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On Semiparacompactness and z-paracompactness in Bitopological Spaces

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Summary

We find some properties of semi paracompactness and z-paracompactness in bitopological spaces and give the relation between these concepts. Throughout the present paper m will denote infinite cardinal numbers.

Keywords: Paracompact, z-paracompact and bitopological spaces

1. Introduction

The concept of Paracompactness is due to Dieudonne [6] . The concept of paracompact with respect to three topologies is due to Martin [5] . The term space (X , τ, μ) is referred to as a set X with two generally nonidentical topologies τ and μ .

A cover (or covering) of a space (X , τ) is a collection of subsets of X whose union is all of X . A τ -open cover of X is a cover consisting of τ -open sets , and other adjectives applying to subsets of X apply similarly to covers . If $\coprod$ and $\prod$ are covers of X , we say $\prod$ refines $\coprod$ if each members of $\prod$ is contained in some member of $\coprod$. Then, we say $\prod$ refines (or is a refinement of) $\coprod$. A collection $\prod$ of subsets of X is called locally finite if each x in X has a neighborhood meeting only finitely many member of $\prod$, and is called σ -locally finite if it is a countable union of locally finite collection in X . Note that , every locally finite collection of sets is σ -locally finite . A subset of a topological space (X , τ) is an Fσ if it is a countable union of τ - closed sets , and written by τ - F σ .

1.1. Lemma [6]

Let U be a cover of a topological space X , and let V be a refinement of U . If W refines V , then W refines U .

1.2. Lemma [6]

Let (Y , τ_Y) be a subspace of (X , τ). If a collection $V = \{V_\gamma : \gamma \in \Gamma\}$ of sets is a (σ -)locally finite with respect to τ , then so is $\{V_\gamma \cap Y : \gamma \in \Gamma\}$ with respect to τ_Y .

1.3. Lemma [6]

1. If $U = \{U_\lambda : \lambda \in \Delta\}$ is locally finite collection of sets in (X , τ) . Then any subcollection of U is locally finite .
2. If $U = \{U_\lambda : \lambda \in \Delta\}$ is locally finite collection of sets in (X , τ) , then so is $\{cl_\tau(U_\lambda) : \lambda \in \Delta\}$ and $\bigcup_{\lambda \in \Delta} cl_\tau(U_\lambda) = cl_\tau(\bigcup_{\lambda \in \Delta} U_\lambda)$.
3. The union of a finite number of locally finites collections of sets is locally finite.

1.4. Definition [3]

A bitopological space (X, τ, μ) is called pairwise Hausdorff , if for every two distinct points x and y of X , there exists τ -open set U and μ -open set V such that $x \in U, y \in V$ and $U \bigcap V = \phi$.

1.5. Definition [3]

A bitopological space (X, τ, μ) is (τ, τ, μ)-regular , if every point x of X and every τ -open set U containing x there exists a τ -open set V containing x such that $cl_\mu(V) \subset U$.

2. Main Results

2.1. Definition

A bitopological space (X, τ, μ) is called $(m-)(\tau - \mu)$ semiparacompact with respect to μ [5] , if each τ -open cover of X (with cardinality $\leq m$) has a μ -open refinement which is σ-locally finite with respect to μ .

2.2. Definition

A bitopological space (X, τ, μ) is called $(m-)(\tau - \mu)$-a-paracompact with respect to μ , if each τ -open cover of X (with cardinality $\leq m$) has a refinement which is locally finite with respect to μ .

2.3. Theorem

If (X, τ, μ) is $(m-)(\tau - \mu)$ semiparacompact with respect to μ , then the τ -closed subspace (Y, τ_Y, μ_Y) is $(m)(\tau_Y - \mu_Y)$ semiparacompact with respect to μ_Y .

Proof .

Suppose that (Y, τ_Y, μ_Y) be a τ -closed subspace of (X, τ, μ). Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ_Y -open cover of Y with cardinality $\leq m$. Since each U_λ is a τ_Y-open subset of Y, there is a τ -open subset V_λ of X such that $U_\lambda = V_\lambda \bigcap Y$. Let $\prod = \{V_\lambda : \lambda \in \Delta\} \bigcup \{X / Y\}$. Then $\prod$ is τ -open cover of X ,(with cardinality $\leq m$) . By hypothesis $\prod$ has a $\mu - open$ refinement W which is $\sigma - locally \;\; finite$ with respect to μ , hence $W = \bigcup_{n=1}^{\infty} W_n$ where each $W_n = \{W_{n\gamma} : \gamma \in \Gamma\}$ is locally finite with respect to μ . Let $A = \bigcup_{n=1}^{\infty} A_n$, where $A_n = \{W_{n\gamma} \cap Y : \gamma \in \Gamma\}$. We claim that A is

1. $\mu_Y - open$ cover of Y
2. refine U
3. $\sigma - locally \;\; finite$ with respect to μ_Y .

Proof of (1) . Since every $W_{n\gamma}$ is $\mu-open$, then $W_{n\gamma}\bigcap Y$ is μ_Y-open . Let $y\in Y \Rightarrow y\in X \Rightarrow y\in W_{n\gamma}$ for some n , γ , then $y\in W_{n\gamma}\bigcap Y$ for some n , γ . Hence A is a μ_Y-open cover of Y .

Proof of (2) . Let $\bigcup_{n=1}^{\infty}(W_{n\gamma}\bigcap Y)\in A$ *where* $W_{n\gamma}\bigcap Y=\neq\phi$ since W refines $\prod$, then for every $\bigcup_{n=1}^{\infty}W_{n\gamma}\in W$, there is V_λ *of* $\prod$ such that $\bigcup_{n=1}^{\infty}W_{n\gamma}\subset V_\lambda$, so we get that $\bigcup_{n=1}^{\infty}W_{n\gamma}\bigcap Y\subset V_\lambda\bigcap Y=U_\lambda$, hence $\bigcup_{n=1}^{\infty}(W_{n\gamma}\bigcap Y)\subset U_\lambda$. Therefore A refines U .

Proof of (3) . By Lemma 1.2 , A is $\sigma-locally\ finite$ with respect to μ_Y . Therefore the subspace (Y,τ_Y,μ_Y) is a $(m)(\tau_Y-\mu_Y)$ semiparacompact with respect to μ_Y .

2.4. Theorem

Let (X,τ,μ) be a bitopological space , and let $X=\{X_i : X_i\in\tau\bigcap\mu\ ,\ i\in I\}$ be a patition of X . The space (X,τ,μ) is $(m-)(\tau-\mu)$ semiparacompact with respect to μ if and only if the space (X_i,τ_i,μ_i) is $(m-)(\tau_i-\mu_i)$ semiparacompact with respect to μ_i for every i .

Proof .

The " only if " part , since $X_i=X/\bigcup_{j\neq i}X_j$ is $\tau-closed$ then the subspace (X,τ_i,μ_i) is $(m-)(\tau_i-\mu_i)$ semiparacompact with respect to μ_i , for every I , by theorem 2.3 .

The"if part" . Let $U=\{U_\lambda:\lambda\in\Delta\}$ be a τ -open cover of X with cardinality $\leq m$. The collection $\prod=\{U_\lambda\bigcap X_i:\lambda\in\Delta\ \}$ be a τ_i-open cover of X_i with cardinal $\leq m$ for every i .

Since (X_i,τ_i,μ_i) is $(m-)(\tau_i-\mu_i)$ semiparacompact with respect to μ_i , for every i , there is a μ_i-open refinement A_i which is $\sigma-locally\ finite$ with respect to μ_i so $A_i=\bigcup_{n=1}^{\infty}A_{i_n}$, where each $A_{i_n}=\{\ A_{i_{n\gamma}}:\gamma\in\Gamma\ \}$ is locally finite with respect to μ_i .

Let $W=\bigcup_{n=1}^{\infty}W_n$ where $W_n=\{\bigcup A_{i_{n\gamma}}:\gamma\in\Gamma\ \}$. We claim that W is

1. μ -open cover of X .
2. refine U .
3. $\sigma-locally\ finite$ with respect to μ .

Proof of (1) . Since $A_{n\gamma}$ is μ_i-open , and $X_i\in\mu$, then $A_{n\gamma}$ is $\mu\ -open$. Since

$$X=\bigcup_{i\in I}X_i=\bigcup_{i\in I}(\bigcup A_i)=\bigcup_{i\in I}(\bigcup A_i)=\bigcup_{i\in I}(\bigcup_{n=1}^{\infty}(\bigcup A_{i_{n\gamma}}))=\bigcup(\bigcup_{n=1}^{\infty}(\bigcup_{i\in I}A_{i_{n\gamma}}))=\bigcup(\bigcup_{n=1}^{\infty}W_n)=\bigcup W$$

Proof of (2)

Let $\bigcup_{n=1}^{\infty}(\bigcup A_{i_{n\gamma}})\in W$. Since A refine $\prod$, then there is a member G of $\prod$ such that $\bigcup_{n=1}^{\infty}A_{i_{n\gamma}}\subset G$, then there is $U_\lambda\in U$ such that $G=U_\lambda\bigcap X_i$, hence $\bigcup_{n=1}^{\infty}A_{i_{n\gamma}}\subset U_\lambda\bigcap X_i$, *so* $\bigcup_{i\in I}(\bigcup_{n=1}^{\infty}A_{i_{n\gamma}})\subset U_\lambda\bigcap(\bigcup_{i\in I}X_i)$, therefore $\bigcup_{n=1}^{\infty}(\bigcup_{i\in I}A_{i_{n\gamma}})\subset U_\lambda\bigcap X=U_\lambda$. Hence W refine U .

Proof of (3). Let $x \in X$, if $x \in X_i$, then x has a $\mu_i - open$ neighborhood V such that $V \bigcap (\bigcup_{i \in I} A_{i_{n\gamma}}) = \phi$ for all but finitely many γ . Since V is $\mu - open$ neighborhood of x , then W_n is locally finite with respect to μ and consequentely W is $\sigma - locally\ finite$ with respect to μ. Therefore (X, τ, μ) is $(m-)(\tau - \mu)$ semiparacompact with respect to μ.

2.5. Theorem

If (X, τ, μ) is $(m-)(\tau - \mu)$ semiparacompact with respect to μ , then the subspace (Y, τ_Y, μ_Y) is $(m-)(\tau_Y - \mu_Y)$ *semiparacompact* with respect to μ_Y , where Y is $\tau - F_\sigma - set$.

Proof .

Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a $\tau_Y - open$ cover of Y (with cardinality $\leq m$). Since each U_λ is $\tau_Y - open$ subset of Y then there exists a $\tau - open$ set V_λ such that $U_\lambda = V_\lambda \bigcap Y$.

For each fixed n , $\prod_n = \{V_\lambda : \lambda \in \Delta\} \bigcup \{ X / Y_n \}$ form a $\tau - open$ cover of X (with cardinality $\leq m$) . By hypothesis $\prod_n$ has a $\mu - open$ refinement W which is $\sigma - locally\ finite$ with respect to μ . Then $W = \bigcup_{n=1}^{\infty} W_n$ where each $W_n = \{ W_{n\gamma} : \gamma \in \Gamma \}$ is locally finite with respect to μ .

For each n , let $B = \bigcup_{n=1}^{\infty} B_n$, where $B_n = \{W_{n\gamma} \bigcap Y : W_n \bigcap Y \neq \phi \}$ we claim that B is

1. $\mu_Y - open$ cover of X
2. refines U
3. $\sigma - locally\ finite$ with respect to μ_Y .

Proof of (1). Since each $W_{n\gamma}$ is $\mu - open$ set , then $W_{n\gamma} \bigcap Y$ is a $\mu_Y - open$ set , hence B_n is a collection of $\mu_Y - open$ sets .To show that B covers Y. Let $y \in Y$, then $y \in Y_n$ for some n , then $y \in W_{n\gamma}$ for some γ , then $y \in W_{n\gamma} \bigcap Y$ for some γ , hence B covers Y .

Proof of (2). Let $\Im \in B\ so$ there exists $W_{n\gamma} \in W$ such that $\Im = \bigcup_{n=1}^{\infty} W_{n\gamma} \bigcap Y$. Here $W_{n\gamma} \subset X - Y_n$ is impossible, so that $W_{n\gamma} \subset V_\lambda$ for some λ , then $\bigcup_{n=1}^{\infty} W_{n\gamma} \subset V_\lambda$ which implase that $\bigcup_{n=1}^{\infty} W_{n\gamma} \bigcap Y \subset V_\lambda \bigcap Y$, so we get that $B \subset U_\lambda$. Therefore B refines U.

Proof of (3) . By Lemma (1.2) B is $\sigma - locally\ finite$ with respect to μ_Y . Therefore the subspace (Y, τ_Y, μ_Y) is $(m-)(\tau_Y - \mu_Y)$ *semiparacompact* with respect to μ_Y .

2.6. Theorem

Every $(m-)(\tau - \mu)$ semiparacompact with respect to μ bitopological space (X, τ, μ) is $(m-)(\tau - \mu)$-a-paracompact.

2.7. Definition [1]

A bitopological space (X, τ, μ) is called $(m-)(\tau - \mu)$ compact if for every $\tau - open$ cover $U = \{U_\lambda : \lambda \in \Delta\}$ of X (with cardinality $\leq m$) has a $\mu - open$ finite subcover .

2.8. Theorem [1]

If f is a $(\mu-\tau^{'})$ closed and $(\mu-\mu^{'})$ continuous mapping of a bitopological space (X,τ,μ) onto a $(m\_)(\tau^{'}-\mu^{'})$ semiparacompact with respect to $\mu^{'}$ bitopological space $(Y,\tau^{'},\mu^{'})$ such that $z=f^{-1}(y)$, for all $y\in Y$ is $(m-)(\tau-\mu)$ compact , then (X,τ,μ) is a $(m-)(\tau-\mu)$ semiparacompact with respect to μ .

2.9. Definition

A bitopolgical space (X,τ,μ) is called $(m-)(\tau-\mu)$ semiparacompact with respect to μ , if every $\tau-open$ cover of X (with cardinality $\leq m$) has a μ -closed refinement which is locally finite with respect to μ .

2.10. Definition

A bitopological space (X,τ,μ) is called (m-)-z- semiparacompact , if every $\tau-open$ cover of X (with cardinality $\leq m$) has a μ -closed refinement which is $\sigma-locally\ finite$ with respect to μ .

2.11. Theorem

If a bitopolgical space (X,τ,μ) is a $(m-)(\tau-\mu)$-z-paracompact with respect to μ , then the $\tau-closed$ subspace (Y,τ_Y,μ_Y) be an $(m)(\tau_Y-\mu_Y)$-z-paracompact with respect to μ_Y .

Proof . Let $U=\{U_\lambda:\lambda\in\Delta\}$ be a $\tau-open$ cover of X with cardinality $\leq m$, then there is a $\tau-open$ subset V_λ of X such that $U_\lambda=V_\lambda\bigcap Y$ for every λ .

The collection $\prod=\{V_\lambda:\lambda\in\Delta\}\bigcup\{X/Y\}$

form a $\tau-open$ cover of X with cardinality $\leq m$. Since (X,τ,μ) is a $(m-)(\tau-\mu)$-z-paracompact with respect to μ , then $\prod$ has μ -closed refinement $W=\{W_\gamma:\gamma\in\Gamma\}$ which is locally finite with respect to μ .

The collection $\wp=\{W_\gamma\bigcap Y:\gamma\in\Gamma\}$ is a μ -closed refinement of U which is locally finite with respect to μ . Therefore (Y,τ_Y,μ_Y) is a $(m)(\tau_Y-\mu_Y)$-z-paracompact with respect to μ_Y .

2.12. Corollary

If a bitopolgical space (X,τ,μ) is a $(\tau-\mu)$-z-paracompact with respect to μ , then the $\tau-closed$ subspace (Y,τ_Y,μ_Y) is a $(\tau_Y-\mu_Y)$-z-paracompact with respect to μ_Y .

2.13. Theorem

Let (X,τ,μ) be a bitopolgical space and let $X=\{X_i:X_i\in\tau\bigcap\mu, i\in I\}$ be a partition of X .

The bitopolgical space (X,τ,μ) is a (m) $(\tau-\mu)$-z-paracompact with respect to μ if and only if the space (X,τ_i,μ_i) is a (m) $(\tau_i-\mu_i)$-z-paracompact with respect to μ_i for every i .

Proof . The "only if " part . Since $X=X/\bigcup_{j\neq i}X_j$ is a $\tau-closed$ then the subspace (X,τ_i,μ_i) is an m $(\tau_i-\mu_i)$-z-paracompact with respect to μ_i for every i by Theorem (2.11).

The "if" pat . Let $U=\{U_\lambda:\lambda\in\Delta\}$ be a $\tau-open$ cover of X with cardinality $\leq m$ .The collection $\prod=\{U_\lambda\bigcap X_i:\lambda\in\Delta\}$, is a τ_i-open cover of X_i with cardinality $\leq m$ for every I . Since (X,τ_i,μ_i) is an m $(\tau_i-\mu_i)$-z-paracompact with respect to μ_i for every i , there exists a $\mu_i-closed$

refinement $\Re_i = \{ A_{i_\lambda} : \lambda \in \Delta \}$ of $\prod$ which is locally finite with respect to μ_i for every i. Set $W = \{ \bigcup_{i \in I} A_{i_\lambda} : \lambda \in \Delta \}$.

Then W is μ-closed refinement of U which is locally finite with respect to μ . Therefore (X, τ, μ) is an (m) $(\tau - \mu)$-z-paracompact with respect to μ .

2.14. Theorem

If each $\tau - open$ in an m $(\tau - \mu)$-z-paracompact with respect to μ bitopological space (X, τ, μ) is an m $(\tau - \mu)$-z-paracompact with respect to μ , then very subspace (Y, τ_Y, μ_Y) is an m $(\tau_Y - \mu_Y)$-z-paracompact with respect to μ_Y .

Proof . Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a $\tau_Y - open$ cover of Y with cardinality $\leq m$. Since each U_λ is $\tau_Y - open$ in Y , we have $U_\lambda = V_\lambda \cap Y$ where V_λ is a $\tau - open$ subset of X for every $\lambda \in \Delta$. Then $G = \bigcup_{\lambda \in \Delta} V_\lambda$ is a $\tau - open$ set .

Let $V = \{V_\lambda : \lambda \in \Delta\}$ be a $\tau - open$ cover of G with cardinality $\leq m$. Then G is an m $(\tau - \mu)$-z-paracompact with respect to μ . Thus V has a μ-closed refinement $\Re = \{A_\gamma : \gamma \in \Gamma\}$ which is locally finite with respect to μ . Set $\Im = \{B_\gamma : \gamma \in \Gamma\}$, *where* $B_\gamma = A_\gamma \cap Y$.

The collection $\Im$ is μ_Y-closed refinement of U , which is locally finite with respect to μ_Y . Therefore (Y, τ_Y, μ_Y) is an m $(\tau_Y - \mu_Y)$-z-paracompact with respect to μ_Y .

2.15. Corollary

If each $\tau - open$ set in a $(\tau - \mu)$-z-paracompact with respect to μ bitopological space (X, τ, μ) is a $(\tau - \mu)$-z-paracompact with respect to μ , then every subspace (Y, τ_Y, μ_Y) is a $(\tau_Y - \mu_Y)$-z-paracompact with respect to μ_Y .

2.16. Theorem

If (X, τ, μ) be an m $(\tau - \mu)$-z-paracompact with respect to μ ,then the F_σ-subspace (Y, τ_Y, μ_Y) is an m $(\tau_Y - \mu_Y)$ semi-z-paracompact with respect to μ_Y .

Proof . Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a $\tau_Y - open$ cover of Y with cardinality $\leq m$. Since U_λ is a $\tau_Y - open$ subset of Y for every $\lambda \in \Delta$, we have $U_\lambda = V_\lambda \cap Y$ for every $\lambda \in \Delta$. For each fixed n , $E_n = \{ V_\lambda : \lambda \in \Delta \} \bigcup \{ X / Y_n \}$ form a $\tau - open$ cover of X with cardinality $\leq m$, since X is an m $(\tau - \mu)$-z-paracompact with respect to μ , then E_n has a μ-closed refinement $W = \{ W_{\lambda_n} : (\lambda, n) \in \Delta \times \mathrm{N} \}$ which is locally finite with respect to μ .

For each n , let $B_n = \{ W_{\lambda_n} \cap Y : W_{\lambda_n} \cap Y \neq \phi \}$. Then $B = \bigcup B_n$ is μ- closed refinement of U which is $\sigma -$locally finite with respect to μ , therefore (Y, τ_Y, μ_Y) is an m $(\tau_Y - \mu_Y)$ semi-z-paracompact with respect to μ_Y .

2.17. Corollary

If (X, τ, μ) be a $(\tau - \mu)$-z-paracompact with respect to μ ,then the F_σ-subspace (Y, τ_Y, μ_Y) is a $(\tau_Y - \mu_Y)$ semi-z-paracompact with respect to μ_Y .

2.18. Theorem

Let (X,τ,μ) be a $(\tau,\tau,\mu)-regular$ bitopological space .

If (X,τ,μ) is $(\tau-\mu)$-a-paracompact with respect to μ , then it is $(\tau-\mu)$-z-paracompact with respect to μ .

Proof. Let $U=\{U_\lambda : \lambda\in\Delta\}$ be a $\tau-open$ cover of X . With each $x\in X$, associates a $\tau-open$ set U_x containing it and since (X,τ,μ) is $(\tau,\tau,\mu)-regular$, fined a $\tau-open$ set V_x with $x\in V_x\subset cl_\mu(V_x)\subset U_x$. The collection $\prod=\{V_x : x\in X\}$ is a $\tau-open$ covering since (X,τ,μ) is $(\tau-\mu)$-a-paracompact with respect to μ , then $\prod$ has a refinement $A=\{A_x : x\in X\}$ which is locally finite with respect to μ . The collection $\coprod=\{cl_\mu(A_x) : x\in X\}$ is a μ-closed refinement of U which is locally finite with respect to μ by Lemma (1.3)(2) . Therefore (X,τ,μ) is a $(\tau-\mu)$-z-paracompact with respect to μ .

2.19. Theorem

Let (X,τ,μ) be a $(\tau,\tau,\mu)-regular$ bitopological space .If (X,τ,μ) is $(\tau-\mu)$ semiparacompact with respect to μ , then it is $(\tau-\mu)$-z-paracompact with respect to μ .

Proof . Thus follows from Thorem (2.6) and Theorem (2.18) .

2.20. Definition [2]

A collection of sets $U=\{U_\lambda : \lambda\in\Delta\}$ is said to be conservative ina topological space (X,τ) if $\Gamma\subset\Delta$ *implies that* $cl_\tau(\bigcup_{\lambda\in\Gamma}U_\lambda)=\bigcup_{\lambda\in\Gamma}CL_\tau(U_\lambda)$.

2.21. Proposition [2]

The following statements are equivalent to any collection of sets $U=\{U_\lambda : \lambda\in\Delta\}$

1. U is conservative ;
2. If $\Gamma\subset\Delta$, then $\bigcup_{\lambda\in\Gamma}cl\ (U_\lambda)$ is $\tau-closed$;
3. The collection $\{cl_\tau(U_\lambda) : \lambda\in\Delta\}$ is conservative.

2.22. Proposition [2]

Every locally finite collection of sets is conservative.

2.23. Theorem

If (X,τ,μ) is m $(\tau-\mu)$-z-paracompact with respect to μ , then every $\tau-open$ cover of X with cardinality $\leq m$ has a refinement which is a conservative $\mu-closed$ cover .

Proof. Let U be a $\tau-open$ cover of X . Since (X,τ,μ) is a m$(\tau-\mu)$-z-paracompact with respect to μ , then U has a $\mu-closed$ refinement V which is locally finite with respect to μ . Then by Proposition (2.22) V is conservative .Thus the result .

2.24. Corollary

If (X,τ,μ) is $(\tau-\mu)$-z-paracompact with respect to μ , then every $\tau-open$ cover of X has a refinement which is a conservative $\mu-closed$ cover.

2.25. Proposition [2]

Let (X,τ) *and* (Y,μ) be topological spaces .If $f:X\to Y$ a closedmap and $U=\{U_\lambda:\lambda\in\Delta\}$ is a conservative collection consisting of closed sets in (X,τ) , then $\prod=\{f(U_\lambda):\lambda\in\Delta\}$ is a collection in (Y,μ) having the same property .

2.26. Theorem

Let f be a $(\tau_1-\tau_2)$ continuous and $(\mu_1-\mu_2)$ closed mapping of a bitopological space (X,τ_1,μ_1) to a bitopological space (Y,τ_2,τ_2) . If X is a m $(\tau_1-\mu_1)$ -z-paracompact with respect to μ_1 , then Y is an m $(\tau_2-\mu_2)$ -a-paracompact with respect to μ_2 .

Proof . Let $U=\{U_\lambda:\lambda\in\Delta\}$ be a τ_2-open cover of Y with cardinality $\le m$. Since f is $(\tau_1-\tau_2)$ continuous then $\prod=\{f^{-1}(U_\lambda):\lambda\in\Delta\}$ will be τ_1-open cover of X with cardinality $\le m$. By Theorem (2.23) , U has a refinement $V=\{V_\gamma:\gamma\in\Gamma\}$ which is a conservative $\mu_1-closed$ cover . By Proposition (2.23) , the collection $\prod^*=\{f(V_\gamma):\gamma\in\Gamma\}$ is a conservative $\mu_2-closed$ cover of Y , and is evidently a refinement of U ; hence (Y,τ_2,τ_2) is an m $(\tau_2-\mu_2)$ -a-paracompact with respect to μ_2 .

2.27. Corollary

Let f be a $(\tau_1-\tau_2)$ continuous and $(\mu_1-\mu_2)$ closed mapping of a bitopological space (X,τ_1,μ_1) to a bitopological space (Y,τ_2,τ_2) . If X is a $(\tau_1-\mu_1)$ -z-paracompact with respect to μ_1 , then Y is a $(\tau_2-\mu_2)$ -a-paracompact with respect to μ_2 .

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