The Attitude Dynamics of an Axisymmetric Dual-Spin Spacecraft Containing Ring Dampers

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Abstract

The problem of the attitude motion removing of a torque free dual-spin spacecraft using ring dampers is investigated. The system composed of two balanced, axisymmetric rigid bodies and double totally filled viscous ring dampers mounted with offsets center from the spin axis on the rotor section of the spacecraft. It is shown that the fluid motion occurs in two distinct modes, previously named the nutation synchronous mode and spin synchronous mode. Using a Newton-Lagrange approach, the equations of motion are developed resulting in equations in terms of system variables and parameters. An approximate solution for the nutation angle time history is obtained and the time constants for the two modes of motion are given as a function of suitable dimensionless parameters. Comparison is then made with the exact solution obtained by numerical integration of the exact equations of motion.

1-Introduction

A dual spin spacecraft consists of two bodies constrained to relative rotation about a shaft connecting the bodies but otherwise free to rotate in space. The bodies are in general flexible and dissipative, as is the connection between them, and all spacecraft are subject to environmental torques such as the gravity gradient torque. However, as a first approximation it is useful to model dual-spin spacecraft as two rigid bodies connected by a rigid shaft and free for external torques. The first body, referred to as a rotor, spins about the axis of relative rotation providing gyroscopic stiffness that helps to maintain the desired spacecraft attitude. The second body, referred to as the platform, is inertially nonrotating ; it provides a support base for pointing payloads, such as a directional antenna. Such a model is made more tractable by further assuming one of the bodies is axisymmetric about the axis of relative rotation. This model is called a gyrostat. For the special class of axial gyrostats, where the rotor is aligned with the principle axis of the platform.

Spacecraft separation from booster, spinup the rotor motor, reorientation maneuver, and/or external disturbances can produce transverse angular rates with a resulting coning (free precession) motion of the spin axis about the moment of momentum vector (which is constant in magnitude and direction in the absence of external moments). This resulting motion is called "wobble" or "free precession". The wobble motion will be continue infinitely in the absence of energy dissipation, unless it is corrected. Damping internal to the spacecraft will reduce this motion, so that the final motion approaches a pure spin about the axis of rotation. Several methods of reducing the wobble motion have been proposed and implemented. Of these, passive devices are appealing in that they eliminate the need for a sensor and a power source and also provide a high degree of reliability.

Of the various schemes which have been proposed and analyzed the ones, which is called the nutation ring dampers, containing fluid in a closed tube are especially desirable since they do not involve any moving parts, other than the fluid. This type of damper is simply a round tube that is bent into a closed, circular ring, and partially or fully filled with a viscous fluid. In this device, kinetic energy is dissipated by converted into heat when nutational motion causes the fluid to move through the tube. Dampers of this type have a long history, with the mercury ring on the 1958 Pioneer 1 lunar probe being the first nutation damper to be flown in space [(1)].

A partially filled viscous ring damper on a spinning satellite was first analyzed by Carrier [(2)] , then by Carrier and Miles [(3)]. They assumed that the motion of the damper did not appreciably affect the precession rate of the satellite but acted only as a source of energy dissipation. With this assumption the motion of the fluid on the tube was then treated as a fluid mechanics problem. A different approach to the analysis of the mercury ring damper was taken by Cartwright et al [(4)], where the fluid was modeled as a lumped mass in the presence of assumed damping forces. They identified two distinct modes of motion, which they named, the nutation synchronous and spin synchronous modes. Alfriend [(5)] treated with the fluid, not as a point of mass as Cartwright et al [(4)], but as a rigid slug of finite length. Approximate equations for time constants in the two modes have been developed and comparison with numerical integration of the exact equations shows that the approximations are good ones. The problems of the fluid separation into several slugs and spreads out on the internal wall, which occurs in this dampers, are avoided by using a fully-filled ring dampers.

Hameed [(6)] analyzed a completely filled viscous ring damper provided with a rigid ball, to enhance the damping performance, on dual-spin spacecraft as a passive nutation damping system. The ball motion was experienced to occur in two modes of motion as [4]. He used two types of fluids, liquid mixture and neon gas.

In this paper, double totally filled viscous ring dampers were used as a passive nutation damping system to reduce or eliminate the undesirable attitude motion of an axisymmetric dual-spin spacecraft. The dampers are mounted with equal offsets center from the spin axis in the rotor section of the spacecraft. The equations of motion are developed in terms of dimensionless variables and approximate time constants are obtained in terms of a suitable set of dimensionless parameters. These approximate solutions are then compared with those obtained from the numerical integration of the exact equations of motion.

2-Dual-Spin Spacecraft Model

Consider the dual-spin spacecraft of Fig.1 which consists of two symmetrical rigid bodies R_r and P constrained to rotate about the axis of symmetry with a relative angular rate $\dot{\alpha}_r$. The rotor section contained the two ring dampers which fixed, in parallel planes, with the same offsets center from the spin axis, at heights h_1 and h_2, above the center of mass of the spacecraft. Let a set of mutually perpendicular axes $ouvz$ and $ou'v'z'$,fixed in the center of mass of the spacecraft, such that the u and u' axes passing through the center of mass of a portion of the fluid and making an angle α counterclockwise with respect to the x-axis of the local coordinate system xyz, which is fixed in the spacecraft center of mass O.

3-Equations of Motion

For the spacecraft system under consideration (Fig.1), the equations of motion which describe its rotational behavior about the center of mass O in a torque free environment are obtained by using the conservation of angular momentum and Lagrange's equations for the motion of the fluid inside the tubes. The angular momentum of the system about the spacecraft center of mass is

$$\vec{h} = \left(A\omega_u + I_u\ \omega_u - I_{uz}(\omega_z + \dot{\alpha}) + I_{u'}\omega_u - I_{u'z'}(\omega_z + \dot{\alpha})\right)e_u + (A\omega_v + I_v\omega_v + I_{v'}\omega_v)e_v$$

$$+ \left(C\omega_z - I_{uz}\omega_u + I_z(\omega_z + \dot{\alpha}) - I_{u'z'}\omega_u + I_{z'}(\omega_z + \dot{\alpha}) + C_P\omega_{Pz}\right)e_z \qquad (1)$$

Application of the conservation of angular momentum equation $\dot{\vec{h}} = 0$, the equations of motion become

$$(A + I_u + I_{u'})\dot{\omega}_u - (I_{uz} + I_{u'z'})(\dot{\omega}_z + \ddot{\alpha}) - (\omega_z + \dot{\alpha})\left[(A + I_v + I_{v'})\right]\omega_v$$

$$+ \left[C\omega_z + 2I_z(\omega_z + \dot{\alpha}) - (I_{uz} + I_{u'z'})\omega_u + C_p\omega_p\right]\omega_v = 0 \qquad (2)$$

$$(A + I_v + I_{v'})\dot{\omega}_v + (\omega_z + \dot{\alpha})\left[(A + I_u + I_{u'})\omega_u - (I_{uz} + I_{u'z'})(\omega_z + \dot{\alpha})\right]$$

$$- \left[C\omega_z + 2I_z(\omega_z + \dot{\alpha}) - (I_{uz} + I_{u'z'})\omega_u + C_p\omega_{pz}\right]\omega_u = 0 \qquad (3)$$

$$C\dot{\omega}_z + 2I_z(\dot{\omega}_z + \ddot{\alpha}) - (I_{uz} + I_{u'z'})\dot{\omega}_u - \omega_v\left[(A + I_u + I_{u'})\omega_u - (I_{uz} + I_{u'z'})(\omega_z + \dot{\alpha})\right]$$

$$+ \omega_u\left[(A + I_v + I_{v'})\omega_v\right] = 0 \qquad (4)$$

Lagrange's equation expressed in terms of quasi-coordinates α is

$$\frac{d}{dt}\left\{\frac{\partial T}{\partial \dot{\alpha}}\right\} - \omega_v \frac{\partial T}{\partial \omega_u} + \omega_u \frac{\partial T}{\partial \omega_v} = Q_\alpha \tag{5}$$

where Q_α is the generalized moment associated with the quasi-coordinates α, and it is given by:

$$Q_\alpha = 2C_f R^2 \dot{\alpha}$$

C_f is the coefficient of the viscous friction between the viscous fluid and the ring wall.

After substitution the derivatives, Eq.(5) becomes

$$\begin{aligned} 2I_z(\dot{\omega}_z + \ddot{\alpha}) - (I_{uz} + I_{u'z'})\dot{\omega}_u - [(A + I_u + I_{u'})\omega_u - (I_{uz} + I_{u'z'})(\omega_z + \dot{\alpha})]\omega_v \\ + [(A + I_v + I_{v'})\omega_v]\omega_u = -2C_f R^2 \dot{\alpha} \end{aligned} \tag{6}$$

It is advantageous in this problem to express the all equations in suitable dimensionless variables and parameters, thus the equations of motion, Eqs.(2,3,4,6), become (see Appendix A):

$$p' + \frac{(\lambda r - \alpha' + \lambda_s)}{D_1} q + \frac{(A_1 r + A_2 \alpha' + A_3 p)}{D_1} q - \frac{A_4}{D_1} r' + \frac{A_5}{D_1} \alpha' = 0 \tag{7}$$

$$q' - \frac{(\lambda r - \alpha' + \lambda_s)}{D_2} p + \frac{(B_1 r + B_2 \alpha' - B_3 p)}{D_2} p - \frac{B_4}{D_2}(r + \alpha')^2 = 0 \tag{8}$$

$$\alpha'' + C_1\alpha' + C_2 pq + C_3((r + \alpha')q - p') = 0 \tag{9}$$

$$r' - C_4 \alpha' = 0 \tag{10}$$

4-Approximate Analysis (Zero Order Approximation method)

The equations of motion are strongly coupled nonlinear differential equations, an approximate solution is found depended on the zero order approximation method, which is based on the fact that the mass of the fluid is much smaller than that of the spacecraft, or in other wards, the moment of inertia of the fluid is very small compared with that of the spacecraft. The ratio of the fluid moment of inertia (mR^2) to the transverse moment of inertia of the spacecraft is called (ε). Therefore, dependent on the basis of zero order approximation method any term containing the inertia ratio (ε), it is neglected.

Before developing the solution for the motion of the fluid, the equation for the nutation angle will be developed. The nutation angle θ is given by

$$\tan\theta = \frac{h_t}{h_z} \tag{11}$$

where h_t is the transverse component of the angular momentum, $h_t^2 = h_u^2 + h_v^2$ and h_z is the spin axis component of the angular momentum, differentiation of Eq.(11) gives

$$\theta' = -\frac{h_z'}{h_t} = \frac{ph_v - qh_u}{h_t} \tag{12}$$

Substitution of Eq.(1) for h_u and h_v gives

$$\theta' = \frac{\left[2G^2 p + G(b_1 + b_2)(r+\alpha')\right]\varepsilon q}{\sqrt{p^2+q^2} + f(\varepsilon^2)} \tag{13}$$

where $f(\varepsilon)$ represents the term containing the parameter ε.

Now applying the zero order approximation method, by neglecting the terms containing the parameter ε, thus the equations of motion become

$$r = 1 \tag{14}$$

$$p' + (\lambda_n - \alpha')q = 0 \tag{15}$$

$$q' - (\lambda_n - \alpha')p = 0 \tag{16}$$

$$\alpha'' + \frac{\eta}{\zeta}\alpha' + \frac{G^2}{\zeta}pq + \frac{G}{2\zeta}(b_1 + b_2)\big((r+\alpha')q - p'\big) = 0 \tag{17}$$

where $\lambda_n = \sigma - 1 + \lambda_s$ is the nutation frequency. The solution for p and q is

$$p = \omega_t \cos(\lambda_n \tau - \alpha) \tag{18}$$

$$q = \omega_t \sin(\lambda_n \tau - \alpha) \tag{19}$$

where ω_t is the total transverse angular velocity component.

The equation for θ becomes

$$\theta' = \left[2G^2\omega_t \cos(\lambda_n\tau - \alpha) + G(b_1 + b_2)(1+\alpha')\right]\varepsilon \sin(\lambda_n\tau - \alpha) \tag{20}$$

Since

$$\tan\theta = \frac{h_t}{h_z} = \frac{\omega_t}{\sigma_n + f(\varepsilon)} \tag{21}$$

where $\sigma_n = \sigma + \lambda_s$, it can use Eq.(21) to substitute for ω_t. Now, it is need to obtain the solution for α.

5-The Fluid Motion

The fluid inside the ring is subjected to two kinds of forces: centripetal forces and viscous forces. For small nutation angles the viscous forces predominate and the fluid has only a small oscillatory motion about an equilibrium position with respect to the ring. This is referred to as the spin synchronous mode. If the nutation angle is greater than a specific transition angle, the centripetal force predominate and the fluid rotates around at a constant rate with respect to the spacecraft. The fluid is then in a nutation synchronous mode, and dissipates energy at a high rate.

The purpose now is to determine the fluid motion α and consequently θ in these two modes of motion as a function of dimensionless parameters.

5.1-Nutation Synchronous Mode

Let ϕ measure the position of the center of a portion of the fluid with respect to the nutation plane. Assuming that at τ =0, α =0, then

$$\phi = \alpha - \lambda_n\tau \tag{22}$$

Eq.(17) becomes

$$\phi'' + \frac{\eta}{\zeta}\phi' - \left[\frac{G^2}{\zeta}\omega_t \cos(\phi) + \frac{G(b_1 + b_2)}{2\zeta}(1+\lambda_n)\right]\omega_t \sin(\phi) = -\frac{\eta}{\zeta}\lambda_n \tag{23}$$

As mentioned in pervious section, that in the nutation synchronous mode the fluid moves at a constant rate with respect to the spacecraft, the relative position ϕ is constant. Accordingly, the particular solution of Eq. (23), is $\phi = \phi_s$. Therefore, substitution of ϕ_s into Eq. (23), and taking into account that $\phi' = \phi'' = 0$, then

$$\left[2G^2\omega_t \cos(\phi_s) + G(b_1 + b_2)(1+\lambda_n)\right]\omega_t \sin(\phi_s) = 2\eta\lambda_n \tag{24}$$

and the nutation rate equation, Eq.(20), becomes

$$\theta_n' = -\left[2G^2\omega_t \cos\phi_s + G(b_1 + b_2)(1+\lambda_n)\right]\varepsilon \sin(\phi_s) \tag{25}$$

then substituting the left hand side of Eq. (24) into Eq. (25), get:

$$\theta_n{}' = \frac{-2\eta\lambda_n\varepsilon}{\omega_t} \tag{26}$$

where $\omega_t = \sigma_n \tan\theta$ has been used, thus the solution

$$\cos\theta = \cos\theta_0 e^{\frac{\tau}{\tau_n}} \tag{27}$$

where

$$\tau_n = \frac{\sigma_n}{2\eta_n\lambda_n\varepsilon} = \frac{\sigma + \lambda_s}{2\eta_n(\lambda + \lambda_s)\varepsilon} \tag{28}$$

where η_n is the damping constant in nutation-synchronous mode.

Thus in the nutation synchronous mode the cosine of the nutation angle not the nutation angle exhibits exponential behavior. No small angle approximation has been made, hence Eq.(27) is valid for $0 < \theta < \frac{\pi}{2}$. For small θ the nutation angle time history can be approximated by

$$\theta = \sqrt{\theta_\circ^2 - \frac{\tau}{\tau_n}} \tag{29}$$

At the end of the nutation-synchronous mode, the system goes into the spin-synchronous mode and the nutation angle θ_n has minimum value in this mode. So, to satisfy the condition of minimum value of the nutation angle, the angle ϕ_s should be equal to $\pm\frac{\pi}{2}$, substitute this value into Eq. (24), then the transition angle from one mode to other is

$$\tan\theta_t = \frac{2\eta_n\lambda_n}{G(b_1+b_2)\sigma_n^{\ 2}} \tag{30}$$

5.2-Spin Synchronous Mode

Substitution the solution for p and q into Eq.(17), gives

$$\alpha'' + \frac{\eta}{\zeta}\alpha' - \left[\frac{G(b_1+b_2)}{2\zeta}(1+\lambda_n) + \frac{G^2}{\varsigma}\omega_t\cos(\alpha-\lambda_n\tau)\right]\omega_t\sin(\alpha-\lambda_n\tau) = 0 \tag{31}$$

since the spin synchronous mode occurs for the smaller nutation angle ω_t <<1, also G^2<<1, thus the last term within the brackets in Eq.(31) will be dropped.

As it is mentioned previously that the fluid, in this mode, moves with a small variation in its speed, then

$$\alpha = \alpha_\circ + \tilde{\alpha} \tag{32}$$

where $\alpha_\circ$ is the initial value of α and $\tilde{\alpha}$ represent the small change occurs in α such that $\alpha_\circ >> \tilde{\alpha}$. The basis of this assumption is that the change in α is small compared with $\lambda_n\tau$. An approximate steady state solution of Eq. (31), may be given by

$$\tilde{\alpha} = K\tan\theta_s\left[\eta\cos(\alpha_\circ-\lambda_n\tau) - \lambda_n\zeta\sin(\alpha_\circ-\lambda_n\tau)\right] \tag{33}$$

where θ_s referred to the nutation angle in the spin-synchronous mode, and the constant K is given by

$$K = \frac{\sigma_n}{\lambda_n^{\ 2}}\left[\frac{G(b_1+b_2)(1+\lambda_n)}{2\varsigma^2\left(\lambda_n + \frac{\eta^2}{\varsigma^2\lambda_n}\right)}\right]$$

the nutation angle equation is

$$\theta' = -\left[2G^2\omega_t\cos(\alpha-\lambda_n\tau)+G(b_1+b_2)(1+\alpha')\right]\varepsilon\sin(\alpha-\lambda_n\tau) \tag{34}$$

Assuming that θ_s is small enough, so that the terms of $\theta_s^{\ 2}$ can be neglected and as mentioned above that the change in α is small, so that $\sin(\alpha-\alpha_\circ)=(\alpha-\alpha_\circ)$ and $\cos(\alpha-\alpha_\circ)=1$, then Eq. (34) can be reduced to

$$\theta_s' + \theta_s\left[E_1\cos 2\lambda_n\tau + E_2\sin 2\lambda_n\tau + \frac{1}{\tau_{cs}}\right] = -G\varepsilon(b_1+b_2)\sin(\alpha_\circ-\lambda_n\tau) \tag{35}$$

where E_1 and E_2 are constants and the time constant in spin-synchronous mode τ_{cs} is given by

$$\tau_{cs} = \frac{4\varsigma^2\lambda_n\left[\lambda_n^{\ 2}+\dfrac{\eta_s^{\ 2}}{\varsigma^2}\right]}{G^2\varepsilon(b_1+b_2)^2(1+\lambda_n)^2\eta_s\sigma_n} \tag{36}$$

where η_s is the damping constant in the spin-synchronous mode.

The E_1 and E_2 terms contribute nothing to the exponential decay of the solution, so that the important part of the solution of Eq.(35) is

$$\theta_s = \left[\theta_{s0} + \left(\frac{\dfrac{1}{\tau_{cs}}\sin(\alpha_0-\lambda_n\tau_0)+\lambda_n\cos(\alpha_0-\lambda_n\tau_0)}{\left(\lambda_n^{\ 2}+\dfrac{1}{\tau_{cs}^{\ 2}}\right)}\right)G\varepsilon(b_1+b_2)\right]e^{\frac{-(\tau-\tau_0)}{\tau_{cs}}}$$
$$-\left(\frac{\dfrac{1}{\tau_{cs}}\sin(\alpha_0-\lambda_n\tau)+\lambda_n\cos(\alpha_0-\lambda_n\tau)}{\left(\lambda_n^{\ 2}+\dfrac{1}{\tau_{cs}^{\ 2}}\right)}\right)G\varepsilon(b_1+b_2) \tag{37}$$

6-Calculation of the Damping Constant η

For a given system all of the required dimensionless parameters are well defined except for the damping constant η or C_f. η is related to the kinematic viscosity, thus a consideration of the fluid dynamics of the problem is necessary. Since the motion of the fluid is different in the two modes. In nutation-synchronous mode the velocity of the fluid is constant with respect of the spacecraft or tube. One of the approaches suggested, that is to model the motion of the fluid as steady laminar flow in a pipe.

Solution of the Navier-Stokes equations with a flux of

$$Q_{flux} = \pi r^2 R\dot{\alpha} \tag{38}$$

where $R\dot{\alpha}$ is the velocity of the fluid relative to the ring and r is the ring radius, gives

$$u_f = 2R\dot{\alpha}\left[1-\left(\frac{r_i}{r}\right)^2\right] \tag{39}$$

where r_i is the distance from the center of the pipe.

The shear stress at any point is

$$\tau_{shear} = \mu\frac{\partial u}{\partial r_i} \tag{40}$$

where μ is the fluid viscosity. Thus the total viscous force is

$$F_v = (2\pi r)(R\delta)\tau_{shear}\Big|_{r_i=r} = 8\pi\mu R^2\dot{\alpha}\delta \tag{41}$$

where δ is the fullness of the ring and its equal to (2π), (the ring is totally filled).

The damping force from dynamical analysis is

$$F_v = C_f V_f = C_f R\dot{\alpha} = \eta m\Omega R\dot{\alpha} \tag{42}$$

After equating this equation with Eq. (41), the damping constant is given by

$$\eta_n = \frac{8\nu}{r^2\Omega} \tag{43}$$

where ν is the kinematic viscosity of the fluid.

In the present study a mixture of glycerin oil with water [(75%) glycerin and (25%) water] is used as a viscous liquid because it gives minimum weight of the damper and maximum energy dissipation [6].

In the spin-synchronous mode the velocity of the fluid is not constant but oscillatory with respect to the ring, so that the approach is used to calculate the damping constant in the nutation-synchronous mode is not available, therefore, the experimental approach is dependent by many researches [5,6]. Hameed (6) calculated the damping constant in this mode by empirical relation and it is found to be equal (0.15) which is adopted in this research.

7-Results & Comparisons

From Fig.(3), it can be seen that the general trend of the nutation angle time history is well predicted in comparison with that given experimentally by [5], Fig.(2). Also, there is a satisfactory agreement between numerical and analytical solutions, in other words, the approximate equations were developed for the nutation angle behavior and corresponding time constant are good.

The effect of the second ring damper can be shown in Fig.(4), where it can be seen that the nutation angle decay in nutation-synchronous mode is decreased by (50%) compared with using only one ring damper as a passive nutation damping system. Analytically, this indicates to the fact that the number of ring damper is equal to the number of times of the time constant which it will contract.

A comparison of the time constant in nutation-synchronous mode given by Eq.(27) and an “exact” time constant is given in Figs.(5-8). The “exact” time constant is obtained by assuming exponential behavior of $\cos\theta$ for the solution obtained by numerical integration of Eqs.(7-10) and calculating the time constant. The figures (5-8) show that the approximate solutions given by Eqs.(27) and (28) are good.

In the spin-synchronous mode, a comparison of the time constant given by Eq.(36) and an "exact" time constant is given in Figs.(9-14). The "exact" time constant was obtained by numerically integrating the exact equations of motion, assuming exponential behavior for maximum values of θ during each oscillation and calculating the time constant. Again, the approximate solution developed is a good one.

Finally, the time history of the *p, q, r* components of the spacecraft angular velocity with time are shown in Figs.(15-17). The three dimensional phase diagram with inertia ratio (1.2) is shown in Fig.(18), with initial conditions for the angular velocity components $(p_o, q_o, r_o)^T = (0.1, 0.32, 0.98)^T$ respectively.

	parameters	*value*
Satellite Parameters	**A** **C** C_p Ω Ω_P σ	**36 kg.m²** → $36\ kg.m^2$ $43.2\ kg.m^2$ $0.2\ kg.m^2$ **10.5 rad/sec** **0.00007272 rad/sec** **1.2**
Rings Parameters	**R** **r** **d** ε	**250 mm** **9.5 mm** **50 mm** **0.0009**
Fluid Parameters	m ρ_f μ ν	**0.5186 kg** $1164.5\ kg/m^3$ $67.9 \left(\frac{N.\sec}{m^2}\times 10^{-3}\right)$ $58.31\ mm^2/sec$

Table (1) Design parameters of the system.

8-Conclutions

In this paper, the effect of addition of double fully filled viscous ring dampers on the rotor section of a dual-spin axisymmetric spacecraft is investigated to eliminate the attitude (nutational) motion. Comparison with the results from numerical integration of the exact equations of motion has shown that the approximate equations developed are good. The agreement between the analytical results and the numerical results indicates that the proposed configuration of nutation dampers is an acceptable one as a passive nutation damping system. Also, the results show that the addition of second nutation ring damper on the spacecraft is greatly affecting the dynamic characteristics of the nutation damping system. In other words, the time constant specially for nutation-synchronous mode decrease by the half compare with using unique nutation ring damper. Also, the problems of the spreading out on the internal wall, separation into several slugs, and sloshing of the viscous fluid which occur in the partially filled viscous ring nutation dampers are avoided in the fully filled with offset center dampers.

Appendix A: Equations of Motion

Let the dimensionless time variable τ be defined as:

$$\tau = \Omega t \tag{A.1}$$

so that, the time derivative with respect to τ is given by:

$$\frac{d}{dt}(\ ) = \Omega \frac{d}{d\tau}(\ ) = \Omega(\ )', \quad d\tau = \Omega dt \tag{A.2}$$

where Ω is the initial spin rate of the rotor, and primes, $(\ )'$, denotes the differentiation with respect to the dimensionless time variable τ. Also, let the dimensionless angular velocity components p , q and r are defined as:

$$p = \frac{\omega_u}{\Omega}, \quad q = \frac{\omega_v}{\Omega}, \quad r = \frac{\omega_z}{\Omega} \tag{A.3}$$

Let the following dimensionless parameters be defined as:

$\sigma = \dfrac{C}{A}$: ratio of the rotor spin axis principal moment of inertia to the total transverse principal moment of inertia of the spacecraft.

$b_1 = \dfrac{h_1}{R}$: ratio of the first ring height above the spacecraft center of mass to the ring mean radius.

$b_2 = \dfrac{h_2}{R}$: ratio of the second ring height above the spacecraft center of mass to the ring mean radius.

$\eta = \dfrac{C_f}{m\Omega}$: dimensionless damping constant.

$\varepsilon = \dfrac{mR^2}{A}$: ratio of the inertia of the fluid to the total transverse principal moment of inertia of the spacecraft.

$G = \dfrac{d}{R}$: ratio of the ring center offset to the ring mean radius.

$\zeta = 1 + \dfrac{d^2}{R^2}$

The moments of inertia of the fluid are given by:

$$\begin{aligned}
I_u &= \frac{1}{2}mR^2 + mh_1^{\,2} & I_{u'} &= \frac{1}{2}mR^2 + mh_2^{\,2} \\
I_v &= \frac{1}{2}mR^2 + m(h_1^{\,2} + d^2) & I_{v'} &= \frac{1}{2}mR^2 + m(h_2^{\,2} + d^2) \\
I_z &= mR^2 + md^2 & I_{z'} &= mR^2 + md^2 \\
I_{uz} &= mdh_1 & I_{u'z'} &= mdh_2
\end{aligned} \tag{A.4}$$

The equations of motion are

$$\begin{aligned}
&(A + I_u + I_{u'})\dot{\omega}_u - (I_{uz} + I_{u'z'})(\dot{\omega}_z + \ddot{\alpha}) - (\omega_z + \dot{\alpha})\left[(A + I_v + I_{v'})\right]\omega_v \\
&\quad + \left[C\omega_z + 2I_z(\omega_z + \dot{\alpha}) - (I_{uz} + I_{u'z'})\omega_u + C_p\omega_p\right]\omega_v = 0
\end{aligned} \tag{A.5}$$

$$(A+I_v+I_{v'})\dot{\omega}_v+(\omega_z+\dot{\alpha})\left[(A+I_u+I_{u'})\omega_u-(I_{uz}+I_{u'z'})(\omega_z+\dot{\alpha})\right]$$

$$-\left[C\omega_z+2I_z(\omega_z+\dot{\alpha})-(I_{uz}+I_{u'z'})\omega_u+C_p\omega_{pz}\right]\omega_u=0 \tag{A.6}$$

$$C\dot{\omega}_z+2I_z(\dot{\omega}_z+\ddot{\alpha})-(I_{uz}+I_{u'z'})\dot{\omega}_u-\omega_v\left[(A+I_u+I_{u'})\omega_u-(I_{uz}+I_{u'z'})(\omega_z+\dot{\alpha})\right]$$

$$+\omega_u\left[(A+I_v+I_{v'})\omega_v\right]=0 \tag{A.7}$$

$$2I_z(\dot{\omega}_z+\ddot{\alpha})-(I_{uz}+I_{u'z'})\dot{\omega}_u-[(A+I_u+I_{u'})\omega_u-(I_{uz}+I_{u'z'})(\omega_z+\dot{\alpha})]\omega_v$$

$$+[(A+I_v+I_{v'})\omega_v]\omega_u=-2C_fR^2\dot{\alpha} \tag{A.8}$$

Subtracting Eq. (A.8) from Eq. (A.7), to give:

$$C\dot{\omega}_z=2C_fR^2\dot{\alpha} \tag{A.9}$$

Re-arrenging Eq. (A.9) to obtian

$$\dot{\omega}_z=\frac{2C_fR^2\dot{\alpha}}{C} \tag{A.10}$$

Substituting Eq. (A.10) into Eq. (A.8), and results in:

$$\alpha''+C_1\alpha'+C_2pq+C_3((r+\alpha')q-p')=0 \tag{A.11}$$

using α'' from Eq. (A.13), then Eqs. (A.5) and (A.6) become

$$p'+\frac{(\lambda r-\alpha'+\lambda_S)}{D_1}q+\frac{(A_1r+A_2\alpha'+A_3p)}{D_1}q-\frac{A_4}{D_1}r'+\frac{A_5}{D_1}\alpha'=0 \tag{A.12}$$

$$q'-\frac{(\lambda r-\alpha'+\lambda_s)}{D_2}p+\frac{(B_1r+B_2\alpha'+B_3p)}{D_2}p-\frac{B_4}{D_2}(r+\alpha')^2=0 \tag{A.13}$$

By the same way Eq. (A.10) will be

$$r'-C_4\alpha'=0 \tag{A.14}$$

where $\lambda=\sigma-1,\quad \lambda_s=\dfrac{C_P\Omega_P}{A\Omega},\quad \lambda_n=\lambda+\lambda_s$

$$A_1=\varepsilon\left(1-(b_1^2+b_2^2)+\frac{G^2(b_1+b_2)^2}{2\zeta}\right),\qquad A_2=\varepsilon\left(1-(b_1^2+b_2^2)+\frac{G^2(b_1+b_2)^2}{2\zeta}\right)=A_1$$

$$A_3=\varepsilon G(b_1+b_2)\left(\frac{G^2}{\zeta}-1\right),\qquad A_4=\varepsilon G(b_1+b_2),\qquad A_5=\varepsilon G(b_1+b_2)C_1$$

$$B_1 = \varepsilon\left(b_1^2 + b_2^2 - 2G^2 - 1\right), \qquad B_2 = \varepsilon\left(b_1^2 + b_2^2 - 2G^2 - 1\right) = B_1$$

$$B_3 = \varepsilon(b_1 + b_2)G \ , \qquad B_4 = \varepsilon(b_1 + b_2)G = B_3$$

$$C_1 = \frac{\eta}{\zeta}\left(1 + \frac{2\varepsilon\zeta}{\sigma}\right), \qquad C_2 = \frac{G^2}{\zeta}, \qquad C_3 = \frac{G(b_1 + b_2)}{2\zeta} \ , \qquad C_4 = \frac{2\eta\varepsilon}{\sigma}$$

$$D_1 = 1 + \varepsilon\left(1 + b_1^2 + b_2^2 - \frac{G^2(b_1 + b_2)^2}{2\zeta}\right), \qquad D_2 = 1 + \varepsilon\left(1 + b_1^2 + b_2^2 + 2G^2\right)$$

References

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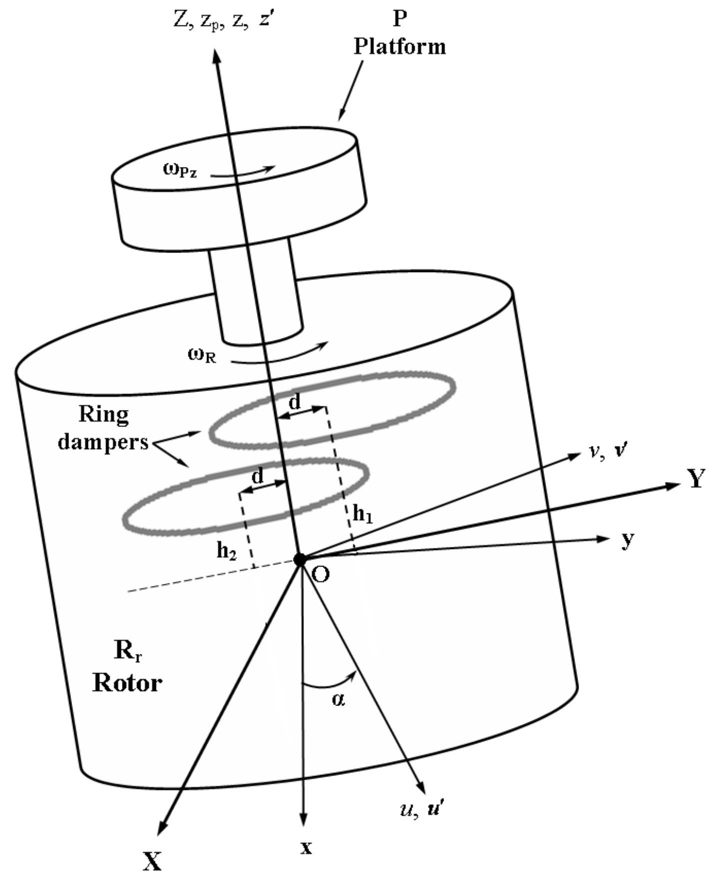

Figure (1) Dual-Spin Spacecraft Containing Nutation Dampers.

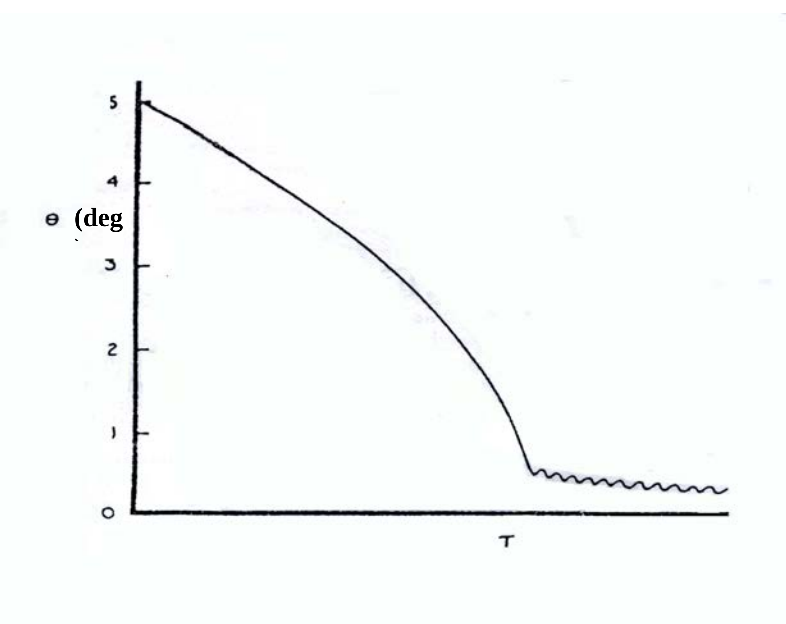

Figure (2).The typical nutation angle decay of Ref. [5].

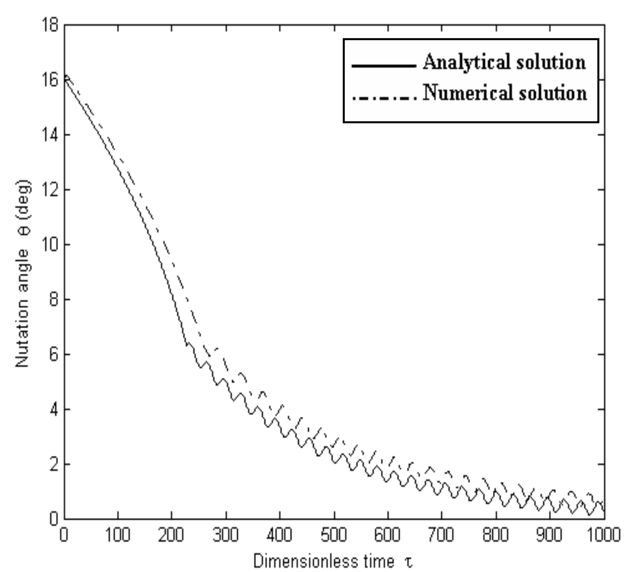

Figure (3). Comparison between analytical and numerical solutions of the nutation angle time history.

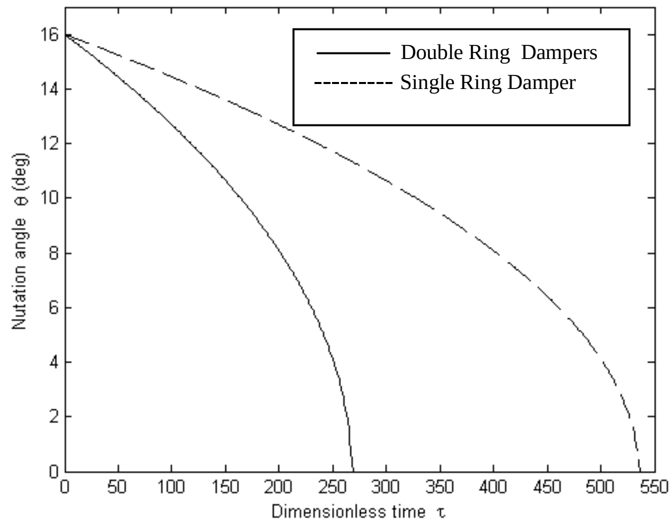

Figure (4). Comparison of nutation angle time history of nutation-synchronous mode using single and double nutation dampers.

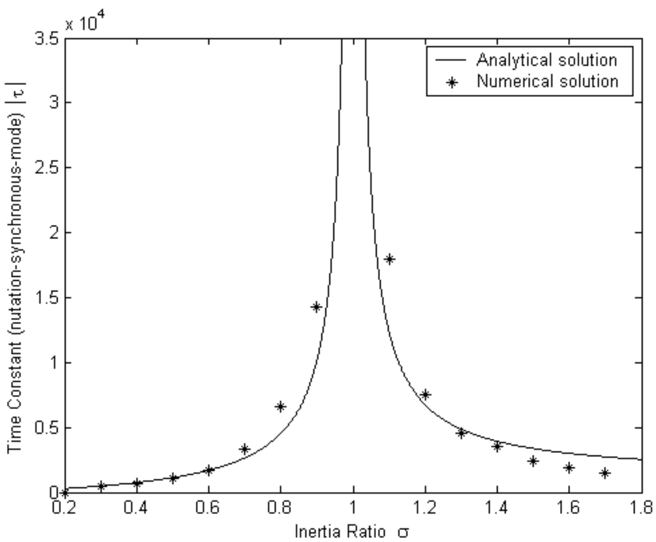

Figure (5). The variation of the time constant of nutation synchronous mode with the inertia ratio (σ).

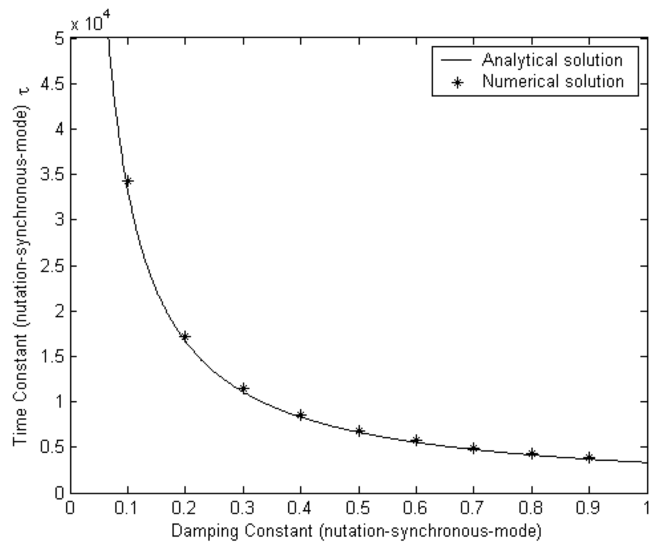

Figure (6). The variation of the time constant with the damping Constant for nutation-synchronous mode .

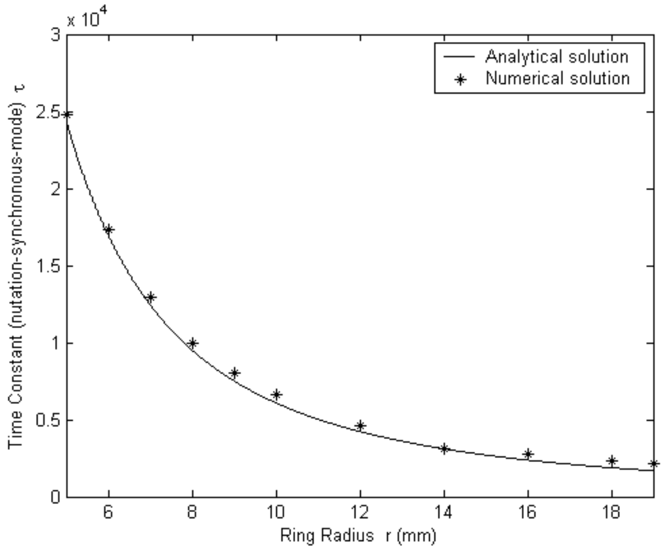

Figure (7). The variation of the time constant of nutation synchronous mode with the Ring radius.

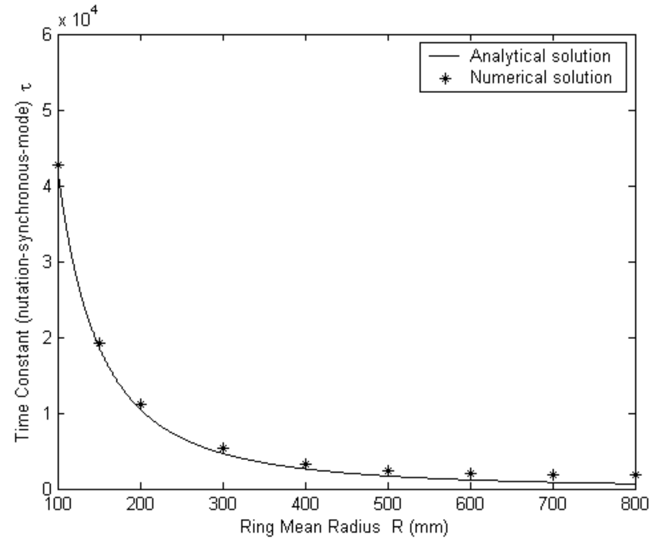

Figure (8). The variation of the time constant of nutation synchronous mode with the Ring mean radius.

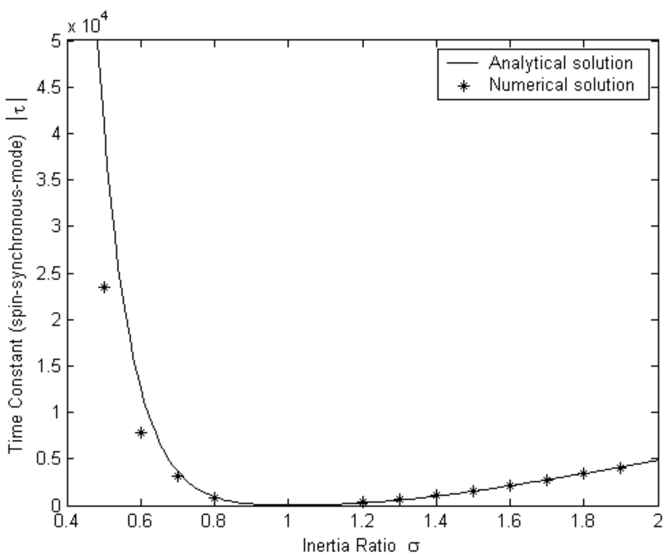

Figure (9). The variation of the time constant of spin synchronous mode with the inertia ratio (σ).

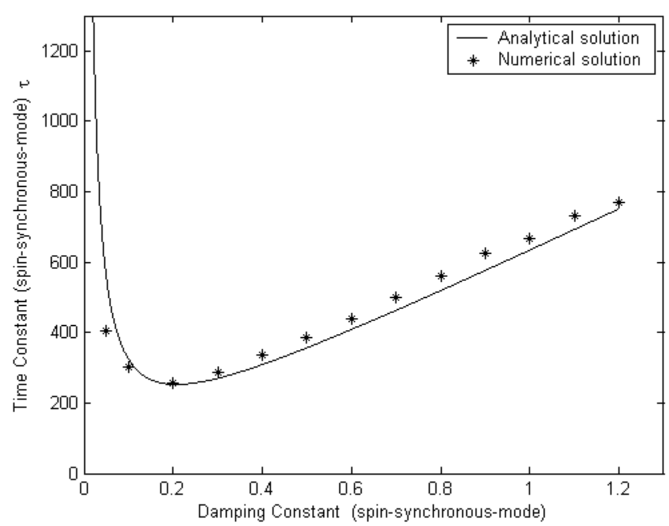

Figure (10). The variation of the time constant with the damping Constant for spin-synchronous mode.

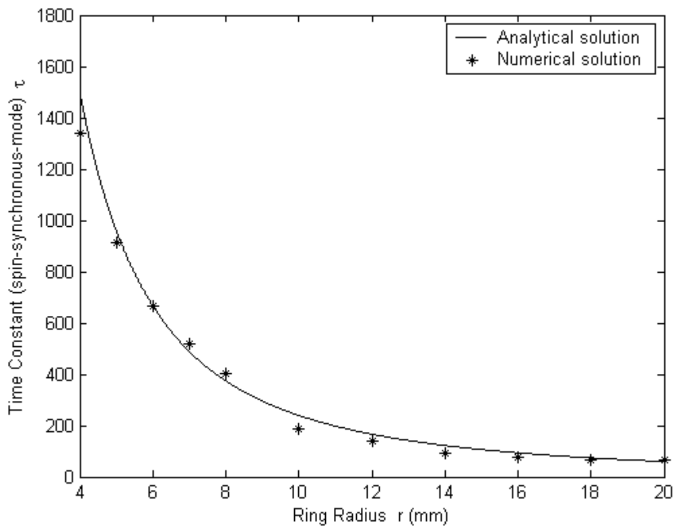

Figure (11). The variation of the time constant of spin synchronous mode with the Ring radius.

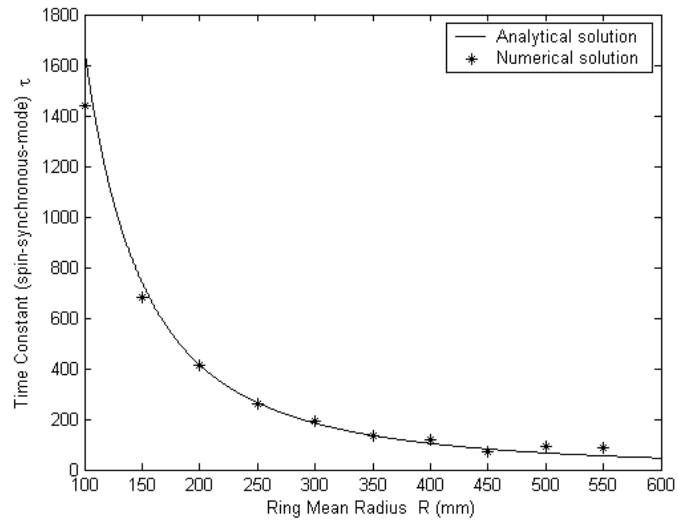

Figure (12). The variation of the time constant of spin synchronous mode with the Ring mean radius.

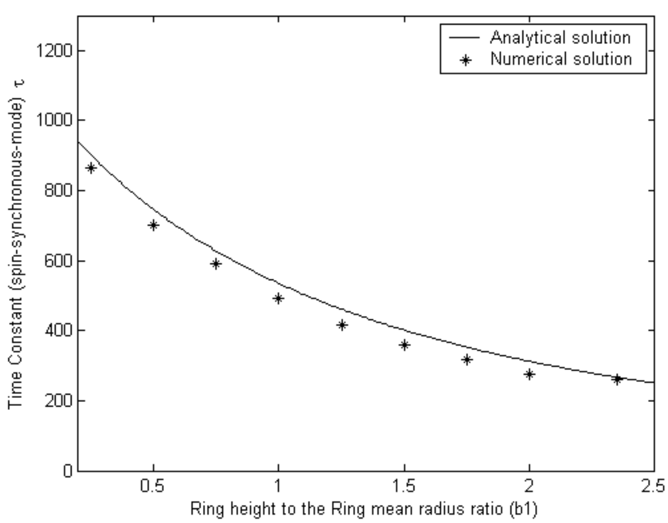

Figure (13). The variation of the time constant of spin-synchronous mode with the Ring height to the ring mean radius ratio (b_1).

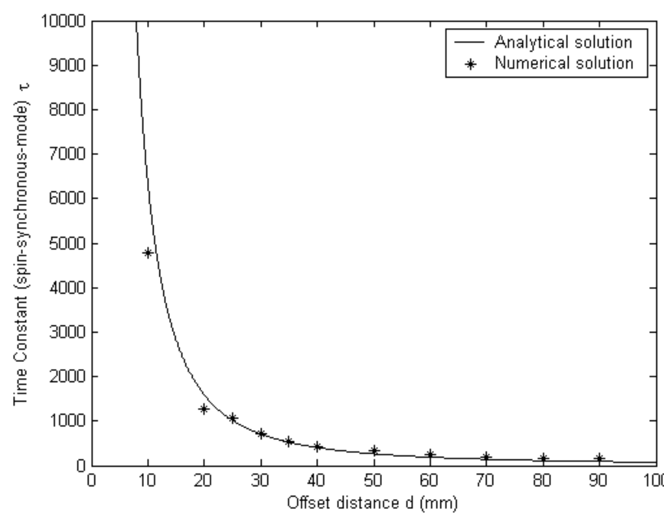

Figure (14). The variation of the time constant of spin synchronous mode with the center offset distance (d).

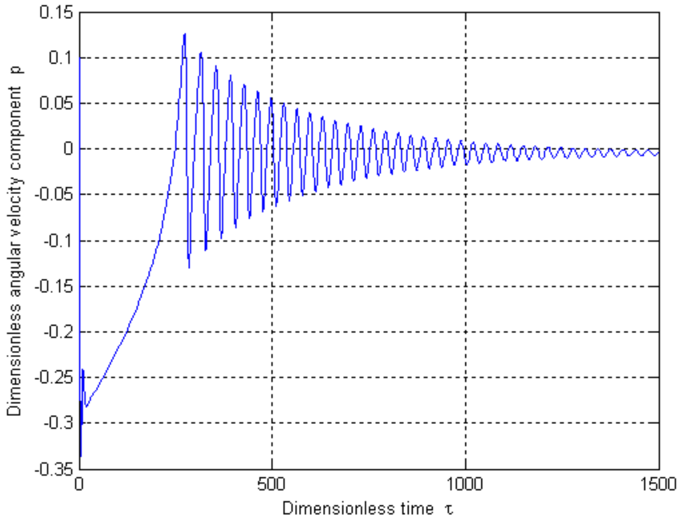

Figure (15). The time history of the p component of the spacecraft angular velocity.

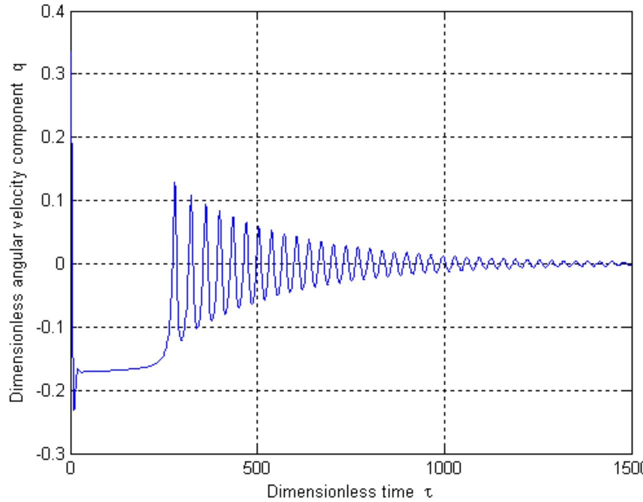

Figure (16). The time history of the q component of the spacecraft angular velocity.

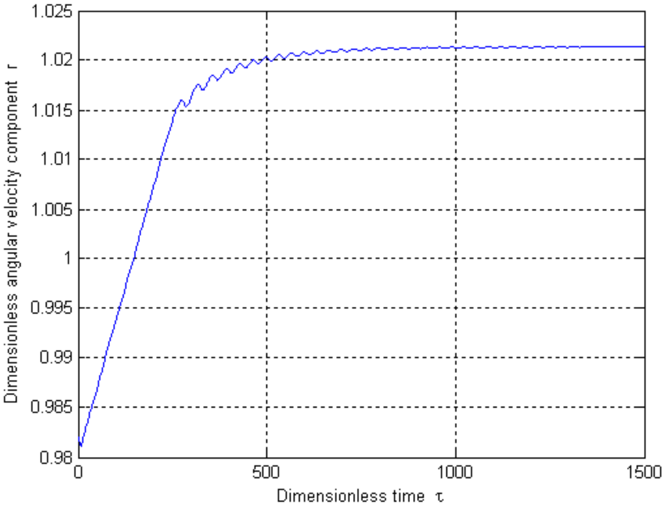

Figure (17). The time history of the r component of the spacecraft angular velocity.

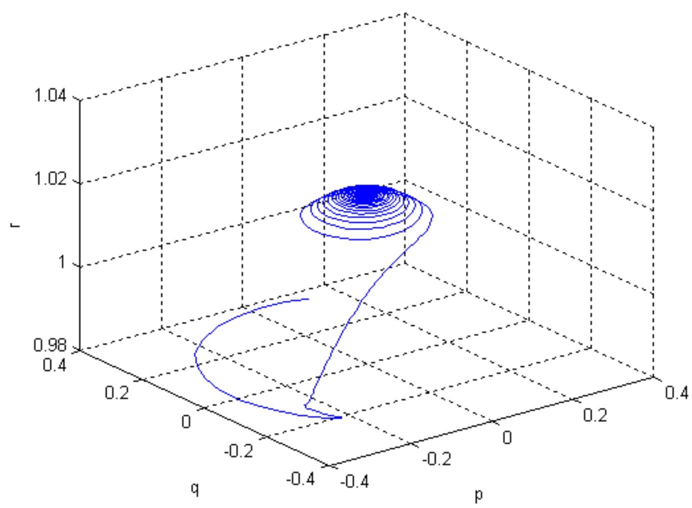

Figure (18). Three dimensional phase diagram.