Weak forms of ω-open sets in bitopological spaces and connectedness

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Abstract:
The aim of this paper is to introduce a new classes of weak ω-open sets in bitopological spaces then study the relations between those classes and some properties . Other aim is to introduce certain type of connectedness in bitopological spaces relative to the new classes of sets introduced in the first part, and get some results .

Keywords: ω pre open set , pre ω open set, ω semi open set, semi ω open set, ω α open set, α ω open set, ω β open set, β ω open set, ω b open set, b ω open set.

1. Introduction

The concepts of pre open sets, semi open sets, α open sets, β open sets, and b-open sets introduced by many authors in topological spaces (cf. **[2, 4, 6, 8, 10])** and extended to bitopological spaces by others (cf. **[9, 11])** . The concept of ω-open sets was introduced and studied by many authors (cf. **[3,12])** , and extended to bitopological spaces in **[5] ,** by defining the concept of $\tau_1\ \tau_2$ –generalized ω-closed set.

In this paper many types of weak open sets in bitopological spaces will be defined, Relations between those sets will be discussed, properties such as supra and infra topological structures will be determined.

Also a new type of connectedness for bitopological spaces will be defined and preserving that type of connectedness under certain type of map between bitopological spaces will be proved , many other results and counter examples ,also will be showed.

Throughout this paper the following notation will be used: $\subset$ denotes subset (not necessarily proper), A^c denotes the complement of A in the space (that A is subset of). If (X, τ_1, τ_2) is a bitopological space, $A \subset X$, i-int A and j-cl A denote the interior and closure of A relative to τ_i and τ_j respectively , i-open(closed) set denotes τ_i open(closed) set (i,j $\in$ {1,2}).

1.1 Definition [4, 11]

Let (X, τ_1, τ_2) be a bitopological space, $A \subset X$, A is said to be :

(i) ij- p open set if $A \subset$ i-int (j-clA).
(ii) ij- s open set if $A \subset$ j-cl(i-int A).
(iii) ij- α open set if $A \subset$ i-int(j-cl(i-int A)).
(iv) ij- β open set if $A \subset$ j-cl(i-int (j-clA)).
(v) ij- b open set if $A \subset$ i-int (j-clA) $\cup$ j-cl(i-int A).

(p-open denotes pre open, and s- open denotes semi open) .

1.2 Remark

It is clear from definition that in any bittopological space the following hold:

- **(i)** Every i-open set is ij- p open , ij- s open, ij- α open, ij- β open and ij- b open set.
- **(ii)** Every ij- p open set is ij- β open.
- **(iii)** Every ij- α open set is ij- s open.
- **(iv)** Every ij- p open(ij- s open) set is ij- b open set.
- **(v)** The concepts of ij- p open and ij- s open sets are independent.
- **(vi)** The concepts of ij- α open and ij- β open sets are independent.

2. Weak forms of ω-open sets in bitopological spaces

First recall the following definition from topological spaces.

2.1 Definition [3]

Let (X , τ) be a topological space, $A \subset X$, $x \in X$ is called a condensation point if for each $U \varepsilon \tau$ with $x \varepsilon U$, the set $U \cap A$ is uncountable . A is said to be an ω-closed if it contains all its condensation points. The complement of ω-closed set is said to be ω-open set. Equivalently a set W is ω-open if for each $x \in W$, there exist $U \in \tau$ with $x \in U$ and U-W is countable .

The family of all ω-open sets in (X , τ) , denoted by τ_ω , forms a topology on X finer than τ . The ω-closure and ω- interior of a set A, will be denoted by $cl_\omega A$ and $int_\omega A$ resp. , are defined by:

$$cl_\omega A = \cap \{F \subset X | F \text{ is } \omega\text{-closed and } A \subset F \}$$

$$int_\omega A = \cup \{G \subset X | G \text{ is } \omega\text{-open and } G \subset A \}$$

2.2 Note The following notes are clear:

(i) If (X , τ) is any topological space , $A \subset X$, then

$$int\, A \subset int_\omega A \quad \text{and} \quad cl_\omega A \subset clA$$

(ii) If X is a countable set and τ is any topology on X , then all the subsets of X are ω-closed and ω-open , i.e. , $\tau_\omega = P(X)$.

In what follows, let $i,j \in \{1,2\}$ and $i \neq j$ **.**

2.3 Definition

Let (X, τ_1, τ_2) be a bitopological space , $A \subset X$,. A is said to be :

ij- ω pre open, if $A \subset i\text{-}int_\omega(j\text{-}cl\, A)$.
ij- pre ω open, if $A \subset i\text{-}int\, (j\text{-}cl_\omega A)$.
ij- ω semi open, if $A \subset j\text{-}cl_\omega(i\text{-}int\, A)$.
ij- semi ω open, if $A \subset j\text{-}cl\, (i\text{-}int_\omega A)$.
ij- ω α open, if $A \subset i\text{-}int_\omega (j\text{-}cl(i\text{-}int_\omega A))$.
ij- α ω open , if $A \subset i\text{-}int\, (j\text{-}cl\omega(i\text{-}int\, A))$.
ij- ω β open, if $A \subset j\text{-}cl_\omega (i\text{-}int(j\text{-}cl_\omega A))$.
ij- β ω open, if $A \subset j\text{-}cl\, (i\text{-}int_\omega(j\text{-}cl\, A))$.
ij- ω b open, if $A \subset i\text{-}int_\omega(j\text{-}cl\, A) \cup j\text{-}cl_\omega(i\text{-}int\, A)$.
ij- b ω open , if $A \subset i\text{-}int\, (j\text{-}cl_\omega A) \cup j\text{-}cl\, (i\text{-}int_\omega A)$.

The set ij-ω pre open (ij-pre ω open; ij-ω semi open; ij-semi ω open) will be denoted briefly ij- ω p open (ij- p ω open; ij- ω s open; ij- s ω open).

2.4 Remark

If (X, τ_1, τ_2) is a bitopological space , A is a countable subset of X, then;

- **(i)** A is ij- p ω (ij- ω s ; ij- α ω; ij- ω β) open if and only if it is i- open.
- **(ii)** A is ij- ω b (ij- b ω) open, if it is i- open.
- **(iii)** A is ij- ω b (ij- b ω) open implies it is ij- ω p (ij- s ω)open.

Proof: (i) and **(ii)** since $cl_\omega A = A$ (relative to any topology),when A is countable.

(iii) since $cl_\omega A = A$(when A is countable) and int $A \subset int_\omega A$ in general**.**

2.5 Remark

If (X, τ_1, τ_2) is a bitopological space , and if A is a subset of X such that A^c is countable, then A is ij – ω h open for h=p, α, b and ij- h ω for h=s, β, b.

Proof: if A^c is countable then $(j\text{-}cl\ A)^c$ is countable too, which implies that $i\text{-}int_\omega(j\text{-}cl\ A) = j\text{-}cl\ A$ and $j\text{-}cl(i\text{-}int_\omega\ A)= j\text{-}cl\ A$. Now the fact that $A \subset j\text{-}cl\ A$ completes the proof.

2.6 Remark

If X is countable , τ_1 and τ_2 are any two topologies on X , $A \subset X$, then;

- **(i)** A is ij – ω h open for h=p, α, b and ij- h ω for h=s, β, b.
- **(ii)** A is ij- p ω (ij- α ω; ii- ω s ; ij- ω β) open if and only if A is i- open.

Proof: (i) since $cl_\omega A = A$ and $int_\omega A= A$ (when X is countable).

(ii) By 2.4 and 2.5.

2.7 Theorem

If X is a countable set , τ_1 and τ_2 are any two topologies on X , then; the family of all ij- ω h (ij- h ω) open subsets of X, h= p, α, s, β, b, form a topology on X

Proof: By 2.6 :

(i) It is P(X) for the cases ij- ω h with h= p, α, b and the cases ij- h ω with h= s, β, b .

(ii) It is τ_i for the cases ij- p ω , ij- α ω, ij- ω s and ij- ω β .

2.8 Remark

Let (X, τ_1, τ_2) be a bitopological space , the following relations between the sets defined in 2.3 hold:

- **(i)** Every ij- h ω open is ij- h open set (h=p, α) but not the converse**.**
- **(ii)** Every ij- ω h open is ij- h open set (h= s, β) but not the converse**.**
- **(iii)** Every ij- h open is ij- ω h open set (h=p, α) but not the converse**.**
- **(iv)** Every ij- h open is ij- h ω open set (h= s, β) but not the converse**.**

Proof: straightforward by definitions.

2.9 Examples

The following examples show that the converse in the previous remark are not true:

Let X={a, b, c, d} ; $\tau_1=\tau_2=$ {φ,{a},{a, b},{a, b, c}} :

- **(i)** A={a, d} is 12- p but not 12- p ω open set.
  B={a, b, d} is 12- α but not 12- α ω open set.
- **(ii)** A is 12-s but not 12- ω s open set.
  A is 12- β but not 12- ω β open set.

(iii) $C=\{b, c\}$ is 12- ω p but not 12- p open set.
$D=\{c, d\}$ is 12- ω α but not 12- α open set.
(iv) **C** is 12- s ω but not 12- s open set.
C is 12- β ω but not 12- β open set.

2.10 Remark

Let (X, τ_1, τ_2) be a bitopological space ,then every ij- α ω open set is ij- β ω open set but not the converse, where the following pairs of concepts are independent;

(i) ij- ω p open and ij- ω s open .
(ii) ij- p ω open and ij- s ω open **.**
(iii) ij- ω α open and ij- ω β open **.**

Proof: i-int A $\subset$A implies j- cl (i-int A) $\subset$ j-cl A, so j-cl$_\omega$(i-intA) $\subset$j-cl A and i-int(j-cl$_\omega$(i-intA) $\subset$i-int(j-cl A) $\subset$i-int$_\omega$(j-clA) $\subset$j-cl(i-int$_\omega$(j-clA)).
Hence an ij- α ω open set is an ij- β ω open set.

Before giving examples to verify the other parts of the remark, the following note is needed.

2.11 Note

If X is uncountable, $\tau=\{\varphi, X\}$,A is an uncountable subset of X, we have:

$$\text{int}_\omega A = \begin{cases} \varphi & \text{if } X\text{-}A \text{ is uncountable} \\ \mathbf{A} & \text{if } X\text{-}A \text{ is countable} \end{cases}$$

$$\text{cl}_\omega A = \begin{cases} A & \text{if } A \text{ is countable} \\ X & \text{if } A \text{ is uncountable} \end{cases}$$

2.12 Examples

Let X be uncountable , A an uncountable subset of X and B a countable subset of A. Suppose that $\tau_1=\{\varphi, X\}$, $\tau_2=\{\varphi, X ,A, B\}$, then the set A-B is:

(i) 12- ω p but not 12- ω s open set.
(ii) 12- s ω but not 12- p ω open set (if A^c is countable).
(iii) 12- ω α but not 12- ω β open set (if A^c is countable).
(iv) 12- β ω but not 12- α ω open set.
(v) The set A is 12- p ω open but not 12- s ω open(if A^c is uncountable).

Now take $X=A\cup B\cup C\cup D$, where A,B,C,D are pair wise disjoint uncountable sets and suppose that $\tau_1=\tau_2=\{\varphi, X, A, B, A\cup B, A\cup B\cup C\}$,then the set $A\cup C$ is:

(vi) 12- ω s but not 12- ω p open set.
(vii) 12- ω β but not 12- ω α open set.

2.13 Remark

Let (X, τ_1, τ_2) be a bitopological space ,then the following relations hold:

$$\begin{array}{ccccc} \text{ij- ω α open} & \xrightarrow{1} & \text{ij- ω p open} & \xrightarrow{2} & \text{ij- ω b open} \\ & & & & \uparrow 3 \\ & & \text{ij- ω β open} & \xleftarrow{4} & \text{ij- ω s open} \end{array}$$

$$\begin{array}{ccccc} \text{ij- α ω open} & \xrightarrow{5} & \text{ij- p ω open} & \xrightarrow{6} & \text{ij- b ω open} \\ & & & & \uparrow 7 \\ & & \text{ij- β ω open} & \xleftarrow{8} & \text{ij- s ω open} \end{array}$$

The above arrows are not reversible , the set A-B in the previous example is a counter example for the arrows 1,3,5,and 6 (with A^c countable), for 7 and 8 (with A^c uncountable) ; the set A^c (with i=2 and j=1) for the arrow 4 and finally the set $A \cup C$ of the second part of the example for the arrow 2, verifying this fact.

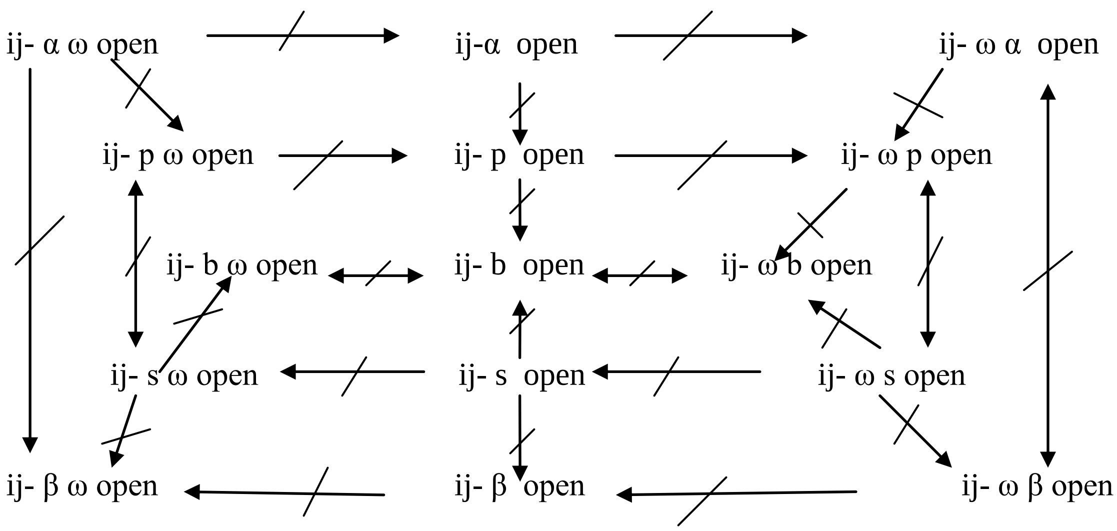
2.14 Remark

Let (X, τ_1, τ_2) be a bitopological space , then the concepts :ij-b open; ij- ω b open; ij- b ω open are pair wise independent as shown in the following examples.

2.15 Example

- **(i)** the set D in example 2.9 (iii) is 12- ω b open and 12- b ω open but not 12-b open set.
- **(ii)** If X=R , $\tau_1=\tau_2=$ the usual topology ,then Q is 12-b open but not 12-b ω open.
- **(iii)** If {A, B, C, D} is a partition of a set X ,where A , C are countable and B, D are uncountable subsets of X and if $\tau_1=\tau_2=\{ \varphi, X, A, B, A\cup B, A\cup B\cup C\}$,then $A \cup C$ is 12-b open set but not 12- ω b open set.

The following diagram summarizes the results of 2.8,2.10,2.13 and 2.14:



Where $\not\longrightarrow$ means the arrow is not reversible and $\not\longleftrightarrow$ means independent.

2.16 Definition

If (X, τ_1, τ_2) is a bitopological space , a subset A of X is said to be ij- ω h closed (ij- h ω closed) set if its complement is ij- ω h open (ij- h ω open) where h= p, s, α, β, b .

Notation

Let (X, τ_1, τ_2) be a bitopological space , the family of all ij- ω h open (ij- h ω open; ij- ω h closed ; ij- h ω closed) subsets of X will be denoted by ij- ω hO(X) (ij- h ω O(X); ij- ω hC(X); ij- h ω C(X)), where h=p, s, α, β, b .

Recall ,a family μ of subsets of X is called supra topological structures on X if μ contains X, φ and is closed under arbitrary union. And it is called infra topological structures on X if μ contains X, φ and is closed under finite intersections [7].

2.17 Theorem

Let **(** X, τ_1, τ_2) be a bitopological space , ij- ω hO(X) and ij- h ω O(X) where h=p, s, α, β, b are supra topological structures on X.

Proof: Since int $\cup A_\lambda = \cup$ int A_λ and $\cup$ cl$A_\lambda \subset$ cl $\cup A_\lambda$, the proof is straightforward .

2.18 Definition

A new closure operations on a subset A of a bitopological space **(** X, τ_1, τ_2) can be defined as follows:

ij- ω h cl(A)= $\cap\{F \mid F \in$ ij- ω h C(X) and A $\subset F\}$

ij- h ω cl(A)= $\cap\{F \mid F \in$ ij- h ω C(X) and A $\subset F\}$

where h= p, s, α, β, b .

2.19 Remark

Let **(** X, τ_1, τ_2) be a bitopological space.

(i) If A and B are two disjoint ij- ω h open subsets of X, then A$\cap$ ij- ω h cl(B)= B$\cap$ ij- ω h cl(A) =φ.

(ii) If A and B are two disjoint ij- h ω open subsets of X, then A$\cap$ ij- h ω cl(B)= B$\cap$ ij- h ω cl(A) =φ.

h= p, s, α, β, b .

Proof: (i) will be proved ,and the proof of (ii) is similar.

A$\cap$B=φ implies A$\subset B^c$, which implies ij- ω h cl(A) $\subset B^c$ (since B^c is ij- ω h closed) . Hence B$\cap$ ij- h ω cl(A) =φ. The proof of A$\cap$ ij- h ω cl(B)= φ is similar.

3. ij-ω h (ij-h ω) connectivity

3.1 Definition

A bitopological space **(**X, τ_1, τ_2) is said to be ij-ω h disconnected (ij-h ω disconnected) if it has a subset (other than φ and X) which is both ij- ω h open and ij- ω h closed (ij- h ω open and ij- h ω closed) , otherwise it is called ij-ω h connected (ij- h ω connected), h= p, s, α, β, b .

3.2 Theorem

If X is a countable set (with more than one point), τ_1, τ_2 are any two topologies on X , then **(**X, τ_1, τ_2) is :

(i) ij- ω h disconnected**,** for h= p, α**,** b.

(ii) ij- h ω disconnected, for h= s, β, b .

(iii) ij- p ω (ij- α ω, ij- ω s, ij- ω β) disconnected if and only if **(**X, τ_i) is disconnected.

Proof: By 2.6.

3.3 Remark

In general , i- disconnectivity implies ij- ω h (ij- h ω disconnectivity) h= p, s, α, β, b . But not the converse.

3.4 Example

(i) (X, τ_1, τ_2) in the first part of example 2.12 is 12- p ω disconnected (since A-B is both 12- p ω open and 12- p ω closed) ,where(X, τ_1) is connected.

(ii) (X, τ_1, τ_2) in the second part of example 2.12 is 12- ω s disconnected

(iii) and 12- ω β disconnected, since $A \cup C$ is both 12- ω s(12- ω β) open and 12- ω s(12- ω β) closed, where (X, τ_1) is connected.

(iv) Let X= R, τ_1= the usual topology and $\tau_2=\{\varphi, X\}$, then (X, τ_1, τ_2) is 12- α ω disconnected ,since the set (0,1) is both 12- α ω open and 12- α ω closed (many other sets exist) ,where (X, τ_1) is connected.

(v) Finally (X, τ_1, τ_2) of example 2.9 is 12- h ω disconnected (h= s, β, b) and 12- ω h disconnected(h= p, α, b), where (X, τ_1) is connected.

3.5 Definition

Let **(** X, τ_1, τ_2) be a bitopological space**,** $Y\subset X$, Y is said to be ij- ω h disconnected (ij- h ω disconnected) set if there exist two nonempty ij- ω h (ij- h ω**)** open subsets of X G and H such that $Y\subset G\cup H$, $G\cap H=\varphi$, $Y\cap G\neq\varphi$, and $Y\cap H\neq\varphi$.
Otherwise Y is called ij- ω h connected (ij- h ω connected). (h= p, s, α, β, b)

3.6 Lemma

Let **(** X, τ_1, τ_2) be a bitopological space**,** $Y\subset X$. If Y is ij- ω h connected (ij- h ω connected) set and if G and H are two ij- ω h (ij- h ω**)** open subsets of X such that $G\cap H=\varphi$ and $Y\subset G\cup H$, then either $Y\subset G$ or $Y\subset H$. (h= p, s, α, β, b)
Proof: Obvious by Definition 3.5.

3.7 Theorem

If **(** X, τ_1, τ_2) is a bitopological space**,** $\{Y_\lambda\}$ a family of ij- ω h (ij- h ω**)** connected subsets of X that have a point in common, then $\cup\ Y_\lambda$ is ij- ω h (ij- h ω**)** connected. (h= p, s, α, β, b)
Proof: Assume that $\{Y_\lambda\}$ is ij- ω h connected sets but $\cup\ Y_\lambda$ is ij- ω h is disconnected, let G and H be disjoint ij- ω h open subsets of X with $\cup\ Y_\lambda\subset G\cup H$, $(\cup\ Y_\lambda)\cap G\neq\varphi$ and $(\cup\ Y_\lambda)\cap H\neq\varphi$. Let $x \in \cap\ Y_\lambda$,then either $x \in G$ or $x \in H$ (since G and H are disjoint) . Now by lemma 3.6 each Y_λ either subset of G or subset of H and since $x \in Y_\lambda$ for each λ then either $Y_\lambda\subset G$ for each λ or $Y_\lambda\subset G$ for each λ, that is, either $\cup\ Y_\lambda\subset G$ or $\cup\ Y_\lambda\subset H$ (a contradiction with the assumption , $(\cup\ Y_\lambda)\cap G\neq\varphi$ and $(\cup\ Y_\lambda)\cap H\neq\varphi$). Hence $\cup\ Y_\lambda$ is ij- ω h connected set.
The proof of the other case is similar.

3.8 Theorem

Let **(** X, τ_1, τ_2) be a bitopological space**,** Y an ij- ω h connected subset of X. If $Y\subset Z\subset$ ij- ω h cl Y, then Z is ij- ω h connected**.(** h= p, s, α, β, b)
Proof: Assume that $Z\subset G\cup H$, where G and H are disjoint ij- ω h open subsets of X , $Y\subset Z$ implies $Y\subset G\cup H$, and by 3.6 either $Y\subset G$ or $Y\subset H$.
If $Y\subset G$, then ij- ω h cl $Y\subset$ ij- ω h clG, so $Z\subset$ ij- ω h clG, but by 2.19 $H\cap$ ij- ω h clG=φ, hence $Z\cap H=\varphi$.
Similarly, $Y\subset H$ implies $Z\cap G=\varphi$. Therefore Z is ij- ω h connected.

3.9 Theorem

Let **(** X, τ_1, τ_2) be a bitopological space**,** Y an ij- h ω connected subset of X. If $Y \subset Z \subset$ ij- h ω cl Y, then Z is ij- h ω connected**.** (h= p, s, α, β, b)

Proof: is similar.

3.10 Remark

Let **(** X, τ_1, τ_2) be a bitopological space, if **(** X, τ_i) is disconnected, then **(** X, τ_1, τ_2) is ij- ω h(ij- h ω) disconnected (h= p, s, α, β, b).

Proof: Obvious since every i-open set is ij- ω h(ij- h ω) open set, (h= p, s, α, β, b).

In what remaining of this section , the question about preserving ij- ω h (ij- hω) connectivity between two bitopological spaces under certain maps will be discussed . First some notation is needed ; let **(** X, τ_1, τ_2), (Y, σ_1, σ_2) be two bitopological spaces and f: X → Y. f is i-continuous means f: (X, τ_i) → (Y, σ_i) is continuous, ;and f is j-open means f: (X, τ_j) → (Y, σ_j) is open.i,j $\in$ {1, 2}.

Also the following lemmas are needed:

3.11 Lemma

If f: (X, τ) → (Y, σ) is continuous and injective , $B \subset Y$, then $f^{-1}(\text{int}_\omega B) \subset \text{int}_\omega (f^{-1}(B))$.

Proof: $x \in f^{-1}(\text{int}_\omega B)$ implies $f(x) \in \text{int}_\omega B$,that is, there is $V \in \sigma$ such that $f(x) \in V$ and $V \cap B^c$ is countable ,hence $x \in f^{-1}(V) \in \tau$, and $f^{-1}(V \cap B^c)= f^{-1}(V) \cap (f^{-1} B)^c$ is countable (since f is injective), therefore $x \in \text{int}_\omega (f^{-1}(B))$.

Note: if f is continuous ,it is clear that, $f^{-1}(\text{int } B) \subset \text{int } (f^{-1}(B))$.

3.12 Lemma

If f: (X, τ) → (Y, σ) is open and bijective , $B \subset Y$, then $f^{-1}(\text{cl}_\omega B) \subset \text{cl}_\omega (f^{-1}(B))$.

Proof: $x \notin \text{cl}_\omega (f^{-1}(B))$ implies there is $U \in \tau$, $x \in U$ and $U \cap f^{-1}(B)$ is countable , hence $f(U \cap f^{-1}(B))= f(U) \cap B$ is countable (since f is bijective) , but $f(x) \in f(U) \in \sigma$ (since f is open). Therefore $f(x) \notin \text{cl}_\omega B$, so $x \notin f^{-1}(\text{cl}_\omega B)$.

Note: if f is open ,it is clear that, $f^{-1}(\text{cl } B) \subset \text{cl } (f^{-1}(B))$.

3.13 Lemma

Let **(** X, τ_1, τ_2), (Y, σ_1, σ_2) be two bitopological spaces, f: X → Y bijective, i-continuous and j-open. If B is an ij- ω p open subset of Y then $f^{-1}(B)$ is ij- ω p open subset of X.

Proof: $B \subset \text{i-int}_\omega(\text{j-cl } B)$ implies $f^{-1}(B) \subset f^{-1}(\text{i-int}_\omega(\text{j-cl } B))$, by 3.11 $f^{-1}(\text{i-int}_\omega(\text{j-cl } B)) \subset \text{int}_\omega (f^{-1}(\text{j-cl } B))$,since $f^{-1}(\text{j-cl } B) \subset \text{j-cl}(f^{-1}(B))$, hence, $f^{-1}(B) \subset f^{-1}(\text{i-int}_\omega(\text{j-cl } B)) \subset \text{int}_\omega(\text{j-cl}(f^{-1}(B))$. There fore $f^{-1}(B)$ is ij- ω p open subset of X.

Note: ij- ω p in the above lemma can be replaced by ij- ω h (ij- h ω) , h= p, s, α, β, b, and the proof will stay similar.

3.14 Theorem

Let **(** X, τ_1, τ_2), (Y, σ_1, σ_2) be two bitopological spaces, f: X → Y bijective, i-continuous and j-open.

if **(** X, τ_1, τ_2) is ij- ω h (ij- h ω) connected then (Y, σ_1, σ_2) is ij- ω h (ij- h ω) connected too. (h= p, s, α, β, b)

Proof: One case will be proved ,other cases are similar.
Assume that **(** X, τ_1, τ_2) is ij- ω p connected, f is bijective , i-continuous and j-open, and assume that (Y, σ_1, σ_2) is ij- ω p disconnected .Let A and B be two non empty disjoint ij- ω p open subsets of Y such that Y=A $\cup$B. By 3.13 $f^{-1}(A)$ and $f^{-1}(B)$ are ij- ω p open subsets of X, they are nonempty and disjoint (since f is bijective) and X= $f^{-1}(A) \cup f^{-1}(B)$,which contradicts the assumption that **(** X, τ_1, τ_2) is ij- ω p connected. Therefore (Y, σ_1, σ_2) is ij- ω p connected .

3.15 Lemma

If f: (X, τ) $\rightarrow$ (Y, σ) is open and bijective**,** A$\subset$X, then f(int_{ω} A) $\subset \mathrm{int}_{\omega}$ (f(A)) .

Proof: y $\in$ f(int_{ω} A) implies y=f(x) , x$\in$ int_{ω} A, so there is U $\in$ τ, x $\in$U such that $U \cap A^c$ is countable, hence $f(U \cap A^c)= f(U) \cap (f(A))^c$ is countable (since f is bijective), where f(U) $\in$ σ (since f is open), and y $\in$ f(U).
Therefore y $\in$ int_{ω} (f(A)) .

Note: if f is open ,it is clear that, f(int A) $\subset$int (f(A)).

3.16 Lemma

If f: (X, τ) $\rightarrow$ (Y, σ) is continuous and bijective**,** A$\subset$X, then f(cl_{ω} A) $\subset \mathrm{cl}_{\omega}$ (f(A)) .

Proof: y$\notin$ cl_{ω} (f(A)) implies there is V $\in$ σ, y $\in$V and V$\cap$f(A) is countable, which implies $f^{-1}(V \cap f(A))= f^{-1}(V) \cap f^{-1}(f(A))= f^{-1}(V) \cap A$ is countable (since f is bijective), where $f^{-1}(y)$=x $\in$ $f^{-1}(V)$ $\in$ τ (since f is continuous).
Hence x$\notin$ cl_{ω} A, so y$\notin$ f(cl_{ω} A) (since f is bijective) .

Note: if f is continuous ,it is clear that, (cl A) $\subset$cl (f(A)) .

3.17 Lemma

Let **(** X, τ_1, τ_2), (Y, σ_1, σ_2) be two bitopological spaces, f: X $\rightarrow$ Y bijective, i-open and j-continuous. If A is an ij- ω p open subset of X, then f(A) is ij- ω p open subset of Y.

Proof: A$\subset$ i-int_{ω}(j-cl A) implies f(A) $\subset$f (i-int_{ω}(j-cl A)), by 3.15 f (i-int_{ω}(j-cl A)) $\subset$ int_{ω} (f (j-cl A)) ,by 3.16 f (j-cl A) $\subset$j-cl(f (A)), hence, f (A) $\subset$ f (i-int_{ω}(j-cl A)) $\subset$ int_{ω}(j-cl(f (A)). There fore f (A) is ij- ω p open subset of Y.

Note: ij- ω p in the above lemma can be replaced by ij- ω h (ij- h ω) , h= p, s, α, β, b, and the proof will stay similar.

3.18 Theorem

Let **(** X, τ_1, τ_2), (Y, σ_1, σ_2) be two bitopological spaces, f: X $\rightarrow$ Y bijective, i-open and j-continuous.
if (Y, σ_1, σ_2) is ij- ω h (ij- h ω) connected then **(** X, τ_1, τ_2) is ij- ω h (ij- h ω) connected too. (h= p, s, α, β, b)

Proof: One case will be proved ,other cases are similar.
Assume that (Y, σ_1, σ_2) is ij- ω p connected, f is bijective , i-open and j-continuous, and assume that **(** X, τ_1, τ_2) is ij- ω p disconnected .Let A and B be two non empty disjoint ij- ω p open subsets of X such that X=A $\cup$B. By 3.17 f (A) and f (B) are ij- ω p open subsets of Y, they are nonempty and disjoint (since f is bijective) and Y= f (A) $\cup$ f (B) ,which contradicts the assumption that (Y, σ_1, σ_2) is ij- ω p connected. Therefore **(** X, τ_1, τ_2) is ij- ω p connected .

REFERENCES

[1] A. A. El-Atik, A study of some types of mappings on topological spaces, M. Sc. Thesis, Tanta Univ.,(1997).

[2] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, On precontinuous and weak precontinuous functions, Proc. Math. Phys. Soc. Egypt 51:47-53 (1982).

[3] A. Z. Hdeib, ω-closed mappings, Rev. Colomb. Mat. 16 (3-4): 65-78 (1982).

[4] D. Andrijevic, On b-open sets, Mat. Vesnik 48: 59-64 (1996).

[5] K. Chandrasekhara Rao and D. Narasimhan, Semi Star Generalized ω- closed Sets in Bitopological Spaces, Int. J. Math. Sciences Vol. 4, 2009, no. 12, 587-595

[6] M. E. Abd El-Monsef, S.N. El-Deeb, and R. A. Mahmoud, β-open Sets and β-continuous Mappings, Bull. Fac. Sci., Assiut Univ., 12 (1983), pp.77-90.

[7] M. E. Abd El-Monsef, A. A. El-Atik and M.M. El-Sharkasy, Some Topologies Induced by b-open Sets, KYUNGPOOK Math. J. 45(2005), 539-547

[8] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[9] O. A. El-Tantawy and H. M. Abu-Donia, Generalized Separation Axioms in Bitoplogical Spaces, The Arabian Journal for Science and Engineering, Vol. 30. Number 1A.

[10] O. Njasted, On some Classes of Nearly Open Sets, Pacific J. Math., 15 (1965), pp. 961-970.

[11] S. Sampath Kumar, On a Decomposition of Pairwise Continuity, Bull. Cal. Math. Soc., 89(1997), pp. 441-446.

[12] T. Noiri, A. Al-Omari and M. S. M. Noorani, Weak forms of ω-open sets and Decompositions of continuity, European J. of Pure and Applied Math. Vol.2, No. 1, 2009, (73-84).