Springer

KSME

Journal of Mechanical Science and Technology 25 (11) (2011) 2827~2834
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-011-0725-0

Mathematical representation of bolted-joint stiffness: A new suggested model †

Nawras Haidar*, Salwan Obeed and Mohamed Jawad
Mechanical Engineering Department, College of Engineering, University of Babylon, Babel, Iraq

(Manuscript Received May 27, 2011; Revised July 8, 2011; Accepted July 16, 2011)

Abstract

Joint member stiffness in a bolted connection directly influences the safety of a design in regard to both static and fatigue loading, as well as in the prevention of separation in the connection. This work provides a new simple model for computing the member stiffness in bolted connections for both fully and partially developed stress envelope fields. The new model is built using a stress distribution polynomial of third order. Finite element analysis (FEA) is performed for some joints geometries, and the results are used to estimate the best analytical envelope angle in the proposed analytical model that gives suitable convergence between the compared results. An experimental effort is exerted to validate the accuracy of a suggested model. When analytical results are compared with FEA results and experimental data, the maximum absolute percentage errors are found to be 2.69 and 14.69, respectively. Also, a good agreement is obtained when the analytical results are compared with other researchers' results.

Keywords: Bolted-joint; Member stiffness; Third order polynomial; FEA

1. Introduction

Due to the large impact in technology, especially for large structures, taking into consideration unit cost and installation, there is an increased need to develop high performance undamaging joint connections. A bolt represents one of the most common methods to connect joints and various parts in structures. Therefore, bolted-joint connections involve a wide range of interest, especially its stiffness. Fig. 1 presents a typical model of a bolted-joint member. Failure of such a joint may cause disastrous failure of the system and can lead to economical and human losses. While joining the members by a bolted joint, a tensile preload is applied to the bolt such that the members are in the state of initial compression. The initial compression in the members will help to keep the members in contact and share a fraction of the external load acting on the joint. Variations in the magnitude of the tensile preload on a bolted joint can produce dramatic differences in the cyclic life of the connection. Accurate predictions of member stiffnesses are essential for determining proper preloads.

When the external load P_e is applied to the bolted joint under initial preload F_i, the resultant force in the bolt is equal to

$$F_b = \left(\frac{k_b}{k_b + k_m}\right) P_e + F_i \tag{1}$$

and that of the connected members is

$$F_m = \left(\frac{k_m}{k_b + k_m}\right) P_e - F_i \tag{2}$$

where the stiffness of the bolt kb is given by

$$k_b = \frac{F_b}{\delta_b} = \frac{A_b E_b}{L} . \tag{3}$$

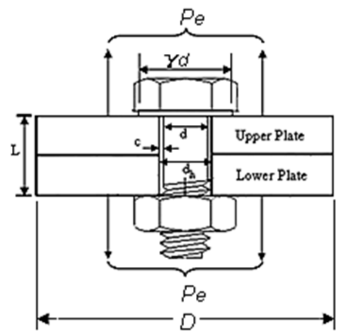


Fig. 1. Typical model of a bolted-joint member.

† This paper was recommended for publication in revised form by Associate Editor Kyeongsik Woo

*Corresponding author. Tel.: +964 7808446567
E-mail address: nawras1980@gmail.com

Similarly, the relation for the member stiffness is given as

$$k_m = \frac{F_m}{\delta_m} = \frac{A_m E_m}{L} . \tag{4}$$

Several authors have proposed both theoretical and experimental techniques to determine the pressure distribution between the members of bolted joints and their corresponding stiffnesses. Some of these works considered, instead of two bolted plates, a thick plate with a symmetric circular or annular pressure loading. Theoretical solutions suggested by Sneddon [1] Fernlund [2], Nelson [3], Greenwood [4], and Lardner [5], etc. Bradley et al. [6] used a three-dimensional photoelastic analysis to guess the interface pressure distribution between the members. Gould and Mikic [7] and Tang and Deng [8] have used finite element analysis (FEA) to find the pressure distribution between the members. They also noted that there was a radius at which flat and smooth members become separated. The computations were performed for models of steel plates with various thicknesses. In their studies, the bolts were replaced by uniformly distributed axisymmetric loads on the connected parts of the bolted joint. Osman et al. [9] discussed a design method for calculating an optimal bolt diameter required for a specific fatigue loading situation. He has suggested that a hollow cylinder whose outside diameter is 1.5 times the bolt diameter be used to determine the area under compression and, thus, the member stiffness. Edwards and McKee [10] and Bickford [11] cited suggestion of the Association of German Engineers to determine the member stiffness using an equivalent cylindrical area dependent on the size of the joint. Ito et al. [12] have used ultrasonic techniques to determine the pressure distribution between the members of bolted joints for various surface topographies, materials, and thicknesses of the members. They suggested the use of a pressure-cone method developed by Rotscher [13] for stiffness calculation with variable cone angles that are generally larger than the cone angles thus far theoretically calculated by other authors.

Rasmussen et al. [14] expressed an equation for estimating the effective area A_m based on their finite element analysis. However, they recommended not using this equation for L/d greater than 5. Their equation has the following form:

$$A_m^* = \frac{\pi}{4}(1 - d_h^{*2}) + 0.5(D_o^{*2} - 1)\tan^{-1}\left\{\frac{0.35\sqrt{L^*} + \sqrt{1+2L^{*2}} - 1}{2(D_o^{*2} - d_h^{*2})}\right\} \tag{5}$$

where

$$D_o^* = D_o/(\gamma d)$$
$$d_h^* = d_h/(\gamma d)$$
$$L^* = L/(\gamma d) \quad .$$
$$A_m^* = A_m/(\gamma d)^2$$

Shigley and Mischke [15] have proposed a simpler method by using a fixed standard cone angle of 30° as the best value of the joint material stiffness, such as

$$k_m = \frac{0.577\pi E_m d}{2\ln\left[(5)\dfrac{(0.577L + 0.5d)}{(0.577L + 2.5d)}\right]} . \tag{6}$$

Wileman et al. [16] performed axisymmetric finite element analysis (FEA) and proposed an exponential expression for member stiffness. In the analysis, the washer diameter was assumed to be 1.5 times the diameter of the bolt. They considered the displacement of the uppermost node located on the center line of the washer for stiffness calculation. Finite element analysis for various aspect ratios (d/L) was carried out and finally an exponential relation for member stiffness evaluation was proposed. Their equation has the following form:

$$k_m = dE_m A\exp(bd/L) . \tag{7}$$

The coefficient (A) and the exponent (b) in the above equation will vary slightly with Poisson's ratio of the material. Wileman et al. [16] caution that the use of their equation should be limited to cases with similar geometry and boundary conditions. The distance from the bolt axis to the edge of clamped members should be at least several times the bolt diameter to avoid the presence of edge effects.

Lehnhoff et al. [17] proposed an analytical model to calculate the member stiffness and the stress distribution of bolted joints with various bolt sizes. They assumed a uniform pressure with conical envelope under the bolt head. Their work provides the following equation:

$$k_m = \frac{\pi d E_m \tan(\alpha)}{2\ln\left[\dfrac{(L\tan(\alpha) + \gamma d - d)(\gamma d + d)}{(L\tan(\alpha) + \gamma d + d)(\gamma d - d)}\right]} . \tag{8}$$

Juvinall and Marshek [18] provided an equation for estimating the effective area of the clamped member, which is given by

$$A_m = \frac{\pi}{4}\left[\left(\frac{d_3 + d_2}{2}\right)^2 - d_1^2\right] \tag{9}$$

where $d_1 \approx d$, $d_2 = 1.5d$, *and* $d_3 = 1.5d + L\tan(30)$.

Pedersen and Pedersen [19], performed contact finite element analysis to evaluate the member stiffness bases on elastic energy in the structure. They found the following relations for member stiffness:

for $\gamma d < D < \gamma d + L$

$$k_m = \frac{k_o - k_{\max}}{\exp\left(\dfrac{\pi E_m \gamma d(D - \gamma d)}{2L(k_{\max} - k_o)}\right)} + k_{\max} , \tag{10a}$$

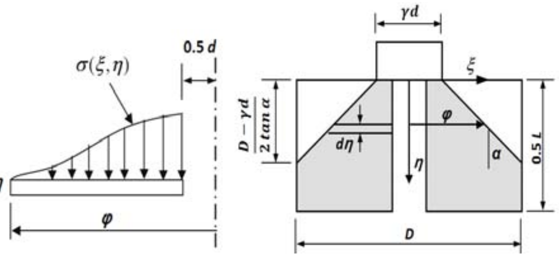


Fig. 2. Stress distribution on element of the bolted joint.

for $D \geq \gamma d + L$

$$k_m = k_{\max}, \tag{10b}$$

for $D \leq \gamma d$

$$k_m = \frac{\pi E_m}{4L}(D^2 - d^2) \tag{10c}$$

where

$$k_o = \frac{\pi E_m d^2}{4L}(\gamma^2 - 1)$$

$$k_{\max} = E_m d \left\{ 0.59(\gamma^2 - 1)\frac{d}{L} + 0.2(\gamma + 1) \right\}.$$

Most of the methods presented in the literature review take different forms. This difference is mainly due to the assumptions made during the model development. The differences caused by various assumptions need higher safety factors for reliable design. The present study proposes a new analytical model for calculating the bolted-joint stiffness. Finite element results will be used to find the appropriate envelope cone angle that gives the best fitness of analytical results. Also, an experiment is performed for a few bolt-joint geometry cases to provide an assurance for the suggested model.

2. Method of approach

The stress distribution within the material under the bolt has a complex geometry. This problem has been studied by a number of investigators and an accurate computation of the distribution of the stresses volume is quite complicated. The compressive stress in the material is highest directly under the bolt and falls off as laterally from the bolt centerline as shown in Fig. 2. At some lateral distance from this centerline, the compressive stress at the joint interface goes to zero, and beyond that point the joint tends to separate since it cannot sustain a tensile stress.

The first step for building the analysis is to guess the pressure distribution through the member as a cone with an envelope angle (α). The third order polynomial in the ξ -direction is assumed for stress distribution as shown in Fig. 2. Thus,

$$\sigma(\xi,\eta) = \bar{A}\xi^3 + \bar{B}\xi^2 + \bar{C}\xi + \bar{D} \tag{11a}$$

where the constants $\bar{A}, \bar{B}, \bar{C}$, and $\bar{D}$ are functions of η. These constants will be determined by the application of the boundary conditions and static equilibrium as follows:

at $\xi = \varphi$, setting $\sigma = \partial\sigma/\partial\xi = 0$ gives the following two equations:

$$\bar{A}\varphi^3 + \bar{B}\varphi^2 + \bar{C}\varphi + \bar{D} = 0 \tag{11b}$$

$$3\bar{A}\varphi^2 + 2\bar{B}\varphi + \bar{C} = 0. \tag{11c}$$

At $\xi = 0.5d$, setting $\partial\sigma/\partial\xi = 0$, gives:

$$\frac{3}{4}\bar{A}d^2 + \bar{B}d + \bar{C} = 0 \tag{11d}$$

and we have

$$P_e = \iint \sigma \, dA = 2\pi \int_{0.5d}^{\varphi} \sigma\xi \, d\xi$$

$$= 2\pi \left\{ \frac{\bar{A}}{5}\left(\varphi^5 - \frac{d^5}{32}\right) + \frac{\bar{B}}{4}\left(\varphi^4 - \frac{d^4}{16}\right) + \frac{\bar{C}}{3}\left(\varphi^3 - \frac{d^3}{8}\right) + \frac{\bar{D}}{2}\left(\varphi^2 - \frac{d^2}{4}\right) \right\}. \tag{11e}$$

Solving Eqs. (11a)-(11e) for the constants $\bar{A}, \bar{B}, \bar{C}$, and $\bar{D}$ will give the following:

$$\bar{A} = 4\beta$$

$$\bar{B} = -3\beta(2\varphi + d)$$

$$\bar{C} = 6\beta d$$

$$\bar{D} = \beta(2\varphi - 3d)\varphi^2$$

where

$$\beta = \frac{160P_e}{\pi(96\varphi^5 - 80d\varphi^4 - 80d^2\varphi^3 + 120d^3\varphi^2 - 50d^4\varphi + 7d^5)}.$$

These constants are substituted in Eq. (11a) to find the stress distribution inside the member which caused by the external load P_e.

2.1 Elastic joint deflection

In elastic range, the strain of member material is obtained by using Hook's law. The average joint deflection represents the average change in disk thickness, taking into account the pressure distribution that is represented by Eq. (11a). Thus, it is easy to write the following equation:

$$d\delta_{average} = \int_{0.5d}^{\varphi} \left(\frac{\delta}{\varphi - 0.5d}\right) d\xi = \left(\frac{1}{\varphi - 0.5d}\right) \int_{0.5d}^{\varphi} \left(\frac{\sigma}{E_m}\right) d\xi \quad d\eta. \tag{12}$$

This leads to

$$\delta_{average} = \int_{0}^{0.5L} (\frac{1}{\varphi - 0.5d}) \int_{0.5d}^{\varphi} (\frac{\sigma}{E_m}) d\xi \quad d\eta \tag{13}$$

and by taking the advantage of symmetry about $\eta = 0.5L$, which give us

$$\delta_{overall} = 2\delta_{average} . \tag{14}$$

This will lead to the following joint deflection

$$\delta_{overall} = \frac{2P_e}{\pi E_m \tan(\alpha)} \left\{ \frac{1}{d} \ln\left[\frac{(3\gamma+7)(D-d)}{(3D+7d)(\gamma-1)} \right] + \left[\frac{10(L\tan(\alpha)-D+\gamma d)}{(3D+7d)(D-d)} \right] \right\} \tag{15}$$

for $\gamma d < D < (L\tan(\alpha)+\gamma d)$,
and

$$\delta_{overall} = \frac{2P_e}{\pi E_m \tan(\alpha)} \left\{ \frac{1}{d} \ln\left[\frac{(3\gamma+7)(L\tan(\alpha)+\gamma d - d)}{(\gamma-1)(3L\tan(\alpha)+3\gamma d+7d)} \right] \right\} \tag{16}$$

for $D \geq (L\tan(\alpha)+\gamma d)$.

If the two plates are made of different thicknesses or different materials, the deflection of each plate is given as follows:

$$\delta_j = \frac{2P_e}{\pi E_m \tan(\alpha)} \left\{ \frac{1}{d} \ln\left[\frac{(3\gamma+7)(D-d)}{(3D+7d)(\gamma-1)} \right] + \left[\frac{10(2t_j\tan(\alpha)-D+\gamma d)}{(3D+7d)(D-d)} \right] \right\} \tag{17}$$

for $\gamma d < D < [(t_1+t_2)\tan(\alpha)+\gamma d)]$,

$$\delta_j = \frac{P_e}{\pi E_j \tan(\alpha)} \left\{ \frac{1}{d} \ln\left[\frac{(3\gamma+7)(2t_j\tan(\alpha)+\gamma d - d)}{(\gamma-1)(6t_j\tan(\alpha)+3\gamma d+7d)} \right] \right\} \tag{18}$$

for $D \geq [(t_1+t_2)\tan(\alpha)+\gamma d]$,
and

$$\delta_{overall} = \sum_{j=1}^{2} \delta_j \tag{19}$$

where j=1, 2 for upper and lower plates, and (t) is the thickness of the plate joint.

It is important to state that Eq. (15) is used when the stress envelope is partially developed through the joint thickness. This will be happen because the joint size D is not large enough to allow the stress envelope to be fully developed while Eq. (16) satisfies the condition to create the fully stresses envelope.

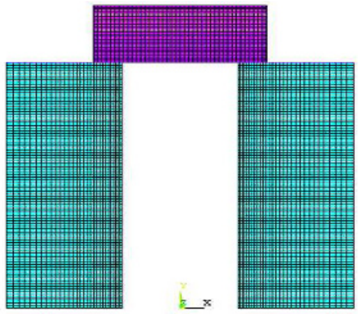


Fig. 3. The meshed finite element of bolted-joint model.

2.2 Member stiffness

The member stiffness k_m is found from the following linear relation as

$$k_m = \frac{P_e}{\delta_{overall}} = \frac{0.5\pi E_m \tan(\alpha)}{\left\{ \frac{1}{d} \ln\left[\frac{(3\gamma+7)(D-d)}{(3D+7d)(\gamma-1)} \right] + \left[\frac{10(L\tan(\alpha)-D+\gamma d)}{(3D+7d)(D-d)} \right] \right\}} \tag{20}$$

for $\gamma d < D < (L\tan(\alpha)+\gamma d)$,
and

$$k_m = \frac{P_e}{\delta_{overall}} = \frac{0.5\pi d E_m \tan(\alpha)}{\ln\left[\frac{(3\gamma+7)(L\tan(\alpha)+\gamma d - d)}{(\gamma-1)(3L\tan(\alpha)+3\gamma d+7d)} \right]} \tag{21}$$

for $D \geq (L\tan(\alpha)+\gamma d)$, while the individual member stiffness for different thicknesses or different materials is equal to

$$(k_m)_j = \frac{P_e}{\delta_j} . \tag{22}$$

The overall member stiffness is represented by linear springs connected in series manner, which leads to

$$k_m = \frac{\prod_{j=1}^{2} (k_m)_j}{\sum_{j=1}^{2} (k_m)_j} . \tag{23}$$

3. Finite element model

A finite element model is created for a bolt diameter d at varying grip lengths L. Fig. 3 shows the finite element mesh used to represent the general joint geometry of Fig. 1. Since only the stiffness of the members is to be considered, the shank of the bolt has been removed from the model. The finite element analysis method is used to calculate the member de-

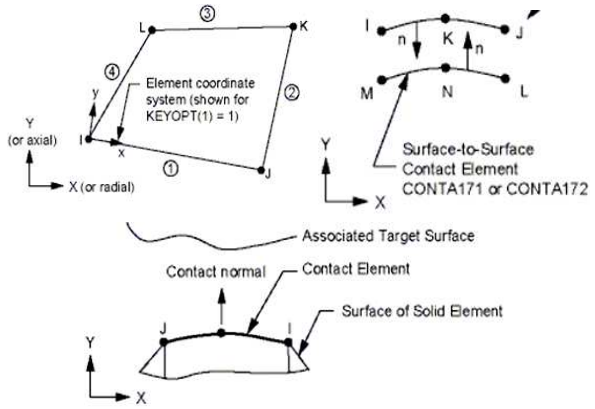


Fig. 4. Element characteristics (ANSYS [20]).

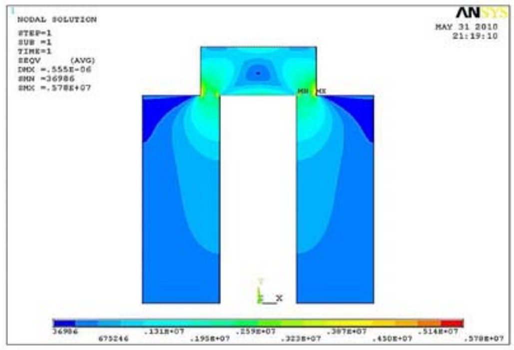
flection in the η-direction, which is taken as the average nodal displacement under the bolt head.

The stiffness of the bolt head is set to be about three orders of magnitude greater than the member material, so the bolt head would not deflect a significant amount relative to the member material stiffness. This will lead to the use of a steel bolt with an aluminum member to give the best fit with the required accuracy. The member and bolt materials are assumed to be isotropic, homogenous, and linearly elastic for all the analyses. Commercial finite element software ANSYS® [20] is used for modeling and analysis. The model geometry is meshed by four-noded axisymmetric quadrilateral elements (PLAN42). Contact and target elements (TARGE169 and CONTA172), shown in Fig. 4, with a coefficient of static friction equal to 0.2 are used to model the contact that occurs between the bottom face of the bolt head and the member.

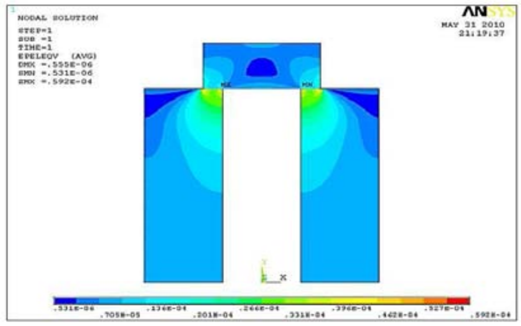
Uniform pressure distribution is applied to the top of the bolt head. Thus, stiffness of the member k_m is calculated using the following simple relation:

$$k_m = \frac{P_e}{\delta_{average}} . \tag{24}$$

Convergence study is carried out on the initial finite element model by decreasing the element size near the bolt. The number of elements used in the converged analysis ranges from 3000 to 5400. The smallest element size was 0.3 mm by 0.3 mm and there was no significant improvement in accuracy by using smaller elements. Fig. 5 shows some results of using ANSYS software for equivalent elastic stress and strain fields.



(a)



(b)

Fig. 5. Finite element results: (a) equivalent stress; (b) equivalent strain.

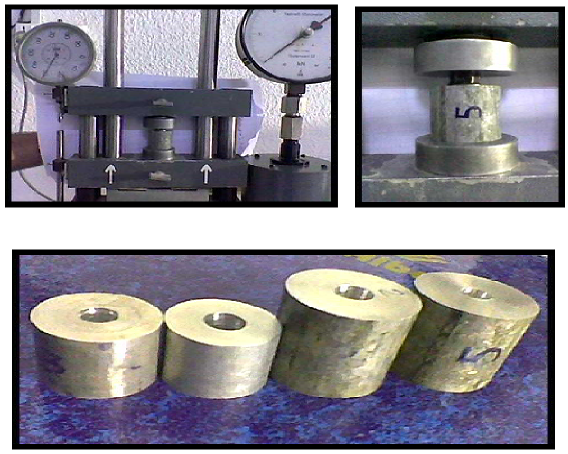


Fig. 6. Cylindrical specimen with different sizes and grip lengths under test.

4. Experimental test

The universal test machine (UTM), shown in Fig. 6, is used to measure the load-deflection data of the bolted joint member, where the slope of load-deflection curve represents the joint stiffness. The tested samples are made from standard aluminum material. Four different joint sizes are used in the test. The maximum applied compressive load is equal to 15 kN. For trustworthy test data, ten tests were performed for each specimen size and the average data for each case is considered.

5. Results and discussion

As the ultimate goal of the study is a simple general expression to be used in routine design problems, it is necessary to generalize the data obtained from the specific models for

Table 1. Analytical versus experiment and FEA results of member stiffness in (MN/m).

Geometry				Member stiffness (MN/m)				
d (mm)	*D* (mm)	*L* (mm)	α (deg.)	Present study	FEA	Exp.	$Erorr_{FEA}$ %	$Erorr_{Exp}$ %
8	25	20	36	556	554	590	-0.36	5.76
8	25	30	36	453	445	485	-1.79	6.59
8	25	40	36	383	380	443	-0.79	13.54
8	25	50	36	331	325	388	-1.84	14.69
12	30	20	36	995	991	927	-0.40	-7.33
12	30	30	36	772	766	811	-0.78	4.81
12	30	40	36	630	618	666	-1.94	5.40
12	30	50	36	533	519	511	-2.69	-4.30

which the new member stiffness expression and finite element solutions were performed.

Table 1 presents the member stiffness for different values of joint size, bolt diameter, and grip length. After getting the member stiffness by performing finite element analysis, the best envelope angle decision is nearly equal to 36° which gives the minimum compared percentage error. The maximum experiment percentage error is about 14.69. This error value is practically acceptable, if we consider the difficulty of controlling the contact frictions between the bolt head and member part and also between the joint member assembly and test machine jaws.

Table 2 lists the results of the present study beside other researchers' formulas. From this table, it is very clear that convergence in the results between the present study and Shigley's formula is inspiring in cases of fully developed pressure envelopes. While for partially developed cases, the studies that have been done by Pederson [19] and by Rasmussen [14] give the best results fitness. In standard bolts, increasing bolt diameter leads to larger bolt heads which in turn increases the contact area under the head, i.e. increasing the member stiffness. Furthermore, the member deflection increases with increasing the member grip length under application of constant external load. Thus, any increase in grip length leads to decrease in member stiffness. An alternative way to present and compare the present work with other researchers is executed in Figs. 7 and 8 for cases of fully and partially developed pressure envelopes respectively. In Fig. 7, the ratio of grip length (*L*) to bolt diameter (*d*) is held constant, which is numerically equal to 4. This substitution will make the denominator part (the nonlinear part) in Eq. (21) constant. Thus, the curves are linearized to give the best results comparison. In Fig. 8, the same procedure has been performed for a partially developed case with holding the ratios (*L/d*) and (*D/d*), in Eq. (20), equal to 4 and 2, respectively. It is important to indicate that Shigley's, Juvinall's, and Wileman's studies did not take into account the joint size (*D*) in their formulas, i.e. their studies are restricted to fully developed cases and disregard partially developed ones.

Table 3 lists the member stiffnesses when different plate materials and thickness ratios (t_1 and t_2) are used. Good agreement is obtained when the present study results are compared with FEA and Lehnhoff's results.

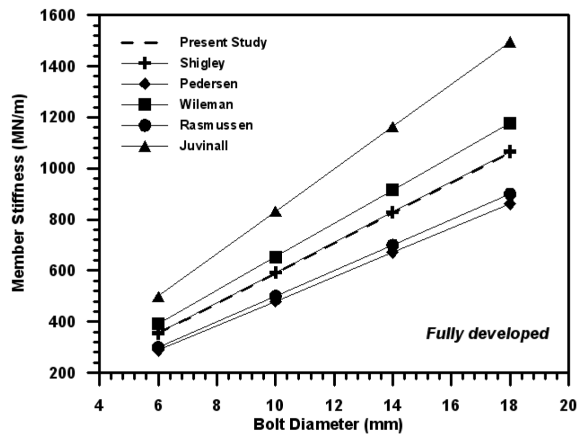


Fig. 7. Comparison of present study's member stiffness with other studies for fully developed case.

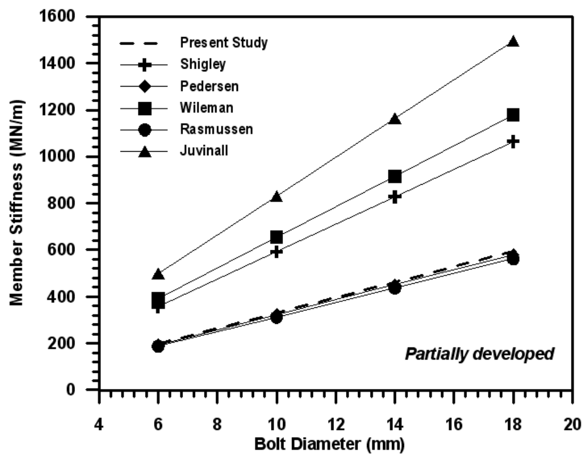


Fig. 8. Comparison of present study's member stiffness with other studies for partially developed case.

Table 2. Present study versus other researchers' results of member stiffness in (MN/m).

d (mm)	*L/d*	*D/d*	Present study*	FEA	Shigley	Pedersen	Wileman	Rasmussen	Juvinall
6	2	2	**367.66**	365.37	466.04	331.48	460.38	302.32	546.81
6	2	4	460.38	458.15	466.04	364.87	460.38	372.02	546.81
6	2	6	460.38	458.03	466.04	364.87	460.38	381.27	546.81
6	4	2	**197.95**	196.28	355.22	193.19	392.49	187.24	498.71
6	4	4	**352.20**	350.43	355.22	281.00	392.49	287.03	498.71
6	4	6	353.88	351.26	355.22	287.43	392.49	300.07	498.71
6	6	2	**135.43**	135.44	317.01	135.82	372.16	137.03	519.32
6	6	4	**296.63**	292.72	317.01	238.87	372.16	255.29	519.32
6	6	6	316.48	314.36	317.01	257.51	372.16	273.37	519.32
10	2	2	**612.78**	609.31	776.73	552.47	767.30	503.87	911.36
10	2	4	767.30	765.03	776.73	608.12	767.30	620.04	911.36
10	2	6	767.30	764.66	776.73	608.12	767.30	635.45	911.36
10	4	2	**329.91**	324.46	592.03	321.99	654.15	312.06	831.18
10	4	4	**587.00**	583.07	592.03	468.33	654.15	478.38	831.18
10	4	6	589.80	585.61	592.03	479.06	654.15	500.12	831.18
10	6	2	**225.72**	222.34	528.36	226.37	620.27	228.39	865.54
10	6	4	**494.39**	490.33	528.36	398.12	620.27	425.48	865.54
10	6	6	527.47	525.53	528.36	429.18	620.27	455.62	865.54
14	2	2	**857.89**	851.80	1087.4	773.46	1074.2	705.41	1275.9
14	2	4	1074.20	1072.0	1087.4	851.37	1074.2	868.06	1275.9
14	2	6	1074.20	1071.7	1087.4	851.37	1074.2	889.63	1275.9
14	4	2	**461.88**	458.05	828.85	450.78	915.81	436.89	1163.7
14	4	4	**821.81**	818.71	828.85	655.67	915.81	669.74	1163.7
14	4	6	825.72	823.48	828.85	670.68	915.81	700.16	1163.7
14	6	2	**316.01**	314.34	739.70	316.92	868.38	319.75	1211.8
14	6	4	**692.15**	689.02	739.70	557.36	868.38	595.68	1211.8
14	6	6	738.46	736.27	739.70	600.85	868.38	637.87	1211.8
18	2	2	**1103.03**	1098.6	1398.1	994.45	1381.1	906.96	1640.5
18	2	4	1381.10	1378.3	1398.1	1094.6	1381.1	1116.1	1640.5
18	2	6	1381.10	1377.9	1398.1	1094.6	1381.1	1143.8	1640.5
18	4	2	**593.85**	590.12	1065.7	579.58	1177.5	561.72	1496.1
18	4	4	**1056.60**	1054.4	1065.7	843.00	1177.5	861.09	1496.1
18	4	6	1061.60	1060.0	1065.7	862.31	1177.5	900.21	1496.1
18	6	2	**406.29**	404.42	951.04	407.47	1116.5	411.11	1558.0
18	6	4	**889.91**	885.33	951.04	716.61	1116.5	765.87	1558.0
18	6	6	949.44	947.18	951.04	772.53	1116.5	820.11	1558.0

* The underlined bolded numbers in this column means that a cone pressure envelope is partially developed else that it is fully developed.

Table 3. Comparison of member stiffness in (MN/m) for different material plates.

d (mm)	*D* (mm)	t_1/t_2	Aluminum /Steel		
			Present study	FEA	Lehnhoff
8	50	12/20	733	712	868
		16/20	683	675	801
12	50	12/20	1283	1271	1297
		16/20	1175	1160	1185
16	50	12/20	1947	1944	1977
		16/20	1763	1755	1784

6. Conclusions

The new suggested member stiffness model is a simplified expression that gives best guessed results for both fully and partially developed pressure envelope fields. The proposed third order polynomial of pressure field gives member stiffness results that fit very well with FEA and existing researchers' results. Also, an experiment for a few bolted joint geometries is performed. The validated data and the verified results indicated that the presently suggested bolted-joint stiffness model is reliable.

Nomenclature

D : Joint diameter
d : Bolt diameter
$d\delta$: Differential element of joint deflection
E_m : Young's modulus of elasticity
k_m : Member (Joint) stiffness
L : Member grip length
P_e : External applied load
t : Plate thickness
α : Envelope pressure angle
η : Coordinate in axial direction of bolted joint
ξ : Coordinate in radial direction of bolted joint
γ : Contact radii ratio
φ : Distance from bolt axis to the farthest surface of stress envelope field
σ : Stress
δ : Joint deflection

References

[1] I. N. Snedden, The elastic stresses produced in a thick plate by the application of pressure to its free surfaces, *Proc. of the Cambridge Philosophical Society*, Cambridge, England, 42 (1946) 260-271.
[2] I. Femlund, A method to calculate the pressure between bolted or riveted plates, *Transaction of Charmers University of Technology*, Gothenburg, Sweden, 245 (1961).
[3] C. W. Nelson, Further consideration of the thick-plate problem with axially symmetric loading, *ASME J. Appl. Mech.*, 29(1962) 91-98.
[4] J. A. Greenwood, The elastic stresses produced in the mid-plane of a slab by pressure applied symmetrically at its surface, *Proc. of the Cambridge Philosophical Society*, Cambridge, England, 60 (1964) 159-169.
[5] T. J. Lardner, Stresses in a thick plate with axially symmetric loading, *J. Appl. Mech.*, 32 (1965) 458-459.
[6] T. L. Bradley, T. J. Lardner and B. B. Mikic, Bolted joint interface pressure for thermal contact resistance, *ASME J. Appl. Mech.*, 38 (2) (1971) 542-545.
[7] H. H. Gould and B. B. Mikic, Areas of contact and pressure distribution in bolted joints, *Technical report, Massachusetts Institute of Technology, Heat Transfer Laboratory*, Cambridge, no. 68 (1970).
[8] T. Jinsong and D. Zhaoyi, Better stress and stiffness estimates for bolted joints, *J. Mach. Des.*, November 24 (1988).
[9] M. O. Osman, W. M. Mansour and R. V. Dukkipati, On the design of bolted connections with gaskets subjected to fatigue loading, *Design Engineering Technical Conference*, Montreal, Canada (76-DET-57) (1976) 26-29.
[10] K. J. Edwards and R. B. McKee, *Fundamentals of Mechanical Component Design*, McGraw-Hill, First Ed., New York, USA (1991).
[11] J. H. Bickford, *An Introduction to the Design and Behavior of Bolted Joints*, Second Ed., Dekker, New York, USA (1990).
[12] Y. Ito, J. Toyota and S. Nagata, Interface pressure distribution in a bolt-flange assembly, *ASME J. Mech. Des.*, 101 (2) (1977) 330-337.
[13] F. Rotscher, *Die Maschineelemente* (*The machine elements*), Springer-Verlage, Berlin, Germany (1927).
[14] J. Rasmussen, I. B. Norgaard, O. Haastrup and J. Haastrup, Bolted joints. a two body contact problem with friction, *Euromech Colloquium* (110) (1978) 115-120.
[15] J. E. Shigley and C. R. Mischke, *Mechanical Engineering Design*, Eighth Ed., McGraw-Hill, New York, USA (2006).
[16] J. Wileman, M. Choudhury and I. Green, Computation of member stiffness in bolted connections, *ASME J. Mech. Des.*, 113 (4) (1991) 432-437.
[17] T. F. Lehnhoff, K. I. Kwang and M. L. McKay, Member stiffness and contact pressure distribution of bolted joints, *ASME J. Mech. Des.*, 116 (1994) 550-557.
[18] R. C. Juvinall and Kurt M. M., *Fundamentals of Machine Component Design*, Fourth Ed., John Wiley & Sons Pte Ltd, *Hoboken,* New Jersey, USA (2006).
[19] N. L. Pedersen and P. Pedersen, On prestress stiffness analysis of bolt plate contact assemblies, *Arch. Appl. Mech.*, 78 (2008) 75-88.
[20] ANSYS®, *Engineering Analysis System Theoretical Manual*, "http:\\ www.Ansys.Com", Ansys V9 (2004).

Nawras Haidar was born in Basra, Iraq in 1980. He completed his Bachelor's degree in Mechanical Engineering in 2002 from the University of Babylon, and received his Master's degree from the same university in 2005. He is interested in studying linear and nonlinear vibration of mechanical systems. Now, he is teaching in Mechanical Engineering Department, College of Engineering, University of Babylon.

Salwan Obeed was born in Babel, Iraq in 1979. He completed his Bachelor's degree in Mechanical Engineering in 2002 from the University of Babylon, and received his Master's degree from the same university in 2006. He is interested in structural analysis, finite element analysis, and dynamic analysis of structures. Now, he is teaching in Mechanical Engineering Department, College of Engineering, University of Babylon.