The Second International Conference on Next Generation Mobile Applications, Services, and Technologies

TURBO CODES WITH INTERNAL PILOT INSERTION

Abdulkareem A. Kadhim
Al-Ahlyyia Amman University
Amman, Jordan
ak_kadhim@ieee.org

Ahmed A. Hamad
Engineering College
Babylon University, Iraq
ahmedbabel@yahoo.com

ABSTRACT

In this paper we present a simple modification of a classical turbo code allowing for improved distance properties and rate adaptation. The proposed scheme suggests the insertion of pilot bits in the data sequence in such a manner that low weight codewords are eliminated or their weight multiplicity being reduced. In this way, pilots are utilized to increase the minimum distance of the code effectively. Furthermore, the rate can be made adaptive, almost arbitrarily according to the channel conditions under fixed interleaver length constraint. The resulting code uses fixed length codeword, can support a wide range of rates and even outperform the original system that uses external pilot insertion, in terms of error performance specifically at high signal to noise ratio. The proposed scheme does not add any complexity to the encoder neither to the decoder. Two recursive systematic convolutional (*RSC*) encoders together with S-random interleaver are used at the encoder side. At the decoder, soft-input soft-output Viterbi algorithm (*SOVA*) to accomplish iterative decoding with a maximum of five iterations is used.

Key words: *Turbo code, Pilot insertion, SOVA,* Interleaving.

1. INTRODUCTION

Continuous quality variations of the channels occupied by wireless packet transmission impose the employment of different data rates and code structures. Practically, the transmitter tries to combat the deterioration in channel parameters by switching to longer code length or lower code rate. Turbo codes are interesting coding techniques for noisy channels. Their performance allows transmitting the signal with very low power which is one of the most crucial demands in mobile systems. Actually, a number of problems facing the practical use of turbo codes in systems operating over fading channels. The usual rate for the typical parallel concatenated turbo codes is 1/2. This rate may be reduced to combat degradation in channel quality, demands transmitting of additional packets when the interleaver size is fixed. This causes additional delays and requires, larger buffering at the receiver [1,2].

Another problem appears when pilot symbols are used for channel estimation. Insertion of external pilots into the packet reduces the payload and shortens the interleaver length of turbo code which decreases the error performance [2].

In this paper, we propose a simple algorithm for optimum pilot insertion to alleviate the above problems. The algorithm is based on employing the insertion of pilot bits in certain positions such that low distance codewords are expurgated from the code space. Rate adaptation and channel estimation are provided by changing the number of internally inserted pilots instead of, usually, external insertion. In this way, codewords can span the whole packet, and the interleaver length can be kept constant which simplifies the hardware design and allows for backward compatibility with the existing systems.

2. PILOT INSERTION TECHNIQUES

Since channel codes are usually designed to combat sparse errors, coding alone appears insufficient to reduce the bit error rate (*BER*), in particular, over time varying multipath fading channels. The main characteristic of these channels is the production of burst errors, which reduces the benefit of using error control codes. Therefore, it is desirable to continuously provide the receiver with the amplitude and phase variations of the channel. A usual method of coping with such channels is inserting a pilot sequence into the packets. A channel is estimated during the pilot transmission and is assumed to be almost constant for a specified period of time after that. The estimated channel properties are used to synchronize the incoming signal in order to recover the useful data sequence followed by transmission of new pilot sequence and so on. In real systems, for example GSM [3], such pilot signals occupy 20-30% of the time slot. Since turbo codes are used with lower signal-to-noise ratios, it is reasonable to assume that channel estimation will be more difficult and require special techniques of coping with bad quality channels.

An obvious solution to improve the channel estimation is to increase the number of pilot symbols. Unfortunately, such solution not only decreases the available bandwidth but also reduces the performance of turbo codes. It is a known fact that increasing the size of the block length of a turbo code improves its *BER* performance [4]. If the synchronization symbols not belong to the code structure, the effective payload length decreases (i.e., the effective length of the packet becomes shorter due to the presence of the pilot symbols). This means that the length of the interleaver must be decreased (so the codeword will fit into the remaining slot space) and the resulting *BER* of the code will increase as

978-0-7695-3333-9 /08 $25.00 © 2008 IEEE
DOI 10.1109/NGMAST.2008.83

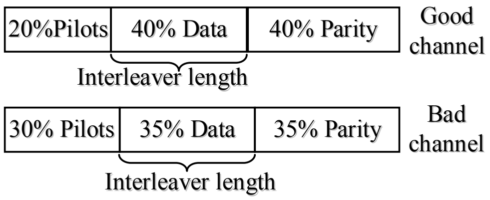

Fig.1. Typical turbo coded frame structure with half coding rate and external pilot insertion.

well [2]. Figure 1 shows a typical turbo coded frame structure for external pilot insertion.

In this work, an algorithm is proposed for optimal internally insertion of pilot symbols. The pilots are injected in certain positions in the data stream so that the resulting distance spectrum of turbo code is improved. The resulting code will have different code weight spectrum, resulting in a significant improvement in terms of *BER* over *AWGN* and multipath fading channel. The minimum distance and/or the error coefficients of the code are improved with each additional inserted pilot. This compensates the degradation in performance expected due to the reduction in bandwidth efficiency resulted by the pilot insertion. No additional complexity is required at the encoder or at the decoder side. The conventional *SOVA* decoder is used with an exception that the log likelihood ratios (*LLR*) for pilots are given large values considering that they are well known to the decoder.

Figure 2 shows a typical structure of the turbo coded frame utilized by the proposed internal pilot insertion scheme.

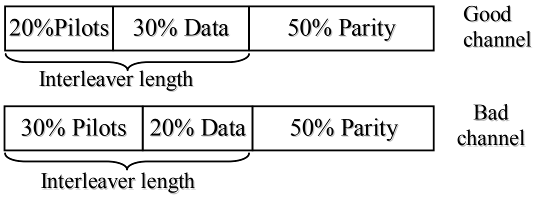

Fig.2. Typical turbo coded frame structure with half coding rate and internal pilot insertion.

It is obvious that the length of interleaver remains constant in spite of increasing the number of inserted pilots. Therefore, the internal pilot insertion algorithm solves the problem of performance degradation, experienced in external pilot insertion that resulted from the reduction of interleaver length.

The terms *external* and *internal* are utilized to imply the influence of pilots in turbo code structure. Internal insertion of pilots improves the code performance, whereas the use of external pilot insertion has no such effect. Figure 3 shows a general block diagram representing the two pilot insertion schemes.

3. THE PROPOSED ALGORITHM

The proposed system relies on new method to improve the distance properties of turbo code having fixed frame length with variable rates. The pattern of pilot is chosen in such a way so that the minimum distance of the resulting code is increased. Coding rate adaptation can be obtained by changing the number of embedded pilots. Thus the pilots in this way, increase minimum distance of the code, achieve rate adaptation, assist in channel estimation, and increase interleaver gain due to the larger frame size.

It is well known that the parallel turbo codes have sparse weight spectrum, which means that their low Hamming weight codewords have low multiplicities [6]. Moreover, the typical turbo code may have relatively low minimum Hamming distance (d_{min}) which manifests itself in the *error floor* region at high signal- to-noise power ratios (*SNR*). The *BER* of turbo codes is determined by a large number of the medium distance spectral lines at low *SNR* and the first several spectral lines at high *SNR* [6]. To allow rate adaptation with fixed delay and interleaver size, the present method exploit the above properties in inserting pilots in certain positions that expurgate low weight codewords from the code space or reduce their weight multiplicity. The resulting coding rate (effective rate R_{eff}) is determined by;

$$R_{eff} = r_o(1 - \frac{\upsilon + p}{N}) \tag{7}$$

where r_o is the original code rate, N is the interleaver length, υ is the constituent code memory order, and p is the number of inserted pilots. It is obvious that, when using different values of p it is possible to change the code rate. The positions and the number of pilots must also be known at the receiver. The header of the transmitted frame is used to carry this information.

To obtain the pilots insertion pattern (for given code with two *RSC* constituent encoders, interleaver, and puncturing pattern), the weight spectral lines having distances less than certain threshold weight are needed. Weight spectral lines means the set of all pairs (d, A_d), where d is the Hamming distance and A_d is the error coefficient which determines the contribution of the codewords with the same weight d to the bit error probability [7, 8]. Thus A_d can be defined as;

$$A_d = \sum_{d=w+z} \frac{w}{N} A_{w,z} \tag{8}$$

where $A_{w,z}$ is the number of the codewords with the input information weight w and parity check information weight z. The overall Hamming weight of the codeword is $d=w+z$.

Computation of spectrum for higher weights would be computationally prohibitive. Many authors resort to simplifications such as considering only the codewords corresponding to input sequences of Hamming weight w=2 [8]. Here, we consider all codewords corresponding to input sequences having weight up to w=5 for N=56, since for short codes (which are typically used in real-time traffic), higher

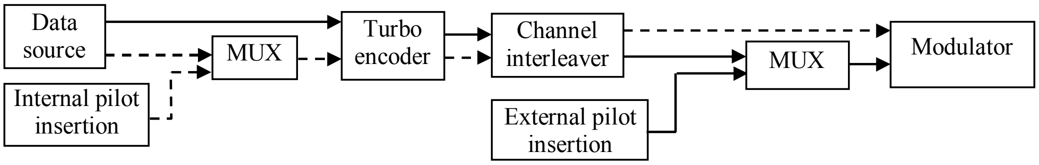

Fig.3. A general block diagram representing of external and internal pilot insertion.
——— External pilot insertion
- - - - - Internal pilot insertion

order input sequences must be considered in order to determine the true free distance of the code. For an equivalent (*n, k*) linear systematic block code of a turbo code (where *n* and *k* are the number of bits in codeword and data block, respectively) the upper bound of bit error probability over an *AWGN* channel is [8];

$$P_b \le \sum_{d=d_{free}}^{n} A_d \, Q\left(\sqrt{d\, R_{eff} \frac{E_b}{N_o}}\right) \qquad (9)$$

A rate r_o=1/3 turbo code consists of two identical rate half *RSC* encoders with memory order υ=3 and generator matrix of the component code g=$(13,15)_8$ is considered. The two encoders are separated by S-random interleaver of length *N*=56.

4. LOCATING PILOT'S POSITIONS

To find the optimum positions for a given number of pilots, the following steps may be followed [9]:

Step1. Generate the partial distance spectrum considering all combination of input sequences having weight less than or equal *w*.
Step2. Determine the number of pilots *p* to be inserted. This depends on the channel condition and designed minimum distance.
Step3. Starting with the sequence(s) that have minimum distance; find the new error coefficient A'_d corresponding to each bit in the input sequence having a value of "1" as it is selected as a pilot.
Step4. Find the relative contribution to *BER* (*RC*) of the first spectral lines over the range of E_b/N_o under consideration

$$RC = \frac{\sum_{d=d'_{min}}^{q} A'_d \, Q\left(\sqrt{d\, R_{eff} \frac{E_b}{N_o}}\right)}{P_b} \qquad 10$$

where d'_{min} is the new minimum distance after pilot insertion and *q* is a threshold weight. From the extensive computer simulation tests conducted, it is found that the value of *q* should be within $2\,d'_{min}$ to consider the effective spectral lines.

Step5. Select the pilot position which gives the least *RC* to the *BER* over the considered range of E_b/N_o.
Step6. Remove all input sequences having ones with index equal to the index of the selected pilot and resulting weight greater than threshold weight *q*.
Step7. If the number of selected pilots *p* is reached then stop, otherwise go back to *Step 3*.

To explain the above algorithm, the first spectral lines and the corresponding input sequences with their indices of bits having value of '1' are illustrated in Table 1. It is clear that the minimum distance of the code d_{min}=9, w_9=2 (w_d weight of input sequence(s) which produces codeword having distance *d*), and weight multiplicity N_9=1. This is acquired by the self terminating sequence having two 1's in positions 23 and 30.

Figure 4 shows the two curves of *RC* corresponding to the selection of positions 23 and 30 as pilots. The curve also

Table 1: First spectral lines of the g = $(13,15)_8$, *N*=56, unpunctured turbo code.

d	*Wd*	*Ad*	Indices of 1's in input sequence
9	2	0.0357	23, 30
13	3	0.1071	22, 23, 27
13	3		35, 36, 40
14	2	0.1071	13, 20
14	2		17, 24
14	2		29, 36
15	2	0.2500	16, 23
15	3		24, 25, 29
15	4		16, 17, 23, 24
15	5		23, 24, 29, 35, 36
16	2	0.2678	3, 17
16	3		8, 11, 17
16	3		24, 30, 35
16	3		46, 48, 49
6	4		35, 38, 39, 40

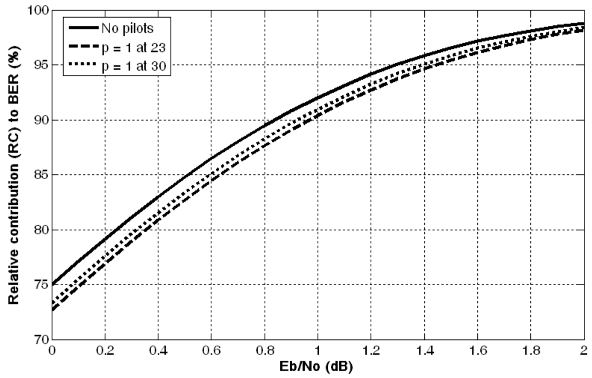

Fig.4. Relative contribution to BER versus E_b/N_o.

represents the *RC* of unmodified code (with no pilots). It is clear that position 23 gives less contribution to *BER* than 30, therefore it is selected as the first pilot. Since the new Hamming distance of the codeword produced by the input sequence (23,30) becomes 48, this sequence is expurgated from Table 1 and the resulting code has a minimum distance $d'_{min}=13$, $N_{13}=2$, and $w_{13}=6$. It is worth mentioning that the new error coefficients must be calculated with the insertion of pilots at position 23 and 30 independently. The sequence of all zero is no longer valid and the path of all zero in the hyper trellis is cancelled [8].

The steps of the proposed algorithm are repeated until all pilot positions are covered. For the case of four pilots $p=4$, the pilot pattern is $P=[23, 36, 24, 13]$ and the resulting code have $d'_{min}=16$, $w_{16}=12$ and $N_{16}=4$. The partial distance spectrum for the original code (without pilot insertion $p=0$) and modified code with $p=8$ with pilot pattern $P=[23, 36, 24, 13, 17, 39, 49, 15]$ is shown in Figure 5. The figure shows the improvement in d_{min} and A_d that could be achieved when the proposed algorithm is adopted.

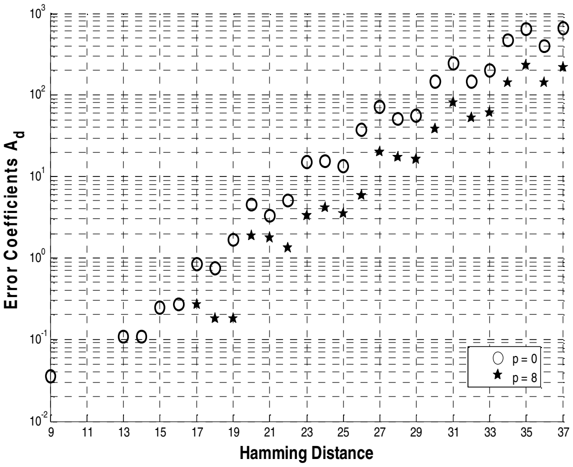

Fig.5. The partial distance spectrum for the $g=(13,15)_8$ & $N=56$ unpunctured turbo code.

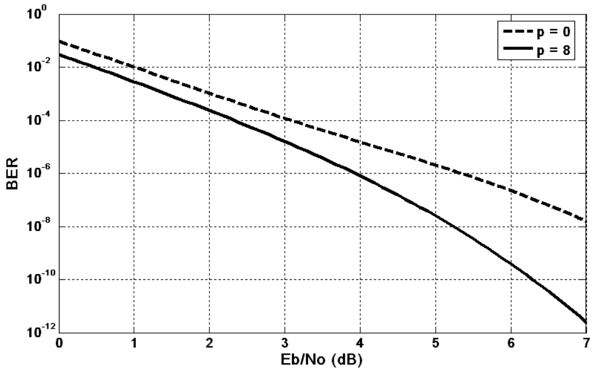

Fig.6. Upper bound performance of unmodified and modified turbo code.

Figure 6 shows the upper bound of the modified code with $p=8$ compared with the original code $p=0$, by applying equation 9 and using the partial distance spectrum obtained in step-1. The figure shows an improvement in *BER* especially at high *SNR*.

To achieve good outage probability, pilot symbols must be uniformly spread in small clusters of sizes equal to the delay spread of the channel [10,11]. Obviously the resulting pilot's pattern is irregular. This problem can be solved at the channel interleaver and the pilots may be redistributed uniformly along the packet with some constraints on the interleaver length.

5. SIMULATION TESTS AND RESULTS

The system presented in this paper was simulated for different lengths of pilot patterns over *AWGN* channel and multipath fading channel (perfect estimation is assumed) with normalized Doppler spread $f_D T_s=0.0001$, where f_D is the maximum Doppler offset and T_s is the symbol duration. The *BER* values in the simulations are calculated following transmission of about 10,000,000 packets and the number of decoder iterations was set to 5. Figure 7 shows that an improvement may be achieved when the proposed algorithm is adopted over *AWGN* channel. Improvement in performance of about 0.7 dB at *BER* of 10^{-6} is obtained by using internal pilot insertion.

The systems are also tested over multipath Rayleigh fading channel as shown in Figure 8. In general, all systems have shown performance degradation compared with their counterparts tested over *AWGN* channel. Systems utilize the proposed pilot insertion technique provide a significant improvement over the ones using the conventional technique. On average, the improvement in terms of required *SNR* for given *BER* increased at relatively lower error rates. This is about 4dB at *BER* equal or below 10^{-6}.

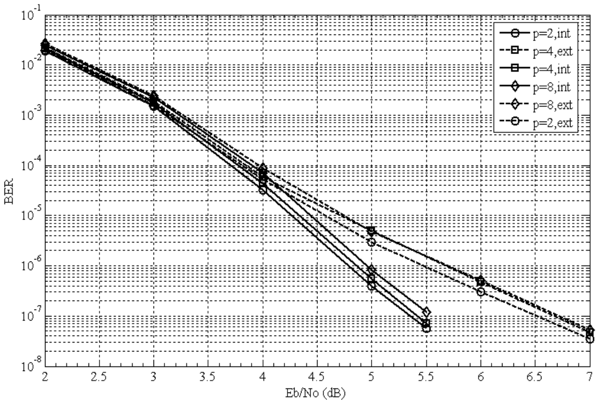

Fig.7. *BER* performances of internal and external pilot insertion techniques with different number of pilots over *AWGN channel* (K=4, g=$(13,15)_8$, N=56, S-random interleaver, 5 iterations, R_c=1/3).

6. CONCLUSION

A simple scheme to improve the distance properties of turbo code using internal pilot insertion is introduced. The resulting code has better error performance when compared to the classical ones in particular at high SNR. The results of the simulation tests have shown that the improvement in error performance at very low error rates may be better than 4 dB. It is also possible to make the rate of the coding scheme adaptive under fixed frame length constraint. This may assist in operating over severely distorted channels. The proposed coding scheme needs no extra complexity at both the encoder and decoder.

7. REFERENCES

[1] C. Berrou, A. Glavieux & P. Thitimajshima, "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes," IEEE International Conference on Communications ICC'93, Geneva, Switzerland, Vol-2, pp. 1064–1070, May 1993.

[2] B. Mielczarek & A. Svensson. "Joint Adaptive Rate Turbo Decoding and Synchronization on Rayleigh Fading Channels", Proceedings of IEEE Vehicular Technology Conference VTC 2001, Rhodes, Greece, Vol-2, pp. 1342-1346, May 2001.

[3] L. Hanzo and R. Steele, "The Pan-European Mobile Radio System, Parts i and ii," European Transactions on Telecommunications, Vol-5, pp. 117–148, Mar 1994.

[4] S. Dolinar, D. Divsalar, and F. Pollara, "Turbo code performance as a function of code block size," Proceedings of IEEE International Symposium on Information Theory, Cambridge, Massachusetts, Pages 32, August 1998.

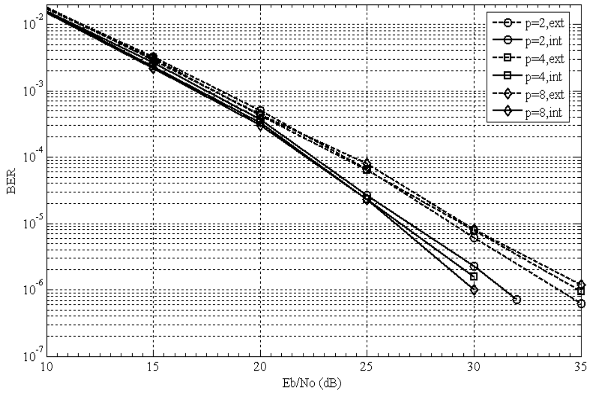

Fig.8. *BER* performances of internal and external pilot insertion techniques with different number of pilots over *multipath fading channel (K*=4, g=$(13,15)_8$, N=56, S-random interleaver, 5 iterations, R_c=1/3) .

[5] J. Hagenauer and P. Hoeher, "A Viterbi Algorithm with Soft-decision Outputs and its Applications," Proceedings of Global Communications Conference GLOBECOM'89, Dallas, Texas, Vol-3, pp. 1680-1686, November 1989.

[6] L. Perez, J. Seghers, and D. Costello, Jr. "A Distance Spectrum Interpretation of Turbo Codes", IEEE Transactions on Information Theory, Vol. 42, No. 6, pp. 1698–1709, November 1996.

[7] J. Yuan, B. Vucetic, and W. Feng, "Combined Turbo Codes and Interleaver Design", IEEE Transactions on communications, Vol. 47, No. 4, pp. 484-487, April 1999.

[8] S. Benedetto and G. Montorsi, "Unveiling Turbo Codes: Some Results on Parallel Concatenated Coding Schemes," IEEE Transaction on Information Theory, Vol. 42, No. 2, pp. 409–428, Mar. 1996.

[9] A. Hamad, "Improved Turbo Coded Signals Using Pilot Insertion for Single Carrier and OFDM Transmission," Ph.D Thesis, University of Technology, Baghdad, Iraq, July 2007.

[10] S. Adireddy & L. Tong, "Optimal Placement of Known Symbols for Nonergodic Broadcast Channels," Proceedings of Information Sciences & systems Conferences CISS 2002, Princeton, NJ, March 2002.

[11] L. Tong, B. Sadler & M. Dong, "Pilot-assisted wireless transmissions: general model, design criteria, and signal processing", IEEE signal processing magazine, Vol. 21, No. 6, pp.12-25, November 2004.