On Fourth Combining of Wavelet from RASP Family with Wavelet from SLOG Family and Membership Function

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الملخّص: التركيب بين دوال المويجات مِنْ عوائلِ مختلفةِ مثل عائلة RASP و دالة او دوال اخرى من عائلة SLOG مع دالةِ عضوية اعتيادية، مثل دالة العضوية لكاوس من خلال التركيب حتى الرتبة الرابعة ، والتي تحقق بعض شروط المويجات. وذلك للحصول على دالة عضوية جديدة سوف نسميها دالة العضوية GaSLR من الكلمات (**Ga**ussian **SL**OG **R**ASP) ، مع صفات مجتمعة من الدوال التي انشأت منها ، وهذه الدالة يمكن ان تستخدم كدالة تنشيط في بعض الانظمة مثل :الشبكات العصبية المضببة ، شبكات المويجات ، والشبكات العصبية الاخرى .

الكلمات المفتاحية: المويجات ، مويجات SLOG ، مويجات RASP ،المنطق المضبب ،دالة العضوية ، دالة العضوية لكاوس.

Abstract: The compose among the wavelets functions from different families, such RASP family with other(s) from SLOG family with normal membership function, such Gaussian membership function, that apply some condition for wavelets. That through fourth order composes to get on new membership function we will call it (GaSLR)membership function(from words **Ga**ussian **SL**OG **R**ASP), with combined features from the functions that it were constructed from it, this function could be used as activation function in several models such as; Fuzzy Neural Networks, Wavenets, and other Neural Networks .

Key words: wavelets, SLOG wavelets, RASP wavelets, fuzzy logic, membership function, Gaussian membership function .

1. Introduction;

Most Artificial Intelligent Models such as; Neural Networks(NNs), Wavenets, Fuzzy logic(FL), Fuzzy Controller …, have important applications in real life, but, also have some drawbacks (Jain L.,1997). The trying to resolve ineffective point(s) in some previous methods is to complete its false programming but not as alternatives on it. Such techniques named “Hybrid Systems”. Hybridizing techniques is compiling two (more) techniques models that enhances with other .

A hybrid methods that compiling soft computing methods with wavelet theory have probability to composing two brain's abilities; ability to select an appropriate resolution to the problem description and to somewhat tolerant of imprecision(Synergy,2000). i.e. Computationally, NNs are approaching significantly than others in matching complicated, recognition, vague, or incomplete patterns (Daniel Klerfors,1998). So it has advantage in signal processing, pattern recognition, classifications and predications. In spite of that it have some drawbacks such; generalization, convergence, local minima, to be not efficient in numerous applications **.**So many attempts to resolve these drawbacks for example; adding a momentum term into adjustment equation to increase the converging speed and to avoid a local minima as in reference(Zurada J.,1997), or using a wavelet transform (WT) in neural networks in feature extraction, or to reduce the dimension of the input space. Also has been combined with NNs in wavenets, fuzzy wavenets, and fuzzy neural networks (Meng Joo,2016)(L.M.Reyneri,1999).

Using wavelet functions as universal approximations, introduced in function approximation and process modeling. That is; any function in $L^2(R)$ can be approximated(with dilation and translation are get from mother wavelet) in some accuracy with a finite sum of wavelets(Ossar Y.,2000) .

Wavelet has been combined with fuzzy logic to improve features of both as in flame detectors for on_line signal processing(*Thuillard M.,2000), and fuzzy wavelet which is a multiresolution fuzzy technique, first fuzzy wavelet concept was defined as a normal fuzzy set (its maximum 1), whose membership function satisfy the admissibility condition for wavelet (Mao Z. H.,1998) .

Hybrid systems opened the gates to more and more works for combination of fuzzy wavenets, fuzzy wavelets and activation function .

From modification of the activation function is as summation of two functions such as a summation of two sigmoidal functions(Leondes C.,1998), or in the modified method in construct the fuzzy rules by use wavelet transform and neural networks with fuzzy max_min inference(Daniel Klerfors,1998).

In 2000 March Thuillard, suggested a form for activation function as a summation of a wavelet and MF, implement a multiresolution analysis through architecture present a series of a networks and then repression a coefficients under validation operations as in (Thuillard M.,2000).

Here in this paper work, we will present a new concept which is construct activation function as compose of two wavelets from different families of wavelets with the a normal fuzzy set (its maximum degree= 1), whose membership function satisfy the admissibility condition for wavelet which is a Gaussian membership function (GMF) .

2. RASP Wavelets :

These **Rational functions** with **Second-order Poles** wavelets are real valued, odd functions with mean zero. The distinction among these mother wavelets is their rational form of functions being strictly proper and having simple/double poles of second order.

$RASP_1$ and $RASP_2$ Wavelet with form see the reference for more on these wavelets (Lekuati G,1997) or any reference on wavelets .

$$h_{RASP_1}(x) = \frac{x}{(x^2+1)^2} \quad \text{.... (1)}$$

$$h_{RASP_2}(x) = \frac{x\cos(x)}{x^2+1} \quad \text{.... (2)}$$

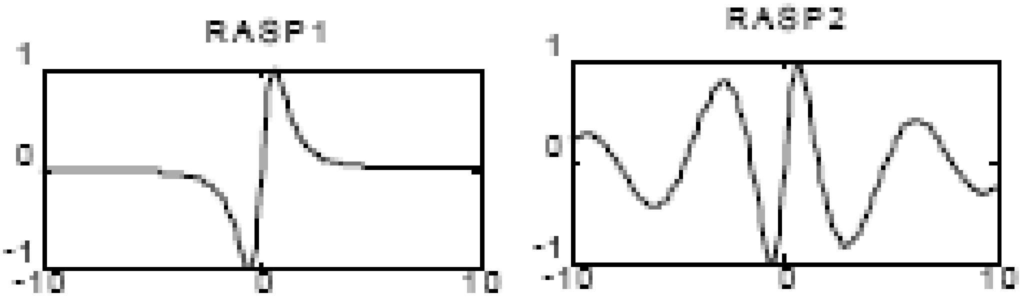

Figure (1):First and Second RASP Wavelets.

3. SLOG Wavelets :

This family of mother wavelets result from finite term sums of weighted and delayed logistic functions(Lekuati G,1997) .A logistic function which is a type of monotonically increasing, smooth, asymptotic sigmoid (*S*-shaped) function which usually represents the threshold function at the neuron output of the neural networks model .

Sigmoid function which centered at zero

$$\int_{-\infty}^{+\infty} \frac{dx}{e^x+1} = 0 \qquad \text{.... (3)}$$

The first SLOG mother wavelet exhibiting the following Superposition LOGistic sigmoid;

$$h_{SLOG_1}(x) = \frac{1}{1+e^{-x+1}} - \frac{1}{1+e^{-x+3}} - \frac{1}{1+e^{-x-3}} + \frac{1}{1+e^{-x-1}} \qquad \text{…. (4)}$$

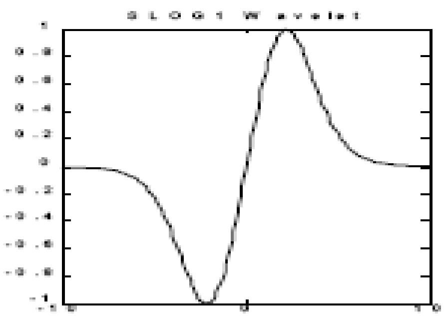

Figure (2) $Slog_1$ Wavelet

4. Composes Assumption ;

The assumed functions will be as following forms;

Put

$$f(x) = F_{RASP_1}(x) = \frac{x}{(x^2+1)^2},$$

$$g(x) = F_{RASP_2}(x) = \frac{x.\cos x}{x^2+1}$$

$$h(x)=F_{SLOG_1}(x)=\frac{1}{1+e^{-x+1}}-\frac{1}{1+e^{-x+3}}-\frac{1}{1+e^{-x-3}}+\frac{1}{1+e^{-x-1}}\ ,$$

And $l(x)$ is a (GMF) defined to be monotonic, and symmetric. Continuity and Normalization properties are implemented since has a maximum MF degree =1 normal fuzzy set ,and satisfy the admissibility condition for wavelet, and it is defined as;

$$l(x)=e^{-x^2/2} \qquad \text{.... (5)}$$

$$\vartheta=(l\circ h\circ f\circ g)(x)=l(h(f(g(x))))=l\circ h(f(g(x))) \qquad \text{.... (6)}$$

$$=l\circ h\left(\frac{\frac{x.\cos x}{x^2+1}}{\left(\left(\frac{x.\cos x}{x^2+1}\right)^2+1\right)^2}\right)$$

$$=l\circ h\left(\frac{x.\cos x}{x^2+1}.\frac{1}{\left(\frac{x^2.\cos^2 x}{(x^2+1)^2}+1\right)^2}\right)$$

If we simplify the compose $f\circ g(x)$ as;

$$f(g(x))=\frac{x.\cos x}{x^2+1}.\frac{1}{\left(\frac{x^2.\cos^2 x+(x^2+1)^2}{(x^2+1)^2}\right)^2}$$

$$=\frac{x.\cos x}{x^2+1}.\frac{(x^2+1)^4}{(x^2.\cos^2 x+(x^2+1)^2)^2}$$

$$=\frac{(x.\cos x)(x^2+1)^3}{(x^2.\cos^2 x+(x^2+1)^2)^2}$$

$$=\frac{(x.\cos x)(x^2+1)^3}{(x^4.\cos^4 x+2x^2.\cos^2 x(x^2+1)^2+(x^2+1)^4)^2}$$

By supposing $u=x^2+1$, $v=x.cox(x)$, are functions with independent variable x , then;

$$f(g(x))=\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)^2} \qquad \text{….(7)}$$

So here compute the compose $l\circ h(x)$, then ;

$$l\circ h(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)^2})=l(\frac{1}{1+e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})+1}}-\frac{1}{1+e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})+3}}$$

$$-\frac{1}{1+e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})-3}}+\frac{1}{1+e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})-1}})$$

$$=e^{-\left[(\frac{1}{1+e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})+1}}-\frac{1}{1+e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})+3}}-\frac{1}{1+e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})-3}}+\frac{1}{1+e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})-1}})^2\right]/2}$$

2. Second Attempt :Through suppose the compose as;

$$h\circ l(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})=h(e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})^2/2}) \quad .... (8)$$

$$=\frac{1}{1+e^{-(e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})^2/2})+1}}-\frac{1}{1+e^{-(e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})^2/2})+3}}$$

$$-\frac{1}{1+e^{-(e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})^2/2})-3}}+\frac{1}{1+e^{-(e^{-(\frac{(v)(u)^3}{(v^4+2v^2u^2+(u)^4)})^2/2})-1}}$$

5. Numerical Example for Assumption :

The Table (1) below shows us the supposed values for the independent variable x to calculate the values of composition functions, positive and negative values were given in the closed interval $[-100,100]$, and the resulted values for composed functions until fourth order and the gained MF with values within the closed interval $[0,1]$.

Table (1): Resulted values for MF from compose for First Attempt .

x	*u*	*v*	*f(g(x))*	*h(f(g(x)))*	*l(h(f(g(x))))*
-100	**10001**	**-86.2319**	**-0.00862**	**-0.00261**	**0.999997**
-90	**8101**	**40.32663**	**0.004978**	**0.001508**	**0.999999**
-80	**6401**	**8.83098**	**0.00138**	**0.000418**	**1**
-70	**4901**	**-44.3323**	**-0.00905**	**-0.00274**	**0.999996**
-60	**3601**	**57.14478**	**0.015869**	**0.004806**	**0.999988**
-50	**2501**	**-48.2483**	**-0.01929**	**-0.00584**	**0.999983**
-40	**1601**	**26.67752**	**0.016663**	**0.005047**	**0.999987**
-30	**901**	**-4.62754**	**-0.00514**	**-0.00156**	**0.999999**
-20	**401**	**-8.16164**	**-0.02035**	**-0.00616**	**0.999981**
-10	**101**	**8.390715**	**0.083071**	**0.025147**	**0.999684**
0	**1**	**0**	**0**	**0**	**1**
10	**101**	**-8.39072**	**-0.08307**	**-0.02515**	**0.999684**
20	**401**	**8.161641**	**0.020353**	**0.006164**	**0.999981**
30	**901**	**4.627543**	**0.005136**	**0.001556**	**0.999999**
40	**1601**	**-26.6775**	**-0.01666**	**-0.00505**	**0.999987**
50	**2501**	**48.2483**	**0.019292**	**0.005843**	**0.999983**
60	**3601**	**-57.1448**	**-0.01587**	**-0.00481**	**0.999988**
70	**4901**	**44.33234**	**0.009046**	**0.00274**	**0.999996**
80	**6401**	**-8.83098**	**-0.00138**	**-0.00042**	**1**
90	**8101**	**-40.3266**	**-0.00498**	**-0.00151**	**0.999999**
100	**10001**	**86.23189**	**0.008622**	**0.002611**	**0.999997**

The resulted MF is normal has three maxima values =1, and all values are high larger than the crossover point 0.5, and monotonic closer value to center on each side of the center of range of fuzzy set, and symmetric around the center value 0 (y-axis's) That is number equally in left and right of center value, these can be express as center of fuzzy set. Continuous satisfy the condition in the fuzzy set theory as; for any fuzzy set A.

$$\lim_{x \to c} \mu_A(x) = \mu_A(c) \qquad \text{.... (9)}$$

such that x and $c \in A$.

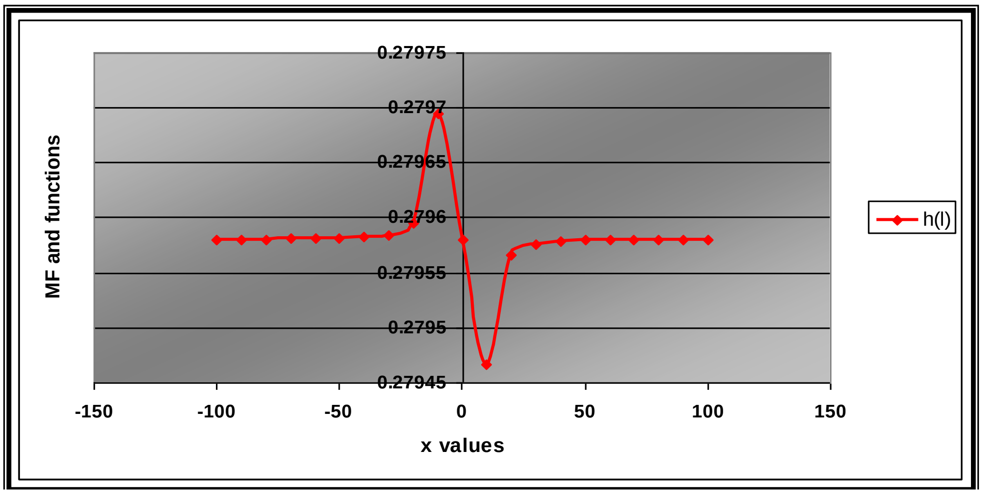

Figure (3): Gaussian MF for values in table(1)

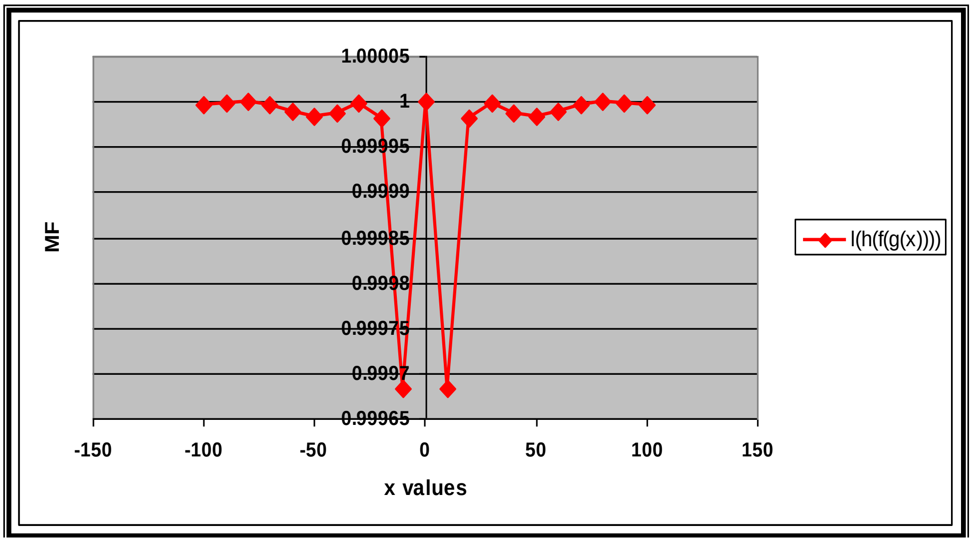

Figure (4): The gained MF shape for values in table(1)

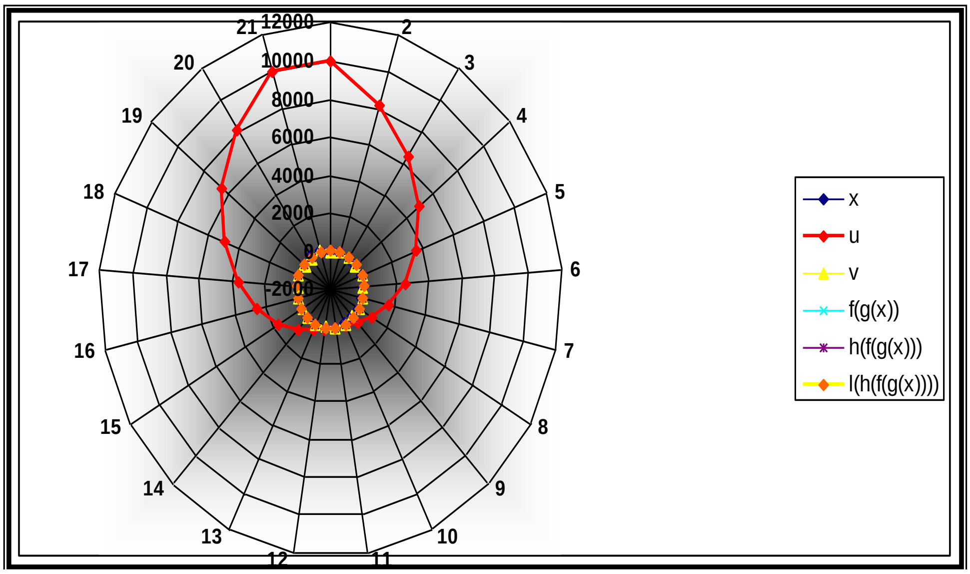

Figure (5): Drawing of gained MFs for values in table(1) through polar form.

6. Conclusion:

Through the presented idea of this work that involves combining three wavelets from different families with MF, with conditions and features for each one individual, these wavelets with features to be real valued, odd functions with mean zero, and having simple or double pole(s), monotonically increasing, smooth, asymptotic, while the MF must be normal fuzzy set (its maximum 1), satisfy the admissibility condition for wavelets. The MF that resulted is continuous and has three maximum values =1 (and 5 poles through domain of chosen values for independent variable), and all values are larger than the crossover point 0.5 and symmetric around the center value 0(y-axis's) ,and monotonic closer value to center on each side of the center of range of fuzzy set, That is number equally in left and right of center value ,these can be express as center of fuzzy set. A features are compiling features of wavelets and MF.

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