Using ψ-Operator to Formulate a New Definition of Local Function

[1]Prof.Dr.Luay A.A.AL-Swidi [2] Ass.prof.Dr.Asaad M.A.AL-Hosainy [3]Msc.St. Reyam Q.M.AL-Feehan

drluayha11@yahoo.com zsene12@yahoo.com reyamqays27@yahoo.com

[1,2,3] Department of Mathematics, College of Education for Pure Science Babylon University

ABSTRACT: In this paper, we use ψ-operator in order to get a new version of local function. The concepts of maps, dense, resolvable and Housdorff have been investigated in this paper, as well as modified to be useful in general.

الخلاصة: استخدمنا في هذا البحث العملية ابساي من أجل الحصول على نسخه جديده من الدالة المحلية.وقد تم التحقيق في مفاهيم الدوال, الكثافة,المجموعات القابلة للحل و فضاء الهاوسدورف,وكذلك تعديلها لتكون مفيدة بشكل عام.

Keywords: ψ*-operator, ψ-dense, ψ-resolvable, ψ-map and ψ-$\mathcal{I}$-Housdorff.

1. INTRODUCTION AND PRILMINARIES

As requirements for our work, we define here the following concepts sequentially: Ideal space, local function, Kuratowski closure, dense, $\mathcal{T}$*-dense, $\mathcal{I}$-dense, codense, ψ-operator, resolvable, $\mathcal{I}$-open, pre-$\mathcal{I}$-open, scattered set and Housdorff space.

We start define the ideal space. Let (X, $\mathcal{T}$) be a topological space with no separation properties assumed. The subject of ideal topological space has been studied by (Kuratowski, 1966) and (Vaidyanathaswamy, 1960). An ideal $\mathcal{I}$ on a topological space (X, $\mathcal{T}$) is a nonempty collection of subsets of X which satisfies the following two conditions:

(1) If $A \in \mathcal{I}$ and $B \subseteq A$, then $B \in \mathcal{I}$ (heredity).

(2) If $A \in \mathcal{I}$ and $B \in \mathcal{I}$, then $A \cup B \in \mathcal{I}$ (finite additivity).

Moreover a σ-ideal on a topological space (X, $\mathcal{T}$) is an ideal which satisfies (1), (2) and the following condition:

3.If $\{A_i : i = 1,2,3,....\} \subseteq \mathcal{I}$, then $\bigcup \{A_i : i =1,2,3,....\} \in \mathcal{I}$ (countable additivity).

An ideal space is a topological space (X, $\mathcal{T}$) with an ideal $\mathcal{I}$ on X and is denoted by (X, $\mathcal{T}$, $\mathcal{I}$).For a subset $A \subseteq X$,

$A^*(\mathcal{I})=\{x \in X : U \cap A \notin \mathcal{I}$ for every $U \in \mathcal{T}(x)\}$ is called the local function of A with respect to $\mathcal{I}$ and $\mathcal{T}$ (Kuratowski, 1933).We simply write A* instead of $A^*(\mathcal{I})$ in case there is no chance for confusion.

It is well known that $Cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for a topology $\mathcal{T}^*(\mathcal{I})$ which is finer than $\mathcal{T}$. During this paper, for a subset $A \subseteq X$, Cl (A) and Int (A) denote the closure and the interior of A ,respectively. A subset A of an ideal space (X, $\mathcal{T}$, $\mathcal{I}$) is said to be dense (resp, $\mathcal{T}^*$-dense (Dontcher, Gansster and Rose, 1999), $\mathcal{I}$–dense (Dontcher, Gansster and Rose,1999) if Cl (A) = X (resp Cl*(A) = X, A*= X).An ideal $\mathcal{I}$ on a space (X, $\mathcal{T}$) is said to be codense (Devid, Sivaraj and Chelvam, 2005) if and only if $\mathcal{T} \cap \mathcal{I} = \{\Phi\}$.For an ideal space (X, $\mathcal{T}$, $\mathcal{I}$) and for any $A \subseteq X$, where $\mathcal{I}$ is codense. Then: dense, $\mathcal{T}^*$-dense and $\mathcal{I}$-dense are equivalent (Jankovic and Hamlett, 1990). (Natkanies, 1986) used the idea of ideals to define another operator known as ψ-operator defined as follow: For a subset $A \subseteq X$, ψ (A) = X-(X-A)*.Equivalently $\psi(A) = \{M \in T : M-A \in I\}$.It is obvious that ψ(A) for any A is a member of T.

For an ideal space (X, $\mathcal{T}$, $\mathcal{I}$) and $Y \subseteq X$ then (Y, $\mathcal{T}_y$, $\mathcal{I}_y$) is an ideal space where $\mathcal{T}_y = \{U \cap Y : U \in \mathcal{T}\}$ and $\mathcal{I}_y = \{U \cap Y : U \in \mathcal{I}\} = \{U \in \mathcal{I} : U \subseteq Y\}$.

In 1943, Hewitt introduced the concept of a resolvable space as follows: A nonempty topological space (X, $\mathcal{T}$) is called resolvable (Hewitt, 1943) if X is the disjoint union of two dense subsets. Given a space (X, $\mathcal{T}$) and $A \subseteq X$, A is called $\mathcal{I}$-open (Jankovic and Hamlett, 1990)(resp per- $\mathcal{I}$–open(Donkhev,1996)if $A \subseteq Int(A^*)$(resp $A \subseteq Int\, Cl^*(A)$.A set $A \subseteq X$ is said to be scattered (Jankovic and Hamlett, 1990) if A contains no nonempty dense-in-itself subset. A space (X, $\mathcal{T}$, $\mathcal{I}$) is called $\mathcal{I}$-Housdorff (Dontcher,1995) if for each two distinct points **x** ≠ **y**, there exist $\mathcal{I}$-open sets U and V containing **x** and y respectively, such that $U \cap V = \Phi$. Throughout this paper we define a new local function and generalizations .Many characterizations, proprieties and relation between them are obtained.

2- ψ-Operator*

In this section we introduce a new type of local function by using ψ-operator, in order to do this we have to have a deep looking in(Jankovic and Hamlett, 1992) work of the local function in ideal space.

Definition 2.1

Let (X, $\mathcal{T}$, $\mathcal{I}$) be an ideal space. An operator ψ *(.): $\mathcal{P}(x) \rightarrow \mathcal{P}(x)$, called ψ -local function of A with respect to $\mathcal{I}$ and $\mathcal{T}$, is define as follow: for any A$\subseteq$X,

ψ *(A) ($\mathcal{I}$, $\mathcal{T}$) = {x$\in$X : ψ (U)$\cap$A$\notin\mathcal{I}$, for every open subset U$\in\mathcal{T}$(x) }, where $\mathcal{T}$(x)={U$\in$ $\mathcal{T}$, x$\in$U}.

When there is no chance for confusion ψ *(A) ($\mathcal{I}$, $\mathcal{T}$) is briefly denoted by ψ *(A).

Fact

Let (X, $\mathcal{T}$, $\mathcal{I}$) be an ideal space .Then A*$\subseteq$ ψ *(A), for any A $\subseteq$ X.

Now we define a new closure operator in terms of ψ -local function.

Definition 2.2

Let (X, $\mathcal{T}$, $\mathcal{I}$) be an ideal space. For any A$\subseteq$X, we define

1. ψ -Cl (A) = {x $\in$ X: ψ (U)$\cap$A ≠ Φ for every U$\subseteq$X, x $\in$ ψ (U)}. It is clear that Cl (A) $\subseteq$ ψ -Cl (A).

2. ψ -Cl*(A) ($\mathcal{I}$, $\mathcal{T}$) = A $\cup$ ψ *(A)

When there is no chance for confusion ψ -Cl*(A) ($\mathcal{I}$, $\mathcal{T}$) is briefly denoted by ψ -Cl*(A).

Note

Let (X, T, $\mathcal{I}$) be an ideal space. Then ψ -Cl*(A) $\subseteq$ ψ -Cl (A), for any A $\subseteq$ X.

We discuss the properties of ψ -local function in following theorem:

Theorem 2.3

Let (X, $\mathcal{T}$) be a topological space, $\mathcal{I}$ and $\mathcal{J}$ be two ideals on X, and let A and B be subsets of X. Then the following properties hold:

1. If A $\subseteq$ B, then ψ *(A) $\subseteq$ ψ *(B).

2. If $\mathcal{I} \subseteq \mathcal{J}$, then ψ *(A) ($\mathcal{J}$) $\subseteq$ ψ *(A) ($\mathcal{I}$).

3. ψ *(A) =Cl (ψ *(A)) $\subseteq$ ψ -Cl (A).

4. (ψ *(A))* $\subseteq$ ψ *(A).

5. ψ *(A $\cup$ B) = ψ *(A) $\cup$ ψ *(B).

6. ψ *(A $\cap$ B) $\subseteq$ ψ *(A) $\cap$ ψ *(B).

7. For every I ϵ $\mathcal{I}$, then ψ *(A $\cup$ I) = ψ *(A) = ψ *(A \ I).

8. If G ϵ $\mathcal{T}$, then G $\cap$ ψ *(A) $\subset$ ψ *(G $\cap$ A).

9. If Uϵ$\mathcal{T}$, then U $\cap$ ψ -Cl*(A) $\subset$ ψ -Cl*(U $\cap$ A).

10. If A ϵ $\mathcal{I}$, then ψ *(A) = Φ.

Proof.

Straight from Definition2.1 and Definition 2.2

3. ψ - *Dense and Generalizations:*

Definition 3.1

A subset A of an ideal space (X, $\mathcal{T}$, $\mathcal{I}$) is called:

(1) ψ -dense, if ψ -Cl (A) = X.

(2) ψ -$\mathcal{T}$*–dense, if ψ -Cl*(A) = X.

(3) ψ -$\mathcal{I}$-dense, if ψ *(A) = X.

(4) nowhere ψ -dense, if Int ψ -Cl(A) = Φ.

(5) nowhere ψ *-dense, if Int* ψ -Cl(A)=Φ.

The following remark is immediate from above definitions:

Remark 3.2

(1) If A is ψ –$\mathcal{I}$-dense, then A is ψ -$\mathcal{T}$*-dense.

(2) If A is ψ -$\mathcal{T}$*-dense, then A is ψ -dense.

(3) If A is nowhere ψ *-dense, then A is nowhere ψ -dense.

The study of ideal got new dimension when Codense ideal [10] has been incorporated in ideal space. In the following definition we introduce a new concept of condense by using ψ-operator.

Definition 3.3

An ideal $\mathcal{J}$ on a topological space (X, $\mathcal{T}$) is said to be ψ -codense if, ψ ($\mathcal{T}$)$\cap$ $\mathcal{J}$ = {Φ}.

Lemma 3.4

Let (X, $\mathcal{T}$, $\mathcal{J}$) be an ideal space and for every subset A$\subseteq$X. If A $\subseteq$ ψ *(A), then the following relations hold:

ψ *(A) = Cl (ψ *(A)) = ψ –Cl (A) = ψ -Cl*(A).

Proof.

By Definition 2.2: (1), and Theorem 2.3: (3).

Theorem 3.5

Let (X, $\mathcal{T}$, $\mathcal{J}$) be an ideal space, and for every subset A$\subseteq$ X. If $\mathcal{J}$ is ψ -codense, then the following relations hold:

(1) If A is ψ -dense, then it is ψ -$\mathcal{T}$*-dense.

(2) If A is ψ -$\mathcal{T}$*-dense, then it is ψ -$\mathcal{J}$-dense.

Proof.

This is an immediate consequence of Lemma 3.4.

The following theorem related to ψ -codense ideal:

Theorem 3.6

If X is finite, then X=X* if and only if $\mathcal{J}$ is ψ -codense.

Proof.

Necessity. By Theorem 2.5,[1]. we have $\mathcal{T}\cap\mathcal{J}$= {Φ}. Since ψ ($\mathcal{T}$)$\cap\mathcal{J}\subseteq\mathcal{T}\cap\mathcal{J}$. It follow that ψ ($\mathcal{T}$)$\cap$ $\mathcal{J}$ = {Φ}.So $\mathcal{J}$ is ψ -codense.

Sufficiency. Assume that $\mathcal{I}$ is ψ -codense. Since X-X* is open, let $x \in$ X-X*. Then there exists $U_x \subseteq$ X-X*, and $x \notin X^*$.Then there exists $V_x \in \mathcal{T}(x)$, such that $U_x \cap V_x \in \mathcal{I}$. Assume that $W_x = U_x \cap V_x \in \mathcal{I}$. But X-X* = $\bigcup W_x$. Hence X-X* $\in \mathcal{I}$ and ψ $(\mathcal{T}) \cap \mathcal{I} \neq \{\Phi\}$, a contradiction. Therefore X = X*.

Corollary 3.7

Let X be any nonempty set .If $(X, \mathcal{T}, \mathcal{I})$ is an ideal space where $\mathcal{I}$ is σ-ideal, then the following statement are equivalent:

(1) X = X*;

(2) $\mathcal{T} \cap \mathcal{I} = \{\Phi\}$;

(3) ψ $(\mathcal{T}) \cap \mathcal{I} = \{\Phi\}$.

Proof.

Similar to the proof of Theorem 3.6.

By Remark 3.2, and Theorem 3.5, the following relations hold:

ψ -dens $\overset{♀}{\rightleftarrows}$ ψ -$\mathcal{T}$*-dense $\overset{♀}{\rightleftarrows}$ ψ -$\mathcal{I}$-dense.

♀: $\mathcal{I}$ is ψ -codense.

By Theorem 2.3 :(2), we have the following remark:

Remark 3.8

(1)Let (X, T, $\mathcal{I}$) and (X, T, $\mathcal{J}$) be two ideal spaces, with $\mathcal{I} \subseteq \mathcal{J}$, if A is ψ -$\mathcal{I}$-dense with respect to $\mathcal{J}$,then A is ψ -$\mathcal{I}$-dense with respect to $\mathcal{I}$ for any $A \subseteq X$.

(2) Let $(X, \mathcal{T}, \mathcal{I})$ and (X, σ, $\mathcal{I}$) be two ideal spaces, with $\mathcal{T} \subseteq \sigma$, if A is ψ -$\mathcal{I}$-dense with respect to σ ,then A is ψ -$\mathcal{I}$-dense with respect to $\mathcal{T}$ for any $A \subseteq X$.

By Remark 3.2 and Remark 3.8: (1), (2) we get the following corollary:

Corollary 3.9

(1)Let (X, $\mathcal{T}$, $\mathcal{I}$) and (X, $\mathcal{T}$, $\mathcal{J}$) be two ideal spaces, with $\mathcal{I} \subseteq \mathcal{J}$, if A is ψ -$\mathcal{T}$*-dense (ψ -dense) with respect to $\mathcal{J}$, then A is ψ -$\mathcal{T}$*-dense (ψ -dense) with respect to $\mathcal{I}$ for any A $\subseteq$ X.

(2) Let (X, $\mathcal{T}$, $\mathcal{I}$) and (X, σ, $\mathcal{I}$) be two ideal spaces, with $\mathcal{T} \subseteq$ σ, if A is ψ -$\mathcal{T}$*-dense (ψ -dense) with respect to σ, then A is ψ -$\mathcal{T}$*-dense (ψ -dense) with respect to $\mathcal{T}$ for any A $\subseteq$ X.

Definition 3.10

A function f: (X, $\mathcal{T}$, $\mathcal{I}$) → (Y, σ) is called ψ -map if, f (ψ -Cl (A)) $\subseteq$ Cl f (A), for any A $\subseteq$ X.

Note

Where f:(X,$\mathcal{T}$,$\mathcal{I}$)→(Y,σ) is a bijection map ,then f($\mathcal{I}$) will be an ideal on Y.

Theorem 3.11

If f: (X, $\mathcal{T}$, $\mathcal{I}$) →(Y, σ) is ψ - bijection map .Then the image of any ψ -dense subset A of X is dense in Y.

Proof.

Assume that A is ψ -dense in X. Since f is ψ -map, then f (ψ -Cl (A)) $\subseteq$ Cl f (A).

Implies that f (X) $\subseteq$Cl (f (A)) and since f is surjection .Therefore, Y= Cl (f (A)). Hence f (A) is dense in Y.

Definition 3.12

A function f:(X,$\mathcal{T}$,$\mathcal{I}$)→(Y,σ,$\mathcal{J}$) is called :

(1) ψ -($\mathcal{I}$,$\mathcal{J}$) map ,if f (ψ -Cl*(A)) $\subseteq$Cl* f (A). .

(2) ψ_0- ($\mathcal{I}$,$\mathcal{J}$) map ,if f (ψ *(A)) $\subseteq$ (f(A))*.

(3) ψ_1- ($\mathcal{I}$,$\mathcal{J}$) map, if f (ψ *(A)) $\subseteq$ ψ * f(A).

(4) ψ_2- ($\mathcal{I}$,$\mathcal{J}$) map ,if f (ψ -Cl*(A)) $\subseteq$ ψ -Cl* f(A).

(5) ψ_3- ($\mathcal{I}$,$\mathcal{J}$) map, if f(ψ -Cl(A)) $\subseteq$ ψ -Cl f(A).

For any A$\subseteq$X.

The following remark is immediate from above definitions:

Remark 3.13

(1) ψ_0- ($\mathcal{I}$, $\mathcal{J}$) map $\rightarrow$ ψ - ($\mathcal{I}$, $\mathcal{J}$) map

(2) ψ_1 - ($\mathcal{I}$, $\mathcal{J}$) map $\rightarrow$ ψ_2 - ($\mathcal{I}$, $\mathcal{J}$) map $\rightarrow$ ψ_3 - ($\mathcal{I}$, $\mathcal{J}$) map.

Theorem 3.14

If f: (X, $\mathcal{T}$, $\mathcal{I}$) $\rightarrow$(Y, σ) is ψ_0-($\mathcal{I}$, $\mathcal{J}$) bijection map .Then the image of any ψ -$\mathcal{I}$-dense subset A of X is f ($\mathcal{I}$)-dense in Y.

Proof.

Assume that A is ψ -$\mathcal{I}$-dense in X. Since f is ψ_0-($\mathcal{I}$, $\mathcal{J}$) map, then f (ψ *(A)) $\subseteq$ (f (A))*. Implies that f(X) $\subseteq$(f (A))*, and since f is surjection .Therefore, Y= (f (A))*.Hence f (A) is f ($\mathcal{I}$)-dense in Y.

By Remark 3.13(1), and Theorem 3.14we get the following corollary:

Corollary 3.15

If f: (X, $\mathcal{T}$, $\mathcal{I}$) $\rightarrow$ (Y, σ) is ψ - ($\mathcal{I}$, $\mathcal{J}$) bijection map .Then the image of any ψ -$\mathcal{T}$*-dense subset A of X is $\mathcal{T}$*-dense in Y.

Theorem 3.16

If f: (X, $\mathcal{T}$, $\mathcal{I}$) $\rightarrow$ (Y, σ, $\mathcal{J}$) is ψ_1 - ($\mathcal{I}$, $\mathcal{J}$) bijection map. Then the image of any ψ -$\mathcal{I}$-dense subset of X is ψ -f ($\mathcal{I}$)-dense in Y.

Proof.

Assume that A is ψ -$\mathcal{I}$-dense in X. Since f is ψ_1-($\mathcal{I}$, $\mathcal{J}$) map then f (ψ *(A)) ⊆ ψ *(f (A)), and since f is surjection .Therefore, Y = ψ *(f (A)).Hence f (A) is ψ -f ($\mathcal{I}$)-dense in Y.

Corollary 3.17

If f: (X, $\mathcal{T}$, $\mathcal{I}$) → (Y, σ, $\mathcal{J}$) is ψ_2-($\mathcal{I}$, $\mathcal{J}$) [ψ_3-($\mathcal{I}$, $\mathcal{J}$)] bijection map. Then the image of any ψ -$\mathcal{T}$*-dense (ψ -dense) subset of X is Ȓ -$\mathcal{T}$*-dense (ψ -dense) in Y.

Proof. This is an immediate consequence of Remark 3.13(2), and Theorem 3.16.

4: ψ -Resolvability and Generalizations:

In this section we define ψ- resolvable space and recall some of the generalizations of ψ-resolvable and their relationship with Housdorff space.

Definition 4.1

A nonempty topological space (X, $\mathcal{T}$) is called:

(1) ψ -resolvable, if X is the disjoint union of two ψ -dense subsets.

(2) ψ -$\mathcal{T}$*-resolvable, if X is the disjoint union of $\mathcal{T}$*-dense and ψ -dense subsets.

(3) ψ -$\mathcal{I}$-resolvable, if X is the disjoint union of $\mathcal{I}$-dense and ψ -dense subsets.

It is clear that: ψ -$\mathcal{I}$-resolvable → ψ -$\mathcal{T}$*-resolvable → ψ -resolvable.

Theorem 4.2

Let (X, $\mathcal{T}$) be a nonempty topological space. Then if X is ψ -$\mathcal{T}$*-resolvability, then X is ψ -$\mathcal{I}$-resolvability.

Question

We claim that ψ-resolvability does not imply ψ-$\mathcal{T}^*$-resolvability, but we could not find an example.

Remark 4.3

Let (X, T, $\mathcal{I}$) and (X, T, $\mathcal{J}$) be two ideal spaces, with $\mathcal{I} \subseteq \mathcal{J}$, if X is ψ -resolvable with respect to $\mathcal{J}$,then X is ψ -resolvable with respect to $\mathcal{I}$.

This is an immediate consequence of Corollary 3.9.

Definition 4.4

A function f:(X,$\mathcal{T}$,$\mathcal{I}$)→(Y,σ,$\mathcal{J}$) ,is said to be bi*-map if :

(1) f: (X,$\mathcal{T}$,$\mathcal{I}$)→(Y, σ) is ψ_3-($\mathcal{I}$,$\mathcal{J}$) map;

(2) f: (X,$\mathcal{T}^*$)→(Y, σ*) is a continuous map.

Theorem 4.5

Let f :(X, $\mathcal{T}$, $\mathcal{I}$) → (Y, σ) be bi*- bijection map, and if X is ψ -$\mathcal{T}^*$-resolvable space .Then Y is ψ -$\mathcal{T}^*$-resolvable with respect to the ideal f ($\mathcal{I}$).

Proof.

Assume that X is ψ -$\mathcal{T}^*$-resolvable space, then there exists two disjoint ψ -dense and $\mathcal{T}^*$-dense subsets such that ψ-Cl (A) = Cl*(B) = X. It is clear by Corollary 3.19, and Definition 4.4 that Y is ψ -$\mathcal{T}^*$-resolvable.

Lemma 4.6

Let U$\in\mathcal{T}$ and ψ *(D) = X, then U$\subseteq$ ψ *(D$\cap$ ψ (U)).

Proof.

Let $x \in U$, and suppose if possible that $x \notin \psi^*(D \cap \psi(U))$ Then $(D \cap \psi(U)) \cap \psi(V) \in \mathcal{I}$, and since $\psi(U) \cap \psi(V) \in \mathcal{T}$. Therefore, $x \notin \psi^*(D) = X$. This is a contradiction .That is $U \subseteq \psi^*(D \cap \psi(U))$.

Corollary 4.7

(1) Let $U \in \mathcal{T}$ and ψ -Cl*(D) = X, then $U \subseteq \psi$ -Cl* $(D \cap \psi(U))$.

(2) Let $U \in \mathcal{T}$, and ψ -Cl (D) = X. Then $U \subseteq \psi$ -Cl $(D \cap \psi(U))$.

Using Remark 3.2: (1), (2), and similar to the proof of Lemma 4.6.

Theorem 4.8

Let $y \in \mathcal{T}$, and $\psi^*(A) = X$. Then $\psi_y^*(A_1) = Y$, where $A_1 = A \cap y$.

Proof.

By lemma 4.6 $Y \subseteq \psi_x^*(A \cap Y) = \psi_x^*(A_1)$.

Then $Y = \psi_x^*(A_1) \cap y = \psi_y^*(A_1)$.

Using Remark 3.2:(1),(2), and Theorem 4.8we get the following corollary;

Corollary 4.9

(1)Let $y \in \mathcal{T}$, and ψ -Cl*(A) = X. Then ψ - $Cl_y^*(A_1) = Y$, where $A_1 = A \cap Y$.

(2) Let $y \in \mathcal{T}$, and ψ -Cl (A) = X. Then ψ - $Cl_y(A_1) = Y$, where $A_1 = A \cap Y$.

Theorem 4.10

Let $(X, \mathcal{T}, \mathcal{I})$ be ψ -$\mathcal{I}$-resolvable space, if Y is open .Then $(Y, \mathcal{T}_y, \mathcal{I}_y)$ is ψ -$\mathcal{I}_y$-resolvable.

Proof.

Assume that $(X, \mathcal{T}, \mathcal{I})$ is ψ -$\mathcal{I}$-resolvable space .Then there exists two disjoint ψ -dense, and $\mathcal{I}$-dense subsets such that: $A \cup B = X$, and ψ -Cl (A) = B* = X.

Note that $Y = Y \cap X = Y \cap (A \cup B) = (Y \cap A) \cup (Y \cap B)$, and $(Y \cap A) \cap (Y \cap B) = Y \cap (A \cap B) = \Phi$.

Put $A_1 = A \cap Y$ and $B_1 = B \cap Y$. Then $Y = A_1 \cup B_1$, $A_1 \cap B_1 = \Phi$.

By corollary 3.10: (2), we have ψ -Cl $_y$ (A_1) = Y, and by lemma we have $B^*_1 y$ = Y. Hence $(Y, \mathcal{T}_y, \mathcal{I}_y)$ is ψ -$\mathcal{I}_y$-resolvable.

Corollary 4.11

(1) Let $(X, \mathcal{T}, \mathcal{I})$ be ψ -$\mathcal{T}^*$-resolvable space, if Y is open .Then $(Y, \mathcal{T}_y, \mathcal{I}_y)$ is ψ -$\mathcal{T}^*_y$- resolvable.

(2) Let $(X, \mathcal{T}, \mathcal{I})$ be ψ -resolvable space, if Y is open .Then $(Y, \mathcal{T}_y, \mathcal{I}_y)$ is ψ -resolvable.

Definition 4.12

A subset A of an ideal space $(X, \mathcal{T}, \mathcal{I})$ is said to be ψ -pre-open if $A \subseteq$ Int ψ -Cl (A).

Definition 4.13

(1) An ideal space $(X, \mathcal{T}, \mathcal{I})$ is said to be ψ -$\mathcal{I}$ -Housdorff, if for every distinct point $\mathbf{x}, \mathbf{y} \in X$, there exist two disjoint ψ -pre-open, and $\mathcal{I}$-open sets containing each respectively.

(2)An ideal space $(X, \mathcal{T}, \mathcal{I})$ is said to be ψ -$\mathcal{T}^*$-Housdorff, if for every distinct point $\mathbf{x}, \mathbf{y} \in X$, there exist two disjoint ψ -pre-open, and pre-$\mathcal{I}$-open sets containing each respectively.

(3)An ideal space $(X, \mathcal{T}, \mathcal{I})$ is said to be ψ -Housdorff, if for every distinct point $\mathbf{x}, \mathbf{y} \in X$, there exist two disjoint ψ -pre-open sets containing each respectively.

Lemma 4.14

If ψ ({x}) $\in \mathcal{I}$. Then ψ *({x}) = Φ.

Proof.

Assume that ψ *({x}) ≠ Φ, then there exist y $\in$ ψ *({x}) such that ψ (U) $\cap$ {x} $\notin \mathcal{I}$ for every U $\in \mathcal{T}$(x). Implies that ψ ({x}) $\notin \mathcal{I}$. This is a contradiction.

Theorem 4.15

If an ideal space (X, $\mathcal{T}$, $\mathcal{I}$) is ψ -$\mathcal{I}$-resolvable, and the scattered set of (X, $\mathcal{T}$) are in $\mathcal{I}$, then (X, $\mathcal{T}$, $\mathcal{I}$) is ψ -$\mathcal{I}$-Housdorff.

Proof.

Let A and B be disjoint ψ -dense, and I-dense subsets of X such that X = A $\cup$ B. Since ψ -Cl (A) = X and B* = X. Therefore, A $\subseteq$ Int ψ -Cl (A) and B $\subseteq$ Int B*, then A and B are ψ -pre-open and $\mathcal{I}$-open sets respectively. Let x, y be any two element of X and both x and y are in A .Take U = A\{y} and V = B $\cup$ {y}. It is easily observed (see [6], Theorem 2.3 (h)]) that V also $\mathcal{I}$-open set and y in V. Now to show that U = A $\cup$ {x} is ψ -pre-open.

ψ - Cl (A $\cup$ {x}) = ψ -Cl (A) $\cup$ ψ -Cl ({x} = X $\cup$ ψ -Cl ({x}) = X .Hence A $\cup$ {x} $\subseteq$ Int ψ -Cl (A $\cup$ {x}), so U is ψ -pre-open. Hence (X, $\mathcal{T}$, $\mathcal{I}$) is ψ -$\mathcal{I}$-Housdorff.

From the above theorem we get the following corollary:

Corollary 4.16

(1)If an ideal space (X, $\mathcal{T}$, $\mathcal{I}$) is ψ -$\mathcal{T}$*-resolvable, and the scattered set of (X, $\mathcal{T}$*) are in $\mathcal{I}$, then (X, $\mathcal{T}$, $\mathcal{I}$) is ψ -$\mathcal{T}$*-Housdorff.

(2) If an ideal space (X, $\mathcal{T}$, $\mathcal{I}$) is ψ -resolvable, and the scattered set of (X, $\mathcal{T}$) are in $\mathcal{I}$, then (X, $\mathcal{T}$, $\mathcal{I}$) is ψ -Housdorff.

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