Republic of Iraq

Ministry of Higher Education & Scientific Research

University of Babylon

College of Education for Pure Sciences

Department of Mathematics

Using Innovative Methods to Increase the Reliability of complex - Series Networks

A Thesis

Submitted to the Council of College of Education for Pure Sciences, University of Babylon in Partial Fulfillment of the Requirements for the Degree of Master in Education / Mathematics.

By

Asia Malik Mohammed

Supervised by

Prof.Dr. Zahir Abdul Haddi Hassan

2023 A.D. **1445 A.H.**

بِسْمِ ٱللَّهِ ٱلرَّحْمَٰنِ ٱلرَّحِيمِ

وَيَرَى الَّذِينَ أُوتُوا الْعِلْمَ الَّذِي أُنزِلَ
إِلَيْكَ مِن رَّبِّكَ هُوَ الْحَقَّ وَيَهْدِي إِلَىٰ
صِرَاطِ الْعَزِيزِ الْحَمِيدِ (6)

صدق اللَّهُ العظيم

سورة سبأ

Dedication

To those in whom I have seen the path of my life...and from whom I have drawn my strength and my self-esteem: my dear mother and dear father.

To whom has given me the greatest support my dear husband.

To the seed of the heart and hope for tomorrow, my beloved children.

To my brothers and sisters and everyone who supported me to achieve this success.

To all my teachers who taught me to come to this stage of learning.

I hope from God that it will be a flag window and a card of knowledge.

Asia M M.

Acknowledgements

In the name of Allah, the most merciful the most compassionate all praise be to Allah the lord of the worlds and prayers and peace be upon Mohamed his servant and messenger and to his good and pure household.

I would like to express my deep and sincere gratitude to my supervisor Prof. Dr. Zahir Abdul Haddi Hassan for his continuous support to my research,for his patience,encouragement and invaluable suggestions throughout the entire period of my work.

Also, I would like to thank all the members of Mathematics Department, College of Education for Pure Sciences, University of Babylon, who are credited for my arrival at this stage.

I am extremely and sincerely thankful to my husband for his love, care and sacrifice that enabled me to reach this level of learning.

Finally, I would like to thank my colleagues, Dr. Fouad Hamza Abd Nasser, Haider Salih Huwaidi Jawad, and Hatem Kareem Sulaiman Ali Aljobory, for their help, support, and encouragement.

And to everyone who helped me in one way or another, I owe my sincere thanks and gratitude.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

Table 1: List of abbreviations and symbols

Symbol	Description
G	A graph defined by an ordered pair (V,E)
V	Vertex set
E	Edge set
S	Complex - series network
G	Subsystem of S
H	Subsystem of S
R_s	Reliability of the system
R_i	Reliability of the component i
RBD	Reliability block diagram
MPS	Minimul path set
MCS	Minimul cut set
CM	Connection matrix of the system
PTM	Path tracing method
MCM	Minimal cut method
R^*_{US}	Unit redundancy
R_{ES}	Element redundancy
R_{sub}	Reliability of subsystem
R^*_i	Redundancy of unit i
m_i	The number of modules in subsystem i.

Abstract

This thesis describes a new technique to solve the problem of complex networks in addition to the methods of network reliability that have been studied in the past decades, which is the method of partition complex - series networks into several parts and describing the relationship between them and the partial systems of them. We will study the reliability of some well-known systems such as complex and mixed systems, which are often defined by minimum path set (MPS) and minimum cut set (MCS) to evaluate the network reliability of sub-systems. And the generalization of the method of partitioning complex - series networks to include an infinite number of networks and an infinite number of MPS intertwined with each other. This method has been generalized on different networks and similar (repeated) networks. We shed light on the redundancy methods (element and unit redundancy), as these methods are very effective for increasing the reliability of complex networks, then the results of the series network were compared with the partial systems of them. In addition, we have studied some techniques to assess the importance of reliability (Birnbaum's scale of importance) to improve the reliability of the system, and a comparison has been made between the results of the importance of series networks and partial systems to find out the relationship between them. Also, the Birnbaum importance scale was applied to a mixed system to see the strength of the results obtained by partition the system.

CHAPTER 1

INTRODUCTION AND SOME BASIC MATHEMATICAL CONCEPTS

1.1. Introduction

Reliability remained undeveloped until the 1940s. As a result of World War II needs, the military began using a lot of innovative electrical items. These included electronic detonators, portable radios with vacuum tubes, electronic switches, and portable radios with radar. The electronic tube computers began near the conclusion of the war but were not finished until after the war [42]. The larger reliability issue was discovered at the beginning of the 1950s, and solutions were being developed [59]. The study of large-scale graphs known as complex networks has gained in popularity over the past ten years as interest in small-scale graph analysis has declined. In many scientific fields, such networks have become a coherent representation of a complicated system. Often, they are utilized to represent systems with nontrivial topologies and plenty of vertices [1]. Up until the 1970s, nothing was known about using graph theory to analyze reliability. The popular 1968 book Probabilistic Reliability was the only significant work on the topic. The creation of signal flow graphs is regarded as a significant advancement in the assessment of network reliability. Many algorithms, strategies, and methodologies were then put out in the literature. Graph theory is now a crucial component of determining the reliability of a network [10, 75].

From the 1980s (Albert et al. 1999). Several academics from a variety of disciplines are interested in researching complex networks. The research's findings not only explain the traits and causes of complex systems but also explain why a system is complicated and practical applications [43].

Complex network-based machine learning and data mining methods are gaining popularity. This is because many real-world issues may be directly modeled as networks since networks are so pervasive in nature and daily life. A suitable network shaping technique can also be used to create network representations from a wide variety of different data sets. A complex network representation also integrates the dynamics, functions, and structure of the system it depicts. He explains how vertices interact (the structure) and how these interactions change over time (the dynamics), as well as how these interactions affect how a complex network functions as a whole [28, 70]. Network reliability, then, is a probabilistic metric that indicates whether a network can continue to run even if one or more of its components fail randomly [50]. network reliability presents significant potential as a unifying factor. framework for investigating a variety of problems that occur in complicated network situations [16].Can distinguish complex networks and more complex networks, as the degree of complexity of the network is not limited to the size of the network, but rather to how the components are interconnected, ramified, and intertwined with each other.

Now in our thesis, used a complex-series network and tried to extract the reliability of this network, so thought if split the complex-series network into two subsystems and extract the reliability of each subsystem, what results will get.

The current study consists of six chapters.

The first chapter, present several parts, in which the first part includes the most important basic definitions of the graph and its contents, the second part includes the basics of polynomial reliability, the third the structure functions, the fourth includes the contents of reliability, and the fifth includes the complex reliability system.

The second chapter contains some important methods for evaluating the reliability of complex-series network, such as the methods of MPS, MCS, and PTM.

The third chapter contains the method of partition of the complex - series network and the application of the methods of MPS and MCS on partial systems and obtaining a relationship linking the complex - series network and partial systems.

The fourth chapter in which used one of the methods of improving reliability, which is the redundancy method (element redundancy and unit redundancy), and applied it to the network of complex - series and subsystems

The fifth chapter used the method of the Birnbaum scale to find the importance of the component for the network of complex-series as well as for the sub-systems, as well as the importance of applying the scale of Birnbaum on a mixed system before and after partition.

The sixth chapter contains the conclusions that got from the method of partition of the complex - series network and includes future studies

1.2. Objective of Thesis

The goal of this thesis is to find a new method for the complex network problem (the complex-series network partition method) and thus generalize this method, and then calculate the increase in network reliability using redundancy and importance methods and apply them to the partition method.

1.3. Contributions

1. Developed new method to simplify the analysis of complicated networks and the reliability evaluation of complex systems.

2. The network partition method is a new method whose implementation requires the design of a network consisting of two systems connected in series.

3. Through this advanced method, the reliability assessment of the complex network of series is obtained by means of partition systems, on which some methods are applied, such as the MPS and MCS.

4. Generalize the partition method to take N of complex systems.

5. The researcher generalized calculating the total number of minimum path sets from complex-series networks with different sub networks.Is also generalized in the case of similar complex networks.

6. The researcher generalized calculating the total number of minimum cut sets from

complex-series networks with different sub networks.Is also generalized in the case of similar complex networks.

7. Applied the two redundancy methods to the sub-systems and studied the relationship between them and the network of complex - series.

8. Used the Birnbaum scale of importance on the sub-systems and concluded the relationship between them and the original network, also studied the relationship between the importance of each component in the network of complex -series and the importance of the components in the sub-systems.

9. Applied the Birnbaum scale on a mixed system that was taken in the form of three systems linked in a series and applied Birnbaum on the partial systems, to produce a relationship between the original system and the partial systems and also between the importance of the components in the partial systems and the importance of each component in the original system, and this results in the effectiveness of the method of partition in networks complex.

1.4. Related Works

One of the primary concerns of the early computer circuit designers was reliability. The issue of building trustworthy circuits was initially raised by John von Neumann in 1952, although it is fair to claim that the work of Moore and Shannon from 1956 onward helped the subject become a field of study. E.F. Moore and C.E. Shannon published a ground-breaking paper in 1956 [56]. That marked the beginning of the research on network dependability. They presented a probabilistic model of network reliability in which the links or edges had a chance to fail separately but the network's nodes were assumed to be completely dependable [45]. The challenge is to ascertain the likelihood that the network will remain connected under these circumstances. This results in what is known as the network's dependability polynomial if each edge has the same failure probability. The original challenge has been studied in many different ways over the past 60 years, resulting in a large body of study on the subject. In the sixty years after this foundational work, networks have proliferated across contemporary society, and the ground-breaking theories of Moore and Shannon have been applied to a wide variety of networks besides computer circuits [10, 56]. There have been studies on increasing reliability. The Kuo et albook is among the newest works on the subject. Just a few researchers have focused on series-parallel systems, even though many have been done to enhance parallel-series systems. These studies primarily focus on Jensen's 1968 redundancy allocation system [19]. Van Sylke and Frank assumed that failed nodes were reliable in 1971 and used networks with random components and failed nodes, providing a comprehensive analysis of when the nodes were reliable. Applications for computers, big networks, and decomposition methods for equal components [62, 75]. A method for anticipating the failure rate of parallel networks is introduced by Limnios in 1987. Relationships are created to reach a steady state with a time-dependent failure rate, and it is shown how to leverage these relationships to determine the steady-state failure rate for complex networks [52]. In 1988, Eagle Technology was given a contract by the David Taylor Research Center in Card Rock, Maryland, to create a handbook of reliability prediction procedures for mechanical equipment [45]. Pham goes into great detail about two sorts of failures that can increase network resilience in 2006. Suppose that each component of the network is independent and identical since there is no restriction on the total number of components. Also, it increased the dependability of networks that are series, parallel, parallel-series, and series-parallel [60, 65]. A novel method to determine the reliability of communication networks with similar links was introduced in 2009 by Altiparmak et al. They use two techniques, taboo search, and annealing simulation, to improve network dependability [3]. Padmavathy and Sanjay examined the Ad-Hoc Mobile Wireless Network's (MANET) dependability in 2013, and then they used Monte Carlo simulation to improve the network's dependability [61]. The dependability and failure probability of electric vehicles equipped with wind turbines were researched by Feng, Zhao, and Li in 2018, who also used the Particle Swarming (PSO) approach to boost reliability and identify the main causes of failures [27]. A study of mathematical models in network dependability was published in 2019 by Sulaiman, H.K., and Hassan, Z. A. H, [64]. To calculate the reliability of the electromagnetic system within airplanes and some engineering features, researchers have developed precise methods for complicated systems. Also, the researcher has investigated various methods for allocating reliability to the subsystem. Together with researching reliability allocation, particle swarm optimization, and genetic algorithms are used to optimize the dependability of the electromagnetic system within an airplane [77].

1.5. Some Definitions of Graph Theory

In this section consists of initial concepts that will be used in subsequent chapters. First, the basic concepts, required background, and relevant definitions of this thesis were addressed, and discuss graph theory and related concepts, and also explain the function of the structure and some related concepts in the first part. As for the second part of this chapter, the basic definition of a complex system was dealt with some of the characteristics that we need. Also touched on the importance of reliability. Given the importance of the MPS and the MCS of blocks for the reliability of networks, they were also explained in the second part [63].Graphs are used to model a wide range of scenarios and are seen in many different contexts. Seeing the various, varied ways in which mathematicians have affected the expansion and advancement of computer science may be the simplest approach to adjust to this variety. They first contributed to the creation of computers in order to streamline complicated mathematical processes. The demands of computer scientists started to influence the type of mathematics that was done as a result of the devices' role [66].

Graph theory is a prime illustration of this shift in thought. Graphs are studied by mathematicians due to their inherent mathematical elegance, with topological relations, algebra, and matrix theory sparking their interest. In addition, they investigate graphs' numerous applications in computation, such as data representation and network design. Network-like structures can be found in technical systems such as telecom networks and Electromagnetic systems and computer networks that consist of two main components: computers and computer transmission lines such as copper cables. In view of all of the above, it is necessary to understand the basic ideas of the diagram on which this thesis is based [75].

Definition 1.5.1. [50] **A graph:** G= (V, E) is defined through the ordered pair (V, E), where V is a non-empty set that has elements named nodes (vertices) and E is a set of pairs of elements. The elements are referred E to as the edges or (lines, arcs) of graph G, as shown in Fig 1.1.

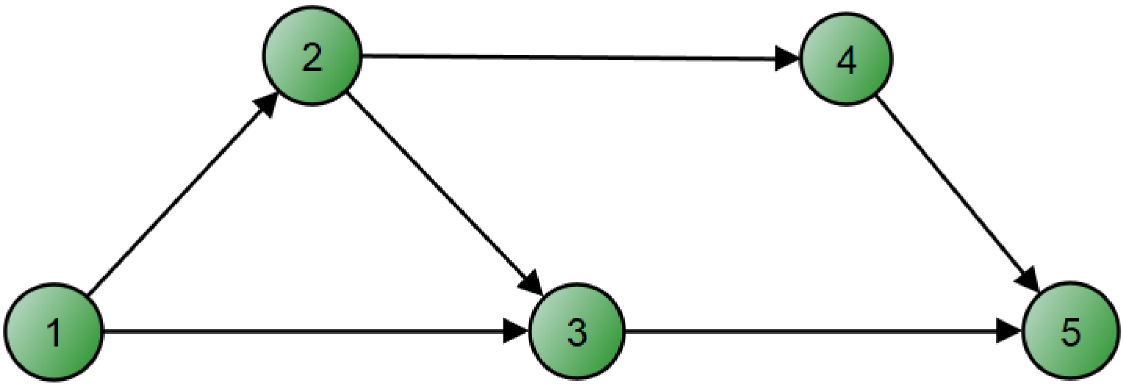

Figure 1.1: Graph G

Definition 1.5.2. [46]. **Subgraph** is a graph whose node set N(S) is a subset of the set of N(G) nodes, that is N(S) $\subseteq$ N(G), and whose edge set E(S) is a subset of the set for edges $\mathbf{E}(\mathbf{G})$, i.e., $\mathbf{E}(\mathbf{S}) \subseteq \mathbf{E}(\mathbf{G})$, see Fig. 1.2.

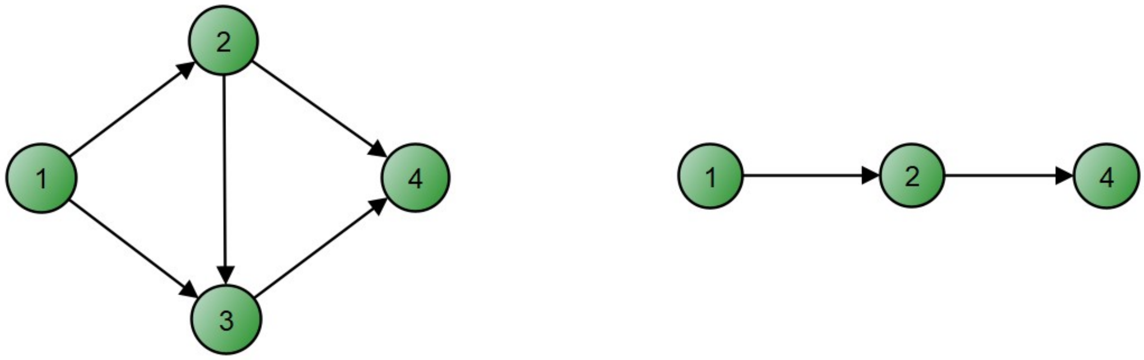

Figure 1.2: Subgraph S of G.

Definition 1.5.3. [30]. Graphs are usually represented by uppercase letters, such as G or H. Edges will be indicated by pairs of lowercase letters such as uv for the edge between nodes, u or v representing nodes.

Definition 1.5.4. [22]. **Adjacent Vertices** are two vertices that connected by an edge.

Definition 1.5.5. [44]. **A directed edge** is an edge between two ordered nodes (u, v) in E, and it is thought to go from nodes u to v.

Definition 1.5.6. [44]. An **undirected edge** is an edge without an arrow used to indicate bidirectional communication links between nodes.

Definition 1.5.7. [75]. If every edge of graph G points in the same direction, then G is called a **Directed graph**, see Fig. 1.3.

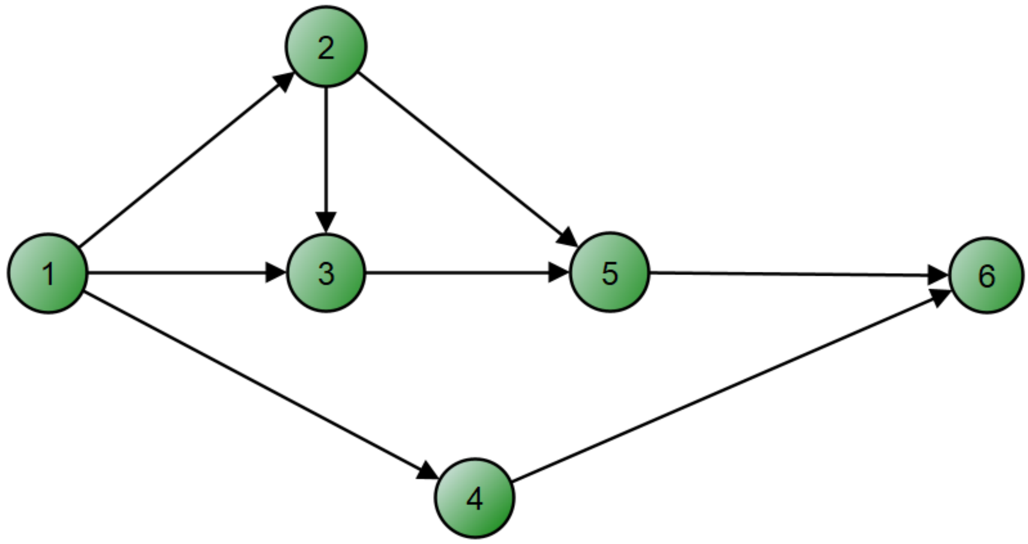

Figure 1.3: Directed Graph.

Definition 1.5.8. [46]. A graph G is undirected if and only if every edge in G is an undirected edge, see Fig. 1.4.

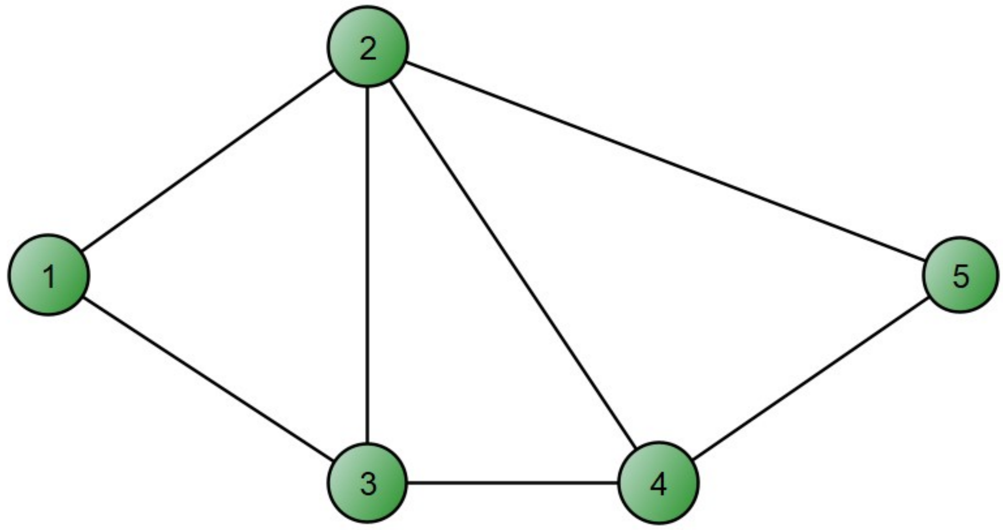

Figure 1.4: Undirected Graph.

Definition 1.5.9. [30]. **A simple graph** is one that does not contain multiple edges and loops, see Fig 1.5.

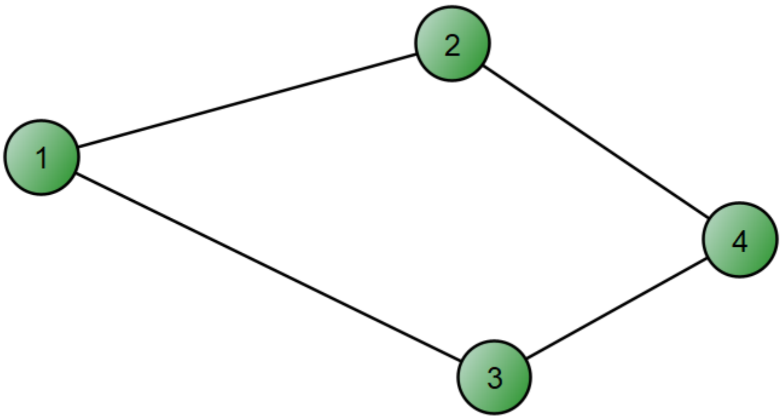

Figure 1.5: Simple Graph.

Definition 1.5.10. [23]. **Multigraph** is a graph with the potential for more than one edge between any pair of vertices, see Fig. 1.6.

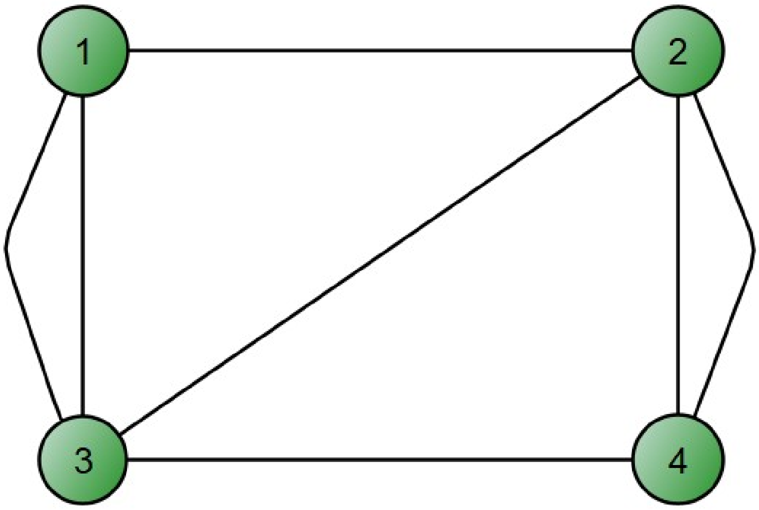

Figure 1.6: Multigraph.

Definition 1.5.11. [31]. **A Mixed graph** is a graph in which some of its edges are directed and others are not. It is introduced by (V, E, D). V denotes the set of nodes, E denotes the set of undirected edges, and D denotes the set of directed edges,see Fig 1.7.

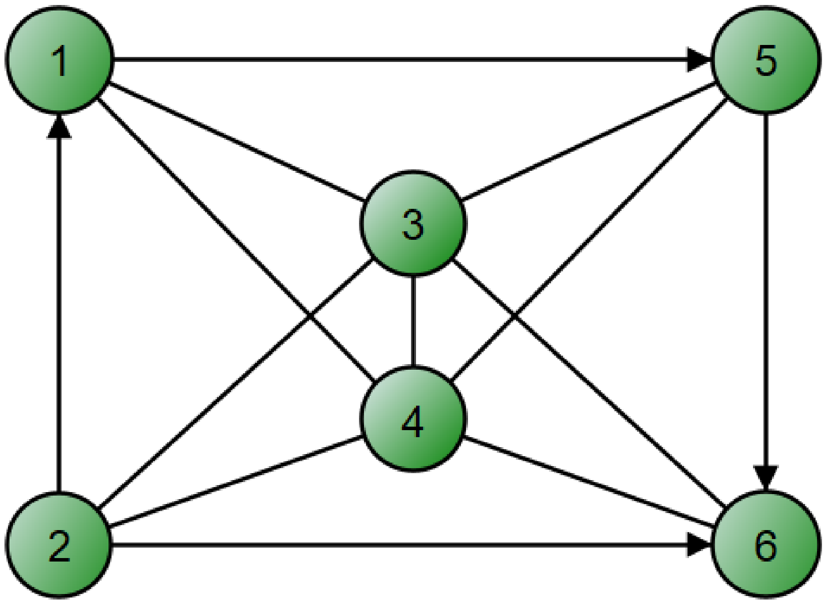

Figure 1.7: Mixed Graph.

Definition 1.5.12. [70]. **A complete graph** is one in which each and every vertex pair is linked to the other pair. The complete graph may also be put into two groups: those with self-loops and those without, see Figs. 1.8 and 1.9.

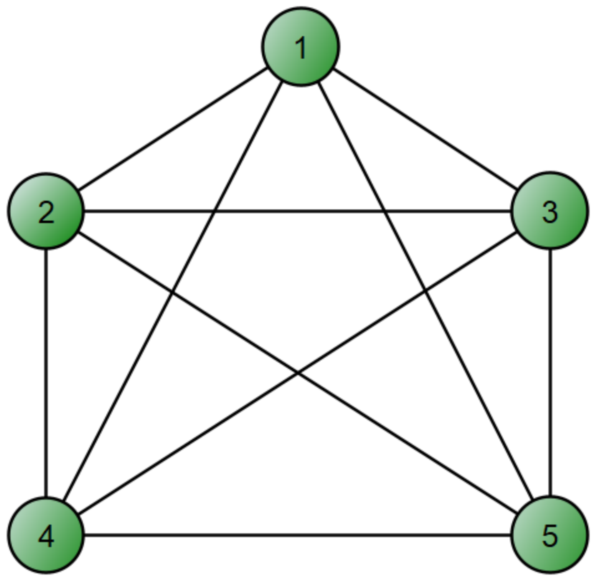

Figure 1.8: Complete Graph.

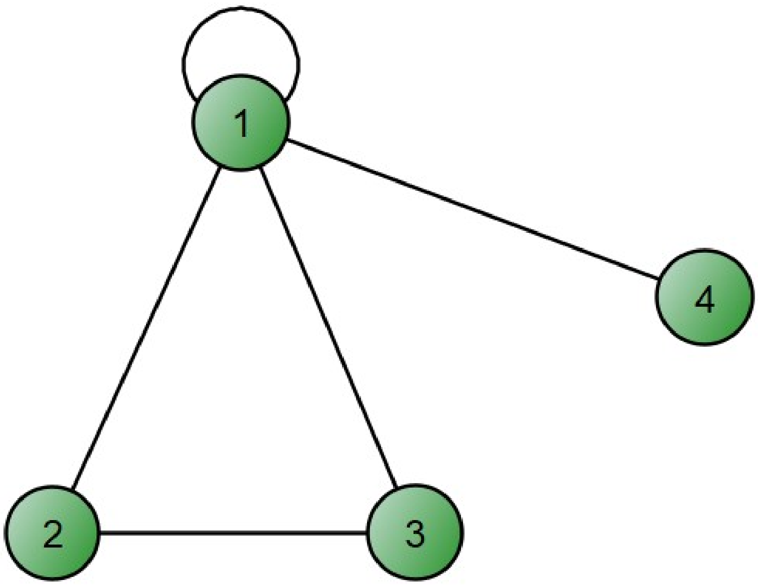

Figure 1.9: Undirected, Self-looping Graph.

Definition 1.5.13. [42]. If a graph has a finite number of edges and vertices, then it is said to be finite.

Definition 1.5.14. [55]. The number of edges of a given graph is indicated by the size of the graph, indicated by E.

Definition 1.5.15. [20]. When two vertices in a graph are connected, they are known as adjacent vertices because there is an edge between them.

Definition 1.5.16. [73]. The adjacency matrix (also called the connection matrix) of a simple graph is a two-dimensional array where each row and column represent a vertex in the graph and a 1 or 0 is placed in the corresponding cell (u, v) based on whether or not the two vertices are adjacent., see Fig. 1.10.

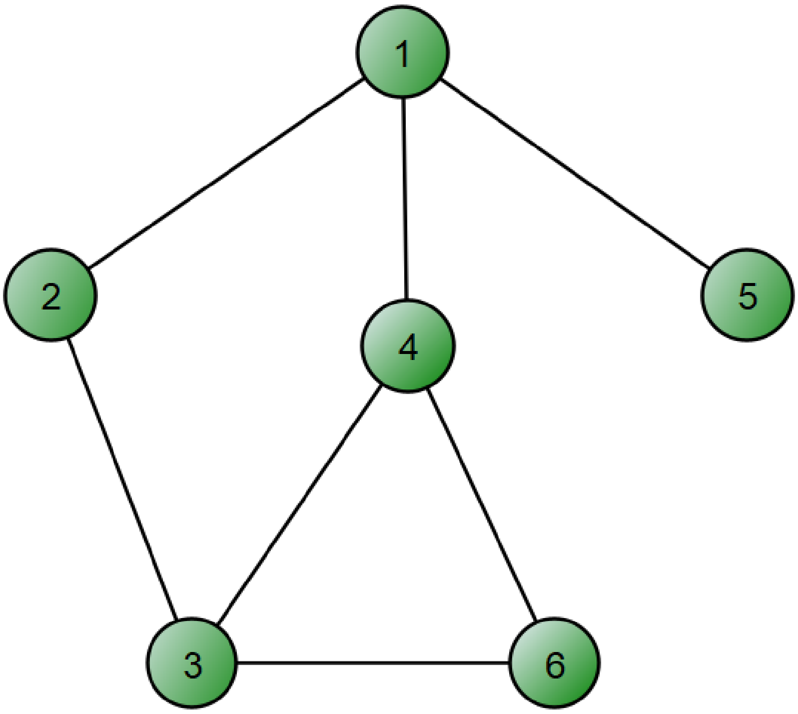

Figure 1.10: A Simple Graph's.

1.6. Fundamentals of Reliability Polynomial

This section includes several initial concepts of reliability polynomials. Of which Polynomial reliability, reliability block diagrams, MPS, and MCS.

Definition 1.6.1. [4]. **The Reliability function** is the probability of successful performance under given conditions of use and time. symbolizes it as R(t), which is also called the survival function. where T is a random variable, which is the time until the system fails:

$$R(t) = Pr\{T > t\}$$

Definition 1.6.2. [15]. **Reliability block diagram RBD** is a graph whose edges are components of the system, and there are a pair of peripheral nodes in the power supply diagram. Connecting them is a path that contains only functional edges, showing the functional relationship between the components. The entire system is functional. Otherwise, it does not work, see Fig 1.11.

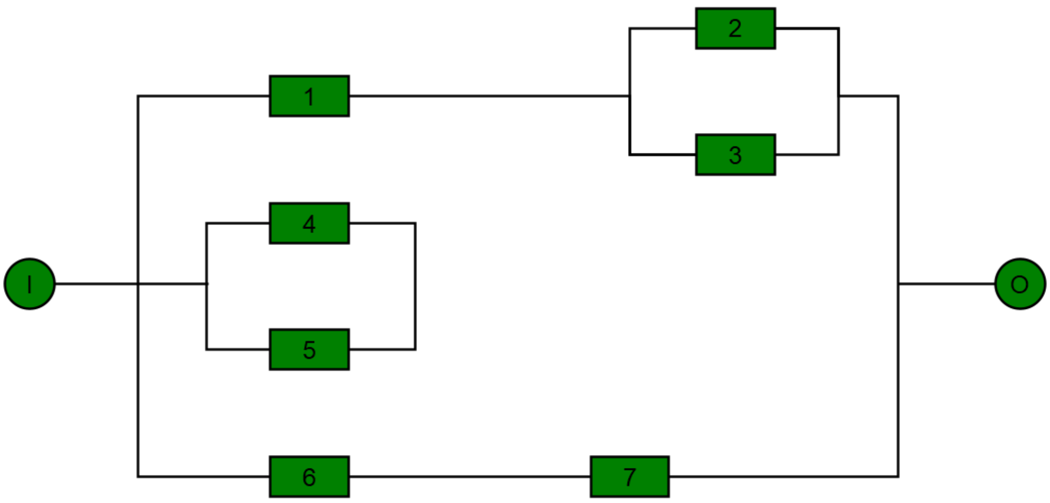

Figure 1.11: Reliability Block Diagram.

Definition 1.6.3. [28]. **A complex network** is a network made up of interconnected or inter- woven pieces (components) that makes it difficult to assess its reliability or solve a specific problem due to the limitations imposed by present techniques, algorithms, and software (such as programming languages and operating networks) , see Fig. 1.12.

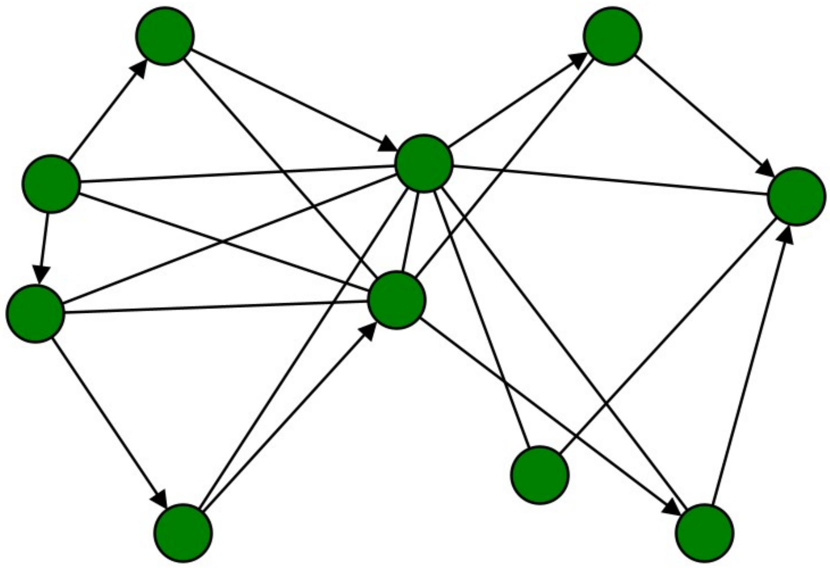

Figure 1.12: Complex Network.

Definition 1.6.4. [63]. **A Path** set is a set of components that, when working together, ensure that the system is working.

Definition 1.6.5. [34]. Path set is called minimum path set **MPS** if they can't have diminished without losing their Path set status.

Definition 1.6.6. [60]. **A Cut** is a collection of components such that the failure of any one of them causes the system to fail.

Definition 1.6.7. [57]. A minimum cut set**MCS** is the set of pieces that cannot be reduced to a minimum lowered without losing its status as a cutting group.

1.7. Structure Function

Let's say we have r component in a system S $e_i = 1, 2, \ldots, r$.

First, we pair a state variable X_i with every component e_i in such a way that

$$X_i = \begin{cases} 1 & \text{If the condition of the component } e_i\text{, is work} \\ 0 & \text{If component } e_i \text{ fails} \end{cases}$$

If $e = \{e_1, e_2, \ldots, e_r\}$ is the set of components, $(X_1, X_2, \cdots, X_r)$ will also be referred to as the "state of the set of components" and represented by the symbol (X) [42].

$$(X) = (X_1, X_2 \ldots \ldots, X_r) .$$

Let Y represent the system's state variable so that

$$Y = \begin{cases} 1 & \text{if the system is functioning properly,} \\ 0 & \text{if the system fails.} \end{cases}$$

it is clear, Y depends on (X), This function will be referred to as a "structure function" within the system.

1.7.1 Series Structure

Let

$$\begin{aligned} Y &= \varphi (X_1, X_2 \ldots, X_r) \\ &= X_1 \cdot X_2 \cdot \cdots \cdots \cdot X_r \\ &= \prod_{i=1}^{r} X_i \end{aligned}$$

This structure function is just compatible with systems that run beneath. The condition is that all of its components are in good working order [70].

$$(\forall_i, X_i = 1) \Rightarrow (Y = 1),$$

$$(\exists_i : X_i = 0) \Rightarrow (Y = 0).$$

This type of construction is known as a (series structure), and the components are said to be in series [57].

1.7.2 Parallel Structure

Let

$$\begin{aligned} Y &= \varphi\left(X_1, X_2 \ldots, X_{\mathbf{r}}\right) \\ &= 1 - (1 - X_1) \cdot (1 - X_2) \cdots (1 - X_r) \\ &= 1 - \prod_{i=1}^{r} \left(1 - X_i\right). \end{aligned}$$

This structure function is only compatible with systems that run beneath. A situation exists when at least one of its components is in excellent working order:

$$(\exists_i : Xi = 1) \Rightarrow (Y = 1),$$

$$(\forall_i, Xi = 0) \Rightarrow (Y = 0).$$

Such a structure is referred to be a (parallel structure), and the components are said to be parallel. [12].

1.8. Some Concepts About Reliability of Systems

A System refers to a collection of components that execute a specific function; how a system is decomposed into components depends on a number of variables [45].

Practically speaking, a component is typically an element that is readily available on the market and may be substituted affordably if it fails. For instance, when a computer system fails due to a malfunctioning computer module, seldom replaces the complete computer unit. Instead, efforts are made to identify the component within the computer device that caused the failure. For instance, they may replace the hard drive to restore the computer's functionality. Hierarchical analytics are frequently used to assess system reliability of complex systems. Consequently, and because the performance for a system is dependent on the performance of its components, the significance of constituents can vary. in reliability analysis of systems [28].

1.8.1 System

A group of compounds arranged in a precise sequence that interacts with one other and with external components or other structures in order to achieve a certain function [47].

1.8.2 Subsystem

A combination of several groups that perform a function within a system and are one of the main subdivisions of the system [47].

1.8.3 Series System

Reliability-wise, a series system (Fig. 1.13) is one in which the entire system fails if even a single component does. If a motorcycle's engine, petrol tank, chain, frame, front wheel, rear wheel, or anything else fails, the machine won't go anywhere. Therefore, these components form a sequential list. Screws and other fasteners are also examples of elements. The dependability of a system is equal to the sum of the reliabilities of its parts if the failure of any part does not affect the operation of any other part [66].

$$R_s = R_1 \times R_2 \times R_3 \times \cdots \times R_n = \prod_{j=1}^{n} R_j \tag{1.1}$$

The practical result is that "the dependability of a series system is always less than that of any of its components"

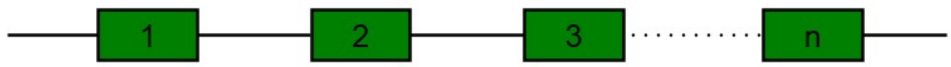

Figure 1.13: Series System.

1.8.4 Parallel System

To ensure the success of a parallel system, at least one of its components must be successful. For a system with n statistically independent, parallel parts, the probability of failure or instability is the chance that part 1 fails, component 2 fails, and all other components in the system fail. In other words, the system is successful if either component 1 or component 2 or any of the n components succeed. In a parallel system, the best part has the biggest effect on how well the whole thing works, because the component with the highest level of security is the least likely to malfunction. As the number of parts in a parallel setup goes up, the stability of the system goes up [58]. See Fig. 1.14.

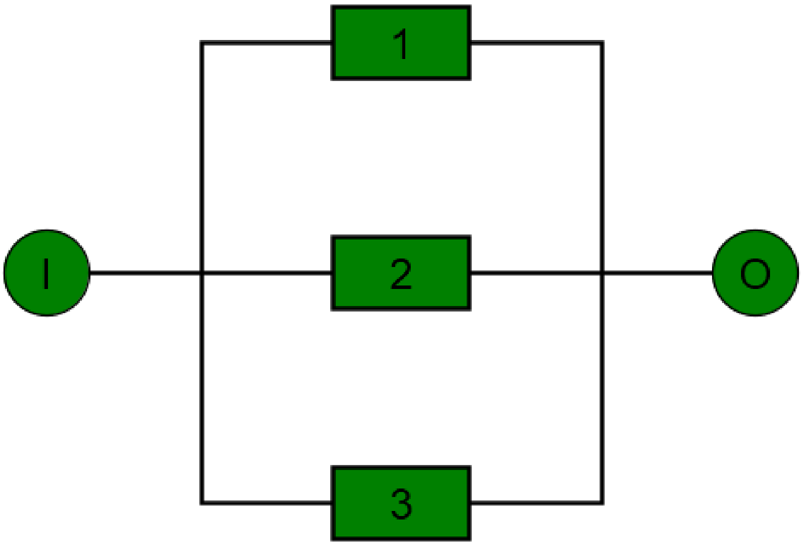

Figure 1.14: Parallel System.

$$R_S = 1 - \prod_{i=1}^{n} (1 - R_i) \, . \tag{1.2}$$

1.8.5 Series-Parallel System

Is made up of m discontinuous modules connected in series, whereas module i for $1 \leq i \leq m$ is made up of ni components connected in parallel [77]. This system's reliability is

$$R_S = R_1 \cdot R_2 \ldots . Rn = \prod_{i=1}^{n} R_i$$

See Fig. 1.15.

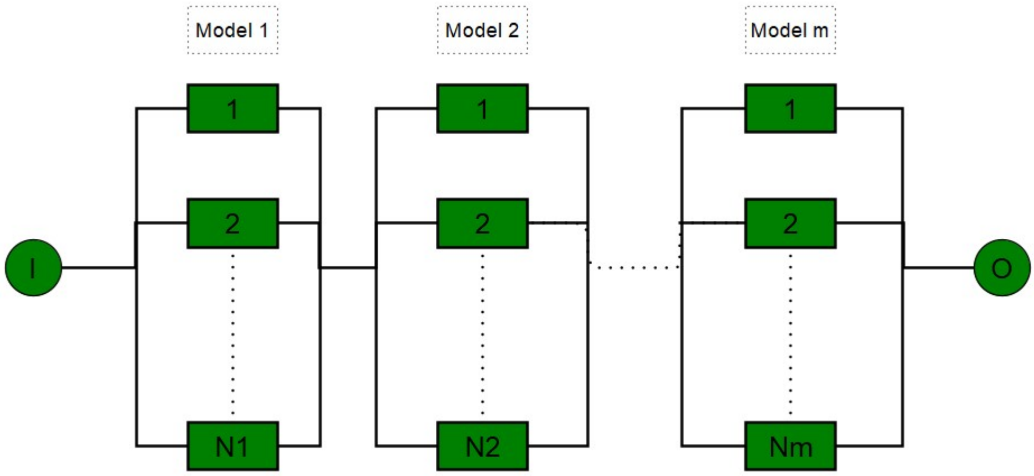

Figure 1.15: Series-Parallel System.

This diagram represents in Fig. 1.15 a series-parallel system where Ri is the single component reliability, if place in sets in parallel, where each has m component in series [37].Thus:

$$R_S = \prod_{i=1}^{m} \left(1 - \prod_{i=1}^{n} (1 - R_i) \right) . \tag{1.3}$$

1.8.6 Parallel-Series System

Is made up of m discontinuous modules linked in parallel [27]. And module i for $1 \leq i \leq m$ is made up of ni components connected in series

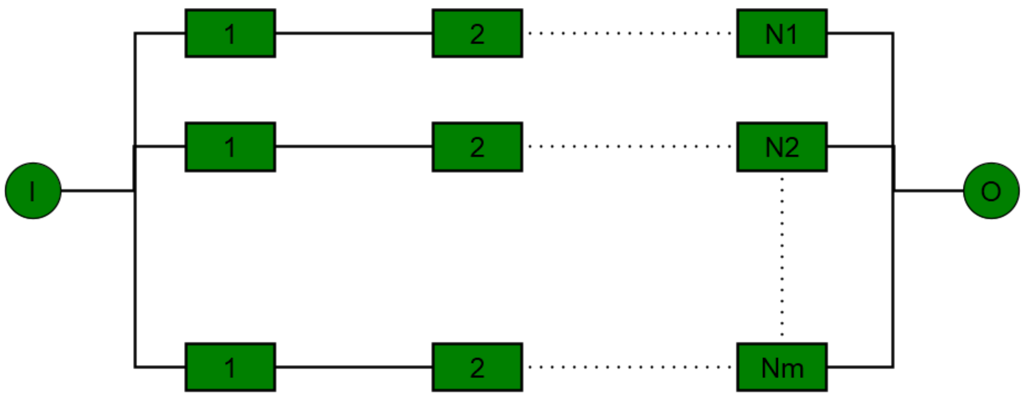

Figure 1.16: Parallel -Series System.

Where module 1 is $(1, 2 \ldots N_1)$,

Module 2 is $(1, 2 \ldots N_2)$,

Module m is $(1, 2 \ldots N_m)$.

This diagram in Fig. 1.16 represents a series-parallel system where Ri is a single component's reliability, if m such sets are connected in series, where each set consists of n parallel components [26]. Then the system's reliability is determined,

$$R_S = 1 - \prod_{i=1}^{m} \left(1 - \prod_{i=1}^{n} R_i\right) \tag{1.4}$$

1.8.7 Complex systems

A complicated system is one that combines series, parallel, R out of N, and standby components, according to the reliability definition. In order to decompose the original system (or subsystem) into an analogous one with a known cumulative distribution function (CDF) or reliability function, relevant mathematical formulations for reliability computations are provided for each of these models. By continuing the decomposition process, the decision-maker can eliminate all but one component from the system, each of which has a known CDF [16, 65]. See (Fig 1.17). There are several methods exist for obtaining the reliability of a complex system:

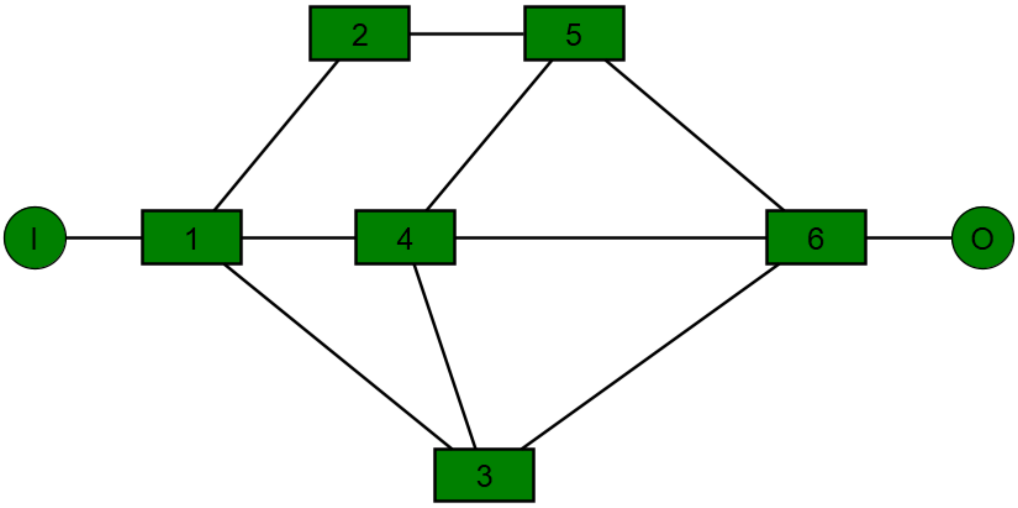

Figure 1.17: Complex Systems.

1.9. Reliability Redundancy

Redundancy is commonly used in systems design to improve system reliability, especially when it is difficult to increase the reliability of the component itself. [37].

1.10. Reliability Importance Measures

The reliability importance, I of component i in a system of n components is given by:

$$Ii = \frac{\partial Rs}{\partial Ri} \tag{1.5}$$

R_S, is the system reliability, and R_i, is the component reliability. The value of the reliability importance given by this equation depends both on the reliability of a component and its corresponding position in the system. [12].

CHAPTER 2

RELIABILITY OF COMPLEX – SERIES NETWORKS

2.1. Introduction

In this chapter, will discuss a new system, which is the complex-series network. This chapter is divided into two parts. The first part talks about the concepts of the minimal path set(MPS) and minimal cut set (MCS) and how they are calculated for complex-series network. As for the second part, it contains the two methods are path tracing method (PTM) and minimal cut method (MCM) [64, 65].

2.2. Complex - Series network

A complex - series network (S) is a structure resulting from a group of complex systems linked together in the form of a series system, see Fig 2.1

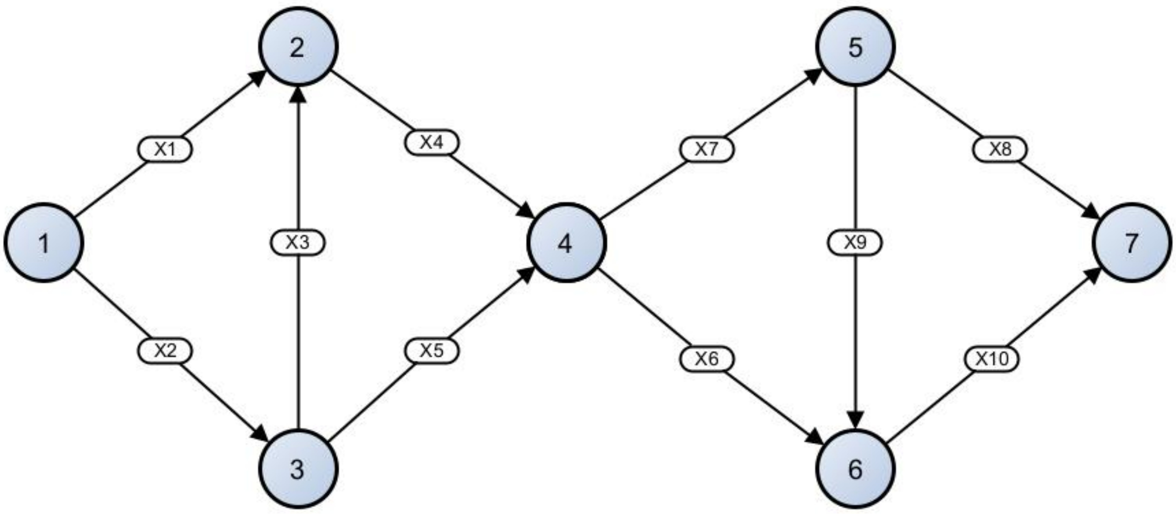

Figure 2.1: Complex- Series Network.

2.3. Extract the MPS of S

By adding an adjacency matrix of size $(n \times n)$ to the identity matrix (unit matrix) of size n, a connection matrix (Cm) is constructed in order to find the MPS to get 1 in the main diagonal [18].

$$Cm = \begin{pmatrix} 0 & a_{12} & \cdots & a_{1n} \\ a_{21} & 0 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

It produces the contact matrix as shown below:

$$Cm = \begin{pmatrix} 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{pmatrix} \tag{2.1}$$

where the collection of vertices is $1,\ 2,\ ...\ ,\ n$, and the edge between two vertices i and j is indicated by $a_{ij} = (ij)$. If vertex i and vertex j are connected, then $a_{ij} = x_{ij}$, otherwise, $a_{ij} = 0$. A system's MPS vector can be produced by deleting source or sink vertices one at a time from the (C_m) matrix until only the source vertices and sink vertices are left in the matrix in order to solve the MPS problem, as a particular example involving two terminal systems shows [72]. The following equation is used to change the connection matrix inputs for the remaining vertices when a vertex is removed:

$$a_{ij} = a_{ij} + (a_{il})(a_{lj}) \tag{2.2}$$

if vertex l is removed, where $i \neq j,\ i \neq l, j \neq l$and $1 \leq i \leq n,\ 1 < j \leq n$ for $i = 1,\ 2,\ \ldots\ ,\ n$. Otherwise $a_{ij} = 1$ iff $i = j$. While employing this technique, the source vertex should be the first, and the sink vertex should be the final [59]. After eliminating each intermediate node individually, a 2×2 matrix with an a_{12} representing the sum of all MPS is left. Thus, removing node i also implies removing row i and column j from the initial connection matrix, where $2 \leq i \leq n-1$ for a network with n perfect nodes. For the complex-series network in Fig.2.1 it contains vertices 1,2,3,4,5,6 and 7 and the edges are numbered from X_1 to X_{10} and the connection matrix (C_m) is 7×7

$$Cm = \begin{pmatrix} 1 & x_1 & x_2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & x_4 & 0 & 0 & 0 \\ 0 & x_3 & 1 & x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_7 & x_6 & 0 \\ 0 & 0 & 0 & 0 & 1 & x_9 & x_8 \\ 0 & 0 & 0 & 0 & 0 & 1 & x_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Now, we remove node 2. The inputs to the original matrix according to the following equation become as follows:

$a^1_{13} = x_2, a^1_{14} = x_1x_4, a^1_{15} = 0, a^1_{16} = 0,$

$a^1_{17} = 0, a^1_{31} = 0, a^1_{34} = x_5 + x_3x_4, a^1_{35} = 0,$

$a^1_{36} = 0, a^1_{37} = 0, a^1_{41} = 0, a^1_{43} = 0, a^1_{45} = x_7,$

$a^1_{46} = x_6, a^147 = 0, a^1_{56} = x_9, a^1_{57} = x_8, a^1_{67} = x_{10}.$

The modified connectivity matrix becomes matrix 6×6,

$$Cm^1 = \begin{pmatrix} 1 & x_2 & x_1x_4 & 0 & 0 & 0 \\ 0 & 1 & x_5 + x_3x_4 & 0 & 0 & 0 \\ 0 & 0 & 1 & x_7 & x_6 & 0 \\ 0 & 0 & 0 & 1 & x_9 & x_8 \\ 0 & 0 & 0 & 0 & 1 & x_{10} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Remove node 2. The modified input of the original matrix becomes as follows according to the following equation:

$a^2_{13} = x_1x_4 + x_2x_5 + x_2x_3x_4,\ a^2_{14} = 0,\ a^2_{15} = 0,\ a^2_{16} = 0,\ a^2_{31} = 0,\ a^2_{34} = x_7,$

$a^2_{35} = x_6,\ a^2_{36} = 0, a^2_{41} = 0,\ a^2_{43} = 0,\ a^2_{45} = x_9,\ a^2_{46} = x_8,$

$a^2_{51} = 0,\ a^2_{53} = 0,\ a^2_{54} = 0\ a^2_{56} = x_{10}.$

The communication matrix becomes matrix 5×5,

$$C^2_m = \begin{pmatrix} 1 & x_1x_4 + x_2x_5 + x_2x_3x_4 & 0 & 0 & 0 \\ 0 & 1 & x_7 & x_6 & 0 \\ 0 & 0 & 1 & x_9 & x_8 \\ 0 & 0 & 0 & 1 & x_{10} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Remove node 2. The modified input of the original matrix becomes as follows according to the following equation:

$a^3_{13} = x_1x_4x_7 + x_2x_5x_7 + x_2x_3x_4x_7,\ a^3_{14} = x_1x_4x_6 + x_2x_5x_6 + x_2x_3x_4x_6,$

$a^3_{15} = 0,\ a^3_{31} = 0,\ a^3_{34} = x_9,\ a^3_{35} = x_8,\ a^3_{41} = 0,\ a^3_{43} = 0, a^3_{45} = x_{10}$

The communication matrix becomes matrix 4×4,

$$C_m^3 = \begin{pmatrix} 1 & x_1x_4x_7 + x_2x_5x_7 + x_2x_3x_4x_7 & x_1x_4x_6 + x_2x_5x_6 + x_2x_3x_4x_6 & 0 \\ 0 & 1 & x_9 & x_8 \\ 0 & 0 & 1 & x_{10} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Remove node 2. The modified input of the original matrix becomes as follows according to the following equation:

$a_{13}^4 = x_1x_4x_6 + x_2x_5x_6 + x_2x_3x_4x_6 + x_1x_4x_7x_9 + x_2x_5x_7x_9 + x_2x_3x_4x_7x_9,$

$a_{14}^4 = x_1x_4x_7x_8 + x_2x_5x_7x_8 + x_2x_3x_4x_7x_8,$

$a_{31}^4 = 0,\ a_{34}^4 = x_{10}$

The communication matrix becomes matrix 3×3,

$$C_m^4 = \begin{pmatrix} 1 & x_1x_4x_6 + x_2x_5x_6 + x_2x_3x_4x_6 + x_1x_4x_7x_9 + x_2x_5x_7x_9 + x_2x_3x_4x_7x_9 & x_1x_4x_7x_8 + x_2x_5x_7x_8 + x_2x_3x_4x_7x_8 \\ 0 & 1 & x_{10} \\ 0 & 0 & 1 \end{pmatrix}$$

Remove node 2. The modified input of the original matrix becomes as follows according to the following equation:

$a_{13}^5 = x_1x_4x_7x_8 + x_2x_5x_7x_8 + x_2x_3x_4x_7x_8 + x_1x_4x_6x_{10} + x_2x_5x_6x_{10} + x_2x_3x_4x_6x_{10} + x_1x_4x_7x_9x_{10} + x_2x_5x_7x_9x_{10} + x_2x_3x_4x_7x_9x_{10}$

The communication matrix becomes a degree matrix 2×2,

$$C_m^5 = \begin{pmatrix} 1 & x_1x_4x_7x_8 + x_2x_5x_7x_8 + x_2x_3x_4x_7x_8 + x_1x_4x_6x_{10} + x_2x_5x_6x_{10} + x_2x_3x_4x_6x_{10} + x_1x_4x_7x_9x_{10} + x_2x_5x_7x_9x_{10} + x_2x_3x_4x_7x_9x_{10} \\ 0 & 1 \end{pmatrix}$$

MPS sets of S is

$$MPS = \{\{x_1, x_4, x_7, x_8\}, \{x_1, x_4, x_6, x_{10}\}, \{x_1, x_4, x_7, x_9, x_{10}\}, \{x_2, x_5, x_7, x_8\}, \{x_2, x_3, x_4, x_7, x_8\}, \{x_2, x_5, x_6, x_{10}\}, \{x_2, x_3, x_4, x_6, x_{10}\}, \{x_2, x_5, x_7, x_9, x_{10}\} \{x_2, x_3, x_4, x_7, x_9, x_{10}\}\}$$

They represent all the MPS of S, it shown in Fig.2.2

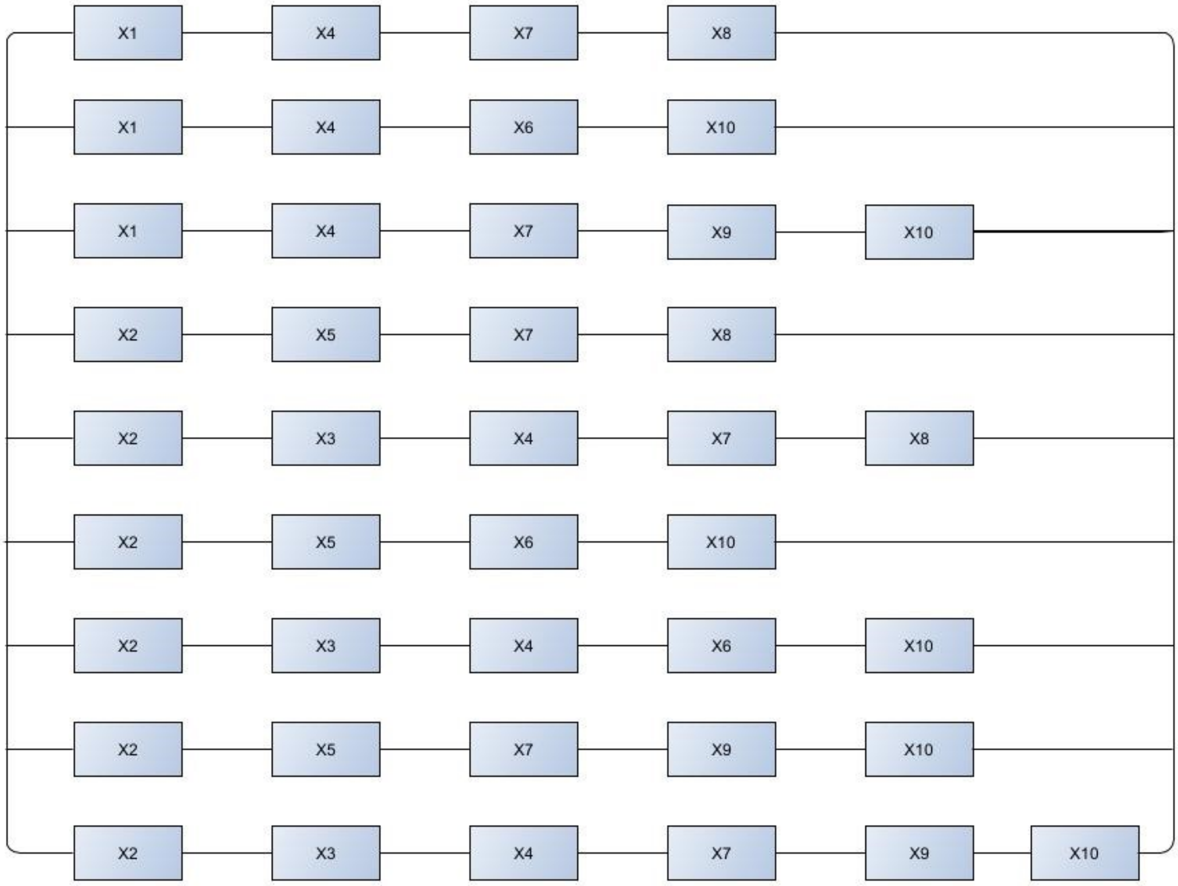

Figure 2.2: All MPS of S.

2.4. Extract the MCS of S

Using this method, we determine polynomial reliability using the following assumptions:

- The term "cut vector" is used to describe a state vector x if $(x) = 0$. Moreover, if $(y) = 1$ for every y greater than x, then x is regarded, as a MCS vector.

- A MCS set, where x is a MCS vector, is the set $C = I, \quad x_i = 0$.
- A MCS set is composed of components whose failure ensures the failure of the entire system.
- The MCS sets of the provided system are indicated by the letters $C_1, ..., C_k$.
- The indicator function for the jth MCS set, $\beta j(x)$, is defined as

$$\beta_j(x) = \begin{cases} 1, & \text{if the } jth \text{ MCS at least one component is operational,} \\ 0 & \text{if none of the } jth \text{ MCS components are working.} \end{cases}$$

There are four procedures that must be followed in order to generate [66]. MCS sets:
Step1: To determine the MCS sets, create (IM).
Step2: Any column in IM that has an $aij \neq 0$ generates a first degree cut.
Step3: By combining two IM columns at once, if $\vee i; a_{ij} + a_{ik} \neq 0$ where $k > j(k = 1, ..., n)$, then $x_i x_k$ constitutes a second degree cut.
Step4: Repeat step (2) with three columns at a time, deleting MCS that involve first- and second-degree MCS [47]. Up until the maximum order of MCS is reached. Now, we begin to implement the above-mentioned steps Fig.2.1, which contains the set of MPS for the complex - series network. The (IM) of this complex - series network is a 9×10 matrix in the following:

$$IM = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

First order MCS are not available if there isn't a single column in the IM were all of the elements are non-zero.

Then add two columns of IM at a time, we get two MCS of order two which are:

$$MCS = \{\{x_1, x_2\}, \{x_2, x_4\}, \{x_3, x_4, x_5\}, \{x_6, x_7\}, \{x_7, x_{10}\}, \{x_8, x_9, x_{10}\}\}$$

Note, when adding three columns, four columns, and five columns, it also produces MCS, but these cuts, if they are partial from the large MCS, these cuts are deleted, leaving only the original cuts that lead to the failure of the system.

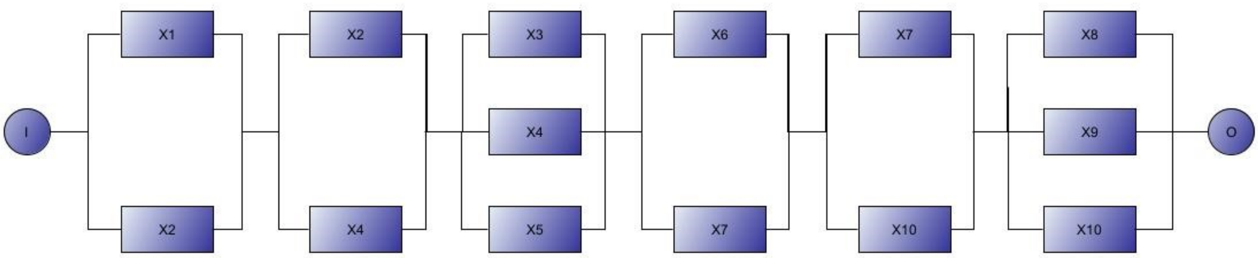

Figure 2.3: All MCS of S.

2.5. Reliability Calculation Methods

There are different methods for calculating the reliability of systems. In this part, path trucing and minimum cut to calculate the complex-series network.

2.5.1 Path tracing method (PTM)

In this method, the following procedures are used to determine a complex system's reliability:

1. List the system's MPS (tie-set).
2. The success of all segments in MPS ways determines the system's success (tie-set).
3. The series' elements are connected, as evidenced by this.
4. The effectiveness of the system is produced by each set of minimal paths.
5. It indicates a parallel connection between the minimal tie sets.
6. Use a parallel and series reduction to draw an equal system and determine the system's dependability [32]. The system's dependability is determined by the likelihood of merging these shortest MPS. The union of MPS can be calculated using the generic equation shown below. P_i:

$$\begin{aligned} p_r\left(p_1+p_2+\cdots+p_n\right) =& p_r(\sum_{i=1}^{n} pi) \\ &= \sum_{i=1}^{n} p_r(p_i) - \sum_{i<j=2}^{n} p_r(p_i.p_j) + \sum_{i<j<k=3}^{n} p_r(p_i.p_j p_k) \\ &= - \cdots + (-1^{n+1}) p_r(p_1.p_2 \cdots p_n) \end{aligned} \tag{2.3}$$

The reliability that at least one MPS will work for n independent MPS [64]. As a result, we end up with.

$$p_r\left(\sum_{i=1}^{n} pi\right) = 1 - \left[1 - p_r\left(p_1\right)\right] \times \left[1 - p_r\left(p_2\right)\right] \times \cdots \times \left[1 - p_r\left(p_n\right)\right] \tag{2.4}$$

$$MPS = \{\{x_1, x_4, x_7, x_8\}, \{x_1, x_4, x_6, x_{10}\}, \{x_1, x_4, x_7, x_9, x_{10}\}, \{x_2, x_5, x_7, x_8\}, \{x_2, x_3, x_4, x_7, x_8\}, \{x_2, x_5, x_6, x_{10}\}, \{x_2, x_3, x_4, x_6, x_{10}\}, \{x_2, x_5, x_7, x_9, x_{10}\} \{x_2, x_3, x_4, x_7, x_9, x_{10}\}\}$$

Then the structure function are:

$$\begin{aligned} Rs =& 1 - [[1 - (x_1x_4x_7x_8)][1 - (x_1x_4x_6x_{10})][1 - (x_1x_4x_7x_9x_{10})][1 - (x_2x_5x_7x_8)] \\ & [1 - (x_2x_3x_4x_7x_8)][1 - (x_2x_5x_6x_{10})][1 - (x_2x_3x_4x_6x_{10})][1 - (x_2x_5x_7x_9x_{10})] \\ & [1 - (x_2x_3x_4x_7x_9x_{10})]] \end{aligned}$$

$$\begin{aligned} Rs =& R_1R_{10}R_4R_6 + R_{10}R_2R_5R_6 + R_1R_4R_7R_8 + R_2R_5R_7R_8 + R_{10}R_2R_3R_4R_6 \\ & + R_1R_{10}R_4R_7R_9 + R_2R_3R_4R_7R_8 + R_{10}R_2R_5R_7R_9 - R_1R_{10}R_2R_3R_4R_6 \\ & - R_1R_{10}R_2R_4R_5R_6 - R_{10}R_2R_3R_4R_5R_6 - R_1R_2R_3R_4R_7R_8 + R_{10}R_2R_3R_4R_7R_9 \\ & - R_1R_2R_4R_5R_7R_8 - R_1R_{10}R_4R_6R_7R_8 - R_1R_{10}R_4R_6R_7R_9 - R_2R_3R_4R_5R_7R_8 \\ & - R_{10}R_2R_5R_6R_7R_8 - R_1R_{10}R_4R_7R_8R_9 - R_{10}R_2R_5R_6R_7R_9 - R_{10}R_2R_5R_7R_8R_9 \\ & + R_1R_{10}R_2R_3R_4R_5R_6 - R_1R_{10}R_2R_3R_4R_7R_9 - R_1R_{10}R_2R_4R_5R_7R_9 \\ & + R_1R_2R_3R_4R_5R_7R_8 - R_{10}R_2R_3R_4R_5R_7R_9 - R_{10}R_2R_3R_4R_6R_7R_8 \\ & - R_{10}R_2R_3R_4R_6R_7R_9 - R_{10}R_2R_3R_4R_7R_8R_9 + R_1R_{10}R_4R_6R_7R_8R_9 \\ & + R_{10}R_2R_5R_6R_7R_8R_9 + R_1R_{10}R_2R_3R_4R_5R_7R_9 + R_1R_{10}R_2R_3R_4R_6R_7R_8 \\ & + R_1R_{10}R_2R_3R_4R_6R_7R_9 + R_1R_{10}R_2R_4R_5R_6R_7R_8 + R_1R_{10}R_2R_3R_4R_7R_8R_9 \\ & + R_1R_{10}R_2R_4R_5R_6R_7R_9 + R_{10}R_2R_3R_4R_5R_6R_7R_8 + R_1R_{10}R_2R_4R_5R_7R_8R_9 \\ & + R_{10}R_2R_3R_4R_5R_6R_7R_9 + R_{10}R_2R_3R_4R_5R_7R_8R_9 + R_{10}R_2R_3R_4R_6R_7R_8R_9 \\ & - R_1R_{10}R_2R_3R_4R_5R_6R_7R_8 - R_1R_{10}R_2R_3R_4R_5R_6R_7R_9 - R_1R_{10}R_2R_3R_4R_5R_7R_8R_9 \\ & - R_1R_{10}R_2R_3R_4R_6R_7R_8R_9 - R_1R_{10}R_2R_4R_5R_6R_7R_8R_9 - R_{10}R_2R_3R_4R_5R_6R_7R_8R_9 \\ & + R_1R_{10}R_2R_3R_4R_5R_6R_7R_8R_9 \end{aligned} \tag{2.5}$$

2.5.2 Minimal cut method (MCM)

In this method the following presumptions guide our use of this technique to determine polynomial dependability [31].

1. Compile a list of all the MCS for the system.
2. With an MCS, the failure of every component leads to system failure. This implies that there are parallel connections between these components.
3. As every MCS causes the system to fail, this demonstrates the connection between the MCS in the series.

$$R = 1 - p_r(c_1 + c_2 + ... + c_i). \tag{2.6}$$

where n represents the overall number of MCS and $C_i(i = 1, 2, ..., n)$ is the situation where all of the MCS components are in a failure state [29]. Eq. 2.6 in order to evaluate, employ the inclusion-exclusion principle, which is

$$p_r(c_1 + c_2 + ... + c_n) = \sum_{i=1}^{n} p_r(c_i) - \sum_{i<j=2}^{n} p_r(c_i.c_j) + \sum_{i<j<k=3}^{n} p_r(c_i.c_j.c_k) + ... + (-1)^{n+1} p_r(c_1.c_2....c_n).$$

The likelihood that at least one MCS will be successful for n independent MCS [62]. Thus, we obtain

$$p_r(\sum_{i=1}^{n} c_i) = 1 - [1 - p_r(c_1)] \times [1 - p_r(c_2)] \times ... \times [1 - p_r(c_n)] \tag{2.7}$$

All MCS of S are:

$MCS_s = \{\{x_1, x_2\}, \{x_2, x_4\}, \{x_3, x_4, x_5\}, \{x_6, x_7\}, \{x_7, x_{10}\}, \{x_8, x_9, x_{10}\}\}.$

Then the structure function are:

$R_s = [1 - (1 - x_1)(1 - x_2)][1 - (1 - x_2)(1 - x_4)][1 - (1 - x_3)(1 - x_4)(1 - x_5)][1 - (1 - x_6)(1 - x_7)][1 - (1 - x_7)(1 - x_{10})][1 - (1 - x_8)(1 - x_9)(1 - x_{10})].$

The result that has been obtained is the same in Eq. 2.6.

CHAPTER 3

PARTITION OF COMPLEX - SERIES NETWORK

3.1. Introduction

In this chapter, we will discuss the method of partitioning the network of complex-series, the application of MPS, MCS, and PTM methods, and the generalization of this method to more complex networks [44].

3.2. Partitioning a complex- series network

Start by partition the complex network in Fig 2.1 into two parts of subsystems, the first part is denoted by the symbol G subsystem, the second part is denoted by the symbol H subsystem, see Fig.3.1.

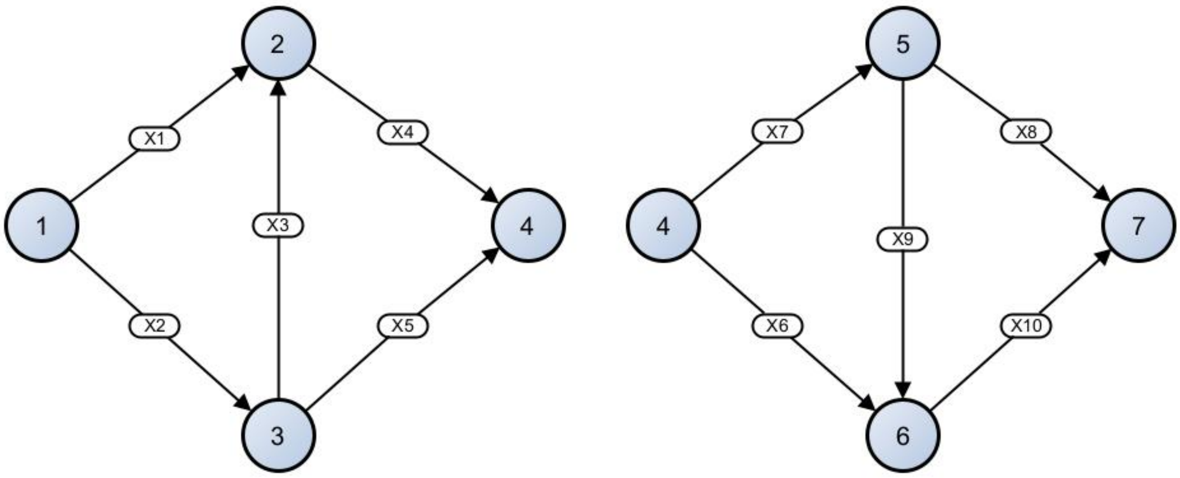

Figure 3.1: Subsystems of S.

3.3. Finding MPS and MCS of Subsystems

By applying the same previous steps in two sections (2.3) and (2.4) to calculate MPS and MCS [63] of subsystems G and H.

3.3.1 Calculating MPS of G

The first part of the subsystem is taken G and perform the steps of calculating the MPS and calculating the MCS, see Fig. 3.2.

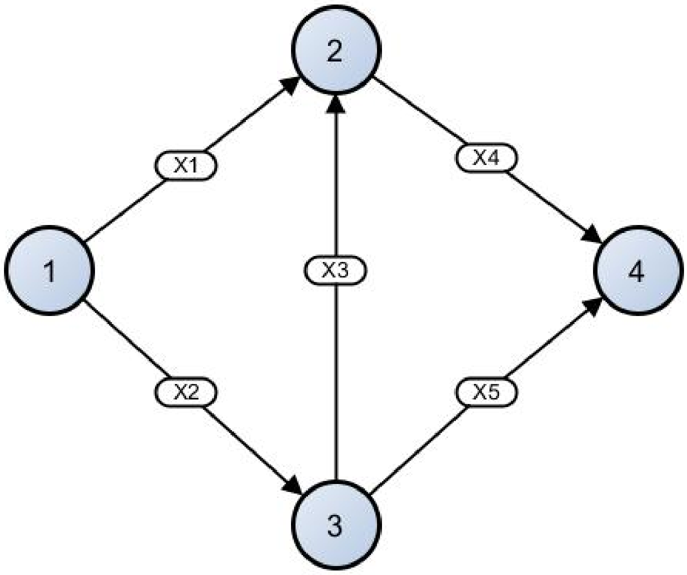

Figure 3.2: Subsystem G.

$$C_m = \begin{pmatrix} 1 & x_1 & x_2 & 0 \\ 0 & 1 & 0 & x_4 \\ 0 & x_3 & 1 & x_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now, remove node 2. The inputs to the original matrix according to the following equation become as follows:

$$a_{14} = x_1x_4, a_{34} = x_5 + x_3x_4$$

The communication matrix becomes a degree matrix 3×3,

$$C_m^1 = \begin{pmatrix} 1 & x_2 & x_1x_4 \\ 0 & 1 & x_5 + x_3x_4 \\ 0 & 0 & 1 \end{pmatrix}$$

Remove node 2 . The inputs to the original matrix according to the following equation become as follows:

$$a_{13} = x_1x_4 + x_2x_5 + x_2x_3x_4$$

The communication matrix becomes a degree matrix 2×2,

$$C_m^2 = \begin{pmatrix} 1 & x_1x_4 + x_2x_5 + x_2x_3x_4 \\ 0 & 1 \end{pmatrix}$$

The MPS of subsystem G is:

$$MPS_G = \{\{x_1, x_4\}, \{x_2, x_5\}, \{x_2, x_3, x_4\}\}$$

All MPS of the subsystem that represent the success of the system, it shown in Fig. 3.3.

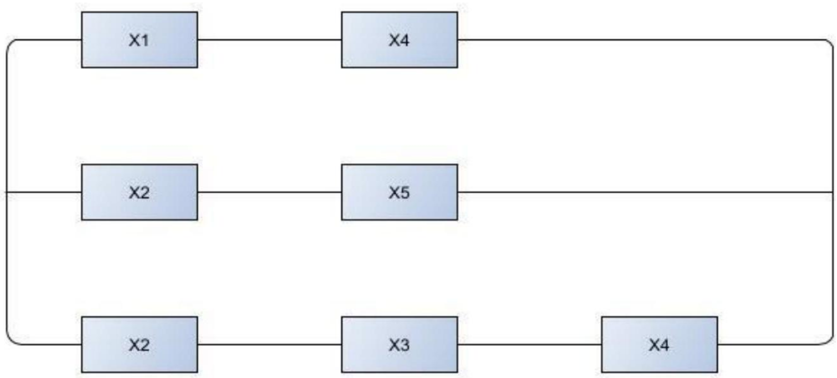

Figure 3.3: All MPS of G.

$$R_G = R_1R_4 + R_2R_5 + R_2R_3R_4 - R_1R_2R_3R_4 - R_1R_2R_4R_5 - R_2R_3R_4R_5 + R_1R_2R_3R_4R_5 \tag{3.1}$$

3.3.2 Calculating MCS of G

$$IM = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

All MCS of G are:

$$MCS_G = \{\{x_1, x_2\}, \{x_2, x_4\}, \{x_3, x_4, x_5\}\}$$

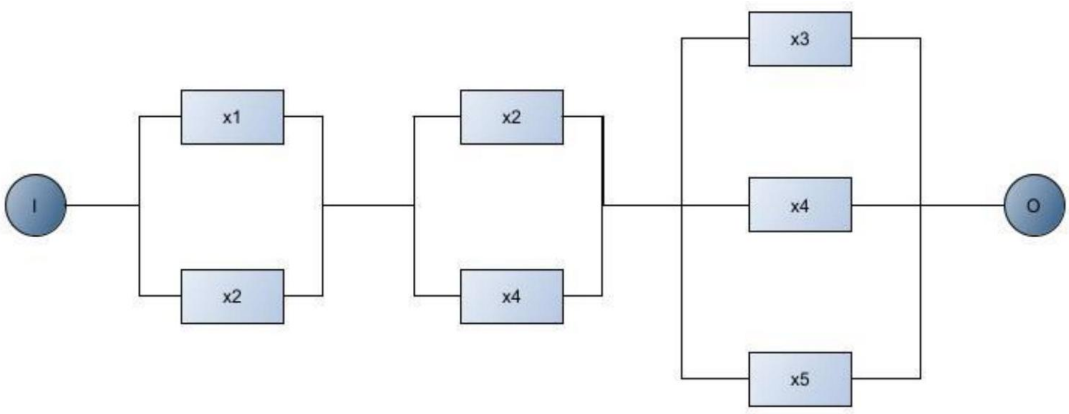

Figure 3.4: All MCS of G.

The result that has been obtained is the same in eq. 3.1

3.3.3 Calculating MPS of H

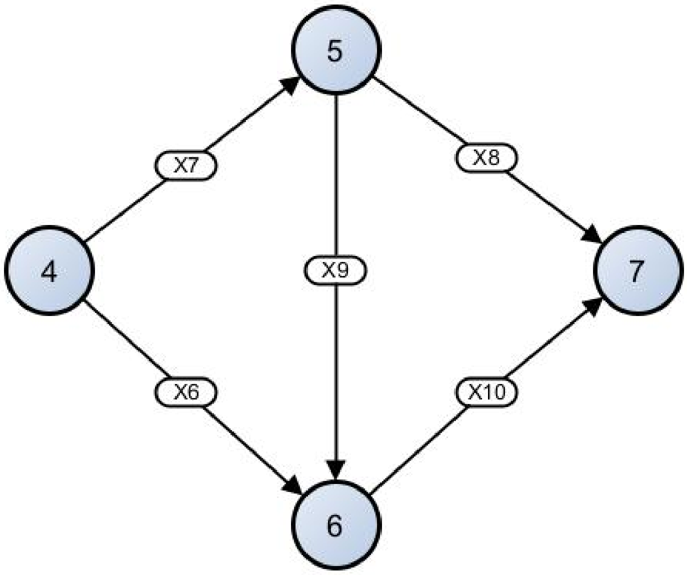

Figure 3.5: Subsystem H.

$$C_m = \begin{pmatrix} 1 & x_7 & x_6 & 0 \\ 0 & 1 & x_9 & x_8 \\ 0 & 0 & 1 & x_{10} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Remove node 2. The inputs to the original matrix according to the following equation become as follows:

$$a_{46} = x_6 + x_7x_9, a_{47} = x_7x_8,$$

$$a_{67} = x_{10}.$$

The communication matrix becomes a degree matrix 3×3,

$$C_m^1 = \begin{pmatrix} 1 & x_6 + x_7x_9 & x_7x_8 \\ 0 & 1 & x_{10} \\ 0 & 0 & 0 \end{pmatrix}$$

Remove node 2. The inputs to the original matrix according to the following equation become as follows:

$$a_{47} = x_7x_8 + x_6x_{10} + x_7x_9x_{10}$$

The communication matrix becomes a degree matrix 2×2,

$$C_m^2 = \begin{pmatrix} 1 & x_7x_8 + x_6x_{10} + x_7x_9x_{10} \\ 0 & 1 \end{pmatrix}$$

The MPS of subsystem H is:

$$MPS_H = \{\{x_7, x_8\}, \{x_6, x_{10}\}, \{x_7, x_9, x_{10}\}\}$$

All MPS of the subsystem that represent the success of the system, it shown in Fig. 3.6.

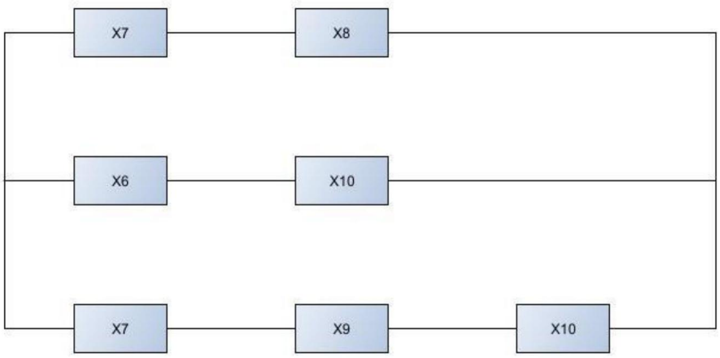

Figure 3.6: All MPS of H

$$R_H = R_{10}R_6 + R_7R_8 + R_{10}R_7R_9 - R_{10}R_6R_7R_8 - R_{10}R_6R_7R_9 - R_{10}R_7R_8R_9 + R_{10}R_6R_7R_8R_9 \quad (3.2)$$

3.3.4 Calculating MCS of H

$$IM = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

All MCS of H are:

$$MCS_H = \{\{x_6x_7\}, \{x_7, x_{10}\}, \{x_8, x_9, x_{10}\}\}$$

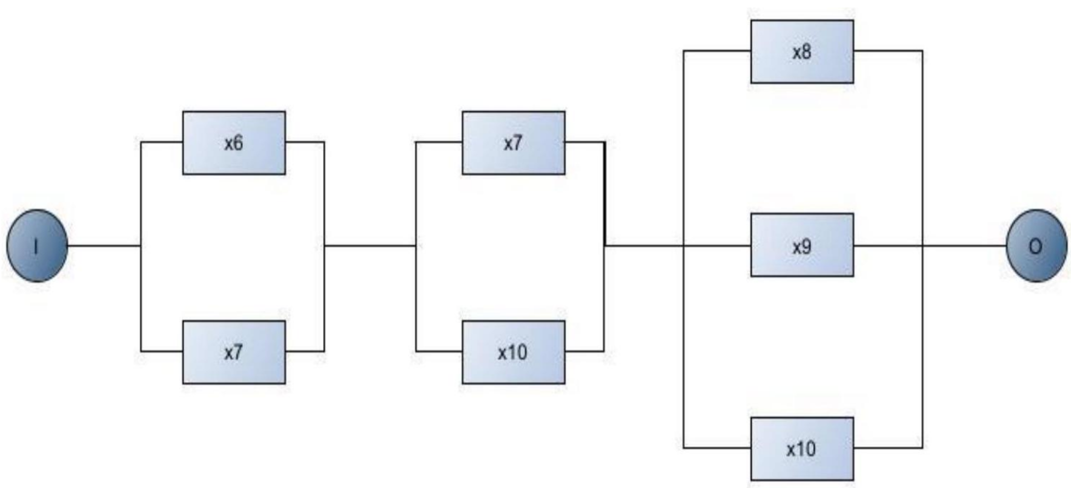

Figure 3.7: All MCS of H.

The result that has been obtained is the same in Eq.3.2

3.4. The most important results of MPS and MCS between G, H, and S

The result of our partitioning study is that we can obtain MPS for a complex-series network through subsystems. This is done by multiplying each of the MPS terms for the first subsystem by all the terms of the second subsystem.

$$\begin{aligned} MPS_G \times MPS_H =& [\{x_1, x_4\}, \{x_2, x_5\}, \{x_2, x_3, x_4\}] \times \\ & [\{x_7, x_8\}, \{x_6, x_{10}\}, \{x_7, x_9, x_{10}\}] \end{aligned} \tag{3.3}$$

We were also able to obtain the MCS for the complex-series network through subsystems by combining the MCS for the first subsystem with the MCS for the second subsystem.

$$MCS_G + MCS_H = \{\{x_1, x_2\}, \{x_2, x_4\}, \{x_3, x_4, x_5\}\} + \quad \{\{x_6 x_7\}, \{x_7, x_{10}\}, \{x_8, x_9, x_{10}\}\} \tag{3.4}$$

3.5. The most important results of G, H and S

Now, consider the partial polynomials R_G and R_H whose products are multiplied

$$\begin{aligned} R_G \times R_H =& [R_1R_4 + R_2R_5 + R_2R_3R_4 - R_1R_2R_3R_4 - R_1R_2R_4R_5 - R_2R_3R_4R_5 + R_1R_2R_3R_4R_5] \times \\ & [R_{10}R_6 + R_7R_8 + R_{10}R_7R_9 - R_{10}R_6R_7R_8 - R_{10}R_6R_7R_9 - R_{10}R_7R_8R_9 + R_{10}R_6R_7R_8R_9] \end{aligned} \tag{3.5}$$

That is the same result of Eq. 2.6.

A complex-series network was generated, and we applied MPS and MCS methods. This network consists of nine MPS, starting upstream and settling downstream. Then partition the network into two sub-systems, and the same methods are used in each sub-system. Each partial system consisted of three MPS, and extracted the reliability of each partial system. Then the polynomial of the subsystem G was multiplied by the polynomial of the subsystem H to conclude that the result is the reliability of the complex-series network. Also, the MCS output of the G subsystem is summed with the MCS output of the H subsystem to form the MCS of the complex-series network.

3.6. Generalization of calculating the total number of MPS

The number of MPS is the result of the product of the sub complex-series network:

$$\begin{aligned}
&\text{Let the number of MPS in a complex subsystem}(1) = N_{p1}\\
&\text{Let the number of MPS in a complex subsystem}(2) = N_{p2}\\
&\text{Let the number of MPS in a complex subsystem}(3) = N_{p3}\\
&\qquad\vdots\\
&\text{Let the number of MPS in a complex subsystem}(n) = N_{pn}
\end{aligned}$$

The total number of MPS in the complex-series network:

$$N_{path} = N_{p1} \times N_{p2} \times N_{p3} \times \cdots\cdots \times N_{pn} \tag{3.6}$$

N_{path}, It represents the total number of MPS in the system, N_{pi} It represents the number of MPS in the sub system of complex-series, i=1,. . . ,n. for example, see Fig 3.8.

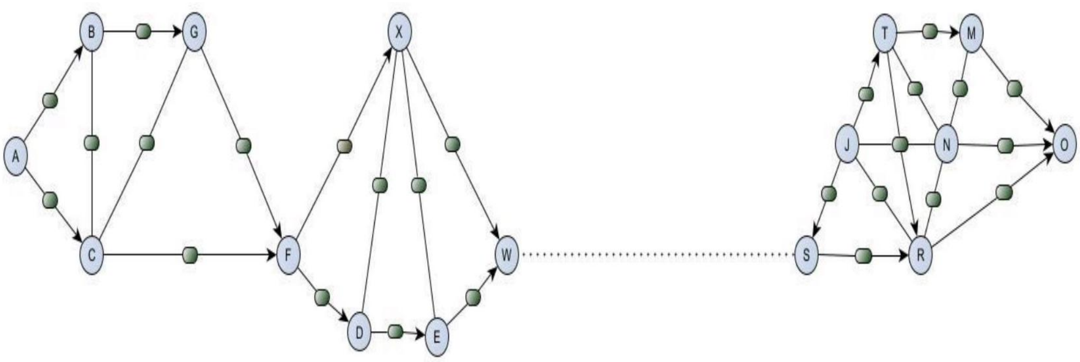

Figure 3.8: S with m subsystems.

When the sub systems of complex-series are symmetrical, the equation becomes:

$$N_{path} = N_p^m \tag{3.7}$$

N_p, It represents the number of MPS in one the sub system of complex-series, m It represents the number of sub systems of S. See Fig. 3.9

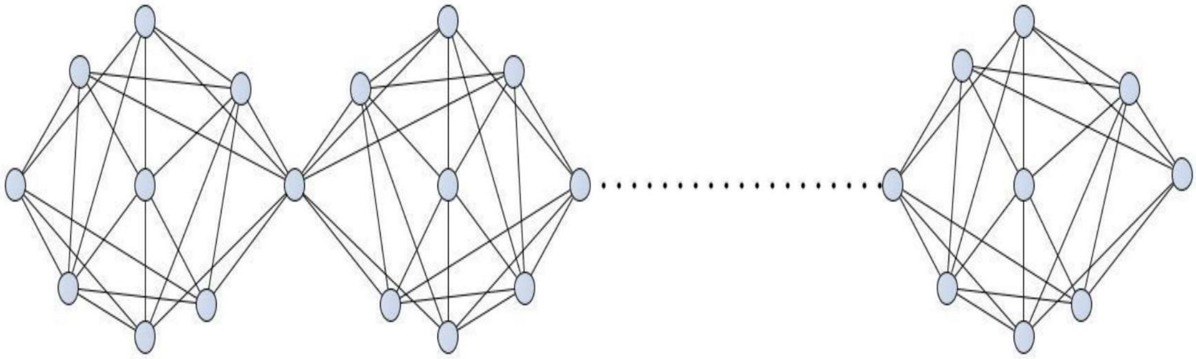

Figure 3.9: Number of MPS of complex-symmetric series network.

3.7. Generalization of calculating the total number of MCS

The number of MCS is the result of the the sub complex-series network:

$$\text{Let the number of MCS in a complex subsystem}(1) = N_{c1}$$
$$\text{Let the number of MCS in a complex subsystem}(2) = N_{c2}$$
$$\text{Let the number of MCS in a complex subsystem}(3) = N_{c3}$$
$$\vdots$$
$$\text{Let the number of MCS in a complex subsystem}(n) = N_{cn}$$

The total number of MCS in the system:

$$N_{cut} = N_{c1} + \ N_{c2} + \ N_{c3} + \ \cdots\cdots + \ N_{cn} \tag{3.8}$$

N_{cut}, represents the total number of MCS in the system, N_{ci} It represents the number of MCS in the sub system of complex-series, i=1,. . . ,n.

When the sub system of complex-series is symmetrical, the equation becomes:

$$N_{cut} = mN_c \tag{3.9}$$

N_c, represents the number of MCS in one sub system of complex-series, m It represents the number sub system of complex-series.

Example: Consider the total number of MPS and MCS in the complex- series network in Fig. 3.10 .

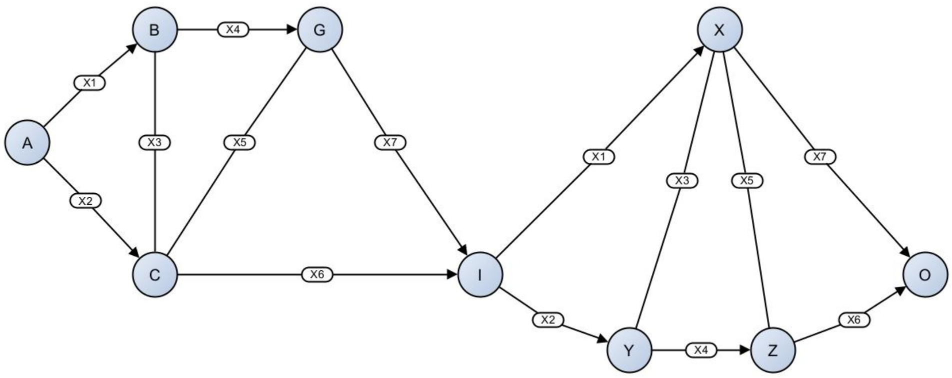

Figure 3.10: Total number of MPS and MCS in S.

The number of MPS in a complex subsystem N_1, , see Fig. 3.11

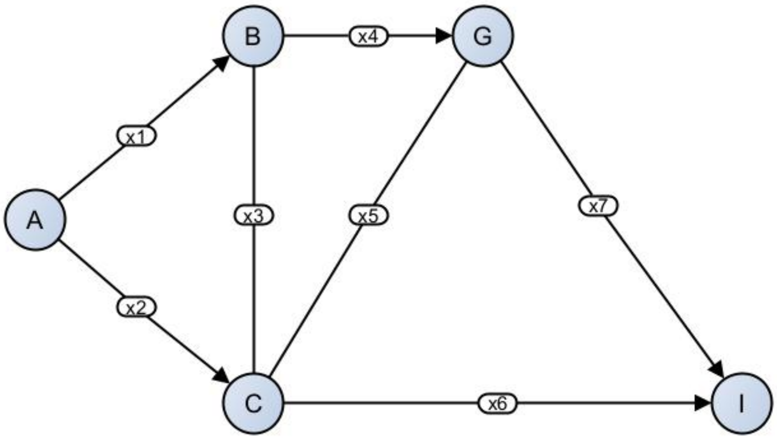

Figure 3.11: (N_1) Subsystem of S.

where when using the method of extracting the MPS, it turns out that he has seven MPS are:

$$MPS_1 = [\{x_2x_7\}, \{x_1x_3x_7\}, \{x_1x_4x_6\}, \{x_2x_5x_6\}, \{x_2x_3x_4x_6\}, \{x_1x_3x_5x_6\}, \{x_1x_4x_5x_7\}].$$

The (N_1) subsystem contains seven MPS.

Also, the number of MPS in a complex subsystem (N_2), see Fig. 3.12

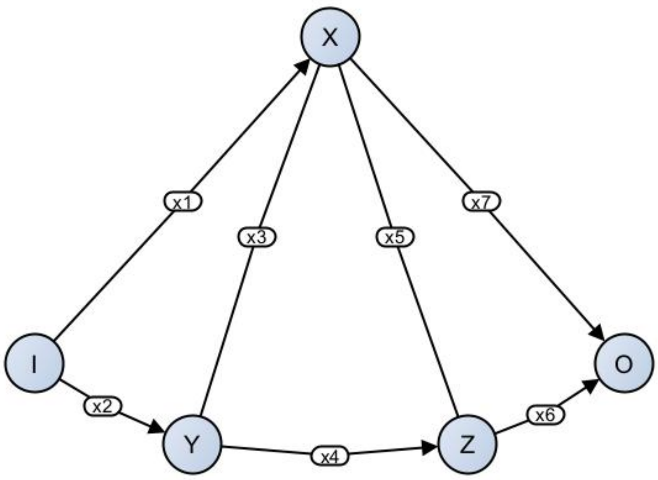

Figure 3.12: (N_2) Subsystem of S.

when one performs the steps of the MPS, we get eight MPS as a result are:

$$MPS_2 = [\{x_1x_4\}, \{x_2x_4x_5\}, \{x_1x_3x_7\}, \{x_5x_6x_7\}, \{x_2x_3x_5x_7\}, \{x_1x_2x_6x_7\}, \{x_3x_4x_5x_6\}\ \{x_1x_2x_3x_4x_6\}]$$

The N_2 subsystem contains eight MPS.

Now, the final number of MPS in a complex- series network product of the two subsystems:

$$N_{path} = N_{p1} \times N_{p2} \tag{3.10}$$

$$7 \times 8 \ = 56_{MPS}$$

The number of MCS in a complex subsystem (N1) where when using the method of extracting the MCS, it turns out that he has six MCS are:

$$MCS_1 = [\{x_1x_2\}, \{x_6x_7\}, \{x_2x_3x_4\}, \{x_4x_5x_7\}\ \{x_1x_4x_6x_7\}, \{x_3x_4x_5x_7\}].$$

Also, the number of MCS in a complex subsystem N_2 when perform the steps of the MCS, we get six MCS as a result are:

$$MCS_2 = [\{x_1x_5\}, \{x_4x_7\}, \{x_1x_2x_6\}, \{x_1x_2x_3x_7\}\ \{x_3x_4x_6\}, \{x_2x_3x_4x_5\}].$$

$$N_{cut} = N_{c1} + N_{c2} \tag{3.11}$$

$$6 \ + 6 \ = 12_{MCS}.$$

CHAPTER 4

RELIABILITY REDUNDANCY

4.1. Introduction

In This chapter, discusses one of the ways to improve reliability, which is the redundancy method. Used two types of redundancy methods: element redundancy and unit redundancy. They were applied to different systems, such as the series system, the parallel system, and the complex-series network. It was also applied to systems after partitioning [37, 66].

4.2. General concepts of reliability redundancy

In many practical applications, redundancy is the only way to provide high dependability, availability, or safety at the equipment or system level. Redundancy is the presence of multiple methods for carrying out a task that must be done by an item. Redundancy can frequently be done through code, at the software level, or in another way [37]. It does not necessarily mean that redundant hardware is used. Redundancy still shows up in parallel on the dependability block diagram, but to prevent common mode failures, redundant pieces should be realized independently from one another. The element that must be connected in series must receive special consideration while setting up the reliability block diagram. While adding more reliability components is challenging, we employed redundancy in the system to increase system reliability. By adding new parallel paths to the system, we were able to increase reliability through redundancy. According to the procedure, each component in the system is multiplied, and after redundancy, the results of the compounds and the reliability are obtained [55].

4.2.1 Element Redundancy

The reliability of each component is improved using the element redundancy method [22]. And according to the following equations:

$$R_i^* = \quad 1 - [1 - R_i]^2 \tag{4.1}$$

Where R_i^* represent the components' redundancy and, R_i represent the component to be redundancy.

4.2.2 Unit Redundancy

It can be calculated unit redundancy from the following relationship:

$$R_{Us}^* = \quad 1 - [1 - Rs]^2 \tag{4.2}$$

Where R_{US}^* the reliability in unit redundancy , Rs is the reliability system [28].

4.3. Applicability of Redundancy Technique to Series Systems

There are two methods of redundancy through which the results are obtained, but there is a better method than the other method [39]. and this is recognized through the results that we obtain after using the two methods on the same system.

4.3.1 Applicability of Element Redundancy to Series Systems

The shape of the system is as shown in Fig. 4.1,and 4.2.

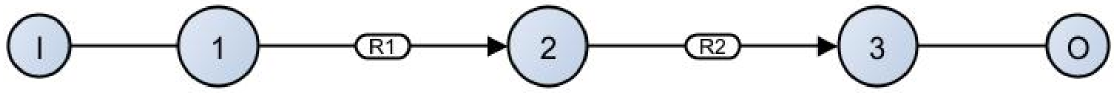

Figure 4.1: Series System.

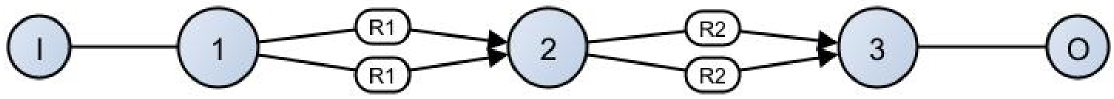

Figure 4.2: Element Redundancy.

As shown in the following example.

Example 4.3.1. Consider series system consist on two paths R_1, R_2 with reliabilities 0.7, 0.6 respectively. Apply Eq. 4.1.The value of reliability in series system

$$R_y = R_1 R_2$$

The value of reliability in series system $R_y = 0.42$ Now find the reliability value for each of R_1^* and R_2^*

$$R_1^* = 0.91, R_2^* = 0.84$$

The system value after redundancy the element is: $R_{Ey} = R_1^* R_2^* = 0.7644$

4.3.2 Applicability of Unit Redundancy to Series Systems

The shape of the system is as shown in Fig.4.3.

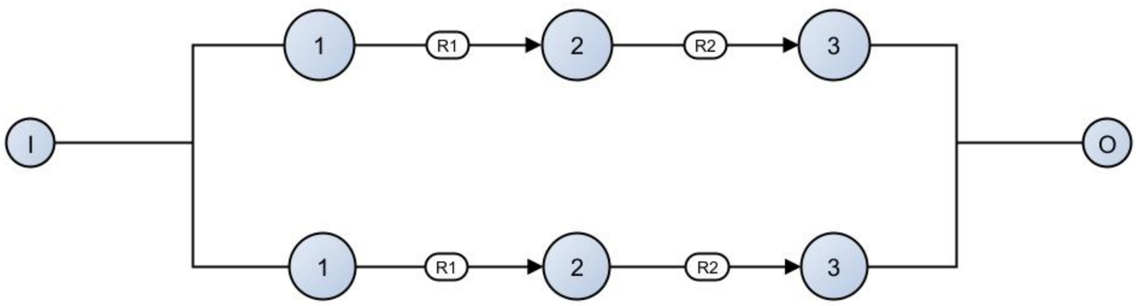

Figure 4.3: Unit Redundancy.

We use Eq. 4.2. to extract the reliability of the system after redundancy units

$$R_{Uy} = 1 - [1 - 0.42]^2 = 0.66$$

4.4. Applicability of Redundancy Technique to Parallel Systems

The parallel system contains three components. This system is applied to the repetition method to obtain improved reliability results, because the function of the repetition of the system is to increase the improvement of the reliability of the system. As shown in the following example.

Example 4.4.1. Consider parallel system consist on three element 1,2,3 with reliabilities $0.4, 0.5, 0.6$ respectively. The reliability of this system:

$$\begin{aligned} R_p &= 1 - [(1 - R_1)(1 - R_2)(1 - R_3)] \\ &= R_3 + R_1 - R_1R_3 + R_2 - R_2R_3 - R_1R_2 + R_1R_2R_3 \end{aligned}$$

The value of reliability in parallel system is: $R_p = 0.88$.

4.4.1 Applicability of Element Redundancy to Parallel Systems

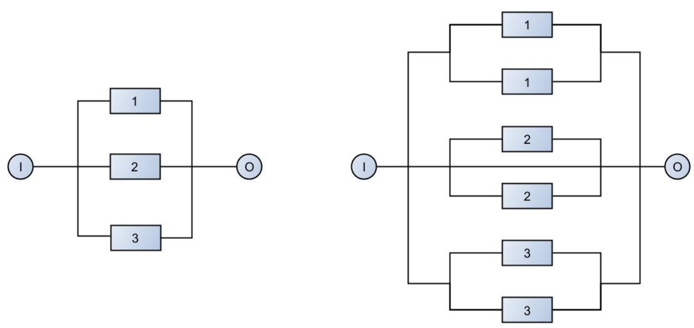

Figure 4.4: Element redundancy parallel system.

Now the researcher uses the laws of the redundancy element Eq. 4.1. In order to obtain new values for the compounds and then substitute these values into the basic law of the redundancy element in order to obtain the new improved system reliability.

$$R_1^* = 0.64, R_2^* = 0.75, R_3^* = 0.84$$
$$R_{Ep} = 0.985$$

4.4.2 Applicability of Unit Redundancy to Parallel Systems

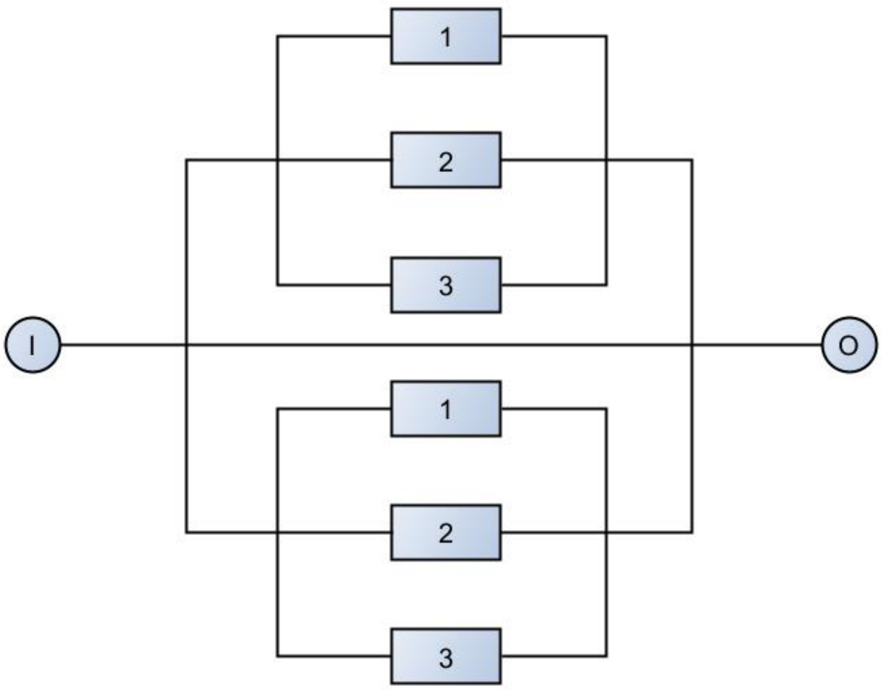

Figure 4.5: Unit redundancy parallel system.

Apply the Eq. 4.2.

$$R_{Up} = 1 - [1 - 0.88]^2 = 0.985$$

4.5. Applicability of Redundancy Technique to S

Example 4.5.1. Consider complex network consists of 10 elements with reliabilities $0.6, 0.7, 0.6, 0.5, 0.4, 0.9, 0.8, 0.85, 0.95$ and 0.9 respectively.

4.5.1 Applicability of Element Redundancy to S

Using a separate path for each component of complex - series network , each vector is repeated in the system from R_1 to R_{10} so the components of the system are the result of the reliability of each component, and as a result the overall system reliability is improved. Where R_1 represents the value of the component and R_1^* represents the new value of the component after repeating the element and also R_2 represents the value of the component and by repeating the element the new value became symbolized by R_2^* and so on up to the value of the last component R_{10} and after repeating the element it became symbolized by R_{10}^* These new values were calculated after iterating the element through the use of the above equations.

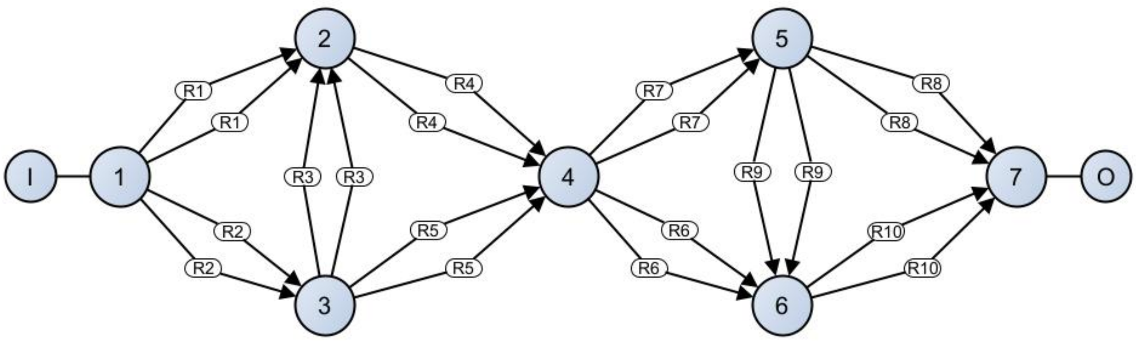

Figure 4.6: Element redundancy of S.

The table 4.1 represents the reliability values of the compounds of the complex - series and the new values after repetition. the reliability value of the complex - series network is Rs =0.5188.

Table 4.1: New values of the components of S after element redundancy.

R_i	value	R_i^*	value
R_1	0.6	R_1^*	0.84
R_2	0.7	R_2^*	0.91
R_3	0.6	R_3^*	0.84
R_4	0.5	R_4^*	0.75
R_5	0.4	R_5^*	0.64
R_6	0.9	R_6^*	0.99
R_7	0.8	R_7^*	0.96
R_8	0.85	R_8^*	0.9775
R_9	0.95	R_9^*	0.9975
R_{10}	0.9	R_{10}^*	0.99

And by substituting the values in equation (2.5) the researcher gets the value of reliability after iteration $R_{Es} = 0.8776$ Also, partition the S into two parts from which the reliability of each is extracted after the redundancy process.

4.5.2 Applicability of Element Redundancy to G

Use the element redundancy method for subsystem G, where redundancy the components from R_1 to R_5. R_i represents the value of the component and R_i^* the new value after redundancy the component.

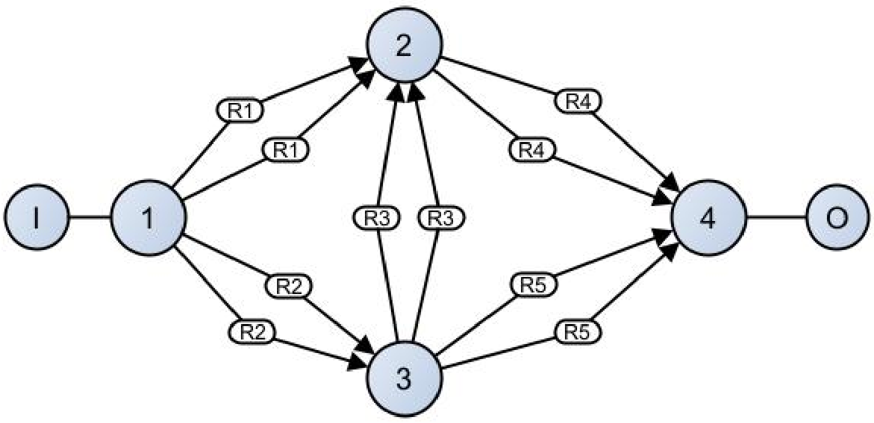

Figure 4.7: Element redundancy of G.

Table 4.2 Represents the reliability values of the compounds of G and the new values after repetition. The reliability value of G is $R_G = 0.5464$.

Table 4.2: New values of the components of G after element redundancy.

R_i	value	R_i^*	value
R_1	0.6	R_1^*	0.84
R_2	0.7	R_2^*	0.91
R_3	0.6	R_3^*	0.84
R_4	0.5	R_4^*	0.75
R_5	0.4	R_5^*	0.64

And by substituting the values in equation (3.1) the researcher comes our with the value of reliability after redundancy $R_{EG} = 0.8785$.

4.5.3 Applicability of Element Redundancy to H

Use the element redundancy method for subsystem H, where redundancy the components from R_7 to R_{10}. R_7 represents the value of the component and R_7^* the new value after redundancy the element.

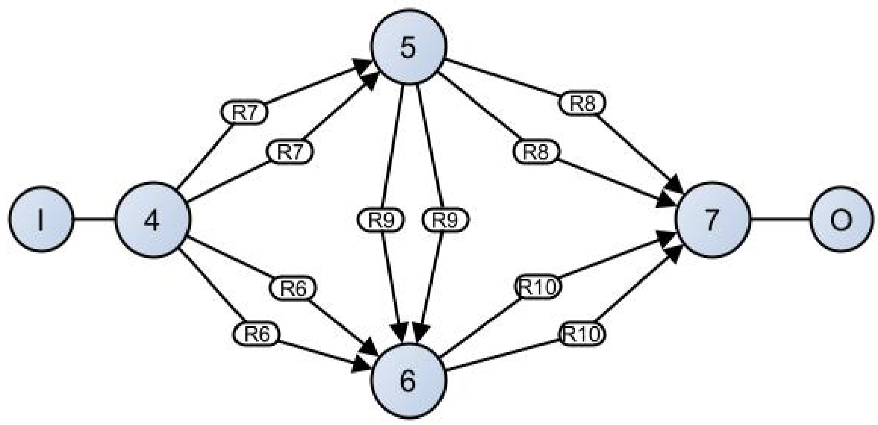

Figure 4.8: Element redundancy of H.

Table 4.3 Represents the reliability values of the compounds of H and the new values after redundancy. The reliability value of H is $R_H = 0.9495$.

Table 4.3: New values of the components of H after element redundancy.

R_i	value	R_i^*	value
R_6	0.9	R_6^*	0.99
R_7	0.8	R_7^*	0.96
R_8	0.85	R_8^*	0.9775
R_9	0.95	R_9^*	0.9975
R_{10}	0.9	R_{10}^*	0.99

And by substituting the values in equation (3.2) the researcher comes our with the value of reliability after redundancy $R_{EH} = 0.9990$.

4.5.4 Applicability of Unit Redundancy to S

Use the unit redundancy for complex-series network and extract the new value.

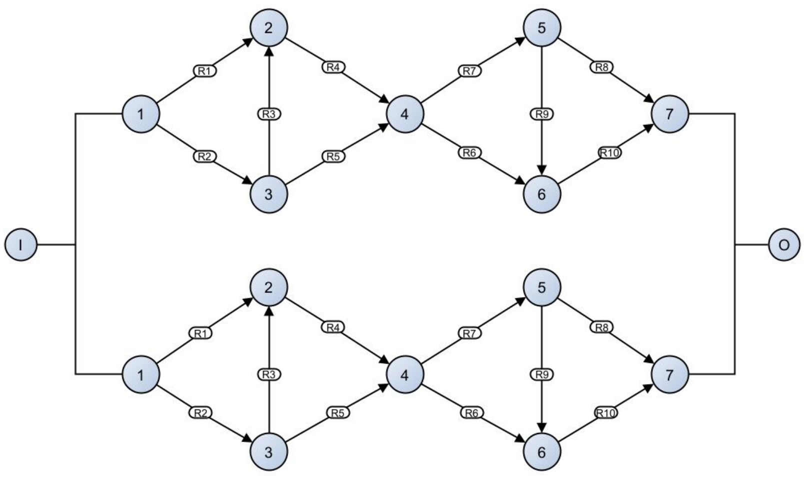

Figure 4.9: Unit redundancy of S.

Use the equation (4.2) to find the reliability of the system after redundancy units.

$$R_{Us} = 1 - [1 - 0.5188]^2 = 0.7684$$

4.5.5 Applicability of Unit Redundancy to G

Use the unit redundancy for subsystem G, and extract the new value.

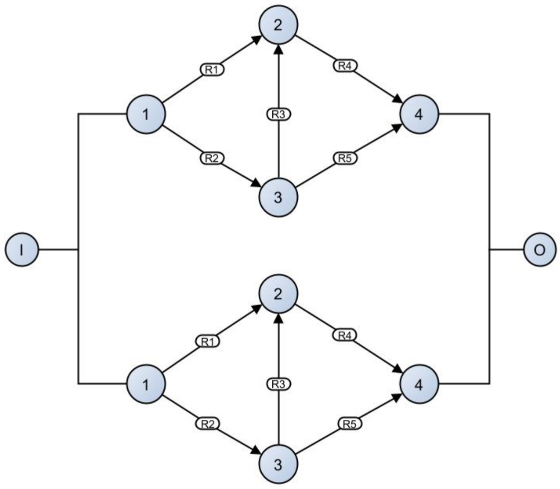

Figure 4.10: Unit redundancy of G.

Apply Equation (4.2) to G subsystem.

$$R_{UG} = 1 - [1 - 0.5464]^2 = 0.7942.$$

4.5.6 Applicability of Unit Redundancy to H

Use the unit redundancy for subsystem H, and extract the new value.

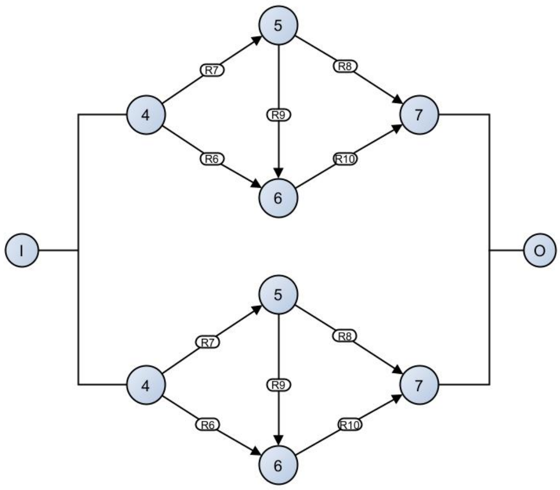

Figure 4.11: Unit redundancy of H.

Apply Equation (4.2) to H subsystem.

$$R_{UH} = 1 - [1 - 0.9495]^2 = 0.9974.$$

4.6. The Relationship Between the G ,H and S After Redundancy

Use the element redundancy method for the network of complex-series, and the result of the reliability value was higher than the original reliability value. partitioned the system and applied the element redundancy method to the subsystem G and the subsystem H. Also, the result of the reliability of each of the subsystems after element redundancy was higher than the original value. After that, used the relationship between them, which is the multiplication of each of the reliability of the partial systems after redundancy the element with each other, so that the result is the reliability of complex-series network after redundancy the element.Apply the redundancy unit method to the network of complex - series and extract the reliability product. Then, after dividing the network into two parts, apply the redundancy unit method to the sub-systems and extract the reliability output for each sub-system after the unit redundancy. The product of the reliability of the sub-systems was multiplied to be higher than the original value of the network of complex - series after redundancy the unit. conclude that the element redundancy method is more improved than the unit redundancy method.

CHAPTER 5

RELIABILITY IMPORTANCE OF THE COMPLEX-SERIES NETWORK AND MIXED SYSTEM

5.1. Introduction

In this chapter, will also discuss one of the methods of improving reliability, which is the importance of the Birnbaum scale for compounds. studied this method on the complex - series network of as well as on its partial systems, and showed the relationship between them. Then studied the method on another system, the gap of the mixed system, and applied it also to the partial systems of the mixed system, and studied the relationship between them.

5.2. Birnbaums Reliability Importance

Birnbaum introduces the reliability importance (B-importance) metric (1969) [14]. as the rate at which system reliability is increasing relative to that of its constituent parts. Measures of component relevance are crucial for enhancing system design and creating the best replacement strategies. This measure can be described by a number of identical formulas if the components are stochastically independent. The independence assumption is unrealistic in a lot of real-world circumstances, though. It also turns out that different definitions of Birnbaum's measure provide distinct conceptions when applied to dependent components [70]. As a result, when component credibility is independent, the Birnbaum scale is created by partially differentiating the system's dependability with regard to pi. Birnbaum establishes the importance of a system with component reliability.
$P = (P_1, P_2, \ldots, P_n)$ through the next.

$$IB_s(1i, p) = P\{\phi(x) = 1 \mid Xi = 1\} - P\{\phi(x) = 1\} \tag{5.1}$$

$$I_{Bf}(0i, p) = P\{\phi(x) = 0 \mid Xi = 0\} - P\{\phi(x) = 0\} \tag{5.2}$$

Component I's reliability value ranges from 0 to 1 [40]. And the importance of reliability

(I_i) is indicated by:

$$I_i = \frac{\partial Rs}{\partial Ri} \tag{5.3}$$

Ii for all the components of the complex - series network shown in Fig 2.1 are the partial derivatives of Rs for the first component of Eq. 2.6 are as follows:

$$\begin{aligned}
I_{1,S} =&R_{10}R_4R_6 + R_4R_7R_8 + R_{10}R_4R_7R_9 - R_{10}R_2R_3R_4R_6 - R_{10}R_2R_4\\
&R_5R_6 - R_2R_3R_4R_7R_8 - R_2R_4R_5R_7R_8 - R_{10}R_4R_6R_7R_8 - R_{10}R_4R_6\\
&R_7R_9 - R_{10}R_4R_7R_8R_9 + R_{10}R_2R_3R_4R_5R_6 - R_{10}R_2R_3R_4R_7R_{9-}\\
&R_{10}R_2R_4R_5R_7R_9 + R_2R_3R_4R_5R_7R_8 + R_{10}R_4R_6R_7R_8R_9+\\
&R_{10}R_2R_3R_4R_5R_7R_9 + R_{10}R_2R_3R_4R_6R_7R_8 + R_{10}R_2R_3R_4R_6\\
&R_7R_9 + R_{10}R_2R_4R_5R_6R_7R_8 + R_{10}R_2R_3R_4R_7R_8R_9 + R_{10}R_2R_4R_5\\
&R_6R_7R_9 + R_{10}R_2R_4R_5R_7R_8R_9 - R_{10}R_2R_3R_4R_5R_6R_7R_8 - R_{10}R_2\\
&R_3R_4R_5R_6R_7R_{9-}R_{10}R_2R_3R_4R_5R_7R_8R_9 - R_{10}R_2R_3R_4R_6R_7R_8\\
&R_9 - R_{10}R_2R_4R_5R_6R_7R_8R_9 + R_{10}R_2R_3R_4R_5R_6R_7R_8R_9.
\end{aligned}$$

$$\begin{aligned}
I_{2,S} =&R_{10}R_5R_6 + R_5R_7R_8 + R_{10}R_3R_4R_6 + R_3R_4R_7R_8 + R_{10}R_5R_7R_{9-}\\
&R_1R_{10}R_3R_4R_6 - R_1R_{10}R_4R_5R_6 - R_{10}R_3R_4R_5R_6 - R_1R_3R_4R_7R_8\\
&+ R_{10}R_3R_4R_7R_9 - R_1R_4R_5R_7R_8 - R_3R_4R_5R_7R_8 - R_{10}R_5R_6R_7R_{8-}\\
&R_{10}R_5R_6R_7R_9 - R_{10}R_5R_7R_8R_9 + R_1R_{10}R_3R_4R_5R_6 - R_1R_{10}R_3\\
&R_4R_7R_9 - R_1R_{10}R_4R_5R_7R_9 + R_1R_3R_4R_5R_7R_8 - R_{10}R_3R_4R_5\\
&R_7R_9 - R_{10}R_3R_4R_6R_7R_8 - R_{10}R_3R_4R_6R_7R_9 - R_{10}R_3R_4R_7R_8R_9\\
&+ R_{10}R_5R_6R_7R_8R_9 + R_1R_{10}R_3R_4R_5R_7R_9 + R_1R_{10}R_3R_4R_6R_7\\
&R_8 + R_1R_{10}R_3R_4R_6R_7R_9 + R_1R_{10}R_4R_5R_6R_7R_8 + R_1R_{10}R_3R_4\\
&R_7R_8R_9 + R_1R_{10}R_4R_5R_6R_7R_9 + R_{10}R_3R_4R_5R_6R_7R_8 + R_1R_{10}R_4
\end{aligned}$$

$$R_5R_7R_8R_9 + R_{10}R_3R_4R_5R_6R_7R_9 + R_{10}R_3R_4R_5R_7R_8R_9 +$$
$$R_{10}R_3R_4R_6R_7R_8R_9 - R_1R_{10}R_3R_4R_5R_6R_7R_8 - R_1R_{10}R_3R_4R_5R_6$$
$$R_7R_9 - R_1R_{10}R_3R_4R_5R_7R_8R_9 - R_1R_{10}R_3R_4R_6R_7R_8R_9 - R_1R_{10}$$
$$R_4R_5R_6R_7R_8R_9 - R_{10}R_3R_4R_5R_6R_7R_8R_9 + R_1R_{10}R_3R_4R_5R_6R_7R_8R_9.$$

$$\begin{aligned}
I_{3,S} =& R_{10}R_2R_4R_6 + R_2R_4R_7R_8 - R_1R_{10}R_2R_4R_6 - R_{10}R_2R_4R_5R_6 \\
& - R_1R_2R_4R_7R_8 + R_{10}R_2R_4R_7R_9 - R_2R_4R_5R_7R_8 + R_1R_{10}R_2 \\
& R_4R_5R_6 - R_1R_{10}R_2R_4R_7R_9 + R_1R_2R_4R_5R_7R_8 - R_{10}R_2R_4R_5 \\
& R_7R_9 - R_{10}R_2R_4R_6R_7R_8 - R_{10}R_2R_4R_6R_7R_9 - R_{10}R_2R_4R_7R_8 \\
& R_9 + R_1R_{10}R_2R_4R_5R_7R_9 + R_1R_{10}R_2R_4R_6R_7R_8 + R_1R_{10}R_2 \\
& R_4R_6R_7R_9 + R_1R_{10}R_2R_4R_7R_8R_9 + R_{10}R_2R_4R_5R_6R_7R_8 \\
& + R_{10}R_2R_4R_5R_6R_7R_9 + R_{10}R_2R_4R_5R_7R_8R_9 + R_{10}R_2R_4R_6R_7 \\
& R_8R_9 - R_1R_{10}R_2R_4R_5R_6R_7R_8 - R_1R_{10}R_2R_4R_5R_6R_7R_9 - R_1R_{10}R_2 \\
& R_4R_5R_7R_8R_9 - R_1R_{10}R_2R_4R_6R_7R_8R_9 - R_{10}R_2R_4R_5R_6R_7R_8R_9 \\
& + R_1R_{10}R_2R_4R_5R_6R_7R_8R_9.
\end{aligned}$$

$$\begin{aligned}
I_{4,S} =& R_1R_{10}R_6 + R_1R_7R_8 + R_{10}R_2R_3R_6 + R_1R_{10}R_7R_9 + R_2R_3R_7R_8 \\
& - R_1R_{10}R_2R_3R_6 - R_1R_{10}R_2R_5R_6 - R_{10}R_2R_3R_5R_6 - R_1R_2R_3R_7R_8 \\
& + R_{10}R_2R_3R_7R_9 - R_1R_2R_5R_7R_8 - R_1R_{10}R_6R_7R_8 - R_1R_{10}R_6R_7R_9 \\
& - R_2R_3R_5R_7R_8 - R_1R_{10}R_7R_8R_9 + R_1R_{10}R_2R_3R_5R_6 - R_1R_{10}R_2R_3R_7 \\
& R_9 - R_1R_{10}R_2R_5R_7R_9 + R_1R_2R_3R_5R_7R_8 - R_{10}R_2R_3R_5R_7R_9 - R_{10} \\
& R_2R_3R_6R_7R_8 - R_{10}R_2R_3R_6R_7R_9 - R_{10}R_2R_3R_7R_8R_9 + R_1R_{10} \\
& R_6R_7R_8R_9 + R_1R_{10}R_2R_3R_5R_7R_9 + R_1R_{10}R_2R_3R_6R_7R_8 + R_1R_{10}
\end{aligned}$$

$$R_2R_3R_6R_7R_9 + R_1R_{10}R_2R_5R_6R_7R_8 + R_1R_{10}R_2R_3R_7R_8R_9$$
$$+ R_1R_{10}R_2R_5R_6R_7R_9 + R_{10}R_2R_3R_5R_6R_7R_8 + R_1R_{10}R_2R_5R_7R_8$$
$$R_9 + R_{10}R_2R_3R_5R_6R_7R_9 + R_{10}R_2R_3R_5R_7R_8R_9 + R_{10}R_2R_3R_6$$
$$R_7R_8R_9 - R_1R_{10}R_2R_3R_5R_6R_7R_8 - R_1R_{10}R_2R_3R_5R_6R_7R_9 - R_1R_{10}$$
$$R_2R_3R_5R_7R_8R_9 - R_1R_{10}R_2R_3R_6R_7R_8R_9 - R_1R_{10}R_2R_5R_6R_7R_8$$
$$R_9 - R_{10}R_2R_3R_5R_6R_7R_8R_9 + R_1R_{10}R_2R_3R_5R_6R_7R_8R_9.$$

$$I_{5,S} = R_{10}R_2R_6 + R_2R_7R_8 + R_{10}R_2R_7R_9 - R_1R_{10}R_2R_4R_6 - R_{10}R_2R_3R_4$$
$$R_6 - R_1R_2R_4R_7R_8 - R_2R_3R_4R_7R_8 - R_{10}R_2R_6R_7R_8 - R_{10}R_2R_6R_7$$
$$R_9 - R_{10}R_2R_7R_8R_9 + R_1R_{10}R_2R_3R_4R_6 - R_1R_{10}R_2R_4R_7R_9$$
$$+ R_1R_2R_3R_4R_7R_8 - R_{10}R_2R_3R_4R_7R_9 + R_{10}R_2R_6R_7R_8R_9 + R_1R_{10}$$
$$R_2R_3R_4R_7R_9 + R_1R_{10}R_2R_4R_6R_7R_8 + R_1R_{10}R_2R_4R_6R_7R_9$$
$$+ R_{10}R_2R_3R_4R_6R_7R_8 + R_1R_{10}R_2R_4R_7R_8R_9 + R_{10}R_2R_3R_4R_6R_7R_9$$
$$+ R_{10}R_2R_3R_4R_7R_8R_9 - R_1R_{10}R_2R_3R_4R_6R_7R_8 - R_1R_{10}R_2R_3$$
$$R_4R_6R_7R_9 - R_1R_{10}R_2R_3R_4R_7R_8R_9 - R_1R_{10}R_2R_4R_6R_7R_8R_9$$
$$- R_{10}R_2R_3R_4R_6R_7R_8R_9 + R_1R_{10}R_2R_3R_4R_6R_7R_8R_9.$$

$$I_{6,S} = R_1R_{10}R_4 + R_{10}R_2R_5 + R_{10}R_2R_3R_4 - R_1R_{10}R_2R_3R_4 - R_1R_{10}$$
$$R_2R_4R_5 - R_{10}R_2R_3R_4R_5 - R_1R_{10}R_4R_7R_8 - R_1R_{10}R_4R_7R_9$$
$$- R_{10}R_2R_5R_7R_8 - R_{10}R_2R_5R_7R_9 + R_1R_{10}R_2R_3R_4R_5 - R_{10}R_2$$
$$R_3R_4R_7R_8 - R_{10}R_2R_3R_4R_7R_9 + R_1R_{10}R_4R_7R_8R_9 + R_{10}R_2$$
$$R_5R_7R_8R_9 + R_1R_{10}R_2R_3R_4R_7R_8 + R_1R_{10}R_2R_3R_4R_7R_9 + R_1$$
$$R_{10}R_2R_4R_5R_7R_8 + R_1R_{10}R_2R_4R_5R_7R_9 + R_{10}R_2R_3R_4R_5R_7$$

$$R_8 + R_{10}R_2R_3R_4R_5R_7R_9 + R_{10}R_2R_3R_4R_7R_8R_9 - R_1R_{10}R_2$$
$$R_3R_4R_5R_7R_8 - R_1R_{10}R_2R_3R_4R_5R_7R_9 - R_1R_{10}R_2R_3R_4R_7R_8R_9$$
$$- R_1R_{10}R_2R_4R_5R_7R_8R_9 - R_{10}R_2R_3R_4R_5R_7R_8R_9 + R_1R_{10}$$
$$R_2R_3R_4R_5R_7R_8R_9.$$

$$I_{7,S} = R_1R_4R_8 + R_2R_5R_8 + R_1R_{10}R_4R_9 + R_2R_3R_4R_8 + R_{10}R_2R_5R_9$$
$$- R_1R_2R_3R_4R_8 + R_{10}R_2R_3R_4R_9 - R_1R_2R_4R_5R_8 - R_1R_{10}R_4R_6R_8$$
$$- R_1R_{10}R_4R_6R_9 - R_2R_3R_4R_5R_8 - R_{10}R_2R_5R_6R_8 - R_1R_{10}R_4$$
$$R_8R_9 - R_{10}R_2R_5R_6R_9 - R_{10}R_2R_5R_8R_9 - R_1R_{10}R_2R_3R_4R_9$$
$$R_1R_{10}R_2R_4R_5R_9 + R_1R_2R_3R_4R_5R_{8-} - R_{10}R_2R_3R_4R_5R_9$$
$$- R_{10}R_2R_3R_4R_6R_8 - R_{10}R_2R_3R_4R_6R_9 - R_{10}R_2R_3R_4R_8R_9$$
$$+ R_1R_{10}R_4R_6R_8R_9 + R_{10}R_2R_5R_6R_8R_9 + R_1R_{10}R_2R_3R_4R_5R_9$$
$$+ R_1R_{10}R_2R_3R_4R_6R_8 + R_1R_{10}R_2R_3R_4R_6$$
$$R_9 + R_1R_{10}R_2R_4R_5R_6R_8 + R_1R_{10}R_2R_3R_4R_8R_9 + R_1R_{10}R_2R_4$$
$$R_5R_6R_9 + R_{10}R_2R_3R_4R_5R_6R_8 + R_1R_{10}R_2R_4R_5R_8R_9 + R_{10}R_2$$
$$R_3R_4R_5R_6R_9 + R_{10}R_2R_3R_4R_5R_8R_9 + R_{10}R_2R_3R_4R_6R_8R_9$$
$$- R_1R_{10}R_2R_3R_4R_5R_6R_8 - R_1R_{10}R_2R_3R_4R_5R_6R_9 - R_1R_{10}R_2R_3$$
$$R_4R_5R_8R_9 - R_1R_{10}R_2R_3R_4R_6R_8R_9 - R_1R_{10}R_2R_4R_5R_6R_8R_9$$
$$- R_{10}R_2R_3R_4R_5R_6R_8R_9 + R_1R_{10}R_2R_3R_4R_5R_6R_8R_9.$$

$$I_{8,S} = R_1R_4R_7 + R_2R_5R_7 + R_2R_3R_4R_7 - R_1R_2R_3R_4R_7 - R_1R_2R_4R_5$$
$$R_7 - R_1R_{10}R_4R_6R_7 - R_2R_3R_4R_5R_7 - R_{10}R_2R_5R_6R_7 - R_1R_{10}$$
$$R_4R_7R_9 - R_{10}R_2R_5R_7R_9 + R_1R_2R_3R_4R_5R_7 - R_{10}R_2R_3R_4R_6R_7$$

$$
\begin{aligned}
&- R_{10}R_2R_3R_4R_7R_9 + R_1R_{10}R_4R_6R_7R_9 + R_{10}R_2R_5R_6R_7R_9 \\
&+ R_1R_{10}R_2R_3R_4R_6R_7 + R_1R_{10}R_2R_4R_5R_6R_7 + R_1R_{10}R_2R_3R_4 \\
&R_7R_9 + R_{10}R_2R_3R_4R_5R_6R_7 + R_1R_{10}R_2R_4R_5R_7R_9 + R_{10}R_2R_3 \\
&R_4R_5R_7R_9 + R_{10}R_2R_3R_4R_5R_7R_9 + R_{10}R_2R_3R_4R_6R_7R_9 \\
&R_1R_{10}R_2R_3R_4R_5R_6R_7 - R_1R_{10}R_2R_3R_4R_5R_7R_9 \\
&- R_1R_{10}R_2R_3R_4R_6R_7R_9 - R_1R_{10}R_2R_4R_5R_6R_7R_9 \\
&R_{10}R_2R_3R_4R_5R_6R_7R_9 + R_1R_{10}R_2R_3R_4R_5R_6R_7R_9.
\end{aligned}
$$

$$
\begin{aligned}
I_{9,S} =&R_1R_{10}R_4R_7R_9 + R_{10}R_2R_5R_7 + R_{10}R_2R_3R_4R_7 \\
&- R_1R_{10}R_4R_6R_7 - R_1R_{10}R_4R_7R_8 - R_{10}R_2R_5R_6R_7 \\
&- R_{10}R_2R_5R_7R_8 - R_1R_{10}R_2R_3R_4R_{7-}R_1R_{10}R_2R_4R_5R_7 \\
&- R_{10}R_2R_3R_4R_5R_7 - R_{10}R_2R_3R_4R_6R_7 - R_{10}R_2R_3R_4R_7R_8 \\
&- R_1R_{10}R_4R_6R_7R_8 + R_{10}R_2R_5R_6R_7R_8 + R_1R_{10}R_2R_3R_4 \\
&R_5R_7 + R_1R_{10}R_2R_3R_4R_6R_7 + R_1R_{10}R_2R_3R_4R_7R_8 + R_1R_{10}R_2R_4 \\
&R_5R_6R_7 + R_1R_{10}R_2R_4R_5R_7R_8 + R_{10}R_2R_3R_4R_5R_6R_7 + R_{10}R_2R_3 \\
&R_4R_5R_7R_8 + R_{10}R_2R_3R_4R_6R_7R_8 - R_1R_{10}R_2R_3R_4R_5R_6R_7 \\
&- R_1R_{10}R_2R_3R_4R_5R_7R_8 - R_1R_{10}R_2R_3R_4R_6R_7R_8 - R_1R_{10}R_2R_4 \\
&R_5R_6R_7R_8 - R_{10}R_2R_3R_4R_5R_6R_7R_8 + R_1R_{10}R_2R_3R_4R_5R_6R_7R_8.
\end{aligned}
$$

$$
\begin{aligned}
I_{10,S} =&R_1R_4R_6 + R_2R_5R_6 + R_2R_3R_4R_6 + R_1R_4R_7R_9 + R_2R_5R_7R_9 \\
&- R_1R_2R_3R_4R_6 - R_1R_2R_4R_5R_6 - R_2R_3R_4R_5R_6 + R_2R_3R_4R_7R_9 \\
&- R_1R_4R_6R_7R_8 - R_1R_{10}R_4R_6R_7R_9 - R_2R_5R_6R_7R_8 - R_1R_4R_7 \\
&R_8R_9 - R_2R_5R_6R_7R_9 - R_2R_5R_7R_8R_9 + R_1R_2R_3R_4R_5R_{6-}R_1R_2 \\
&R_3R_4R_7R_9 - R_1R_2R_3R_4R_7R_9 - R_1R_2R_4R_5R_7R_9 - R_2R_3R_4 \\
&R_5R_7R_9 - R_2R_3R_4R_6R_7R_8 - R_2R_3R_4R_6R_7R_9 - R_2R_3R_4
\end{aligned}
$$

$$R_7R_8R_9 + R_1R_4R_6R_7R_8R_9 + R_2R_5R_6R_7R_8R_9 + R_1R_2R_3$$

$$R_4R_5R_7R_9 + R_1R_2R_3R_4R_6R_7R_8 + R_1R_2R_3R_4R_6R_7R_9$$

$$+ R_1R_2R_4R_5R_6R_7R_8 + R_1R_2R_3R_4R_7R_8R_9 + R_1R_2R_4$$

$$R_5R_6R_7R_9 + R_2R_3R_4R_5R_6R_7R_8 + R_1R_2R_4R_5R_7R_8R_9$$

$$+ R_2R_3R_4R_5R_6R_7R_9 + R_2R_3R_4R_5R_7R_8R_9 + R_2R_3R_4R_6$$

$$R_7R_8R_9 - R_1R_2R_3R_4R_5R_6R_7R_8 - R_1R_2R_3R_4R_5R_6R_7R_9$$

$$- R_1R_2R_3R_4R_5R_7R_8R_9 - R_1R_2R_3R_4R_6R_7R_8R_9$$

$$- R_1R_2R_4R_5R_6R_7R_8R_9 - R_2R_3R_4R_5R_6R_7R_8R_9$$

$$+ R_1R_2R_3R_4R_5R_6R_7R_8R_9.$$

Table 5.1 shows the importance of reliability and the level of units for Fig 2.1.

Table 5.1: The reliability importance and level of units in Fig 2.1

Components	I (i)	Level
R_4	0.5059	1
R_5	0.3855	2
R_2	0.3342	3
R_1	0.2222	4
R_{10}	0.1636	5
R_6	0.1013	6
R_7	0.0953	7
R_3	0.0798	8
R_8	0.0457	9
R_9	0.0059	10

5.2.1 Reliability Importance of G

The importance of the components of the subsystem G are:

$$I_{1,G} = R_4 - R_2R_3R_4 - R_2R_4R_5 + R_2R_3R_4R_5$$

$$I_{2,G} = R_5 + R_3R_4 - R_1R_3R_4 - R_1R_4R_5 - R_3R_4R_5 + R_1R_3R_4R_5$$

$$I_{3,G} = R_2R_4 - R_1R_2R_4 - R_2R_4R_5 + R_1R_2R_4R_5$$

$$I_{4,G} = R_1 + R_2R_3 - R_1R_2R_3 - R_1R_2R_5 - R_2R_3R_5 + R_1R_2R_3R_5$$

$$I_{5,G} = R_2 - R_1R_2R_4 - R_2R_3R_4 + R_1R_2R_3R_4$$

The importance of G is:

$R_G = 0.5464$

Table (5.2) shows the importance of reliability and the level of units in Fig 3.2.

Table 5.2: The reliability importance and level of units of G in Fig 3.2

Components	I (i)	Level
R_4	0.5328	1
R_5	0.4060	2
R_2	0.3520	3
R_1	0.2340	4
R_3	0.0840	5

5.2.2 Reliability Importance of H

The importance of the components of the subsystem H are:

$$I_{6,H} = R_{10} - R_{10}R_7R_8 - R_{10}R_7R_9 + R_{10}R_7R_8R_9$$

$$I_{7,H} = R_8 + R_{10}R_9 - R_{10}R_6R_8 - R_{10}R_6R_9 - R_{10}R_8R_9 + R_{10}R_6R_8R_9$$

$$I_{8,H} = R_7 - R_{10}R_6R_7 - R_{10}R_7R_9 + R_{10}R_6R_7R_9$$

$$I_{9,H} = R_{10}R_7 - R_{10}R_6R_7 - R_{10}R_7R_8 + R_{10}R_6R_7R_8$$

$$I_{10,H} = R_6 + R_7R_9 - R_6R_7R_8 - R_6R_7R_9 - R_7R_8R_9 + R_6R_7R_8R_9$$

Table (5.3) shows the importance of reliability and the level of units in Fig 3.5.

$$R_H = 0.9495$$

Table 5.3: The reliability importance and level of units of H in Fig. 3.5.

Components	I(i)	Level
R_{10}	0.2994	1
R_6	0.1854	2
R_7	0.1743	3
R_8	0.0836	4
R_9	0.0108	5

5.2.3 Relation Ships Among I_G, I_H and I_s

Can extract the importance of the components for the complex-series network, for example, the component R_1 located in the complex - series network is obtained through the R_1 a component in the first subsystem G by multiplying it by the reliability importance of the second subsystem H,

$$
\begin{aligned}
I_{1s} &= I_{1G} \times R_H \\
&= 0.2340 \times 0.9495 \\
&= 0.2222.
\end{aligned}
$$

and also, another example of extracting the importance of the (R_6) a component of the complex - series network by multiplying the importance of the (R_6) component in the second subsystem H by the importance of the reliability of the subsystem The first is G

$$
\begin{aligned}
I_{6S} &= R_G \times I_{6H} \\
&= 0.5464 \times 0.1854 \\
&= 0.1013.
\end{aligned}
$$

and so on, that is, through these partial systems we extracted the importance of the components of the large complex system and also the importance of its reliability accordingly thus this gives us a future dimension We can extract the importance of reliability and the importance of components for a large complex network containing N MPS.

5.3. Importance to Mixed System

A mixed system is composed of three connected systems in series. The system contains 6 tracks, starting from the upstream and ending with the basin, with reliability 0.7, 0.5, 0.9, 0.6, 0.5, and 0.4 respectively, see Fig. 5.1.

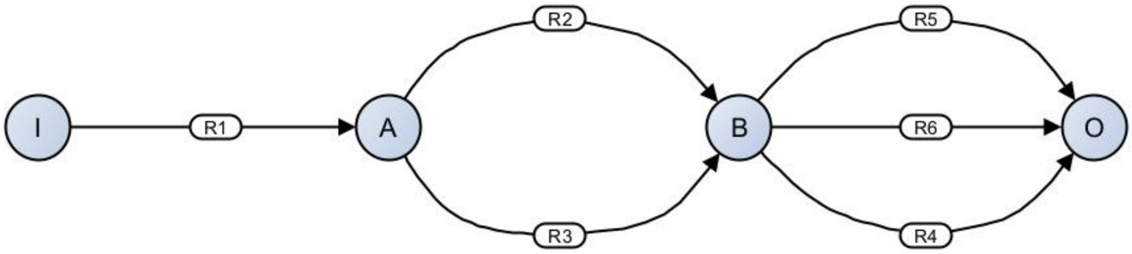

Figure 5.1: Mixed Series System.

Performing the previous MPS steps as they were done in the complex - series network to extract the data, thus ensuring the reliability of the mixed series system.
All MPS of the mixed system are:

$$MPS_m = \{\{x_1x_2x_4\}, \{x_1x_2x_5\}, \{x_1x_2x_6\}, \{x_1x_3x_4\}, \{x_1x_3x_5\}, \{x_1x_3x_6\}\}.$$

$$\begin{aligned}
Rm =& 1 - [1-(x_1x_2x_4)][1-(x_1x_2x_5)][1-(x_1x_2x_6)][1-(x_1x_3x_4)][1-(x_1x_3x_5)][1-(x_1x_3x_6)] \\
Rm =& R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_4 + R_1R_2R_6 + R_1R_3R_5 + R_1R_3R_6 - R_1R_2R_3R_4 \\
& - R_1R_2R_3R_5 - R_1R_2R_3R_6 - R_1R_2R_4R_5 - R_1R_2R_4R_6 - R_1R_3R_4R_5 \\
& - R_1R_2R_5R_6 - R_1R_3R_4R_6 - R_1R_3R_5R_6 + R_1R_2R_3R_4R_5 + R_1R_2R_3R_4R_6 \\
& + R_1R_2R_3R_5R_6 + R_1R_2R_4R_5R_6 + R_1R_3R_4R_5R_6 - R_1R_2R_3R_4R_5R_6.
\end{aligned}$$

We start by gradually deriving the compounds from R_1 to R_6

$$\begin{aligned}
I_{1,m} =& R_2R_4 + R_2R_5 + R_3R_4 + R_2R_6 + R_3R_5 + R_3R_6 - R_2R_3R_4 - R_2R_3R_5 - R_2R_3R_6 \\
& - R_2R_4R_5 - R_2R_4R_6 - R_3R_4R_5 - R_2R_5R_6 - R_3R_4R_6 - R_3R_5R_6 \\
& + R_2R_3R_4R_5 + R_2R_3R_4R_6 + R_2R_3R_5R_6 + R_2R_4R_5R_6 + R_3R_4R_5R_6 - R_2R_3R_4R_5R_6. \\
I_{2,m} =& R_1R_4 + R_1R_5 + R_1R_6 - R_1R_3R_4 - R_1R_3R_5 - R_1R_3R_6 - R_1R_4R_5 - R_1R_4R_6 \\
& - R_1R_5R_6 + R_1R_3R_4R_5 + R_1R_3R_4R_6 + R_1R_3R_5R_6 + R_1R_4R_5R_6 - R_1R_3R_4R_5R_6.
\end{aligned}$$

$$
\begin{aligned}
I_{3,m} =& R_1R_4 + R_1R_5 + R_1R_6 - R_1R_2R_4 - R_1R_2R_5 - R_1R_2R_6 - R_1R_4R_5 - R_1R_4R_6 \\
& - R_1R_5R_6 + R_1R_2R_4R_5 + R_1R_2R_4R_6 + R_1R_2R_5R_6 + R_1R_4R_5R_6 - R_1R_2R_4R_5R_6. \\
I_{4,m} =& R_1R_2 + R_1R_3 - R_1R_2R_3 - R_1R_2R_5 - R_1R_2R_6 - R_1R_3R_5 - R_1R_3R_6 + R_1R_2R_3R_5 \\
& + R_1R_2R_3R_6 + R_1R_2R_5R_6 + R_1R_3R_5R_6 - R_1R_2R_3R_5R_6. \\
I_{5,m} =& R_1R_2 + R_1R_3 - R_1R_2R_3 - R_1R_2R_4 - R_1R_3R_4 - R_1R_2R_6 - R_1R_3R_6 + R_1R_2R_3R_4 \\
& + R_1R_2R_3R_6 + R_1R_2R_4R_6 + R_1R_3R_4R_6 - R_1R_2R_3R_4R_6. \\
I_{6,m} =& R_1R_2 + R_1R_3 - R_1R_2R_3 - R_1R_2R_4 - R_1R_2R_5 - R_1R_3R_4 - R_1R_3R_5 + R_1R_2R_3R_4 \\
& + R_1R_2R_3R_5 + R_1R_2R_4R_5 + R_1R_3R_4R_5 - R_1R_2R_3R_4R_5.
\end{aligned}
$$

Where the importance of reliability in Fig. 5.1 is:

R_m=0.5852

Table 5.4 shows the importance of reliability and unit levels in Fig. 5.1.

Table 5.4: The reliability importance and level of units in Fig. 5.1.

Components	I (i)	Level
R_1	0.8360	1
R_3	0.3080	2
R_4	0.1995	3
R_5	0.1596	4
R_6	0.1330	5
R_2	0.0616	6

5.4. Partition of Mixed System

We divide the system into three parts, they are, in order, N, V, and D.

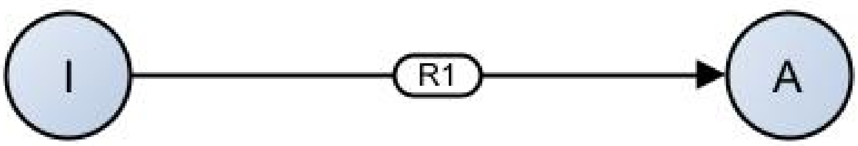

Figure 5.2: N System.

$$R_N = R_1, \qquad I_{1,N} = 1.$$

The importance of reliability for this N system is:

$$R_N = 0.7$$

For the second and third partial systems, in the same way, we extract the importance of the reliability of the system and the compounds

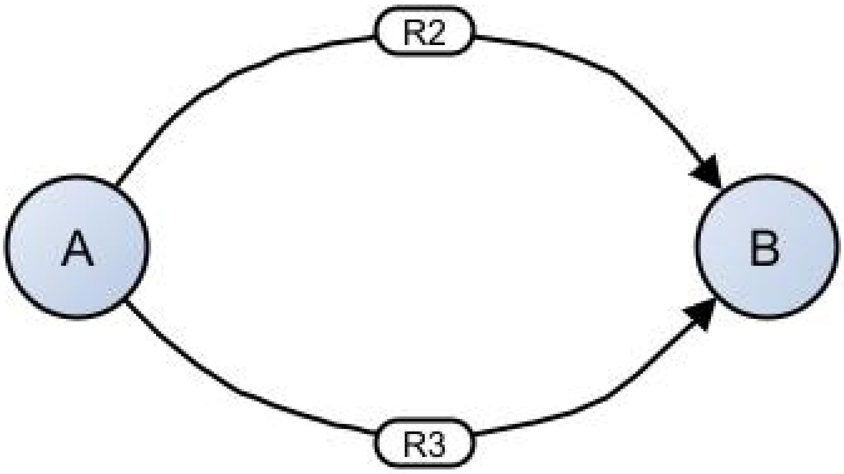

Figure 5.3: V System.

$$R_V = R_2 + R_3 - R_2 R_3$$
$$I_{2,V} = 1 - R_3$$
$$I_{2,V} = 0.1$$
$$I_{3,V} = 1 - R_2$$
$$I_{3,V} = 0.5$$
$$R_V = 0.95$$

Table 5.5: The reliability importance and level of units in Fig. 5.3.

Components	$I(i)$	Level
R_3	0.5	1
R_2	0.1	2

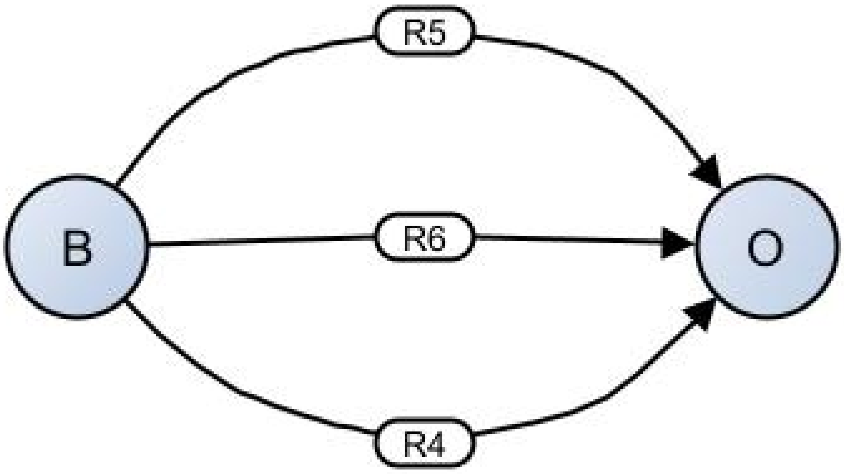

Figure 5.4: D System.

$$R_D = R_4 + R_5 - R_4R_5 + R_6 - R_4R_6 - R_5R_6 + R_4R_5R_6$$

$$I_{4,D} = 1 - R_5 - R_6 + R_5R_6$$

$$I_{4,D} = 0.3$$

$$I_{5,D} = 1 - R_4 - R_6 + R_4R_6$$

$$I_{5,D} = 0.24$$

$$I_{6,D} = 1 - R_4 - R_5 + R_4R_5$$

$$I_{6,D} = 0.2$$

$$R_D = 0.88$$

Table 5.6: The reliability importance and level of units in Fig. 5.4.

Components	I (i)	Level
R_4	0.3	1
R_5	0.24	2
R_6	0.2	3

5.5. Relation ships Among I_N, I_V, I_D and I_s

Took the mixed consecutive system and divided it into three systems. Carried out the previous operations that were mentioned in the complex consecutive system, and thus we extracted the reliability for it and for each system. Then we extracted the importance of reliability for each partial system and the importance for the compounds as well. Then, after that, we began to extract the importance of reliability for the mixed series system, by multiplying the importance of the reliability of the systems. The three parts are as follows

$$R_m = R_N \times R_V \times R_D$$

$$= 0.7 \times 0.95 \times 0.88$$

$$R_m = 0.5852$$

Also, we were able to extract the importance of each component in the mixed series system through the following sub-systems

$$I_{1m} = I_{1N} \times R_V \times R_D$$

$$= 0.7 \times 0.95 \times 0.88$$

$$I_{1m} = 0.8360$$

And

$$I_{2m} = I_{2V} \times R_N \times R_D$$

$$= 0.1 \times 0.7 \times 0.88$$

$$I_{2m} = 0.0616$$

And so on for the rest of the other vehicles. This conclusion gives us a dimension in the future that we can generalize this case to N of mixed systems from which we cannot extract the importance of reliability as well as the importance of its compounds, so this method was formed for such cases that are difficult to extract

$$R_z = M \times A \times F \times \cdots\cdots \times G_N \tag{5.4}$$

CHAPTER 6

CONCLUSIONS AND FUTURE WORKS

6.1. Conclusions

This thesis proves that can develop a complex network consisting of two complex systems linked together in the form of a series, and use some methods to calculate the reliability of the complex network of series, such as the method of minimum paths and minimum cuts. After that, he studied the relationship between the sub-systems and the complex network of after applying the methods of minimum cuts and minimum paths. Equation (3.1) in Figure (3.2) and Equation (3.2) in Figure (3.3) result in Equation (3.3) in Figure (2.1).After that, the division method was generalized to include N of complex networks. The total number of minimum paths for complex-series networks with mismatched partial networks was generalized. Equation (3.4) in Fig. (3.8), and the total number of minimum paths for complex-series network was generalized with Partial networks symmetrical to equation (3.5) in Fig. (3.9). Then he generalized the calculation of the total number of minimum cut sets of complex-series network with different sub-network, equation (3.6).And we get equation (3.7) in the case of the sub-networks are symmetrical. The researcher also used one of the methods to improve the reliability of the systems (element redundancy) and (unit redundancy) methods, and we studied the relationship between subnetworks and the complex-series network. In the fifth chapter, can say that the results of the importance of the network of complex -series coincide with the results of the importance of sub-networks using the Birnbaum scale, and a relationship was found between the results of the levels of importance of the components in the sub-networks and the importance of the components of the network of complex -series.

6.2. Future Works

1. Advise utilizing different techniques to determine the reliability of complex networks to investigate the reliability of the network of complicated series depicted in Fig 3.2
2. To increase system dependability on partition technology, advise using contemporary techniques and examining the outcomes.
3. Propose to apply the partition technique to other systems, such as the parallel system, and compare the outcomes.

REFERENCES

[1] Albert, R., Baribasi, A.L.: Statistical mechanics of complex networks. Rev. Mod. Phys.74(1), 47-97 (2002).

[2] Allan, R. N., Billinton, R. and De Oliveira, M. F., An efficient algorithm for deducing the minimal cuts and reliability indices of a general network configuration, IEEE Transactions on Reliability, vol. 25.4 (1976), pp. 226-233.

[3] Altiparmak, Fulya, Berna Dengiz, and Alice E. Smith." A general neural network model for estimating telecommunications network reliability." IEEE Transactions on Reliability 58.1 (2009): 2-9.

[4] Abd, F. H. and Hassan, Z. A. H.,On Some Approaches to Optimize the Reliability of Complex Networks , Thesis, University of Babylon, (2022).

[5] Abraham, J. A., An improved algorithm for network reliability. IEEE Transactions on Reliability, April (1979), R-28: pp. 58-61.

[6] Aggarwal, K.K., Reliability engineering,Center for Excellemce in Reliability engineering, Regional engineering College ,Kurukshetra, India,(1993).

[7] Allan, R. N., Billinton, R. and De Oliveira, M. F., An efficient algorithm for deducing the minimal cuts and reliability indices of a general network configuration, IEEE Transactions on Reliability, vol. 25.4 (1976), pp. 226-233.

[8] Alghamdi, S. M. and Percy, D. F., Reliability equivalence factors for a series-parallel system of components with exponentiated Weibull lifetimes, IMA Journal of Management Mathematics, (2015), vol. 28.3, pp. 339-358.

[9] Ansell, J. I. and Phillips M. J., Practical Methods for Reliability Data Analysis, Oxford University Press, New York, (1994).

[10] Ball, M.O., Colbourn, C.J. and Provan, J.S., 1995. Network reliability. Handbooks in operations research and management science, 7, pp.673-762.

[11] Beichelt, F. and Tittmann, P., Reliability and maintenance: networks and systems, CRC Press, (2012).

[12] Biegel, J. E., Determination of tie sets and cut sets for a system without feedback, IEEE Transactions on Reliability, vol. 26.1 (1977), pp. 39-42.

[13] Billinton, R. and Allan, R. N., Reliability evaluation of engineering systems: concepts and techniques, Springer Science and Business Media, (2013).

[14] Chelson, P. O. and Eckstein, E., Reliability computation from reliability block diagrams, IEEE Transactions on Reliability, vol. 25 (1975), pp. 283-283.

[15] Chen, W. K., Graph theory and its engineering applications, vol. 5, World Scientific, (1997).

[16] Chithra, M. and Vijayakumar, A, The diameter variability of the Cartesian product of graphs", Discrete Math, vol. Algorithms Appl. 6 (2014), pp. 289 -301.

[17] Chelson, P. O. and Eckstein, E., Reliability computation from reliability block diagrams, IEEE Transactions on Reliability, vol. 25 (1975), pp. 283-283.

[18] Colbourn, C. J., The Combinatorics of Network Reliability, Oxford University Press, New York, Oxford, (1987).

[19] Dhillon, Balbir S. Maintainability, maintenance, and reliability for engineers. CRC press, (2006).

[20] Di Bona G, Forcina A, Petrillo A, et al. A-IFM reliability allocation model based on the multicriteria approach. Int J Qual Reliab Manage. 2016;33(5):676- 698.

[21] Dhillon, B. S., Design Reliability Fundamentals and Applications, 1st ed., CRC Press, Taylor, Francis group, Boca Ration, Florida, (1999).

[22] Diestel, R., Graph Theory. Electronic library of mathematics. Springer Science, (2006).

[23] Dod, M., Domination in graphs with application to network reliability, Hinweis Senior Theses (2015).

[24] Dohmen, K., Inclusion-exclusion and network reliability, The Electronic Journal of Combinatorics, Research Paper R36, 5 (1998), pp. 1 - 8.

[25] Elsayed, E. A., Reliability engineering, John Wiley and Sons, (2012).

[26] Faraci, V., Calculating failure rates of series/parallel networks, The Journal of Alion s, System Reliability Center, vol. First Quarter (2006), pp. 1-3.

[27] Feng, X., Zhu, X., Zhao, W. and Li, X., Reliability of Electric Vehicle with Wind Turbine Based on Particle Swarm Optimization, Chemical Engineering Transactions, (2018), 66, pp. 1291-1296.

[28] Gertsbakh, I. and Shpungin, Y., Network Reliability and Resilience, 1st ed., Springer-Verlag Berlin Heidelberg, (2011).

[29] Gilbert, G. T., Positive definite matrices and Sylvester's criterion, The American Mathematical Monthly, vol. 98.1 (1991), pp. 44-46.

[30] Govil, A. K., Reliability Engineering, TaTa Mc-Graw Hill Pub. Com. Ltd., New Delhi, India, (1983).

[31] Howeidi, H. S. and Hassan, Z. A. H., Use innovative methods to increase the reliability of complex and mixed networks, Thesis, University of Babylon, (2022).

[32] Hassan, Z. A. H. and Mutar, E. K., Evaluation the reliability of a high-pressure oxygen supply system for a spacecraft by using GPD method, Al-Mustansiriyah Journal of college of education, vol. special issue. 2 (2017), pp. 993 -1004.

[33] Hassan, Z. A. H. and Mutar, E. K., Geometry of reliability models of electrical system used inside spacecraft, 2017 Second Al-Sadiq International Conference on Multidisciplinary in IT and Communication Science and Applications (AIC-MITCSA). IEEE, (2017), pp. 301-306.

[34] Hassan, Z. A. H. and Udriste, C., Equivalent reliability polynomials modeling EAS and their geometries, Annals of West University of Timisoara Mathematics and Computer Science, vol. 53.1 (2015), pp. 177-195.

[35] Hassan, Z. A. H. and Udriste, C., Geometry of Reliability Models, Ph.D. Thesis, University Politehnica of Bucharest,(2016).

[36] Haynes, T. W., Hedetniemi, S. T. and Slater, P. J., Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York, (1998).

[37] Huang, Chia-Ling, and Wei-Chang Yeh. Simplified Swarm Optimization Algorithm for reliability redundancy allocation problems. International Telecommunication Networks and Applications Conference (ITNAC). IEEE, (2015).

[38] Hoyland, A. and M. Rausand, System Reliability: Models and Statistical Methods, John Wiley Sons, New York, (1994).

[39] Ireson, W. G., Coombs, C. R. and Moss, R. Y., Handbook of Reliability Engineering and Management, 2nd ed., McGraw-Hill Comp., U.S.A., (1995).

[40] Jula, Nicolae, and Cepisca Costin. Methods for analyzing the reliability of electrical systems used inside aircrafts. Recent Advances in Aircraft Technology. Intech open, (2012).

[41] Katiyar, Sapna. A Comparative Study of Genetic Algorithm and the Particle Swarm Optimization. International Journal of Technology, (2010), 2.2: pp. 21-24

[42] Kapur, K.C. and Lamberson, L.R., Reliability in Engineering Design, Wiley, 1977, New York.

[43] Ko lowrocki, K., Reliability of large systems and Complex Systems, Wiley Online Library, (2008).

[44] Kim, M. C., Reliability block diagram with general gates and its application to system reliability analysis, Annals of Nuclear Energy, vol. 38.11 (2011), pp. 2456-2461.

[45] Kim, Y. H., Case, K. E. and Ghare, P., A method for computing complex system reliability, IEEE Transactions on Reliability,vol. 21.4 (1972), pp. 215-219.

[46] Kulli, V. R. and Soner, N.D., Complementary Edge Domination in Graphs, Indian J Pure Appl.Math. 28, 1997, pp. 917-920.

[47] Kuo, S. Y., Lu, S. K. and Yeh, F. M., Determining terminal-pair reliability based on edge expansion diagrams using OBDD, IEEE Transactions on Reliability, vol. 48.3 (1999), pp. 234-246.

[48] Kuo, Way, and Ming J. Zuo. Optimal reliability modeling: principles and applications. John Wiley, Sons, 2003.

[49] Kumar, A.,. Khosla, A., Saini, J.S. and Singh, S., Meta-heuristic range based node localization algorithm for wireless sensor networks, in localization and GNSS (ICLGNSS), 2012 International Conference on IEEE, (2012) , pp. 1-7.

.

[50] Lakey, P. B. and Neufelder, A. M., System and Software Reliability Assurance Notebook, Rome Laboratory, Rome, (1996), pp. 6.1- 6.24.

[51] Lakey,P. B. and Neufelder, A. M. , System and Software Reliability Assurance Notebook, Rome Laboratory, Rome, (1996), pp. 6.1- 6.24.

[52] Limnios, N." Failure rate of non-repairable systems." Reliability Engineering 17.2 (1987): 83-88.

[53] Lazzaroni, M., Cristaldi, L., Peretto, L., Rinaldi, P. and Catelani, M., Reliability Engineering Basic Concepts and Applications in ICT, Springer Verlag Berlin Heidelberg, Germany, (2011).

[54] Leitch, R.D., Reliability Analysis for Engineers: An Introduction, Oxford University Press, New York, (1995).

[55] Limnios, N., Failure rate of non-repairable systems, Reliability Engineering, vol. 17.2 (1987), pp. 83-88.

[56] Moore, E.F., Shannon, C.E.: Reliable circuits using less reliable relays. J. Frankl. Inst. 262(3), 191–208 (1956)

[57] Mutar, E. K. and Hassan, Z. A. H., On the Geometry of the Reliability Polynomials, Thesis, University of Babylon, (2017).

[58] Nedjah, N., dos Santos, L., Coelho, and de- MacedoMourelle, L., Multiobjective swarm intelligent systems: theory experiences. Springer Science Business Media, (2009), vol. 261.

[59] Personal report of Gus Huneke, Failure Analysis Manager who I worked with at Control Data Corporation in the late 1970s. Gus had worked on these early computer systems as a young engineer in the early 1950s at Univac.

[60] Pham, Hoang, ed. Springer Handbook of engineering statistics. Springer Science Business Media, (2006).

[61] Padmavathy, N., and Sanjay K. Chaturvedi." Evaluation of mobile ad hoc network reliability using propagation-based link reliability model." Reliability Engineering System Safety 115 (2013): 1-9.

[62] Page, L. B. and Perry, J. E., Reliability of directed networks using the factoring theorem, IEEE Transactions on Reliability, vol. 38.5 (1989), pp. 556-562.

[63] Rebaiaia, M.L. and AitKadi, D., A new technique for generating minimal cut sets in nontrivial network, AASRI Procedia, vol. 5 (2013), pp. 67-76.

[64] Sulaiman, H. K. and Hassan, Z. A. H., A Study of Mathematical Models in Reliability of Networks, Thesis, University of Babylon, (2019).

[65] Sandler, G. H., System Reliability Engineering, Prentice-Hall Int. Series In Space Technology, Prentice Hall Inc., Englewood Cliffs N.J., (1963).

[66] Shaghaghi, Saba, et al. "Comparative analysis of GMDH neural network based on genetic algorithm and particle swarm optimization in stable channel design." Applied Mathematics and Computation 313 (2017): pp. 271-286.

[67] Simionescu, P. A. and Beale, D., Visualization of hypersurfaces and multivariable (objective) functions by partial global optimization, The Visual Computer, vol. 20.10 (2004), pp. 665-681.

[68] Singh, S., Shivangna, S., Mittal, E., Range Based Wireless Sensor Node Localization Using PSO and BBO and Its Variants. 2013 International Conference on Communication Systems and Network Technologies,(2013), pp. 309-315.

[69] Srinath, L. S., Concepts in Reliability Engineering, East-West Press Private Ltd., (1985).

[70] Silva, T. C., Zhao, L. (2016). Machine learning in complex networks. Springer.

[71] Thoft-Cristensen, P. and Baker, M. J., Structural reliability theory and its applications, Springer Science and Business Media, (2012).

[72] Todinov, M. T., Risk-based reliability analysis and generic principles for risk reduction, Elsevier, (2006).

[73] Van Sylke, R., and H. Frank." Network reliability analysis: Part I." Networks 1 (1972): 279-290.

[74] Xu, L., Chen, Y., Briand, F., Zhou, F. and Givanni, M., Reliability Measurement for Multistate Manufacturing Systems with Failure Interaction, Procedia CIRP, vol. 63 (2017), pp. 242-247.

[75] Yamuna, M. and Karthika, K., Medicine Names as a DNA Sequence using Graph Domination, Scholars Research Library Journals, USA, 5(6), (2014), pp. 2747-2756.

[76] Yang, Guang. Life cycle reliability engineering. John Wiley, Sons, (2007).

[77] Zarei, E., Khan, F., Abbassi, R. (2021). Importance of human reliability in process operation: A critical analysis. Reliability Engineering System Safety, 211, 107607.