Modified Suboptimal Iterative Decoding for Regular Repeat- Accumulate Coded Signals

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Abstract

In this work, two algorithms are suggested in order to improve the performance of systematic Repeat-Accumulate (RA) decoding. The first one is accomplished by the insertion of pilot symbols among the data stream that entering the encoder. The positions where pilots should be inserted are chosen in such a way that to improve the minimum Hamming distance and/or to reduce the error coefficients of the code.

The second proposed algorithm includes the utilization of the inserted pilots to estimate scaling (correction) factors. Two-dimensional $(2-D)$ correction factor was suggested in order to enhance the performance of traditional Minimum-Sum (MS) decoding of regular repeat accumulate codes. An adaptive method can be achieved for getting the correction factors by calculating the mean square difference (MSD) between the values of received pilots and the a-posteriori data of bit and check node related to them which created by the minimum-sum (MS) decoder.

Keyword :repeat –Accumulate code (RA) ,Two-dimensional(2-d)correction Factors , Minimum Sum(ms) decoder , Means Square Difference(msd)

الخلاصة

في هذا العمل، تم اقتراح خوارزميتين لتحسين اداء معيد الترميز التكراري التجميعي. الاول انجز باضافة رموز ارشاد بين سيل البيانات التي تدخل الى المرمز. في هذه الطريقة، رموزالارشاد ترمز مع البيانات المدخلة (رموز الارشاد المرمزه او الداخليه). مواقع المرشدات يجب اختيارها بطريقة معينة لتحسين مسافة هامينج الصغرى و/او تقليل معملات الخطا للمرمز.

الطريقة الثانية المقترحة تتضمن استخدام المرشدات المدخلة لتخمين عوامل التصحيح. تم اقتراح نظام تصحيح ثنائي البعد لتحسين اداء معيد الترميز اقل– جمع الاعتيادي للمرمز التكراري التجميعي النظامي. يمكن الوصول الى طريقة قابلة للتكيف لحساب معاملات التصحيح عن طريق حساب مربع معدل الفرق بين قيم رموز الارشاد المستلمة والمعلومات الخارجة من عقد البت والفحص الخاصة بها والمتولدة من معيد الترميز.

كلمات المفتاحية : رموز الارشاد المرمزة او الداخلية ،(معيد الترميز اقل ، جمع اعتيادي مرمز التكراري التجميعي النظامي)، معاملات التصحيح .

1. Introduction

In coding theory and information theory with implementations in telecommunication and computer science , error control or correction and error detection are techniques which allow credible conveyance of digital information during undependable channels.

The idea of concatenating two or more error correction codes in series is to improve the all decoding performance of the system [Chung *et.al*., 2001]. A new class of turbo-like codes that straddle the gap between $LDPC$ codes and parallel concatenated turbo codes can be generated by applying interleaving and iterative decoding to these codes [Sarah *et.al*., 2010].

A special type of serially concatenated codes is called Repeat–Accumulate (RA) code. This code has an outer code with a rate-$1/q$ repetition code and an inner code with generator $1/(1 + D)$ convolutional code where D is the number of flip flops. An interleaver is taken place between the inner and outer codes [Sarah *et.al*., 2010].

In this work, it is focused on how to improve the performance of RA code without adding excessive complexity. Several literatures suggest combining pilots and information before passing them to the encoder [Khurrum *et.al*., 2005, Amitav *et.al*., 2009]. Although this results a reduction in rate, it has a bright side; in addition to their role in channel estimation, pilots might be utilized to:

1. Improve the decoding process by increasing the minimum Hamming distance of the code.
2. Calculate a correction factor to elevate the over estimation that characterizes the extrinsic information in iterative decoding of sub-optimal codes.

In 2005, Hyuck M. Kwon, Khurram Hassan and Ashutosh Goyal proposed a novel iterative channel estimation and low density parity check ($LDPC$) decoding scheme where the pilot symbols are encoded and can be used for both channel estimation and decoding [Hyuck *et.al.*,2005]. In 2007, Shiva and Aaron Gulliver present rate-compatible systematic RA codes for the $AWGN$ channel [Shiva *et.al.*, 2007]. In addition, they design rate-compatible punctured RA codes for the Additive White Gaussian Noise ($AWGN$) channel [Shiva *et.al.,*2007]. In 2013, Ahmed A. Hamad proposed two-dimensional ($2-D$) correction scheme to improve the performance of conventional Min-Sum (MS) decoding of regular LDPC codes [Ahmed *et.al* 2013].

The paper has the following sections:

Section 2: MS decoding algorithm, section 3: scaling factors for MS decoding, section 4: optimum position for encoded pilots, section 5: practical implementation of RA codes, section 6: simulation and practical results, section 7: conclusions and section 8: the references.

2. Minimum-Sum (MS) Decoding Algorithms

A Tanner graph is a graphical representation for the parity-check matrices. It contains two sets of nodes: X nodes of the codeword bits and Y nodes of the parity-check equations. An edge connects a check node to a bit node if that bit is included in the regarding parity-check equation and therefore the number of edges in the Tanner graph equal to the number of 1s in the parity-check matrix [Sarah *et.al* 2010]. The tanner graph for the parity check matrix in form (1) is shown in figure 1.

$$H=\begin{bmatrix}1&1&0&1&0&0\\0&1&1&0&1&0\\1&0&0&0&1&1\\0&0&1&1&0&1\end{bmatrix} \tag{1}$$

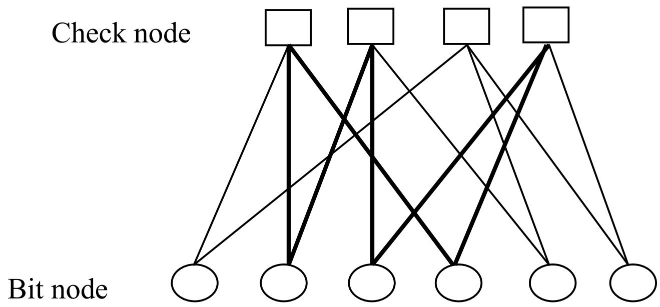


Figure 1 The tanner-graph representation for H in form (1).

The sum–product (SP) algorithm can be modified to reduce the implementation complexity of the decoder. This can be done by altering equation (2) to replace the product term by a sum.

$$E_{j,i}=\log\frac{1+\prod_{i'\in B_j,i'\neq i}\tanh\left(\frac{M_{j,i'}}{2}\right)}{1-\prod_{i'\in B_j,i'\neq i}\tanh\left(\frac{M_{j,i'}}{2}\right)} \tag{2}$$

Where $E_{j,i}$ is the extrinsic information from check node j to bit node i and $M_{j,i}$ is the information transmitted from bit node i to check node j.

The MS algorithm simplifies the calculation more by identifying that the expression corresponding to the minimum $M_{j,i'}$ controls the product term and the multiplication can be approximated by a minimum:

$$E_{j,i} \approx \prod_{i'} sign(M_{j,i'})\, min|M_{j,i'}| \tag{3}$$

Again, the multiplication of the marks be able to compute by utilizing modulo-2 addition of the hard decisions on every $M_{j,i'}$ and the originating MS algorithm only needs additions and computation of minimums [Sarah *et.al* 2010].

3. Scaling Factors for minimum-sum Decoding

The marginal log likelihood ratio (LLR) values E_{ji}^{l} and M_{ji}^{l} are scaled by correction factors a^{l} and b^{l} to make the a-posteriori probabilities close from the transmitted codeword, thereby reducing the probability of error. The corrected values $\acute{E}_{ji}^{l}$ and $\acute{M}_{ji}^{l}$ are given by the following equations:

$$\acute{E}_{ji}^{l} = a^{l}.E_{mn}^{i} \tag{4}$$

$$\acute{M}_{ji}^{l} = b^{l}.M_{ji}^{l} \tag{5}$$

It is possible to derive the correction factor by minimizing the mean square difference MSD $\delta(a)$ between the a-posteriori probability M_{i}^{l} and the modulated codeword Q. Assume that $V^{l} = [M_{i}^{l}]$, then:

$$\delta(a^{l}) = E\left[\left(Q - a^{l}sgn(V^{l})\right)^{2}\right] \tag{6}$$

$E[.]$ denotes the expected value. The value of a^{l} is simply found by derive $\delta(a^{l})$ with respect to a^{l} and equating the result to zero.

$$\frac{d\delta(a^{l})}{da^{l}} = a^{l}E\left[\left(sgn(V^{l})\right)^{2}\right] - E(sgn(V^{l}) \cdot Q] = 0 \tag{7}$$

Considering that $\left(sgn(V^{l})\right)^{2} = 1$, the value of a^{l} is given by:

$$a^{l} = E[sgn(V^{l}.Q)] \tag{8}$$

In similar manner, the value of b^{l} can be found by minimizing the MSD between the modulated codeword and the vector representing the vertical sum of M_{ji}^{l}. Suppose that:

$$U_{i}^{l} = \sum_{j \in A_{i}} M_{ji}^{l} \tag{9}$$

and let the vector $U^{l} = [U_{i}^{l}]$ then:

$$b^{l} = E[sgn(U^{l}.Q)] \tag{10}$$

Actually, for certain code H and channel characteristics, the correction factors are function of the iteration's number (l) and the number of simulated frames (t), $(i.e., a_{s,t}^{l}$ and $b_{s,t}^{l})$. The average values of $a_{s,t}^{l}$, overall simulated codewords is given by [Amitav *et.al* 09]:

$$a_s^l = E_t\left[a_{s,t}^l\right] \tag{11}$$

Further, if the average is done over all iterations, then:

$$a_s = E_l[a_s^l] \tag{12}$$

And finally, the averaging overall simulated values of E_b/N_o produces a constant:

$$a = E_s[a_s] \tag{13}$$

It is worth mentioning that both equations (8) and (10) need to be computed to the transmitted codeword Q and this actually not available at the receiver in real-world systems. It is suggested to solve this problem in three different ways.

- First, it is possible to compute the values of a^l and b^l offline and store their values in memory to be used latter by the decoder online.
- Second, pilots and headers are known signals to the decoder, so they offer online estimation for the correction factors.
- Third, from the simulation of different systems, it is observed that the values of a^l and b^l have nearly stationary values for certain number of iterations, code parameters and channel characteristics. In this work, the correction factor was estimated as the average value of a^l and b^l:

$$\gamma = E(a^l, b^l) \tag{14}$$

4. Optimum Position for Encoded Pilots

It is suggested that the first two optimum pilot positions are selected using brute force search using all combination of data bits with all combination of pilot's positions. For example, in the systematic (50,15) RA code with repetition of $q = 7$ and a combiner have $a = 3$, pilots can take any positions from (1,2) to (14,15) inside data that enters the RA encoder. For each pilot's positions, all combination of data should be considered to refill the remaining data positions. The codewords and the values of multiplicity $(A_{w,j})$ are recorded during the tests. Thereby, the values of error coefficient A_d are calculated using equation (15) and employed in the bit error rate that is upper bounded by the formula presented by equation (16). For each combination of pilots the upper bounded bit error rate curve is depicted against the signal to noise power ratio (E_b/N_o).

$$A_d = \sum_{d=w+j} \frac{w}{k} A_{w,j} \tag{15}$$

$$P_b = \sum_{d=d_{min}} A_d Q\left(\sqrt{\frac{2Rd_{min}E_b}{N_0}}\right) \tag{16}$$

Where A_d is the error coefficient, $A_{w,j}$ is the number of codewords with w input information weight and j parity bits weight, k is the message length, P_b is the bit error probability for Maximum Likelihood soft decoding of the code, R is the code rate,

d_{min} is the minimum Hamming distance and $\frac{E_b}{N_0}$ is the energy per bit over noise spectral density.

Figure 2 shows samples (to avoids curves congestion) for two encoded pilot systems (i.e., [8,9] and [2,3]). Unencoded pilot system is presented for comparison.

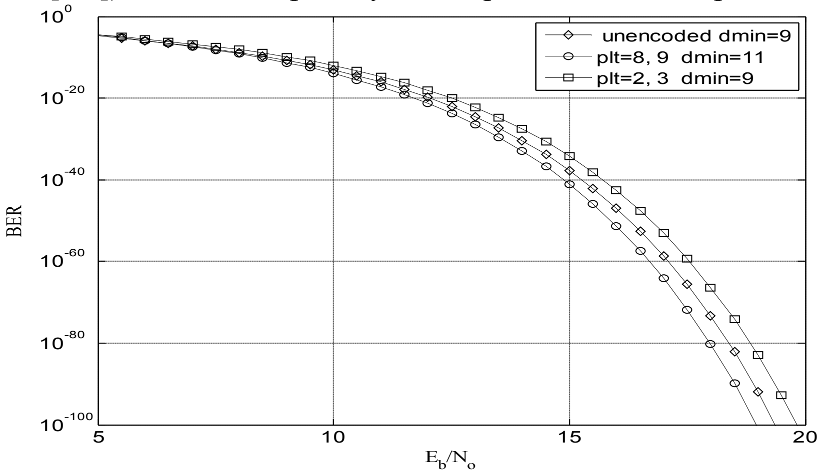


Figure 2 The upper bound for systematic (50,15) *RA* code with two encoded pilots.

It is worth to mention that position 8 &9 is not the only one that produces $d_{min} = 11$ but it is the best in terms of A_d that reflected on its performance at the required E_b/N_0. This is obvious from figure 2, where the performance of signal with encoded pilots at position 8 &9 outperform other encoded and unencoded ones

For signals with 3 pilots, excessive tests show that the third pilot is one of the data bits that appear in the codewords that have the optimum two pilot positions. Therefore, it is enough to search within these bits thereby saving the time. Also it is noted that all codeword with $d_{min} = 11$ sharing the same sets of data bits.

5. Practical Implementation of Repeat-Accumulate Transmission

Software Defined Radio (SDR) approach is proposed as a practical mean to implement the proposed system. Implementation of this approach needs only the most common PC sound card and $MATLAB$ software.

Figure 3 illustrates the general block diagram of the proposed wireless communication system. The k data bits are generated randomly and fed to the RA encoder

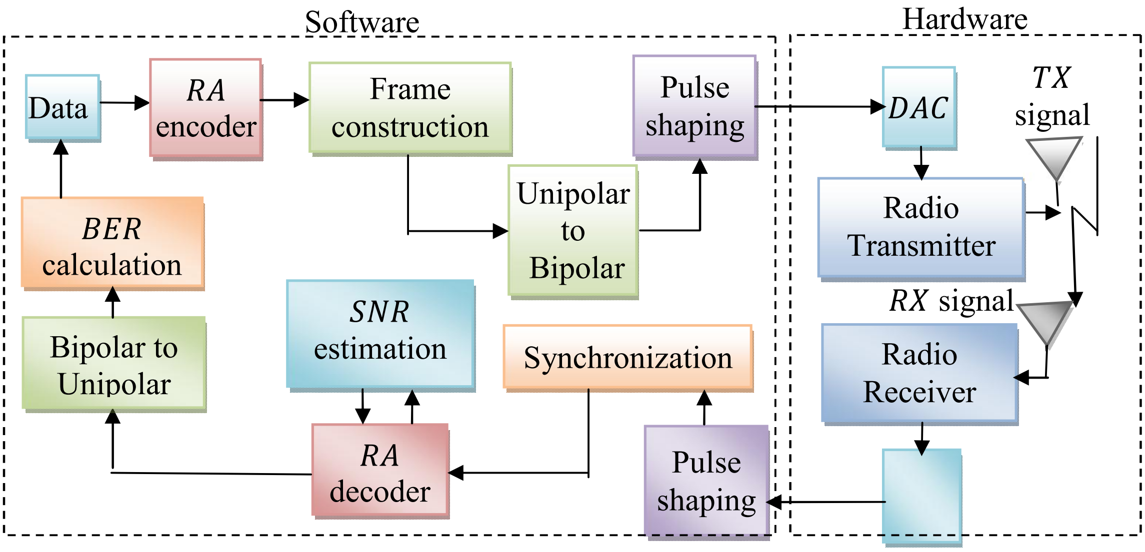


Figure 3 Transmitter and receiver of the proposed *RA* coded system.

The signal to noise ratio (SNR) estimation technique is originated from a method for measuring channel distortion errors in wideband telemetry systems [Chano *et.al.*, 2008]. Assume that x_i be the samples of reference signal and y_i be the measurement signal, such that:

$$y_i = A\, x_{i-\tau} + w_i + d_i \qquad (17)$$

Where A is the gain and $\tau = nT_s$ is the group delay (multiple of sampling time T_s) to the point in the system at which the SNR is to be defined. w_i represents the external additive noise and d_i is the distortion induced by the system, which could result from intersymbol interference or nonlinearity. The SNR is given by the following equations:

$$\frac{S}{N} = \frac{R_{xy}^2\,(\tau_m)}{P_x\, P_y - R_{xy}^2\,(\tau_m)} \qquad (18)$$

Where the maximum correlation between the actual and the desired header signals indicates the system time delay (τ_m) and the value of correlation at this point is denoted by $R_{xy}(\tau_m)$. P_x and P_y indicate the power of the desired header and power of actual received header.

Figure 4 illustrates the arrangement for the connection of the proposed hardware.

Figure 4 Hardware arrangement.

To ensure a reliable data transmission across wire or wireless channel using asynchronous packet transfer, a simple frame structure is assumed. Figure 5 presents the structure of the proposed transmitted frame.

Gold Header 97 bits	Padding (3-bit)	Codeword (data+parity) (Variable length)

Figure 5 Frame construction.

Figure 6 demonstrates a part from the transmitted frame, received frame, and received frame after filtration at the junction between the header and data.

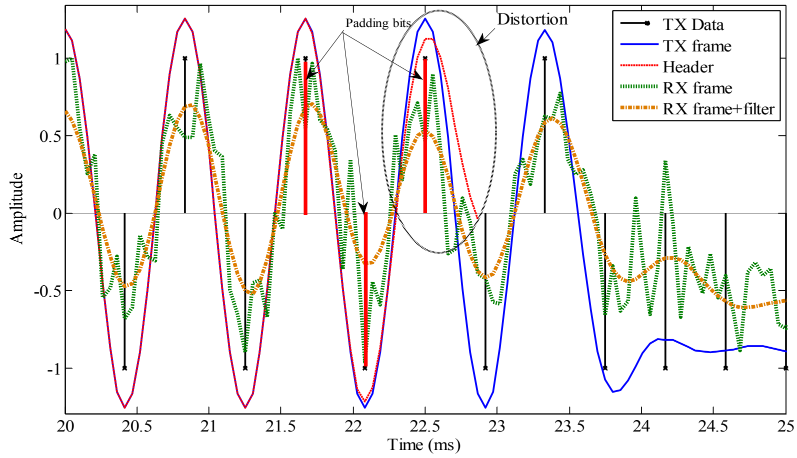


Figure 6 Part of the transmitted and the received frame

The frame synchronization shown in figure 7 can be accomplished by the cross correlation between the samples of the desired header (which is already available at the receiver) and the actual received frame at the output of the pulse shaping filter

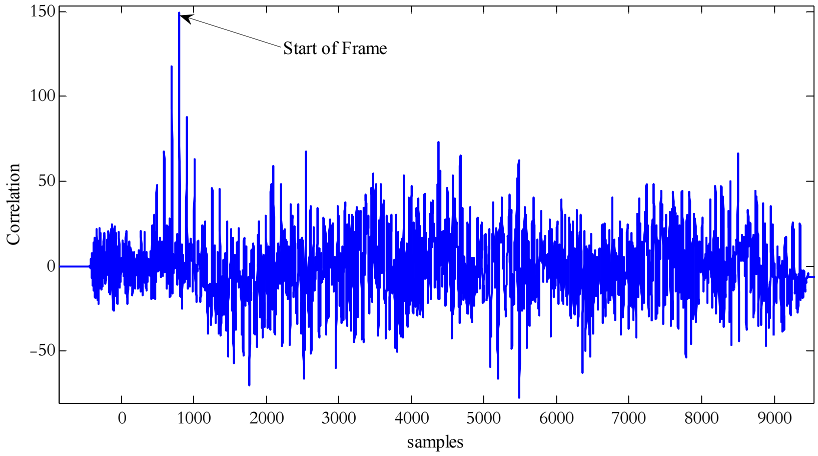
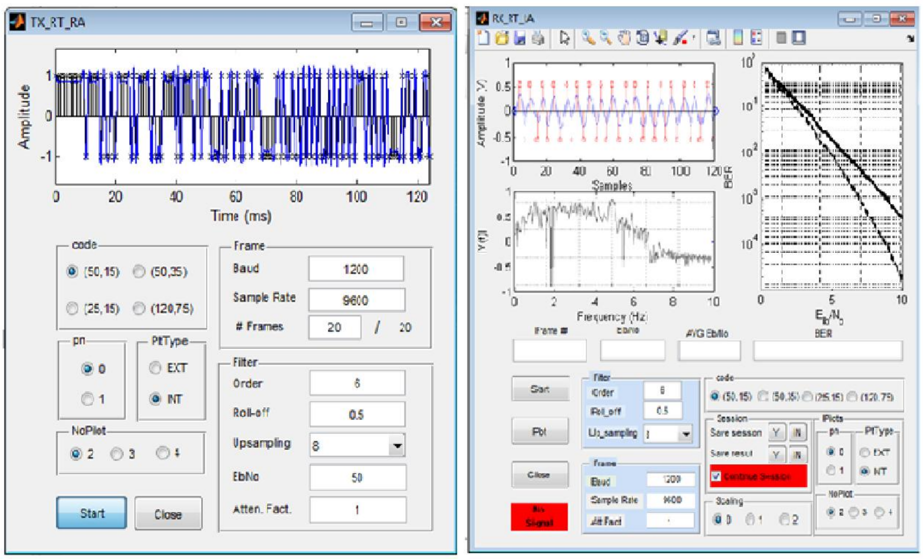


Figure 7 The cross correlation between the transmitted frame and the frame generated at the receiver.

Figure 8 shows the *GUI* panels for the transmitter and receiver that used for practical tests.



(a) (b)

Figure 8 *GUI* panel for (a) the transmitter and (b) the receiver.

6. Simulation and Practical Results

Different simulation tests are carried out to prove the effectiveness of considering pilots as part of information (encoded pilots). Simulation results for the modified RA coding scheme are presented. Unmodified systems are also simulated to display the improvement that achieved by considering modified systems.

Figure 9 shows the performance of modified system (50,35,3,7) with 2 encoded pilots that inserted at position 4 and 6 inside the information sequence. Unmodified system with 2 unencoded pilots is simulated for comparison.

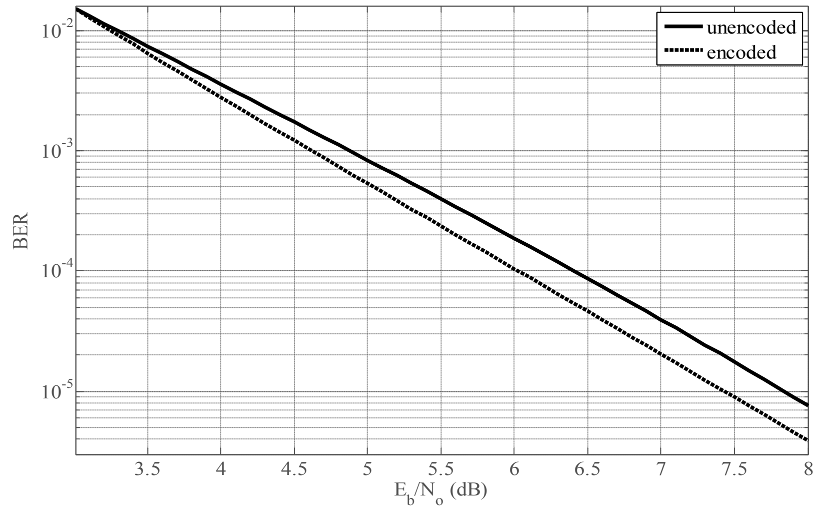


Figure 9 BER performance of systematic (50,35,3,7) RA code with 2 encoded and unencoded pilots over $AWGN$ channel.

It is obvious from figure 9 that modified system outperforms the unmodified one by about 0.4 dB at $BER = 10^{-5}$. The two systems have the same d_{min} but the error coefficient A_d of codewords at the lower weight spectrum of proposed system is low compared with unmodified one.

Figure 10 shows the performance of unmodified system (120,75,3,5) with 2 unencoded pilots and the modified version with 2 encoded pilots that inserted at position 12 and 55 inside the data.

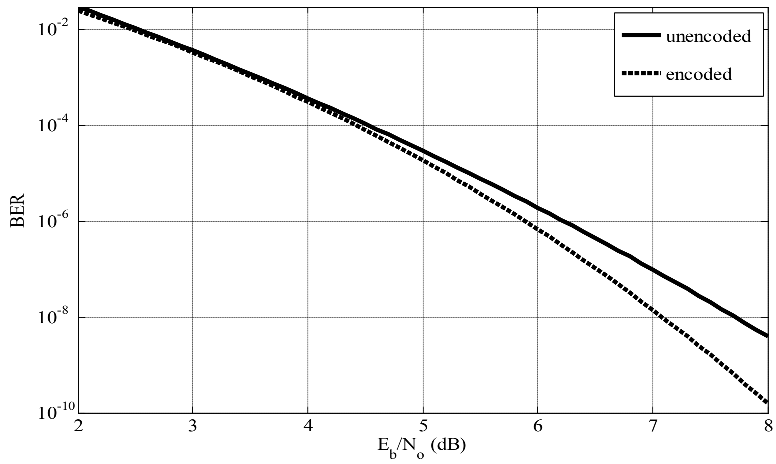


Figure 10 BER performance of systematic (120,75,3,5) RA code with 2 encoded and unencoded pilots over $AWGN$ channel.

From figure 10, it is clear that modified system outperforms the unmodified system by about 0.3 dB at $BER = 10^{-6}$. A number of different practical tests are performed in order to show the effect of encoded pilots. Figure 11 shows the BER

performance of the system (50,35,3,7) with no scaling, 2 encoded and 2 unencoded pilots.

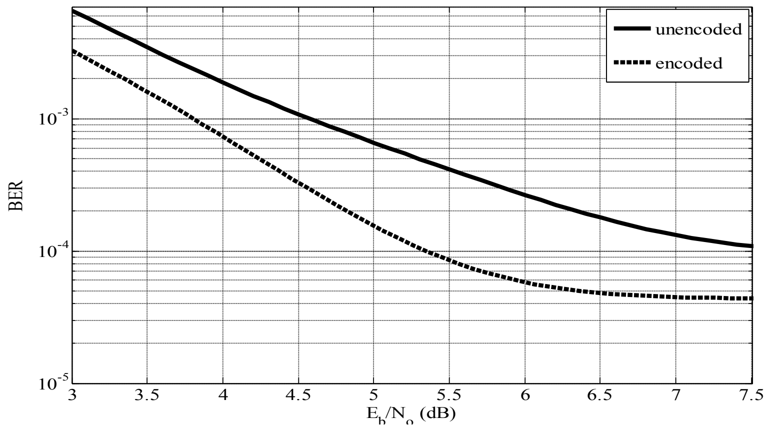


Figure 11 BER performance of systematic (50,35,3,7) RA code with 2 encoded and 2 unencoded pilots over fading channel.

From figure 11 the modified system outperforms the unmodified one by about 2.5 dB at $BER = 10^{-4}$. Figure 12 shows the BER performance of the system (120,75,3,5) with no scaling, 2 unencoded and 2 encoded pilots.

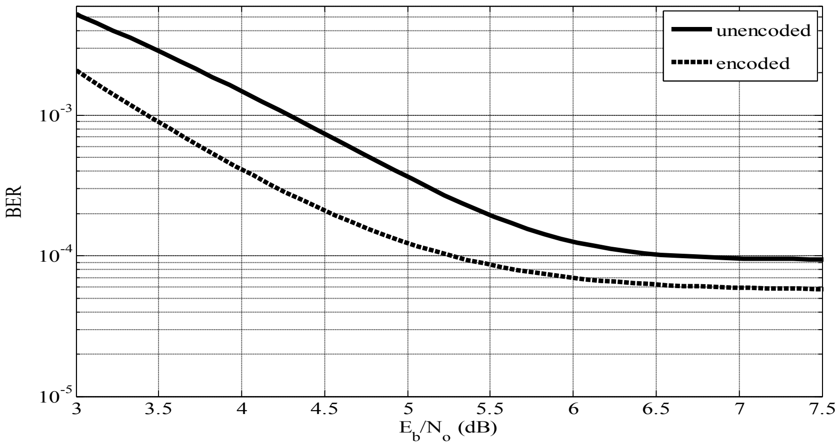


Figure 12 BER performance of systematic (120,75,3,5) RA code with 2 encoded and 2 unencoded pilots over fading channel.

7. Conclusions

In this work, a method that combines RA codes and encoded pilot insertion technique is presented. The method provides a high level of adaptivity in coding rate, minimum Hamming distance and multiplicity. RA codes are extremely simple compared to turbo or $LDPC$ codes. To reduce the overestimation of the extrinsic information, it is suggested that they are scaled by correction factors. The number and position of pilots are selected to increase the Hamming distance and/or to reduce the error coefficients of the code. The modified codes have shown improvement in terms of BER over unmodified codes having comparable data rates.

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