Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

Some Topics on Convex Optimization

Some Topics on Convex Optimization

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Abstract. The aim of this paper is to clarify several important points, including a brief and adequate explanation of loving improvement, as well as laying out a number of important mathematical formulas that we need, supported with graphs.

1. Introduction

Each sub-field of mathematical optimization can be called a convex optimization and a sub-field gap that addresses and studies the problem of reducing convex functions on other convex groups alike.Likewise, There are many classes and classes of convex improvement problems accept polynomial mathematical algorithms and formulas, [1] while mathematical optimization is generally a problem or problem of difficulty to solve, which is one of the difficult NP problems, [1] [2] [3]. The problem of these systems can only be solved directly by approximation and finding the result by estimation, estimation .It also has a number of applications, including signal processing, communications and offices of all kinds, design of private electronic circuits, [3] and its uses also include information investigation and demonstrating, self-financing, measurements (ideal test plan), [3] and basic improvement, where the idea of approximation has proven its high effectiveness. [2], [3] With late advances in figuring calculations and improvement, so curved programming has become nearly as clear as straight programming.

2. Some concept and definition:

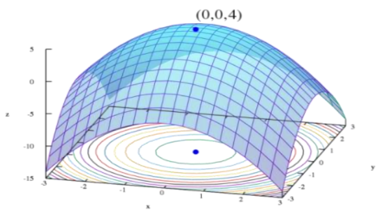

Figure 1. Optimization problem

Mathematical optimization is the choice of the best segment (concerning a few rules) from a bunch of options accessible. [1] Optimization issues of sorts show up in all quantitative orders from software engineering constantly to activities examination and financial aspects, and the advancement of arrangement strategies has been of revenue in arithmetic for quite a long time. [4] In the most straightforward case, the enhancement issue comprises of boosting or limiting a genuine employment by deliberately choosing input esteems from inside a passable set and figuring the estimation of the work. Summing up streamlining hypothesis and procedures to different recipes establishes an enormous zone of applied arithmetic. By and large, streamlining incorporates discovering "best accessible" values for some topic positions (or information sources), including an assortment of subject positions and various kinds of fields.

The standard and basic form of a continuous optimization problem is .

$$\begin{aligned} &\text{Min} \quad G(x) \\ &\text{S.t.} \quad h_j(x) \le 0, \forall\, j \in \{1, \dots, l\} \\ &\qquad\quad g_j(x) = 0, \forall\, j \in \{1, \dots, k\} \end{aligned} \qquad (1)$$

whereas

- $G_0(x)$ is the Is the goal function .
- $\boldsymbol{h_j(x) \le 0}$ we are called inequality constraints.
- $\boldsymbol{g_j(x) = 0}$ we are called equality constraints.
- $\boldsymbol{x = (x1, \dots, xn)}$: optimization variables
- $\boldsymbol{l, k \ge 0}$

If $l = k = 0$ The problem is the unrestricted problem of optimization. By convention, the Standard Model defines the problem of miniaturization. The problem of maximization can be addressed by eliminating the objective function.

Example1:
Scaling of the device in electronic circuits

- Variables: width and lengths
- Constraints: producing limits, timing prerequisites, most extreme territories
- Goal: energy utilization

Example 2:

$$\begin{aligned} &\text{Min} \quad f(x) = 4x^2 + k^2 \\ &\text{S. t.} \quad 2x + 3k \le 1 \\ &\qquad x - k \le 2 \\ &\qquad \text{For all } x, k \ge 0 \end{aligned}$$

2.1. convex optimization:

Convex optimization is the problem of optimization where the target and each convex set is a feasible set.The f function specifies a subset of $\mathbb{R}^n$ .Into$\mathbb{R}\cup\{\mp\infty\}$ if its domain is convex is convex and for all $\tau \in [0,1]$ for all m, n In its field, the following condition is fulfilled:

$$f(\tau m + (1-\tau)n) \le \tau f(m) + (1-\tau) f(n), \forall m, n \in S, 0 \le \tau \le 1 \qquad (2)$$

Therefore, we conclude that S is a convex set of all elements , $n \in S$, $\forall\, 0 \le \tau \le 1$.
then we get $\tau m + (1-\tau)n \in S$ Concretely, The problem of convex optimization is a problem of finding some $m^* \in K$ attaining $inf\{f(m): m \in K\}$.
Where the target function $F: D \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$ is convex, as in the feasible combination K [1], [2].As for this point, we can call it the best solution; And we call the set of all optimal points as optimum set. Iff

is not restricted below on K or infimum not reached then optimization issue is said to be unlimited. Conversely, if K is the empty set, then we say about the problem as infeasible [2].
Here is the basic model to the problem of convex optimization:

$$\begin{aligned} \text{Min} \quad & f(x) \\ \text{S.t.} \quad & h_j(x) \le 0, \forall j \in \{1, \dots, l\} \\ & g_j(x) = 0, \forall j \in \{1, \dots, p\} \end{aligned} \quad (3)$$

where $\boldsymbol{x} \in \boldsymbol{R^n}$is the optimization variable, the function $\boldsymbol{F}: \boldsymbol{D} \subseteq \boldsymbol{R^n} \longrightarrow \boldsymbol{R}$ is convex and $\boldsymbol{gh_j}: \boldsymbol{R^n} \longrightarrow \boldsymbol{R}\ \boldsymbol{j} = \boldsymbol{1} \dots . \boldsymbol{l}$ is convex and $\boldsymbol{g_j}: \boldsymbol{R^n} \longrightarrow \boldsymbol{R}\ \ \boldsymbol{j} = \boldsymbol{1} \dots \boldsymbol{p}$ isAffine[2]. This coding describes the search problem $\boldsymbol{x} \in \boldsymbol{R^n}$that minimizes $\boldsymbol{f}(\boldsymbol{x})$among all $\boldsymbol{x}$ satisfying $\boldsymbol{h_j}(\boldsymbol{x}) \le \boldsymbol{0}\ \ , = \boldsymbol{1} \dots . \boldsymbol{l}$ and $\boldsymbol{g_j}\ (\boldsymbol{x}) = \boldsymbol{0}\ , \boldsymbol{j} = \boldsymbol{1} \dots \boldsymbol{p}$.

- Where $\boldsymbol{f}$ Is the goal function
- $\boldsymbol{h_j}$ and $\boldsymbol{g_j}$ we are called inequality constraints.

Whereas, the feasible group C for the improvement problem consists of all points x∈ D fulfilling the requirements. This set is convex in light of the fact that D is a convex set, and so are the sublevel sets of convex functions, relative sets, and convex sets all convex sets.[3] An answer for a convex streamlining issue is any point $x \in K$ Investigation $inf\{f(x): x \in K\}$. By and large, a convex refinement issue may have zero, one or a few arrangements. Numerous optimization issues can be detailed proportionally in this standard structure. For instance, the inward function expansion issue can be reworded as the curved function minimization issue $-\boldsymbol{f}$. In general, the problem of a convex enhancement is a problem of developing a curved function on a convex array.

2.2. Level set:

In mathematics, the level set of a function with real value f of real variables m is a set of the model:

$$\mathbf{L_k(f)} = \{(\mathbf{x_1}, \dots\dots, \mathbf{x_m}) | \mathbf{f}(\mathbf{x_1}, \dots\dots, \mathbf{x_m}) = \mathbf{k}\} \quad (4)$$

Any combination where the function takes a certain constant value $\boldsymbol{k}$.

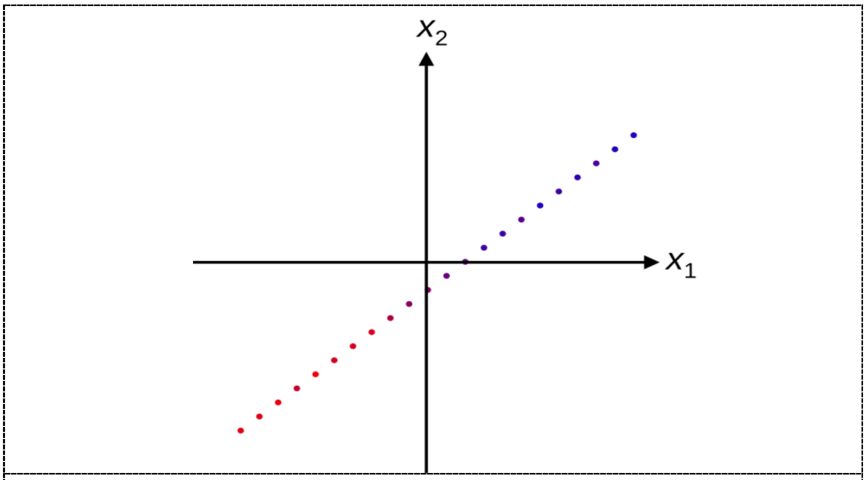

Figure 2. Points when fixed segments of $x_2 = f(x_1)$.

When the number of variables is two variables, then the level set is generally a curve, the level curve is called properties
The most important properties of convex improvement problems:

- Generally speaking, every local minimizer is the global minimizer
- Each set is perfect convex.
- If the target function is carefully convex, the issue contains all things considered one ideal point

Remark: any local ideal for a convex problem is global optimal.

Proof:
Let h is local optimal, then we have ∃ a feasible k such that
$g_0(k) < g_0(h)$
h local optimal means there is an $\mathcal{R} > 0$ such that
w feasible, $\|w - h\|_2 \leq \mathcal{R} \Rightarrow g_0(w) \geq g_0(h)$
Considered
$w = \mu k + (1 - \mu)h$ with $\mu = \mathcal{R}/(2\|k - h\|_2)$

- $\|k - h\|_2 > \mathcal{R}$, so $\mu \in [0, \frac{1}{2}]$
- w It is a convex combination made up of two possible points and thus also possible
- $\|w - h\|_2 = \mathcal{R}/2$ and

$$f_0(w) \leq \mu f_0(k) + (1 - \mu) f_0(h) < f_0(h)$$

This is a contradiction to the assumption h is local optimal

3. Linear programming (optimization) (LP):

It is a strategy for accomplishing the best outcome, (as, maximum benefit or most minimal expense) in a mathematical model whose necessities are addressed by straight associations. Straight composing PC programs is an unprecedented example of mathematical programming (in any case called mathematical improvement). Even more authoritatively, straight composing PC programs is a procedure for improving direct target function, subject to straight balance and constraints of direct imbalance. Its feasible area is a convex polytope, a gathering characterized as the convergence of numerous half spaces, every one of which is characterized by straight disparities. Its target function is a relative (direct) function of genuine worth characterized in this polyhedron. The immediate programming figuring finds a point in a polytope where this capacity contains the tiniest (or greatest) regard if that point is accessible.

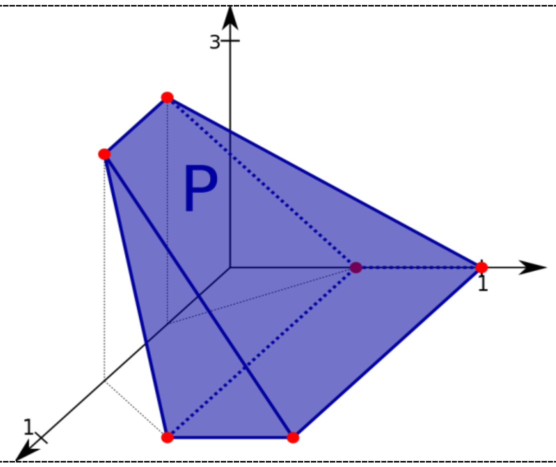

Figure 3. The linear programming problem.

4. Least squares :

The least squares strategy is a standard methodology in relapse examination to approximate the arrangement of exceptionally explicit frameworks (sets of conditions in which there are a larger number of conditions than the obscure) by diminishing the amount of the squares of the excess qualities in the outcomes from each single condition." The main application in information blend. Better fit in the feeling of least squares lessens the amount of the squared residuals (lingering object: the contrast between the noticed worth and the fit worth gave by the model).When the issue contains critical vulnerabilities about the autonomous variable (the variable x), at that point the straightforward relapse and least squares technique experiences issues; In such cases, the philosophy needed for fitting blunder models into factors rather than that of least squares might be thought of. Least squares issues fall into two classifications: straight or normal least squares and nonlinear least squares, contingent upon whether the lingering esteems are direct in all questions or not. The issue of direct least squares happens in factual relapse investigation; It has a shut shape arrangement"[3]. The nonlinear issue is normally explained by iterative refinement; In every emphasis, the framework is approximated by two straight one, so the essential computation is comparative in the two cases.

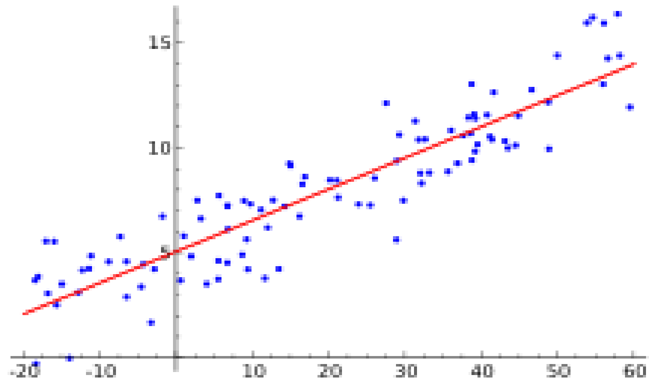

Figure 4. Regression analysis

5. Quadratic programming (QP):

It is a cycle of explaining an exceptional kind of mathematical optimization problem - explicitly, the quadratic optimization problem (linearly compelled), that is, the problem of upgrading (minimizing or maximizing) a quadratic capacity of numerous factors that are dependent upon the linear constraints of these factors. Quadratic programming is a unique kind of nonlinear programming.

Min $(1/2)x^T M x + k^T x + c$ (5)

S.t. $Gx \preccurlyeq h$

$Ax = b$

$M \in S_+^n$ and $x \geq 0$

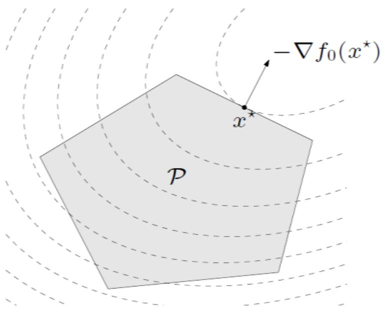

Fig (5). Quadratic programming

6. A second-order cone program (SOCP) :

It is a convex problem of optimization the form and its general form:

$$\begin{aligned} \text{Min} \quad & f^T x \\ \text{S.t.} \quad & \|B_j x + b_j\|_2 \leq k_j{}^T x + d_j \ , j = 1, \ldots, m \\ & Fx = g \end{aligned} \tag{6}$$

Where the problem parameters are $f \in R^n$, $B_i \in R^{n_j \times n}$, $b_j, k_j \in R^n$, $d_j \in R$, $F \in R^{p \times n}$ and $g \in R^p$, $x \in R^n$. is the optimization variable $\|x\|_2$. The "second-order cone" in SOCP emerges from the constraints, which are identical to requiring the relative function $(Bx + b, k^T x + d)$ to lie in the second-order cone in R^{n_j+1} .[1]

SOCPs It can also be solved in several ways, including the internal point method [2]. All in all, it tends to be comprehended preferable and more proficiently over semi-explicit programming problems (SDP). [3] Some of the designing uses of SOCP incorporate channel plan, reception apparatus cluster weight configuration, bracket plan, and ideal force absorption in mechanical technology [4].

7. Semidefinite programming (SDP):

It is a convex optimization subfield stressed over improving the straight objective work (a limit that the customer needs to restrict or augment) through the assembly of a cone of semi-determinant positive grids with a relative space, i.e., a semidefinite programming is a moderately new territory of progress and is of expanding revenue for a few reasons. "Many practical problems can be modeled in operations research and harmonic optimization or approximated as semi- definite programming problems. In automatic control theory, SDPs are used in the context of linear matrix inequality.SDPs are actually a special case of conical programming and can be efficiently solved by endpoint methods. All linear programs can be expressed as SDPs, and it is through hierarchies of SDPs that solutions to polynomial optimization problems can be approximated. Semi- definite programming has been used to improve complex systems. In recent years, some quantitative query complexity problems have been formulated in terms of semi- definite programs".[3]

8. Conic Optimizations:

It is that subfield of convex optimization that reviews most problems comprising of minimizing a convex capacity across the convergence of an affine subspace and a convex cone. Whereas, the category of convex refinement problems incorporates probably the most notable categories of convex optimization problems, and they are also known for linear and semi-unequivocal programming.

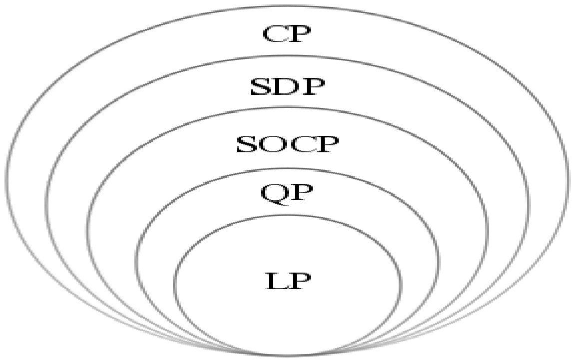

Fig (6). Quadratic programming

Then we get $LP \subseteq QP \subseteq SOCP \subseteq SDP \subseteq CP$

9. Lagrange complications:

Consider that it is a convex decrease problem introduced in standard form by the cost work f (a) and the inequality constraints$g_j(a) \leq 0$ for $1 \leq j \leq n$. Then the domain $\boldsymbol{M}$is:

$$\boldsymbol{M} = \{\boldsymbol{a} \in \boldsymbol{X}: \boldsymbol{g_1}(\boldsymbol{a}), \dots\dots, \boldsymbol{g_n}(\boldsymbol{a}) \leq \boldsymbol{0}\}$$

The formula Lagrangian function for the problem is :

$$L(a, \tau_0, \dots., \tau_n) = \tau_0 f(a) + \tau_1 g_1(a) + \dots\dots\dots\dots + \tau_n g_n(a) \qquad (7)$$

For each point $a \in X$ that limits f over , there exist genuine numbers $\tau_0, \tau_1 \dots., \tau_n$ called Lagrange multipliers, that fulfil these conditions all the while:

1. x minimizes $L(y, \tau_0, \dots., \tau_n)$over all $y \in X$.
2. $\tau_0, \tau_1 \dots., \tau_n \geq 0$with at least one $\tau_k > 0$.
3. $\tau_1 g_1(\boldsymbol{a}) = \dots\dots\dots\dots = \tau_n g_n(\boldsymbol{a}) = 0$ (reciprocal slackness).
4. If there exists a "strictly feasible point", that is, a point w satisfying

$$g_1(w), \dots, g_n(w) < 0$$

at that point the assertion above can be strengthened to require that $\tau_0 = 1$

Conversely, if some $\boldsymbol{a} \in X$ satisfies from(1)to(3) for scalars $\tau_0, \boldsymbol{\tau_1} \dots., \tau_n$with $\tau_0 = 1$ then a is certain to minimize f over X .

10. Conclusion

In this paper, the following points are made:

• We clarified several important definitions of convex enhancement
• Give a brief explanation of the Lagrange Method and its basic form
• We also used the diagrams provided in the Big Mac Library to evaluate methods. Also, these shapes contained various features and characteristics.

11. Recources

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