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RESEARCH

Semiclassical treatment of fusion and breakup processes of ^{6,8}He halo nuclei

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Abstract A semiclassical approach has been used to study the effect of channel coupling on the calculations of the total fusion reaction cross section σ_{fus}, and the fusion barrier distribution D_{fus} for the systems ^{6}He $+^{238}$U and ^{8}He $+^{197}$Au. Since these systems invloves light exotic nuclei, breakup states channel play an important role that should be considered in the calculations. In semiclassical treatment, the relative motion between the projectile and target nuclei is approximated by a classical trajectory while the intrinsic dynamics is handled by time-dependent quantum mechanics. The calculations of the total fusion cross section σ_{fus}, and the fusion barrier distribution D_{fus} are compared with the full quantum mechanical calculations using the coupled-channels calculations with all order coupling using the computer code and with the available experimental data.

Keywords Coupled channels · Fusion reactions · Fusion barrier distribution

Introduction

The effects of channel coupling in fusion reactions induced by light weakly bound projectiles have attracted great interest over the last decade [1–3]. Some theoretical studies predict strong influence of the breakup channel over the complete fusion (CF) cross section [4–10]. This has primarily been motivated by the present availability of radioactive ion beams, some of which exhibit unusual features like halo/skin structure and large breakup probabilities. A critical understanding of the fusion mechanism with radioactive ion beams is very significant for the understanding of reactions of astrophysical interest and for the production of new nuclei near the drip lines [11–19]. The natural theoretical tool for their description is the coupled-channel method. However, when a weakly bound collision partner is involved, breakup states channel play an important role. Since they are in the continuum, an infinite number of channels should be taken into account. To handle this situation, the continuum can be discretized by the continuum discretized coupled-channel method (CDCC). However, the method is very complicated and requires considerable computer power. For practical purposes, it becomes necessary to approximate the continuum by a finite set of states, as in the CDCC method [20–23]. This procedure has been extended to the case of fusion reactions in Refs. [8–10]. A semiclassical treatment alternative based on the classical trajectory approximation of Alder and Winther (AW) [24] has been proposed by Marta et al. [25].

The aim of the present work focuses on using a semiclassical approach by adopting Alder and Winther (AW) theory originally used to treat the Coulomb excitation of nuclei. The approximation of AW consists of using classical mechanics to describe the projectile–target relative motion while the excitation of the extrinsic degrees of the nuclei are described by quantum mechanics. This semiclassical approximation has been implemented and coded in FORTRAN codename (SCF) [26] which has been used to calculate the total fusion cross section σ_{fus} (mb) and the

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fusion barrier distribution $D_{\rm fus}$ (mb/MeV) for the systems involving light exotic nuclei ^{6}He $+^{238}$U and ^{8}He $+^{197}$Au. The results of the present study are compared to the full quantum mechanical calculations using the coupled-channel calculations (CC) using the computer code CCFULL [27] and with the available experimental data.

Theoretical framework

Since this work is mainly concerned with the reactions induced by weakly bound projectiles, two variables have been employed, namely $\mathbf{r}$ and ξ for the projectile–target separation vector and the relevant intrinsic degrees of freedom of the projectile, respectively. The internal structure for the target nucleus have been neglected for simplicity. The Hamiltonian then reads

$$H = H_0(\xi) + V(\mathbf{r}, \xi) \tag{1}$$

where $H_0(\xi)$ is the intrinsic Hamiltonian of the projectile and $V(\mathbf{r}, \xi)$ represents the projectile–target interaction. The eigenvectors of $H_0(\xi)$ are given by the equation

$$H_0|\psi_v\rangle = \epsilon_v|\psi_v\rangle \tag{2}$$

The AW theory [24] is implemented in the following sequence: first, classical mechanics has been used for the time evolution of the variable, r. The trajectory depends on the collision energy, E, the angular momentum, $\hbar\ell$. The Rutherford trajectory $r_\ell(t)$ was used in the original AW method. In our case, the solution of the trajectory is the equations of classical motion with the potential, $V(r) = \langle\psi_0|V(r,\xi)|\psi_0\rangle$ where $|\psi_0\rangle$ is a wave function of the ground state of the projectile. The coupling interaction becomes a time-dependent interaction at using the ξ-space, $V(r_\ell(t), \xi)$. Second, the dynamics were treated in the intrinsic space as a time-dependent quantum mechanics problem. Expanding the wave function in the basis of intrinsic eigen states,

$$\Psi(\xi, t) = \sum_v A_v(\ell, t)\psi_v(\xi)e^{-i\epsilon_v t/\hbar} \tag{3}$$

and inserting this expansion into the time-dependent Schrödinger equation for $\Psi(\xi, t)$, one obtains the AW coupled equations

$$i\hbar\frac{\partial A_v(\ell,t)}{\partial t} = \sum_\mu \langle\psi_v|V_\ell(\xi,t)|\psi_\mu\rangle e^{-i(\epsilon_v-\epsilon_\mu)t/\hbar}A_\mu(\ell,t) \tag{4}$$

These equations are solved with the initial conditions $A_v(\ell, t \longrightarrow -\infty) = \delta_{v0}$, which means that before the collision $(t \longrightarrow -\infty)$ the projectile was in its ground state. The final population of channel v in a collision with angular momentum ℓ is $P_\ell^{(v)} = |A_v(\ell, t \longrightarrow +\infty)|^2]$ and the angle integrated cross section is:

$$\sigma_v = \frac{\pi}{k^2}\sum_\ell (2\ell+1)P_\ell^{(v)} \tag{5}$$

For using this method in the fusion processes, we start with the quantum mechanical calculation of the fusion reaction cross section in a coupled-channel problem. It is assumed that all the channels are bound and have spin zero for simplicity. The fusion reaction cross section is a sum of contributions from each channel. Carrying out partial-wave expansions we get

$$\sigma_F = \sum_v \left[\frac{\pi}{k^2}\sum_\ell (2\ell+1)P_\ell^F(v)\right] \tag{6}$$

with

$$P_\ell^F(v) = \frac{4k}{E}\int |U_{v\ell}(k_v, r)|^2\, W_v^F(r)\mathrm{d}r \tag{7}$$

Above, $U_{v\ell}(k_v, r)$ represents the radial wave function for the ℓ_{th}-partial-wave in channel v and $W_v^F(r)$ is the absolute value of the imaginary part of the optical potential associated to fusion reaction in that channel.

To evaluate the fusion cross section using AW method, we had adopted used the approximation:

$$P_\ell^F(v) \simeq \overline{P}_\ell^{(v)} T_\ell^{(v)}(E_v) \tag{8}$$

$\overline{P}_\ell^{(v)}$ is the probability that the system is in channel v at the point of closest approach on the classical trajectory, and $T_\ell^{(v)}(E_v)$ is the probability that a particle with energy $E_v = E - \epsilon_v$ and reduced mass $\overline{M} = M_P M_T/M_P + M_T$ where M_P, M_T are, respectively, the masses of the projectile and target, tunnels through the potential barrier in channel v.

We now proceed to study the CF cross sections in reactions induced by weakly bound projectiles. For simplicity, we assume that the ground state (g.s.) is the only bound state of the projectile and that the breakup process produces only two fragments, F_1 and F_2. In this way, the labels $v = 0$ and $v \neq 0$ correspond, respectively, to the g.s. and the breakup states represented by two unbound fragments. Neglecting any sequential contribution, the CF can only arise from the elastic channel. In this way, the cross section σ_{CF} can be obtained from Eq. (9), dropping contributions from $v \neq 0$. That is,

$$\sigma_{\rm CF} = \frac{\pi}{k^2}\sum_\ell (2\ell+1)P_\ell^{(\rm Surv)}T_\ell^{(0)}(E) \tag{9}$$

where

$$P_\ell^F(v) \equiv \overline{P}_\ell^{(0)} = |A_0(\ell, t_{\rm ca})|^2 \tag{10}$$

Fusion barrier distribution

The fusion barrier distribution $D_{\rm fus}$ is extracted from the measured data of $\sigma_{\rm fus}$, by taking the second derivative, with respect to center of mass energy ($E_{\rm c.m.}$), of the product ($E_{\rm c.m.}\sigma_{\rm fus}$) [28–30] as

$$D_{\rm fus} = d^2(E_{\rm c.m.}\sigma_{\rm fus})/dE^2_{\rm c.m.} \tag{11}$$

This quantity is calculated numerically using the three-point difference formula,

$$(D_{\rm fus})_{i+1} = \frac{2}{E_{i+2}-E_i}\left[\frac{F_{i+2}-F_{i+1}}{E_{i+2}-E_{i+1}} - \frac{F_{i+1}-F_i}{E_{i+1}-E_i}\right] \tag{12}$$

where

$$F_i = E_i(\sigma_{\rm fus})_i, i = 1, 2, 3, \cdots \tag{13}$$

with equal increments of the energy center of mass $\Delta E = E_{i+2} - E_{i+1} = E_{i+1} - E_i$, the numerical formula of $D_{\rm fus}$ can be written,

$$(D_{\rm fus})_{i+1} = \frac{F_{i+2} + F_i - 2F_{i+1}}{\Delta E^2} \tag{14}$$

and the statistical error $(\delta D_{\rm fus})$ related with the second derivative at energy E is approximately given by,

$$(\delta D_{\rm fus})_{i+1} \cong \frac{E_{i+1}}{\Delta E^2}\sqrt{(\delta\sigma_{\rm fus})^2_{i+2} + 4(\delta\sigma_{\rm fus})^2_{i+1} + (\delta\sigma_{\rm fus})^2_i} \tag{15}$$

where the $(\delta\sigma_{\rm fus})$ are the errors in the fusion interaction cross sections, and $(\delta D_{\rm fus})$ is error of the fusion barrier distribution.

Results and discussion

In this section, the numerical results for the total fusion cross section $\sigma_{\rm fus}$, and the fusion barrier distribution $D_{\rm fus}$ calculated using the semiclassical approach for the systems ^{6}He $+^{238}$U and ^{8}He$+^{197}$Au are explained. Our calculated results of $\sigma_{\rm fus}$ and $D_{\rm fus}$ are compared with the corresponding experimental data and with the full quantum mechanical calculations using the CCFULL code. The Aküz-Winther potential parameters used in the present calculations are listed in Table 1.

In the case of ^{6}He $+^{238}$U system, our calculated results represented by dashed blue and black curves for the semiclassical and coupled-channel calculations without including coupling, respectively. The solid blue and black curves are the calculations including the coupling effects for the semiclassical and coupled-channel calculations, for the total fusion reaction cross section $\sigma_{\rm fus}$, respectively, compared with the respective measured data (solid circles) in Fig.1 panel (a). The experimental data for this system are obtained from Ref. [17]. The coupled-channel calculations using CCFULL performed by considering rotational deformations for both the projectile and target nuclei with deformations parameters β_2 , and β_4 adopted from Moller et al. [31] as listed in Table 2.

The position of the experimental Coulomb barrier V_b indicated by an arrow at the $E_{\rm c.m.}$ axis. In the case of no-coupling both semiclassical and quantum mechanical calculations underestimate the experiential data below the Coulomb barrier. The semiclassical calculations are markedly enhanced over the coupled-channel calculations below the Coulomb barrier when we had considered coupling effects. It is clearly seen that, the semiclassical calculations are in better agreement with the measured data compared to the coupled-channel calculations. Panel (b) of Fig. 1 shows the fusion barrier distribution $D_{\rm fus}$ calculations; we could not construct the second derivative accurately using the three-point difference method from the measured data due to few experimental data points for this system. In the corresponding results of the barrier distribution in the lower panel of Fig. 1 we are unable to make clear judgment, if our semiclassical calculations are more satisfactorily fitting the measured data than the coupled-channel calculations.

In a similar analysis, we compare our calculated results of $\sigma_{\rm fus}$ and $D_{\rm fus}$ with the corresponding data in panels (a) and (b) of Fig. 2, respectively, for the ^{8}He $+^{197}$Au system. The experimental data of this system are obtained from Ref. [32]. In this case also, we had found that the inclusion of the coupling enhances the semiclassical calculations markedly above and below the Coulomb barrier compared with the measured data, while the CC calculations have very slight improvement below the Coulomb barrier, but still far from the measured data. This system also has few experimental data, that the construction of the second derivative is inaccurate; therefore, we cannot make clear judgment if our semiclassical or coupled-channel calculations are able to fit the measured data for the fusion barrier distribution calculations.

Table 1 The parameters of Aküz-Winther potential along with V_b and R_b

Projectile + target	V_0 (MeV)	a_0 (fm)	r_0 (fm)	V_b (MeV)	R_b (fm)
^{6}He $+^{238}$U	157.4	0.67	1.2	19.6	12.8
^{8}He $+^{197}$Au	55.8	0.64	1.17	18.62	11.53

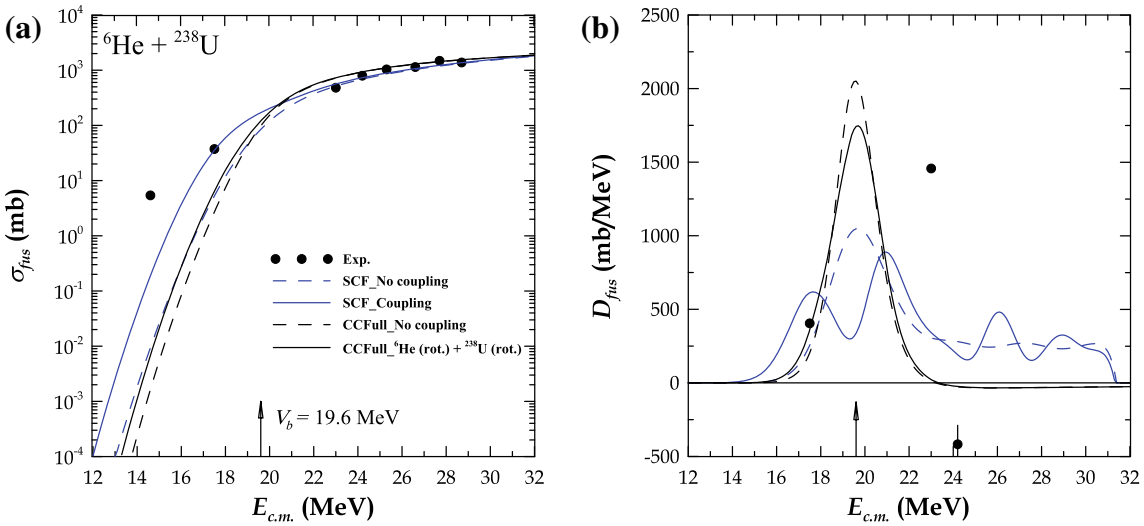

Fig. 1 The comparison of the semiclassical and full quantum mechanical calculations using SCF code (*blue curves*) and CCFull code (*black curves*) with the experimental data (*black filled circles*) [17] for ^{6}He $+^{238}$U system. **a** for the total fusion cross section σ_{fus} (mb), and **b** for the fusion barrier distribution D_{fus}(mb/MeV), and the *arrow* represents the position of the Coulomb barrier V_b

Table 2 Deformation parameters and number of rotational levels used for CCFULL code for the studied systems

System	Projectile nucleus			Target nucleus		
Projectile + target	β_2	β_4	$N_{\mathrm{rot.}}$	β_2	β_4	$N_{\mathrm{rot.}}$
^{6}He $+^{238}$U	0.00	0.00	1	0.242	0.078	4
^{8}He $+^{197}$Au	0.00	0.00	1	−0.131	−0.031	4

Conclusions

The effect of coupled channel between the elastic channel and the continuum is found to be very essential in the semiclassical calculations which leads to improvement in the total fusion reaction cross section σ_{fus} and the fusion barrier distribution D_{fus} around and below the Coulomb barrier and brings the theoretical results closer to the

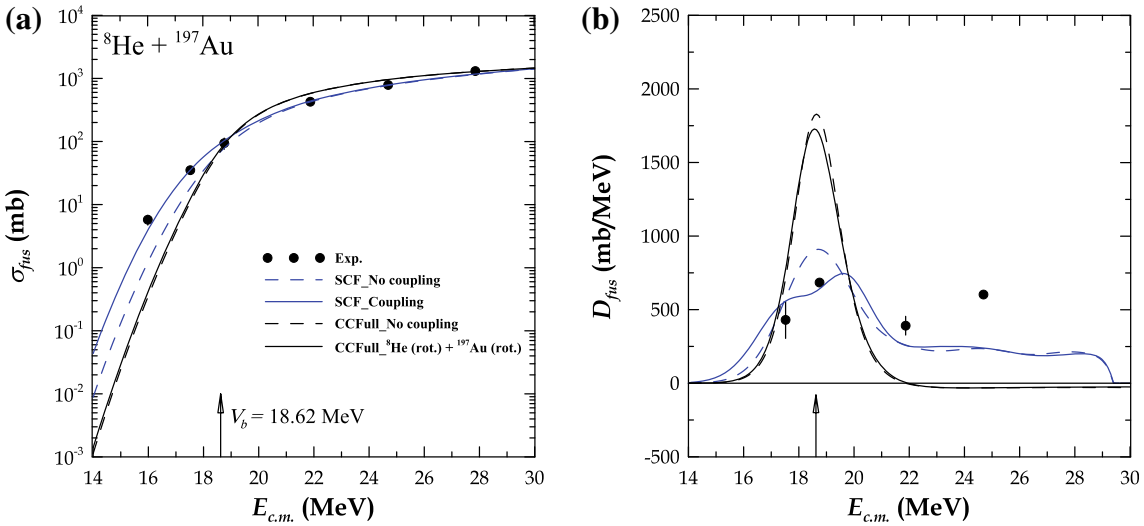

Fig. 2 The comparison of the semiclassical and full quantum mechanical calculations using SCF code (*blue curves*) and CCFull code (*black curves*) with the experimental data (*black filled circles*) [32] for ^{8}He $+^{197}$Au system. **a** for the total fusion cross section σ_{fus} (mb), and **b** for the fusion barrier distribution D_{fus}(mb/MeV), and the *arrow* represents the position of the Coulomb barrier V_b

experimental data. The inclusion of the coupling effects by considering both target and projectile have rotational deformations that enhances the full quantum mechanical calculations around and below the Coulomb barrier, but still have shortfall in reproducing data especially below the Coulomb barrier. The semiclassical approach used in the present work proves to be more adequate to describe light exotic nuclei than the full quantum mechanical ones in comparison with the experiment, because the semiclassical approach has been successfully extended to consider the continuum as a discrete set of channels as in Ref. [8]; therefore, the relative motion between the breakup fragments is more accurately described. The analytical continuation of the time variable is used to extend the trajectories to classically forbidden regions which enhances the calculations below the Coulomb barrier [25]. We could not construct the second derivative to calculate the fusion barrier distribution form the measured data accurately, therefore we could not make clear judgment of whether our semiclassical or coupled-channel calculations agreed with the experimental fusion barrier distribution. This work can be extended to study more systems involving halo nuclei, besides one can test its success or failure in the calculations of fusion reactions involving medium and heavy systems.

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