Rayleigh Pareto Distribution

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Abstract

In this paper Rayleigh Pareto distribution have introduced denote by(R_PD). We stated some useful functions. Therefor we give some of its properties like the entropy function, mean, mode, median , variance , the r-th moment about the mean, the rth moment about the origin, reliability, hazard functions, coefficients of variation, of sekeness and of kurtosis. Finally, we estimate the parameters so the aim of this search is to introduce a new distribution.

Keywords: Rayleigh Pareto distribution, r- th moment about the mean, coefficient of variation, coefficient of skeweness , coefficient of kurtosis , Mode, median, Geometric mean , Harmonic mean, Entropy function, Order statistics, reliability function, reversed hazard function finally the Maximum likelihood estimation.

الخلاصة

في هذه البحث، قدمنا توزيع رايلي باريتو لمتغير وبعض الدوال المستخدمة وبعض الخصائص عليه كـ دالة الكثافة , دالة التوزيع , دالة الغاية , معامل التغيير, معامل الالتواء , معامل التسطح(التفلطح) ,دالة الخطورة , الدالة المعولية, المنوال , الوسيط , المتوسط الحسابي الهندسي , المتوسط الحسابي التوافقي , دالة الامكان الاعظم.

الكلمات المفتاحية توزيع رايلي باريتو , دالة الكثافة , دالة التوزيع , دالة الغاية , معامل التغيير, معامل الالتواء , معامل التسطح(التفلطح) ,دالة الخطوره , الدالة المعوليه , المنوال , الوسيط , المتوسط الحسابي الهندسي , المتوسط الحسابي التوافقي , دالة الامكان الاعظم .

1. Introduction

Sometimes in real life we need to invent a new distribution or discover a new developed way useful in more students so the importance of lifetime distribution is used in the many real life fields such as the biostatistics , reliability and survival analysis ,we try to shed light on the a new Rayleigh Pareto distribution.

The aim of this search is to present anew Rayleigh Pareto distribution (R_PD) by using the similar method to those used by Al-Kadim ,K. and Boshi ,M.,(2013a). exponential Pareto distribution, Mathematical Theory and Modeling, ,Eugene, N., Lee, C. and Famoye, F. (2002)**.** Beta-normal Distribution and its applications.

The rest of the search is order as follows: in section 2 we describe Rayleigh Pareto distribution and discuss some of the properties and we introduce the estimate of the parameters by using the maximum likelihood estimation

2. The Rayleigh Pareto I Distribution (R_PD)

The cumulative distribution function (cdf) of Rayleigh Pareto distribution is given by as following way :

$$F(x) = \int_0^{\frac{1}{1-F^{\#}(x)}} f^*(x)dx$$

in this formula we companied the Rayleigh and Pareto distribution . where $F^{\#}(x)$ is the cdf of Pareto distribution, $(X) = 1-(\frac{p}{x})^{\theta}$, and $f^{*}(x)$ is the pdf of Rayleigh distribution, While in this paper we introduce the Rayleigh Pareto distribution of the form

$$F(x) = \int_{0}^{\frac{1}{1-F^{\#}(x)}} f^{*}(x)dx$$

$$F_{R.p}(X;p,b,\theta) = \int_{0}^{\frac{1}{1-[1-(\frac{p}{x})^{\theta}]}} \frac{x}{b^2}\, exp\left(-\frac{x^2}{2b^2}\right)dx = 1-exp\left(\frac{-(\frac{x}{p})^{2\theta}}{2b^2}\right)$$

Where P is a constant of the Pareto distribution **.**

$Let \quad 2\theta = \propto$

So that the the cumulative distribution function (cdf) of the Rayleigh Pareto distribution (R_PD) is given by

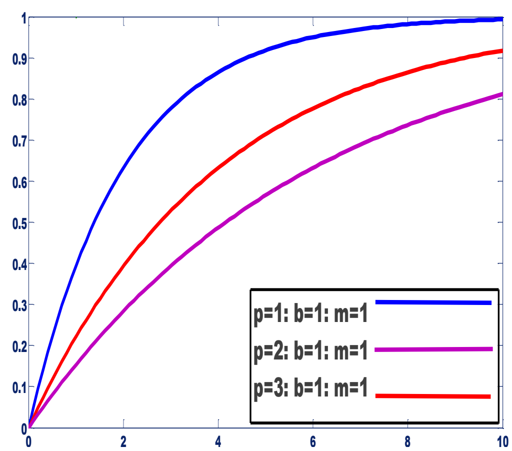
$$F_{R.p}(X;p,b,\theta) = 1-\exp\left(\frac{-1}{2b^2}\right)(\frac{x}{p})^{\alpha} \qquad (1)$$

Also the probability density function (p.d.f) of this distribution is given by :

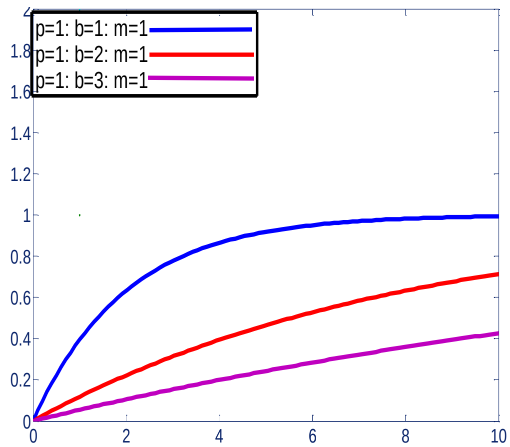
$$f_{R.p}(X;p,b,\propto) = \frac{d\,F_{R.p}(X;p,b,\propto)}{dx}$$

$$f_{R.p}(X;p,b,\propto) = \frac{\propto}{2b^2 p}\,(\frac{x}{p})^{\propto -1}\, e^{-\frac{1}{2b^2}(\frac{x}{b})^{\alpha}}\, I_{(0,\infty)}(x) \qquad (2)$$

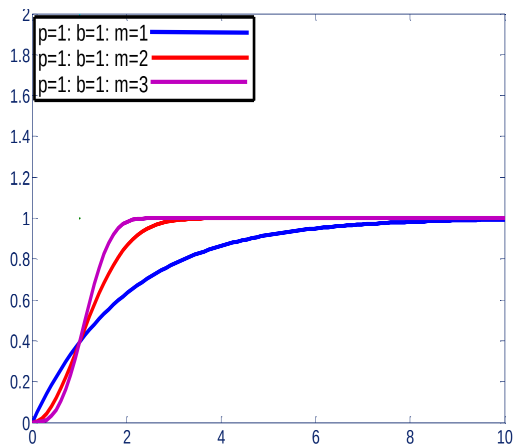
The plot of the p.d.f and c.d.f for the (R_PD) is given by of the form as :



Figure(1). Plot of the c.d.f. of the (R_PD) with the parameter p=1,2,3 ; b=1, m=α=1

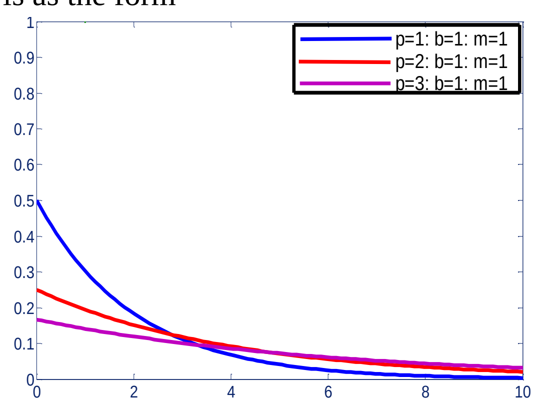


Figure(2). Plot of the c.d.f. of the RPD with the parameter p=1 ; b=1,2,3; m=$\alpha = 1$

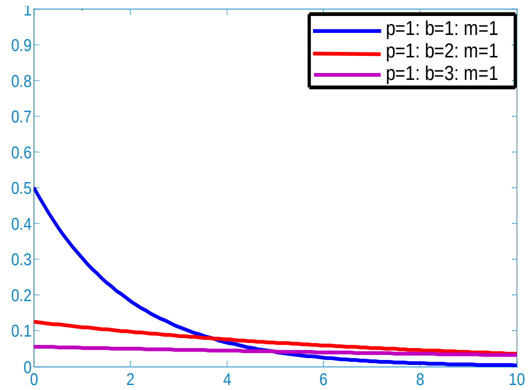


Figure(3). Plot of the c.d.f. of the RPD with the parameter p=1 ; b=1; m=α=1,2,3

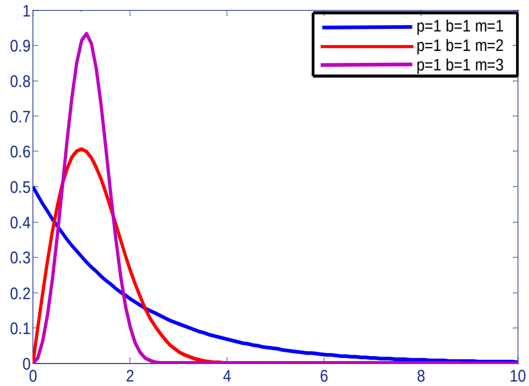
The plot of the pdf is as the form



Figure(4). Plot of the pdf. of the (R_PD) with the parameter p=1,2,3 ; b=1; m=α=1



Figure(5). Plot of the pdf. of the (R_PD) with the parameter p=1 ; b=1,2,3; m=α=1



Figure(6). Plot of the pdf. of the (R_PD) with the parameter p=1 ; b=1; m=α=1,2,3

2.1- Limit of p.d.f. and c.d.f.

The limit of (R_PD) is given by the form as:

$$\text{Lim}_{x\to 0}\ f_{R.p}(X;p,b,\propto) = 0 \qquad (3)$$

This is

$$\lim_{x\to 0} \frac{\propto}{2b^2}\left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}$$

$$= \frac{\propto}{2b^2}\lim_{x\to 0}\left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}} = 0$$

Also $\lim_{x\to\infty} f_{R.P}(x;p,b,\propto) = \lim_{x\to\infty}\frac{\propto}{2b^2}\left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}} = \text{NAN}$ $\qquad (4)$

Where NAN is not integer we know because

$$\lim_{x\to\infty}\left(\frac{x}{p}\right)^{\propto-1} = \infty \quad \text{and} \quad \lim_{x\to\infty} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}} = 0$$

And $0 \times \infty = NAN$

Also since the c.d.f. of this distribution is

$$F_{R.p}\ (X; b, p, \propto) = 1 - e^{-\frac{1}{2b^2}(\frac{x}{p})^{\propto}}$$

So

$$\lim_{x\to 0} F_{R.p}(X\,; p\,, b\,,\ \propto) \ = 0 \qquad (5)$$

Because $\lim_{x\to 0}\ [1 - e^{-\frac{1}{2b^2}(\frac{x}{p})^{\propto}}] = 1 - 1 = 0$

Where $\lim_{x\to 0}\ e^{-\frac{1}{2b^2}(\frac{x}{p})^{\propto}} = e^{-\frac{1}{2b^2}(\frac{0}{p})^{\propto}} = 1$

Also

$$\lim_{x\to\infty} F_{R.p}(X\,; p\,, b\,,\ \propto) = 1 \qquad (6)$$

since

$$\lim_{x\to\infty} F_{R.p}(X\,; p\,, b\,, \propto) = \lim_{x\to\infty}\ [1 - e^{-\frac{1}{2b^2}(\frac{x}{p})^{\propto}}]$$

$$= 1 - e^{-\frac{1}{2b^2}(\frac{\infty}{p})^{\propto}}] = 1 - 0 = 1$$

That is mean $0 \le F_{R.p}(X\,; p\,, b\,,\ \propto) \le 1$ ■

Proposition 1

The r-th central moment about the origin , and the r-th central moment about the mean of (R_PD) are as follows:

$$E\ (X-\mu)^{r} = \sum_{j=0}^{r} c_j^r\ (p\ \sqrt[\propto]{2b^2}\ )^j\ (-\mu)^{r-j}\ \Gamma\ (\frac{j}{\alpha} + \ 1)\ r = 1, 2\ , 3 \ldots\ldots\ldots \qquad (7)$$

And the r-moment about the origin is

$$E\ X^r = (\ p\ \sqrt[\propto]{2b^2}\ )^r\ \ \Gamma\ (\frac{r}{\alpha} + \ 1)\ , 1,2,3,\ldots\ldots\ldots\ldots \qquad (8)$$

Proof

The r-th moment about the mean is given by

$$E\ (X-\mu)^r = \int_0^{\infty} (\ x-\mu\ )^r\ f_{R.p}\ (\ x\ ; p,, b\ , \propto)\ dx$$

$$= \int_0^{\infty} (\ x-\mu\ )^r\ \frac{\propto}{2b^2\ p}\ (\frac{x}{p})^{\propto-1}\ \ e^{\frac{-1}{2b^2}(\frac{x}{p})^{\propto}}\ \ dx$$

Let $u = \frac{1}{2b^2}\ (\frac{x}{p})^{\propto} \ \Rightarrow du = \frac{\propto}{2b^2\ p}\ (\frac{x}{p})^{\propto-1}\ \ dx$

$x = 0 \ \to u = 0\ \ and\ \ \lim_{u\to\infty} u = \infty$

Also $x = \ p\ \sqrt[\propto]{2b^2}\ u$

Than we get

$$E\ (X-\mu)^{r} = \int_0^{\infty} (\ p.\sqrt[\propto]{2b^2}\ u^{\frac{1}{\propto}} - \mu)^r\ . e^{-u}\ \ du$$

$$= \int_0^{\infty}\ \sum_{j=0}^{r} c_j^r\ (\ p\ \sqrt[\propto]{2b^2}\ u^{\frac{1}{\propto}}\ )^j\ \ (-\mu)^{r-j}. e^{-u}\ \ du$$

$$= \sum_{j=0}^{r} c_j^r\ (\ p\ \sqrt[\propto]{2b^2}\ u^{\frac{1}{\propto}}\ )^j\ \ (-\mu)^{r-j}\ \int_0^{\infty} u^{\frac{j}{\propto}}\ \ e^{-u}\ \ du$$

Thus the e-th moment about the mean is

$$E\,(X-\mu)^r = \sum_{j=0}^{r} c_j^r\,(p\sqrt[\propto]{2b^2})^j\,(-\mu)^{r-j}\,\Gamma\left(\frac{j}{\alpha}+1\right) \quad r = 1,2,3,\ldots\ldots$$

Where $\mu = EX$

Also the r-th moment about origin is

$$EX^r = (p\sqrt[\propto]{2b^2})^r\,\Gamma\left(\frac{j}{\alpha}+1\right), \quad r = 1,2,3,\ldots\ldots\ldots\ldots \qquad ■$$

Result 1

The coefficient of variation , the coefficient of skewedness and coefficient of kurtosis is given as

$$CV. = \frac{\sqrt{\Gamma\left(\frac{2}{\alpha}+1\right)-\left[\Gamma\left(\frac{1}{\alpha}+1\right)^2\right]^2}}{\left\lceil\left(\frac{1}{\propto}+1\right)\right.} \qquad (9)$$

$$CS. = \frac{\left\{\Gamma\left(\frac{3}{\alpha}+1\right)-3\Gamma\left(\frac{1}{\alpha}+1\right)\Gamma\left(\frac{2}{\alpha}+1\right)+2\left[\Gamma\left(\frac{1}{\alpha}+1\right)\right]^3\right\}}{\left\{\Gamma\left(\frac{2}{\alpha}+1\right)-\left[\Gamma\left(\frac{1}{\alpha}+1\right)\right]^2\right\}^{\frac{3}{2}}} \qquad (10)$$

$$CK. = \frac{\left\{-3\left(\Gamma\left(\frac{1}{\alpha}+1\right)\right)^4+6\left[\Gamma\left(\frac{j}{\alpha}+1\right)\right]^2\Gamma\left(\frac{2}{\alpha}+1\right)-4\Gamma\left(\frac{1}{\alpha}+1\right)\Gamma\left(\frac{3}{\alpha}+1\right)+\Gamma\left(\frac{4}{\alpha}+1\right)\right\}}{\left\{\Gamma\left(\frac{2}{\alpha}+1\right)-\left[\Gamma\left(\frac{1}{\alpha}+1\right)\right]^2\right\}^2}-3 \qquad (11)$$

Proposition 2

The Harmonic mean is given by

$$H = E\left(\frac{1}{X}\right) = \frac{1}{p\sqrt[\propto]{2b^2}}\,\Gamma\left(\frac{-1}{\alpha}+1\right) \qquad (12)$$

Proof

$$E\left(\frac{1}{X}\right) = \int_0^\infty \frac{1}{x} f_{R.p}\,(x;p,b,\propto)\,dx$$

$$= \int_0^\infty \frac{1}{x}\,\frac{\propto}{2b^2p}\left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}$$

$$We\ have\ \ u = \frac{\propto}{2b^2}\left(\frac{x}{p}\right)^{\propto}\ \&\ du = \frac{\propto}{2b^2p}\left(\frac{x}{p}\right)^{\propto-1}$$

$$Also\ we\ have\ \ x = P\sqrt[\propto]{2b^2\,u}$$

So that

$$E\left(\frac{1}{X}\right) = \int_0^\infty \frac{1}{P\sqrt[\propto]{2b^2}\,u^{\frac{1}{\propto}}}\,e^{-u}\,du$$

$$= \frac{1}{P\sqrt[\propto]{2b^2}}\int_0^\infty u^{-\frac{1}{\propto}}\,e^{-u}\,du$$

$$E\left(\frac{1}{X}\right) = \frac{1}{p\sqrt[\propto]{2b^2}}\,\Gamma\left(\frac{-1}{\alpha}+1\right) \qquad ■$$

Proposition 3

The Geometric mean is given as the form :-

$$G = E\left(\sqrt{X}\right) = P^{\frac{1}{2}} \sqrt[r]{2b^2}\, \Gamma\left(\frac{1}{r} + 1\right) \qquad (13)$$

Proof

$$E\left(\sqrt{X}\right) = \int_0^\infty \sqrt{x}\, f_{R.p}\ (x; p, b, \propto) dx$$

$$= \int_0^\infty x^{\frac{1}{2}} \frac{\propto}{2b^2 p} \left(\frac{x}{p}\right)^{\propto - 1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}} dx$$

$$since\ x = p \sqrt[\propto]{2b^2 u}$$

$$x^{\frac{1}{2}} = \left(p^{\frac{1}{2}}\left((2\, b^2\, u)\right)^{\frac{1}{2\propto}}\right.$$

Let $2\alpha = r$

$$x^{\frac{1}{2}} = p^{\frac{1}{2}} (2\, b^2\, u)^{\frac{1}{r}} \Rightarrow \sqrt{x} = P^{\frac{1}{2}} \sqrt[r]{2b^2}\ u^{\frac{1}{r}}$$

$$So\ that\ E\left(\sqrt{X}\right) = \int_0^\infty P^{\frac{1}{2}} \sqrt[r]{2b^2}\ u^{\frac{1}{r}}\ e^{-u}\ du$$

$$= P^{\frac{1}{2}} \sqrt[r]{2b^2} \int_0^\infty u^{\frac{1}{r}}\ e^{-u}\ du$$

$$= P^{\frac{1}{2}} \sqrt[r]{2b^2}\ \Gamma\left(\frac{1}{r} + 1\right)$$

$$\therefore G = P^{\frac{1}{2}} \sqrt[r]{2b^2}\, \Gamma\left(\frac{1}{r} + 1\right) \qquad ■$$

Proposition 4

The reliability and Hazard function are given as :

$$R(x) = e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}} \qquad (14)$$

$$h(x) = \frac{\propto}{2b^2 p} \left(\frac{x}{p}\right)^{\propto - 1} \qquad (15)$$

Proof

The reliability function of the R_PD is given by as the form :

$$R(x) = 1 - F(x) = 1 - \left[e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right]$$

$$= e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}$$

The hazard function is given by

$$h(x) = \frac{f(x)}{1 - F(x)} = \frac{\frac{\propto}{2b^2 p}\left(\frac{x}{p}\right)^{\propto - 1} e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}}{1 - \left[1 - e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right]} = \frac{\propto}{2b^2 p} \left(\frac{x}{p}\right)^{\propto - 1} \qquad ■$$

Proposition 5

The mode and median of the(R_PD) are given

$$x_{Mode} = p\left(\frac{(\propto -1)(2b^2)}{\propto}\right)^{\frac{1}{\propto}} \qquad (16)$$

$$x_{Median} = p\left(2b^2\, In2\right)^{\frac{1}{\propto}} \qquad (17)$$

Proof

The mode of the(R-PD) is given as :

Mode = arg max (f(x))

We have $f_{R.p}\ (x\,; p\,, b\,, \propto) = \frac{\propto}{2b^2 p}\left(\frac{x}{p}\right)^{\propto -1} e^{\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}$

So that $ln f_{R.p}\ (x_{Mode}\,; p\,, b\,, \propto) = ln\left[\frac{\propto}{2b^2 p}\left(\frac{x_{Mode}}{p}\right)^{\propto -1} e^{\frac{1}{2b^2}\left(\frac{x_{Mode}}{p}\right)^{\propto}}\right.$

$$= ln\left(\frac{\propto}{2b^2 p}\right) + (\propto -1)\, ln\left(\frac{x_{Mode}}{p}\right) - \frac{1}{2b^2}\left(\frac{x_{Mode}}{p}\right)^{\propto}$$

$$\frac{df_{R.p}\ (x_{Mode}\,; p, b\,, \propto)}{dx_{Mode}} = (\propto -1)\frac{\left(\frac{1}{p}\right)}{\left(\frac{x_{Mode}}{p}\right)} - \frac{\propto}{2b^2}\left(\frac{x_{Mode}}{p}\right)^{\propto}$$

$$For = \frac{df_{R.p}\ (x_{Mode}\,; p, b\,, \propto)}{dx_{Mode}} = 0$$

$$So\ (\propto -1)\frac{\left(\frac{1}{p}\right)}{\left(\frac{x_{Mode}}{p}\right)} - \frac{\propto}{2b^2}\left(\frac{x_{Mode}}{p}\right)^{\propto} = 0$$

$$(x_{Mode})^{\propto} = \frac{P^{\propto}\,(\propto -1)\,(2b)^2}{\propto}$$

$$So\ that\ x_{Mode} = p\left(\frac{(\propto -1)(2b^2)}{\propto}\right)^{\frac{1}{\propto}}$$

The median of the(R_PD) is given by

$F\left(x_{Median;}\,;.\right) = 0.5$

So that

$$0.5 = \int_0^{x_{Median}} f_{R.p}\ (x_{Median}\,; p\,, b\,, \propto)dx$$

$$e^{-\frac{1}{2b^2}\left(\frac{x_{Median}}{p}\right)^{\propto}} = 0.5$$

$$\frac{1}{2b^2}\left(\frac{x_{Median}}{p}\right)^{\propto} = \text{In2}$$

$$So\ that\ x_{Median} = p\left(In2\,(2b^2)\right)^{\frac{1}{\propto}}$$ ■

Proposition 6

The entropy function is given by

$$E_p(X) = E\left(-\log f(X)\right) = -\log\left(\frac{\propto}{2b^2p}\right) - (\propto -1)\left[\log\left(p\sqrt[\propto]{2b^2}\right) + \Gamma\left(\frac{1}{\alpha} + 1\right)\right]$$

$$-\log(p) + 1 \qquad (18)$$

Proof

We have $f_{R.P}(x;.p,b,\alpha) = \frac{\propto}{2b^2p}\left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}$

$$-\log f(x) = -\log\left[\frac{\propto}{2b^2p}\left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right]$$

$$= -\log\left(\frac{\propto}{2b^2p}\right) - \left[(\propto -1)\left(\log(x) - \log(p)\right)\right] + \left[\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}\right]$$

So that

$$E\left(-\log f(X)\right) = -\log\left(\frac{\propto}{2b^2p}\right) - \left[(\propto -1)\left(E\left(\log(x)\right) - \log(p)\right] + \frac{1}{2b^2p^{\propto}} E(x^{\propto})\right)$$

so $E\left(\log(X)\right) = \int_0^{\infty} \log x\ f_{R.p}(x;p,b,\propto)\,dx$

$$= \int_0^{\infty} \log(x)\frac{\propto}{2b^2p}\left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\,dx$$

We have $x = p\sqrt[\propto]{2b^2u}$

$$\log(x) = \log\left(p\sqrt[\propto]{2b^2u}\right)$$

$$so\ that \int_0^{\infty} \log\left(p\sqrt[\propto]{2b^2}\ u^{\frac{1}{\propto}}\right) e^{-u}\,du$$

$$= \int_0^{\infty}\left[\log\left(p\sqrt[\propto]{2b^2}\right) + \log u^{\frac{1}{\propto}}\right] e^{-u}\,du$$

$$= \int_0^{\infty}\left[\log\left(p\sqrt[\propto]{2b^2}\right) e^{-4}\,du + \int_0^{\infty} \log u^{\frac{1}{\alpha}} e^{-u}\,du\right.$$

$$= \log\left(p\sqrt[\propto]{2b^2}\right) - \Gamma\left(\frac{1}{\alpha}\log u + 1\right)$$

And $E(x^{\propto}) = (2b^2p^{\propto})$

So that

$$E_p(X) = E\left(-\log f(x)\right) = -\log\left(\frac{\propto}{2b^2p}\right) - (\propto -1)\log\left(p\sqrt[\propto]{2b^2}\right) - \Gamma\left(\frac{1}{\alpha}\log u + 1\right) + (\alpha - 1)\log(p) + 1$$

■

Proposition 7

the Order statistics

Let $x_1 \dots \dots x_n$ be denote a random sample from a R_PD distribution the pdf of ith order statistic is given as:

$$f_{i,n}(x) = \frac{n!}{(i-1)!(n-i)!} f_{R.P}(x;p,b,\propto)\left(F_{R.P}(x;p,b,\propto)\right)^{i-1}$$

$[\,1- f_{R.P}\ (x\ ;p\ ,b\ ,\propto)\ ]^{n-i}$

Form the pdf of (R_PD) we have the p.d.f of the r-th order statistics

$$f_{i,n}\ (x) = \frac{n!}{(i-1)!(i-r)!}\ \frac{\propto}{2b^2 p}\ \left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}$$

$$\left(1- e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right)^{i-1}\left[1-\left(1- e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right)^{n-i}\right.$$

$$= \frac{n!}{(i-1)!(n-i)!}\ \left[\frac{\propto}{2b^2 p}\ \left(\frac{x}{p}\right)^{\propto-1} e^{-\frac{1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right]$$

$$\left[1- e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right]^{i-1}\left[e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right]^{n-i} \qquad (19)$$

There for the pdfs of the minimum , the maximum and the median

That is if i = 1 , the pd.f of the minimum

i = n , the p.df of the maximum

and i = m + 1 , the p.df of the median with $m = \frac{n}{2}$ if n is even number

Then the pdfs of the minimum , maximum and the median are

$Where\ n,m \in R$

When i = 1 we have the p.d.f of minimum

$$f_{1,n}\ (x;.) = \frac{n\propto}{2b^2 p}\ \left(\frac{x}{p}\right)^{\propto-1} e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\left[e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right]^{n-1} \qquad (20)$$

1- When i = n we have the p.d.f of the maximum

$$f_{n,n}\ (x;.) = \frac{n\propto}{2b^2 p}\ \left(\frac{x}{p}\right)^{\propto-1} e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\left[1- e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right]^{n-1} \qquad (21)$$

2- When i = m+ 1 we have the p.d.f of the median

$$f_{m+1,n}(x;.) = \frac{n!}{m!(n-m-1)!}\left(\frac{\propto}{2b^2 p}\ \left(\frac{x}{p}\right)^{\propto-1} e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right)\left(1- e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right)^{m}\left(e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}\right)^{n-m-1} \qquad (22)$$

2.2_Maximum Likelihood Estimation

In this section we can estimate the unknown parameters of the R_PD distribution, by using the method of maximum likelihood

We have $f_{R.P}\ (x\ ;p\ ,b\ ,\propto) = \frac{\propto}{2b^2 p}\ \left(\frac{x}{p}\right)^{\propto-1} e^{\frac{-1}{2b^2}\left(\frac{x}{p}\right)^{\propto}}$

So that the log of the pdf $f_{R.P}\ (x\ ;p\ ,b\ ,\propto)$ is given by :

$$l(f_{R.P}\ (x\ ;p\ ,b\ ,\propto)) = \frac{\propto^n}{2b^n\ p^n}\ \left(\frac{\pi_{i=1}^{n} x_i}{p^n}\right)^{\propto-1} .\, e^{\frac{-1}{2b^2}\sum_{i=1}^{n}\frac{x_i^{\propto}}{p^{\propto}}}$$

$$\ln l = nIn\propto - In\,(2b^2) - nInp + (\propto-1)In\,\pi_{i=1}^{n} x_i - n\,(\propto -1)Inp - e^{\frac{1}{2b^2}\sum_{i=1}^{n}\frac{x_i^{\propto}}{p^{\propto}}}$$

$$\frac{\partial InL}{\partial \alpha} = \frac{n}{\alpha} - \; In\, \pi_{i=1}^{n}\, x_i - nlnp - \frac{1}{2b^2}\sum_{i=1}^{n}\left(\left(\frac{x_i}{p}\right)^{\alpha} \ln\left(\frac{x_i}{p}\right)\right) \qquad (23)$$

$$\frac{\partial InL}{\partial b} = \frac{-2n}{b} + \frac{1}{b^3}\sum_{i=1}^{n}\frac{x_i^{\propto}}{p^{\propto}} \qquad (24)$$

$$\frac{\partial InL}{\partial p} = \frac{-n}{p} - \frac{n\,(\propto -1)}{p} + \frac{\propto}{2b^2}\,\frac{\sum_{i=1}^{n} x_i^{\propto}}{p^{\propto}+1} \qquad (25)$$

For $\frac{\partial InL\,(x;p,b,\propto)}{\partial \propto} = \frac{\partial InL\,(x;p,b,\propto)}{\partial b} = \frac{\partial InL\,(x;p,b,\propto)}{\partial p} = 0$

So that

$$\frac{n}{\propto} - In\,\pi_{i=1}^{n}\, x_i - nInp - \frac{1}{2b^2}\;\sum_{i=1}^{n}\left(\frac{x_i}{p}\right)^{\propto}\, In\left(\frac{x_i}{p}\right) = 0$$

To find the parameter Alfa using numerical method .to solve the above equation estimate the parameter (b) when ($\propto$) and (p) is known

$$b^{\wedge} = \frac{-2n}{b} + \frac{1}{b^3}\sum_{i=1}^{n}\frac{x_i^{\propto}}{p^{\propto}} = 0$$

$$= \frac{1}{b^3}\;\sum_{i=1}^{n}\frac{x_i^{\propto}}{p^{\propto}} = \frac{2n}{b}$$

$$b^{\wedge} = \sqrt{\frac{\frac{1}{2}\;\sum_{i=1}^{n}\frac{x_i^{\propto}}{p^{\propto}}}{n}} \qquad (26)$$

The estimate of parameter $p^{\wedge}$ when ($\propto$) and (b) are known

$$\frac{-n}{p} - \frac{n\,(n-1)}{p} + \frac{\propto}{2b^2}\,\frac{\sum x_i^{\propto}}{p^{\propto+1}} = 0$$

$$p^{\wedge} = \sqrt[\propto]{\frac{1}{2b^2}\sum\frac{x_i^{\propto}}{n}} \qquad (27)$$

3.Conclusions

In this paper we have presented a new Rayleigh Pareto distribution denote by (R_PD) . We introduced the cumulative distribution function (cdf) , probability density function (pdf) and some statistical properties of it like The entropy function, mean, mode, median , variance , the r-th moment about the mean, the rth moment about the origin, reliability, hazard functions, coefficients of variation, of sekeness and of kurtosis. Finally, we estimate the parameters . . they observed that the MLE of the unknown parameters can be obtained numerically

4.References

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