Journal of Babylon University

Vol(25)No.7(2017)

a-$soft$ $separation$ $axioms$ in $soft$ $topological$ $space$

Prof . Dr. Luay Abd –Al-Hani Al-Sweedi

Zahraa Mohammad Najem Al-rekabi

University of Babylon ,College of Education for pure science, Department of mathematics .

Drluayha11@yahoo.com

Zahraa90@yahoo.com

الخلاصة

ان اهم ادوات العلوم ومختلف اختصاصاتها وانواعها والتي تعتبر الركيزة الاساسية هي (نظرية المجموعات) ونظراً للتطور الهائل في جميع مجالات الحياة وهذا بدوره يولد مشاكل كبيرة ويحتاج الى حل والى ادوات موازية لتلك التطورات من اجل حلها ، لذا أخذ العلماء على عاتقهم تطوي نظرية الاعداد وفتح افاق جديدة ، حيث ظهر علم جديد هو (نظرة الأعداد الطرية) التي أُعتبرت الاداة المهمة لحل معظم المشاكل المستعصية او تجاوزها في تلك العلوم ومختلف اختصاصاتها الحياتية و الاقتصادية والهندسية والطبية وغيرها وكذلك دخلت نظرية الاعداد الطرية في علم التوبولوجيا العامة بصورة قوية وفعالة في السنوات الاخيرة وظهر علم جديد هو (الفضاء التوبولوجي الطري) .

ان الفكرة الاساسية من هذا البحث هي تعريف (بديهيات الفصل في الفضاء التوبولوجي الطري)وبوجه الخصوص عند نقطة معينة في المعلمات ودراسة اهم الخصائص والنتائج لها .

الكلمات المفتاحية:

المجموعة الطرية ، التبولوجي الطري ، الجوار الطرية- a ، الانفصال الطري- a ، الاستمرارية الطرية-a ، بديهيات الفصل الطرية -a .

Key words

Soft set , soft topology , $a-$ soft neighborhood , a $-$soft disjoint , a $-$soft continuous ,a $-$soft separation axioms .

Abstract

The important science tools in a different kinds and specialties , that considered the basic mainstay of (the set theory) and because of huge development in all life fields. This causes great problems , that need solution and parallel tools for those developments , so the scientis become responsible to work on the development of number theory and open new horizons , that a new science had appeared which is

(soft figures theory) which is considered the important tool to solve most difficult problems or overcome them ,in these sciences and their specific life specialization , economy , medicine , geometry and others . Also the theory of soft numbers had entered in general topology in power full and active way . The last years a new science has appeared is (soft topological space).

The main idea of this research is to define the separation axioms in (soft topological space) and practically in certain point , and to study the most important properties and results of it .

Introduction

Life problems are many and various and need to be solved by human to make his life easier . Scientist didn't stop to find these solutions or trying to do that , in which scientist Molodtsov in 1999 , defined (the soft set) which was regarded quality change and important mathematic tool , to solve these problems that it can't be solved using ordinary methods such as , fuzzy set , especially (mathematical function) membership . Also the problem that existed in most branches of Math., Geometry , economy and computer science and other sciences this field attracts many researchers to work deeply in studying features and laws of (soft sets theory) and developed it by , groups of researches

[P.K.Maji , R.Biswas and A.R.Roy (2003)] the scientist begin to define a (topological space) based on (soft set theory) namely (soft topological space) . Many groups of researchers and scientis have joined to study the new properties of this space and depending of the family of soft sets , every family depends on the concepts which were found (soft sets) or symbolizes (F_a) it is .

Through this definition , many types of families have appeared (i.e.g soft set) . In [Cagman & Englinglu , (2011)] the researcher depended on (soft set) to confirm function (F) and change its space which it is a subset of the set E of parameters . In[S.Yuksel , N.Tuzlu and Z. G.Ergul , 2014] , The Turkish researcher made depend on the changing of function F and its domain is a subset of E , when he defined a soft set in [G.Xuechong , (2015)] , this Chinese researcher try to defined a new type of a soft sets called it (central soft sets) who present a new concepts that depend on set of (soft sets) basic map (F) on the a subset of (E) , in [Molodtsov , 1999 , W.Min , 2011 and Aygunoglu and HAygun , 2012 , the researchers are defined the last kind of these sets which is going to the base of our study .This type depends on the change of a function F and confirming sets of parameters which is a subset of E . That's mean all functions on A to opposite $IP(X)$, that $A \subseteq E$ where the concept of (separation axioms) in soft topological spaces of last type were studies .

1. Soft sets

Definition 1.1[D.A.Molodtsov (1999)**]**

A pair (F, A) is called soft set over X , where F is a mapping defined as : $\mathrm{F}: \mathrm{A} \longrightarrow \mathrm{IP(X)}$ such that $(F, A) = \{(a, F(a)), F(a) \in \mathrm{IP(X)}\}$, where

1) X be the initial universal set , and E_{X} be the set of all possible parameters with respect to X .
2) IP(X) is denoted to the power set of X (I . e IP(X) the family of all subset of X .
3) $A \subseteq \mathrm{IP(X)}$, s-imply we denoted for E_{X} by E .
4) The set of all soft sets over the universe X is denoted by $S(\mathrm{X})$.

Definition 1.2 [D.A.Molodtsov (1999)]

If $F(a) = \mathrm{X}$ for all $a \in A$ then , the soft set F_A is called the absolute soft set and it is denoted by $\tilde{X}_A$. If $F(a) = \varphi$ for all $a \in A$ then the soft set F_A is called the null soft set . and it is denoted by $\tilde{\Phi}_A$.

Note 1.3

let F_A is any soft set over the universe X , then :

1. The point of the soft set F_A at $a \in A$ is denoted by F_a such that $F_a = \{(a, F(a))\}$
2. for $a \in A$ and $\forall\, x \in \mathrm{X}$, a soft point , x_a is of the form: $x_a = \{(a, x)\} \cup \{(p, \varphi);\ \forall p \in A\, , p \neq a\}$, simply we write the soft point by $x_a = \{(a, x)\}$.
3. A soft point x_a belong to the soft set F_A and denoted by $x_a \tilde{\in} F_A$ or $x_a \tilde{\subseteq} F_A$ if and only if $x \in F(a)$ and $x_a \tilde{\notin} F_A$ if and only if $x \notin F(a)$.
4. An a $-$ absolute soft set is of the form $\tilde{X}_a = \{(a, \{X\})\}$, and is called the element of the absolute soft set $\tilde{X}_A$ at $a \in A$.
5. An a $-$null soft set is of the form $\tilde{\Phi}_{\mathrm{a}} = \{(\mathrm{a}, \{\varphi\})\}$, and is called the element of the null soft set $\tilde{\Phi}_A$ at $a \in A$.
6. An e $-$soft complement is an e $-$soft set over χ defined as :
$\tilde{X}_A -_a F_A = H_a \text{ such that } H_a = \{(a, X - F(a))\}$.
7. We denoted to the soft set $\mathrm{F_A}$ at the point $\mathrm{a} \in \mathrm{A}$ by F_a .

Note 1.4 [D.N .Georgion 2013]

Let $F_A \tilde{\in} S(X)$, $a \in A$ and $x \in X$, then we write $x \in_a F_A$ and $x \notin_a F_A$ resp. if and only if $\mathrm{x} \in F(a)$ and $\mathrm{x} \notin \mathrm{F(a)}$.

Note 1.5

If $\mathrm{F_A}$, $\mathrm{G_A}$ be are soft sets over the universe X , then:

(1) F_A is an a $-$ soft subset of G_A if and only if $F(a) \subseteq G(a)$ and we write by the form $F_A \tilde{\subseteq}_a G_A$. for $a \in A$ **.**

(2) The a-soft intersection of $\mathrm{F_A}$ and $\mathrm{G_A}$, is an a $-$soft set defined as follows :

$F_A \tilde{\cap}_a G_A = H_a$ *such that* $H_a = \{(a, F(a) \cap G(a))\}$.

(3) The a $-$soft union of F_A *and* G_A, is an a $-$soft set defined as follows :

$F_A \tilde{\cup}_a G_A = H_a$ such that $H_a = \{(a, F(a) \cup G(a))\}$.

(4) $\mathrm{F_A}$ and $\mathrm{G_A}$ are called a $-$soft equal iff $\mathrm{F(a) = G(a)}$, and denoted by $\mathrm{F_A =_a G_A}$.

(5) The a $-$soft disjoint of F_A *and* G_A is defined by $F_A \tilde{\cap}_a G_A = \tilde{\Phi}_\mathrm{a}$.

(6) A point x is a $-$belong to the soft set F_A is denoted by $x \tilde{\in}_a F_A$, that is $x \in F(a)$.

Let $\wedge$ be any arbitrary index set, and let F_{λ_A} be any soft set over X, we define :

(1) $\bigcap_{a\, \lambda \in \wedge} F_{\lambda_A} = H_a = \{(a, \bigcap_{a\, \lambda\in\wedge} F_\lambda(\mathrm{a}))\}$
(2) $\bigcup_{a\, \lambda \in \wedge} F_{\lambda_A} = H_a = \{(a, \bigcup_{a\, \lambda\in\wedge} F_\lambda(\mathrm{a}))\}$

Proposition1.6

F_A, G_A be any two soft sets over the universe X, for each a $\in$ A the following statements are hold :

(1) $F_A \tilde{\cap}_a \tilde{\Phi}_\mathrm{A} = \tilde{\Phi}_\mathrm{a}$

(2) $\mathrm{F_A} \tilde{\cap}_\mathrm{a} \tilde{\mathrm{X}}_\mathrm{A} = \mathrm{F_a}$

(3) $\mathrm{F_A} \tilde{\cup}_\mathrm{a} \tilde{\Phi}_\mathrm{A} = \mathrm{F_a}$

(4) $F_A \tilde{\cup}_a \tilde{X}_A = \tilde{X}_a$

We can conclude these facts directly from [Note 1.5] it can be considered as a special case of [proposition 2.3] in [P. k . Maji , R. Bis was and A .R . Roy (2003)] .

Proposition1.7

If $\mathrm{F_A}, \mathrm{G_A}$ and $\mathrm{H_A}$ are any soft sets over the universe X, for each a $\in$ A the following are true :

1) $F_A \tilde{\cap}_a (G_A \tilde{\cap}_a H_A) = (F_A \tilde{\cap}_a G_A) \tilde{\cap}_a H_A$
2) $F_A \tilde{\cup}_a (G_A \tilde{\cup}_a H_A) = (F_A \tilde{\cup}_a G_A) \tilde{\cap}_a H_A$
3) $F_A \tilde{\cap}_a (G_A \tilde{\cup}_a H_A) = (F_A \tilde{\cap}_a G_A) \tilde{\cup}_a (F_A \tilde{\cap}_a H_A)$
4) $F_A \tilde{\cup}_a (G_A \tilde{\cap}_a H_A) = (F_A \tilde{\cup}_a G_A) \tilde{\cap}_a (F_A \tilde{\cup}_a H_A)$.

Remark1.8

All of theorems and properties that are true in [N . Cagman ,F . Cltak and S . Enginolu (2001), N.Cagman and S .S . Englinglu (2011), N.Cagman and S .S . Englinglu (2011)] are also be true in this research (i .e when we considered the point a $\in$ A as a base of this work) .

Definition 1.9

Let X and Y be two universal sets and A ,B be a sets of parameters , u:X⟶Y and p:A⟶B , then the mapping :

f: $(\widetilde{X}_A, \tilde{\tau}, A) \longrightarrow (\widetilde{Y}_B, \tilde{\rho}, B)$ (i . e f: $S(X) \longrightarrow S(Y)$) on A and B respectively is denoted by f_{pu} and it can shows as :

$$f_{pu} = \left\{\left(\mathrm{f_{pu}}(\mathrm{F_A}), \mathrm{p(A)}\right), \mathrm{p(A)} \subseteq \mathrm{B}\right)\} .$$

$$\text{Where} \;:\; f_{pu}(\mathrm{F_A})(\beta) = \begin{cases} u\left\{\bigcup_{\alpha\in p^{-1}(\beta)} \left(\mathcal{F}(\alpha)\right)\right\}, & \text{if } \mathrm{P}^{-1}(\mathcal{B}) = \varphi \\ \varphi & \text{other wise} \end{cases}$$

For $\mathcal{B} \in \mathrm{B}\ \exists\ \mathrm{a} \in \mathrm{p(A)}$ such that $\mathrm{p(a)} = \mathcal{B}$. that is $\mathrm{p}^{-1}(\mathcal{B}) \neq \varphi$

A $\mathrm{p}^{-1}(\beta) \subseteq \mathrm{A}$, hence $\mathrm{p}^{-1}(\beta) \cap \mathrm{A} \neq \mathrm{p}^{-1}(\beta)$, hence we get that

$$f_{pu}(\mathrm{F_A})(\beta) = \mathrm{u}\left\{\bigcup_{\alpha\in p^{-1}(\beta)} \mathrm{F}(\alpha)\right\}$$

Constructing :

Since p is a mapping , so $p(A) \neq \varphi$,$\forall A \neq \varphi$, that is $\forall\, \mathcal{B} \in p(A) \exists\ a \in A$ such that $\mathrm{P}(a) = \mathcal{B}$ $\;and\;$ $\mathrm{P}^{-1}(\mathcal{B}) \neq \varphi$ now , $a \in \mathrm{P}^{-1}(\mathrm{P(a)})$ so ,

$$f_{pu}(F_A)(\mathcal{B}) = u\left\{\bigcup_{\alpha\in p^{-1}(\beta)} \left(F(\alpha)\right)\right\} \forall\, \beta \in \mathrm{P}(a)\} .$$

- If p is a one to one , then $\mathrm{P}^{-1}\left(\mathrm{P}(A)\right) = \mathrm{A}$, $that\ is$ $\forall\, \mathcal{B} \in \mathrm{P}(A)\ \exists\ a \in A$ such that $\mathrm{P}(a) = \mathcal{B}$ $\;and\;$ $f_{pu}(F_A)(\mathcal{B}) = u\left(F(a)\right)$
- If G_B $S(Y)$ then the inverse image of G_B under f_{pu} is denoted by ${f_{pu}}^{-1}(G_B)$ is a soft set $F_A \in S(X)$ such that:
  $\mathrm{P}(a) = u^{-1}(G\left(\mathrm{P}(a)\right)$ for every $a \in A$.

Remark 1.10

For each $a \in A$ and $x \in X$, then we can defined the soft mapping f_{pu} at a soft point x_a , is :

$$f_{pu}(x_a)(\mathcal{B}) = u(\bigcup_{\alpha\in p^{-1}(\mathcal{B})} x_a(\alpha)) = u(x)$$

$$I\,.\,e\ \left(f_{pu}(x_a)\right)\left(p(a)\right) = u(x)$$

$I\,.\,e\ \left(f_{pu}(x_a)\right)_{p(a)}$ ={(P(a), {u(x)})}

Now , for $b \in B$ and $y \in Y$, ${f_{pu}}^{-1}(y_b)(a) = u^{-1}(y)$ for $b = p(a)$.

proposition 1.11 [I . Zorlutuna , M . Akdag , W- K - Min and S - A smaka(2012)]

let $F_{1_A}, F_{2_A} \in S(X)\ and\ G_{1_B}\ , G_{2_B}\ \in S(Y)$, the following statements are true

i) $if\ F_{1_A} \tilde{\subseteq}\ F_{2_A}$, $then\ f_{pu}(\,F_A)\ \tilde{\subseteq}\ f_{pu}(F_{1_A})$.

ii) $G_{1_B}, \tilde{\subseteq}\ G_{2_B}$, $then\ {f_{pu}}^{-1}(G_{1_B}\,)\ \tilde{\subseteq}\ {f_{pu}}^{-1}(\,G_{2_B})$

iii) $F_{1_A} \tilde{\subseteq} {f_{pu}}^{-1}\left(f_{pu}(F_{1_A})\right)$

iv) if p is a one to one map of X into Y and u is a one to one map of A into B , then $F_{1_A} = {f_{pu}}^{-1}\left(f_{pu}(F_{1_A})\right)$.

v) $f_{pu}\left({f_{pu}}^{-1}(G_{1_B})\right) \tilde{\subseteq} G_{1_B}$.

Remark 1.12

For any $x_1, x_2 \in X$ and $y_1, y_2 \in Y\ and\ a \in A\ , b\ \in B$, the flowing statements are true :

(1) $x_{1_a} \tilde{\subseteq} x_{2_a}$, $f_{pu}(x_{1_a}) \tilde{\subseteq} f_{pu}(x_{2_a})$

(2) $y_{1_b} \tilde{\subseteq} y_{2_b}$, ${f_{pu}}^{-1}(y_{1_b}) \tilde{\subseteq} {f_{pu}}^{-1}(y_{2_b})$

(3) $x_{1_a} \tilde{\subseteq} {f_{pu}}^{-1}(x_{1_a})$

(4) $f_{pu}\left({f_{pu}}^{-1}(y_{1_b})\right) \tilde{\subseteq} y_{1_b}$

(5) If $p\ is$ a map from X into Y , and u is a map of A into B , then:
$f_{pu}\left({f_{pu}}^{-1}(y_{1_b})\right) = y_{1_b}$.

(6) ${f_{pu}}^{-1}(\tilde{X}_b - y_{1_b}) = \tilde{X}_a - {f_{pu}}^{-1}(y_{1_b})$

2. Soft topology

In this section we defines some of important concepts of a soft topological spaces with some of counter examples .

Definition 2.1

Let X be an initial universal set , and A ⊆ E be a set of parameters ,

Let $\tilde{\tau}$ be a subfamily of a the family of all soft sets $S(X)$, we say that

the family $\tilde{\tau}$ is a soft topology on X if the following axioms are holds :

(1) $\tilde{\Phi}_A$, $\tilde{X}_A \tilde{\in} \tilde{\tau}$

(2) if F_A ,$G_A \tilde{\in} \tilde{\tau}$, then then $F_A \tilde{\cap} G_A \tilde{\in} \tilde{\tau}$

(3) For any index set I $G_{i_A} \tilde{\in} \tilde{\tau}$, for any i ∈ I , then $\tilde{\cup}\left\{G_{i_A},\ i \in I\right\} \tilde{\in} \tilde{\tau}$.

*T*he triple $(\tilde{X}_A, \tilde{\tau}, A)$ is called soft topological space or (soft space) .

*T*he *m* embers *o*f $\check{\tau}$, *a*re *c*alled *s*oft *o*pen *s*ets , A *s*oft set F_A is called *s*oft *c*losed set *i*f *a*nd *o*nly *i*f *i*ts *c*omplement *i*s *s*oft *o*pen , *T*he family *o*f all *s*oft *c*losed set *i*s *d*enoted *b*y :

$$C(\tilde{X}_A) = \{\tilde{X}_A - F_A\ , F_A \tilde{\in} \tilde{\tau}\}$$

Definition 2.2[Georgion 2014]

Let $(\tilde{X}_A, \tilde{\tau}, A)$ *b*e *a* *s*oft *t*opological *s*pace ,a∈A , x ∈ X ,*w*e *s*ay that *a* *s*oft set $F_A \tilde{\in} \tilde{\tau}$ is an a−*s*oft *o*pen nhd *o*f x *i*n $(\tilde{X}_A, \tilde{\tau}, A)$ *i*f x ∈ F(a) , simply we denoted by $G_{(a,x)}$.

Simply we write the (neighborhood) by nhd .The set of all soft (nhd) of a point x ∈ X at a ∈ A is denoted by $N_{\tilde{\tau}(a,x)}$, that is $N_{\tilde{\tau}(a,x)} = \{G_{(a,x)}; G_{(a,x)}$is soft nhd of a soft point $x_a\}$.

Definition 2.3

Let $(\tilde{X}_A, \tilde{\tau}, A)$ *b*e a *s*oft *t*opological *s*pace , a∈A , *w*e *s*ay *t*hat the *s*oft *s*et F_A is a-soft nhd of x ∈ X *if t*here *e*xists *a* *s*oft *o*pen set $G_{(a,x)}$ *s*uch *t*hat x $\in_a G_{(a,x)} \tilde{\subseteq}_a F_A$.

A soft set F_A is called a −soft conhd of $x \in X$ if and only if $\tilde{X}_A - F_A$ is a −soft nhd of $x \in X$.

A soft set F_A is called a −soft locally open set if and only if F_A is a −soft nhd of each $x \in_a F_A$.

A soft set F_A *i*s *c*alled a−*s*oft locally *c*losed set *if and only if* $\tilde{X}_A - F_A$ *i*s *a* − *s*oft locally *o*pen .

Proposition2.4

Let $(\tilde{X}_A, \tilde{\tau}, A)$ *b*e a *s*oft *t*opological *s*pace , The a−soft nhd system

of a point x has the following properties :

(1) if $G_{(a,x)} \in N_{\tilde{\tau}(a,x)}$ then x $\in_a G_{(a,x)}$.

(2) if $G_{(a,x)} \in N_{\tilde{\tau}(a,x)}$ and $G_{(a,x)} \tilde{\subseteq}_a H_A$, then $H_A \tilde{\in} N_{\tilde{\tau}(a,x)}$.

(3) if $G_{(a,x)}$ and $H_{(a,x)} \in N_{\tilde{\tau}(a,x)}$, then $G_{(a,x)} \tilde{\cap} H_{(a,x)} \in N_{\tilde{\tau}(a,x)}$.

The proof of them is directly from definition [٢.٢]

3. Soft separation axioms

Definition ٣.١

Let $(\tilde{X}_A, \hat{\tau}, A)$ be a soft topological space, let $a \in A$, the soft space is called a−soft $T_\circ -$ space if for each $x_1 \neq x_2$ in X, there is a a−soft open set $G_{(a,x_1)}$ containing x_1 but not containing x_2 or there is a soft open set $G_{(a,x_2)}$ containing x_2 but not containing x_1 .

The soft topological space is called soft $T_\circ -$ space if $\forall a \in A$, the soft space is a −soft $T_\circ -$ space..

Note ٣.2

If $(\tilde{X}_A, \tilde{\tau}, A)$ is a−soft $T_\circ -$ space for some point $a \in A$, then the soft space need not be a soft $T_\circ -$ space .

Example 3.3

Let $A \subseteq E$ such that $A = \{a_1, a_2\}$ and let X be a universal set such that : $X = \{x_1, x_2, x_3\}$, note that :

$\hat{\tau} = \{F_{1_A}, F_{2_A}, F_{3_A}, F_{4_A}, F_{5_A}, F_{6_A}, \tilde{\Phi}_A, \tilde{X}_A\}$ *where :*

$F_{1_A} = \{(a_1, \{x_2\}), (a_2, \{\varphi\})\}$, $F_{2_A} = \{(a_1, \{x_1, x_2\}), (a_2, \{x_3\})\}$,

$F_{3_A} = \{(a_1, \{x_1, x_3\}), (a_2, \{x_1, x_2, x_3\})\}$, $F_{4_A} = \{(a_1, \{x_2\}), (a_2, \{x_1, x_2\}\}$

$F_{5_A} = \{(a_1, \{x_1\}), (a_2, \{x_3\})\}$, $F_{6_A} = \{(a_1, \{x_1, x_3\}), (a_2, \{x_1, x_2\})\}$

then $(\tilde{X}_A, \tilde{\tau}, A)$ is $a_1 -$ soft $T_\circ -$ space .

Now :$\{(a_2, \{x_1\})\} \neq \{(a_2, \{x_2\})\}$ and $\nexists$ a_2 −soft open set such that its contain one and not contain the other .

Thus $(\tilde{X}_A, \tilde{\tau}, A)$ is not soft $T_\circ -$ space .

Definition ٣.٤ [D.N .Georgion 2013]

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space, for $a \in A$, the soft space is called a −soft $T_1 -$ space if for each $x_1 \neq x_2$ in X, there is two soft open sets $G_{(a,x_1)}$ and $H_{(a,x_2)}$ such that : $x_1 \in_a G_{(a,x_1)}$ and $x_2 \notin_a G_{(e,x_1)}$ so $x_2 \in_a H_{(a,x_2)}$ and $x_1 \notin_a H_{(a,x_2)}$.

The soft topological space is called soft $T_1 - space$ if and $only$ if $\forall a \in A$ the $soft$ space $(\tilde{X}_A, \tilde{\tau}, A)$ is an $a -$ soft $T_1 -$ space , [D.N .Georgion 2013] .

Note ٣.٥

$a -$ soft $T_1 -$ space need not be soft $T_1 - space$.

Example ٣.٦

the example (3.3) elucidate that the soft space is $a_1 -$ soft $T_1 -$ space but not $a_2 -$ soft $T_1 -$ space , so it is not soft $T_1 -$ space ..

Definition ٣.٧ [D.N .Georgion 2013]

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a $soft$ $topological$ $space$, and $a \in A$, the $soft$ $space$ is called $a -$soft $T_2 -$ space if for each $x_1 \neq x_2$ in X , there is two $a -$ disjoint soft open sets $G_{(a,x_1)}$, $H_{(a,x_2)}$.

The soft space $(\tilde{X}_A, \tilde{\tau}, A)$ is called strongly soft $T_2 -$ space if it is $a -$soft $T_2 -$ space if for each $a \in A$.

Note ٣.٨

$a -$soft $T_2 -$ space need not be a soft $T_2 -$ space .

Example ٣.٩

The example (3.3) elucidate that the soft topology is $a_1 -$soft $T_2 -$ space , but it is not $a_2 -$soft $T_2 -$ space , so it is not soft $T_2 -$ space .

Definition3.10 [Georgion 2014]

Let $(\tilde{X}_A, \tilde{\tau}, A)$and $(\tilde{Y}_B, \tilde{\rho}, A)$ are two $soft$ $topological$ $spaces$ over X and Y respectively , $x \in X$ and p:A⟶B .A map u:X⟶Y is $called$ $soft$ $p-continuous$ at the $point$ x if $every$ $a \in A$ and $p(a)-soft$ $open$ nhd G_B of u(x) $there$ exist an $a-soft$ $open$ nhd F_A of x $such$ $that$ $f_{pu}(F_A) \tilde{\subseteq} G_B$.

Proposition 3.11 [Georgion 2014]

Let $(\tilde{X}_A, \tilde{\tau}, A)$and $(\tilde{Y}_B, \tilde{\rho}, A)$ are two $soft$ $topological$ $spaces$ over X and Y respectively , and $p: A \longrightarrow B$ and $u: X \longrightarrow Y$, then u is soft $p-continuous$ iff $f_{pu}{}^{-1}(G_B) \tilde{\in} \tilde{\tau}$, for every $G_B \tilde{\in} \tilde{\rho}$.

Definition3.12

Let $(\tilde{X}_A, \tilde{\tau}, A)$and $(\tilde{Y}_B, \tilde{\rho}, A)$ are two soft topological spaces over X and Y respectively , then for $a \in A$ and $x \in X$, the map $f_{pu}(\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\rho}, A)$ is called an $a -$soft continuous at x iff for each $p(a) -$soft open set G_B containingg $u(x)$ there exists an $a -$soft open set H_A containing x, such that $f_{pu}(H_A) \tilde{\subseteq}_{p(a)} G_B$.

f_{pu} is called soft continuous if it is a −soft continuous $\forall a \in A$.

Definition 3.13 [Zorlutuna 2012**]**

A soft mapping f_{pu} from a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ into a soft topological space($\tilde{Y}_B, \tilde{\rho}, A$) is a soft continuous iff ${f_{pu}}^{-1}(G_A) \tilde{\in}\ \tilde{\tau}$ for any $G_A \tilde{\in} \tilde{\rho}$.

Definition 3.14[Nazmul 2012]

Let $(X_A, \tilde{\tau}, A)$ and ($\tilde{Y}_B, \tilde{\rho}, A$) are two soft topological spaces over X and Y respectively , the map f_{pu}: $(\tilde{X}_A, \tilde{\tau}, A) \longrightarrow$ ($\tilde{Y}_B, \tilde{\rho}, A$) is called soft open map if for each soft open set $G_A in\, \tilde{X}_A$, then $f_{pu}(G_A)$ is soft open set in $\tilde{Y}_B$.

Definition 3.15

Let $(X_A, \tilde{\tau}, A)$ and ($\tilde{Y}_B, \tilde{\rho}, A$) are two soft topological spaces over X and Y respectively , the map f_{pu}: $(\tilde{X}_A, \tilde{\tau}, A) \longrightarrow$ ($\tilde{Y}_B, \tilde{\rho}, A$) is :

(a) One to one if p and u are ono to one maps .
(b) Onto if p and u are onto maps .

Theorem ٣.16

Let $(\tilde{X}_A, \tilde{\tau}\,, A)$ and $(\tilde{Y}_B, \tilde{\sigma},\ \mathrm{B})$ be $t$wo $s$oft $t$opological $s$paces and $\tilde{X}_A$ be $\mathrm{a}-\mathrm{soft}-\mathrm{T}_\circ-\mathrm{space}$ for some $\mathrm{a} \in \mathrm{A}$, if the map f_{pu}: $(\tilde{X}_A, \tilde{\tau}\,, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is soft open map and u, p are onto maps , then $\tilde{Y}_B$ is a $p(a) - \mathrm{soft} - \mathrm{T}_\circ - \mathrm{space}$.

Proof //

Let $b \in B$ and $y_1 \neq y_2$ in Y then there exist $e \in A$ and $x_1 \neq x_2$ $in\, X$ such that $p(e) = b$, $u(x_1) = y_1$ and , $u(x_2) = y_2$ because u, p are onto maps .Now ,

Since $(\tilde{X}_A, \tilde{\tau}\,, A)$ is $\mathrm{a} - \mathrm{soft} - \mathrm{T}_1 - \mathrm{space}$, then there exist two soft open sets ${G_1}_A$ $and\, {G_2}_A$ such that : $x_1 \in_a {G_1}_A$ and $x_2 \notin_a {G_1}_A$ or $x_2 \in_a {G_2}_A$ $x_1 \notin_a {G_2}_A$ and , then
$y_1 = u(x_1) \in_b f_{pu}\left({G_1}_A\right)$

$y_2 = u(x_2) \notin_b f_{pu}\left({G_1}_A\right)$. or
$y_2 = u(x_2) \in_b f_{pu}\left({G_2}_A\right)$ and , $y_1 = u(x_1) \notin_b f_{pu}\left({G_2}_A\right)$,

So $\left(\tilde{Y}_B, \tilde{\sigma}, B\right)$ is $b - \mathrm{soft} - \mathrm{T}_\circ - \mathrm{space}$.

Theorem 3.17

Let $(\tilde{X}_A, \tilde{\tau}\,, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, \mathrm{B})$ be $t$wo $s$oft $t$opological $s$paces and u:X⟶Y , is a one to one soft is p − continuous map , if $\tilde{Y}_B$ is a b−$s$oft−T∘ −space $f$or $s$ome b ∈ B and p is onto mapping , then there exist a ∈ A such that $b = p(a)$ and $\tilde{X}_A$ is a−soft − T∘ − space .

Proof//

Since p is onto map and $b \in B$, so $\exists a \in A$ such that $b = p(a)$, now let $x_1 \neq x_2$ in X , since u is a one to one map , then $u(x_1) \neq u(x_2)$ in Y , but $\tilde{Y}_B$ *is a* b−*s*oft−T∘ −space , so *t*here *e*xist a *s*oft *o*pen set $G_{(b,u_b(x_1))}$ containing $u_b(x_1)$ but not contain $u_b(x_2)$, hence $f_{pu}{}^{-1}(u_b(x_1)) \tilde{\in}_e f_{pu}{}^{-1}\left(G_{(b,u_b(x_1))}\right)$ and $f_{pu}{}^{-1}(u_b(x_2)) \tilde{\notin}_a f_{pu}{}^{-1}\left(G_{(b,u_b(x_1))}\right)$, but $f_{pu}{}^{-1}(u_b(x_1)) = x_{1_a}$ and $f_{pu}{}^{-1}(u_b(x_2)) = x_{2_a}$ because u is a one to one map , also f_{pu} is *p*-soft continuous , so by [definition 3.11] $f_{pu}{}^{-1}\left(G_{(b,u_b(x_1))}\right)$ is soft open set containing x_{1_a} but not x_{2_a} therefore $(\tilde{X}_A, \tilde{\tau}, A)$ is a−soft−T∘ − space .

Theorem ٣.18

*E*very *s*oft *s*ubspace *o*f a−soft − T∘ −space *i*s a − *s*oft−T∘ −space ∀ a ∈ A .

Proof//

Suppose that $\tilde{Y}_A$ is a soft subspace of the of the a − soft − T∘ − space $(\tilde{X}_A, \tilde{\tau}, A)$, and $x_1, x_2 \in_a \tilde{Y}_A$ with $x_1 \neq x_2$.

Since $\tilde{Y}_A \tilde{\subseteq} \tilde{X}_A$, then $x_1, x_2 \in_a \tilde{X}_A$, but $\tilde{X}_A$ *i*s a−soft − T∘ −space , *t*hen *t*here *e*xist *a s*oft *o*pen *s*et $G_{(a,x_1)}$ *s*uch *t*hat $x_1 \in_a G_{(e,x_1)}$ *and*

$x_2 \notin_a G_{(a,x_1)}$ or *t*here *e*xist *a s*oft *o*pen *s*et $H_{(a,x_2)}$ *s*uch *t*hat $x_2 \in_a H_{(a,x_2)}$ *a*nd $x_1 \notin_a H_{(a,x_2)}$, since $x_1 \in_a G_{(a,x_1)}$ and $x_1 \in_a \tilde{Y}_A$, then

$x_1 \in_a G_{(a,x_1)} \tilde{\cap} \tilde{Y}_A$ and since $x_2 \notin_a G_{(a,x_1)}$ then $x_2 \notin_a G_{(a,x_1)} \tilde{\cap} \tilde{Y}_A$or

$x_2 \in_a H_{(a,x_2)}$ and $x_2 \in_a \tilde{Y}_A$, then $x_2 \in_a H_{(a,x_2)} \tilde{\cap} \tilde{Y}_A$, and since $x_1 \notin_a H_{(a,x_2)}$, then $x_1 \notin_a H_{(a,x_2)} \tilde{\cap} \tilde{Y}_A$so , $\tilde{Y}_A$ *i*s a−soft − T∘ −space .

Definition ٣.19

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces , we say that $\tilde{\tau}$ is finer than $\tilde{\sigma}$ if and only if $\tilde{\sigma} \subseteq \tilde{\tau}$.

Theorem ٣.20

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces such that $\tilde{\sigma}$ is finer than $\tilde{\tau}$, if $\tilde{\tau}$ is a − soft − T∘ − space, then $\tilde{\sigma}$ is e − soft − T∘ − space for some $a \in A$.

Proof // the prove is directly .

Theorem ٣.21

A soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ is a −soft-$T_1 -$ space , $a \in A$ iff there is a sodt point x_a is a −soft locally closed set for each $x \in X$.

Proof //

Let $x \in X$ we must prove that x_a is a−$s$oft locally $c$losed set , now let y $\in_a \tilde{X}_A - x_a$, y $\in X - \{x\}$, thus $y \neq x$, then there exists $t$wo $s$oft $o$pen set $G_{(a,y)}$ and $H_{(a,x)}$ $s$uch $t$hat y $\in_a$ $G_{(a,y)}$ and $x \notin_a G_{(a,y)}$ and $x \in_a H_{(a,x)}$ and $y \notin_a H_{(a,x)}$, hence $y \in G(a) \subseteq X - \{x\}$, implies that $G_{(a,y)} \tilde{\subseteq} \tilde{X}_A - x_a$, hence $\tilde{X}_A - x_a$ is a −soft nhd of each $y \in_a \tilde{X}_A - x_a$, that is $\tilde{X}_A - x_a$ is a −soft locally open set , hence x_a is a −soft locally open set .

Conversely : suppose that for each $x \in X$ x_a is a −soft locally open set , we want to prove that $(\tilde{X}_A, \tilde{\tau}, A)$ is a −soft-$T_1 -$ space , let $x, y \in X$, such that $x \neq y$, that is x_a and y_a are −soft locally closed sets , this implies that $\tilde{X}_A - x_a$ and $\tilde{X}_A - y_a$ are a −soft locally open sets and evidently that $x \in_a \tilde{X}_A - y_a$ and $y \notin_a \tilde{X}_A - y_a$ and $y \in_a \tilde{X}_A - x_a$ and $x \notin_a \tilde{X}_A - x_a$ so there exists a soft open sets G_A and H_A such that $x \in_a G_A \tilde{\subseteq}_a \tilde{X}_A - y_a$ and $y \in_a H_A \tilde{\subseteq}_a \tilde{X}_A - x_a$ that is $x \notin_a G_A$ and $x \notin_a H_A$, hence $\tilde{X}_A$ is a −soft-$T_1 -$ space .

Theorem ٣.22

Let $(\tilde{Y}_B, \tilde{\sigma}, B)$ is b −soft-$T_1 -$ space for $b \in B$ and let $(\tilde{X}_A, \tilde{\tau}, A)$ be any soft topological space , such that the maps $u: X \longrightarrow Y$ be onto and $p: A \longrightarrow B$ is a one to one , then there exists $a \in A$ with $p(a) = b$ and $\tilde{X}_A$ is a −soft-$T_1 -$ space if $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is a −soft continuous.

Theorem ٣.23

A soft topological space$(\tilde{X}_A, \tilde{\tau}, A)$ is an a −soft-$T_1 -$ space if and only the soft point x_a is a −soft locally closed set for each $x \in X$.

Theorem ٣.24

A soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ is a−soft − $T_\circ$ −space for some $a \in A$ if and $o$nly if ∀ x≠y in X we have $Cl(x_a) \neq Cl(y_a)$.

Theorem 3.25

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}$,B) be $t$wo $s$oft $t$opological $s$paces and $\tilde{X}_A$ be a − soft − T_1 − space for some a ∈ A , if the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is e −soft open map and u, p are onto maps , then $\tilde{Y}_B$ is a p(a) − soft − T_1 − space .

Theorem ٣.26

$Every$ soft $s$ubspace of a $-$ $s$oft T_1 $-$ $space$ is a$-$ $s$oft T_1 $-$ $space$.

Proof // the proof is directly from Definition ٣.٤

Theorem 27

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft $topological$ space , then the following statements are equivalent :

a) $(\tilde{X}_A, \tilde{\tau}, A)$ is a $-$ soft $-$ T_1 $-$ space.
b) an a $-$ soft intersection of all the a $-$soft open nhd of an arbitrary soft point of x_a is singleton point $\{x\}$.

Proof//

The prove is directly by Note 1.5 and Definition ٣.٤ .

Note 3.28

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_A, \tilde{\sigma}, A)$ be two soft topological spaces such that $\tilde{\sigma}$ is finer than $\tilde{\tau}$, for some point $a \in A$ if $(\tilde{X}_A, \tilde{\tau}, A)$ is a $-$ soft $-$ T_2 $-$ space , then $(\tilde{Y}_B, \tilde{\sigma}, B)$ is a $-$ soft $-$ T_2 $-$ space .

Theorem 3.29

Every soft subspace of a $-$soft a $-$ soft $-$ T_2 $-$ space is a $-$ soft $-$ T_2 $-$ space

Proof // the prove is directly by Definition ٣.٧ .

Theorem ٣.30

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a $s$oft $t$opological $s$pace , for a $\in$A $t$he $f$ollowing $s$tatements $a$re $e$quivalent :

a) $(\tilde{X}_A, \tilde{\tau}, A)$ is a $-$ soft $-$ T_2 $-$ space .
b) The a$-$ $s$oft $i$ntersection of all a $-$soft closed nhds of an arbitrary a $-$soft point of x_a is a $-$ singleton point .

proof// The proof is directly from definition Note 1.5 and Definition ٣.٤

Theorem ٣.31

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, \mathrm{B})$ be $t$wo $s$oft $t$opological $s$paces and $\tilde{X}_A$ be a $-$ soft $-$ T_2 $-$ space for some e $\in$ A, if the maps $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is soft open and u , p are onto maps , then $\tilde{Y}_B$ is $p(a)$ $-$ soft $-$ T_2 $-$ space .

Proof // simply combine .

Theorem ٣.32

Let ($(\tilde{Y}_B, \tilde{\sigma}, B)$ be a $\mathrm{b}-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$ for $b \in B$ and let $(\tilde{X}_A, \tilde{\tau}, A)$ be any soft topological space such that the map u be onto and p is a one to one , then there exists $a \in A$ with $p(a) = b$ and $\tilde{X}_A$ is $\mathrm{a}-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$, if f_{pu} is a −soft conyinuous .

Theorem ٣.33

Let ($(\tilde{Y}_B, \tilde{\sigma}, B)$ be a $\mathrm{b}-\mathrm{soft}-\mathrm{T}_\circ-\mathrm{space}$ for $b \in B$ and let $(\tilde{X}_A, \tilde{\tau}, A)$ be any soft topological space such that the map $u: X \longrightarrow Y$ be onto and $p: A \longrightarrow B$ is a one to one , then there exists $a \in A$ with $p(a) = b$ and $\tilde{X}_A$ is $\mathrm{a}-\mathrm{soft}-\mathrm{T}_\circ-\mathrm{space}$, if f_{pu} is a −soft conyinuous .

Theorem ٣.34

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces such that the maps u is one to one soft p −continuous and p is onto map .If for some $b \in B$, $\tilde{Y}_B$ is $\mathrm{b}-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$, then there exists $a \in A$ such that $b = p(a)$ and $\tilde{X}_A$ is $\mathrm{a}-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$.

References

- A .Aygunoglu and H.Aygun , (2012), some notes on soft topological spaces , journal computer and applied , 21 ,113-119 .
- B.Ahmad and Athar Kharal ,2009 , on fuzzy soft sets , Hindawi publishing corporation in fuzzy system , Article ID 586507 , 6 pages .
- D.A.Molodtsov , (1999), soft set theory- first results ,computers and mathematics with applications , 37 ,19-31.
- D.N.Georgiou , A.C.Megaritis and V.I . Pertropoulos (2013) , on soft topological spaces , Applied math. And inf . science 7 , No .5 , 1889-1901.
- D.N.Georgiou and A.C.Megaritis , (2014) , soft set theory and topology , App. Gen. top.1 , 93-109 .
- G.Xuechong , (2015) , Study on central soft sets :definitions and basic operations (2015) .
- I.Zorlutuna , M.Akdag , W.K.Min and S.Asmaka(2012) , Remarks on soft topological spaces , annals of fuzzy mathematics and informations , 3,171-185 .
- M . I .Ali etal , (2009), on some new operations in soft set theory , computers and mathematics with applications , 75 ,1547-1553 .

- M.Shabir and M.Naz (2011), on soft topological spaces , computers and mathematics with applications , 611780-1799 .
- N.Cagman and S .S . Englinglu , (2011), soft topology , computers and mathematics with applications , 62 ,351-358 .
- P.K.Maji . R.Biswas and A.R.Roy (2003) , soft set theory , computers and mathematics with applications , 45, 555-562.
- S.K.Nazmuland S.K.Samanta , (2012) , Neighborhood of soft topological spaces , Ann.fuzzy Math . inf. 1-16 .
- S.Yuksel , N.Tuzlu and Z. G.Ergul , 2014 , soft regular generalized closed sets in soft topological spaces , Int. journal of math .Analysis , 8 (355-367) .
- W.K.Min , (2011),a note on soft topological spaces , computers and mathematics with applications , 62 ,3524-3028 .