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Navier-Stokes Equations for Elliptic Complexes

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We continue our study of invariant forms of the classical equations of mathematical physics, such as the Maxwell equations or the Lamé system, on manifold with boundary. To this end we interpret them in terms of the de Rham complex at a certain step. On using the structure of the complex we get an insight to predict a degeneracy deeply encoded in the equations. In the present paper we develop an invariant approach to the classical Navier-Stokes equations.

Keywords: Navier-Stokes equations, classical solution.
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Introduction

The problem of describing the dynamics of incompressible viscous fluid is of great importance in applications. In 2006 the Clay Mathematics Institute announced it as the sixth prize millennium problem, see [5]. The dynamics is described by the Navier-Stokes equations and the problem consists in finding a classical solution to the equations. By classical we would mean here a solution of a class which is good motivated by applications and for which a uniqueness theorem is available. Essential contributions are published in the research articles [9, 13, 16, 19, 20, 27] as well as surveys and books [6, 17, 18, 21, 22, 29], etc.

In physics by the Navier-Stokes equations is meant the impulse equation for the flow. In the computational fluid dynamics the impulse equation is enlarged by the continuity and energy equations.

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