ON THE DYNAMICS OF DUAL-SPIN SPACECRAFT CONTAINING A NUTATION DAMPER

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ABSTRACT

The dynamics of dual-spin spacecraft which containing a proposed nutation damper which consisting of a ring totally filled with a viscous liquid with offset center, to improve damping, is investigated. The equations of motion were developed using Newton-Lagrange approach resulting equations in terms of spacecraft's and damper's parameters , which are given in dimensionless form. The expression of the nutation angle and time constant in both modes are developed using zero-order approximation technique. The equilibria states and stability condition, and the analytical expression for residual nutation angle were derived. The analytical results were compared with those found numerically using computer simulation program named MATLAB, ver. 7.
Also the effect of various spacecraft's and damper's parameters on the dynamic and damping characteristics are discussed. The three dimensional graphical representation of the first and the second relative equilibria states are introduced. The numerical results are compared with the analytical for both modes of motion, where the percentage error of the time constant for nutation mode is less than ($\approx$3.6%), and for spin mode is less than ($\approx$8%). As an important result its concluded that the proposed damper works better than that used by Alfriend[2].

الخلاصة

تم تحليل ديناميكية الأقمار الصناعية ذات البرم المزدوج والتي تحوي على مخمد ترنحي مقترح يتألف من حلقة مملوءة كليا بسائل وبمركز مزاح عن محور الدوران للحصول على أحسن تخميد للحركة الترنحية للأقمار الصناعية. استخدمت طريقة نيوتن-لاگرانج لاشتقاق معادلات الحركة وتم الحصول على معادلات بدلالة متغيرات القمر الصناعي ومتغيرات المخمد الترنحي، حيث تم جعل هذه المتغيرات بحيث تكون لا بعدية. وجدت التعابير الخاصة بزاوية الترنح (Nutation angle) وثابت الزمن في كلا الطورين باستخدام طريقة تقريب المرتبة الصفرية (zero-order approximation) وكذلك نوقشت حالات الاتزان وشروط الاستقرارية الخاصة بهذه الأقمار؛ وكذلك تم الحصول على التعبير الرياضي لزاوية الترنح المتبقية. تم مقارنة النتائج التحليلية المستحصلة

في هدا البحث مع النتائج العددية المستحصل عليها باستخدام المحاكاة بواسطة برنامج MATLAB 7 نوقشت أيضا تأثير المتغيرات المختلفة الخاصة بالقمر الصناعي ومنظومة التخميد على الخواص الديناميكية والتخميدية للمنضومة. وأخيرا وضَحت حالة الاستقرارية النسبية الأولى والثانية من خلال رسوم ثلاثية الأبعاد (Three Dimensional Representation). تم مقارنة النتائج العددية مع التحليلية لكلا الطورين، حيث وجدت نسبة الخطأ لثابت الزمن لطور التزامن الترنحي اقل من (3.6%) ولطور التزامن التدويمي اقل من (8%)

NOMENCLATURE

Symbol	Description	Unit
A	Principal transverse moment of inertia of the spacecraft	$kg.m^2$
b	Ratio of the ring height to the ring mean radius	-
C, C_p	Moment of inertia of the rotor and the platform along z-axis	$kg.m^2$
d	Distance of the offset.	M
F_d	Drag force	N
h_t	Transverse angular momentum	
h_{tt}	Total angular momentum	
$[I_{s/c}]$	Spacecraft inertia matrix	$kg.m^2$
I_u, I_v, I_z	Moments of inertia of the damping viscous liquid measured along u, v, and z axes	$kg.m^2$
I_{uz}	Product moment of inertia of the fluid	$kg.m^2$
m	Mass of the fluid	kg
p, q, r	Dimensionless angular velocity components of the spacecraft about x,y and z, respectively.	
Q_α	Generalized moment associated with the generalized coordinate α	N.m
{Q}	Column matrix of the moment component of the non conservative forces	N.m
R	Ring mean radius of the nutation damper	m
T	Total kinetic energy of the spacecraft equipped with nutation damper	N.m
t	Time	sec
x, y, z	Body fixed frame.	

Greek Symbol

Symbol	Description	Unit
α	Relative angular displacement between the fluid and the spacecraf	rad
α_o	Initial value of α in the spin-synchronous mode	rad
$\tilde{\alpha}$	Small variation in α in the spin-synchronous mode	rad
$\dot{\alpha}$	Relative angular velocity of the fluid	rad/s
ε	Inertia ratio of the fluid to the transverse inertia of the spacecraft	-
η	Damping constant of the damping fluid	-
θ	Nutation angle of the spacecraft	rad
θ_n	Nutation angle in the nutation-synchronous mode	rad

θ_r	Residual nutation angle of the spacecraft	rad
θ_s	Nutation angle in the spin-synchronous mode	rad
σ	Inertia ratio of the spacecraft (σ=C/A)	-
τ	Dimensionless time	-

INTRODUCTION

The attitude control system of the spacecraft is to control the attitude and position of the spacecraft as it performs its mission. The techniques that provide attitude stabilization and control of spacecraft are; passive control system, semi-passive and active control system(15). The type of the system adopted in the present study is the passive type system. Passive system does not require any external power source, once they are in place, they use gravity or momentum to create the necessary control forces and moments(19).

Dual-spin stabilization type is the method of attitude stabilization adopted in the present study. A spin and Dual-spin stabilized spacecraft, or spinners, utilizes its own spinning motion to keep it's self aligned in a certain inertial direction. The spinning motion creates stiff angular momentum vector, which tend to resist external disturbance torques. A spinner is stable if it is spun about the axis of largest principal moment of inertia, if it is spun about a different axis, any disturbance could cause the spin axis to shift to the major axis.

In single spin stabilization the whole body rotates about the axis of maximum principal moment of inertia. Early communication satellites, such Syncom I, ATS I, II and Inelsat I, II were single spin stabilized. Its advantages are simple, reliable, and long life time but the main limitations of these satellites are that they could not use earth oriented antennas. These limitations are overcome in a dual spin spacecraft. Whereas dual spin spacecraft consisting of spinning rotor producing gyroscopic stiffness and a platform rotating at a much slower rate in accordance with the desired attitude of the spacecraft. There are two types which are commonly known as the external rotor and body stabilized spacecraft, each type employs a different method of attitude stabilization. The external rotor type or "Gyrostat" uses spin stabilization where the rotor of relatively large moment of inertia rotates to provide gyroscopic stiffness, while the platform usually containing communication equipment and antennas are despun.

Chang, and Liu (4), studied the dynamic and stability of an inertially symmetric, spinning, rigid body with a partial filled viscous-ring damper mounted normal to the spin axis. They used the nonlinear equations directly by using center manifold theory; they generated the stability criteria and the decay time constant. Then Alfriend(2), studied the attitude stability of

Dual-spin spacecraft with a partial filled viscous-ring damper utilizing zero order approximation technique. Hamed [31], studied the ball in ring nutation damper, he utilized neon gas and many percentage of glycerin water mixture as damping fluid in his study.

The present work represents an attempt to study the full filled viscous ring damper mounted normally to the spin axis with offset centre (d) from the spin axis.

-EQUATION OF MOTION

The total angular momentum can be written in terms of angular velocity component:

$$\vec{h} = [(A+I_u)\omega_u - I_{uz}(\omega_z+\dot{\alpha})]e_u + [(A+I_v)\omega v]e_v + [C\omega_z + I_z(\omega_z+\dot{\alpha}) - I_{uz}\omega_u + C_p\omega_{pz}]e_z \qquad 1$$

When the external torque components are zero, the system referred to as a freely precessing system, then the principle of conservation of angular momentum can be applied, such that[21]:

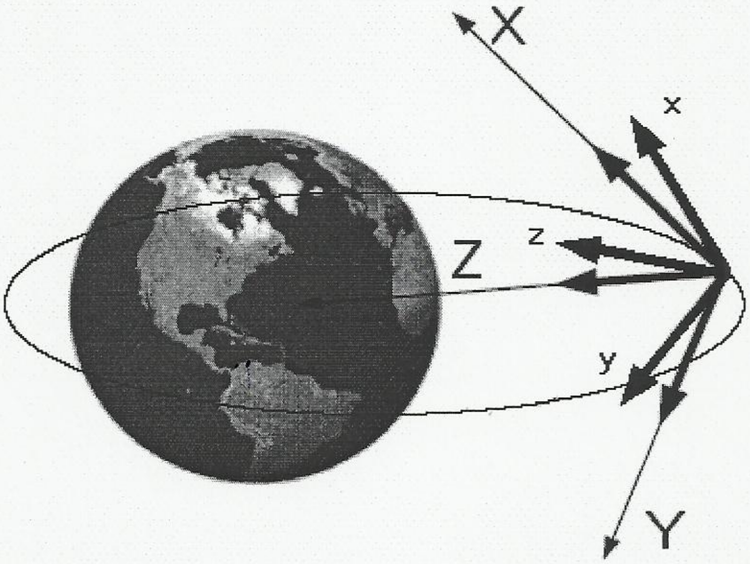

Fig. A: Body fixed coordinate system.

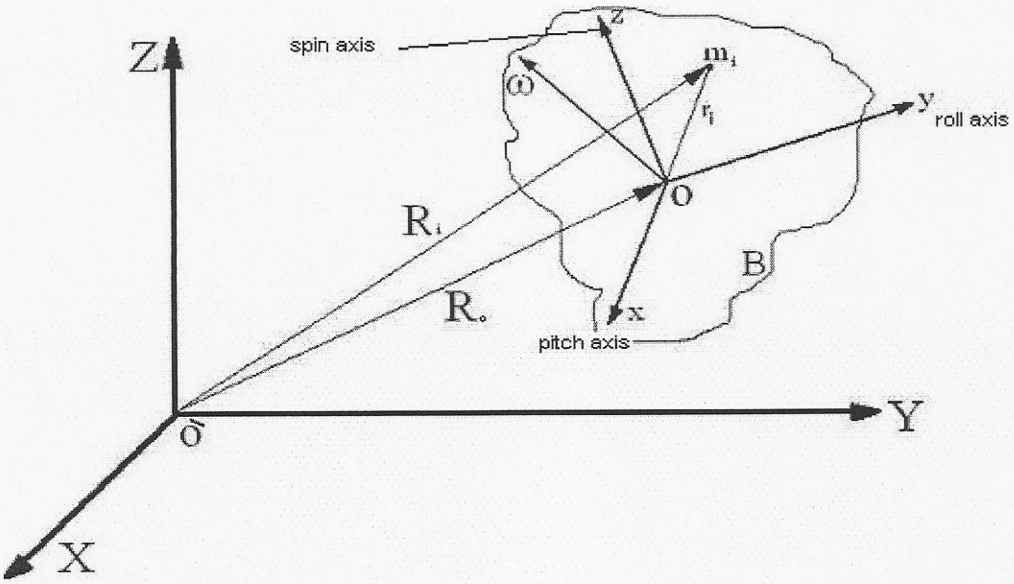

Fig. B: Rigid body angular momentum.

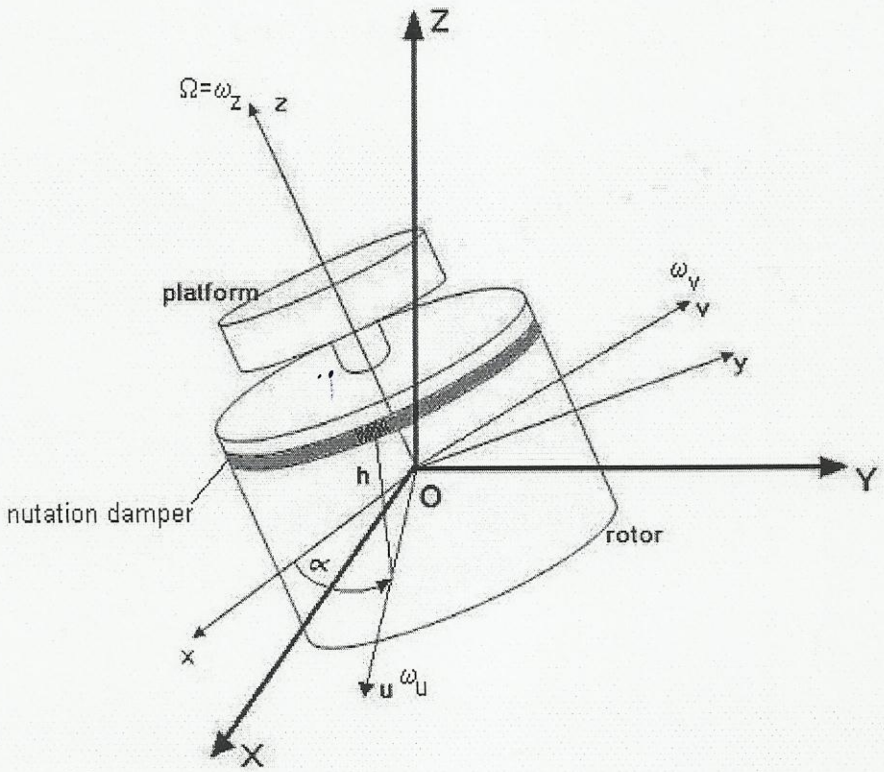

Fig.C: Spacecraft model with viscous ring damper.

$$\vec{M} = \left(\frac{d\vec{h}}{dt}\right)_{oxyz} = \left(\frac{d\vec{h}}{dt}\right)_{ouvz} + \vec{\omega}\times\vec{h} = 0 \qquad 2$$

the equation which describes the motion of the fluid inside the ring, can be obtained by using Lagrange's equation expressed in terms of quasi-coordinate.

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} - \omega_v \frac{\partial T}{\partial \omega_u} + \omega_u \frac{\partial T}{\partial \omega_v} = Q_\alpha \qquad 3$$

where Q_α is the generalized moment associated with the generalized coordinate α, and it is given by:

$$Q_\alpha = C_f R^2 \dot{\alpha} \qquad 4$$

C_f: coefficient of viscous friction between fluid and ring wall.(N s/m)

The kinetic energy (T) of the system is in the form:

$$T = \frac{1}{2}\left[A\left(\omega_u^{\,2} + \omega_v^{\,2}\right) + C\omega_z^{\,2} + C_p\Omega_p^{\,2} + I_u\omega_u^{\,2} + I_v\omega_v^{\,2} + I_z\left(\omega_z + \dot{\alpha}\right)^2 - 2I_{uz}\left(\omega_z + \dot{\alpha}\right)\omega_u \right] \qquad 5$$

Using Eqs.1, 4 and 5, then Eqs. 2 and 3 yields:

$$p' + \frac{(\lambda\, r - \alpha' + \lambda_s)}{D_1} q + \frac{(A_1 r + A_2\alpha' - A_3 p)}{D_1} q - \frac{A_4}{D_1} r' + \frac{A_5}{D_1}\alpha' = 0 \qquad 6$$

$$q' - \frac{(\lambda\, r - \alpha' + \lambda_s)}{D_2} p + \frac{(B_1 r + B_2\alpha' + B_3 p)}{D_2} p - \frac{B_4}{D_2}(r + \alpha')^2 = 0 \qquad 7$$

$$\alpha'' + C_1\alpha' + C_2 pq + C_3[(r + \alpha')q - p'] = 0 \qquad 8$$

$$r' - C_4\alpha' = 0 \qquad 9$$

where, ( )' = (d/dt), D_1 & D_2 are given in the appendix

-SOLUTION OF THE PROBLEM

Before developing the solution for the attitude equations of the spacecraft, the equation which is describing the nutation angle in terms of dimensionless parameters and variables, will be developed.

$$\cos\theta = \frac{h_z}{h_{tt}}, \quad \sin\theta = \frac{h_t}{h_{tt}}, \quad h_{tt}^{\,2} = h_t^{\,2} + h_z^{\,2} = \text{constant}, \quad h_t^{\,2} = h_u^{\,2} + h_v^{\,2} \qquad 10a, b, c, d$$

differentiate Eq. 10a and substitute in Eq. 10b gives:

$$\therefore \ \theta' = -\frac{h_z'}{h_t} \qquad 11$$

from Eq. 2, the time derivative of the spin axis angular momentum vector h_z, may be given by (in dimensionless angular velocity variables (p, q)):

$$h_z' = qh_u - ph_v \qquad 12$$

Substitute Eq. 11 into Eq. 12, yields the following equation:

$$\theta' = \frac{ph_v - qh_u}{h_t} \qquad 13$$

Substitute h_u, h_v, h_t and h_{uz} into Eq. 13 yields (details in appendix):

$$\theta' = \frac{\left(-pG^2 + bG(r + \alpha')\right)q\varepsilon}{\sqrt{p^2 + q^2}} \qquad 14$$

apply zero-order approximation procedure to Eqs. 6, 7, 8, and 9 to get:

$$p' + (\lambda\, r + \lambda_s - \alpha')q = 0 \qquad 15$$

$$q' - (\lambda\, r + \lambda_s - \alpha')p = 0 \qquad 16$$

$$r' = 0 \qquad 17$$

$$\alpha'' + \frac{\eta\lambda_n}{\sigma^2\varsigma}\alpha' - \frac{bG}{\varsigma}p' + \left[-\frac{G^2}{\varsigma}p + \frac{bG}{\varsigma}(r + \alpha')\right]q = 0 \qquad 18$$

The solution of the angular velocity component r is obtained from Eq. 17, with the fact that the initial value of the r-components equal to 1, then the solution of Eq. 17 is:

$$r=1 \qquad 19$$

substitute the solution of the r-component Eq. 19 in Eqs. 15 and 16, then one get:

$$p' + (\lambda_n - \alpha')q = 0, \quad q' - (\lambda_n - \alpha')p = 0 \qquad 20, 21$$

where,

$$\lambda_n = \lambda + \lambda_s$$

the solution of p and q are given by:

$$p = \omega_t \cos(\lambda_n \tau - \alpha), \; q = \omega_t \sin(\lambda_n \tau - \alpha) \qquad 22, 23$$

substitute Eqs. 22 and 23 into Eq. 14, then Eq. 14 becomes:

$$\theta' = \left[-G^2\omega_t \cos(\lambda_n\tau - \alpha) + bG(1 + \alpha')\right]\varepsilon \sin(\lambda_n\tau - \alpha) \qquad 24$$

Now, it is required to express for ω_t in terms of the nutation angle θ.

Divide Eq. 10b by Eq. 10a and substitute h_t and h_z then apply zero-order approximation to get:

$$\tan\theta = \frac{\omega_t}{\sigma_n} \qquad 25$$

where, $\sigma_n = \sigma + \lambda_s$

sub. p' given by Eq. 15 into Eq. 18 results:

$$\alpha'' + \frac{\eta\lambda_n}{\sigma^2\varsigma}\alpha' - \left[\frac{bG}{\varsigma}(\alpha' - \lambda_n) + \frac{G^2}{\varsigma}p - \frac{bG}{\varsigma}(1+\alpha')\right]q = 0 \qquad 26$$

DAMPER MOTION

The symmetric rigid body, which is spinning about its axis of symmetry, has a constant nutation angle when no damping is present. The transverse angular velocity ω_t rotates at a rate of $(\sigma\Omega + \sigma_p\Omega_p)\cos\theta$ and the body rotates relative to ω_t at a rate of $((1-\sigma)\Omega + \sigma_p\Omega_p)$, when no damping is present, a plane containing the angular momentum vector $\vec{h}$ or ω_t and the z-axis, called the nutation plane, is formed. the fluid is then moving at a constant rate of $((1-\sigma)\Omega + \sigma_p\Omega_p)$ (relative spin), with respect to the body. At the same time, the fluid subjected to centrifugal force due to the relative rotation (spin) about the z-axis. This type of motion is called "nutation-synchronous" motion. In this mode the fluid is moving at a constant rate with respect to the spacecraft (damper ring); hence the energy dissipation rate is constant. If $\sigma > 1$the nutation angle decreases which cause a decrease in the centrifugal force. Eventually the component of the centrifugal force is not large enough to balance the damping and friction forces, and the fluid begins to decelerate and oscillate until becomes at rest. This type of motion is called "spin-synchronous" motion.

Nutation-Synchronous Mode

Let β measure the position of the center of a portion of the fluid with respect to the nutation plane. Assuming that at $\tau = 0$, $\alpha = 0$, then

$$\beta = \alpha - \lambda_n\tau \qquad 27$$

Substituting for β in Eq. 26 and using Eqs. 22, 23 then Eq. 26 becomes:

$$\beta'' + \frac{\eta_n\lambda_n}{\sigma^2\varsigma}\beta' + \left[\left(\frac{bG}{\varsigma} - \frac{bG}{\varsigma}\right)\beta' + \frac{G^2}{\varsigma}p - \frac{bG}{\varsigma}(1+\lambda_n)\right]\omega_t \sin(\beta) = -\frac{\eta_n\lambda^2{}_n}{\sigma^2\varsigma} \qquad 28$$

where, η_n refer to the damping constant in the nutation-synchronous mode.

Since that, the motion of the fluid in this mode is constant, then the solution of Eq.28 is, $\beta = \beta_s = con.$, so, $\beta'' = \beta' = 0$, substituting in Eq.28 and take into account that $\lambda_n = \lambda + \lambda_s$ and $\sigma = 1 + \lambda$, then

$$-\left[-G^2 p + bG(1+\lambda_n)\right]\omega_t \sin\beta = -\frac{\lambda^2{}_n \eta_n}{\sigma^2} \qquad 29$$

substitute for $\beta = \beta_s$ and $\beta_s' = 0$ into Eq. 24, then 24 becomes:

$$\theta_n' = -\left[-G^2\omega_t \cos\beta + bG(1+\lambda_n)\right]\varepsilon \sin\beta_s \qquad 30$$

where, θ_n refer to the nutation angle in the nutation-synchronous mode.

Substituting for the left hand side of Eq.29 in Eq.30, and using Eq.25 then,

$$\theta' \tan\theta_n = -\frac{\eta_n \lambda^2{}_n}{\sigma^2 \sigma_n}\varepsilon \qquad 31$$

Carrying out the integration, then the nutation angle θ_n is given by:

$$\cos\theta_n = \cos\theta_{n0}\ e^{\frac{\tau}{\tau_n}} \qquad 32$$

where, θ_{n0} is the initial value of θ_n, τ_n is the time constant of the system which it is given by:

$$\tau_n = \frac{\sigma_n \sigma^2}{\eta_n \lambda^2{}_n \varepsilon} \qquad 33$$

At the end of the nutation-synchronous mode, the system goes into the spin-synchronous mode and the nutation angle θ_n gain its minimum value. Referring to Eq.29, to satisfy the condition of minimum value of the nutation angle in this mode, the angle β_s should be equal to $\pm\frac{\pi}{2}$ substituting for this value and for ω_t from Eq.25, then;

$$\tan\theta = \frac{\eta\lambda_n^2}{\sigma^3 bG\sigma_n\left(1+\frac{\lambda_s}{\sigma}\right)} \qquad 34$$

Spin-Synchronous Mode

In the Spin-Synchronous mode, the spacecraft becomes more closely to the state of pure spin about the spin axis (z-axis). Accordingly, the relative speed of the fluid, about the spin axis with respect to the spinning rotor, will be decreased. Eventually the relative speed between the fluid and the spinning rotor becomes zero, then the spacecraft spin axis is aligned with the initial direction of the total angular momentum vector. In the spin-synchronous mode, it can be shown that the fluid is moving with a small variation in its speed with respect to the damper ring therefore it is necessary to find the solution for the motion of the fluid (α) as a function of dimensionless time (τ), substituting for p and q from Eqs.22 and 23 into Eq.26, yields:

$$\alpha'' + \frac{\eta\lambda_n}{\varsigma\sigma^2}\alpha' - \left[\frac{bG}{\varsigma}(1+\lambda_n)\right]\omega_t \sin(\alpha - \lambda_n\tau) = 0 \qquad 35$$

As mentioned above that the fluid moving with a small variation in its speed, then the following equation can be assumed:

$$\alpha = \alpha_0 + \tilde{\alpha} \qquad 36$$

where, α_0 is the initial value of α and $\tilde{\alpha}$ represent the small variation of the speed of the fluid such that $\alpha_0 >> \tilde{\alpha}$, which gives the following expressions:

$$\sin(\alpha - \alpha_0) = \sin\tilde{\alpha} = \tilde{\alpha},\ \cos(\alpha - \alpha_0) = \cos\tilde{\alpha} = 1,\ \sin(\alpha - \lambda_n\tau) \approx \sin(\alpha_0 - \lambda_n\tau),$$
$$\cos(\alpha - \lambda_n\tau) \approx \cos(\alpha_0 - \lambda_n\tau) \qquad 37$$

The basis of the above assumptions is that the change in α is small compared with $\lambda_n\tau$.
Using the above assumption, then Eq.35 becomes:

$$\alpha'' + \frac{\eta\lambda_n}{\varsigma\sigma^2}\alpha' - \left[\frac{bG}{\varsigma}(1+\lambda_n)\right]\omega_t \sin(\alpha_0 - \lambda_n\tau) = 0 \qquad 38$$

assume the forced oscillating solution of Eq.38 is given by

$$\tilde{\alpha} = \alpha - \alpha_0 = A\sin(\alpha_0 - \lambda_n\tau) + B\cos(\alpha_0 - \lambda_n\tau) \qquad 39$$

to find the constants A and B, substituting Eq.39 in Eq.38, and after some mathematical manipulations we get:

$$A = -\left[\frac{bG\sigma^2(1+\lambda_n)}{\varsigma\lambda_n^2\left(\sigma^2 + \frac{\eta^2}{\varsigma^2\sigma^2}\right)}\right]\omega_t,\quad B = \left[\frac{bG\eta(1+\lambda_n)}{\varsigma^2\lambda_n^2\left(\sigma^2 + \frac{\eta^2}{\varsigma^2\sigma^2}\right)}\right]\omega_t$$

substitute A and B in Eq. 39, and substitute for ω_t, then the expression of $\tilde{\alpha}$ becomes:

$$\tilde{\alpha} = F\tan\theta_s\left[-\varsigma\sin(\alpha_0 - \lambda_n\tau) + \frac{\eta}{\sigma^2}\cos(\alpha_0 - \lambda_n\tau)\right] \qquad 40$$

where θ_s referred to the nutation angle in the spin-synchronous mode and the constant F is given by:

$$F = \frac{\sigma_n}{\lambda_n^2}\left[\frac{bG\sigma^2(1+\lambda_n)}{\varsigma^2\left(\sigma^2 + \frac{\eta^2}{\varsigma^2\sigma^2}\right)}\right]$$

The differential equation of the nutation angle rate Eq.24 is given by:

$$\theta_s' = -\left[-G^2\sigma_n \tan\theta_s \cos(\alpha - \lambda_n\tau) + bG(1+\alpha')\right]\varepsilon \sin(\alpha - \lambda_n\tau) \qquad 41$$

it was mentioned that $\tilde{\alpha}$ represents small variation in α such that

$\alpha = \alpha_0 + \tilde{\alpha}$, where $\alpha_0 >> \tilde{\alpha}$ .using the approximation of small angle, then

$$\sin(\alpha - \alpha_0) = \sin\tilde{\alpha} \approx \tilde{\alpha}\,,\ \cos(\alpha - \alpha_0) = \cos\tilde{\alpha} \approx 1,\ \sin(\alpha + \alpha_0) \approx \sin\alpha_0,$$
$$\sin(\alpha + \alpha_0) = \sin(2\alpha_0 + \tilde{\alpha}) \approx \sin 2\alpha_0, \qquad 42$$

using the above mentioned relations, then the expression of $\sin(\alpha - \lambda_n\tau)$ may be given by

$$\sin(\alpha - \lambda_n\tau) = \sin(\alpha_0 + \tilde{\alpha} - \lambda_n\tau) = (\sin\alpha_0 + \tilde{\alpha}\cos\alpha_0)\cos\lambda_n\tau - (\cos\alpha_0 - \tilde{\alpha}\sin\alpha_0)\sin\lambda_n\tau \qquad 43$$

Substituting $\tilde{\alpha}$ from Eq.40 in Eq.43 (θ_s is small in the spin-synchronous mode $\tan\theta_s = \theta_s$), then (by expanding the trigonometric terms) Eq.41 becomes:

$$\theta_s' = -\varepsilon\left\{\begin{array}{l} -\sigma_n\theta_s G^2\left[\sin\alpha_0\cos\alpha\cos^2\lambda_n\tau + \sin\lambda_n\tau\cos\lambda_n\tau(\sin\alpha_0\sin\alpha - \cos\alpha_0\cos\alpha) - \cos\alpha_0\sin\alpha\sin^2\lambda_n\tau\right] \\ +bG\left[\sin(\alpha_0 - \lambda_n\tau) + F\theta_s\left[\begin{array}{l} -\varsigma\left(\begin{array}{l}\sin\alpha_0\cos\alpha_0\cos^2\lambda_n\tau - \cos\alpha_0^2\sin\lambda_n\tau\cos\lambda_n\tau \\ +\sin\alpha_0^2\sin\lambda_n\tau\cos\lambda_n\tau - \sin\alpha_0\cos\alpha_0\sin^2\lambda_n\tau\end{array}\right) \\ +\dfrac{\eta}{\sigma^2}\left(\begin{array}{l}\cos\alpha_0^2\cos^2\lambda_n\tau + 2\sin\alpha_0\cos\alpha_0\sin\lambda_n\tau\cos\lambda_n\tau \\ +\sin\alpha_0^2\sin^2\lambda_n\tau\end{array}\right) \\ +\varsigma\lambda_n'\left(\begin{array}{l}\sin\alpha_0\cos\alpha_0\cos^2\lambda_n\tau - \cos\alpha_0^2\sin\lambda_n\tau\cos\lambda_n\tau \\ +\sin\alpha_0^2\sin\lambda_n\tau\cos\lambda_n\tau - \sin\alpha_0\cos\alpha_0\sin^2\lambda_n\tau\end{array}\right) \\ +\dfrac{\eta\lambda_n}{\sigma^2}\left(\begin{array}{l}\sin\alpha_0^2\cos^2\lambda_n\tau - 2\sin\alpha_0\cos\alpha_0\sin\lambda_n\tau\cos\lambda_n\tau \\ +\cos\alpha_0^2\sin^2\lambda_n\tau\end{array}\right)\end{array}\right]\right]\end{array}\right\} \qquad 44$$

then Eq.44 is written as:

$$\theta_s' = -\theta_s\left[E_1\cos 2\lambda_n\tau + E_2\sin 2\lambda_n\tau + \frac{FbG\eta(1+\lambda_n)}{2\sigma^2}\varepsilon\right] + bG\varepsilon\sin(\alpha_0 - \lambda_n\tau) \qquad 45$$

where,

$$E_1 = \left[-\frac{\varsigma}{2}\sin 2\alpha_0 + \frac{\eta}{2\sigma^2}\cos 2\alpha_0 + \frac{\varsigma\lambda_n}{2}\sin 2\alpha_0 - \frac{\eta\lambda_n}{2\sigma^2}\cos 2\alpha_0\right]FbG\varepsilon + \frac{G^2\sigma_n}{2}\sin 2\alpha_0$$

$$E_2 = \left[\frac{\varsigma}{2}\cos 2\alpha_0 + \frac{\eta}{2\sigma^2}\sin 2\alpha_0 - \frac{\varsigma\lambda_n}{2}\cos 2\alpha_0 - \frac{\eta\lambda_n}{2\sigma^2}\sin 2\alpha_0\right]FbG\varepsilon - \frac{G^2\sigma_n}{2}\cos 2\alpha_0$$

Eq.45 can be written in terms of spin-synchronous time constant as:

$$\theta_s' = -\theta_s\left[E_1\cos 2\lambda_n\tau + E_2\sin 2\lambda_n\tau + \frac{1}{\tau_{cs}}\right] + bG\varepsilon\sin(\alpha_0 - \lambda_n\tau) \qquad 46$$

where,

$$\tau_{cs} = \frac{2\sigma^2}{FbG\eta(1+\lambda_n)\varepsilon} = \frac{2\lambda_n^2\varsigma^2\left(\sigma^2 + \frac{\eta^2}{\varsigma^2\sigma^2}\right)}{\sigma_n b^2 G^2 \eta (1+\lambda_n)^2 \varepsilon} \qquad 47$$

one can see that the nutation angle time history consists of dominant exponential decay super imposed on it an oscillation of small amplitude represents the effect of the trigonometric terms. So that, the general solution of Eq.46 is given by:

$$\theta_s = \theta_{sc} + \theta_{sP.I.} \qquad 48$$

where, θ_{sc} : is the complementary part of the, $\theta_{sP.I.}$: is the particular part of the solution. These solutions may be given by:

$$\theta_{sc} = ce^{-\frac{\tau}{\tau_{cs}}} \qquad 49$$

$$\theta_{sP.I.} = A'\sin(\alpha_0 - \lambda_n\tau) + B'\cos(\alpha_0 - \lambda_n\tau) \qquad 50$$

substitute Eq.50 in Eq.46 to get:

$$A' = \frac{bG\varepsilon}{\tau_{cs}\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2}\right)}, \qquad B' = \frac{\lambda_n bG\varepsilon}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2}\right)}$$

substituting the solution of Eq.50 in Eq.48 and using the initial condition ($\theta_s = \theta_{s0}$ at time $\tau = \tau_0$) then the constant c is given by:

$$c = \theta_{s0} - \frac{\left(\left(\frac{\sin(\alpha_0 - \lambda_n\tau_0)}{\tau_{cs}}\right) + \lambda_n\cos(\alpha_0 - \lambda_n\tau_0)\right)}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2}\right)} bG\varepsilon$$

and from Eq.48 the complete solution of the nutation angle θ_s is given by:

$$\theta_s = \left[\theta_{s0} - \frac{\left(\left(\frac{\sin(\alpha_0 - \lambda_n\tau_0)}{\tau_{cs}}\right) + \lambda_n\cos(\alpha_0 - \lambda_n\tau_0)\right)}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2}\right)} bG\varepsilon\right] e^{\frac{-(\tau-\tau_0)}{\tau_{cs}}} + \frac{\left(\left(\frac{\sin(\alpha_0 - \lambda_n\tau_0)}{\tau_{cs}}\right) + \lambda_n\cos(\alpha_0 - \lambda_n\tau_0)\right)}{\left(\lambda_n^2 + \frac{1}{\tau_{cs}^2}\right)} bG\varepsilon \qquad 51$$

RESULTS AND DISCUSSION

Figures (1) and (2) shows the nutation angle time history prepared in this work and that presented by Alfriend[2] respectively, where figure (2) represent the experimental work of Alfriend[2]. One can see that the trend of results of the present work is acceptable in comparison with Alfriend[2]. Figure (3) shows the comparison of the nutation angle time history of this work compared with that predicted by Alfriend [2], for nutation-synchronous mode. This figure shows that the time constant in this work is decreased compared with the time constant Alfriend[2]. In figure (4), the comparison of the nutation angle time history for both nutation-synchronous mode and spin-synchronous mode is shown. It is seen that the analytical solution very well agrees with the numerical solution. The time constant for nutation-synchronous mode obtained analytically is (99981) and numerically is (96428.5) which means that the percentage error is less than ($\approx 3.6\%$), and this means that the analytical solution predicts the time constant very well, and for spin-synchronous mode, it could be seen the numerical is (866.66) and the analytical is (802.17), i.e. the percentage error is ($\approx 8\%$). Figures (5, 6, 7, 8, 9 and 10) show the variation of the time constant with the inertia ratio σ, ring mean radius (R), and damping constant, respectively. The variation of the time constant with the ratio of the ring height to the ring mean radius (b) for spin-synchronous mode is shown in Fig. (11). In Fig. (12), the variation of the time constant with the distance of offset center (d) is shown. Figure (13) shows the degradation of p component while, the time history of the r component of the spacecraft angular velocity for the first relative equilibrium state is shown in Fig. (14). A three dimensional visualization of the first relative equilibrium state is shown in Fig. (15). It is shown that even the system being at a point in neighborhood of the second relative equilibrium state, it will converge to the first relative equilibrium state. This is because that the system parameters satisfy the stability condition of the first relative equilibrium state.

CONCLUSIONS

From the results shown, it is concluded that a good agreement was obtained between the analytical and numerical solutions. The proposed damper overcomes the problems of the spreading and sloshing which occur in the partially filled nutation dampers. Utilizing fluids with high damping coefficient will decrease the time constant in both modes of motion.

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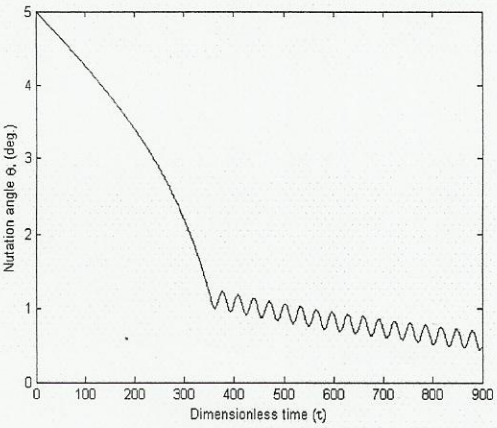

Fig. (1). Nutation angle time history of present work.

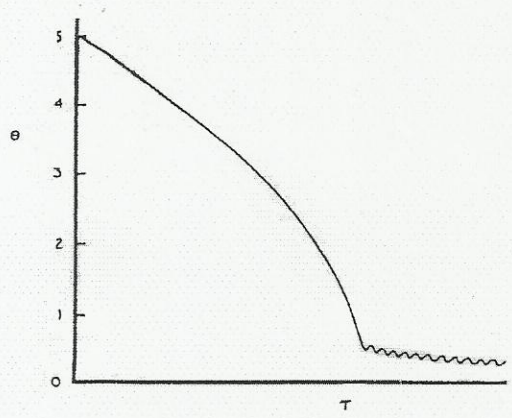

Fig. (2).Nutation angle time history of the ref.(2).

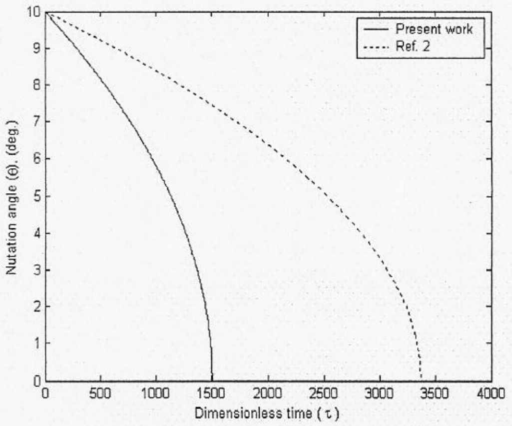

Fig. (3). Comparison of the nutation angle time history of present work with ref.(2) for the nutation-synchronous mode.

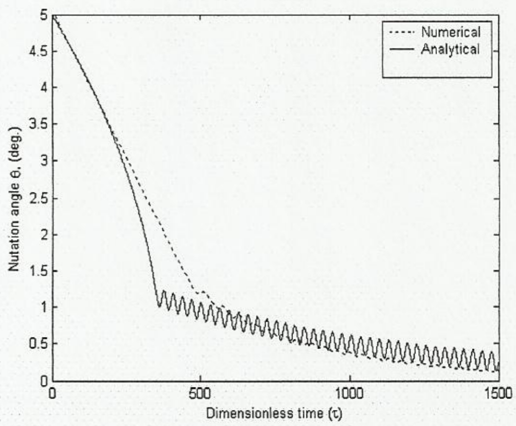

Fig. (4). Comparison between numerical and analytical solution of the nutation angle time history.

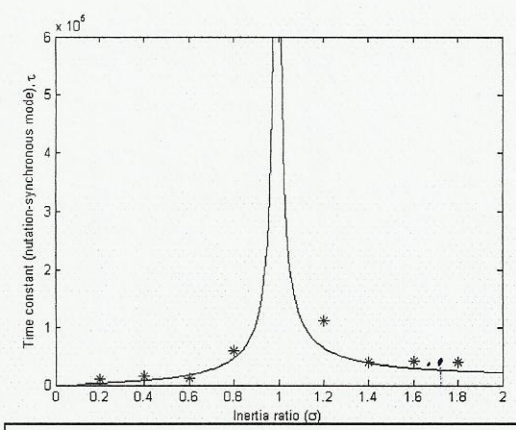

Fig. (5). Influence of the inertia ratio (σ) on the time constant for the nutation-synchronous mode.

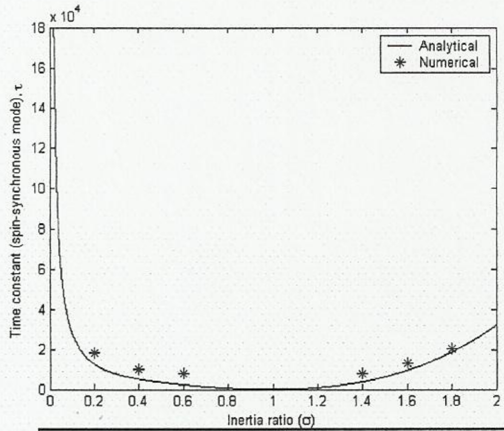

Fig. (6). Influence of the inertia ratio (σ) on the time constant for the spin-synchronous mode.

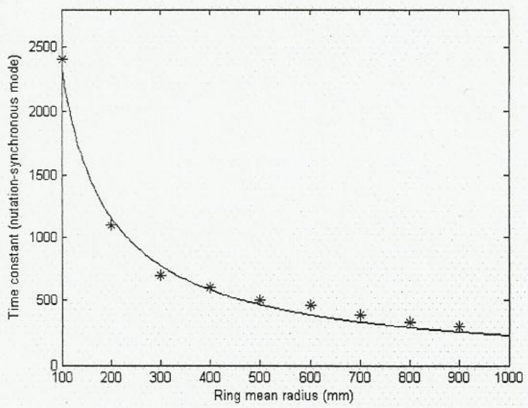

Fig. (7) Influence of the ring mean radius on the time constant for nutation-synchronous mode.

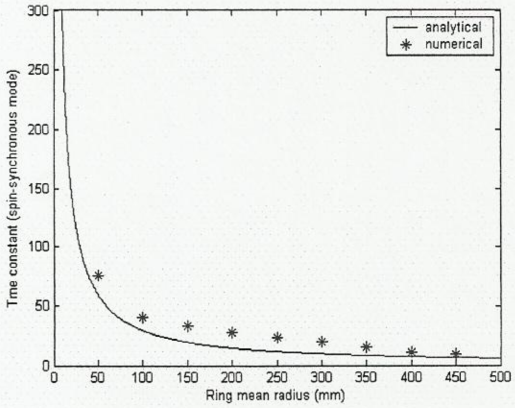

Fig. (8) Influence of the ring mean radius on the time constant for spin-synchronous mode.

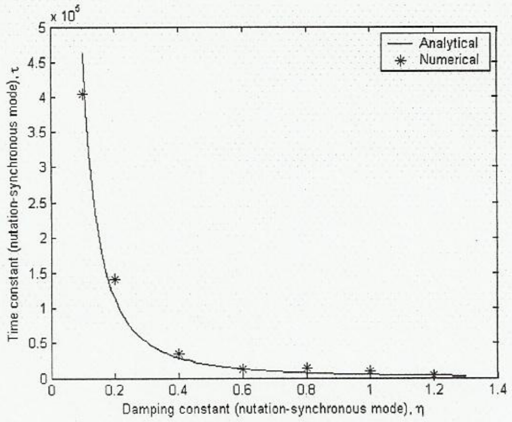

Fig. (9) Influence of the damping constant on the time constant for nutation-synchronous mode.

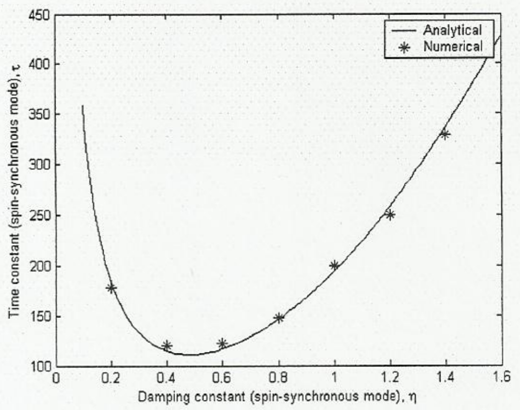

Fig. (10) Influence of the damping constant on the time constant for spin-synchronous mode.

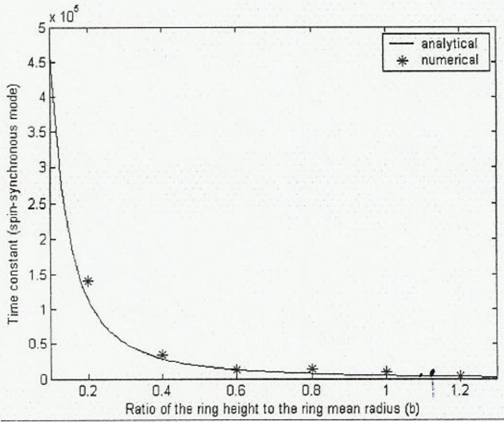

Fig. (11) Influence of the ratio of the ring height to the ring mean radius, b on the time constant for spin-synchronous. mode.

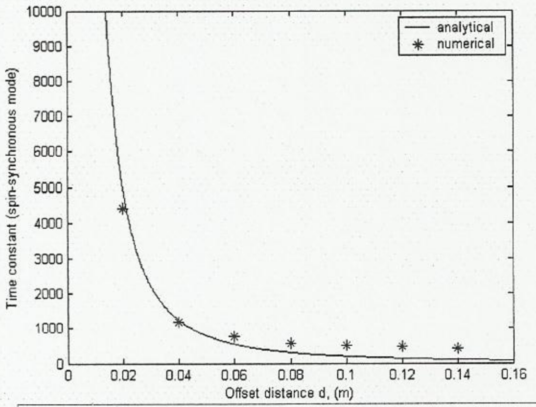

Fig. (12) Influence of the offset distance, d on the time constant for spin-synchronous mode.

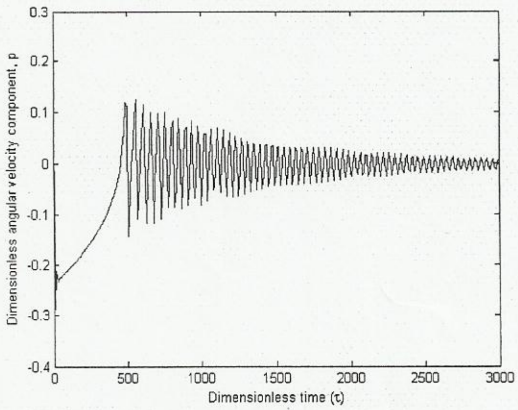

Fig. (13) Time history of dimensionless angular velocity component (p) for condition $(p_0, q_0, r_0)^T=(0.15, 0.3, 0.9)^T$. First relative equilibrium state.

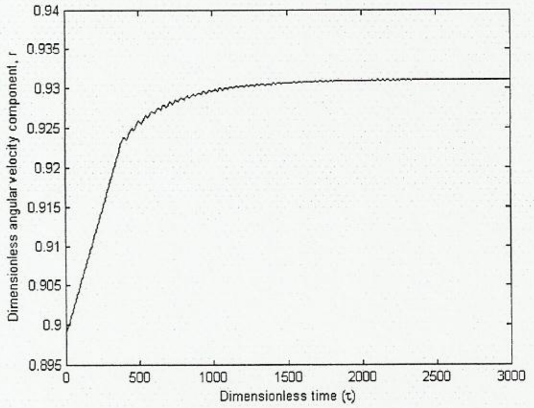

Fig. (14) Time history of dimensionless angular velocity component (r) for condition $(p_0, q_0, r_0)^T=(0.15, 0.3, 0.9)^T$. For the first relative equilibrium state.

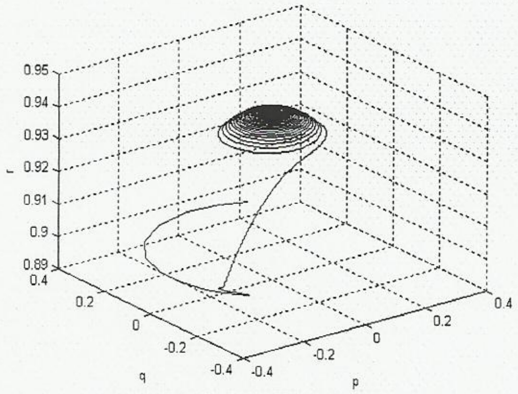

Fig. (15) Three dimensional visualization shows that the spacecraft diverge from second relative stability and reaches to the first relative stability, σ =1.2, $(p_o, q_o, r_o)^T$=(0.15, 0.3, 0.9)^T.

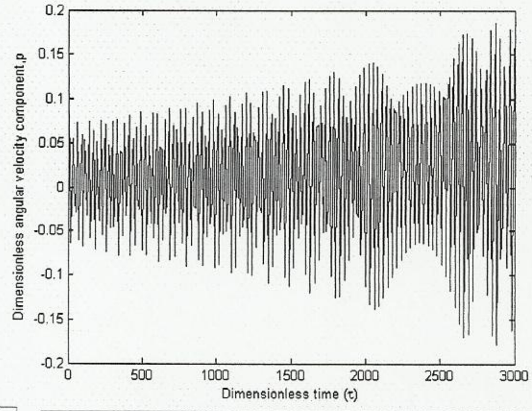

Fig. (16) Time history of dimensionless angular velocity component (p) for condition $(p_0, q_0, r_0)^T$=(0.01, 0.07, 0.95)^T.
Second relative equilibrium state.

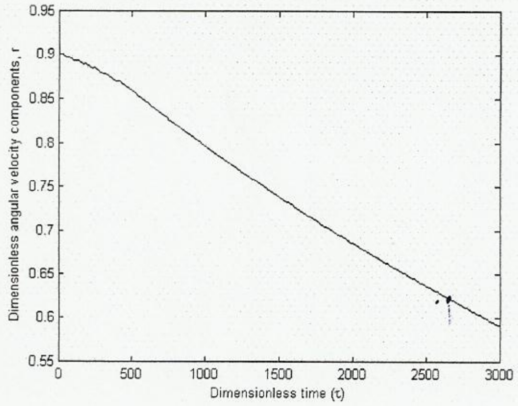

Fig. (17) Time history of dimensionless angular velocity component (r) for condition $(p_0, q_0, r_0)^T$=(0.01, 0.07, 0.95)^T
For the second relative equilibrium state.

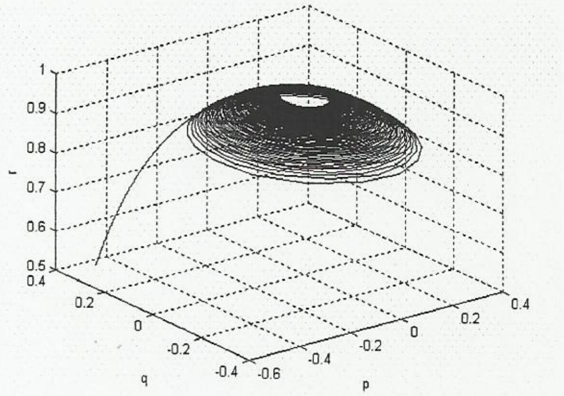

Fig. (18) Three dimensional visualization shows that the spacecraft diverge from first relative stability and reaches to the second relative stability, σ =0.75, $(p_o, q_o, r_o)^T$=(0.01,

Appendix

$$p=\frac{\omega_u}{\Omega},\ q=\frac{\omega_v}{\Omega},\ r=\frac{\omega_z}{\Omega},\ b=\frac{h}{R},\ \varepsilon=\frac{mR^2}{A},\lambda=\sigma-1,\lambda_s=\frac{C_p\Omega_p}{A\Omega},\lambda_n=\lambda+\lambda_s$$

$$D_1=1+\varepsilon\left(\frac{1}{2}+\frac{h^2}{R^2}+\frac{d^2}{R^2}-\frac{b^2G^2}{\varsigma}\right),A_1=\varepsilon\left(\frac{1}{2}+b^2G^2+G^2-b^2\right),$$

$$A_2=\varepsilon\left(\frac{1}{2}+b^2G^2+G^2-b^2\right)=A_1,\ A_3=-\varepsilon\left(bG\left(1+\frac{G^2}{\varsigma}\right)\right),\ A_4=\varepsilon bG,\ A_5=\varepsilon C_1 bG$$

$$D_2=1+\varepsilon\left(\frac{1}{2}+\frac{h^2}{R^2}\right),B_1=\varepsilon\left(\frac{h^2}{R^2}-\frac{1}{2}\right),B_2=\varepsilon\left(\frac{h^2}{R^2}-\frac{1}{2}\right)=B_1,B_3=\varepsilon bG,B_4=\varepsilon bG=B_3$$

$$C_1=\frac{\eta\lambda_n}{\varsigma\sigma^2}\left(1+\frac{\varepsilon\varsigma}{\sigma}\right),\ C_2=-\frac{G^2}{\varsigma},\ C_3=\frac{bG}{\varsigma},\ C_4=\frac{\eta}{\sigma}\varepsilon$$

$$h_u=(A+I_u)\omega_u-I_{uz}\left(\omega_z+\dot{\alpha}\right),\ h_v=(A+I_v)\omega_v,\text{ and } h_t=\sqrt{h_u^{\,2}+h_v^{\,2}}$$