# Thermal Analysis of turbulent Natural Convection phenomenon in an air filled inclined rectangular differential heated cavity

By

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## Abstract:

Heat transfer plays a major role in many processes in living systems. Natural convection in heat transport is one of the main species of heat transfer and is a mechanism, in which the fluid motion is not generated by any external source such as a fan, pump, etc. Its generated only by density differences in the fluid occurring due to temperature gradients. In the present research, natural convection heat transfer in an inclined rectangular cavity filled with air was presented. Top and bottom of vertical walls were adiabatic. The two horizontal walls of the opposing cavity have different temperature. The temperature of left wall was smaller than the right one. Dimension of the cavity **H** x L giving two dimensional flow .The hot and cold walls of the cavity were  $60^{\circ}$ C and  $20^{\circ}$ C respectively. Rayleigh numbers from  $2.7 \times 10^7$  to  $2.16 \times 10^8$ . Low Reynolds number of (k- $\varepsilon$ ) model was used to make the flow very close to the turbulent flow . This study illustrate the effect of the degree of inclination, Rayleigh numbers in the flow field, the dimensions of content and average Nusselt numbers on the thermal heat load. **ANSYS** package was used to construct the results, such as temperature, velocity and pressure distribution. The local and average Nusselt numbers were also presented. From the constructed results, it was found there are good accordance between the results as compared to other researchers.

Keywords: Thermal Analysis, Turbulent flow, Natural convection, Rectangular, Inclined cavity, (k-ε) model, Buoyant flow, and ANSYS.

التحليل الحراري لظاهرة الحمل الطبيعي المضطرب الحاصل في محتوى مائل مستطيل الشكل مملوء بالهواء باختلاف درجة حرارة السطحين

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#### الخلاصة:

أن أنتقال الحرارة يلعب دور آرئيسيآ في عدد من العمليات في الفعاليات الحياتية. الحمل الحراري الطبيعي في نقل الحرارة هو أد المصخة وغيرها . أن الحركة أهم الأنواع لنقل الحرارة وهو الآلية التي لا تحتاج فيها حركة السوائل ألى مصدر خارجي مثل المروحة أو المضخة وغيرها . أن الحركة تتولد فقط بسبب الأختلاف في كثافة السوائل نسبة ألى التدرج في درجة الحرارة. في هذا البحث تم دراسة أنتقال الحرارة بالحمل الحر في محتوى مائل مستطيل الشكل مملوء بالهواء ، القمة وقاع التجويف معزولة ، السطحان العموديان من التجويف لهما درجة حرارة مختلفة ، محتوى مائل مستطيل الشكل مملوء بالهواء ، القمة وقاع التجويف معزولة ، السطحان العموديان من التجويف لهما درجة حرارة مختلفة ، درجة حرارة المحتوى مائل مستطيل الشكل مملوء بالهواء ، القمة وقاع التجويف معزولة ، السطحان العموديان من التجويف لهما درجة حرارة مختلفة ، درجة حرارة الجدار الايسر اقل من الجدار الايمن. أبعاد المحتوى (Η . L) مما أعطى جريان ثنائي الأبعاد . أن درجة الحرارة لسطحي المحتوى الحار والبار و البار و الايسر اقل من الجدار الايمن. أبعاد المحتوى (Η . L) مما أعطى جريان ثنائي الأبعاد . أن درجة الحرارة لسطحي المحتوى الحار و البعاد والي تتراوح مابين (2001) إلى درجة الحرارة لسطحي المحتوى الحار و البار و البور د (٤ - ٤٥) ، (20°C) على التوالي ولأعداد رالي تتراوح مابين (2001) إلى درجة الحرارة لسطحي المحتوى الحار والبرد ثابتة عند (20°C) ، (20°C) على التوالي ولأعداد رالي تتراوح مابين (2015 الترقي و لأعداد رالي تشراوح مابين (2016 مالة) . و محال التدفق وجعله قريب جدا من الجريان المضطرب . أوضحت هذه الدراسة تأثير كل أستخدام عدد رينولد منخفض لنموذج (٤ - ٤٩) لمحاكاة التدفق وجعله قريب جدا من الجريان المصطرب . أوضحت هذه الدراسة تأثير كل معدر ميدار والبرد نابي في مجال التدفق ، أبعاد المحتوى ومتوسط أعداد بسلت على المصلوب . والحراري في هذه الدراسة تأثير كل أستخدام عدد رينولد منخفض لنموذج (٤ - ٤٩) لمحاكاة التدفق وجعله قريب جدا من الجريان المحلوب . أوضحت هذه الدراسة من درجة ميلان المحتوى ، أعداد رالي في مجال التدفق ، أبعاد المحتوى ومتوسط أعداد نسلت على الحمل الحر الحراري . في هذه الدراسة من درجة ميلان المحلوم الحرارة ، السرعة ، وتوزيع الضحل مالمحراج المحلوب . أمم مم درجة الحرارة ، السرعة ، وتوزيع الضحل . أمما ورام وماتشر

## Nomenclature:

В	aspect ratio	$\mathbf{B} = \mathbf{H} \mathbf{L}^{-1}$				
Ср	specific heat of air	$J (Kg K)^{-1}$				
g	acceleration due to gravity	$m \sec^{-2}$				
Η	Width of the cavity (m)					
Κ	turbulent Kinetic energy	_				
Kg	thermal conductivity of air	$W (m K)^{-1}$				
L	length of the cavity (m)					
Nu	Nussult number of air					
Pr	Prandtl number of air	$Pr = (\mu Cp) k^{-1}$				
Ra	Rayleigh number of air	$Ra = (g_{\beta} \Delta TL^{3}) (\alpha v)^{-1}$				
Ret	Turbulent Reynolds number	$\operatorname{Re}_{t} = k^{2} (v \varepsilon)^{-1}$				
Т	temperature in Kelvin (K)					
q	Heat flux	$W m^{-2}$				
u	velocity in x-axis					
V	velocity in y-axis					
х, у	coordinates along x-axis and y-axis					
Greek symbols						
α	thermal diffusivity	$m^2 sec^{-1}$				
β	thermal coefficient of expansion	K <sup>-1</sup>				
3	dissipation of kinetic energy of fluctuation	W kg <sup>-1</sup>				
v	molecule kinematic viscosity	$m^2 \sec^{-1}$				
$v_{\rm t}$	turbulent viscosity	kg $(m.sec)^{-1}$				
ρ	density of air	kg m <sup>-3</sup>				
$\sigma_k$ , $\sigma_\epsilon$ turbulence model constants						
μ	dynamic viscosity of air	kg $(m.sec)^{-1}$				

dynamic viscosity of air μ

shear stress at the wall τ

slip coefficient η

mean free path λ

momentum accommodation coefficient σ

## **Subscripts**

- т mean
- hot h
- cold с
- slip S
- change Δ
- 1, 2, 3 notation

## 1.1 - Introduction:

The most important criteria which should be tacking into the consideration for each design process are cost and time. For specific situation, the cost is dominate rather than time. A dramatic development in the technology and industrial is appeared in the recent years. The development in technology means development in the structure and construction. Besides, the best way to reduce the unit cost which be very effective especially for larger constructions is the reduction in the construction materials (constructions volume). The smallest element of the construction material is the brick as a commonly used in the civil construction. The easiest way to reduce its volume is create a small cavity inside the brick itself. Dimension and shape of the cavity depends on the designer itself. Whatever its shape, cavity means a specific reduction in the cost. Beside money saving, another advantages can be noticed; heat and sound isolation. Then it's very important now to study the heat transfer processes through a cavity between walls like our problems in this research.

The important studies around turbulent Natural Convection in an air filled inclined rectangular cavity was increasing because of its presence both in nature and engineering applications. A schematic representation of the system under investigation was shown in Fig. (1), where  $\mathbf{L}$  is the length of the cavity  $\mathbf{H}$  the width of the cavity. The gravity vector was directed in the negative y coordinate direction. A general analysis was presented for transport from horizontal isothermal surface allowing for the variation of all the fluid properties.

W.P. Jones and B.E Lander (1972), reported a new model of turbulence in which the local turbulent viscosity is determined from the solution of transport equations for the turbulence kinetic energy and the energy dissipation rate. A prime components of a suitable form of the model for regions with low turbulence Reynolds number were supported. This work also support the computational economy, range of applicability and physical realism were best served at present by turbulence models in which the magnitudes of two turbulence quantities, the turbulence kinetic energy  $\mathbf{k}$  and its dissipation rate  $\boldsymbol{\epsilon}$ , were calculated from transport equations solved simultaneously with those governing the mean flow behavior.

**N.C. Markatos, et. al (1982),** reported a computational procedure for predicting velocity and temperature distributions in enclosures containing a fire source. The two dimension equations of conservation of mass, momentum, energy, turbulence energy and eddy dissipation rate were solved at each x-constant line for  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $\mathbf{k}$ ,  $\mathbf{\epsilon}$ ,  $\mathbf{h}$ , using the **Jacobi** method for  $\mathbf{u}$  and  $\mathbf{v}$ , and the tri-diagonal matrix algorithm (**TDMA**) for the other variables. An assumed pressure distribution is used to solve these equations.

**O.J. Svec and L.E. Goodrich (1986),** studied natural convection in the cavity of a basement block wall. A full-scale experimental programmed to study the Adfreezel heaving problem of an insulated basement was examined at the **National Research Council of Canada**. Apart from heaving results this experiment has be also yielded interesting findings related to the natural convection in concrete-block wall cavities. Both the field data and a simplified finite element model calculation showed that natural convection in a block basement wall preset a significant factor contributing to heat losses. Insulating only the inside upper portion of the wall, as was current practice, was not be effective in reducing heat losses with block walls, while full length insulation was lead to significant lowering of ground temperatures near footing levels.

**Yen-Ming Chen, and Arme J. Pearlstein (1989),** studied the stability free-convection flows of variable viscosity fluids in vertical and inclined slots. He also studied the stability of the buoyancy driven shear flow of a variable viscosity Newtonian fluid between vertical or inclined plates at different temperatures. The analysis was capable of dealing with arbitrary viscosity temperature relations. They showed that, variable viscosity fluid, non- monotonic dependence of the critical Rayleigh umber on the inclination angle have been a great effect on the stability of the free convection flow in inclined slot.

**R. A. W. M. Henkes and C.J. hoogendoorn (1989)**, examined the boundary-layer equations, and the performance of different turbulence models for the natural convection boundary layer for air along a heated vertical plate using a numerical code for the solving. The algebraic Cebeci-Smith model, the standard  $\mathbf{k}$ - $\boldsymbol{\epsilon}$  model with wall functions for  $\mathbf{k}$  and  $\boldsymbol{\epsilon}$  and different low-Reynolds number  $\mathbf{k}$ - $\boldsymbol{\epsilon}$  models were tested. The Cebeci-Smith model calculates a too low wall-heat transfer and turbulent viscosity. The standard  $\mathbf{k}$ - $\boldsymbol{\epsilon}$  model with wall functions gives a too high wall-heat transfer. The use of low-Reynolds number  $\mathbf{k}$ - $\boldsymbol{\epsilon}$  models gave accurate results in heat transfer from the wall.

G. Barakos and E. Mitsoulis (1994), studied the natural convection flow in a square cavity for laminar and turbulent models with different wall functions. Control volume method was used to solve the conservation equations for laminar and turbulent rows for a series of Rayleigh numbers (**Ra**). The  $\mathbf{k}$ - $\boldsymbol{\epsilon}$  model has been used for turbulence modeling with and without logarithmic wall functions. Uniform and non-uniform (stretched) grids have been employed with increasing density to guarantee accurate solutions, especially near the walls for high (**Ra**) values. Comparisons with experimental data for heat transfer (Nusselt number) clearly demonstrated that the limitations of the standard  $\mathbf{k}$ - $\boldsymbol{\epsilon}$  model with logarithmic wall functions gave significant over predicted values.

Mohammed A. Khan (1996), studied the numerical solutions of the buoyancy driven turbulent flows in an inclined two dimensional rectangular enclosure in which one of the vertical walls was heated and the other was cooled. At low Reynolds number , two equations of  $\mathbf{k} - \boldsymbol{\epsilon}$  model were used to model the turbulent flow. The effect of various parameters such as the number of partitions, the angle of inclination and the Rayleigh number on the flow field and the average Nusselt number were investigated. The results showed that for optimum angle of inclination the average Nusselt number increases as the Rayleigh number increases.

Sundara Vadivelu and Kandaswamy (1999), carried out numerical investigations by maintaining one vertical wall at a uniform temperature and the other vertical wall at a different temperature. He showed that the fluid flow and heat transfer characteristics were significantly modified by the cavity inclination.

Y. S. Tian, T. G. Karayiannis (2000), investigated two-dimensional flow of turbulent natural convection in an air filled vertical square cavity at a low grade. The temperature and velocity distribution were systematically measured at different locations in the cavity, and were nearly anti-symmetrical. In this work, wall shear stress and the average Nusselt numbers were presented. Variation were found at mid-width and in the rate of velocity and temperature changes near the walls.

Man Yeong Ha, et. al (2002), obtained a two dimensional solutions for unsteady natural convection in an enclosure with a square body using an accurate and different chevyshev spectral collocation method. A spectral multi-domain methodology was used to handle a square body located at the center of the computational domain. Hot and cold walls was used as a boundary conditions. It was noted that, for a small values of Rayleigh and temperature difference between hot and cold walls a symmetrical fields and steady pattern. The Rayleigh number for the fluid flow and temperature fields became nonsymmetrical pattern.

**L. Adjlout et. al. (2002),** reported effect of hot wall on the flow structure in a differentially heated inclined square cavity. The results discussed at different inclination angles and Rayleigh number. They showed that hot wall undulation have a great effect on the flow and the heat transfer rate in the cavity.

A.I. Fomichv, et. al (2002), studied the natural convection flow of air in an inclined rectangular cavity for turbulent flow at varied angle of inclination from  $0^{\circ}$  (heated from below) to  $180^{\circ}$  (heated from above) with Rayleigh numbers ranging between  $10^4$  and  $10^{11}$ . The standard  $k - \varepsilon$  model for turbulence was used in the prediction of turbulent flows. Numerical predictions of the heat flux at the hot wall and the influence of the angle of inclination on the Nusselt number were presented. The Nusselt number shows strong dependence on the orientation of the cavity and the power law dependence on the Rayleigh number of the flow. Flow patterns and isotherms are shown to give greater understanding of the local heat transfer.

M. A. R. Sharif and W. Liu (2003), gave an account of turbulent natural convection in a side heated square cavity at various angles of inclination. The flow field and heat transfer characteristics became significantly different for inclination angles greater than  $45^{\circ}$ .

**O. Polat and E. Bilgen (2003),** investigate the conjugate heat transfer in an open shallow rectangular cavity at different Rayleigh numbers. Also they studied effect of angles of inclination and aspect ratio. They showed that the heat transfer was linearly or direct relationship with Rayleigh number.

**S.A.M Said, et. al (2005),** studied numerically, the two dimensional turbulent natural convection between inclined isothermal plate. Validation of the present computational procedures were carried out utilizing experimental and numerical data published in literature. The comparison indicated that very good agreement. The results also showed that the overall average Nusselt number was reduced. The rate of the reduction was increased as the inclination angle increased. The overall Nusselt number was largely depend on the inclination angles.

Manab Kumar Das et. al (2006), computed natural convection flow in a square enclosure having a centered internal conducting square block. Computations performed at different Rayleigh number and inclination angles to analyze conjugate natural convection heat transfer showed that at low values of (**Ra**),

the heat transfer was by conduction. The angle of inclination was found to have nominal effect on the heat transfer.

**K. Bouaraour** (2008), studied the numerical simulation of the turbulent natural convection in a square cavity, with differentially heated vertical walls and adiabatic horizontal ones. He was modeled the turbulence by the low of Reynolds number  $\mathbf{k}$ - $\boldsymbol{\epsilon}$  model. He also used the finite volume method to discretized the equations which controlling the flow.

**Santhosh Kumar M.K (2009)**, investigated numerically and experimentally the natural convective flows in two dimensional channels, opened to ambient conditions at both end sections. A two-dimensional laminar simulations were obtained by solving the governing equations using a **Fluent 6.2.16**. The results for Nusselt number, velocity and temperature profiles and heat transfer rate were computed. The results showed, at low Rayleigh number effect of varying density dominates over effect of thermal conductivity and thus results increase in Nusselt number. However at higher Rayleigh number as density does not have pronounced effect because of varying thermal conductivity, the Nusselt number decreases.

Ayla Dogan et.al (2009), investigated numerically the heat transfer by natural convection from partially open cavities with one wall heated. The equations of conservation of mass, momentum and energy were solved using adequate boundary conditions by means of the **PHOENICS code**. The results show that, the average heat transfer coefficient increases and the average wall temperature decreases, with the increase in opening ratio and decrease in the tilt angle.

**F. A. Munir et.al (2010),** reported that the fluid flow behavior and the characteristic of heat transfer from a differentially heated enclosure walls and tilted at various inclination angles. Two different types of boundary condition were applied at the top and bottom walls of the enclosure. They found that the vortex formation, size and flow characteristics were significantly affected by the magnitude of inclination angles. They also found that the convection mode of heat transfer dominates the heat transfer mechanism for every simulation condition due to relatively high Rayleigh number condition applied in this study.

**Eliton Fontanna et. al (2011),** studied the natural convection through a partially open square cavity with internal heat source. A steady buoyancy driven flow of partially square was accomplished. Adiabatic bottom and top walls, and vertical walls maintained at different constant temperature was investigated numerically. The influence of the temperature gradient between the vertical walls was also analyzed. Their results showed that, the isotherm plots were smooth and follow a parabolic shape indicated the dominance of the heat source. When Rayleigh increase, the flow became dominated by the temperature difference between walls. It was also observed multiple strong secondary circulation formed for fluids with a small Rayleigh whereas these features were absent at higher Rayleigh numbers.

**Hua - Shu Dou et. al (2013),** examined the process of instability of natural convection in an inclined cavity based on numerical simulations. The energy gradient method was employed to analyze the physics of the flow instability in natural convection. It was found that the maximum value of the energy gradient function in the flow field correlated well with the location where flow instability occurred. Meanwhile, the effects of the flow time, the plate length, and the inclination angle on the instability have also been discussed. It is observed that the locations of instabilities migrate right as the flow time increased. With the increase of plate length, the onset time of the instability on the top wall of the cavity was decreased gradually and the locations of instabilities move to the right side. Furthermore, the inclination angle of the cavity have a great effect on the stability and instability the flow pattern.

#### 2 - Mathematical modeling:

We defined two dimensions inclined rectangular cavity. Angle of inclination measured from vertical plate to the hot wall of the cavity. The rectangular cavity has opposite isothermal hot and cold walls and adiabatic horizontal walls. Physical model of the cavity was represented on Fig 1.The dimensions of the cavity are L and H. For x = 0 temperature of the hot wall is  $T_h$  and for x = L temperature of the opposite cold wall is  $T_C$ . Other sides of cavity for y = 0, y = H are adiabatic. We defined aspect ratio B, as relation between height over the length of the cavity B = H/L = 1.

## 2.1 - <u>Assumptions</u>:

- 1- All the physical properties are assumed to be constant except for the density which varies with the temperature.
- 2- The fluid is considered viscous and incompressible.
- **3-** There are no heat generations within the cavity, and all temperatures are low intensity, thereby, the radiation is neglected.
- 4- The left and right ends of the cavity were insulated.

#### 2.2 - The governing equations:

The governing equations with the boundary layer approximations for convection are as follows **Santhosh Kumar M.K**(2009). The continuity equation in X and Y directions is:

#### **Conservation of mass:**

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

## **<u>X - Momentum Equation</u>**:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial}{\partial x}(v + v_t)(2\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(v + v_t)(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) - \beta g(T - T_o)\cos\phi$$
(2)

## **<u>Y- Momentum Equation</u>** :

$$u\frac{\partial \upsilon}{\partial x} + v\frac{\partial \upsilon}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{\partial}{\partial y}(v + v_t)(2\frac{\partial v}{\partial y}) + \frac{\partial}{\partial x}(v + v_t)(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) - \beta g(T - T_o)\sin\varphi \quad (3)$$

The energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial x}\left(\frac{v}{\Pr} + \frac{v_t}{\sigma_T}\right)\frac{\partial T}{\partial x} + \frac{\partial}{\partial y}\left(\frac{v}{\Pr} + \frac{v_t}{\sigma_T}\right)\frac{\partial T}{\partial y}$$
(4)

**Turbulent Kinetic energy transport equation:** 

$$u\frac{\partial K}{\partial x} + v\frac{\partial K}{\partial y} = \frac{\partial}{\partial x}\left(v + \frac{v_t}{\sigma_k}\right)\frac{\partial k}{\partial x} + \frac{\partial}{\partial y}\left(v + \frac{v_t}{\sigma_k}\right)\frac{\partial K}{\partial y} + P_k + G_k - \varepsilon$$
(5)

**Dissipation of turbulent Kinetic energy transport equation:** 

$$\mathbf{u}\frac{\partial\varepsilon}{\partial\mathbf{x}} + \mathbf{v}\frac{\partial\varepsilon}{\partial\mathbf{y}} = \frac{\partial}{\partial x}\left(\mathbf{v} + \frac{\mathbf{v}_t}{\sigma_{\varepsilon}}\right)\frac{\partial\varepsilon}{\partial x} + \frac{\partial}{\partial y}\left(\mathbf{v} + \frac{\mathbf{v}_t}{\sigma_{\varepsilon}}\right)\frac{\partial\varepsilon}{\partial y} + C_1 f_1 \frac{\varepsilon}{k} P_k + C_2 f_2 \frac{\varepsilon^2}{k} P_k + C_3 \frac{\varepsilon}{k} G_k + E_{\varepsilon} + S_{\varepsilon}$$
(6)

#### **Rayleigh Number:**

$$Ra = \frac{g\beta\Delta tL^3}{v\gamma}$$
(7)

Where  $\beta$  is thermal expansion coefficient for temperature  $T_0$ .

The production terms due to the buoyancy was not included in this study, because it was founded that buoyancy force was insignificant as a direct source of turbulent generation with the **k**- $\epsilon$  model, Simple Gradient diffusion, and Generalized Gradient diffusion, **Markatos et al.** (1982). Turbulent viscosity was connected to the turbulent Kinetic energy and its function of dissipation by the <u>Prandtl - Kolmogorov</u> relation:

$$v_{t} = C_{\mu} f_{\mu} \frac{K}{\varepsilon}$$
(8)

The stress production terms,  $P_k$ , was modeled by expression:

$$P_{k} = v_{t} \left[ 2 \left( \frac{\partial u}{\partial x} \right)^{2} + \left( \frac{\partial v}{\partial x} \right)^{2} + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \right)^{2} \right]$$
(9)

The buoyancy term  $G_k$ , was modeled by <u>A.I. Fomichv(2002)</u>

$$G_{k} = -g\beta \frac{v_{t}}{\sigma_{t}} \frac{\partial T}{\partial y}$$
(10)

The source term  $E_s$  in the  $\epsilon$  equation was define in LRN k-  $\epsilon$  model of (Jones and Lander 1972)

$$\mathbf{E}_{s} = 2v v_{t} \left[ \left( \frac{\partial^{2} \mathbf{u}}{\partial \mathbf{y}^{2}} \right)^{2} + \left( \frac{\partial^{2} \mathbf{v}}{\partial \mathbf{x}^{2}} \right)^{2} \right]$$
(11)

The source term  $S_s$  in the  $\varepsilon$  equation proposed by Ince and Launder(1989)

$$\mathbf{S}_{\varepsilon} = 0.83 \left( \frac{\mathbf{k}^{1.5}}{\varepsilon c_1 y} - 1 \right) \left( \frac{\mathbf{k}^{1.5}}{\varepsilon c_1 y} \right) \frac{\varepsilon^2}{\mathbf{k}}$$
(12)

**Nusselt Number:** 

$$Nu = \frac{k[\frac{\partial T}{\partial x}]}{k_{m} \left[\frac{T_{h} - T_{c}}{L}\right]}$$
(13)

For free convection flow, density was considered as constant value but for buoyant term it's linearised by relation, <u>Vedat S. Arpaji and Poul S. Larsen (1984)</u>:

$$\rho(T) = \rho(T_o)(1 - \beta(T - T_o))$$
(14)  
In this study, for LRN K-  $\varepsilon$  model, following constant and relationship were used, A.I. Fomichv (2002)  
Cµ=0.09;  $\sigma_t=1$ ;  $\sigma_{\varepsilon}=1.3$ ; C<sub>1</sub>=1.44; C<sub>2</sub>= 1.92; C<sub>3</sub>=tanh  $\left|\frac{v}{u}\right|$ ; f<sub>1</sub>=1;  
f<sub>µ</sub> = exp  $\left[-3.4/(1 + \text{Re}_t/50)^2\right]$ 
(15)

$$f_2 = 1 - 0.3 \exp(-Re_t^2)$$
 (16)

## 2.3 - Boundary conditions:

All velocities on the walls and temperature gradients on the adiabatic walls were equal to zero.  $T_h$  and  $T_c$  refer to hot and cold wall temperature, respectively.

The boundary conditions of the system are :

## **Velocity**:

$$u(x,0) = 0$$
,  $v(x,0) = 0$ ,  $u(x, H) = 0$ ,  $v(x, H) = 0$ ,  $u(0, y) = 0$ ,  $v(0, y) = 0$ ,  $u(L, y) = 0$ ,  $v(L, y) = 0$ 

#### **Temperature:**

$$T(x, 0) = T_h = 60^{\circ}C, T(x, H) = T_c = 20^{\circ}C,$$

T(0, y) and T(L, y) where At 
$$x = 0 \Rightarrow q = -K.A\left(\frac{dT}{dx}\right) = 0$$
 and  
At  $x = L \Rightarrow q = -K.A\left(\frac{dT}{dx}\right) = 0$ 

#### 2.4 - ANSYS Simulations:

**ANSYS11** was used to solve the governing equations based finite element method. Linear quadrilateral elements are used to describe the computational domain. The computation domain was represented rectangular cavity, as shown in Figure (2). Precondition Generalized Minimum Residual (**PGMR**) solver was employed to solve a set of descritization equations of energy and pressure, while Tri-Diagonal Matrix Algorithm (**TDMA**) solver was used to solve the velocity fields. The mesh nearby to walls was fine meshed to cope-up the thermal and velocity boundary layer formation and at the center it was coursed meshed was generated using **ANSYS** because they have greater density near walls.

#### 3 - Result and Discussion:

At different Rayleigh numbers of  $10^7$  to  $10^8$ , the effects of various inclination angles of  $0^{\circ}$  to  $90^{\circ}$  with increments of  $30^{\circ}$  on some of the characteristics of the flow and temperature fields within the cavity were examined. Fig. (3) presents some of the results related to the flow field in terms of streamline profile. At Rayleigh numbers of  $10^7$ , flow pattern is observed at all inclinations. The nature of streamlines did not changed significantly with inclination. At this Rayleigh number, the circulation pattern in the cavity was very weak because the viscous forces were dominated over the buoyant forces. For the original orientation of the cavity ( $\theta = 0^{\circ}$ ), as the Rayleigh number increases to  $10^8$ , the buoyancy forces become stronger. The intensity of recirculation patterns augment and the cells were found to be irregular. The visible peak moved towards the corner of hot and cold walls. At **Ra=10^8** Rayleigh number, with the inclination increases, the strength of the flow field increases and ultimately, at inclination of  $60^{\circ}$ , two peaks are observed in the flow field. Up to now, the researcher was agree with **Y. S. Tian, and T. G. Karayiannis, (2000).** 

## 3.1 - Transition from laminar to turbulence in inclined Cavity:

The most important feature of turbulence is its randomness, and it is assumed that this method can be applied to find the transitional point from laminar flow to turbulence in such a buoyancy-driven cavity. In the range of Rayleigh number that covered in this study, the flow remains laminar for cavity under aspect ratio 20. The results which obtained from figures 3, 4 and 5 present that, the turbulent intensity increased with increase Rayleigh number from  $10^7$  to  $10^8$  because the changing in average Nusselt number and the angle of inclination increase, the Nusselt number decrease as showed clearly in figure-6.

## 3.2 - Temperature and Velocity Distribution:

The Temperature distribution depends on the angle of inclination of cavity and Rayleigh number. The result obtained from figures 3, 4, 5, and 7 illustrate that the distribution of temperature become more homogenous when Rayleigh number and the angle of inclination of cavity are increased. The precise reason was at a large Rayleigh number, the dimension of cavity increase, consequently; the amount of heat transfer was increased in cavity by free convection. Angle of inclination of cavity causes change in buoyancy-driven cavity which means that the gravity force is divided in two components in x and y direction. Figures 3, 4 and 5 present the velocity distribution. It was noted that the velocity distribution is symmetric about the center of cavity when Rayleigh number  $10^7$ . When Rayleigh number increased to  $10^8$  the velocity distribution was change because of the increasing in the dimension of cavity and thereby, increase number of loops (because variation of temperature, start forming new loops increased with increasing of Rayleigh number and changed with inclination of cavity). Here, the researcher were agreeing with **Mohammed A. Khan, (1996).** Figures 8 and 9 carried out effect the angle of inclination on temperature distributions for different Rayleigh number.

## 3.3 - Stream function distribution:

Subfigures I and j for the mentioned figures above present the streamlines of the considered flow. It was noted a presence cavity a clockwise recirculation zone covering the cavity. Stream function was increased with increasing of the Rayleigh number for same inclination angle. Inside of this zone, it was noted that, at  $10^7$  Rayleigh number the presence of vortices located in the lower-left of the cavity and increase in its domain with increasing the angle of inclination. Also, number of vortices seemed to be

increased to two with increase the inclination angle .With increase of Rayleigh number  $to10^8$  the location of vortices were changed with different situation depends on the variation in the angle of inclination. Here the researcher is disagree with **K. Bouaraour** (2008) as regarded to the variation in dimension of cavity. All vortices were rotating not result from instability of the base flow but are a direct consequence of the convective distortion of the temperature field.

## 3.4 - Kinetic energy distribution and energy dissipation:

Subfigures  $\mathbf{e}$ ,  $\mathbf{f}$ ,  $\mathbf{g}$ , and  $\mathbf{h}$  for the mentioned figures above present kinetic energy distribution and energy dissipation. From these figures, one can examine that, the energy was distributed in lower part of cavity more than the upper part of cavity because temperature distribution. The center of cavity has lower energy because this region has lower temperature. When the angle of inclination of cavity increase the energy dissipation in the wall of cavity and more energy in the in high temperature wall.

## 5 - <u>Conclusions</u>:

Thermal analysis of natural convection was accomplished using varying properties and varying boundary conditions in air filled inclined cavity. The following conclusions can be drawn from the analysis and results of the present work:

- 1- Transition from laminar to turbulence in inclined cavity occurs when the Rayleigh number increase and angle of inclination increase.
- 2- Nusselt number is independent of varying effect of viscosity.
- 3- Heat transfer rate increases with higher Rayleigh number .
- **4-** At all Rayleigh numbers, as the inclination angle rises the average temperature in the cavity starts to increase until it reaches its maximum value and then decreases. The rate of decrease is more appreciable at high Rayleigh numbers.
- **5-** The variation of temperature along the centerline of the cavity and the local Nusselt number along the cold wall with Rayleigh number showed different trends at various inclination angles.

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## 6 – <u>Tables and Figures</u>:

- Table (1): Properties of air for Rayleigh number calculation.
- Figure (1): Confined inclined rectangular cavity.
- Figure (2): (a) Grid system of Confined inclined rectangular cavity (Ra= $2.6 \times 10^5$ ), (b) vertical (Ra= $2.35 \times 10^4$ ).
- Figure (3) : Flow fields at different Rayleigh numbers and inclination angles, when  $(\theta=0)$  and  $(Ra=2.7x10^7)$  and  $(Ra=2.16 x10^8)$ .
- Figure (4) : Flow fields at different Rayleigh numbers and inclination angles, when  $(\theta=30^{\circ})$  and  $(Ra=2.7 \times 10^{7})$  and  $(Ra=2.16 \times 10^{8})$ .
- Figure (5): Flow fields at different Rayleigh numbers and inclination angles, when  $(\theta=60^{\circ})$  and  $(Ra=2.7x10^{7})$  and  $(Ra=2.16 x10^{8})$ .
- Figure (6): Effect of angle of inclination on Nusselt number.
- Figure (7) : velocity distribution  $[\theta=0, Ra=2.7 \times 10^7]$ .
- Figure (8): Temperature distribution as function of the angle of inclination  $[Ra=2.7x10^7]$ .
- Figure (9): Temperature distribution as function of the angle of inclination  $[Ra=2.16 \times 10^8]$ .

Description	Symbol	Value		Units
Gravitational acceleration	g	9.81	9.81	m/sec
Density	ρ	1.0137	1.0137	Kg/m <sup>3</sup>
Specific heat	Ср	1007.5	1007.5	J/Kg K
Thermal conductivity	K	0.02917	0.02917	W/m K
Dynamic viscosity	μ	0.00002074	0.00002074	Kg/m sec
Mean temperature	To	313	313	K
Kinematic viscosity	v	0.2415	0.2415	m <sup>2</sup> /sec
Beta	β	0.003169	0.003169	K <sup>-1</sup>
Delta T	ΔΤ	45	45	K
Length	L	0.02	0.04	m
Thermal diffusivity	α	17	17	m <sup>2</sup> /sec

#### Table (1): Properties of air for Rayleigh number calculation



Figure (1): Confined inclined rectangular cavity





Figure (a) : Velocity distribution profiles ( $v_{sum}$ ) through an rectangular cavity  $\theta=0$ , Ra=2.7x10<sup>7</sup>



Figure (c):Temperature distribution profiles through an rectangular cavity  $\theta=0$ , Ra=2.7x10<sup>7</sup>



Figure (e) : Kinetic energy distribution profiles through an rectangular cavity  $\theta=0$ , Ra=2.7x10<sup>7</sup>



Figure (b) : Velocity distribution profiles ( $v_{sum}$ ) through an rectangular cavity  $\theta$ =0, Ra=2.16 x10<sup>8</sup>



Figure (d):Temperature distribution profiles through an rectangular cavity  $\theta=0$ , Ra=2.16 x10<sup>8</sup>



Figure (f) : Kinetic energy distribution profiles through an rectangular cavity  $\theta=0$ , Ra=2.16 x10<sup>8</sup>



Figure (g) : Energy dissipation profiles, through an rectangular cavity  $\theta=0$ , Ra=2.7x10<sup>7</sup>



Figure (i) : Stream function profiles through an rectangular cavity  $\theta=0$ , Ra=2.7x10<sup>7</sup>



Figure (h) : Energy dissipation profiles, through an rectangular cavity  $\theta$ =0, Ra=2.16 x10<sup>8</sup>



Figure(j) : Stream function profiles through an rectangular cavity  $\theta=0$ , Ra=2.16 x10<sup>8</sup>

Figure (3) : Flow fields at different Rayleigh numbers and inclination angles, when  $(\theta=0)$  and  $(Ra=2.7x10^7)$  and  $(Ra=2.16 x10^8)$ 



Figure (a) : Velocity distribution profiles ( $v_{sum}$ ) through an rectangular cavity  $\theta=30^{0}$ , Ra=2.7x10<sup>7</sup>



Figure (b) : Velocity distribution profiles ( $v_{sum}$ ) through an rectangular cavity  $\theta$ =30<sup>0</sup>, Ra=2.16 x10<sup>8</sup>



Figure (d) :Temperature distribution profiles through an rectangular cavity  $\theta=30^{\circ}$ , Ra=2.16 x10<sup>8</sup>



Figure (f) : Kinetic energy distribution profiles through an rectangular cavity  $\theta=30^{\circ}$ , Ra=2.16 x10<sup>8</sup>



Figure (h) : Energy dissipation profiles, through an rectangular cavity  $\theta=30^{\circ}$ , Ra=2.16 x10<sup>8</sup>



Figure (c):Temperature distribution profiles through an rectangular cavity  $\theta=30^{\circ}$ , Ra=2.7x10<sup>7</sup>



Figure (e) : Kinetic energy distribution profiles through an rectangular cavity  $\theta=30^{\circ}$ , Ra=2.7x10<sup>7</sup>



Figure (g) : Energy dissipation profiles, through an rectangular cavity  $\theta=30^{\circ}$ , Ra=2.7x10<sup>7</sup>



Figure (i) : Stream function profiles through an rectangular cavity  $\theta=30^{\circ}$ , Ra=2.7x10<sup>7</sup>



Figure (j) : Stream function profiles through an rectangular cavity  $\theta=30^{\circ}$ , Ra=2.16 x10<sup>8</sup>

Figure (4) : Flow fields at different Rayleigh numbers and inclination angles, when  $(\theta=30^\circ)$  and  $(Ra=2.7\times10^7)$  and  $(Ra=2.16\times10^8)$ 



Figure (a) : Velocity distribution profiles ( $v_{sum}$ ) through rectangular cavity  $\theta$ =60<sup>0</sup>, Ra=2.7x10<sup>7</sup>



Figure (c):Temperature distribution profiles through an rectangular cavity  $\theta=60^{\circ}$ , Ra=2.7x10<sup>7</sup>



Figure (b) : Velocity distribution profiles ( $v_{sum}$ ).through rectangular cavity  $\theta$ =60<sup>0</sup>, Ra=2.16 x10<sup>8</sup>



Figure (d) :Temperature distribution profiles through an rectangular cavity  $\theta = 60^{\circ}$ , Ra=2.16 x10<sup>8</sup>



Figure (e) : Kinetic energy distribution profiles through an rectangular cavity  $\theta=60^{\circ}$ , Ra=2.7x10<sup>7</sup>



Figure (g) : Energy dissipation profiles, through an rectangular cavity  $\theta=60^{0}$ , Ra=2.7x10<sup>7</sup>



Figure (i) : Stream function profiles through an rectangular cavity  $\theta$ =60<sup>0</sup>, Ra=2.7x10<sup>7</sup>



Figure (f) : Kinetic energy distribution profiles through an rectangular cavity  $\theta=60^{0}$ , Ra=2.16 x10<sup>8</sup>



Figure (h) : Energy dissipation profiles, through an rectangular cavity  $\theta$ =60<sup>0</sup>, Ra=2.16 x10<sup>8</sup>



Figure (j) : Stream function profiles through an rectangular cavity  $\theta$ =60<sup>0</sup>, Ra=2.16 x10<sup>8</sup>

Figure (5) : Flow fields at different Rayleigh numbers and inclination angles, when  $(\theta=60^{\circ})$  and  $(Ra=2.7x10^{7})$  and  $(Ra=2.16 x10^{8})$ 



Fig (6) : Effect of angle of inclination on Nusselt number



Fig (7): velocity distribution  $[\theta=0,Ra=2.7x10^7]$ 



Fig (8) :Temperature distribution as function of the angle of inclination  $[Ra=2.7x10^7]$ 

Fig (9) :Temperature distribution as function of the angle of inclination [Ra=2.16 x10<sup>8</sup>]