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Large Angle Bending Behavior of Curved Members Using The Method of Characteristics

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Abstract

This paper deals with the nonlinear large-angle bending dynamic analysis of curved beams which investigated by modeling wave's transmission along curved members. The approach depends on the wave propagation in one-dimensional structural element using the method of characteristics. The method of characteristics (MOC) is found to be a suitable method for idealizing the wave propagation inside structural systems. Timoshenko's beam theory, which includes transverse shear deformation and rotary inertia effects, is adopted in the analysis. Only geometrical non-linearity is considered in this study and the material is assumed to be linearly elastic. Different boundary conditions and loading cases are examined.

From the results obtained, it is found that the geometrical shape, boundary conditions, material properties of the members as well as the load type and direction have considerable effects on the response of the member.

Keywords:

1. Introduction:

Recently, there has been a marked interest in the information of dynamic characteristics of many engineering applications such as high-speed machinery, airplane structure, tall building…etc. The rapid expansion in the study of wave propagation was prompted, in part, by the necessity to understand the transient history of such structures that are subjected to rapidly applied loads. The behavior of structural elements under impact or impulsive loads is a subject of a great interest in the structural dynamic analysis. When forces are applied to an elastic body over a very short period, the response should be considered in terms of the wave propagation theory.

The study of transient wave has an important implication and applications for structures subjected to such loads. With the rapid development of computational capabilities, nonlinear analysis in the structural engineering has become an important field of research. The objective is the realistic assessment of the actual behavior of structures. For this reason a great deal of attention has been given for studying the non-linearity effects. The vibration of planer curved beams, arches and rings have been the subject of numerous studies due to their wide variety of potential applications, such as bridges, aircraft structures, and turbo machinery blades. Therefore, studying the dynamic response, in the elastic region, of this simple structural component under various loading conditions would help in understanding and explaining the behavior of more complex structures under similar loading. filters to be implemented at baseband and thus be integrated.

2. Formulation:

In the present case, the propagation of elastic waves along curved members is considered. Waves in such members behave in more complex manner, because they are composed of a combination of waves causing the overall effect to be dissipative. The propagation of waves along one-dimensional members has been described by

Timoshenko, et al. (1974) [11]. In order to formulate and simplifying the problem, the following assumptions are considered:

1. The material is linearly elastic.
2. Timoshenko's beam theory is used.
3. The effects associated with the change of cross section properties in the neighborhood of the wave front are negligible.
4. The element has a constant cross section area A, constant second moment of area.
5. The material is of constant density ρ.

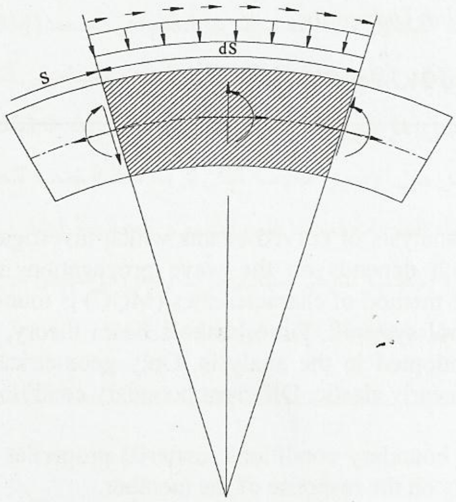

Fig.1. Curved Beam Element.

In the following formulation, an equation of motion in polar coordinates is obtained for each type of waves and material property [Al-Da'ami (1992) [4]]. This leads to pairs of partial differential equations relating a force and a velocity or a moment and an angular velocity. Consider a curved element, of length ds, with a radius of curvature R, and the central angel dθ, as shown in figure (1). The element has a constant cross section area (A), constant second moment of area (I) and material density (ρ). By the characteristic method and material's properties, the following equations are obtained:

i-Tangential, along $\frac{ds}{dt} = \pm C$

$$\frac{dF}{dt} \mp \rho AC \frac{dU}{dt} = EA\frac{V}{R} \mp \frac{QC}{R} \mp quC \quad ...(1)$$

ii-Radial, along $\frac{ds}{dt} = \pm Cs$

$$\frac{dQ}{dt} \mp \rho ACs \frac{dV}{dt} = -K_1 AG \frac{U}{R}$$

$$\pm \frac{FCs}{R} - K_1 AG\psi \pm qvCs \quad ...(2)$$

iii-Bending, along $\frac{ds}{dt} = \pm C$

$$\frac{dM}{dt} \mp \rho IC \frac{d\psi}{dt} = \pm QC \quad ...(3)$$

The above characteristic equations can be integrated along the lines L'P& R'P in case of shear wave and LP & RP in case of tangential wave and bending wave as illustrated in figure (2-a).

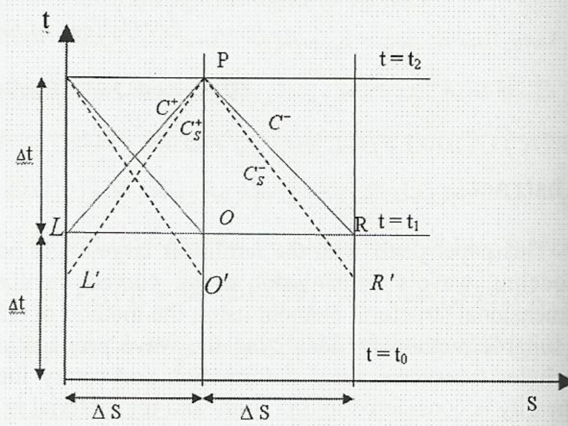

Fig.2-a. Numerical Characteristic Scheme in s-t Plane of Flexural Waves in Curved Member.

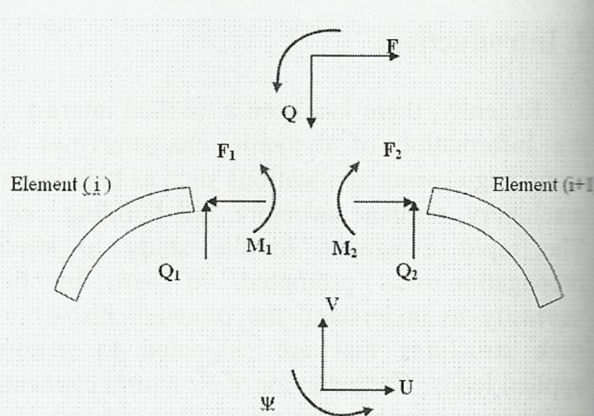

Fig.2-b. Boundary Characteristics with Forces and Velocities.

The method of boundary characteristics can be used to model the dynamic behavior of curved members. For this purpose the following notes are adopted [Chan (1983), Alsarraj (1989)]:

1. The beam is regarded as a prismatic member.
2. The beam capable to transmit axial, shear and moment waves.

3. All displacements are assumed small compared to the overall dimensions of structure.
4. Any node on the beam have two faces, the first face located on the end of element (i) and the second face located on the start of the element (i+1), where the node located between element (i) and element (i+1), as shown in figure (2-b).

Any node on the curved beam has three characteristic equations (three forces and three velocities per node).

The internal forces at either end of any element can be written in matrix form as; from equations (1), (2) &(3) the following relation can be obtained:-

$$\{F_1\} = \{f\}_n + [Ce]_n \{V\}_n \qquad \ldots(4)$$

where, the matrices $\{f\}_n$, [Ce]n , $\{V\}_n$ and $\{F_1\}_n$ are as defined in Appendix A.

Here, (S = -1) for forward characteristic lines [C+ equation] and (S= +1) for backward characteristic lines [C-equation]. The subscript (n) denotes the node on the member in which the values of {F} and {V} are the unknown end forces and velocities can be found for every step.

The force on the start of the next element is found as:

$$\{F_2\}_n = \{F\}_n - \{F_1\}_n \qquad \ldots(5)$$

where, $\{F\}_n$ are the external load on the node which given, and the velocities are the same in the first face. By this procedure the forces and velocities on each node along the curved member can be obtained. Also this method can be applied to the forces and moments at any location on the beam.

In the present study, large rotation due to the bending moment happens in the curved beam causes a change in the shape of the beam. In geometrical non-linearity the length and cross-section of the member are changed after applying the load. But in this study it is assumed that the length and cross-section remain constant, the change of the angle and radius of the beam accrue only, this called (large angle bending behavior). For geometrical non-linearity curved beam problem, the effect of large rotation due to bending can become important. In the present development of the analysis a new value for the angle and radius of the structure is calculated at every time step. The equations of the new angle and radius for each step can be written as:

$$\theta_{new} = \theta_{old} + \Delta t(\Psi_i - \Psi_{i+1}) \qquad \ldots(6)$$

$$R_{new} = \frac{\Delta S}{\theta_{new}} \qquad \ldots(7)$$

where, Ψ is the angular velocity for the node and ΔS is the length of the element.

This could lead to change in the natural period and in the amplitude of vibration, thus producing nonlinear behavior of the member.

3. Risultant and Discussion:

Figure (3-a) shows a comparison between the present work and that given by Ahmed [2] for the bending moment along the beam, while figure (3-b) shows a comparison with Ali [5] for vertical deflection at mid span of the beam. Figures (3-a) and (3-b) show a good agreement between the present work and the given references.

In small angle dynamic analysis, the change in beam angle and radius is regarded small during the analysis. Figure (4) shows a comparison between the large angle bending and small angle bending mode shapes for the first three modes at time of loading (0.08) sec. This could lead to a change in natural period and in amplitude of vibration which producing nonlinear behavior of the member. Also, it can be noticed that the large angle bending analysis lead to larger amplitudes of vibration and larger time periods than small angle bending.

Figure (5) show the small angle behavior and large angle bending dynamic responses of a fixed-fixed curved beam. It is clear that the behavior of the nonlinear wave propagation is seemed to be similar to the linear wave propagation. Moreover, it can be noticed that the large angle bending analysis gives more depressive effects than the large angle bending analysis, due to the change in the internal angle and change in curvature of the member, hence the geometrical shape of the member. Figures (6) & (7) represent for waves propagation of periodic and pulse loadings respectively, from these figures, it is noted that there are differences in the wave amplitude due to differences in load functions. In addition, it is noted that the values of the stresses in the second load function (pulse load) are greater than the first load function (periodic load) in spite of that the two load functions have the same amplitude (4000 N), but the first load function is more dangerous than the second because the load is repeated with the time.

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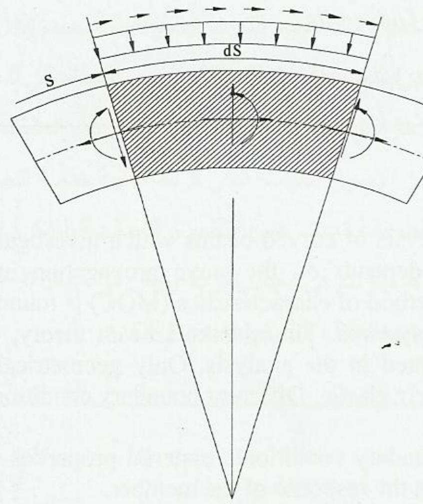

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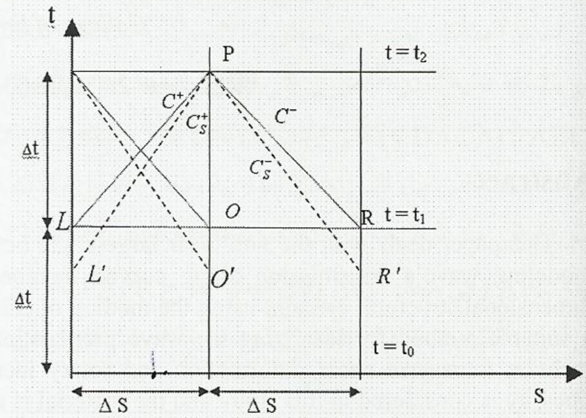

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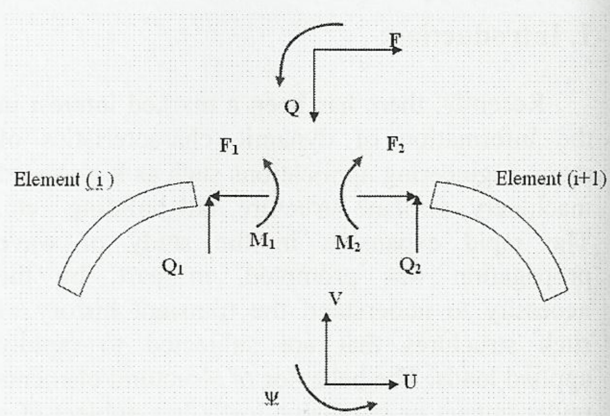

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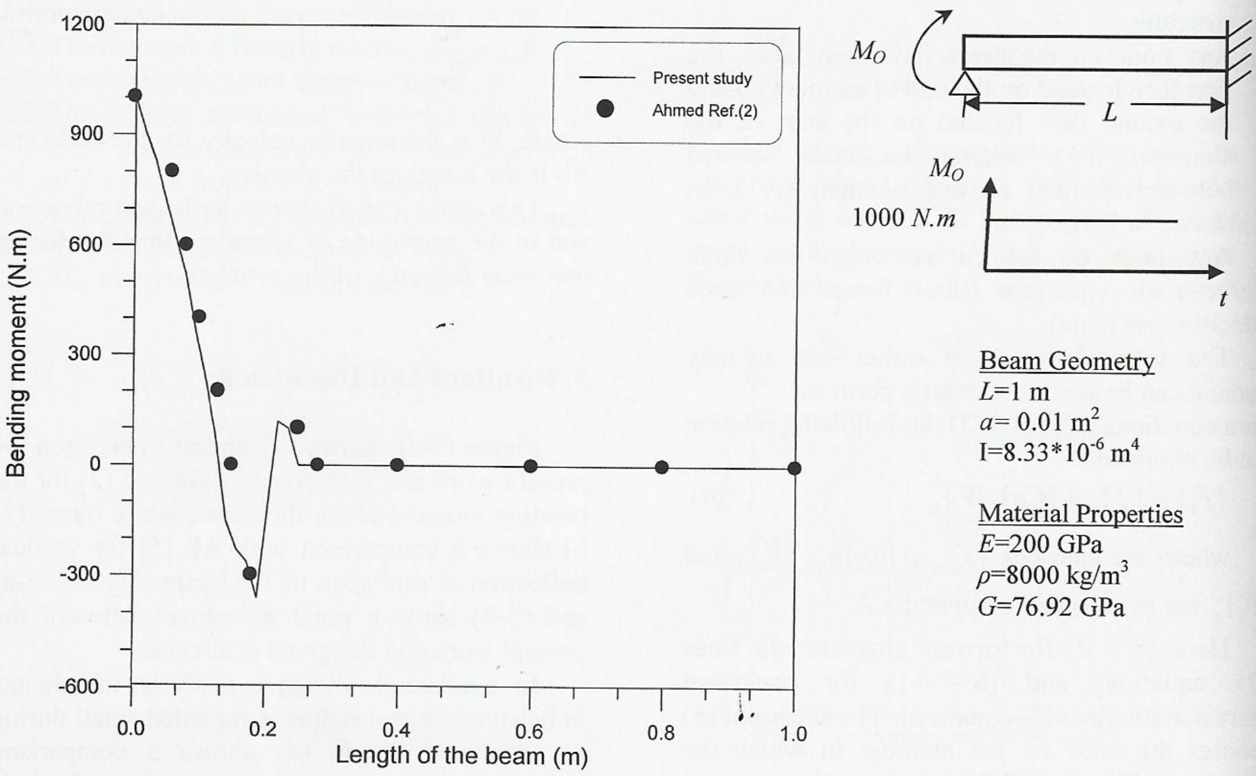


Fig.3-a. Moment Wave Propagation in Hinged-Fixed Beam.

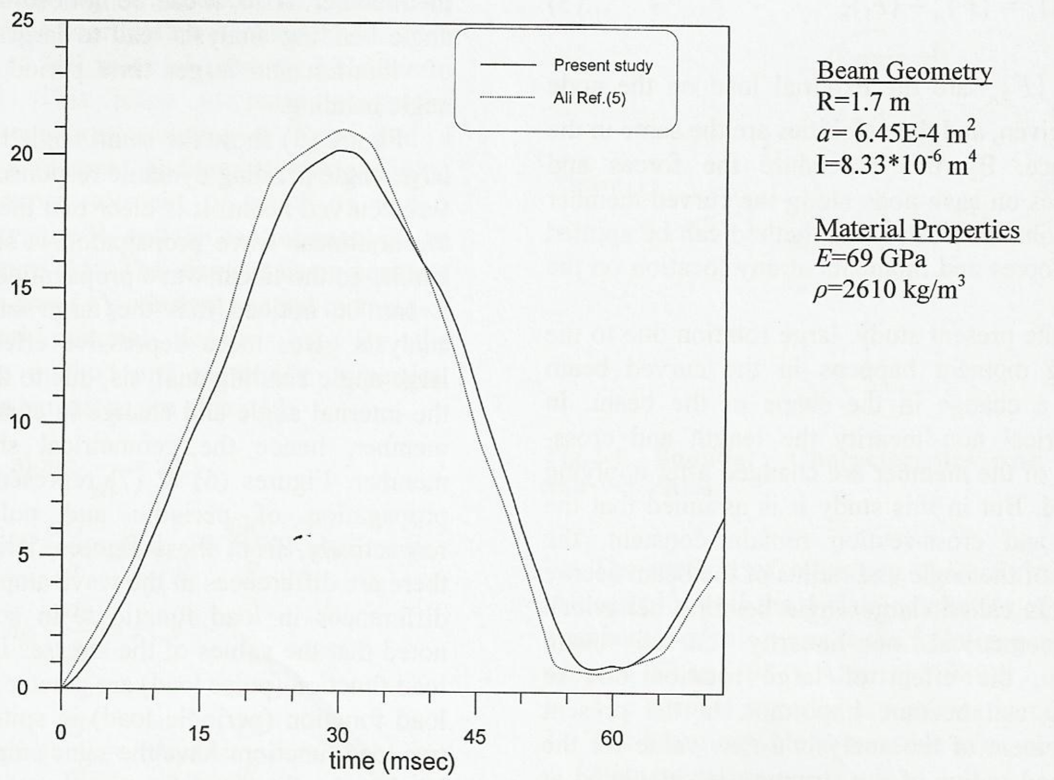
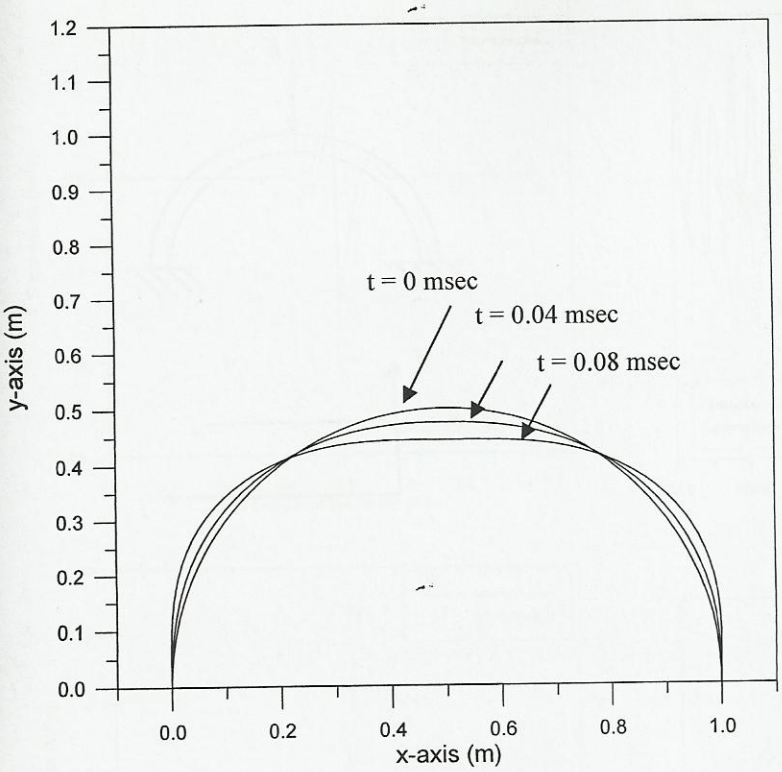
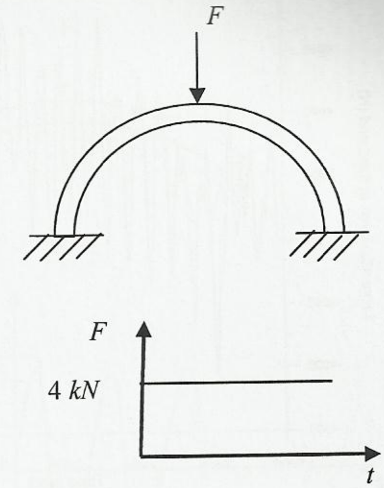
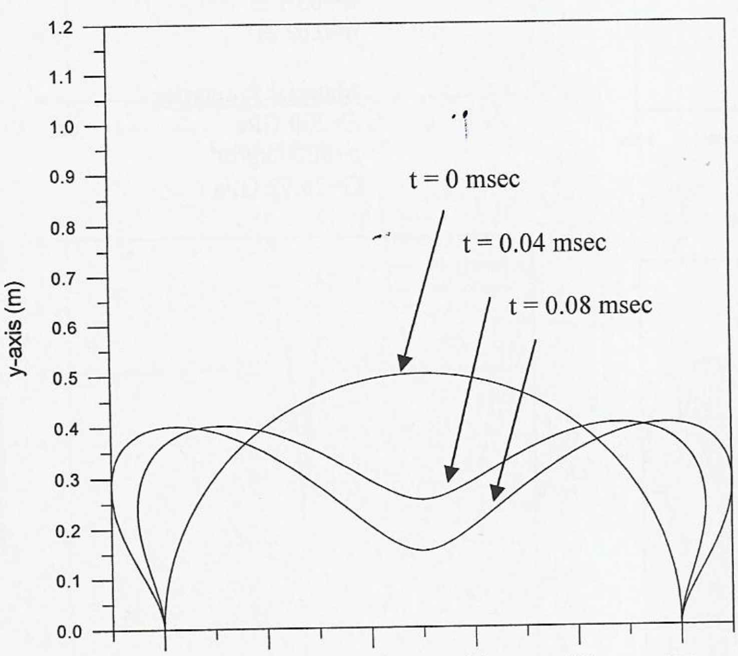


Fig.3-b. Vertical Displacement History in a Curved Beam.





(a) Small-Angle Bending

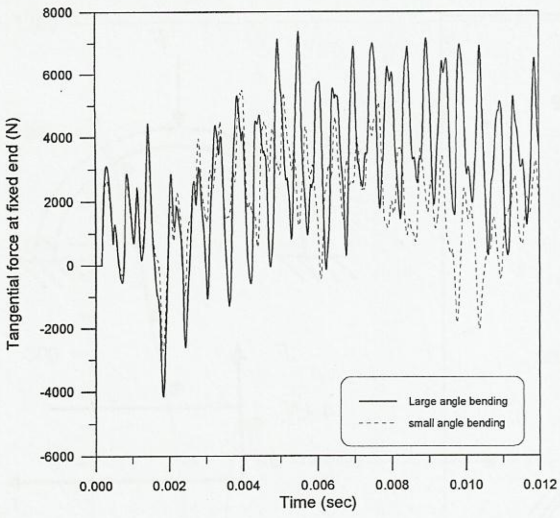


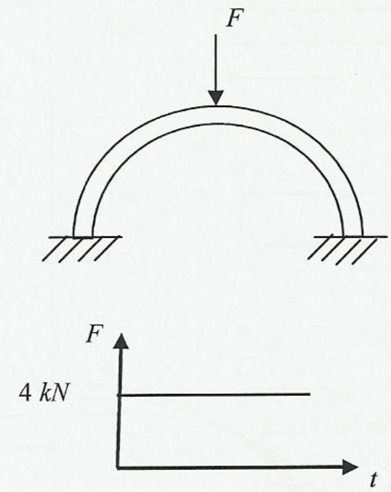
Beam Geometry
Radius=0.5 m
Angle of curvature =180 deg.
w=0.01 m
h=0.01 m
Multiplucation scale=1000

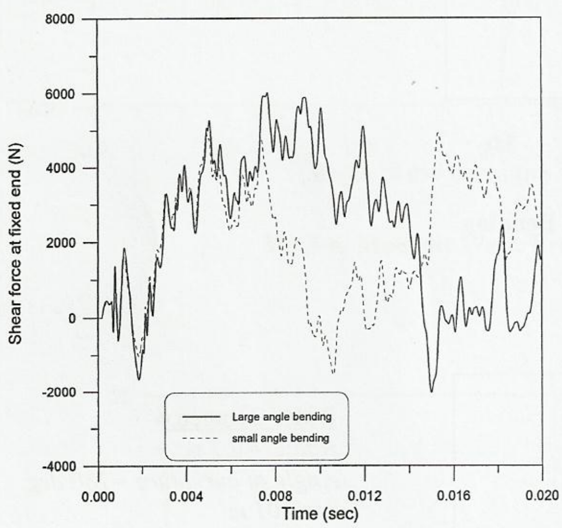
Material Properties
E=200 GPa
ρ=8000 kg/m^3
G=76.92 GPa

(b) Large-Angle Bending

Fig.4. Comparison of Mode Shape Between Small-Angle and Large-Angle Analysis of Curved Beam.







<u>Beam Geometry</u>
Radius=0.5 m
Angle of curvature =180 deg.
w=0.01 m
h=*0.01 m*

<u>Material Properties</u>
E=200 GPa
ρ=8000 kg/m^3
G=76.92 GPa

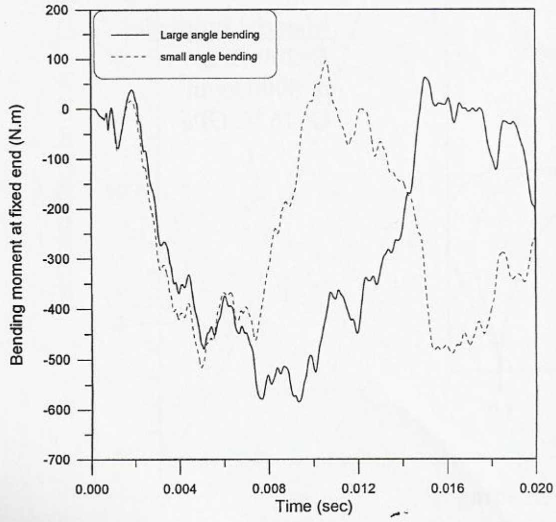
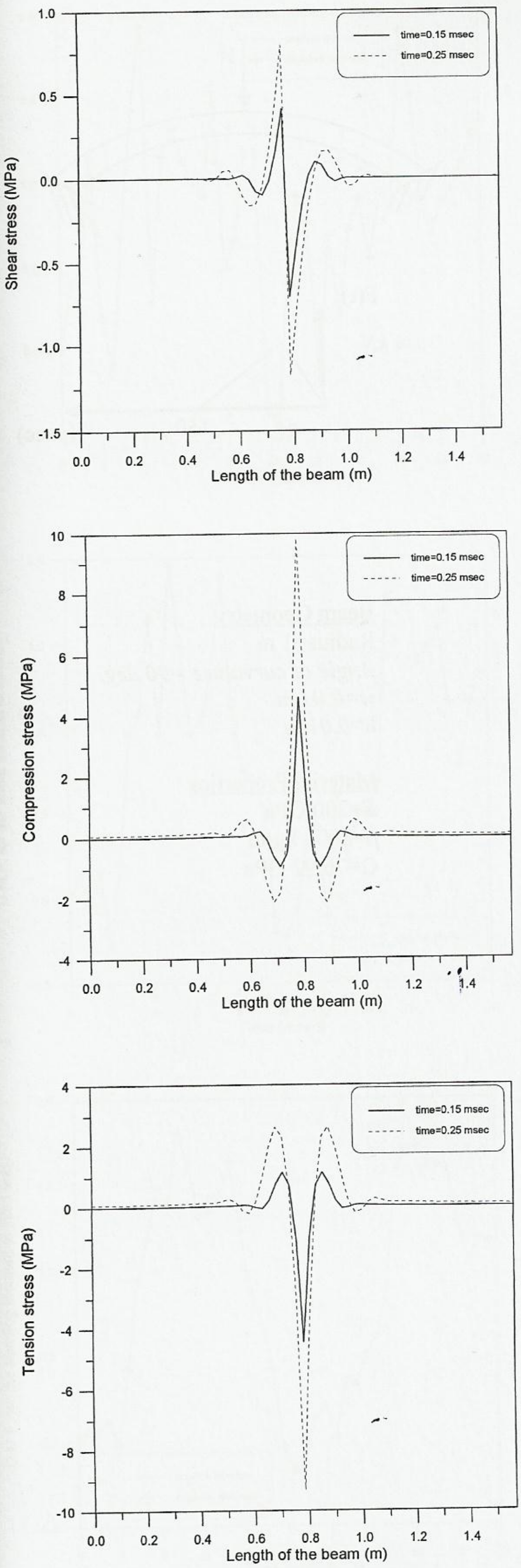
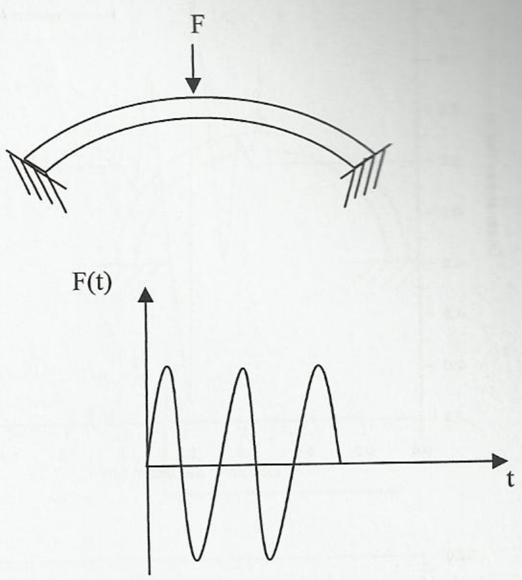


Fig.5. Comparison Between Small and Large Bending Responses of Curved Beam.



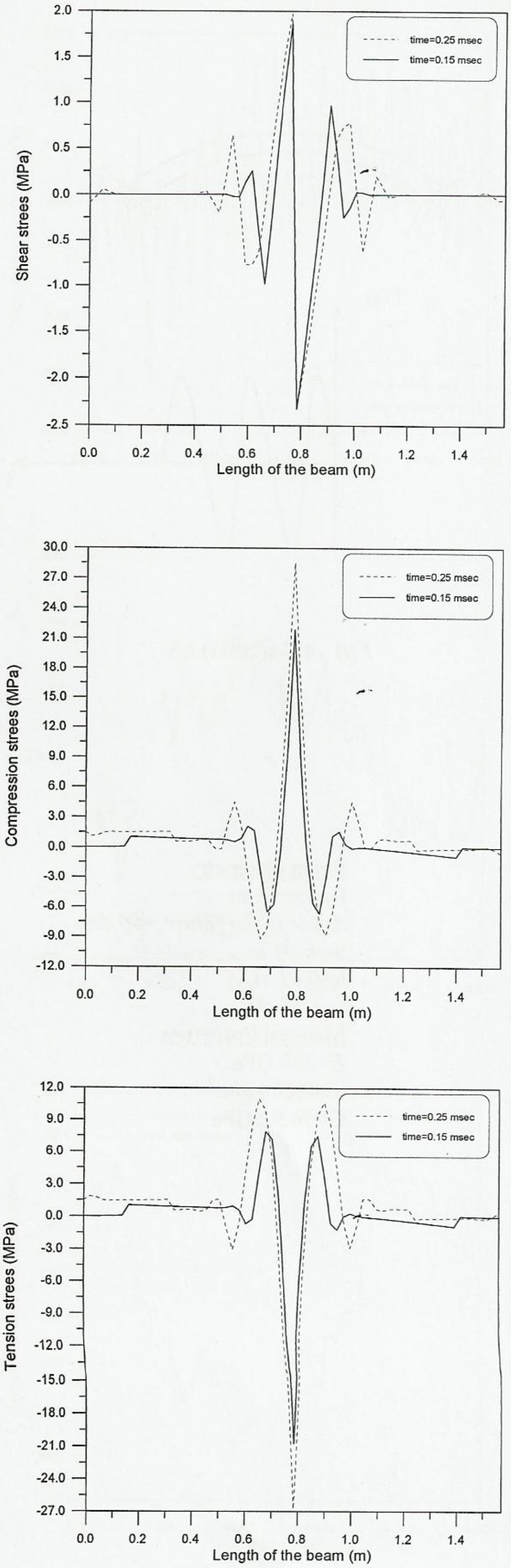


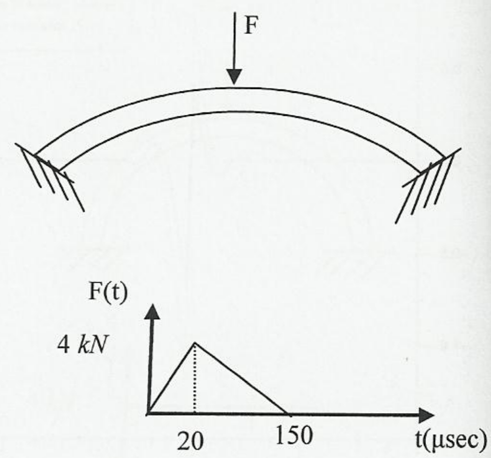
F(t) = **4 sin(200 t)** ***kN***

<u>Beam Geometry</u>
Radius=1 m
Angle of curvature =90 deg.
w=0.01 m
h=*0.01 m*

<u>Material Properties</u>
E=200 GPa
ρ=8000 kg/m^3
G=76.92 GPa

Fig.6. Wave Propagation in Curved Beam in Different Times.

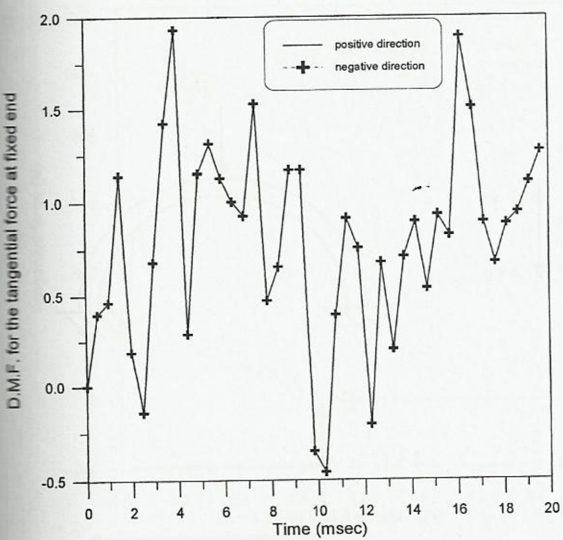


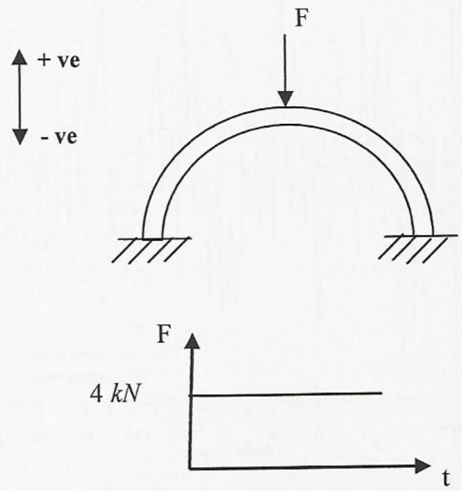


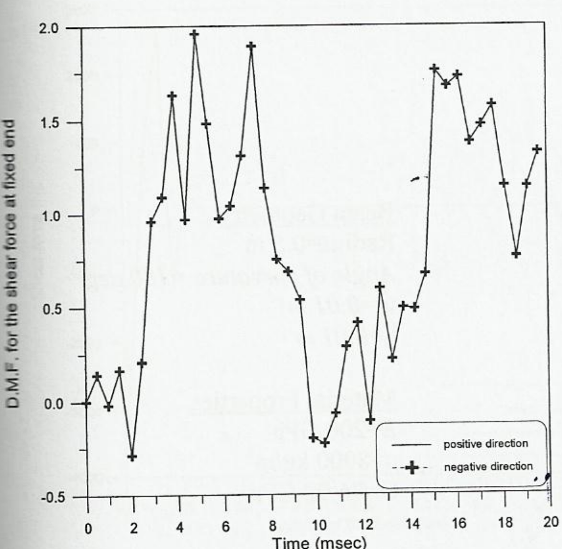
Beam Geometry
Radius=1 m
Angle of curvature =90 deg.
w=0.01 m
h=*0.01 m*

Material Properties
E=200 GPa
ρ=8000 kg/m^3
G=76.92 GPa

Fig.7. Wave Propagation in Curved Beam in Different Times.







Beam Geometry

Radius=0.5 m

Angle of curvature =180 deg.

w=0.01 m

h=*0.01 m*

Material Properties

E=200 GPa

ρ=8000 kg/m^3

G=76.92 Gpa

D.M.F=Dynamic response/ Static value

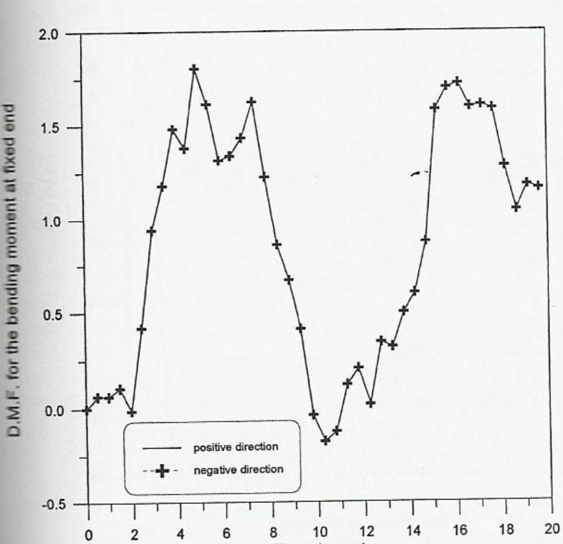


Fig.8. Effect of Load Direction on Dynamic Magnification Factor for Small-Angle Bending Response.

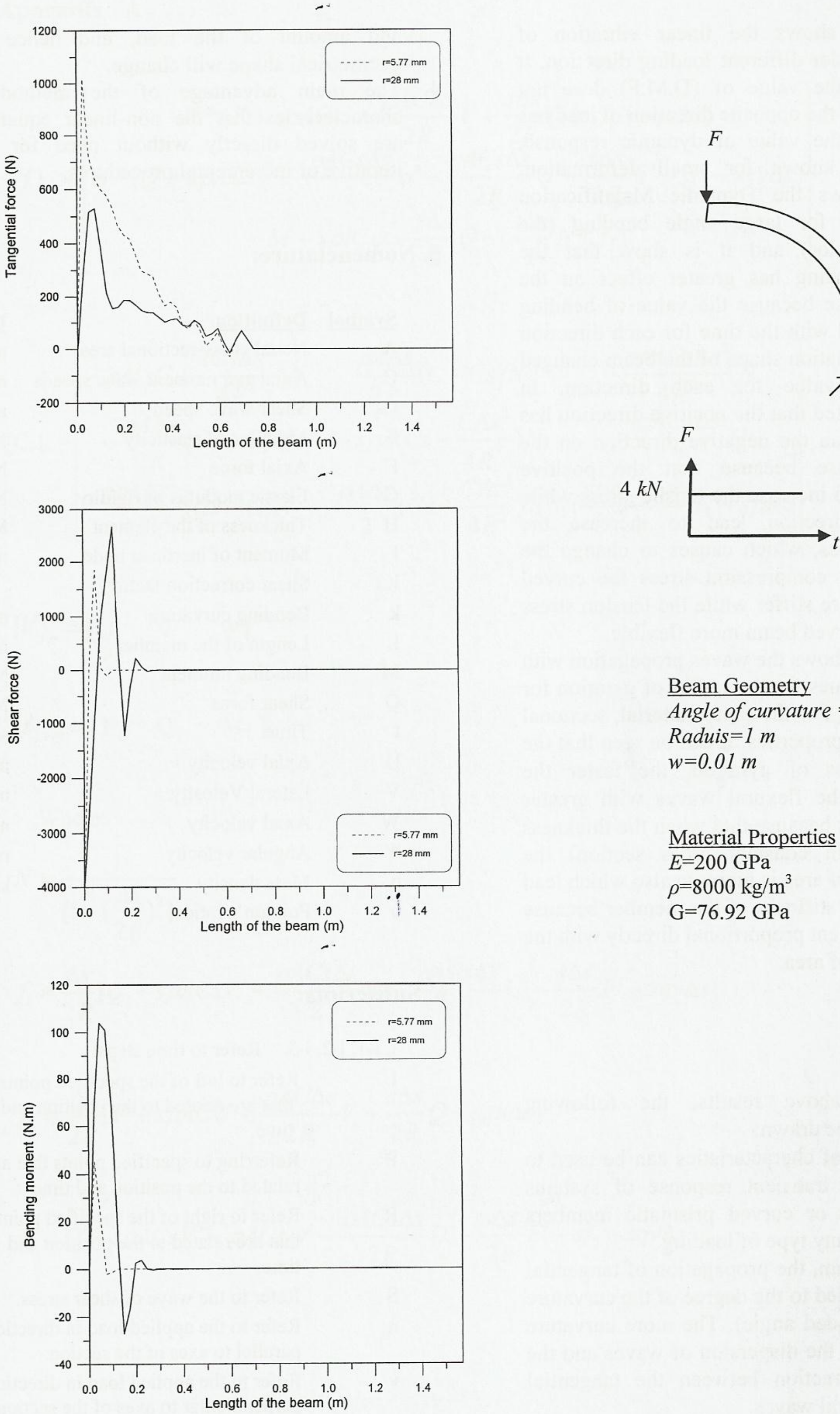


Fig.10. Wave Propagation of Beam with Different Values for Radius of Gyration at (0.15msec).

Figure (8) shows the linear vibration of curved beam under different loading direction. It is shown that the value of (D.M.F) dose not exceed (2.0) and the opposite direction of load has (no effect) on the value of dynamic response, which is well known for small deformation. Figure (9) shows the Dynamic Magnification Factor (D.M.F) for large angle bending (the nonlinear behavior) and it is show that the direction of loading has greater effect on the dynamic response because the value of bending angle is changed with the time for each direction cause the deformation shape of the beam changed with different value for each direction. In addition, it is noted that the positive direction has greater effect than the negative direction on the dynamic response because that the positive direction leads to increase the tensile stress while the negative direction lead to increase the compressive stress, which causes to change the stiffness, and in compression stress the curved beam will be more stiffer while the tension stress will made the curved beam more flexible.

Figure (10) shows the waves propagation with two different values for the radius of gyration for the curved beam. For the same material, sectional and geometrical properties, it can be seen that the larger the radius of gyration, the faster the propagation of the flexural waves with greater amplitude, this is because that when the thickness is increase (with constant cross section) the second moment of area is increase also which lead to increasing the stiffness of the member because the bending moment proportional directly with the second moment of area.

4. Conclution:

From the above results, the following conclusions can be drawn:

1. The method of characteristics can be used to simulate the transient response of systems with straight or curved prismatic members subjected to any type of loading.
2. In curved beam, the propagation of tangential waves is related to the degree of the curvature (beam subtended angle). The more curvature is the greater the dispersion of waves and the stronger interaction between the tangential and the flexural waves.
3. In the large angle analysis, the curvature of the curved member will change with the type and amount of the load, and hence the geometrical shape will change.
4. The main advantage of the method of characteristics that the non-linear equations are solved directly without need for any iterative or incremental procedures.

5. Nomenclature:

Symbol	Definition	Units
A	Nodal cross-sectional area	m^2
C	Axial and moment wave speeds	m/s
Cs	Shear wave speed	m/s
E	Modulus of elasticity	N/m^2
F	Axial force	N
G	Elastic modulus of rigidity	N/m^2
H	Thickness of the element	M
I	Moment of inertia at node	m^4
K1	Shear correction factor	
k	Bending curvature	m^{-1}
L	Length of the member	m
M	Bending moment	N.m
Q	Shear force	N
t	Time	sec
U	Axial velocity	m/s
V	Lateral Velosity	m/s
W	Axial velocity	m
Ψ	Angular velocity	rad/sec
ρ	Mass density	kg/m^3
υ	Poisson's ratio	

6. Subscripts:

i, i-1, i-2, i-3	Refer to time steps.
L	Refer to left of the specified points that are related to the position and time.
P	Referring to specified points that are related to the position and time
R	Refer to right of the specified points that are related to the position and time.
S	Refer to the wave of shear stress.
u	Refer to the applied load in direction parallel to axes of the section.
v	Refer to the applied load in direction perpendicular to axes of the section.
$\dot{}$	Refer to the first derivative with respect to time.

7. Appendix A

$$\{f\}_n = \begin{Bmatrix} N^*(F + s\rho ACU + \dfrac{\rho AC\Delta s}{2R}V + \dfrac{s\Delta s}{2R}Q + sf_1 + squ\Delta s) \\ N^*(Q' + s\rho ACsV' - \dfrac{\rho ACs\Delta s}{2}\psi' - \dfrac{\rho ACs\Delta s}{2R}U' - \dfrac{s\Delta s}{2R}F' - sf_2 - sqv\Delta s) \\ M + s\rho IC\psi - \dfrac{s\Delta s}{2}Q - (\dfrac{s\Delta s}{2}N^* f_3) \end{Bmatrix}$$

$$[C_e]_n = \begin{bmatrix} -\rho A(sC + \dfrac{sCs\Delta s^2}{4R^2})N & \dfrac{\rho A\Delta s}{2R}(C - Cs)N & -\dfrac{s\rho ACs\Delta s^2}{4R^2}N \\ -\dfrac{\rho A\Delta s}{2R}(Cs - C)N & -s\rho A(Cs - \dfrac{C\Delta s^2}{4R^2})N & -\dfrac{\rho ACs\Delta s}{2}N \\ \dfrac{s\rho A\Delta s^2}{4R^2}(Cs - C)N & \dfrac{\rho A\Delta s}{2}(Cs - \dfrac{C\Delta s^2}{4R^2})N & (\dfrac{s\rho ACs\Delta s^2}{4}N - s\rho IC \end{bmatrix}$$

$$\{V\}_n = [\mathrm{U} \quad \mathrm{V} \quad \Psi]_n^T$$

$$\{F_1\}_n = [\mathrm{F}_1 \quad \mathrm{Q}_1 \quad \mathrm{M}_1]_n^T$$

Where:

$$N = \frac{1}{[1 + (\dfrac{\Delta s}{2R})^2]}$$

$$f_1 = \frac{\Delta s}{2R}[Q' + s\rho ACsV' - \frac{\rho ACs\Delta s}{2}\psi' - \frac{\rho ACs\Delta s}{2R}U' - \frac{s\Delta s}{2R}F' - sqv\Delta s]$$

$$f_2 = \frac{\Delta s}{2R}[F + s\rho ACU + \frac{\rho AC\Delta s}{2R}V + \frac{s\Delta s}{2R}Q + squ\Delta s]$$

$$f_3 = Q' + s\rho ACsV' - \rho ACs\psi' - \frac{\rho ACs\Delta s}{2R}U' - \frac{s\Delta s}{2R}F' - sf_2 - sqv\Delta s$$

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