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Chaotic Properties of the Modified Hénon Map

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Abstract: In this study, a dynamical system of modified Hénon map on two dimension with the form $MH_{a,b}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1+ax+\cos 2\pi y\\ by\end{pmatrix}$ is studied. We find some general properties, and we show some chaotic properties of it. The proposed paper prove that the modified Hénon map has positive Lypaunov exponent and sensitivity dependence to initial condition. Fpr applying the suggested scheme, Mat lab programs are used to draw the sensitivity of modified Hénon map and compute the Lyapunov exponent.

Key words: modified the Hénon map, fixed point, attracting- expanding area, Lyapunov exponent, Sensitive dependence on initial conditions

INTRODUCTION

There are several definitions for chaos were proposed .When the system is sensitive to initial condition on its domain or has positive Lyapanov exponent at each point in its domain then this system will be chaotic (Denny,1992). Chaotic behavior of lows dimensional map and flows has been generally considered and described (Sprott and Chaos, 2003). Previously, the French space expert -mathematician Michel–Henon was scanned for simple two-dimensional squeezing extraordinary properties of more complication system the result was family of the form:

$$H_{a,b}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1-ax^2+y\\ bx\end{pmatrix}$$

Where a, b are parameter and real number (Denny ,1992).This is a nonlinear two dimensional map, which can also be written as a two-step recurrence relation:

$$Xn+1 = 1 - ax2n + byn - 1$$

The parameter b is a measure of the rate of area contraction, and the Hénon map is the most general two-dimensional quadratic map with the property that the contraction is Independent of x and y. For b = 0, the Hénon map reduces to the quadratic map which Follows period doubling route to chaos. Bounded solutions exist for the Hénon map Over a range of a and b values. Hénon map had two fixed points. Which can be either attracting, saddle or repelling points depending on the choice of parameters (a, b).

Hénon map had two fixed points. Which can be either attracting, saddle or repelling points depending on the choice of parameters (a, b) (Shameri, 2012) In this research, we introduce a new map in two dimension, we will call it the modified Hénon map as:

$$MH_{a,b}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1+ax+\cos 2\pi y\\ by\end{pmatrix}$$

Preliminaries: Let $I: R^n \to R^n$ be a map, we say I is C^∞if its P-th partial derivatives exist and continuous for all $P \in Z^+$, and it is called diffeomorphism if it is one – to – one , onto $C^\infty$and its inverse is C^∞, let W be subset of R^2, and μ be any element in R^2 concider $G: W \to R^2$ be a map. Furthermore assume that the first partials on R^2 by DG:

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$$(u_\circ) = \begin{pmatrix} \frac{\partial f_1}{\partial x}(u_\circ) & \frac{\partial g_1}{\partial y}(u_\circ) \\ \frac{\partial f_2}{\partial x}(u_\circ) & \frac{\partial g_2}{\partial y}(u_\circ) \end{pmatrix}$$

For all $u_\circ \in R^2$ the determinate of $DG(u_\circ)$ is called Jacobian of G at $u_\circ$ and denoted by $JG(u_\circ) = detDG(u_\circ)$. So G is said to be area expanding at $u_\circ$ if $|Det\, DG(u_\circ)| > 1$, G is said to be area contracting at $u_\circ$ if $|Det\, DG(u_\circ)| < 1$ .Let B be $n \times n$ matrix the real number λ is called Eigen value of B. The point $\binom{p}{q}$ is called fixed point if $G\binom{p}{q} = \binom{p}{q}$ it is repelling fixed point if λ_1 and $\lambda_2 > 1$ in absolute value , and it is an attracting fixed point if λ_1 and $\lambda_2 < 1$ in absolute value $B \in GL(2, Z)$ with det $(B) = \pm 1$ is called hyperbolic matrix if $|\lambda_1| \neq 1$ where λ_1 are the eigenvalue (Denny, 1992).

MATERIALS AND METHODS

General properties of modified Hénon map: In this study, we find the fixed point and study the general properties of modified Henon map (one to one, onto, c^∞, and invertible) which make it diffeomorphism and find the value of a, b which $MH_{a,b}$ has area contracting or expanding.

Proposition (3.1): If $a \neq 1$ and $b \neq 1$ then modified Hénon map $MH_{a,b}$ has unique fixed point.

Proof: By the definition of fixed point, we get:

$$MH_{a,b}\binom{x}{y} = \begin{bmatrix} 1 + ax + \cos 2\pi y \\ by \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Then $by = y$ since $b \neq 1$ then $y = 0$ since $1 + ax + \cos 2\pi(0) = x$ then $bx(a-1) = -2 \quad \Rightarrow x = -2/(a-1)$ then

$$\begin{bmatrix} 2/1 - a \\ 0 \end{bmatrix}$$

is the fixed point Let $\begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} n \\ s \end{bmatrix}$. Also by the definition of fixed point

$$MH_{a,b}\binom{n}{s} = \begin{bmatrix} 1 + an + \cos 2\pi s \\ bs \end{bmatrix} = \begin{bmatrix} n \\ s \end{bmatrix}$$

Since $bs = s$ and $b \neq 1$ then $s = 0$. Also since $1 + an + \cos 2\pi s = x$ and $a \neq 0$ then $n = -2/a - 1$.But this contradiction So:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} n \\ s \end{bmatrix}$$

Such that if $a \neq 0$ then:

$$\begin{bmatrix} 2/1 - a \\ 0 \end{bmatrix}$$

is the unique fixed point.

Proposition (3.2): If $a \neq 1, b = 1$ then $MH_{a,b}$ has infinite fixed point.

Proof: By definition of fixed point:

$$\begin{bmatrix} 1 + ax + \cos 2\pi y \\ by \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Since $b = 1$ then $y = y, 1 + ax - \cos 2\pi y = x \Longrightarrow x = -1 - \frac{\cos 2\pi y}{a} - 1$. Then $MH_{a,b}$ has infinite fixed point:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-1 - \cos 2\pi y}{a - 1} \\ y \end{bmatrix}$$

Proposition (3.3): If $a = 1$, $b \neq 1$ then $MH_{a,b}$ has no fixed point

Proof: By definition of fixed point:

$$\begin{bmatrix} 1 + x + \cos 2\pi y \\ by \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Since $b \neq 1$ then $by = y \Longrightarrow y = o, x - x = 1 + \cos 2\pi by$ then $H_{a,b}$ has no fixed point.

Proposition (3.4): The Jacobian of the modified Hénon map $MH_{a,b}$ is ab

Proof:

$$DH_{a,b}\binom{x}{y} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} a - & -2\pi \sin 2\pi y \\ 0 & b \end{bmatrix}$$

Then $j = detDH_{a,b}(v_\circ) = ab$

Proposition (3.5): Let $MH_{a,b}$ be modified Hénon map

- If $|a| < 1$ and $|b| < 1$ then $MH_{a,b}$ is area contracting map and $|J| < 1$.
- If $|a| > 1$ and $|b| > |j| > 1$.

Proof: if $|a| < 1$ and then $|J| = |ba| = |b||a| < 1$. Therefore, by definition area contracting the Jacobian of modified Hénon map< 1.

Similarity proof (2): By definition area expanding the Jacobian of modified Hénon map> 1.

Proposition (3.6): The modified Hénon map $MH_{a,b}$ is area contracting if

- $|b| > 1, b \neq 0$ and $|a| < 1/|b|$.
- $|a| > 1, a \neq 0$ and $|b| < 1/|a|$.

Proof: If $|b| > 1$, $b \neq 0$ and $|a| < 1/|b|$ then $|j| = |a||b| \Rightarrow |J| < |b|.1/|b|$
$|b| < 1$. So the Jacobian of modified Hénon map< 1 so from definition area contracting.

Similarity proof (2): By definition area contracting the Jacobian of modified Hénon < 1.

Proposition (3.7): The modified Hénon map $MH_{a,b}$ is area expanding if

- $|a| > 1$, $a \neq 0$ and $|b| > 1/|a|$.
- $|b| > 1$, $b \neq 0$ and $|a| > 1/|b|$.

Similarity proof (proposition (3.6))

Proposition (3.8): The eigenvalue of modified Hénon map $MH_{a,b}\binom{x}{y}$ are a, b.

Proof: The eigenvalue of:

$$DMH_{a,b}\binom{x}{y} = Det^{(DH_{a,b}(v)-\lambda I)}$$

$$\det\begin{bmatrix} a-\lambda & -2\pi\sin 2\pi y \\ 0 & b-\lambda \end{bmatrix} = 0 \Rightarrow (a-\lambda)(b-\lambda) = 0$$

Then $\lambda_1 = a$, $\lambda_2 = b$

Proposition (3.9): -Let be modified Hénon map and a≠0, b≠0 then

- If $|a| < 1$ and $|b| < 1$then the fixed point of $MH_{a,b}$ is attracting fixed point.
- If $|a| > 1$ and $|b| > 1$ then the fixed point of $MH_{a,b}$ is repelling fixed point.
- If $|a| > 1$ and $|b| < 1$ then the fixed point of $MH_{a,b}$is saddle fixed map.
- If $|a| < 1$ and $|b| > 1$ then the fixed point of $MH_{a,b}$is saddle fixed map.

Proof: - By proposition (3.5- 3.7) and definition it's satisfying (1- 4).

Proposition (3.10): If $b \neq 0$, $a \neq$ then modified Hénon map $MH_{a,b}$ is diffeomorpism.

Proof: $MH_{a,b}$ Is one - to – one map

Let :

$$\binom{x_1}{y_1},\ \binom{x_2}{y_2} \in R^2$$

Such that:

$$MH_{a,b}\binom{x_1}{y_1} = MH_{a,b}\binom{x_2}{y_2}$$

Then

$$\binom{1 + ax_1 + \cos 2\pi y_1}{by_1} = \binom{1 + ax_2 + \cos 2\pi y_2}{by_2}$$

So

$$by_1 = by_2 \quad \Rightarrow \quad y_1 = y_2$$

$$1 + ax_1 + \cos 2\pi y_1 = 1 + ax_2 + \cos 2\pi y_2$$

$$ax_1 = ax_2 \Rightarrow x_1 = x_2$$

$MH_{a,b}$ is onto: Let $\binom{u}{v}$ any element in R^2 such that $y = v/b$ and :

$$x = \frac{u - 1 - \cos 2\pi \frac{v}{b}}{a}$$

Then:

$$MH_{a,b}\binom{x}{y} = \binom{1 + ax + \cos 2\pi y}{by}$$

Let;

$$\binom{x}{y} = \binom{u}{v} \in R^2$$

$$u = 1 + ax + \cos 2\pi y \Rightarrow ax = u - 1 - \cos 2\pi y$$

$$x = \frac{v-1-\cos 2\pi y}{a} \qquad (1)$$

$$v = by \Rightarrow y = \frac{v}{b}$$

Replacing Eq.2 in 1 we get on.

$$x = \frac{v-1-\cos 2\pi v/b}{a} \qquad (2)$$

Then there exist:

$$\begin{pmatrix} \frac{v-1-\cos 2\pi v/b}{a} \\ \frac{w}{b} \end{pmatrix} \in R^2$$

Such that:

$$=\begin{bmatrix} 1+a\left[\frac{u-1-\cos 2\pi v/b}{a}+\cos 2\pi v/b\right] \\ b\frac{v}{b} \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$MH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

Then $MH_{a,b}$ is onto.

$MH_{a,b}$ is $\mathbf{C}^\infty$ since:

$$MH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 1+ax+\cos 2\pi y \\ by \end{bmatrix}$$

Then All partial derivatives continuous and exist such

$$\frac{\partial f_1}{\partial x_1}=a,\ \frac{\partial^2 f_1}{\partial x^2}=0,\ \ldots\ldots\ldots\ldots\ldots\ldots \frac{\partial^n f_1}{\partial x^n}=0\ n\in N$$

$$\frac{\partial f_1}{\partial y_1}=-2\pi\sin 2\pi y,\ \frac{\partial^2 f_1}{\partial y^2}=4\pi^2\cos 2\pi y \text{ for } \forall n\in N$$

$$\frac{\partial f_2}{\partial x}=0\ \ldots\ldots\ldots\ldots \frac{\partial^n f_2}{\partial x^n}=0\ \ n\in N$$

$$\frac{\partial f_2}{\partial y}=b,\ \frac{\partial^2 f_2}{\partial y^2}=0\ \ldots\ldots\ldots \frac{\partial^n f_2}{\partial y^n}=0\quad \forall n\in N, n\geq 2$$

$MH_{a,b}$ has an inverse*:*

$$MH_{a,b}^{-1}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \frac{u-1-\cos(2\pi v/b)}{a} \\ \frac{v}{b} \end{pmatrix}$$

Such that let:

$$H_{a,b}^{-1}\circ MH_{a,b}\begin{pmatrix} u \\ v \end{pmatrix} = H_{a,b}\circ H_{a,b}^{-1} = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

Then:

$$MH_{\cdot a,b}^{-1}\circ MH_{a,b} = MH_{a,b}^{-1}\begin{pmatrix} 1+au+\cos 2\pi v \\ bv \end{pmatrix}$$

$$=\begin{pmatrix} \frac{1+au+\cos(2\pi v/b)-1-\cos 2\pi v/b}{a} \\ \frac{bv}{b} \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$$

And:

$$MH_{a,b}\circ M\,H_{a,b}^{-1}\begin{pmatrix} u \\ v \end{pmatrix} = MH_{a,b}\begin{pmatrix} \frac{u-1-a\cos 2\pi v/b}{a} \\ \frac{v}{b} \end{pmatrix}$$

Then $MH_{a,b}$ has an inverse and it is invertible.

Remark:

- If a=0, b=0 then $MH_{a,b}$ is not onto
- If a≠ 0 , b= 0 then is not onto
- If a=0, b≠0 and then $MH_{a,b}$ is not onto

Remark: If a= 0 then $MH_{a,b}$ *is* not one to one, so it is not diffeomorphism.

Proposition (3.11): $DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}$ is a Hyperbolic matrix If $|a|\neq 1$, $|b|\neq 0,1$ iff $|ab|=1$.

Proof: Let $DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}$ be a hyperbolic matrix then by definition

$$DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix} \in GL(2,R)$$

Then:

$$det\left(DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}\right) = ba = \pm 1$$

Hence $|ba|=1$.

$\Leftarrow$) Let $|ba|=1$ then:

$$det\left(DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}\right) = ba = \pm 1$$

$$DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix} \in GL(2,R)$$

and by the relation between roots and coefficients $|ba| = |\pm 1| = 1$ so, if $|b|\neq 1$ and $|a|\neq 1$ then $|a| = \frac{-1}{|b|}$ such that $|b|\neq 0$ and by proposition (3.3). $\lambda_1 = a$, $\lambda_2 = b$ are two real number and since: $|\lambda_1| = |a| \neq 1$ and $|\lambda_2| = |b| \neq 1$ and since R is totally order set so either $|a|>1$ or $|a|>1$ or $|b|<1$ if $|a|>1$ then $|b|=1/|a|<1$, and if $|b|<1$ then $|a|=1/|b|>1$

RESULT AND DISCUSSION

Sensitive Dependence on Initial Condition of modified Hénon map $MH_{a,b}$: The $K: X\rightarrow X$ is said to be sensitive dependence on initial conditions if there exist $\eta>0$ such that for any $p_0\in X$ and any open set $W\subset X$ containing p_0 there exists $q_0\in W$ and $m\in z^+$ such that $d(K^m(p_0),K^m(q_0))>\eta$ That is $\exists\eta>0, \forall p, \forall\delta>0, \exists q\in B_\delta(p), \exists m: d(f^m(p_0), f^m(q_0))\geq\eta$ (Elaydi,2000).

.Despite the fact that there is no widespread concurrence on definition of chaos, is for the most part concurred that a chaotic dynamical system should exhibit sensitive dependence on initial conditions as chaotic. Iftichar *et al.* (2013). Let $P = (P_1, P_2, \ldots \ldots, P_n)$ and $q = (q_1, q_2, \ldots \ldots, q_n) \in R^n$ we write if and only if there exist $(\{1, \ldots, n\})$, such that P.

Proposition (4.1): If $|b| > 1 \text{or} |a| > 1$ then $MH_{a,b}$ has sensitive dependence on initial condition

Proof: Let:

$$X = \binom{x_1}{y_1} \in R^2$$

Since:

$$MH_{a,b}(x) = \begin{bmatrix} 1 + ax_1 + \cos 2\pi y_1 \\ by_1 \end{bmatrix}$$

Case1: If $|x| \le 1$ by hypothesis and by definition

$$MH_{a,b}(x) < \begin{bmatrix} 1 + ax_1 \\ by_1 \end{bmatrix}$$

And:

$$MH^2{}_{a,b}(x) < \begin{bmatrix} 1 + a^2x_1 \\ b^2y_1 \end{bmatrix}$$

That is:

$$MH^n{}_{a,b}(x) < \begin{bmatrix} 1 + a^nx_1 \\ b^ny_1 \end{bmatrix}$$

Thus if $|b| > 1$, $n \longrightarrow \infty$

$$MH_{a,b}{}^n{}_{a,b}(x) \longrightarrow \infty$$

Let:

$$y = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \in R^2$$

Such that $d(x, y) < \delta$

$$d\left(MH_{a,b}(x).MH_{a,b}(y)\right) = \sqrt{(1 + ax)^2 + (by)^2}$$

$$d\left(MH^2{}_{a,b}(x).MH^2{}_{a,b}(y)\right) = \sqrt{(1 + a(1 + ax))^2 + (b(by))^2}$$

$$d\left(MH^n_{a,b}(x).MH^n_{a,b}(y)\right) = \sqrt{(1 + a(1 + ax))^{2n} + (b(by))^{2n}}$$

If $|ab| > 1$ and:

$$d\left(H^n_{a,b}(x).H^n_{a,b}(y)\right) \longrightarrow \infty$$

Hence $MH_{a,b}$ has sensitive dependence on initial condition.

Case2: If $|x| > 1$ then the iterates of modified Hénon map are diverge .thus it has sensitive dependence on initial condition.(Fig.1). Then we study the sensitive dependent on initial condition of map by varying the point (as follow (i=1,2) control parameters (a , b) by using (matlab).

The Lyapunov Exponents of modified Hénon map $MH_{a,b}$: Let $F: R^n \to R^n$ be continuous differential map .The map will have n Lyapunov exponents, say

$$L_1^{\pm}(y,v_1), L_2^{\pm}(y,v_2), L_3^{\pm}(y,v_3) \ldots L_n^{\pm}(y,v_n),$$

For a minimum Lyapunov exponent that is

$$L^{\pm}(y, v) = \max\{L_1^{\pm}(y,v_1), L_2^{\pm}(y,v_2), L_3^{\pm}(y,v_3), \ldots, L_n^{\pm}(y,v_n)\}$$

Where $v = (v_1,v_2,\ldots,v_n)$. Where all y in R^n in direction V the Lyapunov exponent was defined of a map F at y by $L^{\pm}(y,v) = \lim_{T(n\to\infty)} 1/n \ln ||DF_y^n v||$ whenever the limit exists. Where $v = (v_1, v2,\ldots,v_n)$" (Sturman et al,2006). The usual test for chaos is calculation of the largest Lyapunov exponent (Bin and Zhang, 2006). A positive largest Lyapunov exponent indicates chaos. When one has access to the equations generating the chaos, and which measure the rates of separation from the current orbit point along m orthogonal directions .The Lyapunov exponent is greater than zero. A quantitative measure of the sensitive dependence on the initial conditions is the Lyapunov exponent it's the average rate of divergence (or convergence) of two neighboring trajectories in the phase space.

Proposition (5.1): If either then the has positive Lyapunov exponents.

Proof: - If $|a| < 1$ *and* $|b| > 1$ by proposition $|\lambda_1| = |a|$, if $|a| < 1$ since

$$L_1\left(\binom{x}{y}, v_1\right) = \lim_{n\to\infty} \frac{1}{n} \ln \left\| DMH_{a,b}\binom{x}{y}, v_1 \right\| < 0$$

But if $|b| > 1$ then:

$$L_2\left(\binom{x}{y}, v_2\right) = \lim_{n\to\infty} \frac{1}{n} \ln \left\| DMH_{a,b}\binom{x}{y}, v_2 \right\| > 0$$

So the Lyapunov exponent

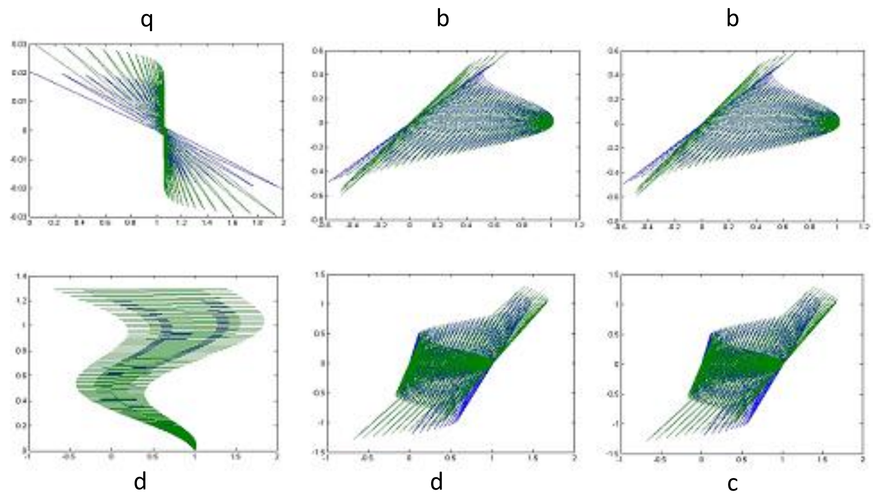

Fig. 1: Modified Henon map is not sensitive dependence on initial condition

Table 1: Negative Lyapunov exponent

Variable	(x , y)	a	b	L_1	L_2
A	(0.04,0.03)	-0.88	-0.79	-0.1278333715	-0.2357223335
B	(0.5,0.4)	-0.98	-0.99	-0.0202027073	-0.0100503359
C	(0.1,0.2)	-0.70	-0.99	-0.3566749439	-0.0100503359
D	(1.3,1.2)	-0.99	0.97	-0.1278333715	-0.0304592075
E	(1.4,1.3)	-0.98	-0.99	-0.0202027073	-0.0100503359
F	(0.7,0.6)	0.77	0.96	-0.2613647641	-0.0408219945

Table 2: Position true

Variable	(x , y)	a	b	L_1	L_2
1	(1.2 , 1.7)	-1.0028	1.0001	0.0027960873	0.0000999950
2	(1.3 , 1.2)	-0.99	1.00019	-0.0100503359	0.0001899820
3	(0.4,0.2)	-1.0088	1.005	0.0087615057	0.0049875415
4	(0.06,0.05)	1.003	-0.99	0.0029955090	-0.0100503359
5	(1.1 , 1.2)	-1.0099	1.0004	0.0098513161	0.0003949200
6	(1.3,1.2)	-1.0016	1.00009	0.0015987214	0.0000899960

Table 3: Arbitrary point

Variable	A	b	L_1	L_2
1	1	1	0	0
2	-1	-1	0	0
3	-1	-1.006	0	0.005982071
4	-1.5	-1	0.4054651081	0
5	1.0022	1	0.0021975835	0
6	1	1.45	0	0.3715

$$L^{\pm}(x,v) = \max\{L_1^{(\pm}(x,v_1), L_1^{(\pm}(x,v_2)$$

That is $MH_{a,b}$ has positive Lyapunov exponent.

Proposition (5.2) (Michael and Garrett, 2002): If $L^{\pm}(x,v) = x > 0$ for some vector, then there is a sequence $\{m_i\}$, $I \to \infty$ Such that for every:

$$\delta > 0 \ \left\| dDMH_{a,b}\binom{x}{y} v \right\| \geq e^{(x-\delta)m_i}\|v\|$$

This implies that, for a fixed; there is a point $\binom{x_1}{y_1} \in R^2$ such that:

$$d(DMH_{(a,b)}(m_i)((x@y),(DMH_{(a,b)}(m_i)((x_2@y_2)) \geq 1/2e^{((x-\delta)m_i)|(d(x@y))|}$$

This does not imply sensitive dependence on initial condition. We use the mat lab program to compute Lyapunov exponent in several value of parameters a and b. (Table 1) show that the points I in Fig. 1 have negative Lyapunov Exponent. Table 2 show that the Proposition (5.1) is true In Table 3 we choose arbitrarily point (0, 0), this table show us that the Lyapunov Exponent equal zero If |b| = 1 and |a| =1, that is, this point is a bifurcation point when |b| = 1 and |a|=1.

CONCLUSIONS

In this research, we have presented a two-dimensional dynamical system. The mathematical properties of the modified Hénon maps

- If $a \neq 1$ and $b \neq 1$ the $MH_{a,b}$ has unique fixed point,2
- If $a = 1$, $b \neq 1$ then $MH_{a,b}$ has no fixed point2)
- The eigenvalues of the

$$MH_{a,b}\binom{x}{y}$$

are λ_1 =a , λ_2=b

- The area contracting and expanding of $MH_{a,b}$
- If $|a| < 1$ and $|b| < 1$ then $MH_{a,b}$ is an area contracting map
- If $|a| < 1$ and $|b| < 1$ then $MH_{a,b}$ is an area expanding map
- If $|a| > 1$, $a \neq 0$ and $|b| < 1/|a|$ or $|b| > 1$, $b \neq 0$ and $|a| < 1/|b|$ then$MH_{a,b}$is an area contracting map
- If $|a| > 1$, $a \neq 0$ and $|b| > 1/|a|$ or $|b| > 1$, $b \neq 0$ and $|a| > 1/|b|$.
- If $|b| > 1$, $b \neq 0$ and $|a|^2 > 1/|b$ and then then $MH_{a,b}$ is an area expanding map

The modified Hénon map are close and they have sensitive dependence on initial condition, they have positive Lyapunov exponents

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