Optimization Dimension of the Finned Surface based on the Logarithmic Temperature Profile

By
Emad D. Aboud
College of Engineering- Babylon University, Iraq, 2010

Abstract:

The use of an extended surface (fin) is one of the important ways to heat transfer from the engines and all kinds of electrical transformers. Therefore, it is so important to study and obtain the best conditions of operation to get the best performance per unit cost. This research presents an analytical study to obtain the temperature distribution through a finned surface. Two new suggested techniques based on the finite element method to obtain the optimum design of finned surface subjected to logarithmic temperature profile were presented. The suggested techniques are based mainly on the temperature difference along fins domain in the longitudinal and transverse directions. A simple apparatus of rectangular fins is studied to obtain the optimum design related to its dimensions. The finite element technique (Ansys 9) is used to simulate and solve the governing differential equations. The problem was considered under free convection mode. It was noted that the suggested optimized criterion give an excellent results as compared with the original fin shape. The temperature response plays an important rule in the selection of suitable optimized criteria. It is so important to present that the profile of the optimized fin length is largely depends on the temperature profile that firstly applied on the fins surface.

Keywords / Optimum Design, Finite Element Method, Finned Surface, Free and Forced convection, ANSYS 9.

الأبعاد المثلى لسطح مزعنف مستند لنموذج حراري لوغاريتمي

عماد داود عبود

قسم الهندسة الميكانيكية–كلية الهندسة – جامعة بابل

الخلاصة:

إن استخدام السطح المزعنف هو من اهم طرق انتقال الحرارة من المحركات وكل انوع المحولات الكهربائية. لذلك، من المهم جدا ايجاد ودراسة افضل ظروف العمل لتعطي افضل النتائج لوحدة الكلفة. يظهر هذا البحث دراسة تحليلية لايجاد التوزيع الحراري خلال سطح مزعنف. اقترحت تقنيتان عددتيان جديدتان اعتمدت على طريقة العناصر المحددة لايجاد التصميم الامثل لسطح مزعنف معرض الى نموذج لوغاريتمي للحرارة. اعتمدت تلك التقنيتان وبشكل اساسي على الاختلاف في درجات الحرارة باتجاه الزعنفه والعمودي عليها. درس نموذج بسيط لسطح مزعنف لايجاد التصميم الامثل المتعلق بابعاد الزعانف. استخدم البنرنامج العددي (الانسز ٩) لصياغة وحل المعادلات الحاكمة. افترضت المشكلة بانها تحت الانتقال الحر للحرارة. لوحظ بان التقنيتان قد اعطت نتائج ممتازة اذا ما قورنت بالنتائج قبل الامثلية. تلعب الاستجابة الحرارية دور اساسي في تعيين المتطابقة المثلى للتصميم. من المهم توضيحه بان نموذج (هيئة) طول الزعانف المثلى يعتمد بشكل كبير على نموذج الحرارة الذي سلط سابقا قبل التصميم الأمثل.

Nomenclature:

A : area, m^2
Cp : specific heat of air, $J / Kg_* K$
g : gravitational acceleration, m / sec^2
h : convection heat transfer coefficient, $W / m^2{}_*K$
K : thermal conductivity, $W / m^2{}_*K$
l : characteristic length, m
L_o : the total fin length, m
L_i : effective (optimized) length of the fin number i, m
N_i : number of node
P : perimeter, m
q : heat transfer rate, W
s : element coordinate
t : element coordinate
T : temperature, K
T_i : temperature of fin at node i, K
u : velocity in x-axes, m/sec
v : velocity in y axis, m/sec

Greek symbols:

P : fluid density, Kg/m^3
ρ_i : ideal gas density, Kg/m^3
β_e : thermal expansion coefficient of fluid (1/K)
υ : kinematics viscosity ,m^2/sec
Φ : degree of freedom

Subscripts:

Conv : convection
r : radiation
i : nodes
j : fin number
∞ : free stream condition

Introduction:

Due to the rapid development in the mechanical and electrical devices which represents the more important in our lives especially that concerned with energy transitions, the requirement to the rapid movement of heat is increased. They produce an expanding demand for high performance heat-transfer components with progressively smaller weights, volumes, costs, or accommodating shapes.

The extended surfaces or fins are used to increase the heat dissipation in many engineering and industrial applications such as the cooling of combustion engines, electronic equipments, compressors, and aircraft and so on. Many papers for the various fin shapes using many kinds of methods have been presented. (Kang and Fijol) **(1)**.

One of the more important feature in the fins are the thermal performance as a basic guide indicates the aim of the extended surfaces typical components are found in air, land and space vehicles and their power source; chemical and refrigeration electrical and electronic circuitry; conventional furnaces and gas turbines; process heat dissipaters and waste-heat boilers; nuclear-fuel modules; direct energy conversion and many more.

True optimization involves many different considerations: thermal, mechanical, fluid, and system environment all play a role, as does manufacturability. In reality, the design process incorporates many different limits in the search for a suitable thermal solution that is easy to manufacture and works in the target system (Biber) **(2)**.

In the design and construction of various type of heat transfer equipment simple shapes are used as cylinder and plates to transfer heat between sources and sink. They provide heat-absorbing or heat-rejecting surfaces, and each is known as a (prime surface). In order to avoid the problem of high convective resistance between the prime surface and the surrounding, the surface area is extended by adding extra surfaces to the prime surface in order to increase the heat transfer area.

The ability of a designer to minimize the thermal resistance between the source of heat dissipation and the thermal sink is essential in controlling maximum operating temperatures and consequently the long term reliability and performance of electronic components. Typical electronic packages can introduce a complex network of resistive paths as heat passes from the integrated circuit through various laminated structures, bonding adhesives, lead frames or sometimes ball grid arrays (Culham and Muzychka) **(3)**.

There are many different simulations that involve combined conduction and convection effects, the most frequent application is one in which an extended surface is used specifically to enhance the heat transfer rate between a solid and adjoining fluid. Such an extended surface is termed as a fin. The surface area of the wall in principle is increased in two ways, in the first one; the extended surfaces are integral parts of the base materials, obtained by casting or extending process. The extended surface in the second way which may or may not made from the base material, are attached to the base by processing, soldiering, and welding.

It is normally more important to maximize the efficiency of fin with respect to the quantity if fin materials (mass, volume and cost) because such an optimization has obvious economic significance. It is more important to note the previous works mentioned in previous papers dealt with the fin optimization and concluding some closing remarks in this search for some facilities.

(Biber) **(2)**, describes the constrained optimization process for a parallel plate heat sink with a dedicated fan for IC cooling. Some of the factors considered are fin thickness and density, fin height, overall envelope, and performance. The constraining factors were chose fan and associated fan curve, weight, cost, and manufacturing technology. The direct iteration method was used to obtain the optimum design of the selected parameters. The emphasis was on illustrating the complexity of the process and the number of factors to consider when pursuing an optimum cooling solution. He concluded that there were a significant relationship between the fin thickness and density, the fin length (more that required) was decreased the fin performance.

(Sfeir) **(3)**, was used the heat balance integral method to solve the heat flow and temperature distribution in extended surfaces for different fin shapes and boundary conditions. He concluded that the integral method that used gives good agreement with the analytical solution.

(Kang and Look) **(5)**, were study the Optimum design of trapezoidal fins in two directions, thermally and geometrically. They concluded that the changes in the geometry of the trapezoidal fin play an essential role in the thermal performance of fin itself.

(Kang) **(6)** , used the analytical two dimensional methods to obtain the optimum design of a triangular fin with variable fin base thickness. The influence of fin base height and fin base thickness on the temperature in the fin is listed. The optimization procedure was based in the results of convection boundary conditions and maximum heat losses. He concluded that the optimum heat losses were increased with increasing in fin volume.

(Ritzer and Lau) **(7)**, analyzed the thermal performance of an initial simple heat sink design and improve cooling. The optimization processes was based on the cost and volume reduction simultaneous. Several changes were examined in an effort to improve the thermal performance and/or to reduce overall cost.

(Do, et al) **(8)**, modified a similarity solutions for velocity and temperature distributions in the heat sink The Brinkman-extended Darcy equations for fluid flow and two-equation model for heat transfer used as the governing equations. Specifically, a method for analytically determining the permeability and the interstitial heat transfer coefficient was presented. Experimental investigations conducted to validate the proposed similarity solutions. From comparison of experimental and analytical results, the analytical results were shown to accurately predict the pressure drop and thermal resistance and they sowed that the formulation can be adopted for optimization processes. The required time was important parameters especially for optimization purposes.

(Naphon) **(9)**, presents theoretical results of the heat transfer characteristics and the fin efficiency of the annular fin. The mathematical models based on the conservation equations of energy and mass were developed and solved by the central finite difference method to obtain temperature distribution along the fin. The required time for solving the equations was effective. He concluded that the fin dimensions were of high importance in increase or decreases the fin performance. There was reasonable agreement between the results obtained from his model and that obtained from the other.

(Shah, et al) **(10)**, presents a numerical analysis of the performance of heat sink. He examines effect of the shape of the heat sink fins, particularly near the center of the heat sink, on the thermal performance of the package. He studied different shape of fins (arbitrary in shapes and their dimensions) for search for an optimal heat sink design that would improve the thermal performance. Parallel plate fins have been studied by removing fin material from the region near the center. It is found that removal of fin material from the central region of the heat sink enhances the thermal performance. The details of selected amount of the fins were not mentioned and he took the dimensions of the fins arbitrary.

(Suk and Dwight) **(11)**, prposed a new approach to a two dimensional for a design of trapezoidal annular fin. The optimization in his research was concentrated on the heat loss, fin tip radius, and fin base height as a function of the ratio of convection characteristic numbers, fin shape factor, dimensionless fin volume, and dimensionless fin base radius. He concluded that the trapezoidal fin shape gave the higher thermal performance.

(Kikkis and Raseloph) **(12)**, attained the optimum dimensions of trapezoidal profile convective circular fin using step by step algorithm. The main problem of the optimization procedure was the long time.

Closing Remarks:

Several closing remarks can be drawn for the literature above.

1- Most of the researcher above used the finite difference method and iterative procedure to solve the governing differential equation and the problem of the long required time and round off error is of high importance.

2- The researcher concluded that remove amount of the fin metals play an essential role to increase or degrease the thermal performance of the fin depending on the location of metal that removed. Therefore, the idea of the metal remove would be adopted in this research.

3- The temperature distribution decides the optimized fin shape through optimization processes.

At last, these important remarks will be taken in the consideration through the domain of the research.

Theoretical Analysis:

Finite Element Method (FEM) can also be useful in determining performance, especially conductivity gradients, temperature distribution, velocity profile.

The Governing Equations:

(1) Heat flow through the fin metal:

(a) Heat transfer analysis in fin as shown in figure (a).

$$dq_{in} - dq_{out} = 0$$
$$dq_x - dq_{x+dx} + dq_y - dq_{y+dy} = 0 \qquad \textbf{(1)}$$

From Fourier law gives

$$dq_{x+dx} = dq_x + \frac{d}{dx}(q_x)dx \qquad \textbf{(2)}$$

$$dq_{y_{+dy}} = dq_y + \frac{d}{dx}(q_y)dy \qquad \textbf{(3)}$$

Substitution of Eq.(8,and 9), in Eq.(7), gives

$$\frac{d}{dx}\left(k.A\frac{dT}{dx}\right)dx+\frac{d}{dy}\left(k.A\frac{dT}{dy}\right)dy=0 \tag{4}$$

$$\left(\frac{d^2T}{dx^2}\right)+\left(\frac{d^2T}{dy^2}\right)=0 \tag{5}$$

(b) Heat transfer analysis at vertical surface of the fin:

$$\begin{aligned} &dq_{in}-dq_{out}=0\\ &dq_x - dq_{x+dx} + dq_y - dq_{y+dy} - dq_{conv}=0 \end{aligned} \tag{6}$$

The convection heat transfer rate may be expressed as

$$dq_{conv} = h\,P.dx.\,(T - T_\infty) \tag{7}$$

Substitution of Eq.(2,3), in Eq.(6), gives

$$\frac{d}{dx}\left(k.A\frac{dT}{dx}\right)dx+\frac{d}{dy}\left(k.A\frac{dT}{dy}\right)dy-2h.p\left(T-T_\infty\right)dx=0 \tag{8}$$

This equation can be rearranged as

$$\left(\frac{d^2T}{dx^2}\right)+\left(\frac{d^2T}{dy^2}\right)=0 \tag{9}$$

(2) continuity, momentum, and energy equations:

The governing equations of continuity, momentum, and energy for a steady, incompressible flow are given below. (Masterson) **(13)**, the continuity equation in X and Y directions are:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \tag{10}$$

The momentum equations in x and y directions are:

Momentum in X axes:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{11}$$

Momentum in y axes:

$$u\frac{\partial \upsilon}{\partial x}+v\frac{\partial \upsilon}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-g(\rho_i-\rho)/\rho_i \tag{12}$$

And Energy equations in X and Y directions are:

$$\rho\, C_p \left(u\frac{\partial T}{\partial x} + \upsilon \frac{\partial T}{\partial y} \right) = K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \qquad \textbf{(13)}$$

$$\alpha = \frac{k}{\rho c} \qquad \textbf{(14)}$$

(Arpaji and Larsen) **(14)** for free convection flow, the change in density is responsible for the flow and the ideal gas state equation was provided as an input to estimate the fluid density,

$$\rho = \rho_i (1 - \beta_e (T - T_i)) \qquad \textbf{(15)}$$

Assumptions:

Important to the analysis of any fin geometry are the constrained the used to define the problem to simplify thee solution. The present analysis is based upon some assumptions which are common to most the previous investigation (Kraus 1988). The analysis of the finned surface is based on some assumptions. Two dimensions and steady state is assumed for fluid motion and temperature distribution. There is perfect contact between the fins and the surface and the material is homogenous. All the physical properties are assumed to be constant except for the density variation with temperature. The temperatures in the rectangular fins are constant. The fluid is considered viscous and incompressible.

Finite Element Discretization:

A mesh is generated from the node of a “grid”. The term grid is used in this work to define the set of nodal points, which puts up the respective mesh. Figure (1) shows proposed our finite element model (finned surface), created by using two dimensional fluid elements (Fluid 141) of quadrilateral shape as shown in Fig (3).

Element Parameters:

Fluid 141 can be used to model a transient or steady state fluid/thermal systems that involve fluid and / or non-fluid regions (fin metal). The conservation equations for viscous fluid flow and energy are solved in the fluid region, only the energy equation will be solved in the non-fluid region. For the FLOTRN CFD element, the velocities are obtained from the conservation of momentum principle, and the pressure is obtained from the conservation of mass principle. The temperature is obtained from the law of energy conservation. The specification of FLUD 141 according to ANSYS program can be shown as follow:

Element name	FLUID 141.
Nodes	I, J, K, L.
Degree of freedom	U, V, P, T, Kinetic energy .For more information, one can see about the element in element manual thought Ansys help.

And the shape function can defined as flow:

$$\Phi = N_1\Phi_1 + N_2\Phi_2 + N_3\Phi_3 + N_4\Phi_4$$

Where Φ :- the degree of freedom.

$$\begin{bmatrix} N_1 = (1-s)(1-t)/4 \\ N_2 = (1+s)(1-t)/4 \\ N_3 = (1+s)(1+t)/4 \\ N_4 = (1-s)(1+t)/4 \\ \end{bmatrix}$$

The finned surface is presented in Fig(1) and primary dimensions of fin thickness, lengths, and distance between them. The fins are equally distributed along the surface with surface thickness is 5 mm and total surface length of 13.5 cm. A convergence criterion to obtain the suitable mesh size was studied for more times and it was noted that the best convergence occurs at 10 elements along fin length and 3 elements along fin thickness.144 elements along the surface length and five in the thickness direction, as shown in Fig (1a). The temperature is distributed at the surface in a logarithmic profile as shown in Fig (1b).

Results and Discussions:

Due to the dependency of the suggested techniques that presented in this research on the temperature difference of the fins; therefore, the concentration of the results will be on the temperature difference and the changing in the temperature profiles.

Temperature Distribution of Constant Fin Length:

Fig(4) represents the temperature distribution along the domain of the problem (surface and fins) of the fin at constant fins length (the lengths of the fins are equal), its noted that the temperature is maximum at the fins bases and decreases in the direction of the fins ends due to the convection and conduction heat transfer that presence by thermal conductivity which affect the heat transfer through fins material by conduction and heat transfer coefficient that affect the heat transfer between fins and the surrounding by convection. The temperature difference from node to node in the fin domain play an important rule in the amount of heat that transferred or removed from the fin itself .Therefore, it will be depends on the temperature differences between the base and fin end to obtain a simple relationship. In the same time, surface of the fins have a great effect to decide and define the performance of the fin. Also, it is noted that temperature difference is different from each fin to other so that one can see that the temperature difference is maximum at the upper section of the fins and reduced gradually up to the final or last section of the fins.

Now, in this search, it is not severe important to obtain values of temperature or temperature difference occurs at each fins but its severe important to obtain optimum criterion. One of the important goals is to reduce the temperature at location of maximum values to a specific level. One of excellent method to doing that is the increasing fin surface area and thereby an increasing in heat transfer to the surroundings and means also maximum temperature difference or maximum fins performance. Then, to increase the fin performance, it is important to increase the fin length at the region of maximum temperature differences and substitutes these increasing in fin length from the fins at the region of temperature differences are not important conditional keeping the total fins length is constant in order to recognize the differences would be present due to suggested optimum criterions. In the other way, it is also to obtain the optimum area or optimum fin configuration. In the next section a new suggested techniques to obtain the optimum fin length and thickness will presented.

Optimum Criteria:

The design of a fin thus becomes an open-ended matter of optimizing, subject to many factors. Some of the factors that have to be considered include the weight of the materials added by the fins, the geometrical configuration of the channel that fin lies on it, the cost and complexity of the manufactured. The optimum dimensions of the fin can be then defined in either minimum weight (material) for dissipated a given heat amount or maximum amount of heat dissipation for a given quantity of weight. In this research, the concentration will be on the second optimum dimension.

Surely, to increase the rate of heat transfers to the surrounding one can increase fin length or replace the rectangular area to a new configuration one corresponds each other. But the major problem here is firstly, what the ratio of the fins length that increase or decrease? and secondly, what are the parameters that indicate the new configuration of area?. Therefore, two suggested techniques are created to solve these problems.

Optimum Fin Length:

The criteria of optimization are so simple and with excellent results. The first is to obtain optimum fin length and the other to obtain the optimum fin configuration (area). They mainly depend on the temperature differences at each fin along the surface. Table (1) presents the temperature difference of fins. Fin length is firstly (2 cm), the total fin lengths is (2cm*11cm =22cm). The criteria are purely based on basics of the finite element method and can be represented by the following suggested (suggested in this research by the researcher itself), relationship;

$$L_i = L_0 * \left(\Delta T_i / \Delta T_{total}\right) \quad \textbf{(16)}$$

Where :-

L_i is the effective (optimized) length of the fin number i

L_0 is the total fin length (22 cm)

$\left(\Delta T_i\right)$ is the temperature difference of fin number i

$\left(\Delta T_{total}\right)$ is the overall temperature differences of all fins which represents the summation of all temperature differences of all fins.

$$\left(\Delta \mathrm{T}_{\mathrm{total}}\right) = \sum_{\mathrm{i}=1}^{11} \Delta \mathrm{T}_{\mathrm{i}} \quad \textbf{(17)}$$

In the finite element, the term $(\Delta T_i / \Delta T_{total})$ can be defined as temperature shape function for fin number i and can be denoted by N_i. According to this criterion, one can obtain the effective fin length to get good performance. This means that the fins of higher temperature difference will be of higher length and those of minimum temperature difference will be minimum length. The length of the upper fin group is increased and of the last group is decreased (which may be considered ineffective as compared with case at which some of its length will add to the upper fins group). After application the optimum criterion, a new fins length is obtained and illustrated in table (1) and can be shown in Fig (3).

After applying same free convection boundary condition again and solve the problem with new fins length (optimized fin lengths), it was noted that the temperature difference is decrease in the direction of cold section (lower section) and increasing in the direction of the hot section (upper section). This is caused by the increasing in fin length and which means an increasing in the contribution of fin lengths (which represents one of the important method to increase heat transfer rate).

It was noted that the values of temperature difference is decreased in the direction of the lower section of the surface which is wanted up to now. As comparison between the tables (1 and 2) of constant fin length and optimized fin length, respectively, it was noted that the overall temperature difference for all fins in two cases are $424K^o$ and $708.2K^o$, respectively. It is so easy to conclude that the new optimized fin length gives an excellent results in temperature gain ($708.2K^o$-$424\ K^o$ =$280\ K^o$) taken in the consideration the total fin length is the same (22cm) for two cases above.

It is so easy to note that as an example the first fin reduced the temperature by 101.85 in optimized fin length rather that the constant fin length. The low length of the fins have a lower values near the cold end of the fin array y=0 where the length reaches to ratio of (1.232/2) percent or 1.232 cm of the length. Therefore, the fin length increases towards the hot end of array y=22 where the length towards the hot end of array where the length reaches to (4.06/2) percent or 4.06cm.

It is important to note that from tables, the temperature difference in the optimized fin length at the upper section is so clear as compared with constant fin length. In the middle section, (fin number 6, 7 and 8), there are a closeness in the temperature difference. This means that the decreasing in the middle fin length has a small effect (may be neglected) on the temperature distribution (which means ineffective fin length). This is another guide to prove the validity of the suggested numerical technique or optimum criteria. But, the behavior is different for the lower section of fins where the temperature distribution in the optimized fin length is larger than that corresponding in constant fin length. Mainly there are two very important reasons, the first is because of the velocity in the vertical direction (v) and (u) in the horizontal direction at these fins caused by the difference in the fluid density which leads to increase the heat transfer rate and thereby an increasing in the temperature difference. The second is due to thermal resistance through the material itself or conduction resistance because in free convection mode, the heat transfer by conduction is important. This behavior can be explained by the temperature or heat is transfer to the regions of minimum thermal resistance.

Optimum Fin Area or Configuration:

In the previous section, the optimum fin length criterion was used to obtain optimum fin length for all fins simultaneously and depends on the temperature differences over all fins (in the longitudinal fin direction). In this section, effect of changing in fin area only (for fin area) for each fin on the temperature difference and heat transfer rate will be presented. Taken into the consideration the dependency on the temperature differences especially along fins itself rather that on overall temperature difference along the surface. The changing will involve the fin itself which means the changing in the fin configuration for same fin area (areas are equal before and after changing) keeping fin length constant. Each fin will be divided into six points equally spaced. Table (3) presents the temperature difference along each fin. The accuracy of the results is largely depends on the number of selected points.

The goal here is to obtain optimized thickness at each selected points to draw the new configuration (new area form) for each fins. Depending on the temperature distribution tabulated above, the thickness at each selected points can be obtained using the following ***suggested*** equation (suggested in this research by the researcher itself),

$$\mathbf{t_{f(i,j)} = (T_{(i,j)}/T_{meanj}) * 0.00}3 \qquad \mathbf{(18)}$$

Where :-

i indicate to point location or (x/l), (i=1 to 6 step 1).

j indicate to fin index or fin number, (j=1 to 11 step 1).

$t_{f(i,j)}$ represents thickness at point i of fin number j.

$T_{(i,j)}$ represent temperature of fin i at number j.

T_{meanj} represents the mean temperature values for the six selected points for fin number j.

After the application of the suggested technique, the new optimized thickness of each selected points at each fin were tabulated in table (4).

Depending on the values of optimized thickness tabulated above, the new fin configuration is drawn but with illogical fin form which presented fitness loss of the curve lines of the fin area which may be leads to other difficulties especially during the manufacturing. Therefore, it is so important to obtain the nearest tapered thickness (good smoothing or curve fitting) configuration using surface topology technique and calculus of variation. After application the calculus of variation and surface topology, a good smoothing in area curves was obtained as shown in Figure (4).

Temperature profile:

Figures (5, 6, and7) present temperature profile along fins for constant fin, optimized fin length, and optimum area configuration, respectively. From Fig(5) at which temperature is logarithmic profile on the surface, its so easy to note that the natural convection response of temperature along the fins is mainly depend on the temperature profile that applied on the surface of the fins and can be denoted by the connected bold circles presented in figures above. The temperature response gives excellent information about the optimum criteria should be used for optimum purposes.

In Fig (6), it was noted that there is some similar in the temperature response as compared with the first figure with some changes in the temperature difference. These changes are largely affected by the changing in thermal resistance due to increasing and decreasing in the area of fin (maximum thickness at the root and minimum thickness at the end of fin. The values of temperature of the fin tapered thickness are smaller than that corresponding in rectangular fin. Therefore, it is noted that the effectiveness of the optimized fin length is better than that related corresponding in the optimized fin length but surely they are better than that corresponding in the rectangular fins.

Conclusions:

Some conclusions can be drawn from the results obtained in this research:-

1- Finite element method is suitable method to predict the temperature distribution and temperature response in the finned surface.

2- Rearranging in the arrangement and configuration of the structure (finned surface) keeping all other parameters constant gives good results as compared with the origin structure.

3- The suggested techniques give an excellent results in optimization cased of finned surface and one can be adopted for this purpose.

4- The profile of temperature distribution through fins give an excellent indication about the criteria must be used for optimization purposes.

5- The profile of optimized fin lengths is largely depends on the original temperature profile that applied firstly on the surface as showed in Fig (1b) and Fig (3) of logarithmic temperature profile and logarithmic fin lengths.

6- The effectiveness of the optimized fin length criterion is better than the corresponding in the optimized fin configuration area.

What is the difference between the present work and the others?

At last, there are several differences between this work and the other, but the more distinct one is related with optimization equations. The optimizations equations are so simple, direct, don't need to iterative procedure and thereby, no time consumption to solve the equations and can be neglected as compared to the other optimize procedure.

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Tables:

Table (1): presents temperature differences at each fin and optimum fin length.

Fin index	Base temperature (K)	End temperature (K)	Temperature difference (K)	Optimized fin length (cm)
F1	468	398.34	79	4.06
F2	456.61	389.3	66	4.05
F3	416.66	367.74	49	2.5186
F4	408.9	363.38	45	2.313
F5	377.57	344.58	34	1.747
F6	370	340.58	30	1.54
F7	367.13	338.35	29	1.49
F8	362.85	338.92	27	1.387
F9	355.93	331.217	24	1.232
F10	351.10	328.35	24	1.232
F11	347.37	326.28	21	1.07

Table (2): shows the temperature difference in the two cases (constant fin length and tapered fin lengths).

Temperature difference constant fin length ($\Delta t_{i\ constant}$) (K)	Temperature difference in optimized fin length ($\Delta t_{i\ optimized}$) (K)	($\Delta t_{i\ optimized}$) - ($\Delta t_{i\ constant}$) (K)
79	180.85	101.85
66	149	83
49	82	3
45	70.7	34.3
34	41.7	7.7
30	35.6	5
29	30.18	1.18
27	29.93	2.98
24	37	13
24	30.8	6.8
21	23.3	2.3

Table(3): presents the temperature at each fins at different points.

Fin number	Temperature value at selected points (K).					
	x/l=1/6	x/l=2/6	x/l=3/6	x/l=4/6	x/l=5/6	x/l=1
F1	458	444	425	411	401	390
F2	456.6	431.1	413	400.2	392	389
F3	416.6	400.2	386.9	376.5	370	367.2
F4	410.5	394.5	381.9	372.2	366	363.2
F5	379.5	367.5	357.8	350.6	346.1	344.5
F6	371.8	361.22	352.2	346.1	342	340.2
F7	368.1	358.1	349.94	343.6	339.7	338.3
F8	364.6	354.41	346.8	340.98	337.2	336
F9	356.1	347.8	340.9	335.7	332	331.2
F10	351.7	343.9	337.66	332.74	329	328.5
F11	344	340.7	334.8	330	327	326.28

Table (4): presents fins and their corresponding thickness at each selected point.

	Fin thickness at each selected points (x/l) in mm						
Fin number	T_{meanj} (K)	x/l=1/6	x/l=2/6	x/l=3/6	x/l=4/6	x/l=5/6	x/l=1
F1	421.5	3.25	3.16	3	2.897	2.823	2.749
F2	413.6	3.316	3.13	3	2.91	2.841	2.821
F3	386.3	3.29	3.109	3	2.93	2.88	2.85
F4	381.2	3.23	3.104	3	2.931	2.88	2.86
F5	358.6	3.178	3.03	3	2.939	2.9	2.88
F6	252.25	3.166	3.07	3	2.95	2.91	2.89
F7	349.3	3.163	3.07	3	2.97	2.93	2.93
F8	346.5	3.151	3.07	3	2.95	2.925	2.92
F9	340.3	3.14	3.06	3	2.95	2.93	2.25
F10	336.9	3.131	3.06	3	2.96	2.95	2.92
F11	335.5	3.07	3.04	3	2.96	2.93	2.92

Figures:

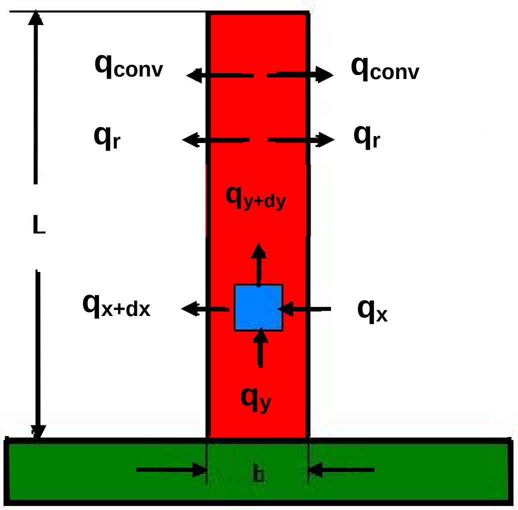
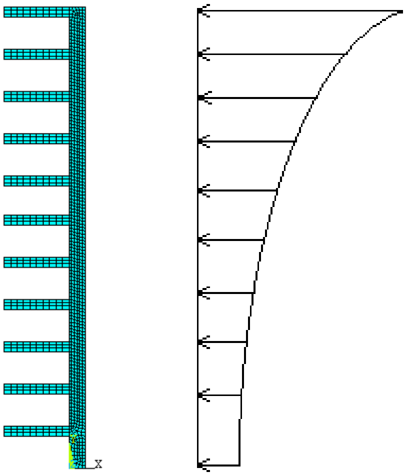


Fig. (a) : Heat balance in fins



a b

Fig. (1a) : mesh generation, Fig(1b), logarithmic temperature profile.

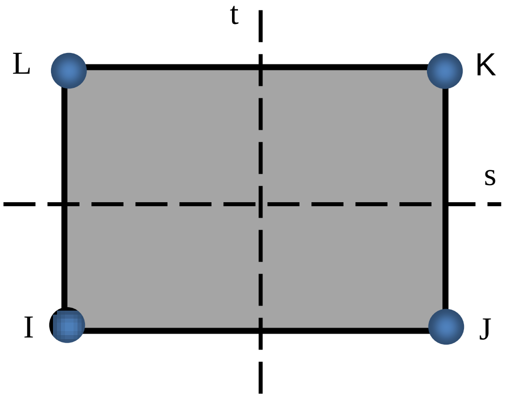


Fig. (2) : Quadrature two-dimension element

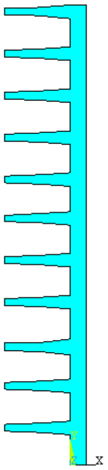


Fig. (3): presents optimized area configuration.

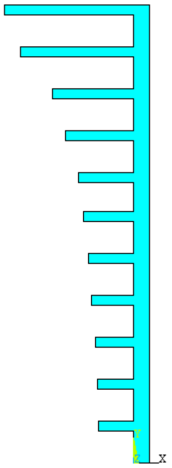
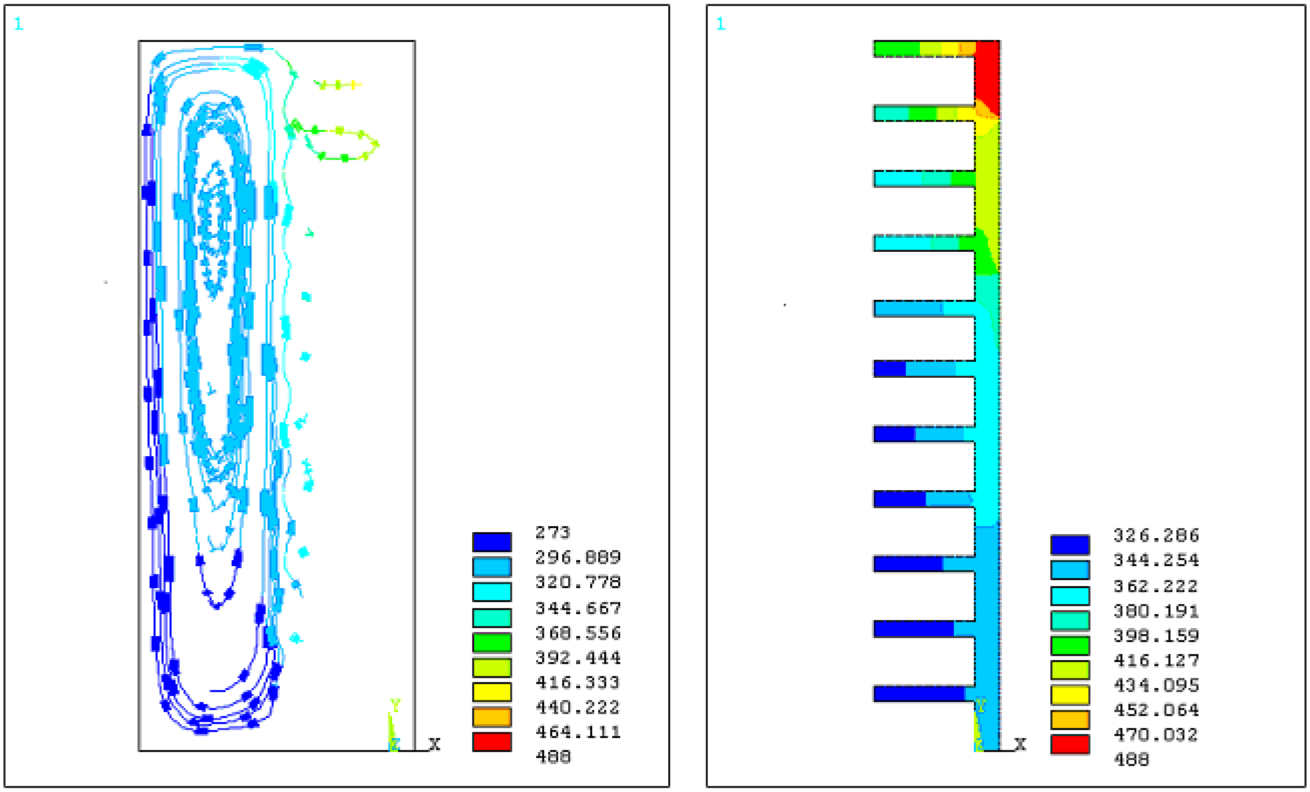


Fig. (4): presents optimized fin length.



(a) **(b)**

Fig. (5): Temperature distribution, (a): flow traces of temperature, (b): contour plot of temperature.

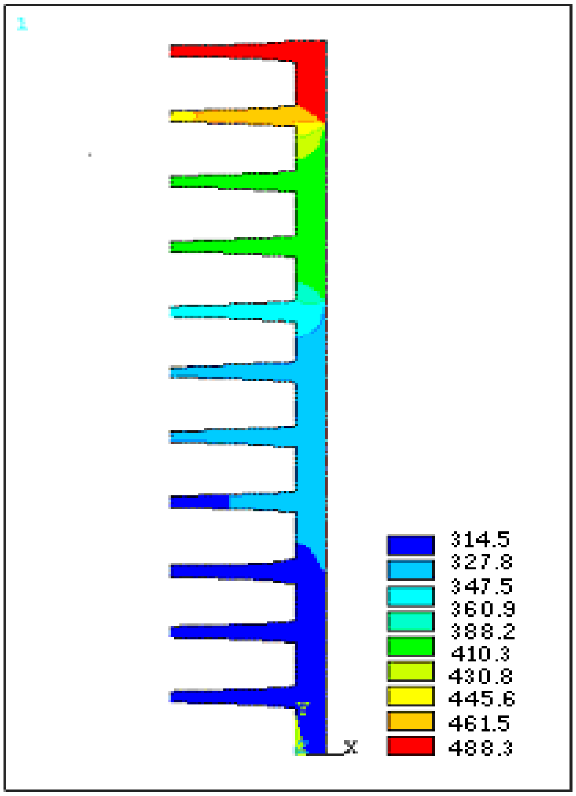


Fig. (6): Presents temperature distribution on optimized area configuration.

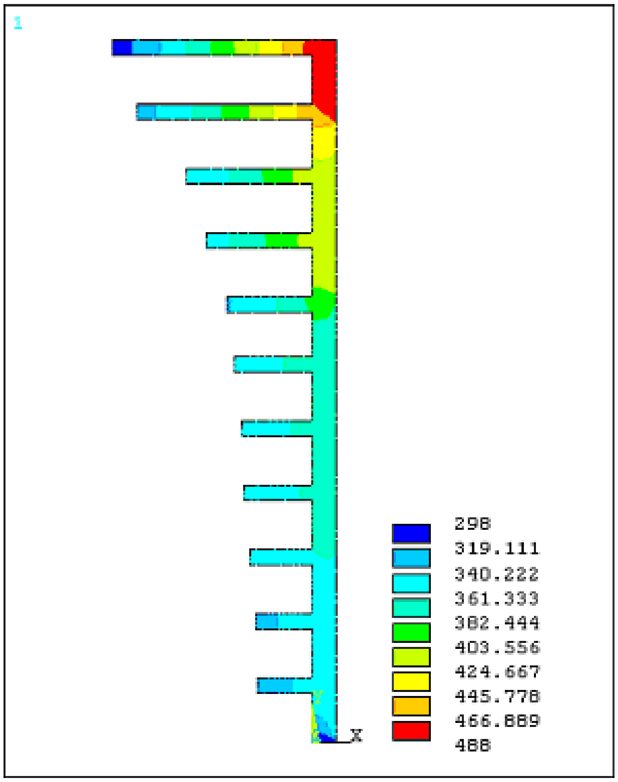


Fig. (7): Presents temperature distribution on optimized fin length.