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Study Kc – Spaces Via ω – Open Sets

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Abstract **: This paper tackles , studying and introducing a new types of Kc – spaces that are called be $\beta Kc_i -$ *spaces* where (i = 1 , 2 , 3) . As well as discussion the important results which are arriving to during this studying.**

Keywords: **βω − open , βω − closed , −*closed map* , *βω* − compact*β Kc_i − spaces*.**

1. INTRODUCTION

In 1965 Njastad , O . [6] defined the β − open set as : The subset $\mathcal{A}$ of the space $\mathcal{L}$ is called β − open if and only if $\mathcal{A} \subseteq$ Cl (Int (Cl ($\mathcal{A}$))) .

Where the closure of $\mathcal{A}$ will be denoted by Cl($\mathcal{A}$) and the interior of $\mathcal{A}$ denoted by Int($\mathcal{A}$) . A point p in the space ($\mathcal{L}$, ϑ) is called condensation point [4] of $\mathcal{A}$ if for each $\mathcal{U}$ in ϑ with p in $\mathcal{U}$, the set $\mathcal{U} \cap \mathcal{A}$ is uncountable . In 1982 the ω − closed set was first introduced by Hdeib, H. Z . in [4] , and he defined it as : $\mathcal{A}$ is ω − closed set if it contains

all its condensation points and the ω − open set is the complement of the ω − closed set . A subset $\mathcal{W}$ of a space ($\mathcal{L}$, ϑ) is ω − open if and only if for each p ∈ $\mathcal{W}$ there exists $\mathcal{U}$ ∈ ϑ such that p ∈ $\mathcal{U}$ and $\mathcal{U} \setminus \mathcal{W}$ is countable . The union of all ω − open sets contained in $\mathcal{A}$ is the ω - interior of $\mathcal{A}$ and will denoted by $Int_\omega(\mathcal{A})$. In 1989 Bourbaki . N. [3] study the concept of compact space . In 2007 Noiri , T . , Al- omari , A . and Noorani . M.S.M. [7] introduced other notions called b ω − open and βω − open sets which are weaker than the ω − open set . In 1967 Wilansky .A [8] , studied and introduced the concept Kc – space . In 2011 Hadi . M . H. [5] , studied the concept of βω − compact spaces , also dealt with the concepts βω − continuous function and βω − T_2 − space . This paper consists of three section . In the first section we recall some of the basic definitions that are connected with this research. In the second section we prove some theorems, proposition and results about concept of βω – compact space. In the last section we study a new types of Kc-spaces which are called βKc_i − spaces , as well as we prove some theorems and results about this concept .

2. PRELIMINARIES

The purpose of this section , which is performed some main states and concepts those will need them for our research to proof some theories , proposition and results , these get them .

Definition 1.1 [1]

A topological space ($\mathcal{L}$, ϑ) is called anti- locally countable , if every non empty open set is uncountable .

Definition 1.2 [7]

A space ($\mathcal{L}$, ϑ) is called a door space if every subset of $\mathcal{L}$ is either open or closed.

Definition 1.3

A topological space ($\mathcal{L}$, ϑ) is said to be $\omega\omega$ −space if for every subset $\mathcal{A}$ of $\mathcal{L}$ has empty ω − interior .

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Definition 1.4 [7]

A subset $\mathcal{A}$ of a space $\mathcal{L}$ is called an ω - set if $\mathcal{A} = \mathcal{U} \cap \mathcal{V}$, where $\mathcal{U} \in \vartheta$ and $\text{Int}(\mathcal{V}) = \text{Int}_{\omega}(\mathcal{V})$.

Definition 1.5 [5]

A space $(\mathcal{L}, \vartheta)$ is said to be satisfy ω - condition if every ω - open set is ω – set.

Lemma 1.6

Let $(\mathcal{L}, \vartheta)$ is a topological door $\omega\omega$ –space and has ω – condition , then every βω − open set is open .

Proof :- The proof is directly from Theorem 1.2.16 in [5] , Proposition 2.17 , 2.18 in [7] and Definition 2.13 in [2] ▪

Definition 1.7 [5]

Let $\mathcal{S}$ be a subset of the topological space $(\mathcal{L}, \vartheta)$, then $\mathcal{A} \subset \mathcal{S}$ is called βω − open in $\mathcal{S}$ if $\mathcal{A} = \mathcal{U} \cap \mathcal{S}$, where $\mathcal{U}$ is βω − open in $\mathcal{L}$. And $\mathcal{A}$ is is βω − closed in $\mathcal{S}$, if its complement is βω − open in $\mathcal{S}$.

3. COMPACT SPACES

This section entail the concept βω - compact space to proof some new theories about this concept . Besides we shall study a new concept of mapping we called β − closed map .

Definition 2.1 [5]

Let $\mathcal{L}$ be a topological space . We say that a subset $\mathcal{A}$ of $\mathcal{L}$ is βω – compact if for each cover of βω − open sets from $\mathcal{L}$ contains a finite sub cover for $\mathcal{A}$.

Theorem 2.2

Let $(\mathcal{L}, \vartheta)$ be a topological door $\omega\omega$ –space and satisfies the ω – condition then any compact set is βω – compact .

Proof : - Directly from Theorem 1.9.3 in [5] ▪

Theorem 2.3

Let $(\mathcal{L}, \vartheta)$ be a topological space , then the union of any two βω – compact subset of $\mathcal{L}$ is a βω – compact

Proof :- Let $\mathcal{M}$ and $\mathcal{N}$ are βω – compact and $\mathcal{U}$ be a family of βω − open subset of $\mathcal{L}$ which cover $\mathcal{M} \cup \mathcal{N}$. Then $\mathcal{U}$ covers $\mathcal{M}$ and , so there exist finite sub cover.

$\mathcal{X}_1 , \ldots , \mathcal{X}_m$ of $\mathcal{M}$, and also there is a finite sub cover. $\mathcal{Y}_1 , \ldots , \mathcal{Y}_n$ of $\mathcal{N}$.

But $\{\mathcal{X}_1 , \ldots , \mathcal{X}_m , \mathcal{Y}_1 , \ldots , \mathcal{Y}_n \}$ is a βω − open sub cover of $\mathcal{M} \cup \mathcal{N}$, so $\mathcal{M} \cup \mathcal{N}$ is βω – compact ▪

Corollary 2.4

Let $(\mathcal{L}, \vartheta)$ be a topological space , then the union of finite collection of βω – compact subsets of $\mathcal{L}$ is a βω – compact .

Proof :- The proof is obvious ▪

Theorem 2.5

Let $\mathcal{S}$ be a subspace of $\mathcal{L}$, then $\mathcal{S}$ is βω – compact iff every covering of $\mathcal{S}$ by sets βω − open in $\mathcal{L}$ contains a finite sub collection covering $\mathcal{S}$.

Proof :- Suppose that $\mathcal{S}$ is βω – compact , and $\mathcal{B}^* = \{ \mathcal{A}_\lambda : \lambda \in \Lambda \}$ is cover of $\mathcal{S}$ by the sets βω − open in $\mathcal{L}$. then the collection $\{ \mathcal{A}_\lambda \cap \mathcal{S} : \lambda \in \Lambda \}$ is cover of $\mathcal{S}$ by sets βω − open in $\mathcal{S}$. Thus there exist a finite sub collection $\{\mathcal{A}_{\lambda 1} \cap \mathcal{S}, \ldots \mathcal{A}_{\lambda i} \cap \mathcal{S} \}$

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cover , then { $\mathcal{A}_{\lambda 1}$, ….. , $\mathcal{A}_{\lambda n}$} is sub collection of $\mathcal{B}^*$ cover $\mathcal{S}$. **Conversely** , let the given condition hold ; to prove $\mathcal{S}$ is βω – compact . Let $\mathcal{B}$ = { $\mathcal{A}^*_{\lambda}\}_{\lambda\in\Lambda}$ be any covering of $\mathcal{S}$ by set βω − open in $\mathcal{S}$ for every Λ , choose set $\mathcal{A}_\lambda$ βω − open in $\mathcal{L}$ such that $\mathcal{A}^*_\lambda = \mathcal{A}_\lambda \cap \mathcal{S}$, so the collection $\mathcal{B} = \{\mathcal{A}_\lambda : \lambda \in \Lambda\}$ is cover of $\mathcal{S}$ by set βω − open in $\mathcal{L}$. But by hypothisis some finite sub collection { $\mathcal{A}_{\lambda 1}$, ….. , $\mathcal{A}_{\lambda n}$} covers $\mathcal{S}$, then { $\mathcal{A}^*_{\lambda 1}$, …, $\mathcal{A}^*_{\lambda n}$ } be a sub collection of $\mathcal{B}$ that cover $\mathcal{S}$ ▪

Theorem 2.6

Let ($\mathcal{L}$, ϑ) be a βω – compact and $\boldsymbol{\mu}$ is coarser than **ϑ** , then ($\mathcal{L}$, μ) is βω – compact .

Proof :- Assume that $\mathcal{C}$ = { $\mathcal{A}_\lambda : \lambda \in \Lambda$ **}** be a βω − open with respect , cover of $\mathcal{L}$.

Since ϑ be a finite than μ , thus $\mathcal{C}$ is also a βω − open cover of $\mathcal{L}$ with respect , ϑ since($\mathcal{L}$, ϑ) is βω – compact , then $\mathcal{C}$ has a finite sub cover , therefore ($\mathcal{L}$, μ) is βω – compact ▪

Theorem 2.7

A topological space $\mathcal{L}$ is βω – compact iff every collection of βω − closed subset of $\mathcal{L}$ with the finite intersection property (FIP) whose intersection is non-empty .

Proof :- Let $\mathcal{L}$ is βω – compact , let { $\mathcal{F}_\alpha$: $\alpha \in \Lambda$ } be a family of βω − closed subset of $\mathcal{L}$ with FIP . If possible $\cap_{\alpha\in\Lambda} \mathcal{F}_\alpha = \Phi$, then the family { $\mathcal{L} - \mathcal{F}_\alpha$: $\alpha \in \Lambda$ } is βω − open cover of the βω – compact , there exist a finite subset Λ_0 of Λ such that $\mathcal{L} = \cup \{ \mathcal{L} - \mathcal{F}_\alpha : \alpha \in \Lambda_0 \}$, therefore $\Phi = \mathcal{L} - \cup \{ \mathcal{L} - \mathcal{F}_\alpha : \alpha \in \Lambda_0 \} = \cap \{ \mathcal{L} - (\mathcal{L} - \mathcal{F}_\alpha) : \alpha \in \Lambda_0 \}$

$= \cap \{ \mathcal{F}_\alpha : \alpha \in \Lambda_0 \}$ which contradiction with FIP . Therefore $\cap_{\alpha\in\Lambda} \mathcal{F}_\alpha \neq \Phi$

Conversely , let $\mathbb{U} = \{ \mathcal{U}_\alpha : \alpha \in \Lambda \}$ be an βω − open cover of the space ($\mathcal{L}$, ϑ) , then $\mathcal{L} - \{ \mathcal{U}_\alpha : \alpha \in \Lambda \}$ is a family of βω − closed subset ($\mathcal{L}$, ϑ) with $\cap \{ \mathcal{L} - \mathcal{U}_\alpha : \alpha \in \Lambda \} = \Phi$ by assumption there exist a finite Λ_0 of Λ such that $\cap \{ \mathcal{L} - \mathcal{U}_\alpha : \alpha \in \Lambda_0 \} = \Phi$, so

$\mathcal{L} = \mathcal{L} - \cap \{ \mathcal{L} - \mathcal{U}_\alpha : \alpha \in \Lambda_0 \} = \cup \{ \mathcal{U}_\alpha : \alpha \in \Lambda_0 \}$. Hence $\mathcal{L}$ is βω – compact ▪

Theorem 2.8

Let ($\mathcal{S}$, μ) be a subspace of a topological space ($\mathcal{L}$, ϑ) , and let $\mathcal{A} \subset \mathcal{S}$, then $\mathcal{A}$ is βω – compact relative to $\mathcal{L}$ iff $\mathcal{A}$ is βω – compact relative to $\mathcal{S}$.

Proof :- Suppose that $\mathcal{A}$ is a βω – compact relative to $\mathcal{S}$, and let $\{ \mathcal{U}_\lambda : \lambda \in \Lambda \}$ be βω − open cover of $\mathcal{A}$ relative to $\mathcal{L}$, then $\mathcal{A} \subseteq \cup_{\lambda\in\Lambda} \mathcal{U}_\lambda$. Since $\mathcal{A} \subset \mathcal{S}$ therefore $\mathcal{A} \subseteq \cup \{ \mathcal{S} \cap \mathcal{U}_\lambda : \lambda \in \Lambda \}$, since $\mathcal{S} \cap \mathcal{U}_\lambda$ is βω − open relative to $\mathcal{S}$, then $\{ \mathcal{S} \cap \mathcal{U}_\lambda : \lambda \in \Lambda \}$ is βω − open cover of $\mathcal{A}$ relative to $\mathcal{S}$. We must have $\lambda_1 , \lambda_2 , \dots , \lambda_n$ such that $\mathcal{A} \subseteq \cup_{i=1}^{n} (\mathcal{S} \cap \mathcal{U}_{\lambda i})$ $\subseteq \cup_{i=1}^{n} \mathcal{U}_{\lambda i}$. Therefore $\mathcal{A}$ is βω – compact relative to $\mathcal{L}$. **Conversely** , let $\mathcal{A}$ is βω – compact relative to $\mathcal{L}$, and let $\{ \mathcal{V}_\lambda : \lambda \in \Lambda \}$ be an βω − open cover of $\mathcal{A}$ relative to $\mathcal{S}$. Then $\mathcal{A} \subseteq \cup_{\lambda\in\Lambda} \mathcal{V}_\lambda$, then there exist $\mathcal{U}_\lambda$ is βω − open relative to $\mathcal{L}$ such that $\mathcal{V}_\lambda = \mathcal{S} \cap \mathcal{U}_\lambda$

$\forall \lambda \in \Lambda$, then $\mathcal{A} \subseteq \cup \mathcal{U}_\lambda$ where $\{ \mathcal{U}_\lambda : \lambda \in \Lambda \}$ is βω − open cover of $\mathcal{A}$ relative to $\mathcal{L}$

, since $\mathcal{A}$ is βω – compact set relative to $\mathcal{L}$. Then there exist $\lambda_1 , \lambda_2 , \dots , \lambda_n \in \Lambda$ such that $\mathcal{A} \subseteq \cup_{i=1}^{n} \mathcal{U}_{\lambda i}$ **,** since $\mathcal{A} \subseteq \mathcal{S}$ **,** then $\mathcal{A} \subseteq \mathcal{S} \cap \{ \mathcal{U}_{\lambda 1} \cup \mathcal{U}_{\lambda 2} \cup \dots \cup \mathcal{U}_{\lambda n} \} = (\mathcal{S} \cap \mathcal{U}_{\lambda 1}) \cup \dots \cup (\mathcal{S} \cap \mathcal{U}_{\lambda n})$. Since $\mathcal{S} \cap \mathcal{U}_{\lambda i} = \mathcal{V}_i$, and thus $\mathcal{A}$ is βω – compact relative to $\mathcal{S}$ ▪

Definition 2.9 [5]

Let $\mathcal{L}$ be a topological space , and for each $n \neq m \in \mathcal{L}$, there exist two disjoint sets $\mathcal{U}$ and $\mathcal{V}$ with $n \in \mathcal{U}$ and $m \in \mathcal{V}$, then $\mathcal{L}$ is called :-

1 . βω − T_2 space , if $\mathcal{U}$ is open and $\mathcal{V}$ is βω − open sets in $\mathcal{L}$.

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2 . $\beta\omega^{**} - T_2$ space , if $\mathcal{U}$ and $\mathcal{V}$ are $\beta\omega -$ open sets in $\mathcal{L}$.

Theorem 2.10

Let $(\mathcal{L}, \vartheta)$ is a topological space , if $\mathcal{L}$ is a $\beta\omega - T_2$ space , and let $\mathcal{A}_1$ and $\mathcal{A}_2$

be a $\beta\omega$ – compact subset of $\mathcal{L}$, such that $\mathcal{A}_1 \cap \mathcal{A}_2 = \Phi$. Then there exist $\mathcal{U}_1$ is open and $\mathcal{U}_2$ is $\beta\omega -$ open , such that $\mathcal{A}_1 \subset \mathcal{U}_1$, $\mathcal{A}_2 \subset \mathcal{U}_2$ and $\mathcal{U}_1 \cap \mathcal{U}_2 = \Phi$.

Proof :- It follow from Proposition **2.2.28** part (9) in **[5]** , that for every point a of $\mathcal{A}_1$ then there exist disjoint sets $\mathcal{V}_a$ is open and $\mathcal{U}_a$ is $\beta\omega -$ open , such that

$a \in \mathcal{V}_a$, $\mathcal{A}_2 \subset \mathcal{U}_a$ and $\mathcal{V}_a \cap \mathcal{U}_a = \Phi$. Then there exists a finite set $\{ a_1,\ldots, a_n \}$

of points of $\mathcal{A}_1$ such that $\mathcal{A}_1 \subset \cup_{i=1}^{n} \mathcal{V}_{ai}$ because $\mathcal{A}_1$ is $\beta\omega$ – compact , put $\mathcal{U}_1 = \cup_{i=1}^{n} \mathcal{V}_{ai}$ $\mathcal{U}_2 = \cap_{i=1}^{n} \mathcal{U}_{ai}$. Then $\mathcal{U}_1$ is open set , and $\mathcal{U}_2$ is $\beta\omega -$ open by Proposition **2.12** in

[7] . Therefore $\mathcal{A}_1 \subset \mathcal{U}_1$, $\mathcal{A}_2 \subset \mathcal{U}_2$ and $\mathcal{U}_1 \cap \mathcal{U}_2 = \Phi$ ▪

Corollary 2.11

Let $(\mathcal{L}, \vartheta)$ is a topological space **,** if $\mathcal{L}$ is a $T_2 -$ space **,** and let $\mathcal{A}_1$ and $\mathcal{A}_2$ be $\beta\omega$ – compact subset of $\mathcal{L}$ **,** such that $\mathcal{A}_1 \cap \mathcal{A}_2 = \Phi$ **.** Then there exists $\mathcal{U}_1$ and $\mathcal{U}_2$ are open , such that $\mathcal{A}_1 \subset \mathcal{U}_1$, $\mathcal{A}_2 \subset \mathcal{U}_2$ and $\mathcal{U}_1 \cap \mathcal{U}_2 = \Phi$.

Proof :- Obvious from Theorem **2.10** ▪

Theorem 2.12

For any topological space $(\mathcal{L}, \vartheta)$. Tow disjoint $\beta\omega$ – compact subset of a $\beta\omega^{**} - T_2$ space have disjoint open sets .

Proof :- We can prove this Theorem , by the same way as the proof of Theorem **2.10** ▪

Corollary 2.13

For any topological space $(\mathcal{L}, \vartheta)$. Tow disjoint $\beta\omega$ – compact subset of $T_2 -$ space have disjoint open sets **.**

Proof :- Clear ▪

Definition 2.14

Let g **:** $\mathcal{L} \rightarrow \mathcal{S}$ be a map of a space $\mathcal{L}$ into a space $\mathcal{S}$, then g is said to be $\beta -$ closed map , if g $(\mathcal{A})$ is closed set in $\mathcal{S}$ for each $\beta\omega -$ closed set $\mathcal{A}$ in $\mathcal{L}$.

Proposition 2.15

Let g : $\mathcal{L} \rightarrow \mathcal{S}$ be a map of a space $\mathcal{L}$ into a space $\mathcal{S}$ then . If g is a $\beta -$ closed map , then g is closed map .

Proof :- Let g be a $\beta -$ closed map , and let $\mathcal{A}$ is closed set in $\mathcal{L}$. Then by Lemma **2.2** in [7] **.** $\mathcal{A}$ is $\beta\omega -$ closed set in $\mathcal{L}$, since g is $\beta -$ closed map ,then g $(\mathcal{A})$ is closed set in $\mathcal{S}$, therefore g is closed map ▪

Proposition 2.16

Let $(\mathcal{L}, \vartheta)$ be a topological space . If g : $\mathcal{L} \rightarrow \mathcal{S}$ be a $\beta\omega -$ continuous function from a $\beta\omega$ – compact space $\mathcal{L}$ to a $\beta\omega - T_2$ space $\mathcal{S}$. Then g is $\beta -$ closed map .

Proof :- Let $\mathcal{A}$ is $\beta\omega -$ closed set in $\mathcal{L}$, then by Theorem **1.9.4** in [5] . $\mathcal{A}$ is $\beta\omega$ – compact , also g $(\mathcal{A})$ is $\beta\omega$ – compact by Theorem **1.9.5** in [5] . And by Theorem **2.2.30** part (9) in [5] , we get g $(\mathcal{A})$ is closed in $\mathcal{S}$, hence g is $\beta -$ closed map ▪

3. $\beta Kc_i -$ spaces

In this section , we'll show the concept of $\beta Kc_i -$ spaces . In additional , we'll give some notice , results and important relation that are connected with this concept.

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Definition 3.1

Let $\mathcal{L}$ be a topological space , then $\mathcal{L}$ is called .

1 . βKc_1 – space , if every $\beta\omega$ – compact subset of $\mathcal{L}$ is closed .

2 . βKc_2 – space , if every compact subset of $\mathcal{L}$ is $\beta\omega$ – closed .

3 . βKc_3 – space , if every $\beta\omega$ – compact subset of $\mathcal{L}$ is $\beta\omega$ – closed .

Remarks 3.2

1 . If $\mathcal{L}$ is βKc_1 – space , then $\mathcal{L}$ is βKc_3 – space .

2 . If $\mathcal{L}$ is βKc_2 – space , then $\mathcal{L}$ is βKc_3 – space .

Theorem 3.3

Let $(\mathcal{L}, \vartheta)$ be a topological door $\omega\omega - space$. If $\mathcal{L}$ has $\omega - condition$ and βKc_3 – space then $\mathcal{L}$ is βKc_1 – space .

Proof :- Let $\mathcal{L}$ be a βKc_3 – space , and $\mathcal{A}$ is $\beta\omega$ – compact subset of $\mathcal{L}$. Since $\mathcal{L}$ is βKc_3 – space , then $\mathcal{A}$ is $\beta\omega$ – closed , by Lemma **1.6** and Definition **1.5** , we get $\mathcal{A}$ is closed . therefore $\mathcal{L}$ is βKc_1 – space ▪

Theorem 3.4

Let $(\mathcal{L}, \vartheta)$ be a topological space **,** then every Kc – space is βKc_1 – space **.**

Proof :- By Theorem **1.9.2** part (15) in [5] , the proof is complete ▪

Theorem 3.5

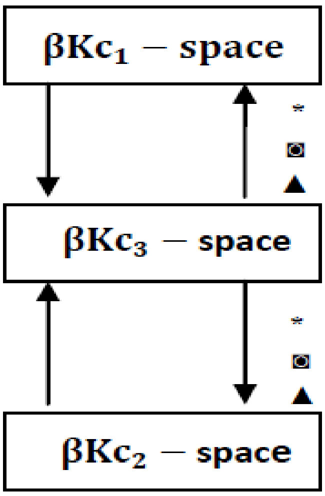
Let $(\mathcal{L}, \vartheta)$ be a topological door $\omega\omega - space$ **.** If $\mathcal{L}$ has ω – condition and

βKc_3 – space **,** then $\mathcal{L}$ is βKc_2 – space .

Proof :- Let $\mathcal{L}$ be a βKc_3 – space , and $\mathcal{A}$ be a compact subset of $\mathcal{L}$ **,** then by Theorem **2.2** , we have $\mathcal{A}$ is $\beta\omega$ – compact . But $\mathcal{L}$ is βKc_3 – space , then $\mathcal{A}$ is $\beta\omega$ – closed , thus $\mathcal{L}$ is βKc_2 – space ▪

Note 3.6

From Theorem **3.3 ,** Theorem **3.5** and Remarks **3.2 .** We have the following sketch



* **door space**

◙ ***ωω – space***

▲ ***ω – condition***

(The sketch above shows the relations among the types of Kc- spaces) .

Theorem 3.7

Let $(\mathcal{L}, \vartheta)$ be a $\beta Kc_1 -$ space , then any subspace of $\mathcal{L}$ is $\beta Kc_1 -$ space .

Proof :- Assume that $\mathcal{L}$ is $\beta Kc_1 -$ space , and let $\mathcal{S}$ be a subspace of $\mathcal{L}$, and $\mathcal{A}$ be subset of $\mathcal{S}$ be $\beta\omega$ – compact relative to $\mathcal{S}$. Then by Theorem **2.8** $\mathcal{A}$ is $\beta\omega$ – compact relative to $\mathcal{L}$, but $\mathcal{L}$ is $\beta Kc_1 -$ space , therefore $\mathcal{A}$ is closed in $\mathcal{L}$, and thus $\mathcal{A} = \mathcal{A} \cap \mathcal{S}$ a closed set in $\mathcal{S}$. Hence $\mathcal{S}$ is $\beta Kc_1 -$ space ▪

Corollary 3.8

The intersection of two $\beta Kc_1 -$ spaces is $\beta Kc_1 -$ space .

Proof :- Let $\mathcal{L}_1$ and $\mathcal{L}_2$ are $\beta Kc_1 -$ spaces , and let $\mathcal{S} = \mathcal{L}_1 \cap \mathcal{L}_2$, so it's a subset of $\mathcal{L}_1$ and $\mathcal{L}_2$. Then $(\mathcal{S}, \vartheta_{\mathcal{S}})$ be a subspace of $\mathcal{L}_1$ and $\mathcal{L}_2$, but $\mathcal{L}_1$ and $\mathcal{L}_2$ are $\beta Kc_1 -$ spaces . Therefore $\mathcal{S}$ is $\beta Kc_1 -$ space ▪

Corollary 3.9

The intersection of finite collection of $\beta Kc_1 -$ space is $\beta Kc_1 -$ space .

Proof :- Clear ▪

REFERENCES

[1] Al – Zoubi , K . and Al-Nashef , B . " The topology of ω- open subsets " , Al- Manareh , 9(2) : 169 – 179 (2003) .

[2] Al-Omari , A . and Noorni , M.S.M . " Contra – ω-continuous and almost Contra – ω-continuous " , Internat . J . Math . Sci . , Vo . 2007 . Article ID 40469 , 13 pages . doi : 10. 1155 / 2007 / 40469 (2007) .

[3] Bourbaki N . , Elements of Mathematics . " General topology " , chapter 1-4 , spring Verloy , Berlin , Heide lberg , New – York , London , Paris , Tokyo , 2^nd Edition (1989) .

[4] Hdeib , H.Z. " ω- closed mappings " , Rev. Colomb . Math. 16(3-4) : 65-78 (1982) .

[5] M. H. Hadi, "Weak forms of ω-open sets and decomposition of separation axioms," Babylon University, M. Sc. Thesis, 2011.

[6] Njastad , O . "On some classes of nearly open sets " , Pacific . J . Math . 15: 961-970 . (1965) .

[7] Noiri , T . , Al-Omari , A . and Noorani , M . S . M . " Weak forms of ω-open sets and decomposition of continuity " , E.J.P.A.M . 2 (1) : 73-84 (2009) .

[8] Wilansky , A . , Between T_1 and T_2 . Amer. Math . Monthly 74 , 261-266 (1967) .