Thermal Analysis of The laminar Free Convection phenomena in an Air Filled Triangular Cavity with Different Boundary Conditions

By

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Abstract :

In this research a numerical study of two-dimensional laminar free convection has been carried out on closed triangular cavities with different wall boundary conditions. Three different boundary conditions were studied **(a) case (A),** the vertical and horizontal wall inside of the triangular cavity are objected to forced convection h=100 W/m **^{0}C** and T=300 K through the intervals $0<x<0.2L$ and $0<y<0.2L$ and the remaining walls are adiabatic . **(b) case (B),** the vertical and horizontal wall inside of the triangular cavity are objected to forced convection h=100 W/m $^\circ$c and T=300 K through the intervals $0.4<x<0.6L$ and $0.4<y<0.6L$ and the remaining walls are adiabatic**. (c) case (C),** the vertical and horizontal wall inside of the triangular cavity are objected to forced convection h=100 W/m $^\circ$c and T=300 K through the intervals $0.8<x<L$ and $0.8< y< L$ and the remaining walls are adiabatic. And the inclined wall of triangular cavity object to heat flux at five location for all these cases and the distance between these five location is adiabatic. Three different boundary conditions for inclined wall were studied (a) case (A), heat flux is 100 W ,(b) case (B), heat flux is 500 W,and (c) case (C), heat flux is 1000 W The fluid considered is air with Prandtl number fixed at 0.7. Equations of continuity, momentum and energy are solved using constant properties is solved by ANSYS . It is found that wall boundary conditions, position the convective heat transfer coefficient between the cavity and the ambient air and effect on the air cavity . At low Rayleigh number, wall boundary conditions have a minimal effect on Nu . However, increasing Ra, the wall boundary conditions have greater effect on the Nusselt number. And the temperature and velocity distribution are variation with location of convection and heat flux also with Rayleigh number **..** The construct result used program(ANSYS) to find temperature, velocity and pressure distribution also the vector of velocity inside the air gap for several cases. The results are presented and discussed in this study and compared with other researches.

Keywords / Heat transfer, Numerical, Laminar, Free convection, Triangular Cavity, Heat Flux, ANSYS

التحليل الحراري لظاهرة الحمل الطبيعي الطباقي الحاصل في محتوى مثلث الشكل مملوء بالهواء باختلاف الفيض الحراري وموقع تعرض للحمل المحتوى

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الخلاصة في هذا البحث تم دراسة التحليل الحراري لظاهرة الحمل الطبيعي الطباقي ثنائي البعد الحاصل في حيز مثلث الشكل مغلق مع تغير الشروط الحدودية، حيث تم دراسة ثلاث من الشروط الحدودية. الحالة الأولى الجداران الأفقي والعمودي معرضان الى حمل حراري قسري **h=100 W/m ^{0}C and T=300 K** خلال الفترة **$0< x< 0.2L$** and **$0< y < 0.2L$** , الحالة ثانية الجداران الأفقي والعمودي معرضان الى حمل حراري قسري **h=100 W/m ^{0}C** and **T=300 K** خلال الفترة . **$0.4 <x< 0.6L$ and $0.4< y< 0.6$** والباقي معزول. الحالة الثالثة الجداران الأفقي والعمودي معرضان الى حمل حراري قسري **h=100 W/m ^{0}C and T=300 K** خلال الفترة **$0.8<x<L$ and $0.8< y< L$.** في كل الحالات الجدار المائل معرض الى فيض حراري في خمسة مواقع المسافات بينها متساوية ومعزولة درست كذلك ثلاث حالات من الشروط الحدودية للجدار المائل الحالة الأولى الفيض الحراري= **100 W** الحالة الثانية الفيض الحراري= **500 W** الحالة الثالثة

الفيض الحراري= **1000 W** .المائع داخل الفجوة هواء .معادلات الاستمرارية والزخم والطاقة تم حلها ضمن الشروط الحدودية المذكورة وبيان تأثير موقع الحمل ومدى تأثيره على الفجوة. في عدد رالي منخفض، الشروط الحدودية للجدار لها تأثير أقل ما يمكن على عدد نسلت. وعند زيادة عدد رالي ، يزداد تأثير الشروط الحدوديةّ للجدار على عدد نسلت. استخدم برنامج (ANSYS) لإيجاد توزيع درجة الحرارة، توزيع الضغط والسرعة لحالات متعددة وبيان تأثير الشروط الحدية . تم إيجاد النتائج ومناقشتها ومقارنتها بالبحوث الأخرى وتم أيضا مناقشة أسباب التشابه والاختلاف.كذلك بينت الدراسة أن معدلات انتقال الحرارة في النوع الثاني عند ارتفاع الفيض المسلط من الشروط الحدودية أعلى من الأنواع الأخرى التي تم دراستها.**:**

Nomenclature

AR	aspect ratio AR =H/L
Cp	specific heat of air J/Kg K
g	acceleration due to gravity m/sec^2
K_g	thermal conductivity of air W/m K
L	length of the cavity (m)
Nu	Nussult number of air
Pr	Prandtl number of air Pr=(µCp)/k
T	temperature in (K)
Q	heat flux W/m^2
u	velocity in x-axis (m/sec)
v	velocity in y-axis (m/sec)
x,y	coordinates along x-axis and y-axis

Greek symbols

α	thermal diffusivity (m^2/sec)
β	thermal coefficient of expansion (K^{-1})
ν	molecule kinematic viscosity (m^2/sec)
ξ	vorticity
ψ	stream function
ρ	density of air (kg/m^3)
μ	dynamic viscosity of air (kg/m.sec)

Subscripts

m	mean
c	cold
x	local in x axes
x,y	local in x and y axes

Introduction:

The important studies around free Convection in Triangular cavity filled with air is increasing , because the cavity is very important for use in thermal control system for low cost , and easy maintenance . Also used in typical applications include the heat exchangers, cooling of electronic equipment and, solar chimneys and nuclear reactors, etc [**Goutam 2007**]. Transient laminar forced convection is the fundamental interest in many industrial situations such as air-conditioning systems, thermal regulation processes or electronic equipment cooling. **[G.polidori 2001]**

El Hassan Ridouane[2005] studied triangular cavity deals with the numerical computation of laminar natural convection in a gamma of right-angled triangular cavities filled with airand the boundary condition as The vertical walls are heated and the inclined walls are cooled and the upper connecting walls are insulated from the ambient air The research used the program FLUENT 6.1to construct the result .

Yasin Varol [2007] studied has been performed to investigate the effects of fin location onto the bottom wall of a triangular cavity filled with porous media whose height base ratio is 1 ,and solve the equation numerically **.** The research studied some parameter as Rayleigh number, location center of fin, dimensionless fin height, and dimensionless fin width and found that the obtained results indicated that the fin can be used as a control element for heat transfer and fluid flow.

Goutam Saha[2007] studied a numerical study of natural convection in tilted isosceles triangular enclosure filled with air is presented by using a finite element based adapting meshing technique and It is found the Optimum heat transport phenomenon is gained for higher value of Grashof number and aspect ratio of the enclosure.

Sun Joo Kwon[2007] studied free convection inside a triangular cavity with porous is It is found that the heat transfer rate is change with the shape of the triangles and all dimension . For the geometries that used , Rayleigh numbers and buoyancy ratio are the parameters that control the heat transfer flow .

Mathematical model:

Cavity geometry:

The cavity is a right-angle triangular cavity filled with air as shown in Fig.1, schematically. Its aspect ratio, AR=H/L ,was taken as 1. The heat flux is subjected on inclined line of triangular cavity at constant distance and the same value at all case , the distance between the area of heat flux is equal at all cases .The fluid in cavity is air see table 1.

Assumptions :

In order to simplify the analyses, the analysis of the triangular cavity is based on the following assumptions:

1- The fluid is Newtonian and the flow is laminar.
2- Two dimensional and steady state for fluid motion in triangular cavity .
3-All the physical properties are assumed to be constant except for the density variation with temperature distribution.
4- Radiation effect is neglected.
5. No heat generations within the triangular cavity,

The governing equations : The mathematical formulation is described by the system of conservation equations , the governing equations of continuity, momentum, and energy for a steady, incompressible flow are given below. **[El Hassan Ridouane (2005)]**,

Mass:

$$(\rho u)_x + (\rho v)_y = 0 \tag{1}$$

Horizontal momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}$$

Vertical momentum:

$$\rho u \frac{\partial \upsilon}{\partial x} + \rho v \frac{\partial \upsilon}{\partial y} = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - g\,\rho(T) \tag{3}$$

The energy equation in X and Y directions **[Y. Zhao, W. Q. Tao(1995)]**:

$$\rho u \frac{\partial T}{\partial x} + \rho \upsilon \frac{\partial T}{\partial y} = \frac{K}{C_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

For free convection flow, the change in density is calculate from [**Karl Bu¨ hler(2003)**]

$$\rho(T) = \rho(T_o) - \beta\,\rho(T_o)(T - T_0) \tag{5}$$

The vorticity equation in terms of stream function and stream function ψ is defined as**[V. Dharma Rao (2007)]**

The vorticity (ξ)

$$\xi = \frac{\partial \mathrm{u}}{\partial \mathrm{x}} - \frac{\partial \mathrm{v}}{\partial \mathrm{x}} \tag{6}$$

$$\xi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}; u = \frac{\partial\psi}{\partial y}\ ; v = -\frac{\partial\psi}{\partial x} \qquad (7)$$

Heat Transfer Analysis:

The fluid in the triangular cavity filled with air of which the Prandtl number is 0.71, The local Nusselt number $Nu_{x,y}$ and the total Nusselt number Nu_x at every X axes is defined as follows. **[W.-S. Fu, J.-C. Perng(1990)]**

$$\mathrm{Nu}_{x,y} = \left[\rho C_p u(T - T_c) - k\frac{\partial T}{\partial x}\right]/Q \qquad (8)$$

$$\mathrm{Nu}_x = \int_0^L \mathrm{Nu}_{x,y}\,dy \qquad (9)$$

Boundary condition:

All velocities on the three walls of triangular cavity and temperature gradients on the adiabatic walls are equal zero.

1.Case (A):

$0 < x < 0.2L,\quad y = 0\quad T(x,0) = 300\ K\ , h = 100\ \mathrm{W/m}\ ^0\mathrm{c}$

$0.2L < x < L,\quad y = 0\quad T(x,0) = 0$

$0 < y < 0.2L,\quad x = 0\quad T(x,0) = 300\ K\ , h = 100\ \mathrm{W/m}\ ^0\mathrm{c}$

$0.2L < y < L,\quad x = 0\quad T(x,0) = 0$

And Q=100 W ,500W, 1000 W

2. Case (B):

$0.4 < x < 0.6L,\quad y = 0\quad T(x,0) = 300\ K\ , h = 100\ \mathrm{W/m}\ ^0\mathrm{c}$

$0.6L < x < L,\quad y = 0\quad T(x,0) = 0$

$0.4L < y < 0.6L,\quad x = 0\quad T(x,0) = 300\ K\ , h = 100\ \mathrm{W/m}\ ^0\mathrm{c}$

$0.6L < y < L,\quad x = 0\quad T(x,0) = 0$

And Q=100 W ,500W, 1000 W

3. Case (C):

$0.8L < x < L,\quad y = 0\quad T(x,0) = 300\ K\ , h = 100\ \mathrm{W/m}\ ^0\mathrm{C}$

$0 < x < 0.8L,\quad y = 0\quad T(x,0) = 0$

$0.8L < y < L,\quad x = 0\quad T(x,0) = 300\ K\ , h = 100\ \mathrm{W/m}\ ^0\mathrm{C}$

$0 < y < 0.8L,\quad x = 0\quad T(x,0) = 0$

And Q=100 W ,500W, 1000 W

ANSYS Simulations:

ANSYS 5.4 is used to solve the governing equations based finite element method. To choose the form of element and number of nodes in all wall of cavity used several element and in all element calculate maximum and minimum stream function and chose the perfect mesh this methods employed in new report for example **[Goutam Saha (2007)]** and this method used in this research the result draw in graph see **figure 11 .** From this figure conclude that Linear quadrilateral elements are used to used the computational domain. The computation domain is represented triangular cavity, as shown in Figure 1. The numerical model consists of elements 150 increment in x axes and 150 increment in y axes. Precondition Generalized Minimum Residual **(PGMR)** solver is employed to solve a set of equations of energy and pressure , while Tri-Diagonal Matrix Algorithm **(TDMA)** solver is used to solve the velocity.

Results and Discussions:

Effect magnitude of heat flux [Q]on Temperature and velocity distribution:

Fluid motion was set up in a right-angle triangular cavities heating the inclined wall by heat flux at five location has the same distance while simultaneously cooling the vertical and horizontal wall at variable convection boundary condition Change along the two wall by distance 0.2 L and the other of walls was insulated.

When examining fig.3 a ,b, and c we observed that contour plots of temperatures for the three typical cavities in a right-angle with different boundary conditions fig3. a were represented Case A with heat flux at inclined wall was 100 W, fig3.b were represented Case A with heat flux at inclined wall was 500 W , fig3.c and were represented Case A with heat flux at inclined wall was 1000 W. From these figures we observed that the range of temperature distributions increase with change figures from (a) to (c) , The reason of this difference that heat always transfer to upper of cavity because the reduction in density this mean as the temperature increase the density decrease therefore rise to upper ,fig3.c .Heat removed from cavity increase with temperature differences (T-T_∞) . Temperature differences in convection region from 320K to340K this range increase with increase of value of L because the heat flux increase and we become near the effect of heat flux**.** From these figure we observed that heat transfer coefficients increase at two corners near inclined wall because the reduction in velocity . fig3.b we observed that temperature differences in convection region from 294 K to 299K. from figure we observed that heat transfer coefficients increase at two corners near inclined wall because the reduction in velocity and increase temperature distributions from 316 K - 322 K. fig3.a we observed that temperature differences in convection region was constant equal 294.5 K because the temperature raised to upper of cavity . but heat flux at inclined wall was 500 W .The temperature increase become 365 K and this value is higher from Case A and case C because conform loop very big inside the triangular cavities .

Effect convection boundary condition on Temperature and velocity distribution:

When examining fig.3,4,and fig.5 we observed that contour plots of temperatures and velocity for the three typical cavities in a right-angle with different boundary conditions fig3. were represented Case A from Case A and case C ,fig4 were represented Case B fig5 were represented Case C. From these figures we observed that the range of temperature distributions increase with change figures from (3) to (5) but temperature in case b is higher, this mean that as the area that heat transfer by convection raised to upper of cavity the temperature distributions increase when heat flux increase .

The reason of this difference that from Case A to case C because buoyancy force, when Q=100W the value of temperature change from (298.5 -299.5) with change in location of area convection, but when Q=500W the value of temperature change from (298.5 -299.5) with change in location of area convection when Q=1000W the value of temperature change from (344.5 -330) with change in location of area convection see Fig.7 we observed that the temperature in case c is higher than that because the location of convection area .from fig.8 we observed that the velocity increase in case B at high heat flux .

When examining fig.6, we observed that contour plots of stream function for the three typical cavities in a right-angle with different boundary conditions .fig9.and fig10 were represented graph represent the change in maximum and minimum stream function with change in heat flux from these figure conclude that at Case A conform two loops at lower heat flux and become one loop when heat flux is 1000 W. Case B conform one circular loop in center of cavity ,and Case C conform one ellipse loops in center of cavity because increase in heat transfer coefficient in this case .

When examining fig.7, fig.8 ,we observed that graph plots of velocity as a function to heat flux for the three typical cavities in a right-angle with different boundary conditions .We observed that Case B with high heat flux was the puffer in temperature and velocity .The reason of this when the convection area in the center of vertical cavity the air raise to upper of cavity and at the area of convection it lose some of heat and the density decrease therefore the air decrease to lower and this operation back to construct big loop in the center of cavity causes good distribution for temperature and velocity .

Conclusions:

1-From the result that get on in result conclude that finite element method is suitable method to predict the temperature and velocity distribution in triangular cavity.
2- As convection boundary condition raise the temperature distribution is briefer.
3- Temperature and velocity distribution increase at two angle near inclined wall.
4- The temperature in case b is higher than that because the location of convection area.
5- The velocity in Case b is higher than that because the location of convection area.

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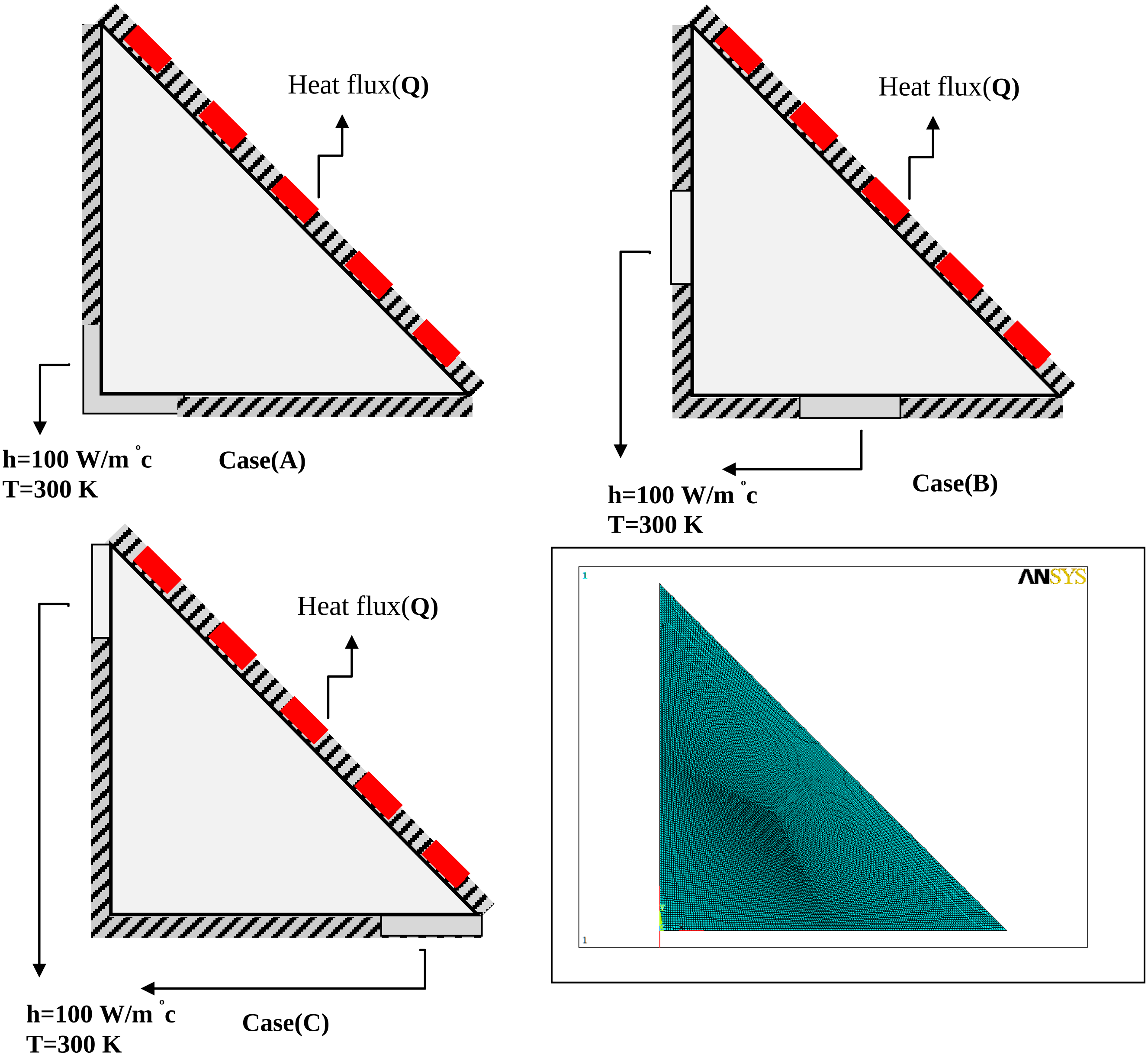
Figures and tables:

Table 1 : Properties of air in a right-angle the triangular cavities

Description	Symbol	Value	Units
Gravitational acceleration	g	9.81	**m/sec**
Specific heat	Cp	1007.5	**J/Kg K**
Thermal conductivity	K	0.02917	**W/m K**
Dynamic viscosity	μ	0.00002074	**Kg/m sec**
Mean temperature	T_o	300	**K**
Kinematic viscosity	ν	0.2415	**m^2/sec**
Horizontal Length	H	0.01	**m**
Vertical Length	L	0.01	**m**
Thermal diffusivity	α	17	**m^2/sec**

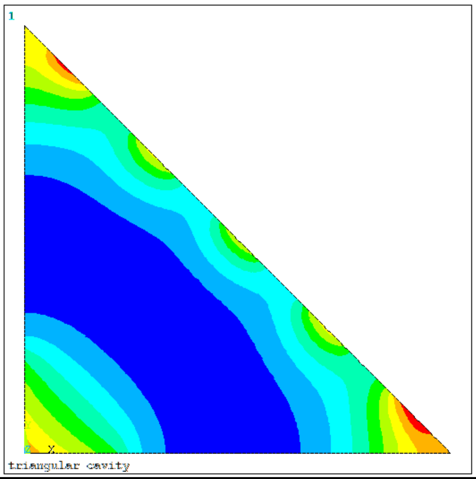


Case(A)

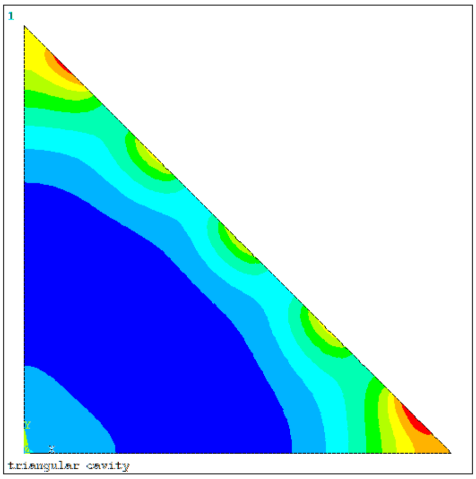
Case(B)

Case(C)

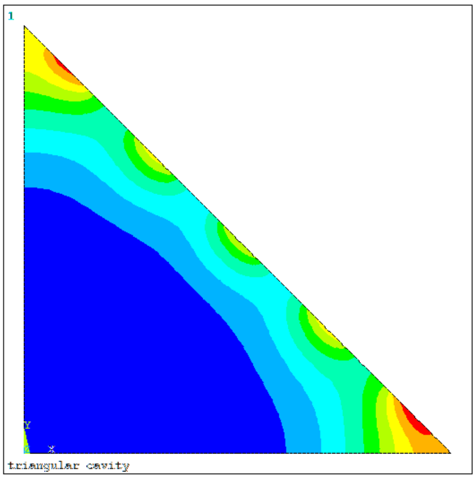
Fig1.Trangular cavity with different boundary surfaces conditions and mesh of cavity



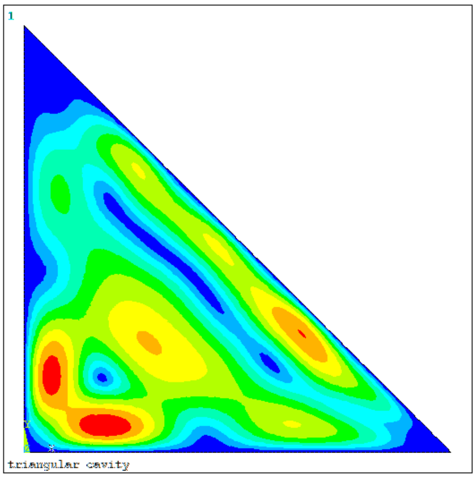
a:Temperture distribution Profiles ,q_r=100 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,



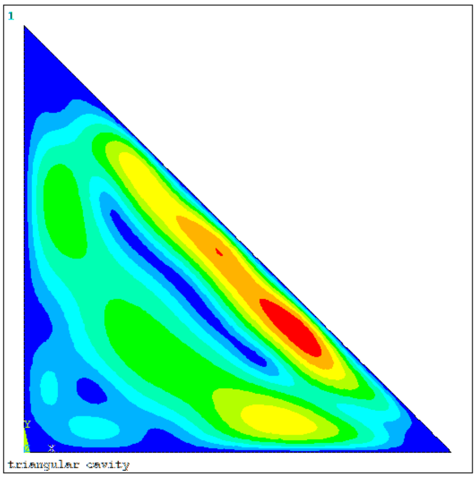
b : Temperature distribution Profiles ,q_r=500 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,



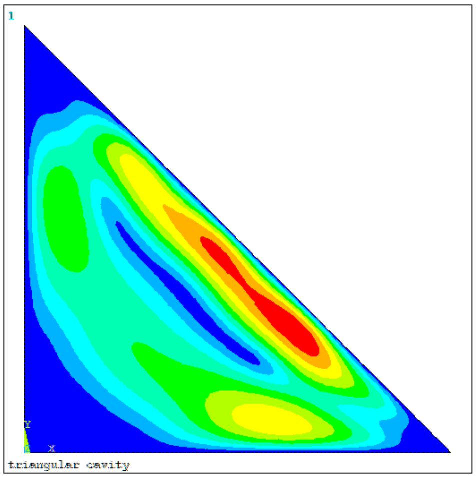
c:Temperture distribution Profiles ,q_r=1000 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,



d:velocity distribution Profiles ,q_r=100 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,

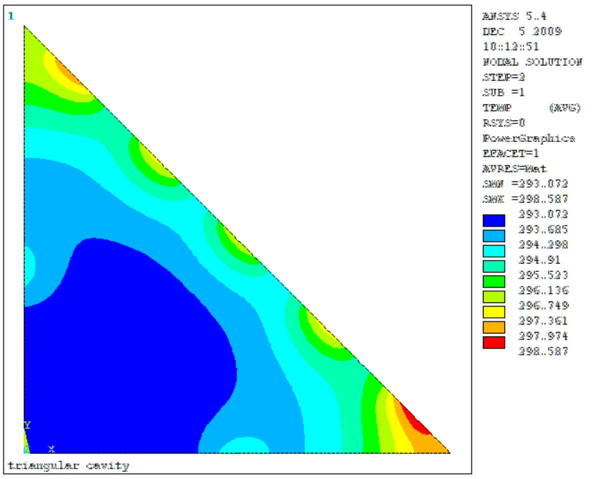


e:velocity distribution Profiles ,q_r=500 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,

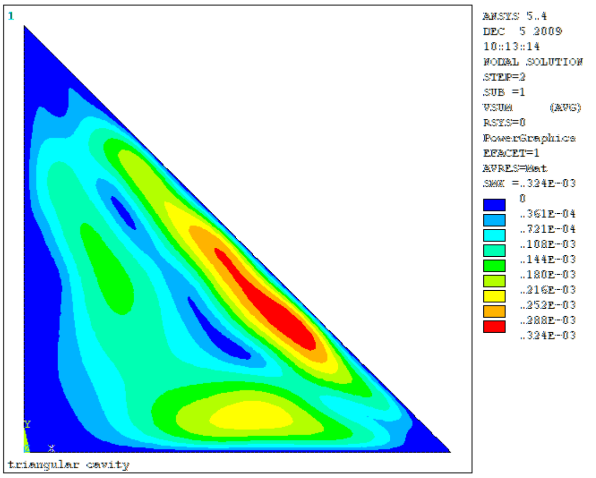


f:velocity distribution Profiles ,q_r=1000 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,

Fig3:Flow fields distribution Profiles through triangular cavity ,Velocity and temperature distributions



a:Temperture distribution Profiles ,q_r=100 W convection at 0.4 L <x <0.6L,y=0 : 0.4L <y <0.6L, x=0,



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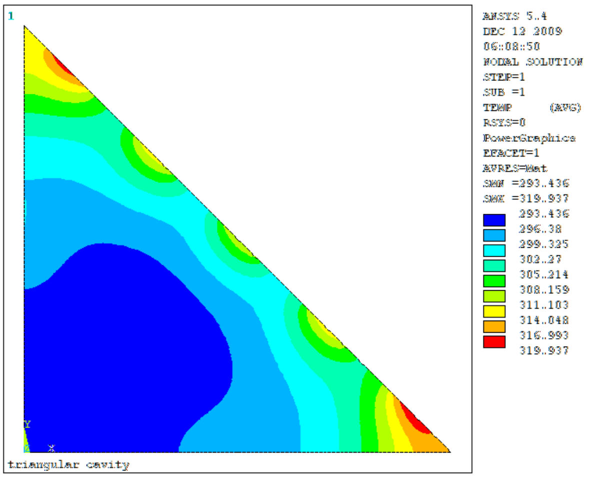
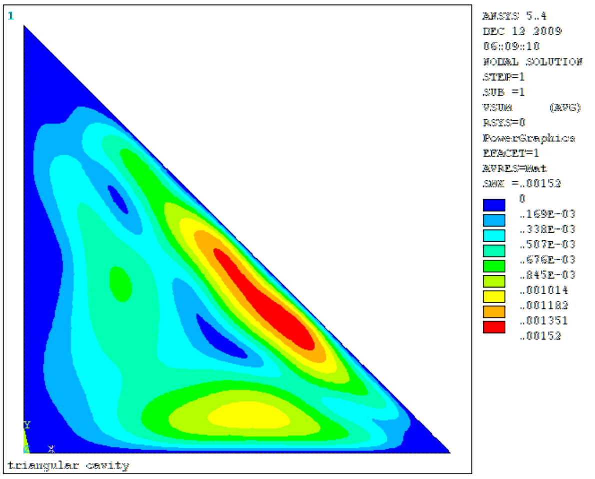
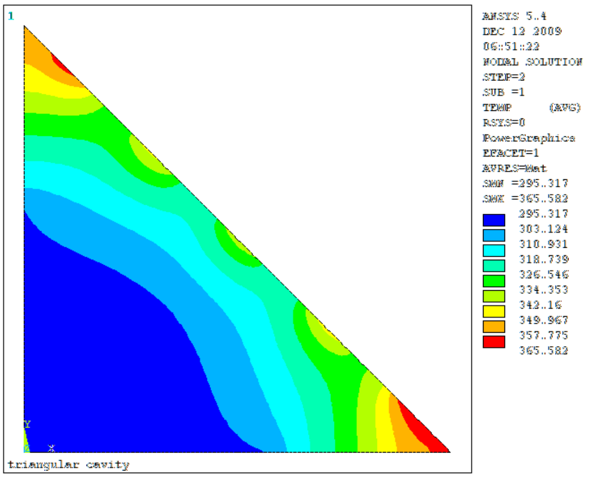


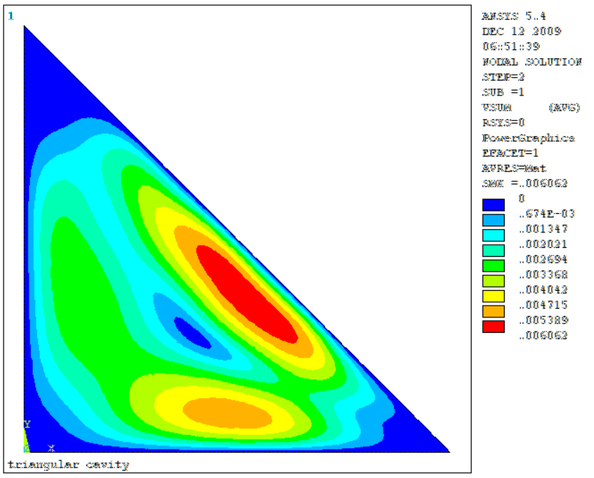
b : Temperature distribution Profiles ,q_r=500 W convection at 0.4 L <x <0.6L,y=0 : 0.4L <y <0.6L, x=0,



e:velocity distribution Profiles ,q_r=500 W convection at 0.4 L <x <0.6L,y=0 : 0.4L <y <0.6L, x=0,

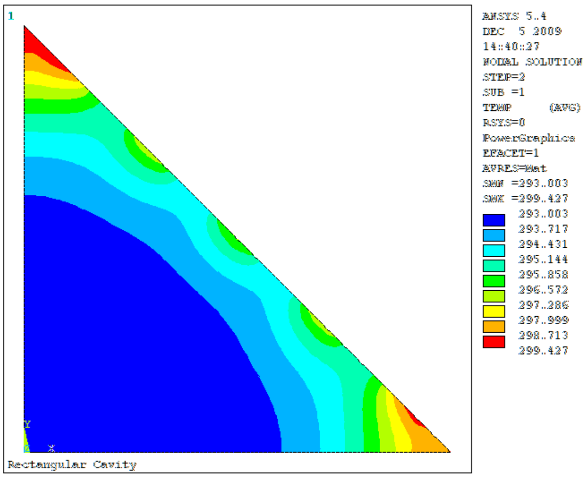


c:Temperture distribution Profiles ,q_r=1000 W convection at 0.4 L <x <0.6L,y=0 : 0.4L <y <0.6L, x=0,

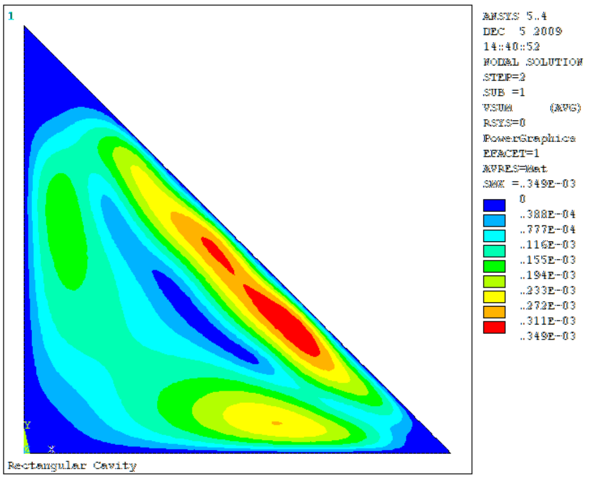


f:velocity distribution Profiles ,q_r=1000 W convection at 0.4 L <x <0.6L,y=0 : 0.4L <y <0.6L, x=0,

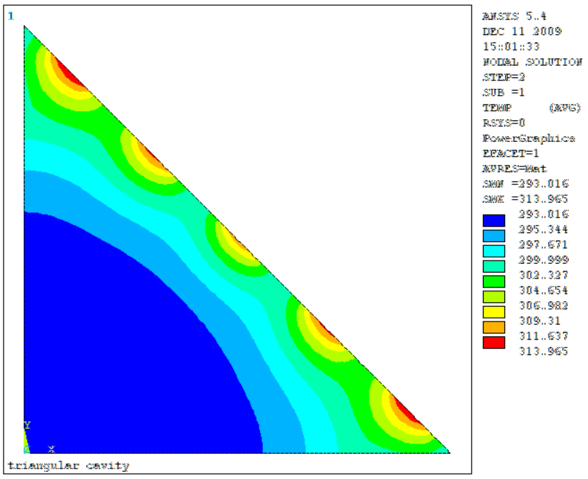
Fig4:Flow fields distribution Profiles through triangular cavity ,Velocity and temperature distributions



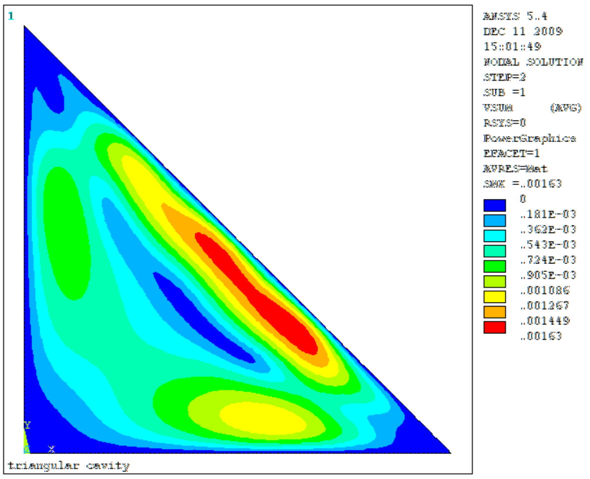
a:Temperture distribution Profiles ,q_r=100 W convection at 0.8 L <x <L, y=0 : 0.8L <y <L, x=0,



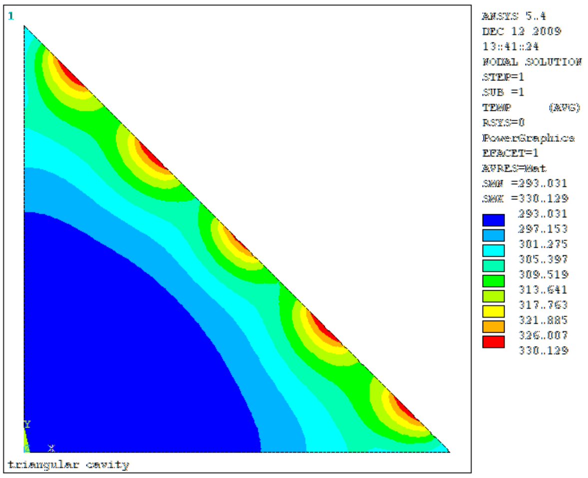
d:velocity distribution Profiles ,q_r=100 W convection at 0.8 L <x <L ,y=0 : 0.8L <y <L, x=0,



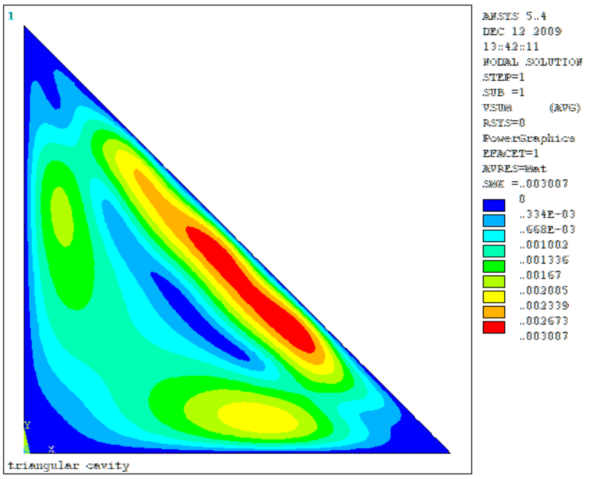
b : Temperature distribution Profiles ,q_r=500 W convection at 0.8 L <x <L, y=0 : 0.8L <y <L, x=0,



e:velocity distribution Profiles ,q_r=500 W convection at 0.8 L <x <L, y=0 : 0.8L <y <L, x=0,

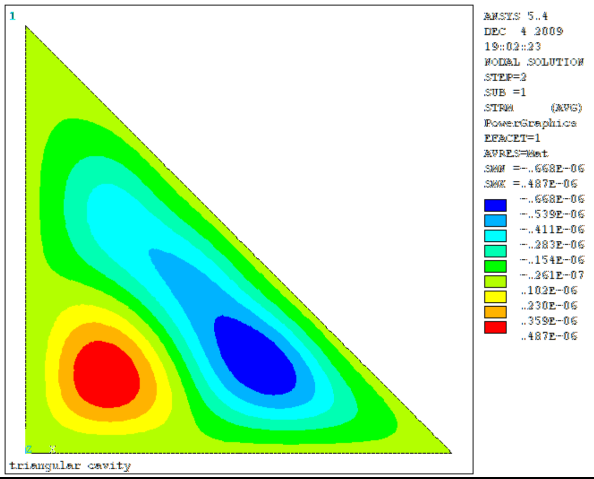


c:Temperture distribution Profiles ,q_r=1000 W convection at 0.8 L <x <L ,y=0 : 0.8L <y <L, x=0,

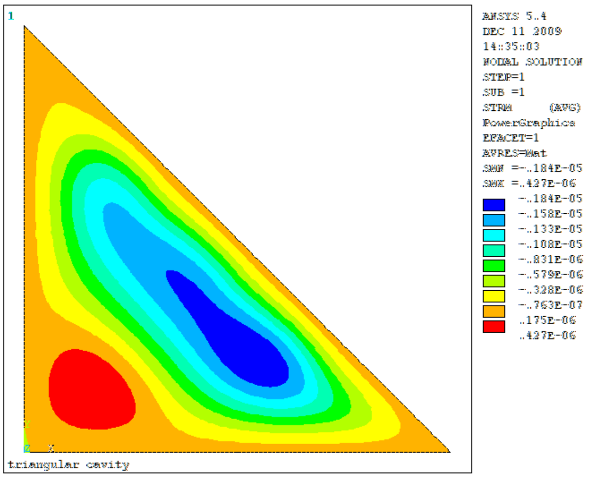


f:velocity distribution Profiles ,q_r=1000 W convection at 0.8 L <x <L, y=0 : 0.8L <y <L, x=0,

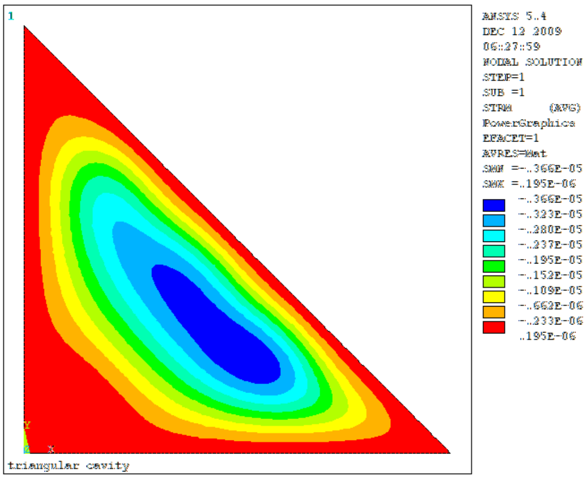
Fig5:Flow fields distribution Profiles through triangular cavity ,Velocity and temperature distributions



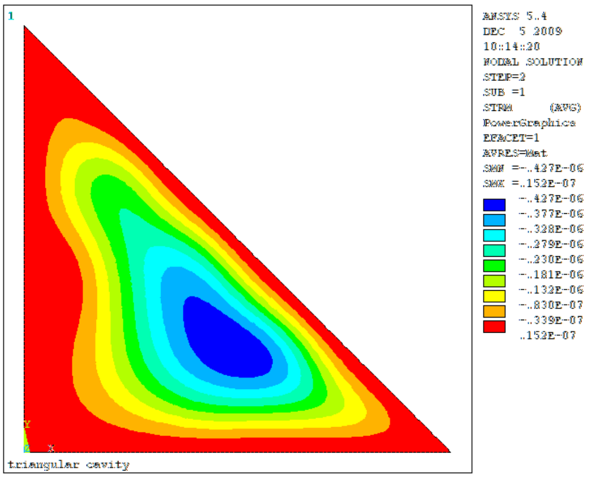
a: Stream function Profiles, q_r=100 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,



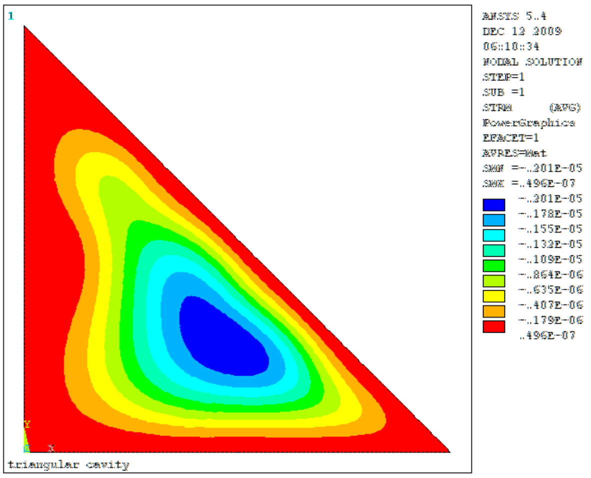
b : Stream function Profiles,q_r=500 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,



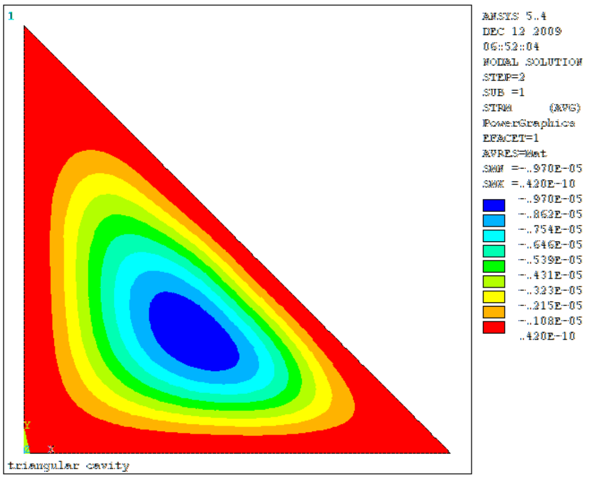
c: Stream function Profiles ,q_r=1000 W convection at 0 L <x <0.2L,y=0 : 0L <y <0.2L, x=0,



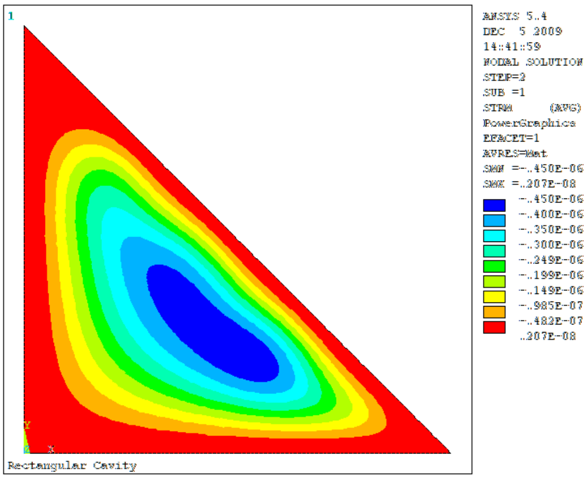
d: Stream function Profiles ,q_r=100 W convection at 0.4 L <x <0.6L,y=0 : 0.4L <y <0.6L, x=0,



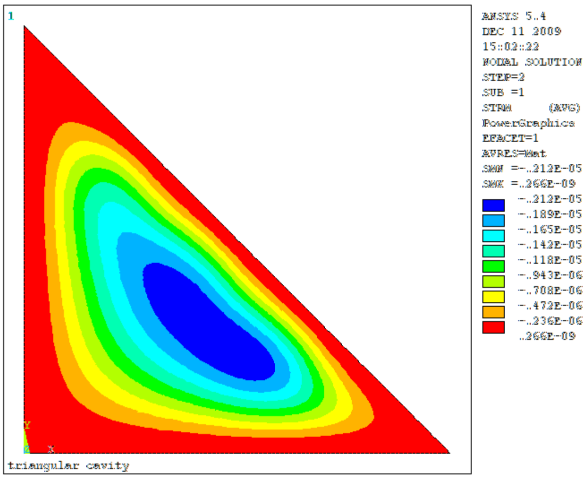
e: Stream function Profiles ,q_r=500 W convection at 0.4 L <x <0.6L,y=0 : 0.4L <y <0.6L, x=0,



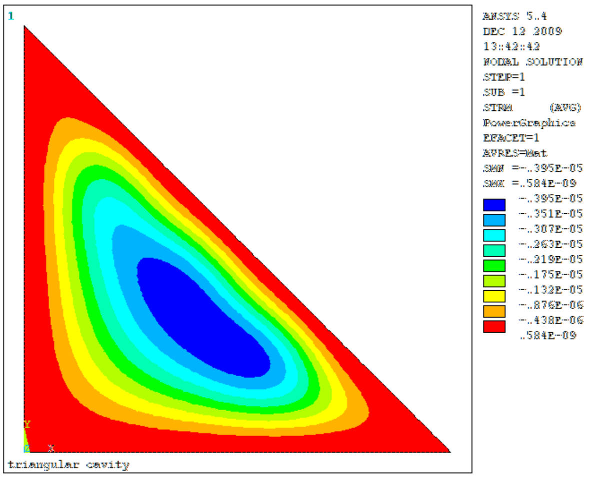
f: Stream function Profiles,q_r=1000 W convection at 0.4 L <x <0.6L,y=0 : 0.4L <y <0.6L, x=0,



a: Stream function Profiles ,q_r=100 W convection at 0.8 L <x <L ,y=0 : 0.8L <y <L, x=0,



b : Stream function Profiles,q_r=500 W convection at 0.8 L <x <L, y=0 : 0.8L <y <L, x=0,



c: Stream function Profiles ,q_r=1000 W convection at 0.8 L <x <L, y=0 : 0.8L <y <L, x=0,

Fig6:Flow fields distribution Profiles through triangular cavity ,Stream function distributions

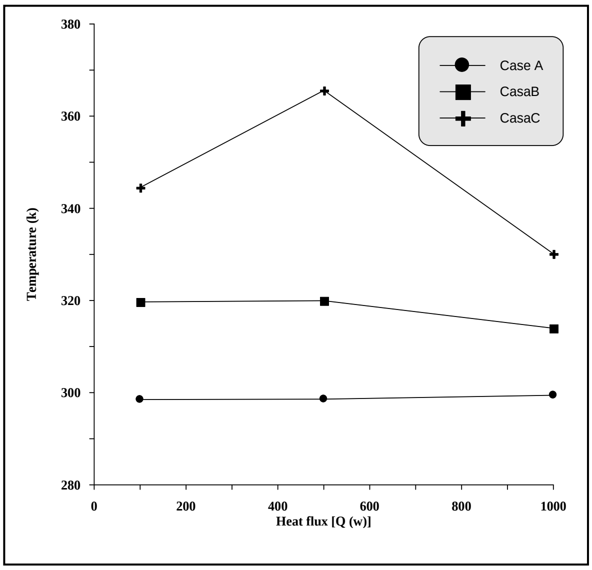


Fig7. Temperature distribution with heat flux

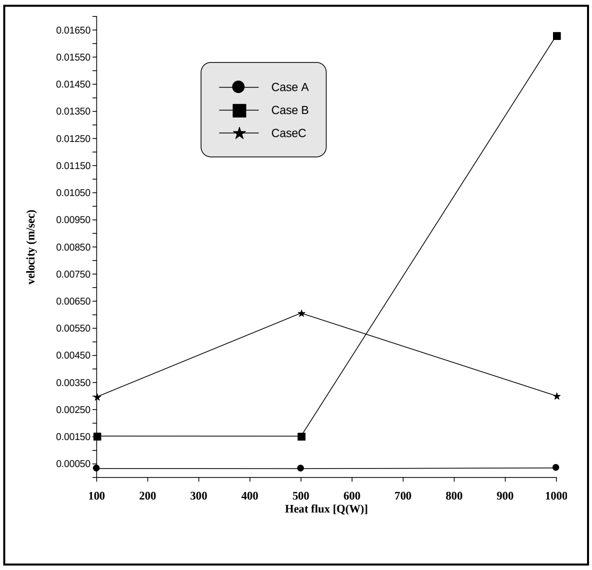


Fig8. Velocity distribution with heat flux

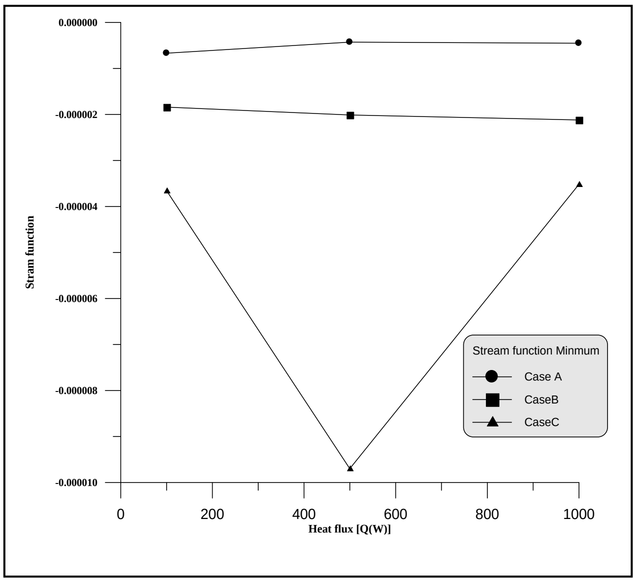


Fig9. Minimum stream function with heat flux

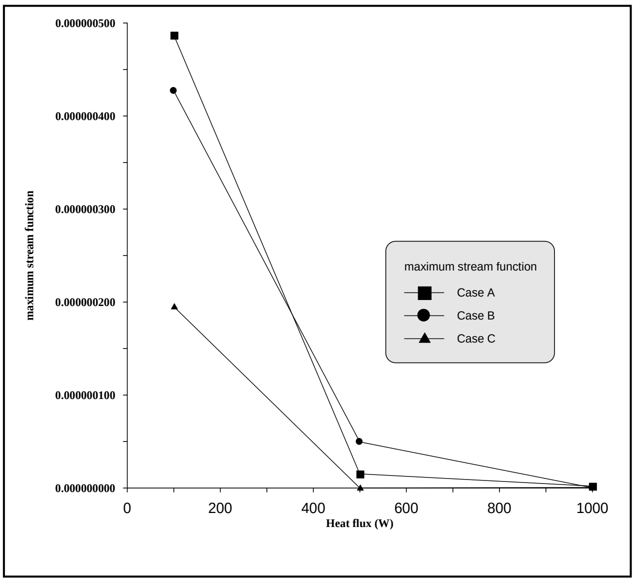


Fig10. Maximum stream function with heat flux

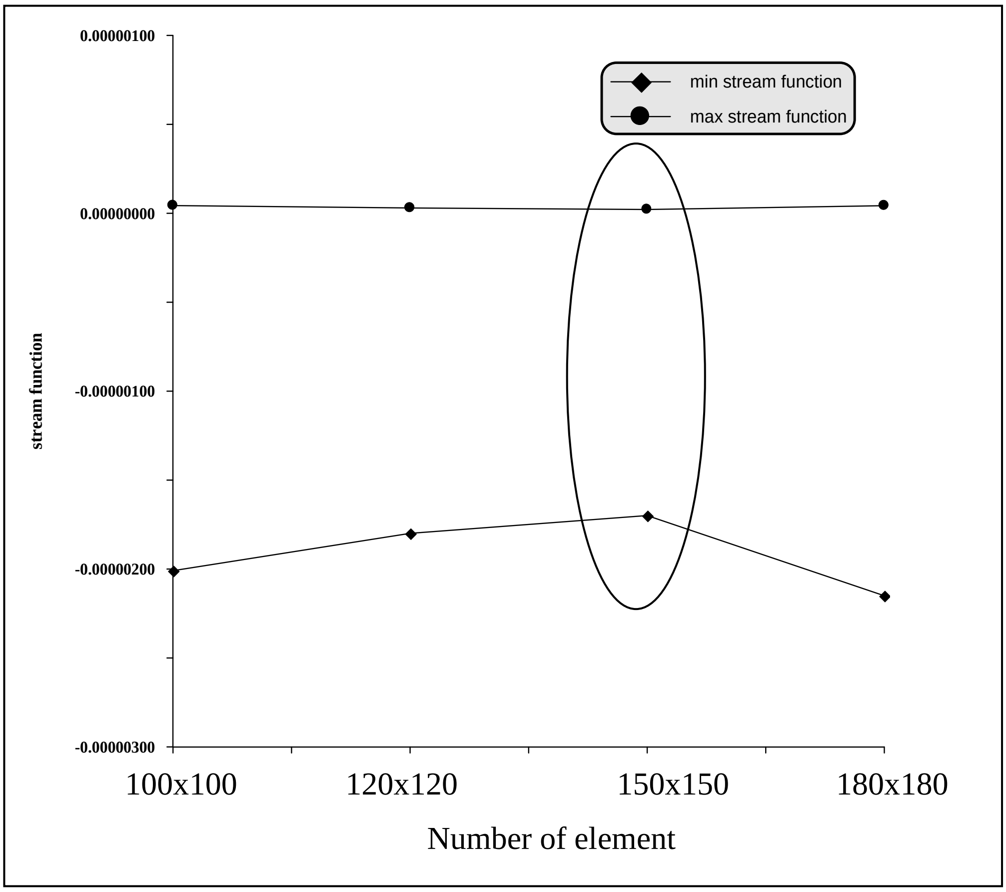


Fig11. Number of element with Stream function