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Various resolvable space in ideal topological spaces

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Abstract:

In this research,we introduce new definitions of resolvable spaces of the depended on the idea:Weakly-ℑ-dense ,ℑ-dense,𝔗*-dense and dense.The definitions are: 𝔚ℑ-resolvable space, 𝔈-resolvable space, 𝔚ℑ-ℑ-resolvable space, 𝔚ℑ-𝔗*-resolvable space, 𝔚ℑ-𝔗-resolvable space, ℑ-𝔗*-resolvable space, ℑ-𝔗-resolvable space and 𝔗*-𝔗-resolvable space. We prove various results in the field resolvable space.

الخلاصة:

في هذا البحث ،نقدم تعاريف جديدة من الفضاءات القابلة للحل والمعتمدة على فكرة كثيفة،*𝔗-كثيفة،-ℑكثيفة،ضعيفة -ℑ-كثيفة وهذه التعريفات هي : 𝔚ℑ-الفضاء القابل للحل، 𝔈-الفضاء القابل للحل ، ℑ -ℑ- 𝔚 - الفضاء القابل للحل، *𝔗-ℑ-𝔚- الفضاء القابل للحل،𝔗-ℑ-𝔚 -الفضاء القابل للحل،*𝔗-ℑ- الفضاء القابل للحل،𝔗-ℑ-الفضاء القابل للحل،𝔗-*𝔗- الفضاء القابل للحل.وحصلنا على النتائج المختلفة في حقل الفضاء القابل للحل.

Keywords: dense,resolvability,Hausdorff and Ψ -operator.

الكلمات الدليلية:- الكثافة ،الفضاء القابل للحل ،هاوسدورف، Ψ – العاملة .

1. Introduction and Preliminaries

In (Hewitt, 1943) introduced the result: If there exists two disjoint union dense subsets, then this means that ℨ is resolvable space. C. Chattopadhyay(1992) have been studied the resolvability, irresolvability space and properties of maximal spaces. Furthermore, the prove of density topology is resolvable such as: J. Dontchev and M. Ganster and Rose(1999). In 1966,Kuratowski define an ideal I on topological space (ℨ,𝔗) is a nonempty collection of subsets of ℨ which satisfies:

1. If $D \in$ ℑ and $G \subseteq D$ implies $G \in$ ℑ.
2. If $D \in$ ℑ and $G \in$ ℑ implies $D \cup G \in$ ℑ.

Moreover, a σ-ideal on a topological space(ℨ,𝔗) is an ideal which satisfies (1),(2) the following condition:

3.If $\{D_i: i=1,2,3,...\} \subseteq$ ℑ, then $\cup\{D_i=1,2,3,...\} \in$ ℑ (countable additively), for further information see(Kuratowski,1966).

For a space $(Ƶ,\mathfrak{T},\mathfrak{I})$ and a subset $D\subseteq Ƶ$, $D^*(\mathfrak{I})=\{z\in Ƶ: W\cap D\notin\mathfrak{I}$ for every $W\in\mathfrak{T}(z)\}$ is called the local function of D with respect to $\mathfrak{I}$ and $\mathfrak{T}$ (Kuratowski,1933).A Kuratowski closure operator cl*(.) for a topology $\mathfrak{T}^*(\mathfrak{I},\mathfrak{T})$ called the $*$-topology, finer than $\mathfrak{T}$,is defined by $cl^*(D)=D\cup D^*$.$D\subseteq Ƶ$ is called $*$-closed (Jankovic and Hamlett,1990) if $cl^*(D)=D$, and D is called $*$-open (i.e.,$D\in\mathfrak{T}^*$) if $Ƶ$-D is $*$-closed. Obviously, D is $*$-open if and only if $int^*(D)=D$. Let $(Ƶ,\mathfrak{I},\mathfrak{T})$ be an ideal space and let $\mathfrak{P}\subseteq Ƶ$. Then $(\mathfrak{P},\mathfrak{T}_{\mathfrak{p}},\mathfrak{I}_{\mathfrak{p}})$ is an ideal space, where $\mathfrak{T}_{\mathfrak{p}}=\{Q\cap\mathfrak{P}:Q\in\mathfrak{T}\}$ and $I_{\mathfrak{p}}=\{\mathfrak{I}\cap\mathfrak{P}:I\in\mathfrak{I}\}=\{I\in\mathfrak{I}:I\subseteq\mathfrak{P}\}$. A subset D of an ideal space $(Ƶ,\mathfrak{T},\mathfrak{I})$ is called dense ($*$-dense ($\mathfrak{I}$-dense (Dontchev, Ganster and Rose,1999))),if $cl(D)=Ƶ$,(resp.$cl^*(D)=Ƶ$,($D^*=Ƶ$)). For an ideal space $(Ƶ,\mathfrak{T},\mathfrak{I})$, $\mathfrak{I}$ is codense (Devi, Sivaraj and Chelvam,2005) if $\mathfrak{T}\cap\mathfrak{I}=\Phi$. A subset D of an ideal space $(Ƶ,\mathfrak{T},\mathfrak{I})$ is said to be $\mathfrak{I}$-open(Jankovic and Hamlett,1992)(pre-$\mathfrak{I}$-open(Dontchev,1996), if $D\subseteq int(D^*)$,(resp. $D\subseteq intcl^*(D)$). A subset D of a space $(Ƶ,\mathfrak{T})$ is said to be preopen (Mashhour, Abd El-Monsef and El-Deeb,1982) if $D\subseteq intcl(D)$.A set $D\subseteq Ƶ$ is said to be scattered(Jankovic and Hamlett,1990) if D contains no nonempty dense-in-itself subset. An ideal space $(Ƶ,\mathfrak{T},\mathfrak{I})$ is called Hausdorff (Hausdorff,1957),if for each two points $z\neq p$, there exist open sets U and V containing z and p respectively, such that $U\cap V=\Phi$.T. Natkaniec (Natkaniec,1986) used the idea of ideals to define another operator known as Ψ-operator. The definition of $\Psi_{\mathfrak{T}}(D)$ for a subset D of $Ƶ$ is as follows: $\Psi_{\mathfrak{T}}(D)=Ƶ-(Ƶ-D)^*$.Equivalently $\Psi_{\mathfrak{T}}(D)=\bigcup\{M\in\mathfrak{T}:M-D\in\mathfrak{I}\}$.It is obvious that $\Psi_{\mathfrak{T}}(D)$ for any D is a member of $\mathfrak{T}$.(Hatir, Keskin and Noiri,2005 for an ideal space $(Ƶ,\mathfrak{T},\mathfrak{I})$ and let $D\subseteq\mathfrak{P}\subseteq Ƶ$. Then $cl_{\mathfrak{p}}^*(D)=cl^*(D)\cap\mathfrak{P}$. (Devi, Sivaraj and Chelvam,2005) for an ideal space $(Ƶ,\mathfrak{T},\mathfrak{I})$ and $D\subset Ƶ$.If $D\subset D^*$,then $D^*=cl(D^*)=cl(D)=cl^*(D)$.In this paper, we define and formulate a new definitions: $\mathfrak{WI}$-resolvable space and its generalizations, we investigate various results in the filed of resolvable space .

2- Weakly-$\mathfrak{I}$-Dense Sets and Generalizations.

In this section, we are using formula M^{**} to define a new type of closure operators and denoted to be $cl^e(M)=M\cup M^{**}$,for any $M\subseteq Ƶ$. This enable us to define a new types of dense.

Definition 2.1: Let $(Ƶ,\mathfrak{T},\mathfrak{I})$ be an ideal space and $M\subseteq Ƶ$. Then M is called.

1. Weakly-$\mathfrak{I}$-dense ,if $(M^*)^*=Ƶ$.
2. Emaciated-dense, if $cl^e(M)=Ƶ$.
3. $\mathfrak{I}$-codense, if $Ƶ$-M is $\mathfrak{I}$-dense.
4. $\mathfrak{WI}$-codense, if $Ƶ$-M is weakly-$\mathfrak{I}$-dense.

Definition 2.2 : An ideal topological space $(Ƶ,\mathfrak{T},\mathfrak{I})$ is called thick if $M\subset M^*$ for every $M\subseteq Ƶ$.

Remark 2.3: Every weakly-$\mathfrak{I}$-dense is $\mathfrak{I}$-dense and $\mathfrak{T}^*$-dense and hence dense.

This tells us that: If M is weakly-$\mathfrak{I}$-dense ,then M is emaciated-dense which leads to:

Lemma 2.4: Let$(Ƶ,\mathfrak{T},\mathfrak{I})$ be an ideal space and $M\subset Ƶ$, then $M^{**}=cl^e(M)$ if $M\subset M^{**}$.

Proof: Since $M^{**} \subset cl^e(M)$ and $M \subset M^{**}$,then $M^{**}=cl^e(M)$.

Remark 2.5:Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space.Then the following statement is hold: If M is emaciated-dense, then M is weakly-$\mathfrak{I}$-dense.

Proof: Get it from Lemma 2.4.

Remark 2.6:Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topolgical space and $\mathfrak{I}$ is codense.Then the following statement is hold: If M is $\mathfrak{I}$-dense, then M is weakly-$\mathfrak{I}$-dense.

Proof: Let $z \in \mathfrak{Z}$ and if possible $z \notin M^{**}$,then there exist $U_z \in \mathfrak{T}(z)$ such that $U_z \cap M^* \in \mathfrak{I}$.Since $M^*=\mathfrak{Z}$,then $U_z \cap M^*=U_z \cap \mathfrak{Z}=U_z \in \mathfrak{I}$ which contradiction.Hence $z \in M^{**}$.Therefore M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$.

This an immediate consequence of Definition 2.2.

Theorem 2.7: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be a thick and an ideal topological space.Then the following statements are hold.

1. If M is $\mathfrak{T}^*$-dense ,then M is $\mathfrak{I}$-dense.
2. If M is dense, then M is $\mathfrak{T}^*$-dense.

Example 2.8 :(1) Let $\mathfrak{Z}=\{a,b,c,d\}$, $\mathfrak{T}=\{\Phi,\{a,c\},\{b,d\},\mathfrak{Z}\}$, $\mathfrak{I}=\{\Phi,\{a\},\{c\},\{a,c\}\}$.Let $M=\{b,c\}$

Then $M^*=\{b,d\}$, $cl^*(M) \neq \mathfrak{Z}$ and $cl(M)=\mathfrak{Z}$.Thus M is dense but not $\mathfrak{T}^*$-dense.

(2) Let $\mathfrak{Z}=\{a,b,c\}$, $\mathfrak{T}=\{\Phi,\{a\},\{a,b\},Z\}$, $\mathfrak{I}=\{\Phi,\{a\}\}$.Let $M=\{a,b\}$.

Then $M^*=\{b,c\}$ and $cl^*(M)=\mathfrak{Z}$.Thus M is $\mathfrak{T}^*$-dense but not $\mathfrak{I}$-dense.

(3) Let $\mathfrak{Z}=\{a,b,c\}$, $\mathfrak{T}=\{\Phi,\{a\},\{a,b\},\mathfrak{Z}\}$, $\mathfrak{I}=\{\Phi,\{a,b\}\}$.Let $M=\{a,c\}$.

Then $M^*=\mathfrak{Z}$, $M^{**}=\{a,c\} \neq \mathfrak{Z}$. Thus M is $\mathfrak{I}$-dense but not weakly-$\mathfrak{I}$-dense.

Note that 1. M is $\mathfrak{I}$-codense, if and only if $\Psi(M)=\Phi$.

2. M is $\mathfrak{W}\mathfrak{I}$-codense, if and only if $\Psi(\Psi(M))=\Phi$.

We focus on the properties of the local function regarding to the formula M** in the following theorem.

Theorem 2.9: Let $(\mathfrak{Z},\mathfrak{T})$ be a topological space with $\mathfrak{I}_1$ and $\mathfrak{I}_2$ are ideals on $\mathfrak{Z}$ and let M subset of $\mathfrak{Z}$. Then

1. If $\mathfrak{I}_1 \subseteq \mathfrak{I}_2$,then $M^{**}(\mathfrak{I}_2) \subseteq M^{**}(\mathfrak{I}_1)$.
2. For every $I \in \mathfrak{I}$,then $(M \cup I)^{**}=M^{**}=(M-I)^{**}$.
3. If $\mathfrak{T} \subset \sigma$,then $M^{**}(\mathfrak{I},\sigma) \subset M^{**}(\mathfrak{I},\mathfrak{T})$.

Proof: (1) Clear .

Proof: (2) Clearly from [(Jankovic and Hamlett,1990), Theorem 2.3(h)].

Corollary 2.10:

1. Let $\mathfrak{I}_1$ and $\mathfrak{I}_2$ being ideals on $\mathfrak{Z}$ such that $\mathfrak{I}_1 \subseteq \mathfrak{I}_2$.If M is weakly-$\mathfrak{I}_2$-dense,then M is weakly-$\mathfrak{I}_1$-dense.
2. Let $\mathfrak{I}_1$ and $\mathfrak{I}_2$ being ideals on $\mathfrak{Z}$ such that $\mathfrak{I}_1 \subseteq \mathfrak{I}_2$.If M is emaciated-dense with respect to $\mathfrak{I}_2$,then M is emaciated-dense with respect to $\mathfrak{I}_1$.

Proof: Get it from Theorem 2.9 .

Corollary 2.11:

1. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ and $(\mathfrak{Z},\sigma,\mathfrak{I})$ be two topological spaces with $\mathfrak{T} \subset \sigma$.If M is σ-weakly-$\mathfrak{I}$-dense, then M is $\mathfrak{T}$-weakly-$\mathfrak{I}$-dense.
2. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ and $(\mathfrak{Z},\sigma,\mathfrak{I})$ be two topological spaces with $\mathfrak{T} \subset \sigma$.If M is emaciated-dense with respect to σ,then M is emaciated-dense with respect to $\mathfrak{T}$.

Proof: Get it from Theorem 2.9.

Lemma 2.12:

1. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space and $M \subset \mathfrak{Y} \subset \mathfrak{Z}$,then $(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^* = (M_{\mathfrak{z}}^*)_{\mathfrak{z}}^* \cap \mathfrak{Y}$ and $\mathfrak{Y} \in \mathfrak{T}$.
2. If M be weakly-$\mathfrak{I}$-dense subsets of $\mathfrak{Z}$ and U is open subset of $\mathfrak{Z}$,then $U \subseteq (U \cap M)^{**}$.

Proof.1. Let $(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^* = (M_{\mathfrak{z}}^* \cap \mathfrak{Y})_{\mathfrak{y}}^* = (M_{\mathfrak{z}}^* \cap \mathfrak{Y})_{\mathfrak{z}}^* \cap \mathfrak{Y} \subset (M_{\mathfrak{z}}^*)_{\mathfrak{z}}^* \cap \mathfrak{Y}$.Hence $(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^* \subset (M_{\mathfrak{z}}^*)_{\mathfrak{z}}^* \cap \mathfrak{Y}$ ......(1).

Now let $a \in (M_{\mathfrak{z}}^*)_{\mathfrak{z}}^* \cap \mathfrak{Y}$,then $a \in (M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*$ and $a \in \mathfrak{Y}$,then $U \cap M_{\mathfrak{z}}^* \notin \mathfrak{I}$ for every $U \in \mathfrak{T}(a)$ and $a \in \mathfrak{Y}$.Since $\mathfrak{I}_{\mathfrak{y}} \subset \mathfrak{I}$,then $U \cap M_{\mathfrak{z}}^* \notin \mathfrak{I}_{\mathfrak{y}}$,for every $U \in \mathfrak{T}(a)$ and $a \in \mathfrak{Y} \in \mathfrak{T}$,then $a \in U \cap \mathfrak{Y} \in \mathfrak{T}(a)$,we have $U \cap \mathfrak{Y} \cap M_{\mathfrak{z}}^* \notin \mathfrak{I}_{\mathfrak{y}}$,for every $U \cap \mathfrak{Y} \in \mathfrak{T}_{\mathfrak{y}}(a)$.Hence $a \in (M_{\mathfrak{y}}^*)_{\mathfrak{y}}^*$.Therefore $(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^* \cap \mathfrak{Y} \subset (M_{\mathfrak{y}}^*)_{\mathfrak{y}}^* \rightarrow (2)$.From (1) and (2),it follows that$(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^* = (M_{\mathfrak{z}}^*)_{\mathfrak{z}}^* \cap \mathfrak{Y}$.

2. Let $z \in U$ and if possible $z \notin (U \cap M)^{**}$,then there exists $V \in \mathfrak{T}(z)$ such that $V \cap (U \cap M)^* \in \mathfrak{I}$.Since $U \cap M^* \subset (U \cap M)^*$,then $V \cap U \cap M^* \subset V \cap (U \cap M)^*$,then $V \cap U \cap M^* \in \mathfrak{I}$ which contradiction with $z \in M^{**}$.Therefore $U \subseteq (U \cap M)^{**}$.

Proposition 2.13: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space. If $\mathfrak{Y} \subset \mathfrak{Z}$ and $(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^* = \mathfrak{Z}$, then $(M_{1\mathfrak{y}}^*)_{\mathfrak{y}}^* = \mathfrak{Y}$ where $M_1 = M \cap \mathfrak{Y}$.

Proof: The proof is clear using Lemma 2.12.

Lemma 2.14:

1. If M be emaciated-dense subsets of $\mathfrak{Z}$ and U is open subset of $\mathfrak{Z}$, then $U \subseteq cl^e(U \cap M)$.
2. let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space and $M \subseteq \mathfrak{Y} \subseteq \mathfrak{Z}$, then $cl^e_{\mathfrak{Y}}(M)=cl^e_{\mathfrak{Z}}(M) \cap \mathfrak{Y}$ and $\mathfrak{Y} \in \mathfrak{T}$.

Proof: Clear.

Proposition 2.15: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space. If $\mathfrak{Y}$ is open subsets of $\mathfrak{Z}$ and $cl^e_{\mathfrak{Z}}(M)=\mathfrak{Z}$, then $cl^e_{\mathfrak{Y}}(M_1)=\mathfrak{Y}$ where $M_1=M \cap \mathfrak{Y}$.

Proof: The proof is clear using Lemma 2.14.

Lemma 2.16:

1. If M be $\mathfrak{T}$*-dense subsets of $\mathfrak{Z}$ and U is $\mathfrak{T}$*-open subset of $\mathfrak{Z}$, then $U \subseteq cl^*(U \cap M)$.
2. If M be $\mathfrak{T}$-dense subsets of $\mathfrak{Z}$ and U is open subset of $\mathfrak{Z}$, then $U \subseteq (U \cap M)^*$.

Proof: Clear.

Proposition 2.17:

1. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space. If $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$ and $M_{\mathfrak{Z}}^*=\mathfrak{Z}$, then $M_{1\mathfrak{Y}}^*=\mathfrak{Y}$ where $M_1=M \cap \mathfrak{Y}$.
2. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space. If $\mathfrak{Y}$ is $\mathfrak{T}$*-open subset of $\mathfrak{Z}$ and $cl_{\mathfrak{Z}}^*(M)=\mathfrak{Z}$, then $cl_{\mathfrak{Y}}^*(M_1)=\mathfrak{Y}$ where $M_1=M \cap \mathfrak{Y}$.
3. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space. If $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$ and $cl_{\mathfrak{Z}}(M)=\mathfrak{Z}$, then
   $Cl_{\mathfrak{Y}}(M_1)=\mathfrak{Y}$, where $M_1=M \cap \mathfrak{Y}$.

   Proof.Get it from Lemma 2.16.

Definition 2.18: A function $\mathfrak{F}:(\mathfrak{Z},\mathfrak{T},\mathfrak{I})\rightarrow(\mathfrak{Y},\sigma,\mathfrak{J})$ is called:

1. $\mathfrak{W}$-$\mathfrak{I}$-map,if $\mathfrak{F}(M^{**}) \subseteq (\mathfrak{F}(M))^{**}$.
2. $\mathfrak{E}$-$\mathfrak{I}$-map,if $\mathfrak{F}(cl^e(M)) \subseteq cl^e(\mathfrak{F}(M))$.
3. $\mathfrak{W}_0$-$\mathfrak{I}$-map,if $\mathfrak{F}(M^*) \subseteq (\mathfrak{F}(M))^*$.

This tells us that: $\mathfrak{W}_{\circ}$-$\mathfrak{I}$-map → $\mathfrak{W}$-$\mathfrak{I}$-map and $\mathfrak{W}$-$\mathfrak{I}$-map → $\mathfrak{E}$-$\mathfrak{I}$-map.

Theorem 2.19: Let $\mathfrak{F}:(\mathfrak{Z},\mathfrak{T},\mathfrak{I})\rightarrow(\mathfrak{Y},\sigma)$ be a $\mathfrak{W}$-$\mathfrak{I}$-map and bijection. If M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$, then $\mathfrak{F}(M)$ is weakly-$\mathfrak{J}$-dense in $\mathfrak{Y}$.

Proof: Suppose that M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$, then $M^{**}=\mathfrak{Z}$, implies that $\mathfrak{Y}=\mathfrak{F}(\mathfrak{Z})=\mathfrak{F}(M^{**}) \subseteq (\mathfrak{F}(M))^{**}$ because $\mathfrak{F}$ is $\mathfrak{W}$-$\mathfrak{I}$-map and bijection. Thus $\mathfrak{Y}=(\mathfrak{F}(M))^{**}$. Hence $\mathfrak{F}(M)$ is weakly-$\mathfrak{J}$-dense in $\mathfrak{Y}$.

Corollary 2.20: Let $\mathfrak{F}:(\mathfrak{Z},\mathfrak{T},\mathfrak{I})\rightarrow(\mathfrak{Y},\sigma,\mathfrak{J})$ be a $\mathfrak{W}_{\circ}$-$\mathfrak{I}$-map and bijection. If M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$, then $\mathfrak{F}(M)$ is weakly-$\mathfrak{J}$-dense in $\mathfrak{Y}$.

Proof: Clear.

Corollary 2.21: Let 𝕱:(Ƶ,𝕿,𝕴)→(𝖄,σ,𝕵) be a 𝖂-𝕴-map and bijection. If M is weakly-𝕴-dense in Ƶ,then 𝕱(M) is emaciated-dense in 𝖄.

Proof: Get it from Theorem 2.19.

Theorem 2.22: Let 𝕱:(Ƶ,𝕿,𝕴)→(𝖄,σ) be a 𝖂₀-𝕴-map and bijection. If M is 𝕴-dense in Ƶ, then 𝕱(M) is 𝕵-dense in 𝖄.

Proof: Proof resemble proof Theorem 2.19.

Theorem 2.23: Let 𝕱:(Ƶ,𝕿,𝕴)→(𝖄,σ,𝕵) be a 𝕰-𝕴-map and bijection. If M is emaciated-dense in Ƶ, then 𝕱(M) is emaciated-dense in 𝖄.

Proof: Suppose that M is emaciated-dense in Ƶ, then cl^e(M)=Ƶ implies that 𝖄=𝕱(Ƶ)=𝕱(cl^e(M))$\subseteq$$cl^e$(𝕱(M)) because 𝕱 is 𝕰-𝕴-map and bijection.Thus 𝖄=cl^e(𝕱(M)). Hence 𝕱(M) is emaciated-dense in 𝖄.

Corollary 2.24: Let 𝕱:(Ƶ,𝕿,𝕴)→(𝖄,σ,𝕵) be a 𝖂-𝕴-map and bijection. If M is emaciated-dense in Ƶ, then 𝕱(M) is emaciated-dense in 𝖄.

3- 𝖂𝕴-Resolvability and Generalizations.

In this section, we define new types of resolvable space in terms of formula M** and indicate its properties and relationships.

Definition 3.1: An ideal space (Ƶ,𝕿,𝕴) is called

1. 𝖂𝕴-resolvable, if Ƶ is the disjoint union of two weakly-𝕴-dense subsets.
2. e-resolvable, if Ƶ is the disjoint union of two emaciated-dense subsets.
3. 𝖂𝕴-𝕴-resolvable, if Ƶ is the disjoint union of two weakly-𝕴-dense and 𝕴-dense subsets.
4. 𝖂𝕴-𝕿*-resolvable, if Ƶ is the disjoint union of two weakly-𝕴-dense and 𝕿*-dense subsets.
5. 𝖂𝕴-𝕿-resolvable, if Ƶ is the disjoint union of two weakly-𝕴-dense and 𝕿-dense subsets.
6. 𝕴-𝕿*-resolvable, if Ƶ is the disjoint union of two 𝕴-dense and 𝕿*-dense subsets.
7. 𝕴-𝕿-resolvable, if Ƶ is the disjoint of two 𝕴-dense and 𝕿-dense subsets.
8. 𝕿*-𝕿-resolvable, if Ƶ is the disjoint union of two 𝕿*-dense and 𝕿-dense subsets.

Theorem 3.2: Let (Ƶ,𝕿,𝕴) be an ideal space. The following statements are holds.

1. Every 𝖂𝕴-resolvable space is 𝖂𝕴-𝕴-resolvable space.
2. Every 𝖂𝕴-𝕴-resolvable space is 𝖂𝕴-𝕿*-resolvable space.
3. Every 𝖂𝕴-𝕿*-resolvable space is 𝖂𝕴-𝕿-resolvable space.
4. Every 𝖂𝕴-𝕿-resolvable space is 𝕴-𝕿-resolvable space.
5. Every 𝕴-𝕿*-resolvable space is 𝕴-𝕿-resolvable space
6. Every 𝕴-𝕿-resolvable space is 𝕿*-𝕿-resolvable space.
7. Every 𝖂𝕴-resolvable space is e-resolvable space.

Proof.Get it from Remark 2.3.

This direct consequence of Remark 2.6.

Proposition 3.3: Let (Ƶ,𝔗,𝔍) be an ideal topological space and 𝔍 is codense.

If Z is 𝔚𝔍-𝔍-resolvable space,then Ƶ is 𝔚𝔍-resolvable space.

This direct consequence of Remark 2.6 and Theorem 2.7.

Proposition 3.4: Let (Ƶ,𝔗,𝔍) be a thick and an ideal topological space and 𝔍 is codense.Then the following statements are holds.

(1) If Ƶ is 𝔍-𝔗*-resolvable space,then Ƶ is 𝔚𝔍-𝔍-resolvable space.

(2) If Ƶ is 𝔍-𝔗-resolvable space,then Ƶ is 𝔚𝔍-𝔗*-resolvable space.

(3) If Ƶ is 𝔗*-𝔗-resolvable space,then Ƶ is 𝔚𝔍-𝔗-resolvable space.

This an immediate consequence of Theorem 2.7.

Proposition 3.5: Let (Ƶ,𝔗,𝔍) be a thick and an ideal topological space.Then the following statements are hold.

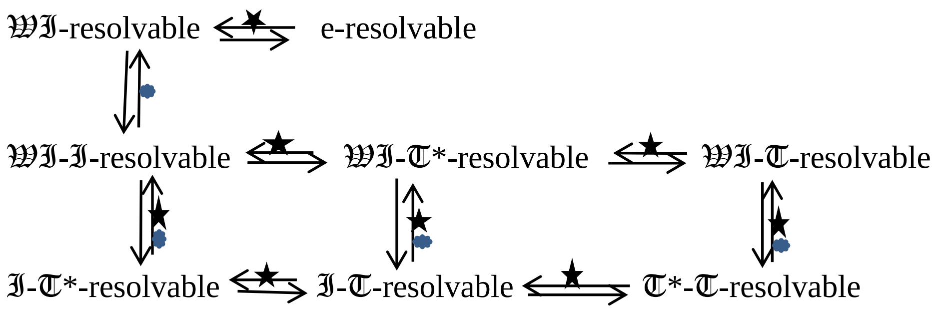
(1) If Ƶ is 𝔚𝔍-𝔗-resolvable space ,then Ƶ is 𝔚𝔍-𝔗*-resolvable space.

(2) If Ƶ is 𝔚𝔍-𝔗* -resolvable space ,then Ƶ is 𝔚𝔍-𝔍-resolvable space.

(3) If Ƶ is 𝔗*-𝔗-resolvable space ,then Ƶ is 𝔍 -𝔗-resolvable space.

(4) If Ƶ is 𝔍-𝔗-resolvable space ,then Ƶ is 𝔍-𝔗* -resolvable space.

Remark 3.6: By the above definitions, we obtain the following applications.



✱ : 𝔍 is codense.

★: Ƶ is thick

Remark 3.7:

1. If $𝔍_1$ and $𝔍_2$ are ideals with $𝔍_1 \subseteq 𝔍_2$,Ƶ is $𝔚𝔍_2$-resolvable implies that Ƶ is $𝔚𝔍_1$-resolvable.
2. If 𝔗 and σ are topological spaces with $𝔗 \subseteq \sigma$,Ƶ is σ-𝔚𝔍-resolvable implies that Ƶ is 𝔗-𝔚𝔍-resolvable.

Theorem 3.8: Let (Ƶ,𝔗,𝔍) be an ideal topological space .Then the following statements are hold:If Ƶ be a 𝔚𝔍-resolvable and 𝔜 is open subset of Ƶ. Then 𝔜 is 𝔚𝔍-resolvable subspace of Ƶ.

Proof: Assume that Ƶ is 𝔚𝔍-resolvable, then M⋃G=Ƶ and M⋂G=Φ,where M**=Ƶ and G**=Ƶ. Note that 𝔜= (𝔜⋂M)⋃ (𝔜⋂G) and (𝔜⋂M)⋂(𝔜⋂G)=Φ.Put M_1=M ⋂𝔜 and G_1=G⋂𝔜 ,then M_1⋃G_1=𝔜 and M_1⋂G_1=Φ.To prove($M_{1𝔜}$*)$_{𝔜}$*=𝔜 and ($G_{1𝔜}$*)$_{𝔜}$*=𝔜. So by Proposition 2.13,it follows that ($M_{1𝔜}$*)$_{𝔜}$*=𝔜 and ($G_{1𝔜}$*)$_{𝔜}$*=𝔜. Hence (𝔜,$𝔗_{𝔜}$,$𝔍_{𝔜}$) is 𝔚𝔍-resolvable.

Theorem 3.9: Let (Ƶ,𝔗,𝔍) be an ideal topological space .Then the following statements are hold:

1. If Ƶ be a 𝔚𝔍-𝔍-resolvable and 𝔜 is open subset of Ƶ. Then 𝔜 is 𝔚𝔍-𝔍-resolvable subspace of Ƶ.
2. If Ƶ be a 𝔚𝔍-𝔗*-resolvable and 𝔜 is open subset of Ƶ. Then 𝔜 is 𝔚𝔍-𝔗*-resolvable subspace of Ƶ.
3. If Ƶ be a 𝔚𝔍-𝔗-resolvable and 𝔜 is open subset of Ƶ. Then 𝔜 is 𝔚𝔍-𝔗-resolvable subspace of Ƶ.
4. If Ƶ be a 𝔍-𝔗*-resolvable and 𝔜 is open subset of Ƶ. Then 𝔜 is 𝔍-𝔗*-resolvable subspace of Ƶ.
5. If Ƶ be a 𝔍-𝔗-resolvable and 𝔜 is open subset of Ƶ. Then 𝔜 is 𝔍-𝔗-resolvable subspace of Ƶ.
6. If Ƶ be a 𝔗*-𝔗-resolvable and 𝔜 is open subset of Ƶ. Then 𝔜 is 𝔗*-𝔗-resolvable subspace of Ƶ.
7. If Ƶ be a e-resolvable and 𝔜 is open subset of Ƶ.Then 𝔜 is e-resolvable subspace of Ƶ.

Proof: The same prove Theorem 3.8.

Definition 3.10: A subset M of Ƶ is called 𝔚𝔍-open, if M⊆int(M**).

Definition 3.11: An ideal space (Ƶ,𝔗,𝔍) is called

1. 𝔚𝔍-Hausdorff, if for every p≠q, ∃ 𝔚𝔍-open sets K and L containing p and q singly, where K⋂L=Φ.
2. 𝔚𝔍-𝔍-Hausdorff,if for every p≠q, ∃ 𝔚𝔍-open and 𝔍-open sets K and L containing p and q singly, where K⋂L=Φ.
3. 𝔚𝔍-𝔓*-Hausdorff,if for every p≠q, ∃ 𝔚𝔍-open and pre-𝔍-open sets K and L containing p and q singly,where K⋂L=Φ.
4. 𝔚𝔍-𝔓-Hausdorff,if for every p≠q, ∃ 𝔚𝔍-open and pre-open sets K and L containing p and q singly, where K⋂L=Φ.
5. 𝔍-𝔓*-Hausdorff,if for every p≠q, ∃ 𝔍-open and pre-𝔍-open sets K and L containing p and q singly, everywhere K⋂L=Φ.
6. 𝔍-𝔓-Hausdorff,if for every p≠q, ∃ 𝔍-open and pre-open sets K and L containing p and q individually, everywhere K⋂L=Φ.

Theorem 3.12: Let (Ƶ,𝔗,𝔍) be an ideal topological space and the scattered sets of (Ƶ,𝔗*) are in 𝔍.The following statement are holds: Every 𝔚𝔍-resolvable space is 𝔚𝔍-Hausdorff space.

Proof: Suppose that Ƶ is a 𝔚ℑ-resolvable, then there exists M,G be disjoint weakly-ℑ-dense subsets of Ƶ such that M⋃G=Ƶ.We get that M and B are 𝔚ℑ-open. Let z,p∈ Ƶ.We have to show that (Ƶ,𝔗,ℑ) is 𝔚ℑ-Hausdorff.So by Theorem 2.9,it follows that (M-{p})**=M**=(G⋃{p})**,then (M-{p})**=Ƶ=(G⋃{p})**.Thus K=M-{p} and L=G⋃{p} are disjoint 𝔚ℑ-open sets having z and p individually.

Theorem 3.13: Let (Ƶ,𝔗,ℑ) be an ideal topological space and the scattered sets of (Ƶ,𝔗*) are in ℑ.The following statements are hold**:**

1. Every 𝔚ℑ-ℑ-resolvable space is 𝔚ℑ-ℑ-Hausdorff space.
2. Every 𝔚ℑ-𝔗*-resolvable space is 𝔚ℑ-𝔓*-Hausdorff space.
3. Every 𝔚ℑ-𝔗-resolvable space is 𝔚ℑ-𝔓-Hausdorff space.
4. Every ℑ-𝔗*-resolvable space is ℑ-𝔓*-Hausdorff space.
5. Every ℑ-𝔗-resolvable space is ℑ-𝔓-Hausdorff space.

Proof: The same prove Theorem 3.12.

Theorem 3.14: Let 𝔉:(Ƶ,𝔗,ℑ)→(𝔜,σ) be 𝔚-ℑ-map and bijection. If Ƶ is 𝔚ℑ-resolvable, then 𝔜 is 𝔚𝔍-resolvable.

Proof: Suppose that Ƶ is 𝔚ℑ-resolvable, then there exists M and G subsets of Ƶ such that M⋃G=Ƶ and M⋂G=Φ,where M**=Ƶ and G**=Ƶ. So by Theorem 2.19,we have f(M) and f(G) are weakly-𝔍-dense in 𝔜. Hence 𝔜 is 𝔍-resolvable.

Theorem 3.15: Let 𝔉:(Ƶ,𝔗,ℑ)→(𝔜,σ,𝔍) be a 𝔚-ℑ-map and bijection. If Ƶ is 𝔚ℑ-resolvable, then 𝔜 is 𝔍-resolvable.

Proof: The same prove Theorem 3.14.

Corollary 3.16: Let 𝔉:(Ƶ,𝔗,ℑ)→(𝔜,σ,𝔍) be a 𝔚-ℑ-map and bijection.If Ƶ is 𝔚ℑ-resolvable,then 𝔜 is e-resolvable.

Proof: Get it from Theorem 3.14.

Theorem 3.17: Let 𝔉:(Ƶ,𝔗,ℑ)→(𝔜,σ) be a 𝔈-ℑ-map and bijection. If Ƶ is e-resolvable, then 𝔜 is e-resolvable.

Proof: Get it from Theorem 2.23.

Here, we will characterize 𝔚ℑ-resolvable spaces by means of Ψ -operator.

Theorem 3.18: Let (Ƶ,𝔗,ℑ) be ℑ-resolvable if and only if there exists M subset of Ƶ such that Ψ (M)= Ψ (M^c)=Φ .

Proof: Necessity. Let (Ƶ,𝔗,ℑ) be ℑ-resolvable, then there exist M and G subsets of Ƶ such that M⋃G=Ƶ and M⋂G=Φ where M*=G*=Ƶ , M^c=G and G^c=M. Since ($(M^*)^c$=Φ,then $(((M^c)^c)^*)^c$=Φ,this implies Ψ (M^c)=Φ and since $(G^*)^c$=Φ,then $((M^c)^*)^c$=Φ,this implies Ψ (M)=Φ .

Sufficiency. Let M⊆Ƶ such that Ψ (M)= Ψ (M^c)=Φ.Since $((M^c)^*)^c$=Φ,then G*=Ƶ. Since $(((M^c)^c)^*)^c$=Φ,then M*=Ƶ,M⋃M^c=Ƶ and M⋂M^c=Φ.Hence Ƶ is ℑ-resolvable.

Theorem 3.19: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be $\mathfrak{WI}$-resolvable if and only if there exist M subset of $\mathfrak{Z}$ such that $\Psi(\Psi(M))=\Psi(\Psi(M^c)=\Phi$.

Proof: The same prove Theorem 3.18.

Proposition 3.20: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be a $\mathfrak{WI}$-resolvable if and only if there exist weakly-$\mathfrak{I}$-dense subset M of $\mathfrak{Z}$ with $\Psi(\Psi(M))=\Phi$.

Proof: Necessity.By Theorem 3.19,then there exist $M\subseteq\mathfrak{Z}$ such that $\Psi(\Psi(M))=\Psi(\Psi(M^c))=\Phi$,then $((((M^c)^c)^*)^*)^c=\Phi$,this implies M**=$\mathfrak{Z}$. Hence M is weakly-$\mathfrak{I}$-dense.

Sufficiency. Let M is weakly-$\mathfrak{I}$-dense with $\Psi(\Psi(M))=\Phi$. Since M**=$\mathfrak{Z}$, then $((((M^c)^c)^*)^*)^c=\Phi$, which implies that $\Psi(\Psi(M^c))=\Phi$.So by theorem 3.19,it follows that $\mathfrak{Z}$ is $\mathfrak{WI}$-resolvable.

Lemma 3.21: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) is $\mathfrak{I}$-resolvable if and only if there exists $\mathfrak{I}$-dense subset M of $\mathfrak{Z}$ with $\Psi(M)=\Phi$.

Proof: Clear.

Theorem 3.22: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal space and $M\subseteq\mathfrak{I}$, then M^c is weakly-$\mathfrak{I}$-dense if and only if $\Psi(\Psi M))=\Phi$.

Proof: Let M^c is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$ iff $((M^c)^*)^*=\mathfrak{Z}$ iff $(((M^c)^*)^*)^c=\Phi$ iff $\Psi(\Psi(M))=\Phi$.

Theorem 3.23: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) is $\mathfrak{I}$-$\mathfrak{T}$*-resolvable, if and only if there exists a subset M of $\mathfrak{Z}$ such that $\Psi(M^c)$=int*(M)=Φ.

Proof: Necessity. Let $\mathfrak{Z}$ be a $\mathfrak{I}$-$\mathfrak{T}$*-resolvable, then there exist M and G subsets of $\mathfrak{Z}$ such that $M\cup G=\mathfrak{Z}$,$M\cap G=\Phi$ where M*=$\mathfrak{Z}$ = cl*(G) , $M=G^c$ and $G=M^c$. Since $(cl^*(G))^c=\Phi$,implies that int*(M)=Φ and since M*=$\mathfrak{Z}$, then $\Psi(M^c)=\Phi$.

Sufficiency. Clear.

Definition 3.24:Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space,let M be any subset of $\mathfrak{Z}$.The e-interior of M is denoted by $int^e(M)$ and is of form $int^e(M)=M\cap\Psi(\Psi(M))$.

Proposition 3.25: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be a $\mathfrak{WI}$-resolvable, then there exist weakly-$\mathfrak{I}$-dense subset M of $\mathfrak{Z}$ such that $int^e(M)=\Phi$.

Proof: By Proposition 3.20 ,we have $int^e(M)=M\cap\Psi(\Psi(M))=\Phi$.

Theorem 3.26: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal toplogical space. The following statements are equivalent.

1. $\mathfrak{Z}$ is $\mathfrak{I}$-resolvable.
2. There exists $\mathfrak{I}$-dense subset M of $\mathfrak{Z}$ such that $\Psi(M)=\Phi$.
3. There exists disjoint union subsets M and G of $\mathfrak{Z}$ such that $\Psi(M)=\Psi(G)=\Phi$.
4. There exists disjoint union $\mathfrak{I}$-codense subsets M and G of $\mathfrak{Z}$.

Proof:

(1)↔(2). Get it from Lemma 3.21.

(2)→(3).By (2) there exist $M \subset \mathfrak{Z}$ such that $M^*=\mathfrak{Z}$ and $\Psi(M)=\Phi$,then $(((M^c)^c)^*)^c=\Phi$ $=\Psi(M^c)=\Phi$,put $M^c=G$.thus, we get $\Psi(G)=\Phi$.

(3)→(4).Let M and G subsets of $\mathfrak{Z}$ such that $M \cup G=\mathfrak{Z}$ and $M \cap G=\Phi$ .By(3) $\Psi(M)=\Psi(G)=\Phi$ .So $((M^c)^*)^c=\Phi$,then $(M^c)^*=\mathfrak{Z}$. Thus M^c is $\mathfrak{I}$-dense. Hence M is $\mathfrak{I}$-codense. Similarity ,we get that G is $\mathfrak{I}$-codense.

(4)→(1). Since $M \cup G=\mathfrak{Z}$ and $M \cap G=\Phi$, $M=G^c$ and $G=M^c$. By(4),we have $(M^c)^*=\mathfrak{Z}$. Therefore $M^*=\mathfrak{Z}$ and $G^*=\mathfrak{Z}$ which means that $\mathfrak{Z}$ is $\mathfrak{I}$-resolvable.

Theorem 3.27: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal space. The following statements are equivalent.

1. $\mathfrak{Z}$ is $\mathfrak{W}\mathfrak{I}$-resolvable.
2. There exist weakly-$\mathfrak{I}$-dense subsets M of $\mathfrak{Z}$ such that $\Psi(\Psi(M))=\Phi$.
3. There exist disjoint union subsets M and G of $\mathfrak{Z}$ such that $\Psi(\Psi(M))= \Psi(\Psi(G))=\Phi$.
4. There exist disjoint union $\mathfrak{W}\mathfrak{I}$-codense subsets M and G of $\mathfrak{Z}$.

Proof:

(1)↔(2). Get it from Proposition 3.20.

(2)→(3). By (2) there exist $M \subset \mathfrak{Z}$ such that $(M^*)^*=\mathfrak{Z}$ and $\Psi(\Psi(M))= \Psi(\Psi(M^c))=\Phi$,put $M^c=G$.thus we get $\Psi(\Psi(G))=\Phi$.

(3)→(4). Let M and G subsets of $\mathfrak{Z}$ such that $M \cup G=\mathfrak{Z}$ and $M \cap G=\Phi$ .By(3) $\Psi(\Psi(M))= \Psi(\Psi(G))=\Phi$,so $((M^c)^*)^c=\Phi$,then $(M^c)^*=\mathfrak{Z}$. Thus M^c is weakly-$\mathfrak{I}$-dense. Hence M is $\mathfrak{W}\mathfrak{I}$-codense.Similarity we get that G is $\mathfrak{W}\mathfrak{I}$-codense.

(4)→(1). Since $M \cup G=\mathfrak{Z}$ and $M \cap G=\Phi$, $M=G^c$ and $G=M^c$. Since M is $\mathfrak{W}\mathfrak{I}$-codense, then $(M^c)^{**}=\mathfrak{Z}$. Therefore $M^{**}=\mathfrak{Z}$ and $G^{**}=\mathfrak{Z}$ which means that $\mathfrak{Z}$ is $\mathfrak{W}\mathfrak{I}$-resolvable.

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