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EFFECT OF ADDITIONAL BOUNDARY CONDITIONS ON THE AXISYMMETRIC FREE VIBRATIONS OF OBLATE SPHEOIDAL SHELLS

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ABSTRACT

This research deals with the effect of using additional boundary conditions as well as the eccentricity ratio on the axisymmetric free vibrational characteristics of thin isotropic oblate spheroid shells. The formulation depends on the boundary matching method using the non – shallow shell theory. The oblate shell was assumed to be constructed from two spherical elements matched along the continuous boundaries. The additional boundary conditions under construction were clamped – clamped, clamped – free and pined – pined. It was found that the natural frequency of the shell with clamped – clamped additional boundaries was greater than other two types of the additional boundaries. On the other hand it was found that in the case of clamped – clamped boundary condition, the eccentricity ratio has an effect on the natural frequency greater than the other two cases, i.e, clamped – free and pined - pined cases, respectively.

Key words: Vibration, Oblate shells, Spheroidal Oblate, Axisymmetric Oblate

تأثير الشروط الحدودية الإضافية على الترددات الطبيعية المتناظرة المحور للقشريات البيضوية المفلطحة

الخلاصة

تناول هذا البحث دراسة تأثير إضافة شروط حدودية إضافة إلى نسبة الحيود على الخصائص الاهتزازية المتناظرة المحور للقشريات البيضوية المفلطحة ذات الخصائص المتماثلة. لقد اعتمدت طريقة تماثل الحدود في عملية التمثيل الرياضي للمسالة وذلك باستخدام نظرية القشريات العميقة . وقد تم افتراض ان القشرية مكونة من جزئين كرويين مثبتين على طول الحدود بينهما. إن الشروط ألحدوديه الاضافية هي: تثبيت محكم – تثبيت محكم ، تثبيت محكم – حر ، تثبيت مفصلي – تثبيت مفصلي . لقد وجد ان الشرط الحدي نوع تثبيت محكم _ تثبيت محكم يزيد من قيمة التردد الطبيعي وان تاثير هذا النوع من الشروط الحدية على التردد الطبيعي اكبر من تاثير باقي النوعين الاخرين من الشروط الحدية. اضافة الى ذلك فان تاثير نسبة الحيود على التردد الطبيعي في حالة استخدام الشروط الحدية نوع تثبيت محكم – تثبيت محكم اكثر من تاثير هذه النسبة في حالة استخدام النوعين الاخرين من الشروط الحدية.

1. INTRODUCTION

One of the commonly used types of elastic thin shells which has a particular interest in the engineering applications is the oblate spheroid shell which is defined as the locus surface resulting by rotating an ellipse around its minor axis. This type of shells has many practical applications such as; the tanks of liquid oxygen used in several upper stages of space vehicles, the housing of the early – warning scanner and others. The nature of these applications of such structures which may cause failure in the structure of these shells. One of the very important dynamic problems is the resonance. Therefore the free vibration of such shells may be studied to present the resonant problems.

In general the free vibration of such shells is effected by thr stiffnes distribution, mass distribution and the type of boundary conditions that used. The main purpose of this paper is to study the effect of using additional boundary conditions on the free vibration of the oblate spheroid shell.

Penzes and **Burgin, 1965** were the first who solve the problem of the free vibrations of thin isotropic oblate spheroid shells by Galerkin's method using membrane theory and harmonic axisymmetric motion. It was shown that Galerkin's method of solution for the oblate spheroid shell yields the exact solution for the closed spherical shell as the eccentricity of the oblate spheroid shell approaches zero. The conditions to be imposed are that the displacements should be single valued and bounded at every point of the oblate spheroid, including the north and south poles. **Penzes, 1969** extended the solution of the above reference to include thin orthotropic oblate spheroid shells. He used the same assumptions and equations of motion in the above reference except that the principal direction of the elastic compliances was assumed to be along parallel of latitude and along meridian. Both of the spheroid and spherical shells were investigated with various orthotropic constants. The discussion was restricted to the axially symmetric torsionless motion of shells. The conditions to be imposed are that the displacements should be single valued and bounded at every point of the oblate spheroid, including the north and south poles. **Irie, 1985** analyzed the free vibration of an elastically or rigidly point supported spherical shell. The deflection displacements of the shell were written in a series of the Legendre functions and the trigonometric functions. The dynamical energies of the shell were evaluated and the frequency equation was derived by Ritz Method. The natural frequencies and mode shapes were calculated numerically for a closed spherical shell supported at equi – spaced four points located along a parallel of latitude. **Fawaz, 1990** in his M.Sc. thesis the Rayliegh variation method was used to obtain natural frequencies and mode shapes of axisymmetric vibrations of thin elastic oblate spheroidal shells with clamped – free for boundary condition and presents the results theoretically and experimentally. He showed that the Rayliegh's method was found to be suitable only for oblate shells with eccentricities less than 0.6. In the paper presented by **Antoine Chaign et. al. 2002,** linear and nonlinear vibrations of shallow spherical shells with free edges are investigated experimentally and numerically and compared to previous studies on percussion instruments such as gongs. The preliminary bases of a suitable analytical model are given. Identification of excited modes is achieved through systematic comparisons between spatial numerical results obtained from a finite element modeling, and spectral information's derived from experiments.

This research deals with the effect of additional boundary conditions such as clamped _ clamped, clamped – free and pined – pined on the free vibration of the oblate spheroid shells.

2. MATHEMATICAL ANALYSIS

The problem of vibration of oblate spheroid shells will be treated as a structure composed of two spherical shells joined rigidly at their ends. Centers of curvature of the two spherical shell elements fall along the minor axis of the proposed oblate spheroid Fig. (1).

Such approximation is not far from reality, as the oblate spheroidal tanks are produced by joining, either by welding or riveting, two spherical shell elements through a toroidal shell element.

The effective radius (R_r) of the spherical shell model represents the radius of curvature at the apex of the shell. This radius can be obtained from the geometrical relation **[Penzes, 1965]**:

$$R_{\Phi} = \frac{a(1-e^2)}{(1-e^2\cos^2\Phi')^{3/2}} \qquad (1)$$

Setting (Φ') to zero results the radius of the shell at the apex as:

$$R_r = \frac{a}{(1-e^2)^{1/2}} \qquad (2)$$

where,

$e = \left[1-\frac{b^2}{a^2}\right]^{\frac{1}{2}}$, a and b represent the major axis and minor axis respectively.

$e = 0$ for sphere , $e = 1$ for plate.

An approximate opening angle (Φ_0) may be obtained by using the following formula:

$$\Phi_o = \cos^{-1}\frac{R_r - b}{R_r} \qquad (3)$$

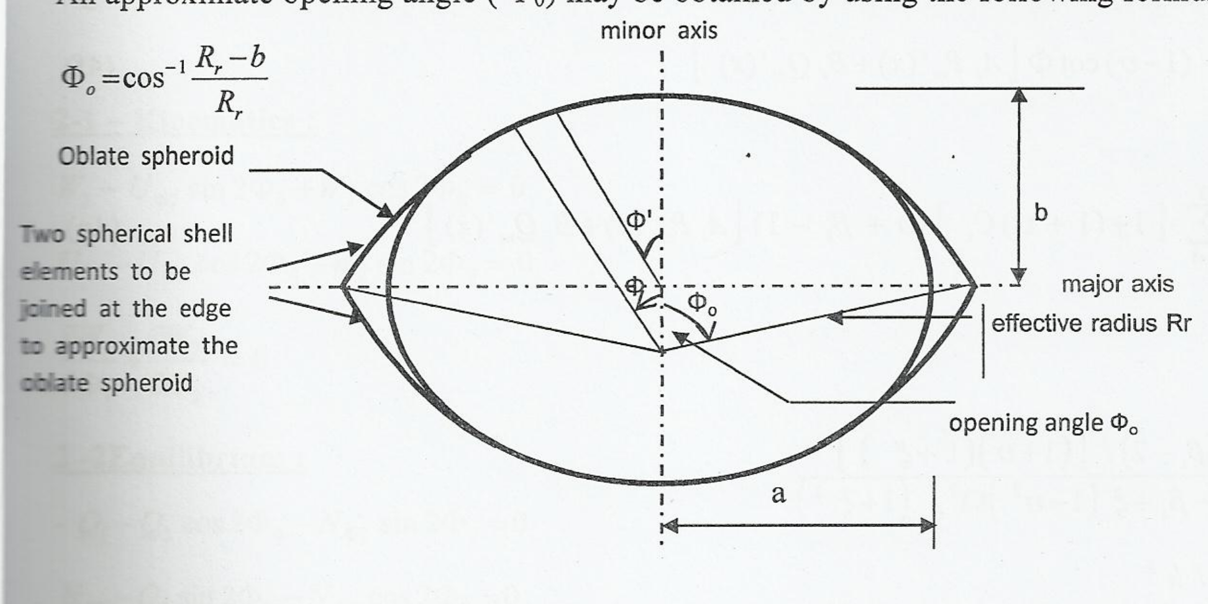

Fig. (1): An oblate spheroid and its approximate of two spherical shell elements joined at the edge

The free vibration of spherical shells was solved analytically by [**Kalinins, 1964**]. In this work considering the actual Φ – dependent coefficient of the variable as those derived in the latter reference which are :

$$W=\sum_{i=1}^{3}\left[A_i\,P_{ni}(x)+B_i\,Q_{ni}(x)\right] \tag{4a}$$

$$U_\Phi=\sum_{i=1}^{3}-(1+\upsilon)\,C_i\left\lfloor A_i\,P_{ni}'(x)+B_i\,Q_{ni}'(x)\right\rfloor \tag{4b}$$

$$N_\Phi=\frac{E.h}{(1-\upsilon)\,R_r}\sum_{i=1}^{3}\left\{(1+C_i\,\beta_i).\left\lfloor A_i\,P_{ni}(x)+B_i\,Q_{ni}(x)\right\rfloor\right. \\ \left.+(1-\upsilon)\,C_i\,\cot\Phi\left\lfloor A_i\,P_{ni}'(x)+B_i\,Q_{ni}'(x)\right\rfloor\right. \tag{4c}$$

$$N_\theta=\frac{E.h}{(1-\upsilon)\,R_r}\sum_{i=1}^{3}\left\{(1+\upsilon\,C_i\,\beta_i).\left\lfloor A_i\,P_{ni}(x)+B_i\,Q_{ni}(x)\right\rfloor\right. \\ \left.-(1-\upsilon)\,C_i\,\cot\Phi\left\lfloor A_i\,P_{ni}'(x)+B_i\,Q_{ni}'(x)\right\rfloor\right. \tag{4d}$$

$$M_\Phi=\frac{D_b}{R_r^{\,2}}\sum_{i=1}^{3}\left[1+(1+\upsilon)\,C_i\right]\left\{\beta_i\left\lfloor A_i\,P_{ni}(x)+B_i\,Q_{ni}(x)\right\rfloor\right. \\ \left.+(1-\upsilon)\,C_i\,\cot\Phi\left\lfloor A_i\,P_{ni}'(x)+B_i\,Q_{ni}'(x)\right\rfloor\right. \tag{4e}$$

$$M_\theta=\frac{D_b}{R_r^{\,2}}\sum_{i=1}^{3}\left[1+(1+\upsilon)\,C_i\right]\left\{\upsilon\,\beta_i\left\lfloor A_i\,P_{ni}(x)+B_i\,Q_{ni}(x)\right\rfloor\right. \\ \left.-(1-\upsilon)\cot\Phi\left\lfloor A_i\,P_{ni}'(x)+B_i\,Q_{ni}'(x)\right\rfloor\right\} \tag{4f}$$

$$Q_\Phi=\frac{D_b}{R_r^{\,2}}\sum_{i=1}^{3}\left[1+(1+\upsilon)\,C_i\right](\upsilon+\beta_i-1)\left\lfloor A_i\,P_{ni}'(x)+B_i\,Q_{ni}'(x)\right\rfloor \tag{4g}$$

where,

$$C_i=\frac{1+(\beta_i-2)/\left[(1+\upsilon)(1+\xi)\right]}{1-\upsilon-\beta_i+\xi\,(1-\upsilon^2)\,\Omega^2/\,(1+\xi^2)}$$

$$\xi=12\,R_r^{\,2}/\,h^2$$

$$n_i = -\tfrac{1}{2} + \sqrt{1/4 + \beta_i}$$

x=cos Φ

The parameters β_i's are the three roots of the cubic equation:-

$$\beta^3 - \left[4 + (1-\upsilon^2)\Omega^2\right]\beta^2 + \left[4 + (1-\upsilon)(1-\upsilon^2)\Omega^2 + (1+\xi)(1-\upsilon^2)\right]$$

$$(1-\Omega^2)\beta + (1-\upsilon)(1-\upsilon^2)\left[\Omega^2 - \frac{2}{1-\upsilon}\right]\left[1 + (1+\upsilon)\left[\Omega^2 - \frac{1}{1+\upsilon}\right]\right] = 0 \qquad (5)$$

and $\Omega^2 = \dfrac{\rho\,\omega^2 R_r^2}{E}$

$$D_b = \frac{E h^3}{12(1-\upsilon^2)}$$

$P_n(x)$, $Q_n(x)$ are Legendre functions of the first and the second kinds, respectively $P_n'(x)$, $Q_n'(x)$ are the derivatives with respect to (Φ) of the Legendre functions of the first and the second kinds, respectively. A_i & B_i are arbitrary constants.

The above solutions can be applied to the study of free vibration of an elastic spherical shell bounded in general by any two concentric openings.

As stated before the two spherical shell elements are assumed to be rigidly connected along their edge $\Phi=\Phi_o$. To guarantee that the continuity of all deflections, slopes, moments and forces along the function is insured, (selecting the coordinates of the top shell as the reference coordinates) the boundary conditions at the junctions may be written as follows [(Fig. 2)]:

2-1 – Kinematics :

$$W_1 - U_{\Phi 2}\sin 2\Phi_0 + W_2\cos 2\Phi_0 = 0 \qquad (6)$$

$$U_{\Phi 1} - U_{\Phi 2}\cos 2\Phi_0 - W_2\sin 2\Phi_0 = 0 \qquad (7)$$

$$\frac{\partial W_1}{\partial \Phi_1} + \frac{\partial W_2}{\partial \Phi_2} = 0 \qquad (8)$$

2 -2Equilibrium :

$$-Q_1 - Q_2\cos 2\Phi_0 - N_{\Phi 2}\sin 2\Phi_0 = 0 \qquad (9)$$

$$N_{\Phi 1} - Q_2\sin 2\Phi_0 - N_{\Phi 2}\cos 2\Phi_0 = 0 \qquad (10)$$

$$M_1 - M_2 = 0 \qquad (11)$$

Substituting the terms of equation. (4a – 4g) into the boundary conditions results in six homogenous simultaneous equations in terms of the constants which can be written as follows : -

$$\sum_{i=1}^{6} C_{i,K}(\Omega).A_{i.K}=0 \qquad , \qquad k=1,....,6$$

where the elements $C_{i,k}$ are functions of Ω. These elements are generated from the applications. For non trivial solution of the simultaneous equations, the determinant of the coefficients $C_{i,k}$ must vanish, thus

$$\begin{vmatrix} C_{11} & & C_{16} \\ & & \\ C_{61} & & C_{66} \end{vmatrix} = 0 \qquad (12)$$

The elements of equation (12) can be found according to the following boundary conditions:

(a) For clamped end : $W(x) = U_\Phi(x) = W'(x) = 0$.

$$C_{1,k} = P_{ni,k}(x), \qquad (13a)$$

$$C_{2,k} = C_i\, P_{ni,k}'(x), \qquad (13b)$$

$$C_{3,k} = P_{ni,k}'(x). \qquad (13c)$$

(b) For free end : $Q(x) = N_\Phi(x) = M_\Phi(x) = 0$.

$$C_{1,k} = [\,1+(1+\upsilon)C_i\,](\upsilon+\beta_i-1)\; P_{ni,k}'(x), \qquad (14a)$$

$$C_{2,k} = \{\,(1+C_i\beta_i).\; P_{ni,k}(x) + (1-\upsilon)C_i\cot\Phi\; P_{ni,k}'(x), \qquad (14b)$$

$$C_{3,k} = [\,1+(1+\upsilon)C_i\,]\{\,\beta_i P_{ni,k}'(x) + (1-\upsilon)C_i\cot\Phi\; P_{ni,k}'(x). \qquad (14c)$$

(c) For hinged edge : $W(x) = U_\Phi(x) = M_\Phi(x) = 0$.

$$C_{1,k} = P_{ni,k}(x), \qquad (15a)$$

$$C_{2,k} = C_i\, P_{ni,k}'(x), \qquad (15b)$$

$$C_{3,k} = [\,1+(1+\upsilon)C_i\,]\{\,\beta_i P_{ni,k}'(x) + (1-\upsilon)C_i\cot\Phi\; P_{ni,k}'(x). \qquad (15c)$$

The calculation of the natural frequency is carried out by specifying an initial guessed then evaluating the determinant $\left| C_{i,j} \right|$. Increasing the frequency by small increments and repeating the same procedure until the value of the determinant changes its sign. This indicates that a natural frequency is expected in the new value. The frequency increment is then minimized and the operation is repeated until the desired accuracy of the natural frequency is obtained when the determinant is vanished. The mode shape associated with any natural frequency is then derived by substituting the value of the natural frequency obtained above in equation. (12) and normalizing the $\left[A \right]$ coefficients and determining the eigenvectors.

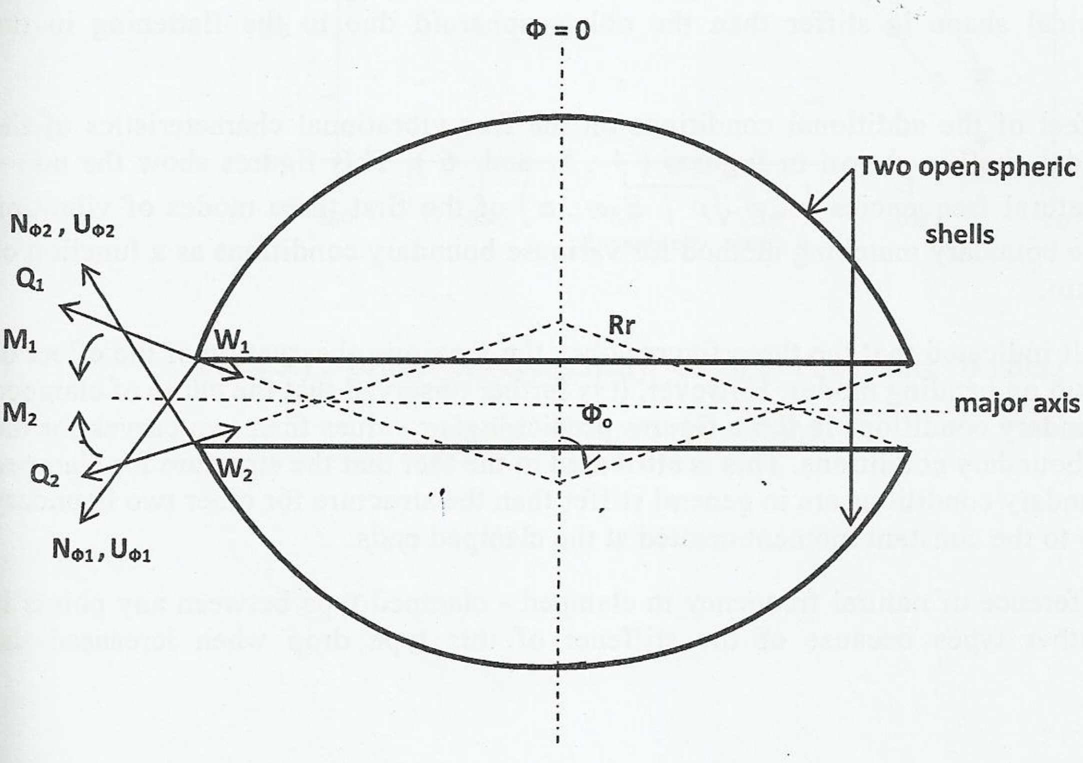

Fig. (2): Matching boundary conditions of two non – shallow spherical shells elements

3. RESULTS AND DISCUSSION:

Figure (3) shows the non–dimensional natural frequencies $\left(\lambda = \sqrt{\rho / E}\,\omega . a\right)$ of the first three modes of vibration as functions of the eccentricity ratio obtained by the matching boundary condition method using the non – shallow shell theory for clamped – free of boundary conditions. This figure shows clearly the tendency of the natural frequencies towards lower values as the eccentricity increases.

This behavior could be explained by the fact that the mode shapes of a closed spherical shell would resemble those of an oblate spheroid up to certain eccentricity ratio. As the eccentricity ratio increases, the oblate spheroid tends to flatten up. Such "flattening" causes the uncoupling of the radial (or transverse motion) and the tangential motion where the latter is minimized and the radial or transverse motion mode shape approaches that of a circular plate (a plate is an oblate spheroid with approach unity eccentricity ratio). Another reason is that the spherical shape is stiffer than the oblate spheroid due to the flattening in the geometry.

The effect of the additional conditions on the free vibrational characteristics of the oblate spheroidal shell is shown in fugures (4 , 5 and 6). This figures show the non – dimensional natural frequencies $\left(\lambda = \sqrt{\rho / E}\,\omega . a\right)$ of the first three modes of vibration obtained by the boundary matching method for variouse boundary conditions as a function of eccentricity ratio.

It is well indicated that the three figures obey the previous observation of the effect of eccentricity ratio on bending modes. However, it is further observed that the curve of clamped – clamped boundary conditions in three figures predict higher values than two curves for the other types of boundary conditions. This is attributed to the fact that the structure for clamped – clamped boundary conditions are in general stiffer than the structure for, other two boundary conditions due to the constant moment created at the clamped ends.

The difference of natural frequency in clamped - clamped type between any points is hihger than other types because of the stiffener of this type drop when icreaased the eccentricity.

4.CONCLUSIONS:

From the results obtained, the main conclusion can be summarized as; the natural frequencies curved of clamped – clamped boundary conditions predict higher values than the other boundary conditions.

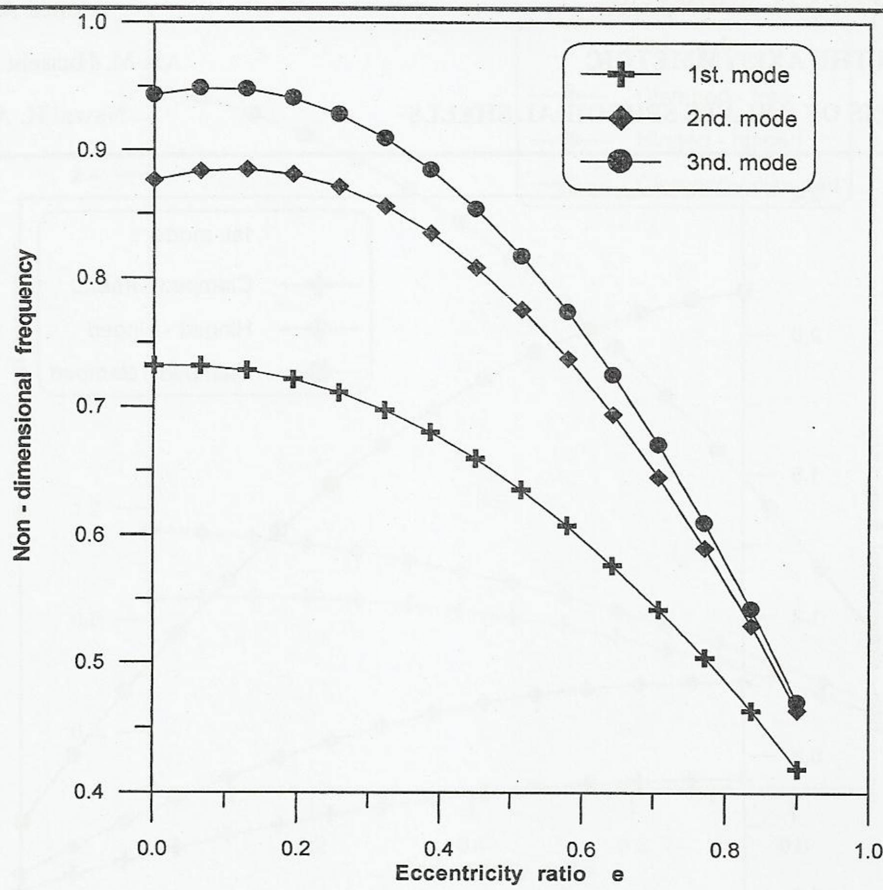

Fig. (3) : Effect of eccentricity on the three first bending modes obtained by BMM

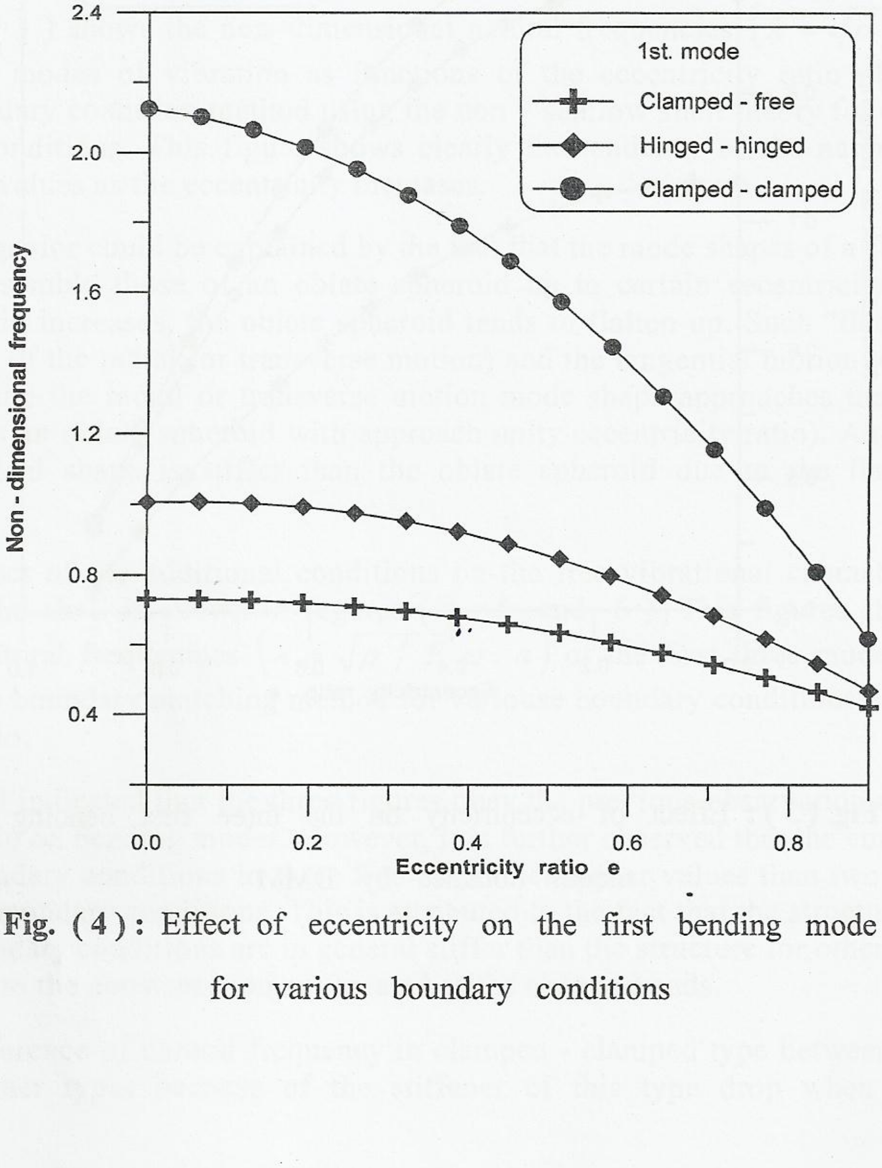

Fig. (4) : Effect of eccentricity on the first bending mode for various boundary conditions

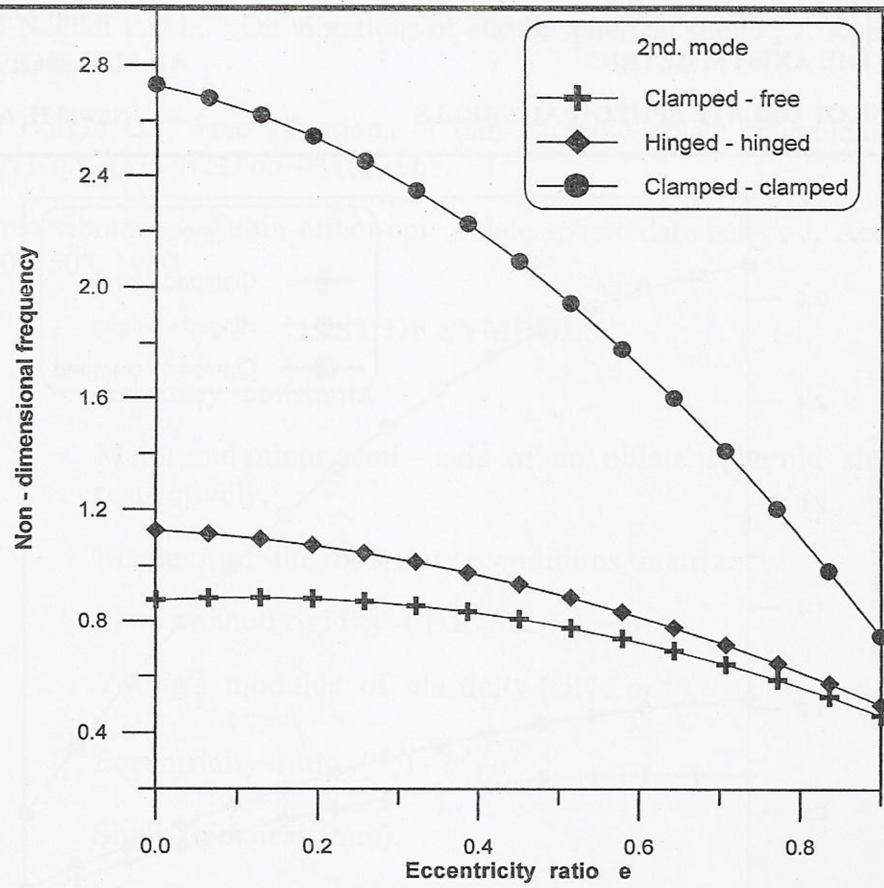

Fig. (5) : Effect of eccentricity on the second bending mode for various boundary conditions

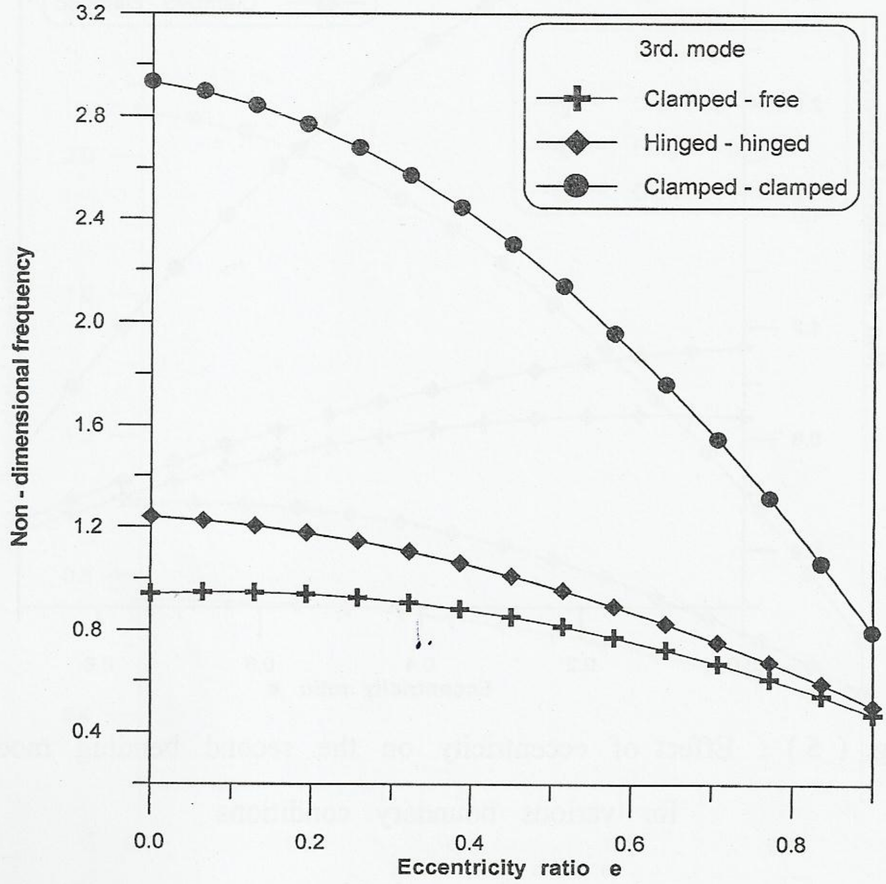

Fig. (6) : Effect of eccentricity on the third bending mode for various boundary conditions

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LIST OF SYMBOLS

A_i , B_i	Arbitrary constants.
a , b	Major and minor semi – axis of an oblate spheroid shell respectively.
$C_{i,j}$	Element of the boundary conditions matrix.
D_b	Plate or shell rigidity ($E.h^3 / 12 (1 - \upsilon^2)$).
E	Young's modulus of elasticity (GN / m^2).
e	Eccentricity ratio ($\sqrt{1-b^2/a^2}$).
h	Shell thickness (mm).
M_Φ , M_θ	Moments per unit length.
N_Φ , N_θ	Membrane forces per unit length.
$P_n(x)$	Legendre function of the first kind.
$P'_n(x)$	First derivative of the Legendre function of the first kind.
$Q_n(x)$	Legendre function of the second kind.
$Q'_n(x)$	Derivative of the Legendre function of the second kind.
$Q_\Phi(x)$	Transverse shearing force per unite length .
R_r	Effective radius.
R_Φ , R_θ	Principal radii of curvatures of an oblate spheroid.
U_Φ	Tangential displacement mode.
u_Φ	Tangential displacement of points on shell middle surface.
W	Transverse or radial displacement mode.
w	Transverse displacement of points on shell middle surface.
β_i	Roots of the non – shallow shell cubic equation.
Φ'	Inclination angle of an oblate spheroid.
Φ	Inclination angle of a spherical shell model.

Φ_0	Opening angle of the approximate spherical shell..
λ	Non – dimensional frequency parameter $\sqrt{\rho/E}\,\omega . a$ (used for oblate spheroid shells).
ρ	Mass density (kg / m ^{3}).
Ω	Non – dimensional frequency parameter $\sqrt{\rho/E}\,\omega . R$ (used for spherical shells).
υ	Poisson ratio.
ω	Circular frequency (rad / sec).