Performance Comparisons of Improved Regular Repeat Accumulate (RA) and Irregular Repeat Accumulate (IRA) Turbo Decoding

Ahmed Abdulkadhim Hamad **Nora Mohammad Hashim**
Department of Electrical Engineering, College of Engineering, University of Babylon
ahmedbabel@yahoo.com nora12012@yahoo.com

Abstract

In this paper, different techniques are used to improve the turbo decoding of regular repeat accumulate (RA) and irregular repeat accumulate (IRA) codes. The adaptive scaling of a-posteriori information produced by Soft-output Viterbi decoder (SOVA) is proposed. The encoded pilots are another scheme that applied for short length RA codes. This work also suggests a simple and a fast method to generate a random interleaver having a free 4 cycle Tanner graph. Progressive edge growth algorithm (PEG) is also studied and simulated to create the Tanner graphs which have a great girth.

Keywords: Low-Density Parity Check (LDPC) Code, Regular and Irregular Repeat Accumulate (RA &IRA) codes.

الخلاصة

في هذه الورقة تم استخدام تقنيات مختلفة من اجل تحسين معيد الترميز من نوع تيربو لمرمزات تراكم التكرار المنتظمة (RA) وغير المنتظمة (IRA). حيث تم تطبيق التحجيم المكيف على المعلومات الناتجة من استخدام خوارزمية معيد الترميز من نوع فيتربي (*SOVA*). كما تم تطبيق رموز الارشاد المرمزة في مرمزات تراكم التكرار ذات الاطوال القصيرة حيث ان رموز الارشاد يمكن ان تلعب دور مزدوج في: أولا، تخمين القناة وهو الواجب الاساسي لها في انظمة الاتصال والثاني، هو تحسين خواص المسافة وتوزيع الاوزان للمرمز. وقد تم في هذا العمل ايضا اقتراح طريقة بسيطة وسريعة لتوليد مبعثر (interleaver) عشوائي خالي من الدورات القصيرة ذات الاربع خطوات في رسم تانر التي تسبب تدهور الاداء. كذلك تم استخدام خوارزمية نمو الحافة التدريجي (PEG) لبناء رسوم تانر البيانية ذات المسارات الكبيرة.

الكلمات المفتاحية: كود منخفض الكثافة ، مرمزات تراكم التكرار المنتظمة (RA) وغير المنتظمة (IRA).

I. Introduction

Turbo and Low Density Parity Check codes (LDPC) are considered two error correcting codes with a performance approaching to Shannon Limit. LDPC codes are like turbo codes but the benefit of using the LDPC code is that the LDPC decoding shows a lower complexity as compared with the turbo decoding. The performance of long irregular LDPC code is close to the Shannon limits but at the same time the drawback of LDPC is that the encoding complexity is quadratic with code lengt (Gallager, 1963).

D. Divsalar suggested "Turbo like code" In 1998, also called Repeat Accumulate Codes (RA) (Divsalar *et.al.*, 1998). Actually, it is a type of " Serial Concatenated Convolutional Codes (SCCC) ". The power of RA codes is similar to that of turbo codes in that it can be encoded by using two simple codes in serial connection and by using sum-product decoding Tanner graph for decoding, as for LDPC codes. This provides two benefits the first is the decoding power and parallelization of LDPC codes and second is the low complexity of turbo codes encoding. Later Hui Jin suggested Irregular Repeat Accumulate (IRA) codes as a type of LDPC codes (Jin *et.al.*, 2000) .

This paper investigates the turbo decoding for Repeat Accumulate code and suggests scaling factors to improve their performance. Also finds the optimum positions for encoded pilots to enhance the code performance. Various computer simulation tests are carried out to reveal the effect of scaling factors, interleaver type, decoding and algorithms.

The rest of this paper is organized as follow: Section II presents the basics of RA and IRA codes. Section III defines the enhanced Turbo decoding for RA and IRA codes. Section IV includes simulation result and discussion. Finally, Section V concludes this paper.

II. Repeat Accumulates Code (RA & IRA)

Repeat accumulates RA codes are considered a class of Serial Concatenated Convolutional Codes. they consist of outer and inner codes linked by an interleaver. The outer code is a repetition code with a rate $1/\mathrm{q}$ whereas the rate of the inner code is 1 convolutional code $\frac{1}{1+\mathrm{D}}$. The outputs of inner code is modulo 2 sum of previous output bit and the current input bit which means that it offers the accumulation for all previous inputs. Moreover the RA code can be constructed with irregular distribution of the degree by the use of irregular repetition codes (Jin *et.al*., 2000).

IRA codes represent a kind of LDPC codes, where an accumulator agrees to weight two columns in parity check matrix (H) while the remaining columns in the parity-check matrix are determined by the interleaver.

With length (N) and rate (R) of IRA, the codes start with a repetition code with a rate $1/q$, and a distribution $[q_1, \dots, q_K]$, which copies the i message bit from length $K = R \times N$ messages q_i times where:

$$q = \frac{1}{K}\sum_{i=1}^{K} q_i \quad (1)$$

The length $\mathrm{n} = \mathrm{qK}$ interleaver, $\Pi = [\pi_1, \pi_2, \dots, \pi_n]$ then the bits $g = [g_1, g_2, \dots, g_n]$ from repetition code permutes to give

$$L = [L_1, L_2, \dots, L_n] = [g_{\pi 1}, g_{\pi 2}, \dots, g_{\pi n}] \quad (2)$$

Then the output bits of interleaver are combined by using modulo 2, in M groups with a_i bits in the i set to provide outputs with length M, $\boldsymbol{v} = [v_1, \dots, v_M]$ where

$$a = \frac{1}{\mathrm{M}}\sum_{\mathrm{i}=1}^{\mathrm{M}} a_{\mathrm{i}} \quad (3)$$

At last, the $M = Kq/a$ parity bits, $\boldsymbol{e} = [e1, \dots, eM]$ in the output of accumulator and it can be defined as:

$$e_i = e_i - 1 \oplus v_i, \quad i = 2, \dots, M \quad (4)$$

Where $e_1 = v_1$, and $\oplus$ denotes modulo 2 addition.

Both message and parity bits are sent to the recipient, and the output codeword becomes as it is shown below:

$$\boldsymbol{c} = [m_1, m_2, \dots, m_k, g_1, g_2, \dots, g_M] \quad (5)$$

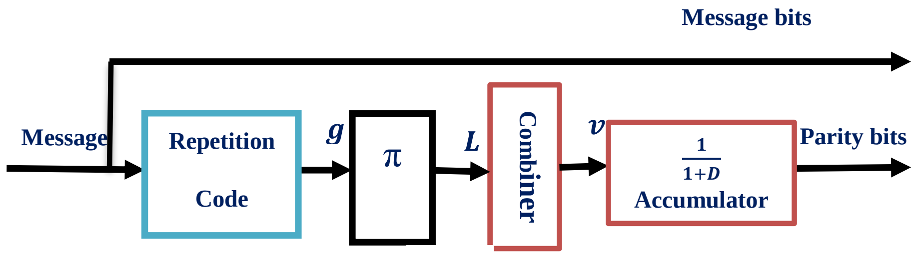
To provide a length ($\mathrm{N} = \mathrm{K}(1 + \mathrm{q}/\mathrm{a})$) and a rate ($R = \frac{a}{a+q}$) code.

H matrix of systematic IRA consists of two parts :

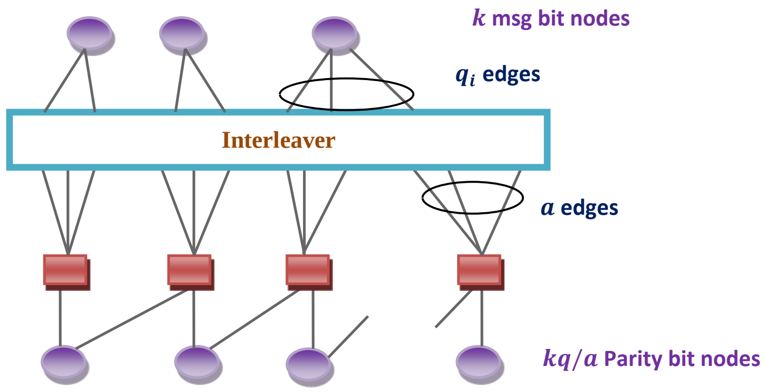
$$\mathrm{H} = [\mathrm{H1}, \mathrm{H2}] \quad (6)$$

Where $H1$ is $M \times K$ matrix, column weights $[q_1, \dots, q_k]$ and row weights $[a_1, \dots, a_M]$ whereas the position of the non-zero entries is stated by interleaver. $H2$ is $M \times M$ matrix which is defined by (7) :

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 \end{bmatrix} \quad (7)$$



(A)



(B)

Figure 1: Systematic (IRA) code observed as turbo code, in figure (A), and as LDPC with Tanner graph shown in figure (B). In the Tanner graph, the filled circles denote the bit nodes, whereas the open squares denote the check nodes.

The Tanner graph of IRA codes is defined by H matrix and it consists from the parity check nodes with length (M) and $(K + M)$ bit nodes.

In this paper we will make the value of a fixed and a change in the value of q depending on the degree distributions in(Sarah *et.al.*, 2005). The variable degree distribution of a code can be represented by:

$$\lambda(x) = \lambda_2 x + .. + \lambda_i\, x^{i-1} + \cdots + \lambda_{q_{max}} x^{q_{max}} - 1, \quad (8)$$

Where λ_i represents a fraction of Tanner graph edges originated from a degree i bit nodes. The row degrees are indicated by $\rho(x)$. The length is N and the rate is R. The IRA code with degree distribution $\lambda(x)$ have:

$$"N \frac{\lambda_i / i}{\sum_j \lambda_j / j}" \quad (9)$$

Message nodes of degree $- i$ for $i > 2$, and

$$"N \frac{\lambda_2 / 2}{\sum_j \lambda_j / j} - (1 - R)N" \quad (10)$$

Message nodes of degree 2.

The best column weight-distribution of IRA codes for a given rate is specified by using density-evolution (Richardson *et.al.*, 2001).

Table I provides degree distributions for rate $\frac{1}{4}$, rate $\frac{1}{2}$ and rate $\frac{3}{4}$ codes on the *(additive white Gaussian noise AWGN channel)* (Sarah *et.al.*, 2005).

Table I

Degree Distributions for Designated Code Rates over *(AWGN)*

	Rate $\frac{1}{4}$ IRA	Rate $\frac{1}{2}$ IRA	Rate $\frac{3}{4}$ IRA
λ_2	0.5001	0.3330	0.2470
λ_3	0.0700	0.3851	0.4321
λ_4	0.0253	0.0002	0.0878
λ_5	0.0163		
λ_6	0.1728		
λ_7		0.1392	
λ_8		0.1425	
λ_{10}	0.2155		0.2331

III Enhanced Turbo Decoding for RA and IRA

A- Scaling Factor Effect

The low performance when used SOVA decoding goes back to the relatively high correlation between the intrinsic and extrinsic information (Papke *et.al.*, 1996). As a result, different methods were proposed to improve SOVA performance. Scaling factors is used to alleviate the distortion of the extrinsic information produced by SOVA (Huang *et.al.*, 2004;Ahmed, 2012).

Scaling factor of the extrinsic information can be used to improve the performance of turbo decoding of RA and IRA codes. Scaling factors are used to alleviate the upbeat nature to the posteriori information and also to reduce the correlation among extrinsic and intrinsic information created by the decoders that utilize suboptimal algorithms like SOVA and MS.

By analogy to the scaling algorithm suggested by (Ahmed,2013) for MS decoding, Figure (2) presents a scaling scheme that can be applied to turbo decoding of both RA and IRA codes.

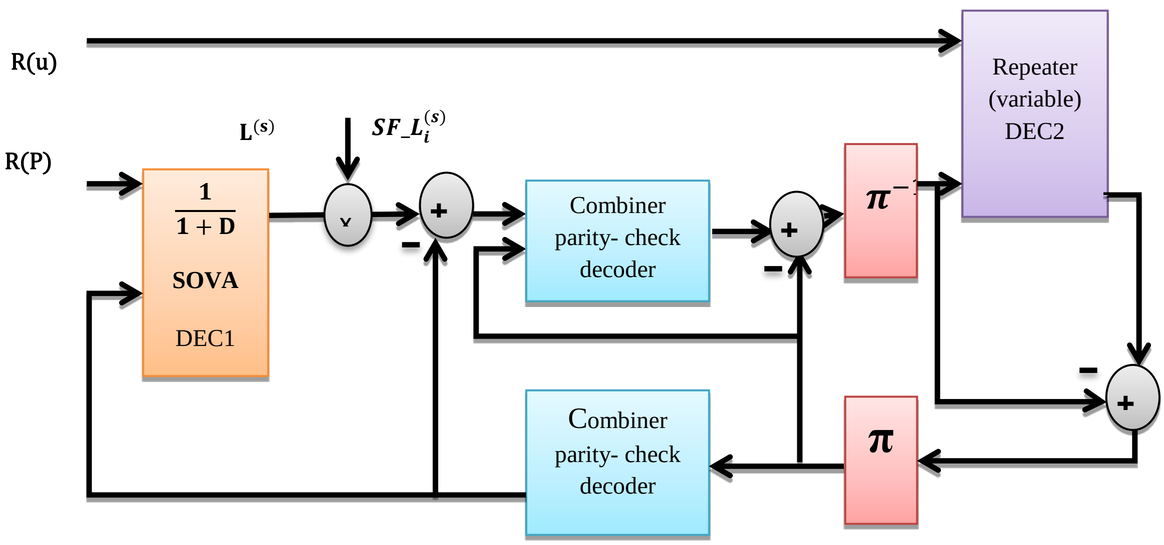


Figure 2 Presentation of Modified RA Turbo Decoding.

The average of scaling factor over all transmitted frames is used to improve the reliability of the a-posteriori LLR information $L^{(s)}$ produced by SOVA decoder. Scaled $L_i^{(s)}$ given below:

$$\hat{L}_i^{(s)} = SF\_L_i^{(s)} \times L_i^{(s)} \tag{11}$$

Where $SF\_L_i^{(s)}$ is the average scaling value of each iteration i over all the transmitted frame. The values of $SF\_L_i^{(s)}$ derived depend on Minimum-Mean-Square Error MMSE (Ahmed, 2012).

$$SF\_L_i^{(s)} = mean\left(sign\left(L_i^{(s)}\right) \times (2v - 1)\right) \tag{12}$$

Where v is the output of the combiner. It is worth mentioning that Eq. (12) need the values of v which are not available at the decoder. The author in (Ahmed, 2013] suggested various ways to solve this problem. One of them is using one of the benefits provided by the encoded pilots. The Pilots are assumed to be known symbols to the receiver and thus they can be used to calculate the scaling factor in Eqn. (12).

B - Cycle Free Effect

The cycle in the Tanner graph is a series of linked nodes which begin and end at the same node and don't include more than one node. Length of this cycle is the number of edges it includes, and the size of the smallest cycle represents the girth of the graph. By making the cycles long enough, the algorithms may run several iterations without being affected by them, in practice they operate only on cycle-free sub graphs of the Tanner graph.

In this paper we present interleavers which guarantee that the Tanner graphs of RA are free of small cycles because these small cycles adversely affect the decoding performance.

C - Encoded Pilots Effect

The Pilot symbols can be encoded along with data in RA and IRA codes and hence they are called encoded (internal) pilots (Hyuck *et.al.*, 2005). The positions of pilots symbols have higher reliability as compare to the data and they can significantly improve the performance of decoding. Furthermore, the studies by (Ahmed, 2013; Amitav *et.al.*, 2009;Hari, 2005) proved that it is possible to increase the distance properties and/or enhance the multiplicity of the code by replacing certain data bits by the pilot symbols. It is important to state that the selection of a position inside the data to be filled with a pilot is not tractable for long codes. This is mainly due to the brute force search approach adopted by the algorithm in (Ahmed, 2013). Therefore only codes with short lengths are tested. Figure (3) shows a schematic diagram of RA encoder using encoded pilots.

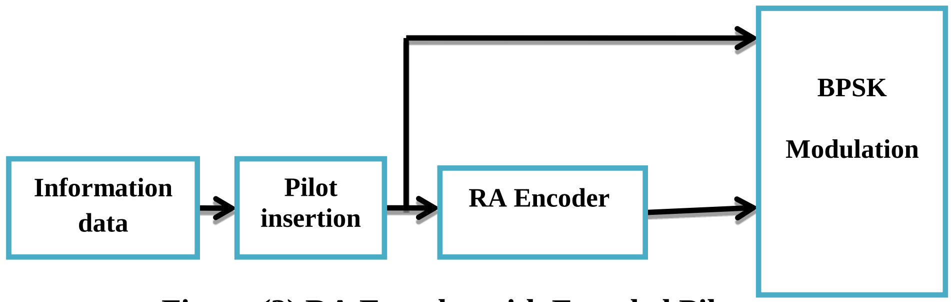


Figure (3) RA Encoder with Encoded Pilots.

D – Progressive Edge Growth Algorithm (PEG)

PEG was suggested by Xiao-Yu Hu (Xiao-Yu Hu, 2005). It represents a general method for the construction of irregular or regular Tanner graphs which have large girth by establishing edges or connections between the check nodes and a variable in the edge by the edge or the progressive manner. The variable node is linked to one of the check node in a way that maximizes the instantaneous girth. Designing the interleaver of IRA or RA code by using PEG algorithm results in a large girth and also a low error floor as compared to other codes constructed by using the random methods (MacKay, 1999;Xiao-Yu Hu, 2005).

Let V_c represents the set of check nodes where $V_c = \{c_0, c_1, \ldots, c_{M-1}\}$ and $V_v = \{v_0, v_1,, \ldots, v_{N-1}\}$ represent the set of variable nodes. E Represents the set of edges such that $E = V_c \times V_v$, with edge$(c_i, v_j) \in E$ if $h_{ij} \neq 0$, where h_{ij} represents the entry of H matrix at i row and j column, where $0 \leq i \leq M-1$ and $0 \leq j \leq N-1$. Algorithm 1 describes building of a Tanner graph with M check nodes and N variable nodes by using the PEG algorithm, where the variable nodes and check nodes are ordered according to their degrees in a no decreasing order. d_{vj} Represents the degree of variable node vj , $N^l_{v_j}$ and $\bar{N}^l_{v_j}$ denotes the set of all check nodes reached by a tree spreading from the variable node vj with a depth l, and its complement, respectively.

Algorithm 1: PEG Algorithm :

For j=0 to n-1
  begin
    For k=0 to d_{vj}-1
      begin

if k=0

$E^0_{v_j} \leftarrow (c_i, v_j)$ where $E^0_{v_j}$ is the first edge incident to v_j, and c_i is check node having the lowest degree under the current graph setting$E_{v0} \cup \ldots E_{vj-1}$.

else

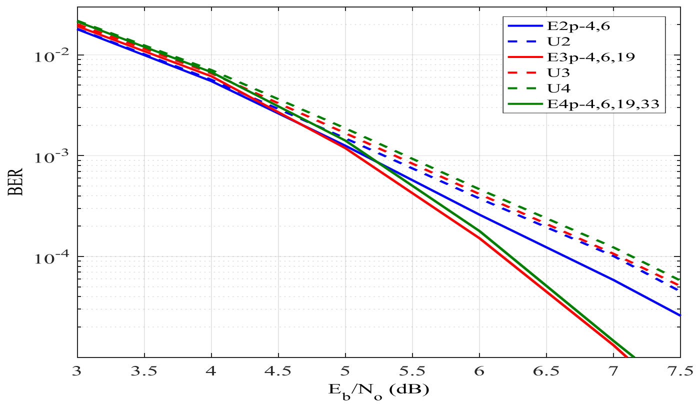
expanding a tree from variable node v_i up to depth l under the current graph setting such that $\bar{N}^l_{v_j} \neq \emptyset$ but $\bar{N}^{l+1}_{v_j} = \emptyset$, or the cardinality of $\bar{N}^l_{v_j}$, stops increasing but is less than m, then $E^k_{v_j} \leftarrow (c_i, v_j)$ where $E^k_{v_j}$ is the k-th edge incident to v_j and c_i is one check node picked from the set $\bar{N}^l_{v_j}$, having lowest check node degree**.**

End

End

IV Simulation Result and Discussion

Figure 4 shows the performance of RA turbo decoding using the unencoded pilots and compares it with the same codes using the encoded pilots. The performance can be improved substantially when a pilot is used in RA decoding for turbo code. The schemes are simulated over *AWGN* channel with a turbo decoder and a random interleaver. MATLAB software version R2011b (7.13.0.564) is used in all simulation tests. The simulations show that the best improvement is achieved when 4 pilot symbols are inserted within data. The BER performance is improved by about 1 dB when N=50 and K=35 at BER of 10^{-4}. Where the letters U, E and P refer to un-encoded pilots, encoded pilots and pilot's position respectively.



Figures 4: BER Performance Comparison of (50, 35) q=3, a=7 Regular RA Turbo Decoding for 2, 3 and 4 Unencoded and Encoded Pilots Code.

Scaling (or normalization) is utilized to alleviate the exaggeration of the a posteriori information generated by *SOVA*. Figure 5 shows the bit error rate (BER) and the frame error rate (FER) performance of the modified IRA turbo codes ($s1$) compared to the unmodified codes ($s0$). The system is simulated over *AWGN* channel with *PEG* interleaver and *SOVA* decoding for the accumulator encoder. Simulation shows that the modified system presents a good behavior especially at the error floor region. An improvement of about 0.25 dB is gained at BER of 10^{-5} when the scaling is applied.

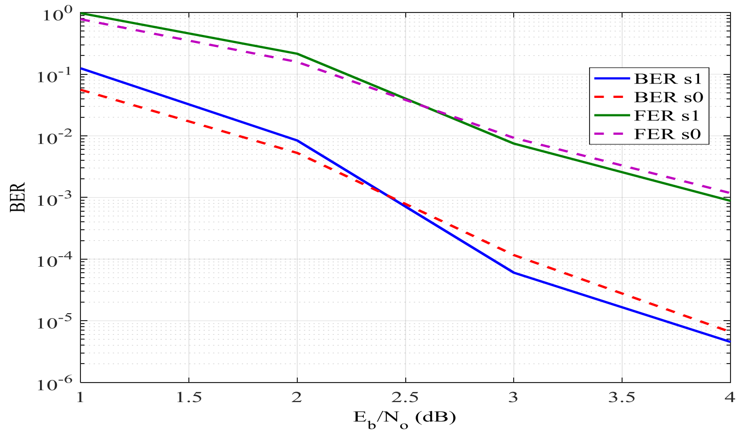


Figure 5: BER and FER Performance of Rate 0.5 (500,250) IRA Codes Using Turbo Decoding Over AWGN Channel.

Figure 6 shows a comparison of the performance of RA turbo codes for different interleaver types (the abbreviations IH, IB, IP, IR and IC symbolize the helical interleaver, block, PEG, random and free 4-cycles interleaver respectively). The curves show the dependency of the performance of turbo codes on the interleaver type. PEG interleaver provides the best performance followed by the free 4-cycles interleaver. This is mainly due to the large girth provided by these two interleavers.

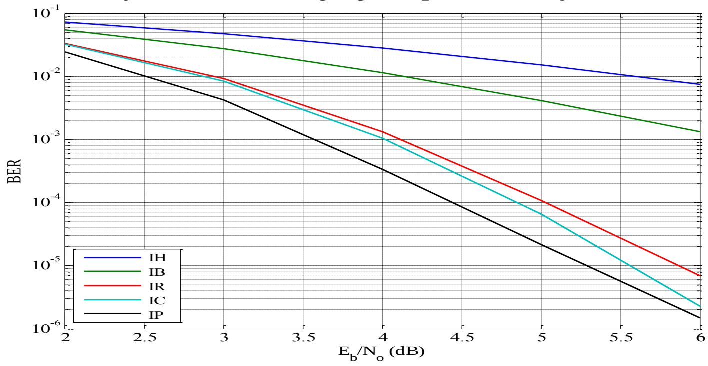


Figure 6: BER Performance Comparison of (120, 75) Regular q= 3, a=5 RA Turbo Codes with Different Interleaver.

Figure 7 shows a comparison of the BER Performance of the regular and irregular IRA repeat accumulates turbo codes with a rate 0.25 and a random interleaver over AWGN. The performance of IRA code outperform the RA counterpart by about 0.8 dB when N=200 at BER of 10^{-4}.

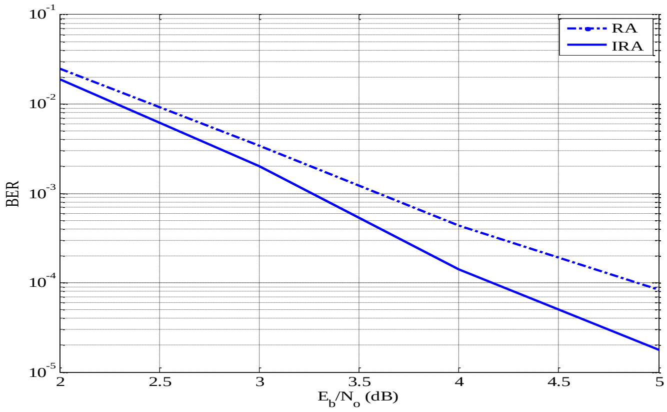


Figure 7: BER Performance Comparison for Regular (RA) and Irregular (IRA) Repeat Accumulates Turbo Codes with Rate 0.25 (200, 50) and Random Interleaver

Figure 8 shows the comparison between the performance of turbo decoding and that of the Sum-Product (SP) and Min-Sum (MS) decoding algorithms for IRA codes with different interleaver type. All schemes are simulated over *AWGN*. A general outlines can be recognized from simulated systems. First; the best performance is achieved by SP decoding followed by MS and then by turbo. Second; the turbo decoding shows a better performance than that of the MS for low SNR. Finally, the systems that utilize PEG algorithm in the design of their parity check matrix shows better performance in the error floor region.

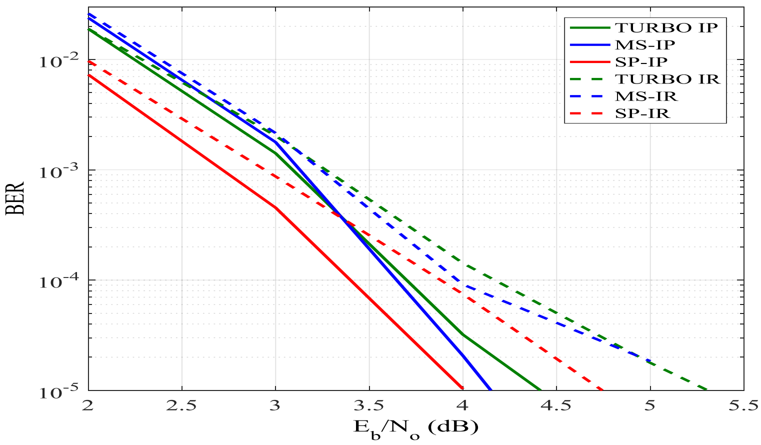


Figure 8: BER Performance Comparison of Rate 0.25 (200, 50) IRA Codes with Different Decoding Algorithms.

V- CONCLUSION

In this paper, different techniques are presented to improve the turbo decoding of regular (RA) and irregular repeat accumulate (IRA) codes. Simulation tests show that a significant improvement is achieved when the scaling is applied to the turbo decoding of RA code also when the encoded pilot scheme applied, the system performance is improved as the number of encoded pilots increased.The encoded pilots can play dual role: firstly, channel estimation which is an essential task in communication systems and secondly it improves the distance properties and the weight distribution of the code. The simulation results also indicate that the PEG interleaver provides the best performance compared with other types of interleaver because of its larger girth. Since the SP is the optimum decoding technique. The performance of the different tested systems using this algorithm reveals a better performance compared with MS and turbo decoding. The simulation result also shows that the performance of irregular repeat accumulates turbo code (IRA) outperform the regular RA code.

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