Large-Scale Shell Model Calculations of ^{134,136}Sn, ^{134,136}Te around doubly-magic ^{132}Sn

Fouad A. Majeed, Sarah M. Obaid

Abstract— Shell model calculations were performed to study the energy levels, binding energy and reduced transition probabilities B(E2;$0^+_{g.s}\rightarrow 2_1^+$) for even-even ^{134,136} Sn,^{134,136}Te around doubly magic core ^{132}Sn by using shell model code Nushellx@MSU for windows by employing the effective interactions jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe. The result of theoretical calculations were compared with recent available experimental data for energy levels, binding energy and reduced transition probabilities.

Index Terms— Shell model, energy levels, nuclear structure, B(E2) values.

1 Introduction

The nuclei around double closures ^{132}Sn becomes recently important for both experimental and theoretical study, nuclei around doubly closed shells play a special role. In fact, they yield direct information on the two basic ingredients of the model: single-particle (SP) energies and matrix elements of the effective interaction [1] . This makes them the best testing ground for realistic shell-model calculations where the effective interaction is derived from the free nucleon-nucleon (NN) potential, as well as to test theoretical shell model description of nuclear structure in this region. Structure properties of some of these nuclei are important inputs for astrophysical r-process model calculations [2]. Some of the structure issues recently studied are the decrease in the spin-orbit interaction at N − Z = 32 above the Z = 50,N = 82 double shell closure [3]. Shell model calculations were performed by F.A. Majeed to study neutron- rich even-even $^{132\text{-}136}$Te using a realistic interaction derived from CD-Bonn nucleon-nucleon potential for the positive and negative parity states and the transition rates B(E2;$0_{g.s}^+\rightarrow 2_1+$) the calculated results were compared with recently available experimental data[4]. Shell model calculation were performed by Brown et al. to study magnetic moments for $^{130\text{-}132}$Sn and $^{132\text{-}134}$Te isotopes due to the development of neutron-rich radioactive beams at the Holifield Radioactive Ion Beam Facility which stimulated experimental and theoretical activity in heavy Sn and Te isotopes[5]. The binding energies, excitation energies, transition probabilities and magnetic moments were studied by M. Saha. Sarkar for neutron-rich isotones N=82-84 nuclei near ^{132}Sn the results theoretical calculations are compared with experimental data [6]. Recently we have performed large

scale shell model calculations to study energy levels and reduced transition probabilities B(E2) for even-even $^{54\text{-}66}$Fe neutron rich isotopes by employing GXPF1, GXPF1A, KB3G and FPD6 effective interactions [7] and very recently we had performed a restricted no-core shell model calculations to study the low-lying energy levels for some selected light halo nuclei lies in the p-shell using *spsdpf* model space with *wbt* effective interaction [8]. The aim of the present work is to study the level energies including the high Jπ-values, binding energy and reduced transition probabilities B(E2;$0_1+\rightarrow 2_1+$) for ^{134}Sn, ^{134,136}Te and B(E2;$4_1^+\rightarrow 6_1^+$) for ^{136}Sn by means of large-scale shell model calculations without any restrictions by employing jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe effective interactions using the recent shell model code Nushellx@MSU [9] and compare the theoretical results with the most recent experimental data.

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2 Shell Model Calculations

The independent-particle Hamiltonian of an A-particle system can be written in terms two-particle interactions as [10].

$$H = \sum_{k=1}^{A} T_k + \sum_{k=1}^{A}\sum_{l=k+1}^{A} W(\vec{r_k},\vec{r_l}) \qquad (1)$$

where $W(\vec{r_k},\vec{r_l})$ is the two-body interaction between the kth and l^{th} nucleons. Choosing an average potential $U(r_k)$, the Hamiltonian becomes [10] .

$$H = \sum_{k=1}^{A}[T_k + U(r_k)] + \sum_{k=1}^{A}\sum_{l=k+1}^{A} W(\vec{r_k},\vec{r_l}) - \sum_{k=1}^{A} U(r_k) \qquad (2)$$

where the first term is identical to the independent-particle Hamiltonian, and the second and third account for the deviation from independent particle motion, known as the residual interaction. Separating the summations into core and valence contributions, eqn.(2) can be re-written [8]

$$H = H_{core} + H_1 + H_2 + V(\vec{r_1},\vec{r_2}) \qquad (3)$$

In equation (3), H_{core} contains all of the interactions of nucleons making up the core, H_1 and H_2 are the single-particle

contributions from particles 1and 2, and $V(\vec{r_1}, \vec{r_2})$ is the residual interaction describing all interactions between particles 1 and 2 as well as any interaction with core nucleons. Inserting this form of the Hamiltonian into the Schrödinger equation yields an analogous expression for the energy [10]

$$E = E_{core} + E_1 + E_2 + \langle \Phi_{J,\tau} | V(\vec{r1}, \vec{r2}) | \Phi_{J,\tau} \rangle \qquad (4)$$

Here, E_{core} is the binding energy of the core nucleus, E_1 and $E_2$are defined as the single-particle energies of orbitals outside the core, and $\langle \Phi_{J,\tau} | V(\vec{r1}, \vec{r2}) | \Phi_{J,\tau} \rangle$ is the residual interaction which needs to be defined by theory. It is important to note that the energy given by eqn. (4) is for pure configurations only. In principle, any close-lying state with the same total angular momentum J and total isospin τ will mix. The mixed eigenstates are given by linear combinations of the unperturbed wave functions [8]

$$(\Psi_{J,\tau})_p \sum_{k=1}^{g} a_{kp} (\Phi_{J,\tau})_p \qquad (5)$$

where g is the number of configurations that mix and the label p =1,2,..., g. The coefficients a_{kp} fulfill the condition [8]

3 Results and Discussions

3.1 Excitation Energies

In order to perform large scale shell model calculation in this mass region the core ^{132}Sn for all nuclei under study with 2,4 particles outside the core for ^{134}Sn, ^{136}Sn, ^{134}Te and ^{136}Te respectively. Figure 1 present comparison between our theoretical work and experimental for 134 Sn isotope. Our theoretical work predicts the differences between 0^+ and 2^+ at 0.846 MeV, 0.846 MeV, 1.245 MeV, 1.245 MeV, 0.778 MeV and 1.636 MeV. By employing the effective interactions jj56pna ,jj56pnb, kh5082, cw5082, jj56cdb and khhe respectively .By comparing them with the experimental differences values is 0.726 MeV. We obtained the best agreement at jj56cdb interaction.

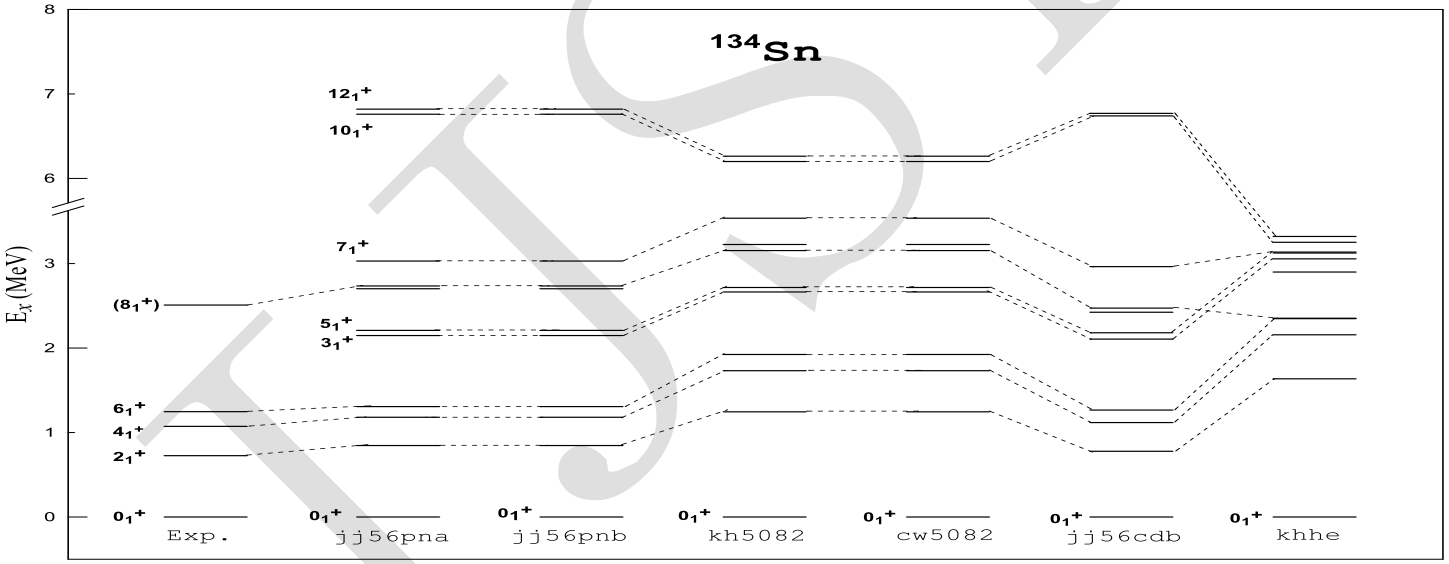

Fig. 1: Comparison of the experimental excitation energies taken from Ref.[11] with the present theoretical work using jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe effective interactions for ^{134}Sn isotope.

$$\sum_{k=1}^{g} |a_{kp}|^2 = 1 \qquad (6)$$

Inserting eqn. (5) into the Schrödinger equation gives,

$$H(\Psi_{J,\tau})_p = E_p(\Psi_{J,\tau})_p \qquad (7)$$

which leads to a system of linear equations[10].

Figure 2 present comparison between our theoretical work and experimental data for ^{136}Sn isotope with 4 particles outside the core. Our calculations predicts the differences between 0^+ and 2^+ at 0.815 MeV, 0.815 MeV, 1.214 MeV, 1.214 MeV, 0.734 MeV and 1.568 MeV for the effective interactions jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe respectively. By comparing them with the experimental differences values between 0^+ and 2^+ is 0.688 MeV. The best agreement with the experimental differences value is achieved at jj56cdb effective interaction.

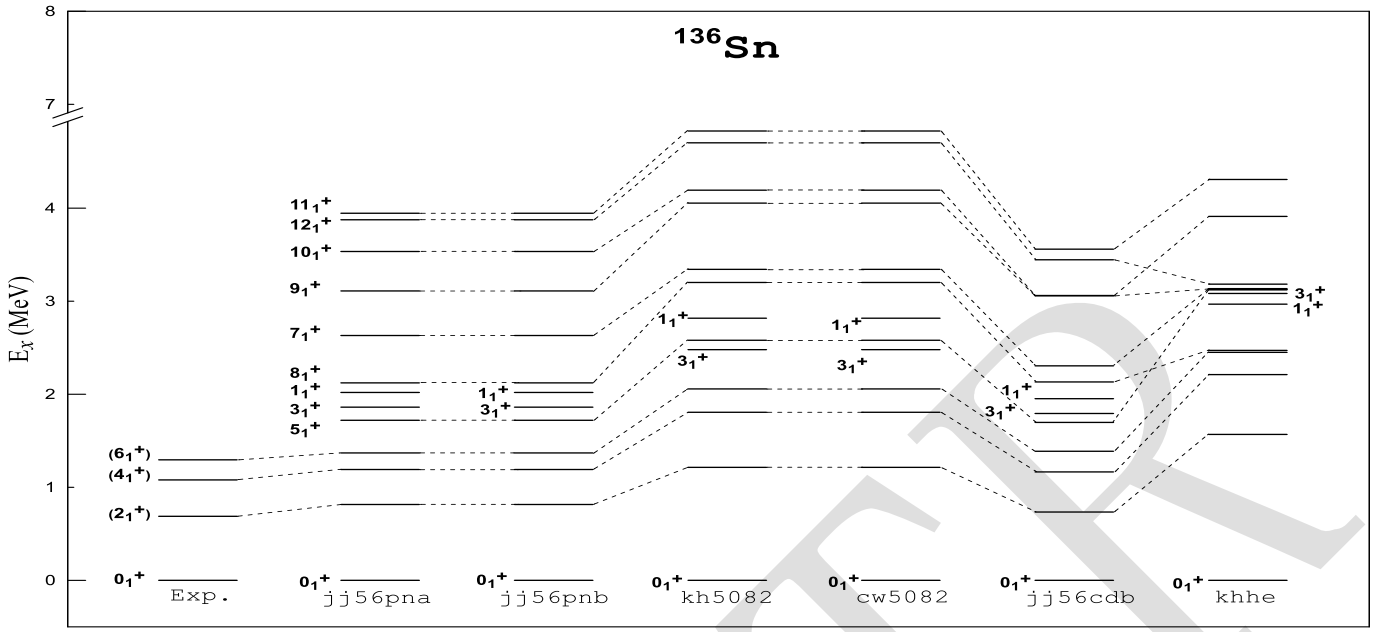

Fig. 2: Comparison of the experimental excitation energies taken from Ref. [11] with the present theoretical work using jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe effective interactions for ^{136}Sn isotope.

Figure 3 presents the calculated excitation energy levels including high Jπ-values for the positive and negative parity for ^{134}Te .Our calculations predicts the differences between 0^+ and 2^+ at 1.627 MeV ,1.205MeV , 1.182 MeV , 1.330 MeV, 1.280 MeV and 1.217 MeV. By employing the effective interactions jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe respectively which compared with the experimental differences value between 0^+ and 2^+ is 1.279 MeV . The better agreement with the experimental differences value is achieved at jj56cdb effective interaction.

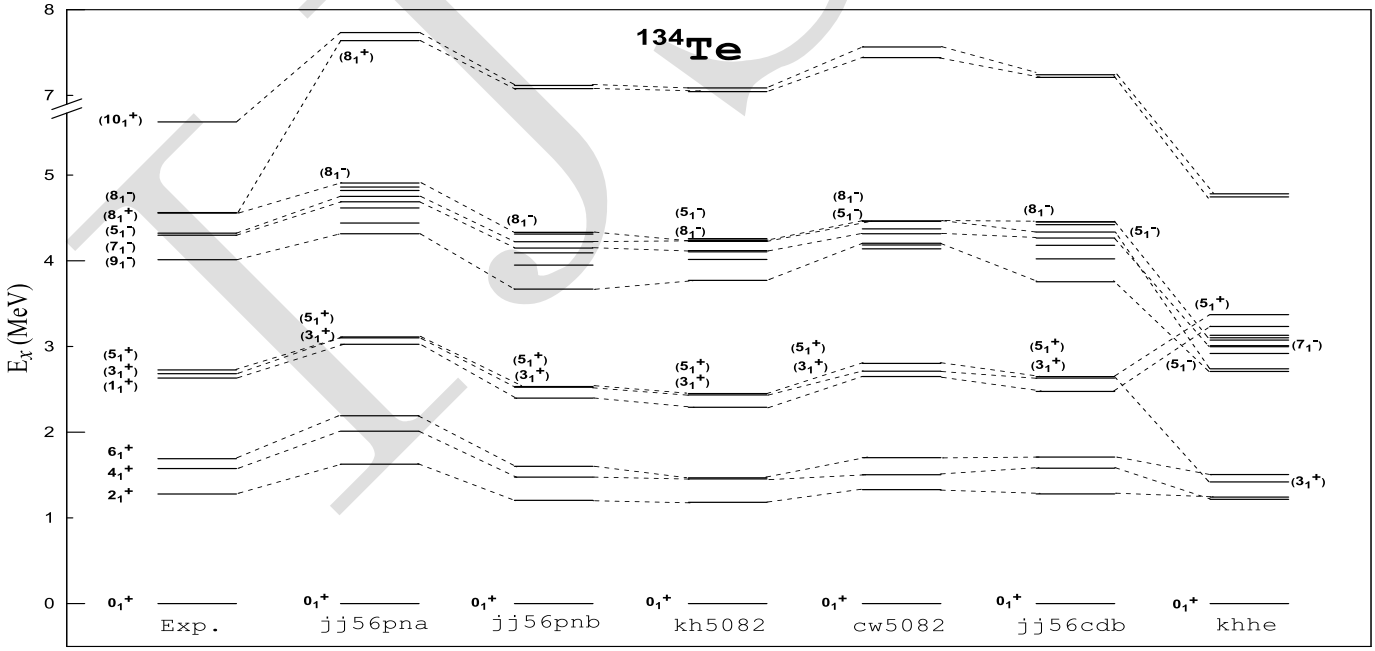

Fig. 3: Comparison of the experimental excitation energies taken from Ref [11] with the present theoretical work using jj56pna, jj56pnb , kh5082 ,cw5082 ,jj56cdb and khhe effective interactions for ^{134}Te isotope.

Figure 4 presents the calculated excitation energy levels including high Jπ-values for the positive and negative parity for 136 Te with 4 particles outside core .Our theoretical predicts the differences between 0^+ and 2^+ at 0.754MeV , 0.645MeV , 0.910 MeV , 0.888MeV,0.558 MeV and 1.000MeV. for the effective interactions jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe respectively which compared with the experimental differences value between 0^+ and 2^+ is 0.607 MeV. The better agreement with the experimental differences value is achieved at jj56cdb effective interaction.

the effective charge for proton and e_π^{eff} =1.6 ,e_υ^{eff} =0.6. We obtained the best agreement at jj56cdb interaction. This interaction is successful in describing the energy levels and Reduced Transition Probabilities for ^{134}Sn. For ^{136}Sn nucleus taking average value of the effective charge for proton and e_π^{eff} =0.2 ,e_υ^{eff} =1.6 for each interactions for each interactions and compared it with the experimental the effective interactions jj56pna and jj56pnbto the are the nearest to the experimental value .for ^{134}Te nucleus taking average value of the effective charge e_π^{eff} =0.8, e_υ^{eff} =0.2. The best agreement with the experimental value at kh5082 interaction.

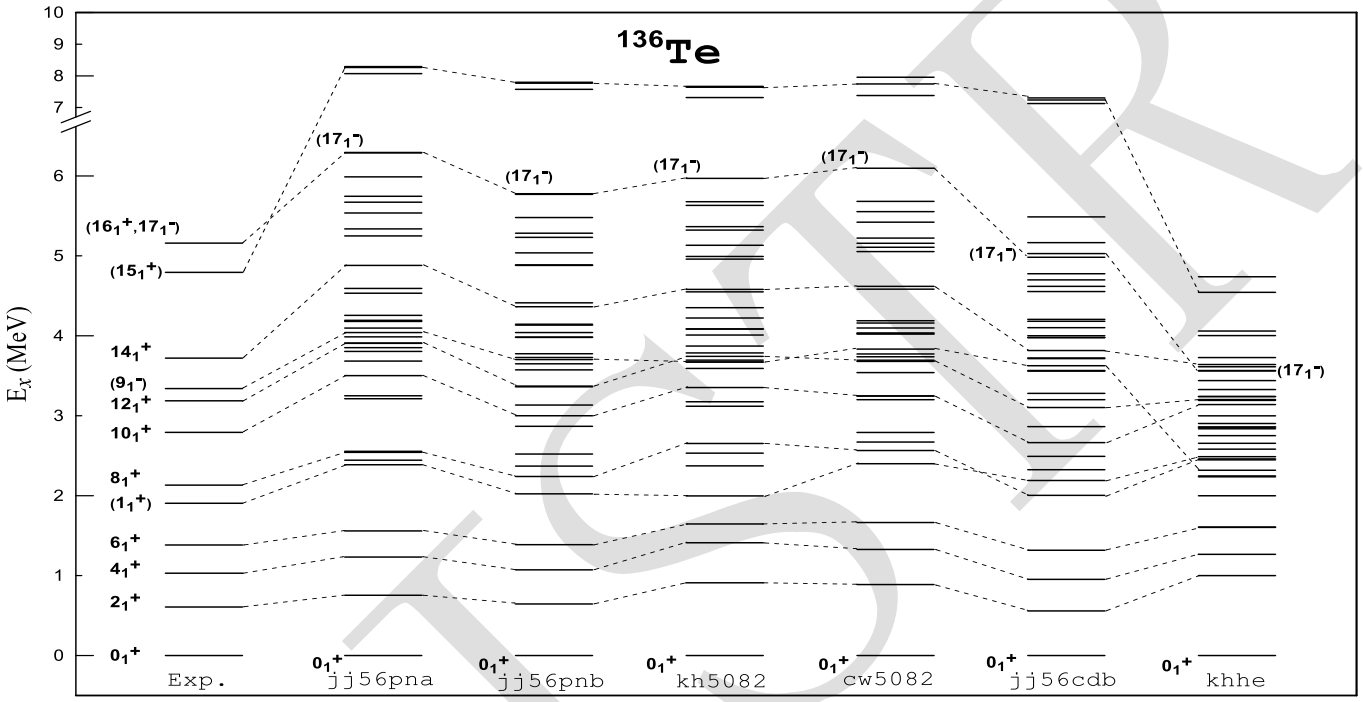

Fig. 4: Comparison of the experimental excitation energies taken from Ref [11] with the present theoretical work using jj56pna, jj56pnb , kh5082 ,cw5082 ,jj56cdb and khhe effective interactions for ^{136}Te isotope.

3.2 Reduced Transition Probabilities

The electromagnetic transition probability $B(E2;0_1^+ \rightarrow 2_1^+)$ and $B(E2;4_1^+ \rightarrow 6_1^+)$ values are calculated for model space and for each interaction are compared with experimental data for ^{134}Sn and ^{134,136} Te and ^{136}Sn respectively. By taking the average value of the effective charge for proton and neutron. Table 1 presents the comparison between the calculated reduced transition probabilities for all interactions and compared it with the experimental data .for 134Sn taking average value of

The ^{136}Te isotope taking average value of the effective charge for proton and e_π^{eff} =0.4, e_υ^{eff} =0.4 for each interactions. We obtained the best agreement at cw5082 interaction

Table 1: Comparison between the calculated B(E2;$2_1^+ \rightarrow 4_1^+$) (e^2fm^4) of the ^{134}Sn ^{134,136}Te and B(E2;$4_1^+ \rightarrow 6_1^+$)(e^2fm^4) of the ^{136}Sn with the experimental data taken from [12].

$J_i^\pi \rightarrow J_f^\pi$	Isotop	Exp.	jj56pn	jj56pn	kh508	cw508	jj56cd	Khhe	e_π	e_υ
$0_1^+ \rightarrow 2$	^{134}Sn	290(5)	276.8	276.8	309.9	309.9	296	281.5	1.6	0.6
$4_1^+ \rightarrow 6$	^{136}Sn	29.004	41.74	41.74	9.394	9.394	113.9	384	0.2	1.6
$0_1^+ \rightarrow 2$	^{134}Te	1040(4	1173	1144	1029	1282	1156	671.5	0.8	0.2
$0_1^+ \rightarrow 2$	^{136}Te	1220(1	1132	1372	1171	1254	1375	1093	0.4	0.4

3.3 Binding Energies

The comparison of the calculated binding energy for the isotopes under study using jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe with their corresponding experimental binding energy [13] are tabulated in Table 2. All the effective interactions used in the present work are able to predict the binding energy of the isotopes under study excellently.

Table 2: Comparison between the calculated the binding energy for each effective interaction (in units MeV) with and compared it experimental data taken from Ref.[13].

Isotope	Exp.	jj56pna	jj56pnb	kh5082	cw5082	jj56cdb	Khhe
^{134}Sn	1109.235	1096.455	1096.455	1097.172	1096.972	1096.395	1098.586
^{136}Sn	1115.064	1102.688	1102.688	1103.829	1103.429	1102.583	1106.61
^{134}Te	1123.434	1111.445	1110.902	1110.78	1110.779	1111.248	1108.18
^{136}Te	1131.442	1119.313	1118.912	1119.415	1119.127	1119.513	1117.861

4 Conclusions

Unrestricted large-scale shell model calculations were performed for to study the energy levels, reduced electric transition probabilities and binding energy for ^{134,136} Sn, ^{134,136} Te isotopes by employing the model space jj56pn with jj56pna, jj56pnb, kh5082, cw5082, jj56cdb and khhe effective interactions by using the shell model code Nushellx@MSU. The comparison of the calculated low-lying energy levels with the most recent experimental data and transition strengths B(E2) for these nuclei are in sophisticated agreement with the experimental data. The experimental binding energy are very well reproduced by the current shell model calculation.

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