On supra λ-open set in bitopological space

By

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Abstract:
in last paper we study a special case of bitopological space consist of T and T^{α} and we define an open set named it λ-open set now we study supra λ-open set in supra topological space and several properties of it .

Keyword :λ-open set , supra topology ,supra quotient map , supra λ-open set , supra λ-continuous , supra λ-quotient map , supra λ-quotient map .

Introduction

in 1983 A.S.Mashhour [1] introduced the supra topological spaces .in 1965 Njastad [5] introduced the notion of α-set in topological space and proved that the collection of all α-set in (X,T) is a topology on X .R.Devi and S.Sampathkumar and M.Caldas [3]introduced and studied a class of sets and maps between topological spaces called supra α-open sets and supra α-continuous maps respectively .H.shaheed and S.
Introduced and study the continuity in bitopological space by using the λ-open set . in this paper we study the λ-open set and λ-continuous function and in supra topology and also we define supra λ-open map and supra λ-closed map and quotient map and supra λ-quotient map and study some theorems and property about them .

The closure and interior of asset A in (X,T) denoted by int(A), cl(A)respectively .A subset A is said to be α-set if $A\subseteq int(cl(int(A)))$.a sub collection $\Omega\subset 2^{x}$ is called supra topological space [4] , the element of Ω are said to be supra open set in (X,Ω) and the complement of a supra open set is called supra closed set . The supra closure of asset A denoted by $cl^{\Omega}(A)$ is the intersection of supra closed sets including A. The supra interior of asset A denoted by $int^{\Omega}(A)$ is the union of a supra open sets included in A . The supra topology Ω on X is associated with T if $T\subset\Omega$. A set A is called supra α-open set if $A\subseteq int^{\Omega}(cl^{\Omega}(int^{\Omega}(A)))$ [7].

A subset A in the bitopological space (X,T,T^{α}) is called λ-open set if there exist α-open set U such that $A\subseteq U$ and $A\subseteq int_{T}(U)$[6].
A mapping from the bitopology (X,T,T^{α}) into (Y,V,V^{α}) is called λ-continuous function iff the inverse image of each open set in Y is λ-open set in X .[2]

1-- Supra λ-open set and supra λ-continuous:

1.1. Definition:
let (X,T,T^{α}) be a bitopological space and Ω is a associated supra topology with T then a subset A of X is said to be supra λ –open set iff there exist a supra α-open set U such that $A\subseteq U$ and $A\subseteq int^{\Omega}(U)$.

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1.2. Remark:

1-Every open set is λ-open set [4]

2-Every α-open set is supra α-open set [3]

3-every supra open set is supra α-open set [3]

1.3. Theorem:
Let (X,T,T^{α}) be a bitopological space and Ω is supra associated with t then :
1- Every λ-open set is supra λ-open set
2- Every supra open set is supra λ-open

Proof:
(1) let A is λ-open set then there exist α-open set U such that $A \subseteq U$ and $A \subseteq int_T(U)$, since $T \subset \Omega$ and every α-open set is supra α-open set then $A \subseteq int^{\Omega}(U)$, this mean that A is supra λ-open set .
(2) let A is supra open set , since every supra open set is supra α-open set then $A \subseteq A$ and $A \subseteq int^{\Omega}(A)$ and then A is supra λ-open set .

1.4. Example:
Let Ω={X,φ, {a,b},{a,c}}
Supra λ-open ={X,φ, {a},{b},{c},{a,b},{a,c}}

The following diagram give us the relation between the above sets:

$$\begin{array}{ccccc} \text{Open set} & \Rightarrow\Rightarrow & \alpha\text{-open set} & \Rightarrow\Rightarrow & \text{supra } \alpha\text{-open set} \\ \Downarrow & & & & \Uparrow \\ \Downarrow & & & & \Uparrow \\ \lambda\text{-open set} & \Rightarrow\Rightarrow & \text{supra } \lambda\text{-open set} & \Leftarrow\Leftarrow & \text{supra open set} \end{array}$$

1.5. Remark:
1- The intersection of two supra λ-open set is supra λ-open set
2- The union of two supra λ-open set is not necessary supra λ-open set

1.6. Example:
Let Ω={X,φ,{a},{a,b},{b,c}}
Supra λ-open ={X,φ,{a},{b},{c}, {a,b},{b,c}} clearly that {a},{c} are two supra λ-open set but the union {a,c} is not supra λ-open set.

1.7. Definition:
let (X,T,T^{α}) and (Y,V,V^{α}) are two bitopological spaces and Ω is a supra topology such that $T \subseteq \Omega$ then a function $f: (X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is supra λ–continuous iff the inverse image of each open set is supra λ-open set .

1.8. Theorem:
if the function $f: (X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is supra continuous then f is supra λ -continuous
Proof:

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Let H is open set, since f is supra continuous then $f^{-1}(H)$ is supra open set and then it is supra λ-open set, Therefore f is supra λ-continuous.

1.9. Theorem:

If the function f: $(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is continuous then f is supra λ-continuous

Proof:

Let H is open set in Y and since f is continuous then $f^{-1}(H)$ is open set in X and then it is supra λ-open set .there for f is supra λ-continuous

1.10. Theorem:

Let f: $(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is λ-continuous then it is supra λ-continuous .

Proof:

By theorem (1-3) No. 1 clearly that f is supra λ- continuous

2--Supra λ-open mapping and supra λ-closed mapping:

2.1. Definition:

a mapping f: $(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is said to be supra λ-open mapping if f(G) is supra λ-open set for each open set G in X.

2.2. Definition:

a mapping f: $(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is said to be supra λ-closed map if f(H) is supra λ-closed set for each closed set H in X.

2.3. Theorem:

let f: $(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is a bijective function then each of the following are equivalent :

1- F is supra λ-continuous
2- F is supra λ-closed map
3- F is supra λ open map

Proof:

(1) $\Rightarrow$(2) ,let B is closed set in X , then X-B is open set and f(X-B)is supra λ-open set in Y, since f is bijective then f(X-B)=Y-f(B) and then f(B) is supra λ-closed set and therefor f is supra λ-closed map.

(2) $\Rightarrow$(3) let B is closed set in X , since f is bijective then $(f^{-1})^{-1}(B)=f(B)$ which is supra closed set and then it is supra λ-closed set in Y. therefor f^{-1} is supra λ-continuous.

(3) $\Rightarrow$(1) let B is open set in X , since f^{-1} is supra λ-continuous then

$(f^{-1})^{-1}(B)$ =f (B) is supra λ-open set in Y and therefor f is supra λ-open map.

2.4. Theorem:

Let (X,T,T^{α}) , (Y,V,V^{α})) are two topological spaces and Ω , μ be the associated supra topologies with T ,V respectively then

f: $(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is supra λ-continuous map if one of the following exist :

1- $f^{-1}(int^{\mu}(A))\subseteq int(f^{-1}(A))$ foe every A in Y
2- $Cl(f^{-1}(A))\subseteq f^{-1}(cl^{\mu}(A))$ for every A in Y
3- $f(cl(A))\subseteq cl^{\Omega}(f(A))$ for every A in X

Proof:

(1) Let A is open set in Y,by assume
$f^{-1}(int^{\mu}(A))\subseteq int(f^{-1}(A))$ and then $f^{-1}(A)\subseteq int((f^{-1}(A)$
Therefor $f^{-1}(A)$ is open set and then it is supra λ-open set .f is supra λ-continuous.

(2) Let A is closed set in Y , by assume $Cl(f^{-1}(A))\subseteq f^{-1}(cl^{\mu}(A))$ we get that $Cl(f^{-1}(A))\subseteq f^{-1}(A))$, therefor $f^{-1}(A)$ is closed set in X and then it is supra λ-closed set in X .f is supra λ-continuous.

(3) let A is open set in Y , then $f^{-1}(A)$ is asset in X , By assume $F(cl(f^{-1}(A))\subseteq cl^{\mu}(f(f^{-1}(A))$ then $F(cl(f^{-1}(A))\subseteq cl^{\mu}(A)$ by (2) we get that f is supra λ-continuous.

2.5. Theorem:

a mapping $f: (X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ is supra λ open mapping iff $f(int(A))\subseteq int^{\mu}(f(A))$.

Proof:

suppose that f is supra λ-open map and let A is open set in X , $int(A)\subseteq A$, then $f(int(A))\subseteq f(A)$, since f is supra λ-open mapping then $f(A)=int^{\Omega}(f(A))$ and then $f(int(A))\subseteq int^{\Omega}(f(A))$.
Now let A is open set in X , by assume $f(int(A))\subseteq int^{\Omega}(f(A))$, then
$f(A) \subseteq int^{\Omega}(f(A))$ and since $int^{\Omega}(f(A))\subseteq f(A)$ this mean that f(A)is supra open set and then it is supra λ-open set .

2.6. Theorem:

let (X,T,T^{α}) , (Y,V,V^{α}) , (Z,W,W^{α}) are three bitopological space and $f:(X,T,T^{\alpha}) \to (Y,V,V^{\alpha})$ and $g:(Y,V,V^{\alpha}) \to (Z,W,W^{\alpha})$are two maps then:

1- if gof is open map and g is continuous and injective then f is supra λ-open mapping
2- if gof is supra λ-open map and f is continuous and surjective then g is supra λ- open map.
3- If gof is open map and g is supra λ-continuous and injective then f is supra λ open map

Proof:

1- let A is open set in X , since gof is open map then g(f(A)) is open set in Z and since g is continuous and injective the $g^{-1}(g(f(A)))=f(A)$ is open set in Y and then it is supra λ-open set there for f is supra λ-open map.
2- Let A is open set in Y ,since f is continuous ,then $f^{-1}(A)$ is open set in X since gof is supra λ-open map and f is surjective then $(gof(f^{-1}(A)))=g(A)$ is supra λ-open set in Z , there for g is supra λ-open map .
3- Since every continuous function is supra λ-continuous function then the result exist by (1)

3--Supra λ-quotient map

3.1. Definition:

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let (X,T,T^{α}) , (Y,V,V^{α}) are two topological spaces and Ω , μ are two associated supra topological space with T , V respectively and f: $(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ is surjective mapping then f is said to be supra quotient map provided a subset A of Y is supra open in Y iff $f^{-1}(A)$ is supra open set in X .

3.2. Definition:

let (X,T,T^{α}) , (Y,V,V^{α}) are two topological spaces and Ω , μ are two associated supra topological space with T , V respectively and f: $(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ is subjective mapping then f is said to be supra λ- quotient map if f is supra λ-continuous and $f^{-1}(V)$ is supra open set in X implies V is supra λ –open set in Y

3.3. Theorem:

Let (X,T,T^{α}) , (Y,V,V^{α}) are two topological spaces and Ω , μ are two associated supra topological space with T , V respectively and

f: $(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ is subjective mapping if f is supra quotient map then it is supra λ-quotient map

Proof:

Let A is supra open set in Y , since f is supra quotient then

$f^{-1}(A)$ is supra open set and then it is supra λ-open set , then f is supra λ-continuous .

Suppose $f^{-1}(A)$ is supra open set, since f is supra quotient map then A is supra open set in Y and then it is supra λ-open set therefor f is supra λ -quotient map.

3.4. Theorem:

let (X,T,T^{α}) , (Y,V,V^{α}) are two topological spaces and Ω , μ are two associated supra topological space with T , V respectively and f: $(X,T,T^{\alpha}) \rightarrow (Y,V,V^{\alpha})$ is subjective mapping then every supra λ quotient then f is supra λ –contiguous

Proof:

Exist by definition.

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