

Using ψ -Operator to Formulate a New Definition of Local Function

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Abstract: In this paper, we use ψ -operator in order to get a new version of local function. The concepts of maps, dense, resolvable and Housdorff have been investigated in this paper, as well as modified to be useful in general

في التحقيق تم وقد المدلية الدالة من جديده ن سخه على الحصول أجل من اب ساي المعدلية ال بحث هذا في اس تخدمنا: الخلاصة
 عام به شكل مفيدة ل تكون ت عدي لها وكذلك،الهوا سدورف ف ضاء ول لحل ال قاب لة المجموعات،الكثافة، الدوال مفاهيم

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1. INTRODUCTION AND PRILMINARIES

As requirements for our work, we define here the following concepts sequentially: Ideal space, local function, Kuratowski closure, dense, T^* -dense, I-dense, codense, ψ -operator, resolvable, I-open, pre-I-open, scattered set and Housdorff space. We start define the ideal space. Let (X, T) be a topological space with no separation properties assumed. The topic of ideal topological space has been considered by (Kuratowski, 1966) and (Vaidyanathaswamy, 1960). An ideal I on a topological space (X, T) is a nonempty collection of subsets of X which satisfies the following two condition:

- (1) If $A \in I$ and $B \subseteq A$, then $B \in I$ (heredity).
- (2) If $A \in I$ and $B \in I$, then $A \cup B \in I$ (finite additivity).

Moreover a σ -ideal on (X, T) is an ideal which settle (1), (2) and the following condition:

3. If $\{A_i : i = 1, 2, 3, \dots\} \subseteq I$, then $\bigcup_{i=1,2,3,\dots} A_i \in I$ (countable additivity).

An ideal space is a topological space (X, T) with an ideal I on X and is denoted by (X, T, I) . For a subset $A \subseteq X$, $A^*(I) = \{x \in X : U \in I \text{ for every } U \in T(x)\}$ is called the local function of A with respect to I and T (Kuratowski, 1933). We simply write A^* instead of $A^*(I)$ in case there is no chance for disorder. It is familiar that $Cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for a topology $T^*(I)$ which is finer than T . During this paper, for a subset $A \subseteq X$, $Cl(A)$ and $Int(A)$ indicate the closure and the interior of A , respectively. A subset A of an ideal space (X, T, I) is said to be dense (resp, T^* -dense (Dontcher, Gansster and Rose, 1999), I-dense (Dontcher, Gansster and Rose, 1999) if $Cl(A) = X$ (resp $Cl^*(A) = X$, $A^* = X$). An ideal I on a space (X, T) is said to be codense (Devid, Sivaraj and Chelvam, 2005) if and only if $T \cap I = \{\Phi\}$. For an ideal space (X, T, I) and for any $A \subseteq X$, where I is codense. Then: dense, T^* -dense and I-dense are comparable (Jankovic and Hamlett, 1990). (Natkanies, 1986) used the idea of ideals to define another operator known as ψ -operator elucidates as follow: For a subset $A \subseteq X$, $\psi(A) = X - (X - A)^*$. Equivalently $\psi(A) = \{M \in T : M - A \in I\}$. It is obvious that $\psi(A)$ for any A is a member of T . For an ideal space (X, T, I) and $Y \subseteq X$ then (Y, T_y, I_y) is an ideal space where $T_y = \{U \in T : U \cap Y = \emptyset\}$ and $I_y = \{U \in I : U \cap Y = \emptyset\}$. In 1943, Hewitt put forward the opinion of a resolvable space as follows: A nonempty topological space (X, T) is said to be resolvable (Hewitt, 1943) if X is the disjoint union of two dense subsets. Given a space (X, T) and $A \subseteq X$, A is called I-open (Jankovic and Hamlett, 1990) (resp per-I-open (Donkhev, 1996) if $A \cap I(A)^*$ (resp $A \cap I \cap Cl^*(A)$). A set $A \subseteq X$ is called scattered (Jankovic and Hamlett, 1990) if A contains no nonempty dense-in-itself subset. A space (X, T, I) is called I-Housdorff (Dontcher, 1995) if for each two distinct points $x \neq y$, there exist I-open sets U and V containing x and y respectively, such that $U \cap V = \emptyset$. Throughout this paper we define a new local function and generalizations. Many characterizations, proprieties and relation between them are obtained.

2. Ψ^* -OPERATOR

In this section we put forward a new type of local function by using ψ -operator, in order to do this we have to have a deep looking in (Jankovic and Hamlett, 1992) work of the local function in ideal space.

Definition 2.1

Let (X, T, I) be an ideal space. An operator $(\cdot)\psi^*: P(x) \rightarrow P(x)$, called ψ -local function of A with respect to I and T , is define as follow: for any $A \subseteq X$, $A\psi^*(I, T) = \{x \in X : \psi(U) \cap A \neq \emptyset \text{ for every open subset } U \in T(x)\}$, where $T(x) = \{U \in T, x \in U\}$. When there is no chance for confusion $A\psi^*(I, T)$ is briefly denoted by $A\psi^*$.

Fact

Let (X, T, I) be an ideal space. Then $A^* \subseteq A\psi^*$, for any $A \subseteq X$.

Now we define a new closure operator in terms of ψ -local function.

Definition 2.2

Let (X, T, I) be an ideal space. For any $A \subseteq X$, we define

(1) $Cl_\psi(A) = \{x \in X : \psi(U) \cap A \neq \emptyset \text{ for every } U \in T, x \in U\}$. It is clear that $Cl(A) \subseteq Cl_\psi(A)$.

(2) $Cl_{\psi^*}(A)(I, T) = A \cup A\psi^*$

When there is no chance for confusion $Cl_{\psi^*}(A)(I, T)$ is briefly denoted by $Cl_{\psi^*}(A)$.

Note

Let (X, T, I) be an ideal space. Then $Cl_{\psi^*}(A) \subseteq Cl_\psi(A)$, for any $A \subseteq X$.

We discuss the properties of ψ -local function in following theorem:

Theorem 2.3

Let (X, T) be a topological space, I and J be two ideals on X , and let A and B be two subsets of X . Then the following properties hold:

1. If $A \subseteq B$, then $A\psi^* \subseteq B\psi^*$.
2. If $I \subseteq J$, then $A\psi^*(J) \subseteq A\psi^*(I)$.
3. $A\psi^* = Cl(A\psi^*) \cap Cl_\psi(A)$.
4. $(A\psi^*)\psi^* \subseteq A\psi^*$.
5. $(A \cup B)\psi^* = A\psi^* \cup B\psi^*$
6. $(A \cap B)\psi^* \subseteq A\psi^* \cap B\psi^*$
7. For every $I \in \mathcal{I}$, then $(A \cap I)\psi^* = A\psi^* \cap (A \cap I)\psi^*$.
8. If $G \in T$, then $G \cap A\psi^* = (G \cap A)\psi^*$.
9. If $U \in T$, then $U \cap Cl_{\psi^*}(A) = Cl_{\psi^*}(U \cap A)$.
10. If $A \subseteq I$, then $A\psi^* = \emptyset$.

Proof.

Straight from Definition 2.1 and Definition 2.2

3. Ψ - DENSE AND GENERALIZATIONS

Definition 3.1

Let (X, T, I) be an ideal space, $A \subseteq X$ is called:

- (1) ψ -dense, if $Cl \psi(A) = X$.
- (2) ψ - T^* -dense, if $Cl \psi^*(A) = X$.
- (3) ψ -I-dense, if $A\psi^* = X$.

The following remark is immediate from above definitions:

Remark 3.2

- (1) If A is ψ -I-dense, then A is ψ - T^* -dense.
- (2) If A is ψ - T^* -dense, then A is ψ -dense.

In the following example we show that the converses of above remark does not hold:

Example 3.3

Let $X = \{a, b, c\}$, $T = \{\Phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\Phi, \{b\}\}$. If $A = \{a, b\}$, then $A\psi^* = \{a, c\}$ but $Cl \psi^* = X$.

The study of ideal got new dimension when Codense ideal [10] has been incorporated in ideal space. In the following definition we introduce a new concept of condense by using ψ -operator.

Definition 3.4

Let (X, T, I) be an ideal space, an ideal I is called ψ -codense if, $\psi(T)I = \{\Phi\}$.

Lemma 3.5

Let (X, T, I) be an ideal space and for every subset $A \subseteq X$. If $A = A\psi^*$, then the following relations hold:

$$A\psi^* = Cl(A\psi^*) = Cl \psi(A) = Cl \psi^*(A).$$

Proof.

Clear from Definition 2.2: (1), and Theorem 2.3: (3).

Theorem 3.6

Let (X, T, I) be an ideal space, and for every subset $A \subseteq X$. If I is ψ -codense, then the following relations hold:

- (1) Where A is ψ -dense, then it is ψ - T^* -dense.
- (2) Where A is ψ - T^* -dense, then it is ψ -I-dense.

Proof.

This is an immediate consequence of Lemma 3.5.

The following theorem related to ψ -codense ideal:

Theorem 3.7

If X is finite, then $X=X^*$ if and only if, I is ψ -codense.

Proof.

Necessity. By Theorem 2.5: [1].we get that $T \cap I = \{\Phi\}$. Since $\psi(T) \cap I \subseteq T \cap I$. It follow that $\psi(T) \cap I = \{\Phi\}$. So I is ψ -codense.

Sufficiency. Assume that I is ψ -codense. Since $X-X^*$ is open, let $x \in X-X^*$. Then there exists $U_x \subseteq X-X^*$, and $x \in U_x$. Then there exists $V_x \subseteq T(x)$, such that $U_x \cap V_x \in I$. Assume that $W_x = U_x \cup V_x \in I$. But $X-X^* = W_x$. Hence $X-X^* \in I$ and $\psi(T) \cap I \neq \{\Phi\}$, a contradiction. Therefore, $X = X^*$.

Corollary 3.8

Let X be any nonempty set .If (X, T, I) is an ideal space where I is σ -ideal, then the following statement are equivalent:

- (1) $X = X^*$;

(2) $T I = \{\Phi\}$;

(3) $\psi(T) I = \{\Phi\}$.

Proof.

Similar to the proof of Theorem 3.7

By Remark 3.2, and Theorem 3.6, the following relations hold:

ψ -dense $\xrightarrow{\ast}$ ψ - T^* -dense $\xrightarrow{\ast}$ ψ - I -dense.

\ast : I is ψ -codense.

By Theorem 2.3 :(2), we have the following remark:

Remark 3.9

(1) Let (X, T, I) and (X, T, J) be two ideal spaces, $I \supseteq J$, if A is ψ - I -dense with respect to J , then A is ψ - I -dense with respect to I for any $A \subseteq X$.

(2) Let (X, T, I) and (X, σ, I) be two ideal spaces, $T \supseteq \sigma$, if A is ψ - I -dense with respect to σ , then A is ψ - I -dense with respect to T for any $A \subseteq X$.

By Remark 3.2 and Remark 3.9: (1), (2) we get the following corollary:

Corollary 3.10

(1) Let (X, T, I) and (X, T, J) be two ideal spaces, $I \supseteq J$, if A is ψ - T^* -dense (ψ -dense) with respect to J , then A is ψ - T^* -dense (ψ -dense) with respect to I for any $A \subseteq X$.

(2) Let (X, T, I) and (X, σ, I) be two ideal spaces, $T \supseteq \sigma$, if A is ψ - T^* -dense (ψ -dense) with respect to σ , then A is ψ - T^* -dense (ψ -dense) with respect to T for any $A \subseteq X$.

Definition 3.11

A function $f: (X, T, I) \rightarrow (Y, \sigma)$ is called ψ -map if, $f(Cl \psi(A)) \subseteq Cl f(A)$, for any $A \subseteq X$.

Note

Where $f: (X, T, I) \rightarrow (Y, \sigma, f(I))$ is a bijection map, then $f(I)$ will be an ideal on Y .

Theorem 3.12

If $f: (X, T, I) \rightarrow (Y, \sigma)$ is bijection ψ -map. Then the image of any ψ -dense subset A of X is dense in Y .

Proof.

Assume that A is ψ -dense in X . Since f is ψ -map, then $f(Cl \psi(A)) \subseteq Cl(f(A))$. Implies that $f(X) \subseteq Cl(f(A))$ and since f is surjection. Therefore, $Y = Cl(f(A))$. Hence $f(A)$ is dense in Y .

Definition 3.13

A function $f: (X, T, I) \rightarrow (Y, \sigma, J)$ is called :

(1) ψ - (I, J) map, if $f(Cl \psi^*(A)) \subseteq Cl^* f(A)$.

(2) ψ_0 - (I, J) map, if $f(A \psi^*) \subseteq (f(A))^*$.

(3) ψ_1 - (I, J) map, if $f(A \psi^*) \subseteq (f(A)) \psi^*$.

(4) ψ_2 - (I, J) map, if $f(Cl \psi^*(A)) \subseteq Cl \psi^* f(A)$.

(5) ψ_3 - (I, J) map, if $f(Cl \psi(A)) \subseteq Cl \psi f(A)$.

For any $A \subseteq X$.

From the above definitions we get the following remark :

Remark 3.14

- (1) ψ_0 - (I, J) map $\rightarrow \psi$ - (I, J) map.
- (2) ψ_1 - (I, J) map $\rightarrow \psi_2$ - (I, J) map $\rightarrow \psi_3$ - (I, J) map.

Theorem 3.15

- (1) If $f: (X, T, I) \rightarrow (Y, \sigma)$ is bijection ψ_0 -(I, J) map .Then the image of any ψ -I-dense subset A of X is f(I)-dense in Y.
- (2) If $f: (X, T, I) \rightarrow (Y, \sigma)$ is bijection ψ - (I, J) map. Then the image of any ψ -T*-dense subset A of X is T*-dense in Y.

Proof.

- (1) Assume that A is ψ -I-dense in X. Since f is ψ_0 -(I, J) map, then $f(A\psi^*) = (f(A))^*$. Implies that $f(X) = (f(A))^*$, and since f is surjection .Therefore, $Y = (f(A))^*$.Hence f (A) is f (I)-dense in Y.
- (2) Assume that A is ψ -T*-dense in X. Since f is ψ -(I, J) map, then $f(Cl\psi^*(A)) = Cl^* f(A)$ Implies that $f(X) = Cl^* f(A)$, and since f is surjection .Therefore, $Y = Cl^* f(A)$.Hence f (A) is T*-dense in Y.

Theorem 3.16

- (1) If $f: (X, T, I) \rightarrow (Y, \sigma, J)$ is ψ_1 - (I, J) bijection map. Then the image of any ψ -I-dense subset of X is ψ -f (I)-dense in Y.
- (2) If $f: (X, T, I) \rightarrow (Y, \sigma, J)$ is bijection ψ_2 -(I, J) map. Then the image of any ψ -T*-dense subset of X is ψ -T*-dense in Y.
- (3) If $f: (X, T, I) \rightarrow (Y, \sigma, J)$ is bijection ψ_3 -(I, J) map. Then the image of any ψ -dense subset of X is ψ -dense in Y.

Proof.

- (1) Assume that A is ψ -I-dense in X. Since f is ψ_1 -(I, J) map then $f(A\psi^*) = (f(A))\psi^*$, and since f is surjection .Therefore, $Y = (f(A))\psi^*$.Hence f (A) is ψ -f (I)-dense in Y.
- (2) Assume that A is ψ -T*-dense in X. Since f is ψ_2 -(I, J) map then $f(Cl\psi^*(A)) = Cl \psi^* f(A)$, and since f is surjection .Therefore, $Y = Cl \psi^* f(A)$.Hence f (A) is ψ -T*-dense in Y.
- (3) Assume that A is ψ -dense in X. Since f is ψ_3 -(I, J) map then $f(Cl \psi(A)) = Cl \psi f(A)$ and since f is surjection .Therefore, $Y = Cl \psi f(A)$.Hence f(A) is ψ -dense in Y.

4. Ψ -RESOLVABILITY AND GENERALIZATIONS

In this section we define ψ -resolvable space and recall some of the generalizations of ψ -resolvable and their relationship with Housdorff space.

Definition 4.1

A nonempty topological space (X, T) is called:

- (1) ψ -resolvable, if X is the disjoint union of two ψ -dense subsets.
- (2) ψ -T*-resolvable, if X is the disjoint union of T*-dense and ψ -dense subsets.
- (3) ψ -I-resolvable, if X is the disjoint union of I-dense and ψ -dense subsets.

It is clear that: ψ -I-resolvable $\rightarrow \psi$ -T*-resolvable $\rightarrow \psi$ -resolvable.

Theorem 4.2

Let (X, T) be a nonempty topological space, where I is ψ -codense. Then if X is ψ -T*-resolvability, then X is ψ -I-resolvability.

Question

We claim that ψ -resolvability does not imply ψ -T*-resolvability, but we could not find an example.

From Corollary 3.9 we get the following remark:

Remark 4.3

Let (X, T, I) , (X, T, J) be an ideal spaces, with $I \subseteq J$, if X is ψ -resolvable with respect to J , then X is ψ -resolvable with respect to I .

Definition 4.4

A function $f: (X, T, I) \rightarrow (Y, \sigma, J)$, is said to be bi*-map if :

- (1) $f: (X, T, I) \rightarrow (Y, \sigma)$ is ψ - (I, J) map;
- (2) $f: (X, T^*) \rightarrow (Y, \sigma^*)$ is a continuous map.

Theorem 4.5

Let $f: (X, T, I) \rightarrow (Y, \sigma)$ be bijection bi*-map, and if X is ψ - T^* -resolvable space. Then Y is ψ - T^* -resolvable with respect to the ideal $f(I)$.

Proof.

Assume that X is ψ - T^* -resolvable space, then there exists two disjoint ψ -dense and T^* -dense subsets such that $Cl \psi(A) = Cl^*(B) = X$. It is clear by Corollary 3.16, and Definition 4.4 that Y is ψ - T^* -resolvable.

Lemma 4.6

If U is an open set and $D \psi^* = X$, then $U \subseteq (D \psi(U)) \psi^*$.

Proof

Let $x \in U$, and suppose if possible that $x \notin (D \psi(U)) \psi^*$. Then $(D \psi(U)) \cap \psi(V) \in I$, and since $\psi(U) \cap \psi(V) \in T$. Therefore, $x \in D \psi^* = X$. This is a contradiction. That is $U \subseteq (D \psi(U)) \psi^*$.

Theorem 4.7

- (1) Let $U \in T$ and $Cl \psi^*(D) = X$, then $U \subseteq Cl \psi^*(D \psi(U))$.
- (2) Let $U \in T$, and $Cl \psi(D) = X$. Then $U \subseteq Cl \psi(D \psi(U))$.

Using Remark 3.2: (1), (2), and similar to the proof of Lemma 4.6.

Theorem 4.8

Let $Y \in T$, and $A \psi^* = X$. Then $A1 \psi y^* = Y$, where $A1 = A y$.

Proof.

By lemma 4.6 $Y \subseteq (A Y) \psi x^* = A1 \psi x^*$.

Then $Y = A1 \psi x^* \cap y = A1 \psi y^*$.

Using Remark 3.2 : (1), (2), and Theorem 4.8 we get the following theorem:

Theorem 4.9

- (1) Let $Y \in T$, and $Cl \psi^*(A) = X$. Then $Cl \psi^* y (A1) = Y$, where $A1 = A Y$.
- (2) Let $Y \in T$, and $Cl \psi(A) = X$. Then $Cl \psi y (A1) = Y$, where $A1 = A Y$.

Theorem 4.10

- (1) Let (X, T, I) be ψ - I -resolvable space, if Y is open. Then (Y, T_y, I_y) is ψ - I_y -resolvable.
- (2) Let (X, T, I) be ψ - T^* -resolvable space, if Y is open. Then (Y, T_y, I_y) is ψ - T^*_y -resolvable.
- (3) Let (X, T, I) be ψ -resolvable space, if Y is open. Then (Y, T_y, I_y) is ψ -resolvable.

Proof.

(1) Assume that (X, T, I) is ψ -I-resolvable space. Then there exists two disjoint ψ -dense, and I-dense subsets such that: $A \cap B = X$, and $\text{Cl}_\psi(A) = B^* = X$. Note that $Y = Y \cap X = Y \cap (A \cap B) = (Y \cap A) \cap (Y \cap B)$, and $(Y \cap A) \cap (Y \cap B) = Y \cap (A \cap B) = \Phi$. Put $A_1 = A \cap Y$ and $B_1 = B \cap Y$. Then $Y = A_1 \cup B_1$, $A_1 \cap B_1 = \Phi$. By corollary 3.10: (2), we have $\text{Cl}_\psi(Y \cap A_1) = Y$, and by lemma we have $B^* \cap Y = Y$. Hence (Y, T_Y, I_Y) is ψ -I $_Y$ -resolvable.

(2) Assume that (X, T, I) is ψ -T*-resolvable space. Then there exists two disjoint ψ -dense, and T*-dense subsets such that: $A \cap B = X$, and $\text{Cl}_\psi(A) = \text{Cl}^*(B) = X$. Note that $Y = Y \cap X = Y \cap (A \cap B) = (Y \cap A) \cap (Y \cap B)$, and $(Y \cap A) \cap (Y \cap B) = Y \cap (A \cap B) = \Phi$. Put $A_1 = A \cap Y$ and $B_1 = B \cap Y$. Then $Y = A_1 \cup B_1$, $A_1 \cap B_1 = \Phi$. By Theorem 2.3.20: (3), we have $\text{Cl}_\psi(Y \cap A_1) = Y$, $\text{Cl}^* \cap Y = Y$. Hence (Y, T_Y, I_Y) is ψ -T*_Y-resolvable.

(3) Assume that (X, T, I) is ψ -resolvable space. Then there exists two disjoint ψ -dense subsets such that: $A \cap B = X$, and $\text{Cl}_\psi(A) = \text{Cl}_\psi(B) = X$. Note that $Y = Y \cap X = Y \cap (A \cap B) = (Y \cap A) \cap (Y \cap B)$, and $(Y \cap A) \cap (Y \cap B) = Y \cap (A \cap B) = \Phi$. Put $A_1 = A \cap Y$ and $B_1 = B \cap Y$. Then $Y = A_1 \cup B_1$, $A_1 \cap B_1 = \Phi$. By Theorem 2.3.20: (3), we have $\text{Cl}_\psi(Y \cap A_1) = \text{Cl}_\psi(Y \cap B_1) = Y$. Hence (Y, T_Y, I_Y) is ψ -T_Y-resolvable.

Definition 4.11

In ideal space (X, T, I) A subset A is called ψ -pre-open if $A \cap I \subseteq \text{Cl}_\psi(A)$.

Definition 4.12

(1) An ideal space (X, T, I) is called ψ -I-Housdorff, if for every distinct point $x, y \in X$, there exist two disjoint ψ -pre-open, and I-open sets containing each respectively.

(2) An ideal space (X, T, I) is called ψ -T*-Housdorff, if for every distinct point $x, y \in X$, there exist two disjoint ψ -pre-open, and pre-I-open sets containing each respectively.

(3) An ideal space (X, T, I) is called ψ -Housdorff, if for every distinct point $x, y \in X$, there exist two disjoint ψ -pre-open sets containing each respectively.

Lemma 4.13

If $(\{x\})_\psi \cap I = \Phi$. Then $(\{x\})_\psi^* = \Phi$.

Proof.

Assume that $(\{x\})_\psi^* \neq \Phi$, then there exist $y \in (\{x\})_\psi^*$ such that $\psi(U) \cap \{x\} \cap I \neq \Phi$ for every $U \in T(x)$. Implies that $(\{x\})_\psi \cap I \neq \Phi$. This is a contradiction.

Theorem 4.14

(1) If an ideal space (X, T, I) is ψ -I-resolvable, and the scattered set of (X, T) are in I, then (X, T, I) is ψ -I-Housdorff.

(2) If an ideal space (X, T, I) is ψ -T*-resolvable, and the scattered set of (X, T^*) are in I, then (X, T, I) is ψ -T*-Housdorff.

(3) If an ideal space (X, T, I) is ψ -resolvable, and the scattered set of (X, T) are in I, then (X, T, I) is ψ -Housdorff.

Proof.

(1) Let A and B be disjoint ψ -dense, and I-dense subsets of X such that $X = A \cup B$. Since $\text{Cl}_\psi(A) = X$, $B^* = X$. Therefore, $A \cap I \subseteq \text{Cl}_\psi(A)$ and $B \cap I \subseteq B^*$, then A and B are ψ -pre-open and I-open sets respectively. Let x, y be any two element of X and both x and y are in A. Take $V = A \setminus \{y\}$ and $U = B \cup \{y\}$. It is easily observed (see [6], Theorem 2.3 (h)) that V also I-open set and y in V. Now to show that $U = A \cup \{x\}$ is ψ -pre-open. $\text{Cl}_\psi(A \cup \{x\}) = \text{Cl}_\psi(A) \cup \text{Cl}_\psi(\{x\}) = X \cup \text{Cl}_\psi(\{x\}) = X$. Hence $A \cup \{x\} \cap I \subseteq \text{Cl}_\psi(A \cup \{x\})$, so U is ψ -pre-open. Hence (X, T, I) is ψ -I-Housdorff.

(2) Let A and B be disjoint T*-dense, and ψ -dense subsets of X such that $X = A \cup B$. Since $\text{Cl}^*(A) = X$ and $\text{Cl}_\psi(B) = X$. Therefore, $A \cap I \subseteq \text{Cl}^*(A)$ and $B \cap I \subseteq \text{Cl}_\psi(B)$, then A and B are pre-I-open and ψ -pre-open sets respectively. Let x, y be any two element of X and both x and y are in A. Take $V = A \setminus \{y\}$ and $U = B \cup \{y\}$. To show that V also pre-I-open set $\text{Cl}^*(A \setminus \{y\}) = \text{Cl}^*(A) \setminus \text{Cl}^*(\{y\}) = X$. Hence $A \setminus \{y\} \cap I \subseteq \text{Cl}^*(A \setminus \{y\})$. Therefore, V is pre-I-open. Now to show that $U = B \cup \{y\}$ is ψ -pre-open. $\text{Cl}_\psi(B \cup \{y\}) = \text{Cl}_\psi(B) \cup \text{Cl}_\psi(\{y\}) = X \cup \text{Cl}_\psi(\{y\}) = X$. Hence $B \cup \{y\} \cap I \subseteq \text{Cl}_\psi(B \cup \{y\})$, so U is ψ -pre-open. Hence (X, T, I) is ψ -T*-Housdorff.

$\{y\}$ is ψ -pre-open. $\text{Cl}_\psi(B \setminus \{y\}) = \text{Cl}_\psi(B) \setminus \text{Cl}_\psi(\{y\}) = X \setminus \text{Cl}_\psi(\{y\}) = X$. Hence $B \setminus \{y\} \in \text{Cl}_\psi(B \setminus \{y\})$, so U is ψ -pre-open. Hence (X, T, I) is ψ - T^* -Housdorff.

(3) Let A and B be disjoint ψ -dense subsets of X such that $X = A \cup B$. Since $\text{Cl}_\psi(A) = \text{Cl}_\psi(B) = X$. Therefore, $A \in \text{Cl}_\psi(A)$ and $B \in \text{Cl}_\psi(B)$, then A and B are ψ -pre-open sets. Let x, y be any two element of X and both x and y are in A . Take $U = A \setminus \{y\}$ and $V = B \setminus \{y\}$. By Theorem 2.1.5: (8) we have V and U are ψ -pre-open sets. To show that U is ψ -pre-open, $\text{Cl}_\psi(A \setminus \{y\}) = \text{Cl}_\psi(A) \setminus \text{Cl}_\psi(\{y\}) = X \setminus \text{Cl}_\psi(\{y\}) = X$, $A \setminus \{y\} \in \text{Cl}_\psi(A)$. Now to show that V is ψ -pre-open. $\text{Cl}_\psi(B \setminus \{y\}) = \text{Cl}_\psi(B) \setminus \text{Cl}_\psi(\{y\}) = X$, $B \setminus \{y\} \in \text{Cl}_\psi(B)$. Hence U, V are ψ -pre-open. Therefore, (X, T, I) is ψ -Housdorff.

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