v.33+v.34, v.22+v.23+v.26+v.27+v.28+v.29, v.22+v.23+v.26+v.27+v.28+v.29+v.34,v.22+v.23+v.26+v.27+v.28+v.29+v.33 ,v.22+v.23+v.26+v.27+v.28+v.29+v.33+v.34, v.22+v.23+v.24+v.28,v.22+v.23+v.24+v.28+v.34, v.22+v.23+v.24+v.28+v.33, v.22+v.23+v.24+v.28+v.33+v.34, v.22+v.23+v.24+v.27+v.28, v.22+v.23+v.24+v.27+v.28+v.34, v.22+v.23+v.24+v.27+v.28+v.33,v.22+v.23+v.24+v.27+v.28+v.33+v.34, v.22+v.23+v.24+v.26+v.28+v.29, v.22+v.23+v.24+v.26+v.28+v.29+v.34, v.22+v.23+v.24+v.26+v.28+v.29+v.33,v.22+v.23+v.24+v.26+v.28+v.29+v.33 +v.34, v.22+v.23+v.24+v.26+v.27+v.28+v.29,v.22+v.23+v.24+ v.26+v.27+v.28+v.29+v.34, v.22+v.23+v.24+v.26+v.27+v.28+v.29+v.33,v.22+v.23+ v.24+v.26+v.27+v.28+v.29+v.33+v.34]]

TotalStiefelWhitneyClass:=sw[1][1]+sw[2][1]+sw[3][1]+sw[4][1]+sw[5][1]

v.1+v.2+v.8+v.10+v.11+v.12+v.17+v.22

StiefelWhitneyClassAsPolynomial :

v.1 + v.2 + v.4*v.4 + v.8 + v.2*v.4*v.4 + v.8*v.2 + v.8*v.4 + v.8*v.2*v.4

****

Group name: SymmetricGroup(7)

Stiefel-Whitney Classes: [[v.1], [v.2, v.2+v.3], [v.8+v.10, v.8+v.9+v.10, v.6+v.8+v.10, v.6+v.8+v.9+v.10], [v.11+v.12+v.17, v.11+v.12+v.17+v.19, v.11+v.12+v.17+v.18, v.11+v.12+v.17+v.18+v.19, v.11+v.12+v.14+v.17, v.11+v.12+v.14+v.17+v.19, v.11+v.12+v.14+v.17+v.18, v.11+v.12+v.14+v.17+v.18+v.19, v.11+v.12+v.13+v.15+v.17, v.11+v.12+v.13+v.15+v.17+v.19, v.11+v.12+v.13+v.15+v.17+v.18, v.11+v.12+v.13+v.15+v.17+v.18+v.19, v.11+v.12+v.13+v.14+v.15+v.17, v.11+v.12+v.13+v.14+v.15+v.17+v.19,v.11+v.12+v.13+v.14+v.15+v.17+v.18 , v.11+v.12+v.13+v.14+v.15+v.17+v.18+v.19], [v.22, v.22+v.34, v.22+v.33, v.22+v.33+v.34, v.22+v.27, v.22+v.27+v.34, v.22+v.27+v.33, v.22+v.27+v.33+v.34, v.22+v.26+v.29, v.22+v.26+v.29+v.34, v.22+v.26+v.29+v.33, v.22+v.26+v.29+v.33+v.34, v.22+v.26+v.27+v.29, v.22+v.26+v.27+v.29+v.34, v.22+v.26+v.27+v.29+v.33, v.22+v.26+v.27+v.29+v.33+v.34, v.22+v.24, v.22+v.24+v.34,v.22+v.24+v.33,

v.22+v.24+v.33+v.34, v.22+v.24+v.27, v.22+v.24+v.27+v.34,
v.22+v.24+v.27+v.33, v.22+v.24+v.27+v.33+v.34, v.22+v.24+v.26+v.29,
v.22+v.24+v.26+v.29+v.34, v.22+v.24+v.26+v.29+v.33,
v.22+v.24+v.26+v.29+v.33+v.34, v.22+v.24+v.26+v.27+v.29,
v.22+v.24+v.26+v.27+v.29+v.34, v.22+v.24+v.26+v.27+v.29+v.33,v.22
+v.24+v.26+v.27+v.29+v.33+v.34, v.22+v.23+v.28, v.22+v.23+v.28+v.34,
v.22+v.23+v.28+v.33, v.22+v.23+v.28+v.33+v.34,
v.22+v.23+v.27+v.28,v.22+v.23+v.27+v.28+v.34, v.22+v.23+v.27+v.28+v.33,
v.22+v.23+v.27+v.28+v.33+v.34, v.22+v.23+v.26+v.28+v.29,
v.22+v.23+v.26+v.28+v.29+v.34, v.22+v.23+v.26+v.28+v.29+v.33,
v.22+v.23+v.26+v.28+v.29+v.33+v.34, v.22+v.23+v.26+v.27+v.28+v.29,
v.22+v.23+v.26+v.27+v.28+v.29+v.34,v.22+v.23+v.26+v.27+v.28+v.29+v.33
,v.22+v.23+v.26+v.27+v.28+v.29+v.33+v.34,
v.22+v.23+v.24+v.28,v.22+v.23+v.24+v.28+v.34, v.22+v.23+v.24+v.28+v.33,
v.22+v.23+v.24+v.28+v.33+v.34, v.22+v.23+v.24+v.27+v.28,
v.22+v.23+v.24+v.27+v.28+v.34,
v.22+v.23+v.24+v.27+v.28+v.33,v.22+v.23+v.24+v.27+v.28+v.33+v.34,
v.22+v.23+v.24+v.26+v.28+v.29, v.22+v.23+v.24+v.26+v.28+v.29+v.34,
v.22+v.23+v.24+v.26+v.28+v.29+v.33,v.22+v.23+v.24+v.26+v.28+v.29+v.33
+v.34, v.22+v.23+v.24+v.26+v.27+v.28+v.29,v.22+v.23+v.24+
v.26+v.27+v.28+v.29+v.34,
v.22+v.23+v.24+v.26+v.27+v.28+v.29+v.33,v.22+v.23+
v.24+v.26+v.27+v.28+v.29+v.33+v.34]]

TotalStiefelWhitneyClass:=sw[1][1]+sw[2][1]+sw[3][1]+sw[4][1]+sw[5][1]

v.1+v.2+v.8+v.10+v.11+v.12+v.17+v.22

Stiefel-Whitney Class As Polynomial :

v.1 + v.2 + v.4*v.4 + v.8 + v.2*v.4*v.4 + v.8*v.2 + v.8*v.4 + v.8*v.2*v.4

We note that Stiefel-WhitneyClasses of $S_7 = \text{Stiefel} - \text{WhitneyClasses of } S_6$

****
Group name: MathieuGroup(9)
Cohomology generators: [v.1, v.2, v.3, v.7]
Generators degree: [0, 1, 1, 4]
StiefelWhitney-Classes:

[[0*v.1, v.1], [0*v.1, v.3, v.2, v.2+v.3], [0*v.1, v.5, v.4, v.4+v.5], [0*v.1, v.6], [0*v.1, v.7], [0*v.1, v.9, v.8, v.8+v.9],

[0*v.1, v.11, v.10, v.10+v.11], [0*v.1, v.12]]
sw[1][2]: v.1

sw[2][1]: 0
sw[2][2]: v.3
sw[2][3]: v.2
sw[2][4]: v.2 + v.3

sw[3][4]: v.2*v.2
sw[3][3]: v.2*v.2 + v.2*v.3
sw[3][2]: v.2*v.3
sw[4][2]: v.2*v.2*v.3
sw[5][2]: v.7
sw[6][2]: v.7*v.2
sw[6][3]: v.7*v.2 + v.7*v.3
sw[6][4]: v.7*v.3
sw[7][2]: v.7*v.2*v.2
sw[7][3]: v.7*v.2*v.3
sw[7][4]: v.7*v.2*v.2 + v.7*v.2*v.3
sw[8][2]: v.7*v.2*v.2*v.3
TotalSteifelWhiteny:=sw[1][2]+sw[2][4]+sw[3][2]+sw[4][2]+sw[5][2]+sw[6][2]+sw[7][2]+sw[8][2];
v.1+v.2+v.3+v.5+v.6+v.7+v.9+v.11+v.12
AsPolynomial(TotalSteifelWhiteny);
v.1 + v.2 + v.3 + v.2*v.3 + v.2*v.2*v.3 + v.7 + v.7*v.2 + v.7*v.2*v.2 + v.7*v.2*v.2*v.3

****

Group name: MathieuGroup(10)
Group([(1,7)(2,3)(4,5)(9,10), (2,4)(3,5)(6,8)(9,10), (1,3,7,2)(4,9,5,10)])
Group Generators: [v.1, v.2, v.3, v.6, v.8]
Generators degree: [0, 1, 1, 3, 4]
Stiefel-Whitney Classes:
[[0*v.1, v.1], [0*v.1, v.3, v.2, v.2+v.3], [0*v.1, v.5, v.4, v.4+v.5], [0*v.1, v.7, v.6, v.6+v.7],[0*v.1, v.10, v.9, v.9+v.10, v.8, v.8+v.10, v.8+v.9, v.8+v.9+v.10],
[0*v.1, v.14, v.13, v.13+v.14, v.12, v.12+v.14, v.12+v.13, v.12+v.13+v.14, v.11, v.11+v.14, v.11+v.13, v.11+v.13+v.14, v.11+v.12,
v.11+v.12+v.14, v.11+v.12+v.13, v.11+v.12+v.13+v.14], [0*v.1, v.18,

v.17, v.17+v.18, v.16, v.16+v.18, v.16+v.17, v.16+v.17+v.18, v.15,
v.15+v.18, v.15+v.17, v.15+v.17+v.18, v.15+v.16, v.15+v.16+v.18,
v.15+v.16+v.17, v.15+v.16+v.17+v.18],
[0*v.1, v.22, v.21, v.21+v.22, v.20, v.20+v.22, v.20+v.21, v.20+v.21+v.22,
v.19, v.19+v.22, v.19+v.21, v.19+v.21+v.22, v.19+v.20,
v.19+v.20+v.22, v.19+v.20+v.21, v.19+v.20+v.21+v.22], [0*v.1, v.27,
v.26, v.26+v.27, v.25, v.25+v.27, v.25+v.26, v.25+v.26+v.27, v.24,
v.24+v.27, v.24+v.26, v.24+v.26+v.27, v.24+v.25, v.24+v.25+v.27,
v.24+v.25+v.26, v.24+v.25+v.26+v.27, v.23, v.23+v.27, v.23+v.26,
v.23+v.26+v.27, v.23+v.25, v.23+v.25+v.27, v.23+v.25+v.26,
v.23+v.25+v.26+v.27, v.23+v.24, v.23+v.24+v.27, v.23+v.24+v.26,
v.23+v.24+v.26+v.27, v.23+v.24+v.25, v.23+v.24+v.25+v.27,
v.23+v.24+v.25+v.26, v.23+v.24+v.25+v.26+v.27]]

Total Steifel-Whetiny class:= sw[1][2]+sw[2][2]+ sw[3][2]+sw[4][2]+
sw[5][3]+sw[6][4]+sw[7][3]+sw[8][2]+sw[9][2]

v.1+v.3+v.5+v.7+v.9+v.13+v.14+v.17+v.22+v.27

Total Steifel-Whetiny class As Polynomial:

v.1 + v.3 + v.2*v.2 + v.2*v.2*v.2 + v.6 + v.6*v.2 + v.2*v.2*v.2*v.2*v.2 +
v.6*v.2*v.2*v.2 + v.2*v.2*v.2*v.2*v.2*v.2*v.2 + v.6*v.2*v.2*v.2*v.2 + v.8\

*v.6 + v.2*v.2*v.2*v.2*v.2*v.2*v.2*v.2

****

Group name: MathieuGroup(11)

Cohomology generators: v.1, v.2, v.3, v.6, v.8

Generators degree: 0, 1, 1, 3, 4

StiefelWhitneyClasses(rho,v,A,true);

[[0*v.1, v.1], [0*v.1, v.3, v.2, v.2+v.3], [0*v.1, v.5, v.4, v.4+v.5],

[0*v.1, v.7, v.6, v.6+v.7],

[0*v.1, v.10, v.9, v.9+v.10, v.8, v.8+v.10, v.8+v.9, v.8+v.9+v.10],

[0*v.1, v.14, v.13, v.13+v.14, v.12, v.12+v.14, v.12+v.13, v.12+v.13+v.14,

v.11, v.11+v.14, v.11+v.13, v.11+v.13+v.14, v.11+v.12, v.11+v.12+v.14, v.11+v.12+v.13, v.11+v.12+v.13+v.14],

[0*v.1, v.18, v.17, v.17+v.18, v.16, v.16+v.18, v.16+v.17, v.16+v.17+v.18,

v.15, v.15+v.18, v.15+v.17, v.15+v.17+v.18, v.15+v.16, v.15+v.16+v.18,

v.15+v.16+v.17, v.15+v.16+v.17+v.18],

[0*v.1, v.22, v.21, v.21+v.22, v.20, v.20+v.22, v.20+v.21, v.20+v.21+v.22,

v.19, v.19+v.22, v.19+v.21, v.19+v.21+v.22, v.19+v.20, v.19+v.20+v.22,

v.19+v.20+v.21, v.19+v.20+v.21+v.22],

[0*v.1, v.27, v.26, v.26+v.27, v.25, v.25+v.27, v.25+v.26, v.25+v.26+v.27,

v.24, v.24+v.27, v.24+v.26, v.24+v.26+v.27, v.24+v.25, v.24+v.25+v.27,

v.24+v.25+v.26, v.24+v.25+v.26+v.27, v.23, v.23+v.27, v.23+v.26,

v.23+v.26+v.27, v.23+v.25, v.23+v.25+v.27, v.23+v.25+v.26,

v.23+v.25+v.26+v.27, v.23+v.24, v.23+v.24+v.27, v.23+v.24+v.26,

v.23+v.24+v.26+v.27, v.23+v.24+v.25, v.23+v.24+v.25+v.27,

v.23+v.24+v.25+v.26, v.23+v.24+v.25+v.26+v.27]]

TotalStiefelWhitneyClass:=
sw[1][2]+sw[2][2]+sw[3][2]+sw[4][2]+sw[5][2]+sw[6][2]+sw[7][2]+sw[8][2]+sw[9][2];

v.1+v.3+v.5+v.7+v.10+v.14+v.18+v.22+v.27

TotalStiefelWhitneyClass AsPolynomial

v.1 + v.3 + v.2*v.2 + v.2*v.3 + v.2*v.2*v.2 + v.2*v.2*v.2*v.2 + v.2*v.2*v.2*v.\2*v.2 + v.6*v.2*v.2 + v.2*v.2*v.2*v.2*v.2*v.2 + v.6*v.2*v.2*v.2 + v.2*v.2*v.2*\v.2*v.2*v.2*v.2 + v.2*v.2*v.2*v.2*v.2*v.2*v.2*v.2

****

Group name: AlternatingGroup(4)
Group([(1,2)(3,4), (1,3)(2,4)])

Group Generators:
[v.1, v.2, v.3]
Generators degree:
[0, 1, 1]
Stiefel-Whitney Classes:
[[v.1], [v.2+v.3], [v.4]]
Total Steifel-Whiteny class:=sw[1]+sw[2]+sw[3];
[v.1+v.2+v.3+v.4]

Total Steifel-Whiteny class As Polynomial:
v.1 + v.2 + v.3 + v.2*v.3

****

Group name: AlternatingGroup(5)
Group([(1,2)(3,4), (1,3)(2,4)])
Group Generators:
[v.1, v.2, v.3]
Generators degree:
[0, 1, 1]
Stiefel-Whitney Classes of A_5 = Stiefel-Whitney Classes of A_4

****

Group name: Alternating Group(6)
Group([(1,2)(3,4), (1,3)(2,4), (1,2)(5,6)])
gap> A:=Mod2SteenrodAlgebra(G,8);;
Group Generators:
[v.1, v.2, v.3, v.4]
Generators degree:
[0, 1, 1, 2]

Stiefel-Whitney Classes:
[[0*v.1, v.1], [0*v.1, v.3, v.2, v.2+v.3], [0*v.1, v.6, v.5, v.5+v.6, v.4, v.4+v.6, v.4+v.5, v.4+v.5+v.6], [0*v.1, v.10, v.9, v.9+v.10, v.8, v.8+v.10, v.8+v.9, v.8+v.9+v.10, v.7, v.7+v.10, v.7+v.9, v.7+v.9+v.10, v.7+v.8, v.7+v.8+v.10, v.7+v.8+v.9, v.7+v.8+v.9+v.10], [0*v.1, v.15, v.14, v.14+v.15, v.13, v.13+v.15, v.13+v.14, v.13+v.14+v.15, v.12, v.12+v.15, v.12+v.14, v.12+v.14+v.15, v.12+v.13, v.12+v.13+v.15, v.12+v.13+v.14, v.12+v.13+v.14+v.15, v.11, v.11+v.15, v.11+v.14, v.11+v.14+v.15, v.11+v.13,

v.11+v.13+v.15,   v.11+v.13+v.14, v.11+v.13+v.14+v.15, v.11+v.12,
v.11+v.12+v.15, v.11+v.12+v.14, v.11+v.12+v.14+v.15, v.11+v.12+v.13,
v.11+v.12+v.13+v.15,   v.11+v.12+v.13+v.14, v.11+v.12+v.13+v.14+v.15]]

Total Steifel-Whieny class:=sw[1][1]+sw[2][2]+sw[3][2]+sw[4][2];
v.3+v.6+v.10
Total Steifel-Whieny class As Polynomial:
v.3 + v.3*v.3 + v.3*v.3*v.3

****

Group name: Alternating Group(6)
Group([(1,2)(3,4), (1,3)(2,4), (1,2)(5,6)])
gap> A:=Mod2SteenrodAlgebra(G,8);;
Group Generators:
[v.1, v.2, v.3, v.4]
Generators degree:
[0, 1, 1, 2]

Stiefel-Whitney Classes of A_7 = Stiefel-Whitney Classes of A_6
****

Conclusion:

The current study computing of Steenrod squares on a a finite 2-groups of order 128. the computation of more than 200 groups of order 128 were conducted. The implementation can compute the Steenrod squares.

• all direct product groups of orders 128 by using Direct product method.

• some small groups of order 128 wich have generators of degree 1, 2, 3 or 4 by using The CohomologicalData(G,n) method or Detection Method as well as using the HAP command CohomologicalDetectedIntersection(G,K,n) on the specified maximal subgroups.

In addition We tried to find the results of computing the largest possible number of SW-classes of real representation of non-prime power groups .

Moreover, fuzzy cofibration and fuzzy serre cofibration were studied as a branch based on our study of the classifying space also, we expansion to study the mixing case of this spaces.

future work:

we can suggest several studies, including:

1. introducing some of point to compute power Steenrod and fuzzy cofibration as a future work.
2. computing power Steenrod square of mod-2 cohomology ring of finite groups.
3. computing power steenrod squer of non-prime power groups .
4. computing Chern classes of mod-2 cohomology ring of finite groups.
5. computing Chern classes of non-prime power groups by using Stiefel-Whitney classes of non-prime power groups and Steenrod square of non-prime power groups .
6. finding structure Triple fuzzy fiber space is called Triple fuzzy Serre (co) fibration (Tri FS(co) fibration , by short), and similarily we find triple fuzzy (co) fibration (Tri F(co) fibration , by short)..

Appendix

GAP Codes:

1) CohomologicalData:

```
########################################

########################################

##

##Input: A Finite 2-groups, Maximal subgroups and and an integer N.
##Output: Print details of the group order, group number, cohomology ring
##generators with degree and relations and the Steenrod square Sq^k for
##each generator x and each positive 2-power k=2^i <degree(x).
##
##
InstallGlobalFunction(CohomologicalData, function(arg)
Local G, N, file, alpha, alpha1, A, gens, gensletters, gensletters1, gensdegrees, d, p,
tmp, tmpdir, x, relabel, relabeltwo, pres, rels, r, s, i, k, w;

G:=arg[1];

if not SSortedList(Factors(Order(G)))=[2] then

    Print("This function is only implemented for 2-groups.\n");

    return fail;

fi;

N:=arg[2];

if Length(arg)=2 then

    tmpdir := DirectoryTemporary();;

    file:=Filename( tmpdir , "cdata.txt" );

    else

    file:=arg[3];

fi;

AppendTo(file,"Group order: ", IdGroup(G)[1],"\n");

AppendTo(file,"Group number: ",IdGroup(G)[2],"\n");

AppendTo(file,"Group description: ",StructureDescription(G),"\n\n");

alpha:= ['1','a','b','c','d','e','f','g','h','p','q','r','s','t','u','v','w','x'];

alpha1:=List(alpha,i->[i]);

A:=ModPCohomologyRing(G,N);

gens:=ModPRingGenerators(A);
```

```
gens:=ModPRingGenerators(A);
 gensletters:=alpha{[1..Length(gens)]};
 gensletters1:=alpha1{[1..Length(gens)]};
 AppendTo(file,"Cohomology generators\n");
gensdegrees:=List(gens,A!.degree);
for d in SSortedList(gensdegrees) do
 if d>0 then
        AppendTo(file, "Degree ",d,": ");
      tmp:=Filtered([1..Length(gens)],i->A!.degree(gens[i])=d);
       tmp:=gensletters1{tmp};
        for x in tmp do
                 if Position(tmp,x)<Length(tmp) then
                      AppendTo(file,x, ",");
                      else
                     AppendTo(file,x, "\n");
                 fi;
           od;
      fi;
  od;
AppendTo(file,x, "\n");
############################################################
Relabel:=function(ss)
Local i, s, us;
s:=String(ss);
s:=List(s,i->i);
 s:=Filtered(s,i->not i='x');
Add(s,' ');
us:=Filtered([1..Length(s)],i->s[i]='_');
for i in us do
```

```
s[i+1]:=gensletters[1+EvalString([s[i+1]])];
 if not s[i+2] in ['^','+','*'] then
  s[i+2]:=' ';
     fi;
od;
s:=Filtered(s,i->not i='_');
 s:=Filtered(s,i->not i=' ');
if Length(s)=0 then return 0;
 fi;
return s;
end;
##########################################################
AppendTo(file,"Cohomology relations\n");
pres:=Mod2CohomologyRingPresentation(A);
pres:=MinimizeRingRelations(pres);
rels:=pres!.relations;
 for r in [1..Length(rels)] do
 s:=relabel(rels[r]);
AppendTo(file,r,": ",s,"\n");
od;
AppendTo(file,"\n");
 AppendTo(file,"Poincare series\n");
 p:=HilbertPoincareSeries(pres);
p:=String(p); p:=List(p,i->i);
for i in [1..Length(p)] do
 if p[i]='_' then p[i]:=' '; p[i+1]:=' ';
fi;
od;
 p:=Filtered(p,i->not i=' ');
 AppendTo(file,p,"\n\n");
N:=Maximum(List(gens,A!.degree));
 A:=Mod2SteenrodAlgebra(G,2*N);
 gens:=ModPRingGenerators(A);
 gensletters:=alpha{[1..Length(gens)]};
 gensletters1:=alpha1{[1..Length(gens)]};
 gensdegrees:=List(gens,A!.degree);
#########################################################
relabeltwo:=function(ss)
local i, s, us, t, ii, l;
 s:=String(ss);
s:=List(s,i->i);
s:=Filtered(s,i->not i='v');
 us:=Filtered([1..Length(s)],i->s[i]='.');
Add(s,' ');
for i in us do
ii:=i+1;
```

```
 l:=[];
while not s[ii] in ['^','+','*', ' '] do
 Add(l,s[ii]);
s[ii]:=' ';
ii:=ii+1;
 od;
t:=Basis(A)[EvalString(l)];
s[i+1]:=gensletters[Position(gens,t)];
 od;
s:=Filtered(s,i->not i='.');
s:=Filtered(s,i->not i=' ');
if Length(s)=0 then return 0;
 fi;
return s;
 end;
###########################################################################
AppendTo(file,"Steenrod squares\n");
for i in [2..Length(gens)] do
 for k in [1..A!.degree(gens[i])-1] do
 if k=2^Log(k,2)then
        AppendTo(file,"Sq^",k,"(",[gensletters[i]],")=");
w:=Sq(A,k,gens[i]);
if IsZero(w) then
AppendTo(file,0,"\n");
else
w:=PrintAlgebraWordAsPolynomial(A,w,1);
w:=relabeltwo(w);
AppendTo(file,w,"\n");
 fi;
fi;
od;
od;
Exec(Concatenation("display ",file));
 if Length(arg)=2 then RemoveFile(file);
fi;
end;
###############################################################################
```

2) CohomologyDetacted:

```
#########################################

#########################################

##

##Input: A Finite 2-groups, Maximal subgroups and and an integer n> 0.

##Output: Print details of the group order, group number, cohomology ring

##generators with degree and relations and the Steenrod square Sq^k for

##each generator x and each positive 2-power k=2^i <degree(x).
```

```
##

##

InstallGlobalFunction(CohomologicalDetected, function(arg)
Local G, N, file, alpha, alpha1, A, gens, gensletters, gensletters1, gensdegrees, d, p,
tmp, tmpdir, x, HG, HLL, K, f, I, iota, j, P. ws, LL, L, ker, KER, relabeltwo, pres, rels,
r, s, i, k, w;

G:=arg[1];
L:=arg[2];
N:=arg[3];
if Length(arg)=3 then
        tmpdir := DirectoryTemporary();;
        file:=Filename( tmpdir , "cdetected.txt" );
        else
        file:=arg[4];
fi;
 LL:=L;
if LL=G then
        return CohomologicalData(G,N);
        AppendTo(file,CohomologicalData(G,N));
        else
        AppendTo(file,"Group order: ", IdGroup(G)[1],"\n");
        AppendTo(file,"Group number: ",IdGroup(G)[2],"\n");
        AppendTo(file,"Group description: ",StructureDescription(G),"\n\n");
        AppendTo(file,"Maximal Subgroup list:",List(L,IdGroup),"\n\n");
        for j in [1..Length(LL)] do
                LL:=L[j];
                f:=GroupHomomorphismByFunction(LL,G,x->x);
                G:=Range(f);
                LL:=Source(f)
                I:=InducedSteenrodHomomorphisms(f,N);
                HG:=I[1];
                HLL:=I[2];
                iota:=I[3];
                alpha:=
                ['1','a','b','c','d','e','f','g','h','p','q','r','s','t','u','v','w','x'];
                alpha1:=List(alpha,i->[i]);
                gens:=ModPRingGenerators(HG);
                gensletters:=alpha{[1..Length(gens)]};
                gensletters1:=alpha1{[1..Length(gens)]};
                AppendTo(file,"Cohomology generators\n");
                gensdegrees:=List(gens,HG!.degree);
                for d in SSortedList(gensdegrees) do
                        if d>0 then
                                AppendTo(file, "Degree ",d,": ");
                                tmp:=Filtered([1..Length(gens)],i-
>HG!.degree(gens[i])=d);
                                tmp:=gensletters1{tmp};
```

```
                    for x in tmp do
                        if Position(tmp,x)<Length(tmp) then
                            AppendTo(file,x,", ");
                            else
                            AppendTo(file,x,"\n");
                        fi;
                    od;
                fi;
            od;
            AppendTo(file,"\n");

    ###########################################################
            relabeltwo:=function(ss)
            local i, s, us, t, ii, l;
                s:=String(ss);
                s:=List(s,i->i);
                s:=Filtered(s,i->not i='v');
                us:=Filtered([1..Length(s)],i->s[i]= '.');
                Add(s,' ');
                for i in us do
                    ii:=i+1;
                    l:=[];
                    while not s[ii] in ['^','+','*',' '] do
                        Add(l,s[ii]);
                        s[ii]:=' ';
                        ii:=ii+1;
                    od;
                    t:=Basis(A)[EvalString(l)];
                    s[i+1]:=gensletters[Position(gens,t)];
                    #if not s[i+2] in ['^','+','*'] then s[i+2]:=' '; fi;
                od;
                s:=Filtered(s,i->not i='.');
                s:=Filtered(s,i->not i=' ');
                if Length(s)=0 or s="[]"  then return 0; fi;
                return s;

            end;

    ###########################################################
    #######
            AppendTo(file,"Steenrod squares\n");
            for i in [2..Length(gens)] do
                for k in [1..HG!.degree(gens[i])-1] do
                  if k=2^Log(k, 2) then
             AppendTo(file,"Sq^",k,"(",[gensletters[i]],")=");
             w:=Sq(HLL,k,Image(iota[HG!.degree(gens[i])+1],gens[i]));
              if IsZero(w) then
                   ker:=Kernel(iota[HG!.degree(gens[i])+1+k]);
                    #KER:=Basis(ker);
```

```
                    #KER:=subspace(HG, Basis(ker));
                     KER:=Elements(ker);
                      KER:=List(KER,x->x);
                              AppendTo(file, "[");
                              for j in [1..Length(KER)] do
                    w:=PrintAlgebraWordAsPolynomial(HG,KEA[j],1);
                    w:=relabeltwo(w);
                   AppendTo(file, "," );
              fi;
          od;
                    AppendTo(file, "]\n" );
                   else
                 P:=List(PreLmagesElm(iota[HG!.degree(gens[i]+1+k], w), x->x);
##ws:=[];
                   AppendTo(file, "[" );
                    for j in [1..Length(P)] do


     w:=PrintAlgebraWordAsPolynomial(HG,P[j],1);
                    w:=relabeltwo(w);
                   #Append(ws,[w]);
                   AppendTo(file, w );
                   if j< Length(P) then
                   AppendTo(file, "," );
                  fi;
                 od;
                 AppendTo(file,"]\n");

                fi;
            fi;
         od;
      od;
  od;
fi;
     Exec(Concatenation("display ",file));
     if Length(arg)=3 then RemoveFile(file);
     fi;
     end;
     #######end of CohomologicalDetected#############################
     ##############################################################
```

3) CohomologyDetactedIntersection:

```
########################################
########################################
##
##Input:A Finite 2-groups, Maximal subgroups and and an integer n> 0.
##Output: Print details of the group order, group number, cohomology ring
```

```
##generators with degree and relations and the Steenrod square Sq^k for
##each generator x and each positive 2-power k=2^i <degree(x).
##
##
CohomologicalDetectedIntersection:=function(arg)
local G,N,file, alpha, alpha1, A, gens, gensletters, gensletters1,gensdegrees,
d, p, tmp, tmpdir, x,HG,HLL, K,f,I,iota,j,P,ws,LL, L,ker,t,IP,IPP,relabeltwo,
pres, rels, r, s,i, k, w;
	G:=arg[1];
	L:=arg[2];
	N:=arg[3];
	if Length(arg)=3 then
		tmpdir := DirectoryTemporary();;
		file:=Filename( tmpdir , "cdetected.txt" );
		else
		file:=arg[4];
	fi;
	if L=G then
		return CohomologicalData(G,N);
		AppendTo(file,CohomologicalData(G,N));
		else
		AppendTo(file,"Group order: ", IdGroup(G)[1],"\n");
		AppendTo(file,"Group number: ",IdGroup(G)[2],"\n");
		AppendTo(file,"Group description:
",StructureDescription(G),"\n\n");
		AppendTo(file,"Subgroup order:",List(L,IdGroup),"\n\n");
		for j in [1..Length(L)] do
			LL:=L[j];
			f:=GroupHomomorphismByFunction(LL,G,x->x);
			I:=InducedSteenrodHomomorphisms(f,N);
			HG:=I[1];
			HLL:=I[2];
			iota:=I[3];
		od;
		alpha:=['1','a','b','c','d','e','f','g','h','p','q','r','s','t','u','v','w','x'];
		alpha1:=List(alpha,i->[i]);
		gens:=ModPRingGenerators(HG);
		gensletters:=alpha{[1..Length(gens)]};
		gensletters1:=alpha1{[1..Length(gens)]};
		AppendTo(file,"Cohomology generators\n");
		gensdegrees:=List(gens,HG!.degree);
		for d in SSortedList(gensdegrees) do
			if d>0 then
				AppendTo(file, "Degree ",d,": ");
				tmp:=Filtered([1..Length(gens)],i-
>HG!.degree(gens[i])=d);
				tmp:=gensletters1{tmp};
				for x in tmp do
					if Position(tmp,x)<Length(tmp) then
```

```
                    AppendTo(file,x,", ");
                    else
                    AppendTo(file,x,"\n");
                fi;
            od;
        fi;
    od;
    AppendTo(file,"\n");

#####################################################
##############
    relabeltwo:=function(ss)
    local i, s, us, t, ii, l;
        s:=String(ss);
        s:=List(s,i->i);
        s:=Filtered(s,i->not i='v');
        us:=Filtered([1..Length(s)],i->s[i]= '.');
        Add(s,' ');
        for i in us do
            ii:=i+1;
            l:=[];
            while not s[ii] in ['^','+','*',' '] do
                Add(l,s[ii]);
                s[ii]:=' ';
                ii:=ii+1;
            od;
            t:=Basis(HG)[EvalString(l)];
            s[i+1]:=gensletters[Position(gens,t)];
        od;
        s:=Filtered(s,i->not i='.');
        s:=Filtered(s,i->not i=' ');
        if Length(s)=0 or s="[]" then return 0;
        fi;
        return s;
    end;

#####################################################
############
    AppendTo(file,"Steenrod squares\n");
    for i in [2..Length(gens)] do
        for k in [1..HG!.degree(gens[i])-1] do
            IP:=[];
            for j in [1..Length(L)] do
                Add(IP,[]);
                LL:=L[j];
            f:=GroupHomomorphismByFunction(LL,G,x->x);
                I:=InducedSteenrodHomomorphisms(f,N);
                HG:=I[1];
                HLL:=I[2];
```

```
                    iota:=I[3];
                    gens:=ModPRingGenerators(HG);
                    if k=2^Log(k,2)then
w:=Sq(HLL,k,Image(iota[HG!.degree(gens[i])+1],gens[i]));
                        if IsZero(w) then
ker:=Kernel(iota[HG!.degree(gens[i])+1+k]);
                            P:=Elements(ker);
                            for t in [1..Length(P)] do
w:=PrintAlgebraWordAsPolynomial(HG,P[t],1);
                                w:=relabeltwo(w);
                                Add(IP[j], w);
                            od;
                            else
P:=List(PreImagesElm(iota[HG!.degree(gens[i])+1+k],w),x->x);
                            for t in [1..Length(P)] do
w:=PrintAlgebraWordAsPolynomial(HG,P[t],1);
                                w:=relabeltwo(w);
                                Add(IP[j], w);
                            od;
                        fi;
                    fi;
                od;
                if k=2^Log(k,2)then
AppendTo(file,"Sq^",k,"(",[gensletters[i]],")=");
                    IPP:=IP[1];
                    for j in[2..Length(L)] do
                        IPP:=Intersection(IPP,IP[j]);
                    od;
                    AppendTo(file,IPP[1]);
                    for j in [2..Length(IPP)] do
                        AppendTo(file, "," ,IPP[j]);
                    od;
                    AppendTo(file,"\n");
                fi;
            od;
        od;
    fi;
    Exec(Concatenation("display ",file));
    if Length(arg)=3 then RemoveFile(file);
    fi;
end;
################end of CohomologicalDetectedIntersection##########
##############################################################
```

4) HAP-StiefelWhitney

```
##################################################################
## Input: Inputs $(G,v)$ or $(G,v,n)$ or $(\phi,v)$ or $(\phi,v,n)$. In the
first ## two cases $G$ must be a matrix group or a permutation group
which we convert to ## a permutation matrix group. In the second case
$\phi:G\longrightarrow Q$ must be ## a homomorphism to a matrix or
permutation group $Q$.
 ##
 ## Output: List of Stief , dimension of polytop, Resolution Prime Power
Group and ## Polytopal Representation Complex.
 ##
## InstallGlobalFunction(HAP_StiefelWhitney,
function(arg)
local G,v,n,P,R,T,homid,ThomR,CRhomCT,CThomCR,CR, CT,trans,
iso,pos, CTmatCR, TRmapping,stief,one,zero,d,u,uu,A,B,B1,B2,i,j;
if  IsGroup(arg[1]) then
        G:=arg[1];
else
        G:=Source(arg[1]);
fi;
v:=arg[2];
P:=PolytopalRepresentationComplex(arg[1],v);
if  IsBound(arg[3]) then
            n:=arg[3];
 else
           n:=0;
          while P!.dimension(n)>0 do
                        n:=n+1; od;
fi;
 T:=FreeGResolution(P,n+1,2);
 R:=ResolutionPrimePowerGroup(G,n+1);
homid:=GroupHomomorphismByImages(G,G,GeneratorsOfGroup(G),
GeneratorsOfGroup(G));
ThomR:=EquivariantChainMap(T,R,homid);
CRhomCT:=HomToIntegersModP(ThomR,2);
CR:=Source(CRhomCT);
 CT:=Target(CRhomCT);
 #########

##Now compute cochain map CThomCR:CT--->CR
CTmatCR:=[];
 for i in [0..n+1] do
```

```
        u:=List([1..R!.dimension(i)],x->0);
        A:=[];
       for j in [1..R!.dimension(i)] do
             u:=0*u;
             u[j]:=One(GF(2));
            Add(A,CRhomCT!.mapping(u,i));
od;
B1:=SemiEchelonMatTransformation(A);
uu:=List([1..Length(B1.heads)],x->0);
 B2:=[];
 for j in [1..Length(B1.heads)] do
uu:=0*uu;
 if B1.heads[j]=0 then
 uu[j]:=One(GF(2));
Add(B2,1*uu);
fi;
od;
B:=SolutionsMatDestructive(One(GF(2))*Concatenation(A,B2),
One(GF(2))*IdentityMat(Length(A[1])));
B:=List(B,x->x{[1..Length(A)]});
Add(CTmatCR,B);
od;
 # The cochain map CT--->CR sends vector v to v*CTmatCR
###############################################
TRmapping:=function(v,n);
 return v*CTmatCR[n+1];
end;
 ################################################
CThomCR:=rec(source:=CT, target:=CR,properties:=CRhomCT!.properties,
mapping:=TRmapping);
CThomCR:=Objectify(HapCochainMap,CThomCR);
##DONE
 ####
#We now construct a function stief(v,k):CR^k--CR^(n+d), v|-->w
 #where d is the dimension of the polytope P
 d:=0;
while P!.dimension(d+1)>0 do
d:=d+1;
od;
zero:=Zero(GF(2));
 one:=One(GF(2));
#####################
 stief:=function(v,k)
```

```
local w;
 w:=zero*[1..T!.filteredDimension(d-1,k+d)];
Append(w,one*v);
 return w*CTmatCR[1+k+d];
 end;
 ######################
 return [stief,d,R,P] ;
end);
 ############################end of HAP-StiefelWhitney #######
###############################################################
```

5) **FundamentalMultiplesOfStiefelWhitneyClasses**

```
###############################################################
###############################################################
########
## Input: either (G,v,A) where G is either a matrix/permutation group or
## a group representation.
## Output: a list $[t_0, t_1,...,t_D]$ where the term $t_i$ is a list of all
## solutions to the equation $\rho^0(1)x=\rho^0(1)\omega_i$ with $x\in
## H^i(G,\mathbb{\Z}_2)$.
##
##
InstallGlobalFunction(FundamentalMultiplesOfStiefelWhitneyClasses,
 function(arg)
local bool,G,v,A,P,N,S,R,stief,d,L,i,k,j,fund,Bas,Bas1,Bas2,swc,swc1,PIRep,
 u,a,b,w;
G:=arg[1];
v:=arg[2];
A:=arg[3];
 if Length(arg)=4 then
     bool:=arg[4];
else
      bool:=false;
 fi;
S:=HAP_StiefelWhitney(G,v);
P:=S[4];
N:=0;
while P!.dimension(N+1)>0 do
         N:=N+1;
od;
stief:=S[1];
d:=S[2];
 R:=S[3];
 Bas:=Basis(A);
```

```
 L:=[];
 for k in [0..N] do for j in [1..R!.dimension(k)] do
 w:=Zero(A);
 if k+d<=N then
 u:=Zero(GF(2))*[1..R!.dimension(k)];
u[j]:=One(GF(2));
 u:=stief(u,k);
 a:=Sum(List([0..d+k-1],s->R!.dimension(s)));
 b:=a+R!.dimension(d+k);
for i in [a+1..b] do
       w:=w+u[i-a]*Bas[i];
              od;
      fi;
     Add(L,w);
   od;
 od;

 fund:=L[1];
 swc:=[];
for i in [0..N] do
             Add(swc,Sq(A,i,fund));
od;
if not bool then
 return swc;
 fi;
 swc1:=[];
 for i in [0..N] do
        Bas1:=Filtered(Basis(A),x->A!.degree(x)=i);
        Bas2:=List(Bas1,i->i*fund);
 L:=LeftModuleGeneralMappingByImages(Subspace(A,Bas1),
 Subspace(A,Bas2),Bas1,Bas2);
 ##########################################
 PIRep:=function(L,x);
          if x=fail then
                 return x;
   fi;
 if x=Zero(A) then
 return Elements(Kernel(L));
 fi;
return Elements(PreImages(L,x));
 end;
 ##########################################
 Add(swc1,PIRep(L,swc[i+1])); od; return swc1; end);
```

```
###############end of FundamentalMultiplesOfStiefelWhitneyClasses###
#################################################################
```

6) **CohomologyHomomorphismOfRepresentation**

```
#################################################################
#################################################################
##
 ## Input: G can be a matrix/permutation group or a group representation.
## A is the Mod2 Steenrod algebra up to some degree.
## v is a vector on which the group (representation) acts.
 ##
 ## Output: a list $[t_0, t_1,...,t_D]$ where the term $t_i$ is a list of all
## solutions to the equation $\rho^0(1)x=\rho^0(1)\omega_i$ with $x\in
## H^i(G,\mathbb{\Z}_2)$.
 ##
##
InstallGlobalFunction(CohomologyHomomorphismOfRepresentation,
function(G,v,A)
local N,S,R,stiefel,d,L,i,k,j,Bas,w,u,a,b;
 N:=Maximum(List(Basis(A),x->A!.degree(x)));
S:=HAP_StiefelWhitney(G,v,N);
stiefel:=S[1];
 d:=S[2];
 R:=S[3];
 Bas:=Basis(A);
 L:=[];
for k in [0..N] do
for j in [1..R!.dimension(k)] do
 w:=Zero(A);
if k+d<=N then
 u:=Zero(GF(2))*[1..R!.dimension(k)];
u[j]:=One(GF(2));
 u:=stiefel(u,k);
a:=Sum(List([0..d+k-1],s->R!.dimension(s)));
 b:=a+R!.dimension(d+k);
 for i in [a+1..b] do w:=w+u[i-a]*Bas[i];
 od;
 fi;
 Add(L,w);
 od;
 od;
 L:=LeftModuleGeneralMappingByImages(A,A,Bas,L);
A!.StiefelWhitneyHomomorphism:=L;
```

```
return L;

end);
 ####end of CohomologyHomomorphismOfRepresentation ###########
###########################################################
```

REFERENCES

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[1] B. Kahn, J. Gunarwardena and C. Thomas,"*Stiefel-Whitney Classes of Real Rerepresentation of Finite Groups*", J. Algebra, pp:327-347,1988.

[2] C.L.Chang, "Fuzzy Topological Spaces", J. Math. Anal. Appl., pp. 182-190,1968.

[3] E. Bredon Glen , "*Topology and Geometry*", Springer, 1993.

[4] E. Graham, HAP - Homological Algebra Programming, Version 1.12.3, 2017.

[5] E. Graham, "*An Invitation to Computational Homotopy*", National University of Ireland, Galway, 2017.

[6] E. Graham, H.James and S.Emil, "*Polytopal resolutions for finite groups*", Journal für die reine und angewandte Mathematik (Crelles Journal), 2006.

[7] F. James Davis, "*Higher Diagonal Approximations and Skeletons of k(π; 1)'s"*, In Algebraic and Geometric Topology,part of Lecture Notes in Mathematics book ,Springer, 1985.

[8] GAP, Groups, Algorithms, and Programming, Version 4.7.2, 2013.

[9] G .Balasubramanian, "Maximal Fuzzy Topologies", *Kybernetika*, pp. 459-464, 1995.

[10] G. Cuvalcioglu, M. Citil, "On Fuzzy Homotopy Theory", Adv. stud. Contemp. Math. Turkey, 2005

[11] G. Pierre, "*The Computation of Stiefel-Whitney Classes*", 2008.

[12] G. Ziegler, "*Lectures on polytopes*", Graduate texts in mathematics, Springer, New York, 1995.

[13] H. Allen, "*Algebraic Topology*", Cambridge University Press, New York, NY, USA, 1994.

[14] H.Dale, "*Fiber Bundles*", Graduate texts in mathematics, Springer, New York, NY, USA, 2001.

[15] J. David Rusin, "The Cohomology of the Group of Order 32"., Mathematics of computation, Vol 53(187, pp. 359-385), 1989.

[16] J. Green David and A. King Simon, "*The Cohomology of Finite p-groups*" , J. of Algebra, pp. 352-363, 2011.

[17] J. Joseph, “*An Introduction to The Theory of Groups*”, Graduate texts in mathematics, Springer, New York, 1995.

[18] K. K .Azad., " *On Fuzzy Semi-Continuity, Fuzzy Almost Continuity and Fuzzy Weakly Continuity*", J. Math. Anal. Appl., pp. 14-32,1981.

[19] L.A zadeh., "*Fuzzy Sets*", information and control, pp.338-365, 1965.

[20] L.Valeri-Elizondo, L. Townsley, J.F.Carlson and M. Zang, "*Cohomology Rings of Finite Groups"*, with an Appendix: Calculations of Cohomology Rings of Groups of Order Dividing 64,Algebra and Applications series, vol. 3, Springer, 2003.

[21] M. Mahmood M, “*The Computation of Steenrod Operation for Non-Prime Power Groups”*, Thesis, 2021.

[22] N. Steenrod, “*Products of Cocycles and Extensions of Mappings*”, Annals of Mathematics, Vol 48(2), 1947.

[23] N. Steenrod and D. B. Epstein, "*Cohomology operations*", Princeton University Press, Ann. of Math. Studies, 1962.

[24] R.E. Mosher and M.C. Tangora, "Cohomology Operations and Applications in Homotopy Theory", Harper and Row, Publishers, New York.1968.

[25] S. Brown Kenneth, "*Cohomology of Groups*", volume 87 of Graduate Texts in Mathematics, Springer, 2nd edition, 1994.

[26] T. Thanh Vo., "*The computation of transfer maps, Evens norm maps and Steenrod operations"*, PhD dissertation, Friedrich Schiller University Jena, 2011

[27] W. Daher Al Baydli, A. Abdulsattar Hussien, "Mixed Cofibration and Mixed Hurewicz Cofibration", University of anbar-Education College of women, Wasit University-College of Science, 2009.

[28] W. Daher Al Baydli, “*Computation of Cohomology Operations for Finite Groups*”, PhD dissertation, 2018.

[29] Z .Y Habeeb, D . Al Baydli , Mixed Serre Fibration ,*Wasit Journal of Pure*, 2022

المستخلص

تهدف هذه الدراسة إلى حساب مربعات ستينرود على mod-2 للحلقات الكهومولوجية لكل الزمر من الرتبة 128 باستخدام برنامج HAP وقد تمكنا من حساب مربعات ستينرود لاكثر من 200 زمرة من الرتبة 128 اعطينا بناء كهومولوجي للزمر و تطبيق ضرب *cup-i* وقدمنا خوارزمية لذلك ومن ثم استخدمنا ضرب *cup-i* لحساب مربعات ستينرود على فضاء التصنيف للزمر من الرتبة 128. اعطينا طرقا ثلاثة لحساب مربعات ستينرود للزمر ذات الرتبة 128 اثنان منهن باستخدام برنامج *HAP* والثالثة يدويا حيث صممنا ونفذنا خوارزمية لحساب كل الزمر التي يمكن تمثيلها كحاصل ضرب زمرتين من رتب اقل من 128..كذلك من المساهمات المهمه في هذه الاطروحة وضحت كيفية استخدام التقنيات الاساسية في حساب صفوف استيفل-وتني للتمثيل الحقيقي للزمر ذات القوى غير الاولية. من المساهمات الاخرى لهذه الاطروحة هوتقديم تعريف التوليف الضبابي و توليف- سيري الضبابي مع حالة الخلط للتوليف(سيري) الضبابي وتضمنت الدراسة الخصائص والنظريات.

جمهورية العراق
وزارة التعلـــيم العـــالي والبحـــث العلمـــي
جامعــــــــــــة بابـــــــــــــــل
كليـــــة التربيـــــة للعلـــــوم الصـــــرفة
قســــــــــم الرياضـــــــــــيات

عمليات الكهومولوجي للزمر ذات الرتبة ١٢٨ الحسابية وصفوف استيفل-وتني للتمثيل الحقيقي للزمر ذات القوى غير الأولية

أطروحة

مقدمة الى مجلس كلية التربية للعلوم الصرفة في جامعة بابل كجزء من متطلبات نيل درجة الدكتوراه فلسفة في التربية / الرياضيات

من قبل

مروه ياسر محسن ناجي

بأشراف

أ.م.د ظاهر والي البيادلي

1445هـ

2023 م