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On preparacompactness in bitopological spaces

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Abstract . J. Dieudonne [8], introduced the notions of paracompactness and Martin M. K. [9] , introduced the notions of paracompactness in bitopological spaces and K. AL-Zoubi and S. AL-Ghour [10], introduced the notions of P3-paracompactness of topological space in terms of preopen sets .In this paper, we introduce paracompactness in bitopological spaces in terms of ij-preopen sets . We obtain various characterizations, properties of paracompactness and its relationships with other types of spaces.

Key words and phrases : ij-preparacompact, ij-precontinuous, separation axioms .

1. Introduction

The concepts of regular open , regular closed , semiopen , semiclosed , and preopen sets have been introduced by many authors in a topological space (cf. [1-4]). These concepts are extended to bitopological spaces by many authors (cf. [5-7]) .

Throughout the present paper (X , τ_1, τ_2) and (Y, μ_1, μ_2) (or simple X and Y) denote bitopological spaces . when A is a subset of a space X , we shall denote the closure of A and the interior of A in (X , τ_i) by τ_i -clA and τ_i -intA , respectively, where i= 1,2 , and i,j = 1,2 ; i$\neq$j .

A subset A of X is said to be ij- preopen (resp. ij-semiopen ,ij-regular open , ij-regular closed and ij-preclosed) if
$A \subseteq \tau_i - \text{int}(\tau_j - clA)$ (*resp.* $A \subseteq \tau_i - cl(\tau_j - \text{int}\, A)$, $A = \tau_i - \text{int}(\tau_j - clA)$,
$A = \tau_j - cl(\tau_i - \text{int}\, A)$, and $\tau_j - cl(\tau_i - \text{int}\, A) \subseteq A$) . The family of all ij-semiopen (resp. ij- regular open and ij- preopen) sets of X is denoted by ij-SO(X) (resp. ij-RO(X) and ij-PO(X)) . The intersection of all ij- preclosed sets which contain A is called the ij- preclosure of A and is denoted by ij-PclA . Obviously , ij-PclA is the smallest ij-preclosed set which contains A .

Definition 1.1 .A bitopological space (X, τ_1, τ_2) is called ij-locally indiscrete if every $\tau_i - open$ subset of X is $\tau_j - closed$.

Definition 1.2 . A collection $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ of subsets of X is called ,

(1) locally finite with respect to the topology τ_i (respectively , ij-strongly locally finite) , if for each $x \in X$, there exists $U_x \in \tau_i$ (respectively,

$U_x \in ij-RO(X)$) containing x and U_x which intersects at most finitely many members of Ξ ;

(2) ij-P-locally finite if for each $x \in X$, there exists a ij- preopen set U_x in X containing x and U_x which intersects at most finitely many members of Ξ.

Definition 1.3 . A bitopological space (X, τ_1, τ_2) is called $(\tau_i - \tau_j)$ paracompact with respect to the topology τ_i , if every $\tau_i - open$ cover of X has $\tau_j - open$ refinement which is locally finite with respect to the topology τ_i [9].

We obtain the following lemmas :

Lemma 1.4. For a bitopological space (X, τ_1, τ_2), the followings are equivalent :

(a) (X, τ_1, τ_2) is ij-locally indiscrete ;

(b) Every subset of X is ij-preopen ;

(c) Every $\tau_j - closed$ subset of X is ij-preopen .

Lemma 1.5 . If A is a ij-preopen subset of X , then $\tau_i - clA$ is ij-regular closed .

Lemma 1.6 . Let A and B be subsets of a space X . Then

(a) If $A \in ij-PO(X)$ and $B \in \tau_i$, then $(A \cap B) \in ij-PO(X)$.

(b) If $A \in ij-PO(X)$ and $B \in ij-SO(X)$, then $(A \cap B) \in ij-PO(B, \tau_{1B}, \tau_{2B})$.

(c) If $A \in ij-PO(B, \tau_{1B}, \tau_{2B})$ and $B \in ij-PO(X)$, then $A \in ij-PO(X)$.

Theorem 1.7 . Let $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ be a collection of ij-semiopen subsets of a bitopological space (X, τ_1, τ_2) , then Ξ is ij-P-locally finite if and only if itis ij-strongly locally finite .

Proof . (Necessity) . Since every ij- regular open subset of X is ij-preopen , so Ξ is ij-P-locally finite.

(Sufficiency) . Let $x \in X$ and W_x be a ij-preopen subset of X such that $x \in W_x$ and $W_x \cap F_{\lambda i} \neq \Phi$ for each i= 1,2,...,n . Put $R_x = \tau_i - \text{int}(\tau_j - clW_x)$, then R_x is a ij-regular open which contains x . We show for every $\lambda \in \Gamma / \{\lambda_1, \lambda_2, ..., \lambda_n\}$, $F_\lambda \cap R_x = \Phi$. For each $\lambda \in \Gamma$, choose $U_\lambda \in \tau_j$ such that $U_\lambda \subseteq P_\lambda \subseteq \tau_i - clU_\lambda$. Now, if $F_\lambda \cap R_x \neq \Phi$, then $\tau_i - clU_\lambda \cap R_x \neq \Phi$ and so $\tau_i - clU_\lambda \cap \tau_i - \text{int}(\tau_j - clW_x) \neq \Phi$, which implies that there exist $p \in \tau_i - clU_\lambda$ *and* $p \in \tau_i - \text{int}(\tau_j - clW_x)$. So for each $\tau_i - open$ set V_p containing p such that $V_p \cap U_\lambda \neq \Phi$, but $\tau_i - \text{int}(\tau_j - clW_x)$ is a $\tau_i - open$ containing p , which implies that $U_\lambda \cap \tau_i - \text{int}(\tau_j - clW_x) \neq \Phi$, since $\tau_i - \text{int}(\tau_j - clW_x) \subseteq \tau_j - clW_x$, so we get that $U_\lambda \cap \tau_j - clW_x \neq \Phi$, then there exists $q \in U_\lambda$ and $q \in \tau_j - clW_x$ .Therefore for each $\tau_j - open$ sets H_q containing q , $H_q \cap W_x \neq \Phi$, but U_λ is $\tau_j - open$ containing q , so $U_\lambda \cap W_x \neq \Phi$, which implies that

$\Phi \neq U_\lambda \bigcap W_x \subseteq P_\lambda \bigcap W_x$, which contradict the hypothesis . Thus $\lambda \in \Gamma / \{\lambda_1, \lambda_2, ..., \lambda_n\}$.

Now using the above theorem and the fact that every ij-regular open is $\tau_i - open$ and every $\tau_j - open$, ij-regular closed set is ij-semiopen set , we obtain the following corollaries :

Corollary 1.8 . Let $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ be a collection of $\tau_j - open$ (ij-regular closed) subset of a bitopological space (X, τ_1, τ_2) . If Ξ is ij-P-locally finite , then Ξ is locally finite with respect to the topology τ_i .

Now by using the above theorem , corollary 1.**8** and the definition 1.3 , we obtain the following corollary :

Corollary 1.9 . A bitopological space (X, τ_1, τ_2) is $(\tau_i - \tau_j)$ paracompact with respect to the topology τ_i if and only if every $\tau_i - open$ cover of X has a $\tau_j - open$ refinement which is ij-p-locally finite .

Theorem 1.10 . Let $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ be a collection of subsets of a bitopological space (X, τ_1, τ_2) . Then

(a) Ξ is ij-P-locally finite if and only if $\{ij - pclF_\lambda : \lambda \in \Gamma\}$ is ij-P-locally finite ;

(b) If Ξ is ij-P-locally finite , then $\bigcup_{\lambda \in \Gamma} ij - pclF_\lambda = ij - pcl(\bigcup_{\lambda \in \Gamma} P_\lambda)$;

(c) Ξ is locally finite with respect to the topology τ_i if and only if the collection $\{ij - pclF_\lambda : \lambda \in \Gamma\}$ is locally finite with respect to the topology τ_i .

Proof . (a) Suppose that , Ξ is ij-P-locally finite . For each $x \in X$, there exists ij-preopen U_x containing x which meets only finitely many of the sets F_λ , say $F_{\lambda 1}, ..., F_{\lambda n}$. Since $F_{\lambda k} \subseteq ij - pclF_{\lambda k}$ for each k=1,2, ... , n , thus U_x meets $ij - pclF_{\lambda 1}, ..., ij - pclF_{\lambda n}$. Therefore $\{ij - pclF_\lambda : \lambda \in \Gamma\}$ is ij-P-locally finite .

Conversely. Let $x \in X$, there exists ij-preopen U_x containing x which meets only finitely many of the sets $ij - pclF_\lambda$. Say $ij - pclF_{\lambda 1}, ..., ij - pclF_{\lambda n}$, we get that $U_x \bigcap ij - pclF_{\lambda k} \neq \Phi$ for each k =1,..., n . Let $q \in U_x$ and $q \in ij - pclF_{\lambda k}$, which implies that , for every ij-preopen set V_q such that $V_q \bigcap F_{\lambda k} \neq \Phi$, but U_x be ij-preopen containing q , so $U_x \bigcap F_{\lambda k} \neq \Phi$ for each k=1,..., n .Thus Ξ is ij-P-locally finite.

(b) Suppose, Ξ is ij-P-locally finite , which can be easily got $\bigcup_{\lambda \in \Gamma} ij - pclF_\lambda \subseteq ij - pcl(\bigcup_{\lambda \in \Gamma} P_\lambda)$. On the other hand , let $q \in ij - pcl(\bigcup_{\lambda \in \Gamma} F_\lambda)$, then every ij-preopen V_q such that $V_q \bigcap (\bigcup_{\lambda \in \Gamma} F_\lambda) \neq \Phi$.By hypothesis , there exists ij-preopen U_q which meets only finitely many of the sets F_λ, say $F_{\lambda 1},......., F_{\lambda n}$.Thus for each ij-preopen set V_q containing q , $V_q \cap (\bigcup_{k=1}^{n} F_{\lambda k}) \neq \Phi$, which implies that $q \in ij - pcl(\bigcup_{k=1}^{n} F_{\lambda k}) = \bigcup_{k=1}^{n} ij - pclF_{\lambda k}$, there exists h such that $q \in ij - pclF_{\lambda h}$. Therefore we get that $q \in \bigcup_{\lambda \in \Gamma} ij - pclF_\lambda$ and hence $\bigcup_{\lambda \in \Gamma} ij - pclF_\lambda = ij - pcl(\bigcup_{\lambda \in \Gamma} P_\lambda)$.

(c) Suppose, Ξ is locally finite with respect to the topology τ_i , which implies that for each $x \in X$, there exists $\tau_i - open$ set U_x which meets only finitely many of the sets F_λ, say $F_{\lambda 1},......., F_{\lambda n}$, but $F_{\lambda k} \subseteq ij - pclF_{\lambda k}$, we get that U_x which meets $ij - pclF_{\lambda 1},......., ij - pclF_{\lambda n}$. Thus $\{ij - pclF_\lambda : \lambda \in \Gamma\}$ is locally finite with respect to the topology τ_i .

Conversely , let $x \in X$, then there exists $\tau_i - open$ set U_x which meets only finitely many of the sets $ij - pclF_\lambda$'s, say $ij - pclF_{\lambda 1},......., ij - pclF_{\lambda n}$. Now choose a point $q \in U_x$
and $q \in ij - pclF_{\lambda k}$. For each k=1,2,...,n , therefore for each ij-preopen set V_q containing q such that $V_q \cap F_{\lambda k} \neq \Phi$, but $q \in U_x$, then we get U_x which meets only finitely many of the sets F_λ's . Hence Ξ is locally finite with respect to the topology τ_i .

Definition 1.11 .A function $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is called :

(1) ij-precontinuous if for each $V \in ij - PO(Y)$, $f^{-1}(V) \in ij - PO(X)$;

(2) ij-strongly precontinuous if for each $V \in ij - PO(Y)$, $f^{-1}(V) \in \tau_i$;

(3) ij-preclosed if for each $V \in ij - PC(X)$, $f(V) \in ij - PC(Y)$;

(4) ij-strongly preclosed if for each $V \in \tau_i - C(X)$, $f(V) \in ij - PC(Y)$;

(5) ij-preopen if for each $V \in ij - PO(X)$, $f(V) \in ij - PO(Y)$;

(6) $(\tau_i - \mu_i)-$continuous ($(\tau_i - \mu_i)-$open and $(\tau_i - \mu_i)-$closed) if $f : (X, \tau_i) \to (Y, \mu_i)$ for i =1,2 are $(\tau_1 - \mu_1)-$ continuous and $(\tau_2 - \mu_2)-$continuous ($(\tau_1 - \mu_1)-$open , $(\tau_2 - \mu_2)-$open and $(\tau_1 - \mu_1)-$closed , $(\tau_2 - \mu_2)-$closed) .

Lemma 1.12 . Let $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ be a function . Then

(1) f is ij-preclosed if and only if for every $y \in Y$ and $P \in ij - PO(X)$ such that $f^{-1}(y) \subset P$, there is $V \in ij - PO(Y)$ such that $y \in V$ and $f^{-1}(V) \subset P$.

(2) f ij-strongly preclosed if and only if for every $y \in Y$ and $P \in \tau_i$ which contains $f^{-1}(y)$, there is $V \in ij - PO(Y)$ such that $y \in V$ and $f^{-1}(V) \subset P$.

(3) If f be $(\tau_i - \mu_i)-$continuous , $(\tau_i - \mu_i)-$open function , then f is ij-preopen function .

Proposition 1.13 .Let $f:(X,\tau_1,\tau_2)\to(Y,\mu_1,\mu_2)$ be a ij-precontinuous function . If $\Xi=\{F_\lambda:\lambda\in\Gamma\}$ is a ij-P-locally finite collection in Y , then $f^{-1}(\Xi)=\{f^{-1}(F_\lambda):\lambda\in\Gamma\}$ is a ij-P-locally finite collection in X .

Proposition 1.14 . Let $f:(X,\tau_1,\tau_2)\to(Y,\mu_1,\mu_2)$ be a ij-strongly precontinuous function . If $\Xi=\{F_\lambda:\lambda\in\Gamma\}$ is a ij-P-locally finite collection in Y , then $f^{-1}(\Xi)=\{f^{-1}(F_\lambda):\lambda\in\Gamma\}$ is a locally finite collection with respect to the topology τ_i .

Definition 1.15 .A subset A of a bitopological space (X,τ_1,τ_2) is called ij-strongly compact if for every cover of A by ij-preopen subsets of X has a finite subcover .

Proposition 1.16 . Let $f:(X,\tau_1,\tau_2)\to(Y,\mu_1,\mu_2)$ be a ij-strongly closed function such that $f^{-1}(y)$ is compact relative to the topology τ_i for every $y\in Y$. If $\Xi=\{F_\lambda:\lambda\in\Gamma\}$ is a locally finite collection with respect to the topology τ_i , then $f(\Xi)=\{f(F_\lambda):\lambda\in\Gamma\}$ is a ij-P-locally finite collection in Y .

Proposition 1.17 . Let $f:(X,\tau_1,\tau_2)\to(Y,\mu_1,\mu_2)$ be a ij-preclosed function such that $f^{-1}(y)$ is ij-strongly compact relative to X for every $y\in Y$. If $\Xi=\{F_\lambda:\lambda\in\Gamma\}$ is a ij-P-locally finite collection in X , then $f(\Xi)=\{f(F_\lambda):\lambda\in\Gamma\}$ is a ij-P-locally finite collection in Y .

2 . ij-preparacompact spaces

In this section, we introduce the generalized paracompact and separation axioms using the notions of ij-preopen sets in bitopoogical spaces, and give some characterization of these types of spaces and study the relationships between them and other well known spaces .

Definition 2.1 . A bitopological space (X,τ_1,τ_2) is called ij-preparacompact if every τ_i-open cover of X has a ij-P-locally finite ij-preopen refinement .

Definition 2.2 . Let (X,τ_1,τ_2) be a bitopological space . The space X is said to be :

(1) pair wise Hausdorff space if and only if for each distinct two points x , y in X , there exists two disjoint τ_i-open U and τ_j-open V such that $x\in U$ and $y\in V$ [11] ;

(2) pair wise regular if and only if for each $\tau_i-closed$ set F and $x\in X$, $x\notin F$, there are two disjoint τ_i-open U and τ_j-open V such that $F\subset U$, $x\in V$ [11];

(3) pair wise P-regular if and only if for each $\tau_i-closed$ set F and $x\in X$, $x\notin F$, there are two disjoint ij-preopen sets U and V , such that $x\in U$ *and* $F\subseteq V$;

(4) pair wise P-normal if and only if whenever A and B are disjoint $\tau_i - closed$ sets in X , there are disjoint ij-preopen sets U and V with $A \subseteq U$ and $B \subseteq V$;

(5) pairwise $P - T_1 - space$ if for each two distincts points x and y in X, there are ij-preopen sets U and V in X such that $x \in U$ *and* $y \in V$;

(6) pairwise $T_1 - space$ if for each two distincts points x and y in X, there are $\tau_i - open$ U and $\tau_j - open$ V in X such that $x \in U$ *and* $y \in V$ [11];

(7) pairwise $P - T_3 - space$ if it's pairwsise P-regular $P - T_1 - space$;

(8) pairwise $P - T_4 - space$ if it's pairwsise P-normal $P - T_1 - space$.

Lemma 2.3 .A bitopological space (X , τ_1 , τ_2) is pairwise p-regular if and only if for each $\tau_i - open$ set U and $x \in U$, then there exists $P \in ij - PO(X)$ such that $x \in P \subseteq ij - pclP \subseteq U$.

Theorem 2.4 . Every ij-preparacompact pairwise Hausdorff bitopological space (X , τ_1 , τ_2) is pair wise p-regular .

Proof : Let A be a $\tau_i - closed$ and let $x \notin A$. For each $y \in A$, choose an $\tau_i - open$ U_y and $\tau_i - open$ H_x such that $y \in U_y$, $x \in H_x$ and $U_y \bigcap H_x = \Phi$, so we get that $x \notin ij - pclU_y$. Therefore the family $U = \{U_y : y \in A \} \bigcup \{X - A\}$ is an $\tau_i - open$ cover of x and so it has a ij-P-locally finite ij-preopen refinement Π . Put $V = \{ H \in \Pi : H \bigcap A \neq \Phi \}$, then V is a ij-preopen set containing A and $ij - pclV = \bigcup \{ij - pclH : H \in \Pi \text{ and } H \bigcap A \neq \Phi \}$ (Theorem 1.10 (b)). Therefore $U = X - ij - pclV$ is a ij-preopen set containing x such that U and V are disjoint subsets of X . Thus X is pairwise p-regular .

From the above theorem, the following corollaries are obtained :

Corollary 2.5 .Every ij-preparacompact pairwise Hausdorff bitopological space (X , τ_1 , τ_2) is pair wise p-normal .

Corollary 2.6 .Every ij-preparacompact paiwise Hausdorff bitopological space (X , τ_1 , τ_2) is

(a) paiwise $p - T_3 - space$;

(b) pairwise $p - T_4 - space$.

Theorem 2.7 . Let (X , τ_1) and (X , τ_2) are regular spaces . Then (X , τ_1 , τ_2) is ij-preparacompact if and only if every $\tau_i - open$ cover Ξ of X has a ij-P-locally finite ij-preclosed refinement Σ .

Proof : To prove necessity, let Ξ be $\tau_i - open$ cover of X . For each $x \in X$, we choose a member $U_x \in \Xi$ and by the regularity of (X , τ_1) and (X , τ_2), an $\tau_i - open$ subsets V_x containing x such that $\tau_i - clV_x \subset U_x$. Therefore $\Psi = \{V_x : x \in X \}$ is an

$\tau_i - open$ cover of X and so by hypothesis it has a ij-P-locally finite ij-preopen refinement, say $\Omega = \{W_\lambda : \lambda \in \Gamma\}$.Now, consider the collection $ij - pcl\Omega = \{ij - pclW_\lambda : \lambda \in \Gamma\}$ as a ij-P-locally finite (Theorem 1.10 (a)) of ij-preclosed subsets of (X, τ_1, τ_2) .Since for every $\lambda \in \Gamma$, $ij - pclW_\lambda \subseteq ij - pclV_x \subseteq \tau_i - clV_x \subseteq U_x$ for some $U_x \in \Xi$, therefore $ij - pcl\Omega$ is a refinement of Ξ .

Conversely, let Ξ be an $\tau_i - open$ cover of X and Ψ be a ij-P-locally finite ij-preclosed refinement of Ξ. For each $x \in X$, choose $W_x \in ij - PO(X)$ such that $x \in W_x$ and W_x intersect at most finitely many members of Ψ. Let Σ be a ij-preclosed ij-P-locally finite refinement of $\Omega = \{W_x : x \in X\}$.For each $V \in \Psi$, let $V' = X / H$, where $H \in \Sigma$ and $H \bigcap V = \Phi$. Then $\{V' : V \in \Psi\}$ is a ij-preopen cover of X . Now, for each $V \in \Psi$, choose $U_V \in \Xi$ such that $V \subseteq U_V$, therefore the collection $\{U_V \bigcap V' : V \in \Psi\}$ is a ij-P-locally finite ij-preopen (Lemma 1.6 (a)) refinement of Ξ and thus (X, τ_1, τ_2) is ij-preparacompact .

Theorem 2.8 .Let A be a ij-regular closed subset of a ij-preparacompact bitopological space (X, τ_1, τ_2). Then $(A, \tau_{1A}, \tau_{2A})$ is ij-preparacompact .

Proof : Let $\Psi = \{V_\lambda : \lambda \in \Gamma\}$ be an $\tau_i - open$ cover of A in $(A, \tau_{1A}, \tau_{2A})$. For each $\lambda \in \Gamma$, choose $U_\lambda \in \tau_i$ such that $V_\lambda = A \bigcap U_\lambda$, then the collection $\Xi = \{U_\lambda : \lambda \in \Gamma\} \bigcup \{X - A\}$ is an $\tau_i - open$ cover of the ij-preparacompact space X and so it has a ij-P-locally finite ij-preopen refinement . Say $\Psi = \{W_\delta : \delta \in \Delta\}$, then by Lemma 1.6(b) and since $ij - RO(X) \subseteq ij - SO(X)$, the collection $\{A \bigcap W_\delta : \delta \in \Delta\}$ is a ij-P-locally finite ij-preopen refinement of Ψ in $(A, \tau_{1A}, \tau_{2A})$. This completes the proof .

Theorem 2.9 .Let A and B be subsets of a bitopological space (X, τ_1, τ_2) such that $A \subseteq B$.

(1) If $B \in ij - PO(X)$ and A is ij-preparacompact relative to $(B, \tau_{1B}, \tau_{2B})$, then A is ij-preparacompact relative to (X, τ_1, τ_2) .

(2) If $B \in ij - SO(X)$ and A is ij-preparacompact relative to (X, τ_1, τ_2), then A is ij-preparacompact relative to $(B, \tau_{1B}, \tau_{2B})$.

Proof : (1) Let $\Xi = \{U_\lambda : \lambda \in \Gamma\}$ be an $\tau_i - open$ cover of A in X . Then the collection $\Xi_B = \{B \bigcap U_\lambda : \lambda \in \Gamma\}$ is an $\tau_{iB} - open$ cover of A in $(B, \tau_{1B}, \tau_{2B})$. Since A is ij-preparacompact relative to $(B, \tau_{1B}, \tau_{2B})$.Hence Ξ_B has a ij-P-locally finite ij-preopen refinement Ω_B in $(B, \tau_{1B}, \tau_{2B})$ .Since $B \in ij - PO(X)$ and $U_\lambda \in \tau_i$,$\forall \lambda \in \Gamma$, so by lemma 1.6 (a) , $B \bigcap U_\lambda \in ij - PO(X)$,$\forall \lambda \in \Gamma$. Then the collection Ω_B is a ij-P-locally finite ij-preopen refinement of Ξ in (X, τ_1, τ_2). Therefore A is a ij-preparacompact relative to (X, τ_1, τ_2).

(2) Let $\Psi=\{V_\lambda : \lambda \in \Gamma\}$ be an $\tau_{iB}-open$ cover of A in $(B, \tau_{1B}, \tau_{2B})$. For every $\lambda \in \Gamma$, choose $U_\lambda \in \tau_i$ such that $V_\lambda = B \bigcap U_\lambda$. Then the collection $\Xi = \{U_\lambda : \lambda \in \Gamma\}$ is an $\tau_i - open$ cover of A relative to (X, τ_1, τ_2) and so it has a ij-P-locally finite ij-preopen refinement of Ξ in (X, τ_1, τ_2), say Σ. Thus, by lemma 1.6 (b), the collection $\Sigma_B = \{P \bigcap B : P \in \Sigma\}$ is a ij-P-locally ij-preopen refinement of Ξ in $(B, \tau_{1B}, \tau_{2B})$.

Corollary 2.10 . Let A be a subset of a bitopological space (X, τ_1, τ_2).

(1) If $A \in ij - PO(X)$ and the subspace $(A, \tau_{1A}, \tau_{2A})$ is ij-preparacompact , then A is ij-preparacompact relative to (X, τ_1, τ_2).

(2) If $A \in ij - SO(X)$ and A ij-preparacompact relative to (X, τ_1, τ_2), then the subspace $(A, \tau_{1A}, \tau_{2A})$ is ij-preparacompact .

Theorem 2.11 .If $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ be $(\tau_i - \mu_i)-$continuous , $(\tau_i - \mu_i)-$open , ij-strongly ij-preopen and surjective function such that $f^{-1}(y)$ is ij-strongly compact relative to (X, τ_1, τ_2) for every $y \in Y$. If (X, τ_1, τ_2) is ij-preparacompact , then so is (Y, μ_1, μ_2).

Proof : Let $\Psi=\{V_\lambda : \lambda \in \Gamma\}$ be an $\mu_i - open$ cover of (Y, μ_1, μ_2). Since f is $(\tau_i - \mu_i)-$continuous , the collection $\Xi = f^{-1}(\Psi)=\{f^{-1}(V_\lambda) : \lambda \in \Gamma\}$ is an $\tau_i - open$ cover of the ij-preparacompact (X, τ_1, τ_2) space and so it has a ij-P-locally finite ij-preopen refinement , say Ω. Since f is $(\tau_i - \mu_i)-$continuous and $(\tau_i - \mu_i)-$open , then by lemma 1.12 (3), f is ij-preopen function and so by Proposition 1.17 , the collection $f(\Omega)$ is a ij-P-locally finite ij-preopen refinement of Ψ. The result is proved .

Theorem 2.12 . Let $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ be a $(\tau_i - \mu_i)-$closed ij-precontinuous surjective function such that $f^{-1}(y)$ is $\tau_i - compact$ in (X, τ_i) for every $y \in Y$. If (Y, μ_1, μ_2) is ij-preparacompact, then so is (X, τ_1, τ_2).

Proof : Let $\Xi = \{U_\lambda : \lambda \in \Gamma\}$ be an $\tau_i - open$ cover of a bitopological space (X, τ_1, τ_2). For each $y \in Y$, Ξ is an $\tau_i - open$ cover of the $\tau_i - compact$ subspace $f^{-1}(y)$ and so there exists a finite subcover Γ_y of Γ such that $f^{-1}(y) \subseteq \bigcup_{\lambda \in \Xi_y} U_\lambda$. Put $U_y = \bigcup_{\lambda \in \Gamma_y} U_\lambda$ is $\tau_i - open$ in (X, τ_i). As f is a $(\tau_i - \mu_i)-$closed function, for each $y \in Y$, there exists $\mu_i - open$ subset V_y of Y such that $y \in V_y$ *and* $f^{-1}(V_y) \subseteq U_y$. Then the collection $\Psi = \{V_y : y \in Y\}$ is an $\mu_i - open$ cover of the ij-preparacompact space (Y, μ_1, μ_2) and so it has a ij-P-locally finite ij-preopen refinement, say $\Omega = \{ W_\gamma : \gamma \in \Delta \}$. Since f is ij-precontinuous , the collection $f^{-1}(\Omega) = \{ f^{-1}(W_\gamma) : \gamma \in \Delta \}$ is a ij-preopen ij-P-locally finite cover of (X, τ_1, τ_2) such that for each $\gamma \in \Delta$, $f^{-1}(W_\gamma) \subseteq U_y$ for some $y \in Y$. Finally , the collection $\{ f^{-1}(W_\gamma) \bigcap U_\lambda : \gamma \in \Delta , \lambda \in \Gamma_y \}$ is a ij-P-locally finite ij-preopen refinement of Ξ. Thus (X, τ_1, τ_2) is ij-preparacompact .

References

[1] M. H. Stone ," Applications of the theory of Boolen rings to general topology ", Trans. Amer. Math. Soc., 41 (1937), 375-381 .

[2] N. Levine ," Semi open sets and continuity in topological spaces ", Amer. Math. Monthly, 70, (1963), 36-41.

[3] S. G. Crossley and S.K. Hildebrand," Semi closure ", Texas J. Sci. , 22 (1971) , 99- 112.

[4] A. S. Mashhour, M.E. Abd EL-Monsef and S.N. EL-Deeb, "On precontinuous and weak precontinuous mappings ", Proc. Math. Phys. Soc. Egypt, 53 (1982), 47-53 .

[5] A. R. Singal and S. P. Arya, "On Pairwise Almost Regular spaces ", Glasnik Math., 6 (1971), 335-343 .

[6] B. Shanta , " Semi open sets , semi continuity and semi open mappings in bitopological spaces ", Bull. Calcatta Math. Sco. , 73(1981), 237-246 .

[7] S. Sampath Kumar, " On a Decomposition of pairwise continuity ", Bull. Cal . Math. Soc. , 28(1997), pp. 441-446.

[8] J. Dieudonne, " Une generalization des spaces compacts ", J. Math. Pures. Appl. 23(1944), 65-76.

[9] Martin M. K. ,"A note on Raghavan – Reilly's Pair wise paracompactness ", Internat. J. Math. And Math. Sci. Vol. 24, No. 2(2001), 139-143 .

[10] K. AL-Zoubi and S. AL-Ghour ," On P3- paracompact spaces ", Int. J. Math. And Math. Sci. Vol. 2007 (2007), Art. ID 80697, PP. 1-16 .

[11] J.C. Kelly, " Bitopological Spaces", Proc. London Math. Soc., 3 (1963), PP. 17-89 .