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A Novel Approach for Model Order Reduction in Discrete Time System

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Abstract: In modern digital control system such as model reference control and model predictive control, where the real time calculation is the main challenge, the model order reduction has become very important issue to minimize the execution time. In this work, our aim is to construct a novel technique for reduction of high order discrete time systems. This could be achieved by computation algorithm model from a given high order pulse transfer function. The proposed model is based on matching the weighting sequence of the original parameters with those adopted in the low-order model. The generalized least squares method is then used to determine the reduced model parameters. The efficiency of the proposed algorithm is validated by using the integral squared error minimization between the original system and the reduced model. An example is presented and discussed to validate the efficiency of the proposed low-order model. Performance comparisons with many recent related works showed that the proposed model is promising in terms of low error indices and time responses.

Keywords: Discrete time systems, Model order reduction, Weighting sequence matching, Pulse transfer function.

1. Introduction

Many large size and complex mathematical models, in real-life operations, show serious challenges in the processes of numerical simulations. The reduction of model order is vital to lower the mathematical model complexity and to decrease its size. Model order reduction receives attention in mathematics community and engineering areas such as control systems [1], electronics [2], and filters in image processing [3].

In electronics, reduced order models are essential in capturing the behavior of complicated electronic systems in the form of small electronic circuits. To achieve better description to the system that contains a circuit and its interconnections, these small circuits are coupled to an existing circuit and co-simulated with it. The complete full order models are mostly unpractical to perform numerical simulations, and then the reduced order models are important to make the full order model feasible in numerical simulations. In the design of control system and in computer simulations, it is vital to use models of low order to minimize the execution time.

The rapid development in engineering sciences made supernumerary research in large-scale systems is vital, and accordingly the mathematical complexity becomes higher. As a result, most computational procedures become more complicated due to increase of the system order. Therefore, simulation of controller design is difficult to be used in case of high order system. Hence, it is essential to represent the complex high order system into satisfactory lower-order model. The aim of the model reduction is to produce a low-order model that can preserve the real properties of the original system as closely as possible.

Many researchers have proposed different approaches for reducing high order models to lower order ones. In shin and Wu [1], a model reduction is performed using continued fraction. Chen and Tsay [4] used the combination of squared magnitude fraction with factorization technique to achieve stable reduced model. The square magnitude Pade approximation has been proposed by Lepschy [5]. The bilinear-transformed domain based on squared-magnitude approximation was used by Hwang and Chow [6, 7] to simplify z-transfer functions. For ensuring the similarity between characteristics of both the simplified model and the high order model, the bilinear transformation is utilized in calculating numerator and the denominator polynomials or is performed on the z-transfer functions.

Hwang and Chow [8] have also investigated the tangent phase Pade approximation for order reduction of pulse transfer function through the bilinear transformation. However this technique is very tedious. The Pade approximation technique, used by Shamash in [9, 10], is very popular among the proposed techniques of model reduction. It is attractive and has significant advantages in terms of computation simplicity over many other related methods. However, there is a major concern in this method that it is unable to obtain stable reduced models even when using stable original system.

Moore et al. [11] developed minimal realization of Kalman's theory in terms of responses of injected signals. In this paper, component analysis is used for analyzing signals. Safonov and Chiang [12] presented the Moore reduction model and proved that the balance of state-space realization is not necessary. This approach showed that the Moore model could be computed without need for projection balancing, which is determined by using the right and left eigenspaces that are related to large eigenvalues. The methods of both Moore and Safonov achieved reduction model with superior stability, but with a drawback of big overshoot and large integral squared error.

In Benner et al. [13], the actual model reduction is achieved by reduction of the stable part, and the technique of state space truncation. The sign function approach is used in all core computational steps. The proposed method efficiency is tested using experiments on Intel Pentium-IV processors. The measurement of the approximation errors and the ways of minimizing these errors are different for different methods of truncated state-space transformations. Gugercin and Antoulas [14] presented a new approach based on compromise between related model reduction systems and their resulting errors. Two proposed methods, for balancing between positive real and frequency weighted are presented. However, this approach has the disadvantage of being computationally complex.

The approximation of large-scale dynamical systems is presented by Antoulas [15], where two families, namely the Krylov-based and SVD-based approximation methods are adopted. The preceding family is based on decomposition of the singular value while the second family is based on moment matching. Since the preceding family has many properties, like an error bound, it is not applicable to be used with high complexity systems. The strength of the second family is that it can be implemented iteratively, and hence it is appropriate to be used with high complexity systems.

Gu [16] investigated the method that is based on approximation of McMillan multivariable transfer. The McMillan degree is set to be smaller than that of discrete-time. The optimal Hankel-norm approximation problem is remedied by using state-space solution with approximations of optimal Hankel-norm. Singh et al. [17] proposed a new method based on factor division and Routh Hurwitz for reduction of model order. For obtaining biased denominators, Routh Hurwitz array and Reciprocal transformation are used. The numerator could be obtained using factor division method. This method obtains stable reduced models, but with large integral time weighted absolute error.

Rozza et al. [18] have used a posteriori error estimation and a hierarchical approximation for linear functional outputs of parameterized elliptic partial differential equations. This method showed interpretative results for convection-diffusion heat conduction, and linear elasticity. The output of the interpretative results includes stress intensity factors, added mass, and transport rates.

The factor division approach has been used by Vishwakarma and Prasad [19] to determine the numerator coefficients and then to produce simple and efficient low order model. The proposed approach is also modified to be applicable for reducing the order of linear stable multivariable system. Dinesh et al. [20] proposed a new approach by combination of Pade approximation methods and pole clustering and these methods are simple and could be oriented computationally. The technique of pole clustering was used to extract lower model denominator. The technique of Pade approximation was utilized to get the numerator coefficients. For a stable original system, this algorithm can generate stable reduced model efficiently.

Kumar et al. [21] proposed uncertain parameters based method for reduction of model order, where the bounds of the uncertain parameters are initially specified. The parameters of the numerator could be obtained using one of the following methods: Pade approximation, moment matching, differentiation, or factor division. The differentiation method is the only suitable method for obtaining the denominator.

Soloklo and farsangi [22] proposed a new approach based on weighted-sum multi-objective approach for reduction of high model. In this approach, Routh–Pade approximation, along with the harmony search is utilized to optimize the parameters of the reduced model. The Routh criterion is used to determine the stability condition, and this condition is adopted to be the constraint in the process of optimization. The drawback of this approach is the very long settling time (53 s).

Huang et al. [23] investigated the setting of the steady-state value using the transfer function matrix and the gain factor of the reduced model. In this work, the balanced truncation method is modified to narrow the deviation of the reduced model. Introducing the gain factor in the lower model does not change the dynamic behavior of the lower model. The defect of this work is that it performs the reduction process with very large integral time weighted absolute error.

Sambariya and Manohar [24] proposed a reduced order approximant using a bat algorithm, where the numerator and denominator coefficients are selected in a way to achieve desired reduced order model. Test results showed that this algorithm is efficient in terms of low error rates compared to the results of Routh-Pade approximation method, but with a defect of long rise time and settling time.

Sikander and Prasad [25, 26] proposed a new technique to simplify high order systems using the technique of single-input single-output and multiple-input multiple-output. This technique is based on applying factor division algorithm and stability equation. The numerator polynomial coefficients are estimated using the approach of factor division, while the denominator polynomial coefficients are calculated using the stability equations. In [26] a modified pole clustering and the algorithm of factor division are used to ensure obtaining satisfactory stable low-order system. In both the proposed and later the modified systems, some examples are presented to prove the superiority of the proposed models compared to the existing recent methods. Anyhow, the drawback associated with these two models is the large integral time weighted absolute error.

Tiwari and Kaur [27] used new indices to determine the dominancy of the transfer function poles efficiently. In this approach poles are selected in such a way that the improved dominant poles could be obtained. The Pade approximation is also adopted to get the numerator polynomial coefficient to the proposed model. Prajapati and Prasad [28] proposed a new approach based on generalized pole clustering for computing the reduced model denominator. The technique of factor division is efficient to extract the numerator polynomial value. In this work, the reduction process includes the translation of all large scale features into the lower order model. This method could preserve the main original system properties in the reduced model, such as initial time moments and the stability.

Tang et al. [29] presented a new system by using absolute nodal coordinate formation with nonlinear finite elements and then followed by local linearized series of quasi-static equilibrium orders. The reduced model is built by using the Craig-Bampton technique which is based on utilizing the incremental displacements of the local linearized system. This method is able to perform with variation of dimension and time which are vital for elaborating and achieving efficient reduction for model order. Anyhow, this method is computationally complex and has long settling time.

Zhu et al. [30] investigated optimization of parameters using the approach of chaotic particle swarm optimization (CPSO) to enhance the performance of vehicle electrical systems. This work adopted CPSO to select optimum power system parameters of the vehicle. The Cruise software was used to carry out performance simulations, and a comparison for the results before and after the optimization was carried out, and showed great improvement in the dynamic performance of vehicles driving range.

Pady et al. [31] presented a combined approach to obtain single-input-single-output model using modified Routh approximation. This approach is followed by matching Markov parameters and then the time moments of interval system for reduced and higher order model. Sun et al. [32] proposed a new model for reducing the linear periodic time-varying systems. In this model, the technique of state space realization has been applied in the form of Fourier-lifted. This could be achieved by using exponentially modulated periodic matrices. The resulting Fourier-lifted system is then expanded by using Laguerre functions.

Abdulla [33] proposed a developed Chaotic Particle Swarm Optimization to accomplish efficient reduced model from a large scale model and to develop a Linear Quadratic Regulator based controller. The modified model is based on combining advantages of Particle Swarm Optimization (PSO) and the Chaotic PSO. This model has the properties of being simple execution, fast convergence, few control parameters, and able to avoid probable local extremes.

The above survey shows that each of the existing proposed methods, in the literature, has important advantages and some limitations. General concerns about most of their limitations are the tedious computational procedures and maintaining the stability of the original high order system.

In this work, we state the following model reduction problem giving the impulse response data for a high order discrete time system of order n. A lower model of order m (m<n) is obtained and the reduced model response is sufficiently very close to that of the original high order system. Our aim, of this work, is to construct an algorithm to obtain stable reduced model from stable original system and to maintain initial time moments of the original system. The model of high order pulse transfer function is performed through two stages. First, the weighting sequence of the original model is computed and then matched with the weighting sequence of the reduced model. Second, a set of linear equations are analyzed using the generalized least square method.

The structure of this article follows the following stages: Section 2 presents the proposed algorithm for model reduction of pulse transfer function. Section 3 introduces the procedure steps of the proposed model. In section 4 an example and simulation results has been presented. Section 5 is associated with the performance analysis to present the goodness and feasibility of the proposed model and finally in Section 5, our conclusions have been presented.

2. Model reduction of pulse transfer function

Consider a high order sampled data system with the following pulse transfer function of order n;

$$G_n(z^{-1}) = \frac{a_0 + a_1 z^{-1} + \ldots + a_n z^{-n}}{1 + b_1 z^{-1} + \ldots + b_n z^{-n}} \qquad (1)$$

where $a_0, a_1, \ldots a_n, b_0, b_1, \ldots . b_n$ are defined as coefficients of the discrete time system and z is the z-transfer variable.

Assume the pulses of the system are included in the z-plane unit circle. The problem is how to obtain the model of order $m<n$ having a pulse transfer function $G_m(z^{-1})$ and given by;

$$G_m(z^{-1}) = \frac{\alpha_0 + \alpha_1 z^{-1} + \ldots + \alpha_m z^{-m}}{1 + \beta_1 z^{-1} + \ldots + \beta_m z^{-m}} \quad (m<n) \qquad (2)$$

which approximate the original system, of Eq. (1).

The problem leads to think of how we can find the coefficients of the reduced mode which are;

$$\alpha_0, \ \alpha_1, \ldots \alpha_m \text{ and } \beta_1, \beta_2 \ldots \beta_m$$

Let $g_n(kT)$ be the impulse response function or weighting function of the system denotes;

$$g_n(KT) = h_K \qquad (k=0, 1, 2, \ldots p) \qquad (3)$$

where h_K refers to the impulse response function of the original system at sampling time k. From the definition of Z-transform, the pulse transfer function of the high order system can be expressed as;

$$G_n(z^{-1}) = \sum_{k=0}^{p} g(kT) z^{-k} \quad (p > 2n)$$

or

$$G_n(z^{-1}) = \sum_{k=0}^{p} h_k z^{-k} \qquad (p > 2n) \qquad (4)$$

The system, of Eq. (1), can be expanded as a weighting sequence $\{h_i, i=0, 1, 2, \ldots, p\}$ as follows;

$$G_n\,(z^{-1}) = h_0 + h_1 z^{-1} + \cdots + h_p z^{-p} \qquad (5)$$

To match the weighting sequence of the original system, represented by Eq. (1), with the that of the model of Eq. (2), it is necessary that;

$$\frac{\alpha_0 + \alpha_1 z^{-1} + \cdots + \alpha_m z^{-m}}{1 + \beta_1 z^{-1} + \cdots + \beta_m z^{-m}} = h_0 + h_1 z^{-1} + \cdots + h_p z^{-p} \qquad (6)$$

Therefore;

$$\alpha_0 + \alpha_1 z^{-1} + \cdots + \alpha_m z^{-m} = (h_0 + h_1 z^{-1} + \ldots + h_p z^{-p})(1 + \beta_1 z^{-1} + \cdots + \beta_m z^{-m}) \qquad (7)$$

Expanding Eq. (7), and equating the coefficients of equal powers of z in both sides of Eq. (7), we can obtain the following linear equations;

$$\begin{aligned} \alpha_0 &= h_0 \beta_0 \\ \alpha_1 &= h_1 \beta_0 + h_0 \beta_1 \\ \alpha_2 &= h_2 \beta_0 + h_1 \beta_1 + h_0 \beta_2 \\ &\cdot \\ &\cdot \\ &\cdot \\ \alpha_m &= h_m \beta_0 + h_{m-1} \beta_1 + h_{m-2} \beta_2 + \cdots + h_0 \beta_m \end{aligned} \qquad (8)$$

Or the following linear algebraic equations can be obtained as follows;

$$\alpha_i = \sum_{j=0}^{i} \beta_j h_{i-j} \qquad (i=0,1, 2,\ldots, m) \qquad (9)$$

Noting that; $\beta_0 = 1$, Eq. (9) can be written in vector matrix form;

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$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix} = \begin{bmatrix} h_0 & & & \\ h_1 & h_0 & & \\ h_2 & h_1 & h_0 & \\ \vdots & \vdots & \vdots & \\ h_m & h_{m-1} & \dots & h_0 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_m \end{bmatrix} \quad (10)$$

Using same mathematical manipulation, we obtain the following linear equations;

$$\begin{aligned} h_m\beta_1 + h_{m-1}\beta_2 + \cdots + h_1\beta_m &= -h_{m+1} \\ h_{m+1}\beta_1 + h_m\beta_2 + \cdots + h_2\beta_m &= -h_{m+2} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ h_{p-1}\beta_1 + h_{p-2}\beta_2 + \cdots + h_{p-m}\beta_m &= -h_p \end{aligned} \quad (11)$$

or in the vector-matrix form;

$$\begin{bmatrix} h_m & h_{m-1} & h_{m-2} & \dots h_1 \\ h_{m+1} & h_m & h_{m-1} & \dots h_2 \\ h_{m+2} & h_{m+1} & h_m & \dots h_3 \\ \vdots & \vdots & \vdots & \dots \vdots \\ h_{p-1} & h_{p-2} & h_{p-3} & \dots h_{p-m} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_m \end{bmatrix} = \begin{bmatrix} -h_{m+1} \\ -h_{m+2} \\ -h_{m+3} \\ \vdots \\ -h_p \end{bmatrix}$$

$$[H] \qquad [\beta] = [h] \quad (12)$$

Eq. (12) can be solved for the β-parameters using the generalized least square method. The least square solution of Eq. (12) is:

$$\beta = [H^T H]^{-1} H^T h \quad (13)$$

where "T" refers to a matrix transpose. Having the reduced model denominator, the parameters of the numerator, α-parameters can be computed using Eq. (9) or Eq. (10).

3. Proposed algorithm for model reduction

The reduction procedure can be easily summarized and presented as follows:

Step 1: Compute the weighting sequence of the original system, Eq. (1),

$$\{h_i,\ i=0, 1\ 2,\dots, p\}, \qquad (p>2n)$$

Step 2: Construct the matrices H and h using Eq. (12).

Step 3: Compute the β-parameters of the reduced model using Eq. (13).

Step 4: Compute the α-parameters of the reduced model using Eq. (9) or Eq. (10).

Step 5: Obtain the reduced model $G_m(z^{-1})$ using Eq. (2).

4. Numerical example

To clarify the application feasibility of the proposed reduction procedure, it is convenient to take the pulse transfer function of 4th order system as follows:

$$G_4(z) = \frac{0.07844z^3 - 0.1556z^2 + 0.1042z - 0.02388}{z^4 - 2.698z^3 + 2.643z^2 - 1.106z + 0.1653} \quad (14)$$

Our aim is to get the second order discrete time model with the following form:

$$\hat{G}_2(z^{-1}) = \frac{\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}{1 + \beta_1 z^{-1} + \beta_2 z^{-2}} \quad (15)$$

Let the impulse response of the system, represented in Eq. (14), to be:

$$h_k = g(kT)\,;\, k = 0, 1, 2 \dots, p \quad (16)$$

The impulse response data of the system of Eq. (16), for $p = 20$ and sampling period $T = 1\,sec$, are:

$h_k =$ 0.0000, 0.0784, 0.0560, 0.0481, 0.0444, 0.0419, 0.0394, 0.0369, 0.0344, 0.0318, 0.0293, 0.0269, 0.0247, 0.0225, 0.0206, 0.0188, 0.0177, 0.0155, 0.0128, 0.0128, 0.0116

The constructions of the matrices H, and h are:

$$H = \begin{bmatrix} 0.0560 & 0.0481 & 0.0444 & 0.0419 & 0.0394 \\ 0.0784 & 0.0560 & 0.0481 & 0.0444 & 0.0490 \end{bmatrix}$$

$$\begin{matrix} 0.0369 & 0.0344 & 0.0318 & 0.0293 & 0.0269 \\ 0.0394 & 0.0369 & 0.0344 & 0.0318 & 0.0293 \end{matrix}$$

$$\begin{matrix} 0.0247 & 0.0225 & 0.0206 & 0.0188 & 0.0171 \\ 0.0269 & 0.0247 & 0.0225 & 0.0206 & 0.0188 \end{matrix}$$

$$\begin{matrix} 0.0155 & 0.0141 & 0.0128 \\ 0.0171 & 0.0155 & 0.0128 \end{matrix}]^T$$

and

$$h = [\ \begin{matrix} 0.0481 & 0.0444 & 0.0419 & 0.0394 \\ 0.0369 & 0.0344 & 0.0318 & 0.0293 \\ 0.0269 & 0.0247 & 0.0225 & 0.0206 \\ 0.0188 & 0.0177 & 0.0155 & 0.0128 \\ 0.0128 & 0.0116 & & \end{matrix}\ ]^T$$

Since

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$$\alpha = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} \quad (17)$$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \quad (18)$$

Then, the β- parameters of the second order model using Eq. (14), are:

$$\beta = \begin{bmatrix} -1.0929 \\ 0.1588 \end{bmatrix}$$

where, $\beta_1 = -1.0929,$ and $\beta_2 = 0.1588.$

Then, the α- parameters of the model is computed as:

$$\alpha = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 \\ h_1 & h_0 & 0 \\ h_2 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0.0784 & 0 & 0 \\ 0.056 & 0.0784 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1.0929 \\ 0.1588 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.0784 \\ -0.0297 \end{bmatrix}$$

where, $\alpha_0 = 0, \alpha_1 = 0.0784, \alpha_2 = -0.0297.$

The resulting second order discrete model of the system of Eq. (15) is:

$$\widehat{G}_2(z^{-1}) = \frac{0.0784z^{-1} - 0.0297z^{-2}}{1 - 1.0929z^{-1} + 0.1588z^{-2}} \quad (19)$$

or

$$\widehat{G}_2(z) = \frac{0.0784z - 0.0297}{z^2 - 1.0929z + 0.1588} \quad (20)$$

A second order model using the proposed method is given as follows:

$$G_2(z) = \frac{0.0448z - 0.03733}{z^2 - 1.822z + 0.8327} \quad (21)$$

5. Performance analysis

Evaluation of the proposed reduction system can be realized by calculating its performance indices and time responses and then comparing them with those of the existing related works. With respect to the performance indices, integral square error (ISE) is calculated for the transient regions of the original system and reduced order model. Integral absolute error (IAE) and integral time weighted absolute error (ITAE) are also calculated to test the goodness of the proposed reduced models approach. Smaller values of ISE, IAE, and ITAE demonstrate that the reduced models are efficient and are closer to the original system.

$$ISE = \int_0^\infty [y(t) - y_r(t)]^2 dt \quad (22)$$

$$IAE = \int_0^\infty |y(t) - y_r(t)| dt \quad (23)$$

$$ITAE = \int_0^\infty t|y(t) - y_r(t)| dt \quad (24)$$

where $y(t)$ and $y_r(t)$ refer to step responses of the original system and the reduced order model respectively.

To verify the robustness of the reduced model, the time responses of the original system and the reduced model are compared to test their proximity. The closer step responses and fast impulse responses are essential evidences of superiority of the proposed model. Lowest values of the overshoot percentage, rise time, and settling time are the prominent signs of reduction approach efficiency. Moreover, calculating the error between the system step response and the model step response is vital in the evaluation of the proposed model.

The error $e(kT)=y(kT)-\hat{y}(kT$, between the step response of the original system $y(kT)$ and the step response of the reduced model $\hat{y}$(kT) is computed for $k=0, 1, 2,\ldots,47$. Fig. 1 shows the error $e(kT)$, and the calculated averaged square error is found to be 1.5643×10^{-4}, which is very small. The performance with respect to error indices for the proposed reduced models and the original system are presented in Table 1. This table is associated with quantitative comparisons between the proposed method and various related works with respect to the ISE, IAE, and ITAE.

Looking into the error indices comparisons, in

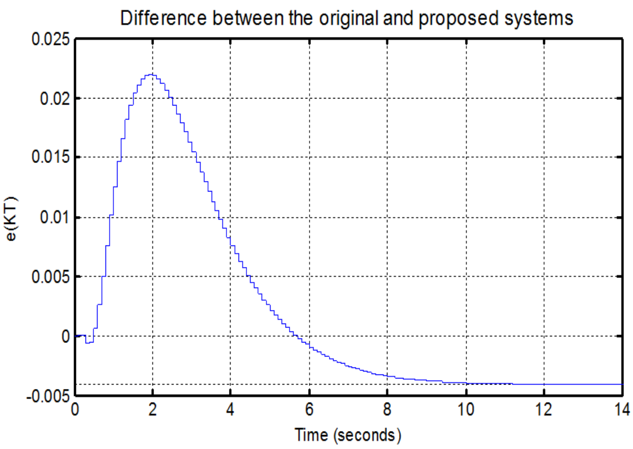

Figure. 1 The error $e(kT)$ between the original system output and the proposed model output

Table 1. Quantitative comparisons of different methods in terms of error indices

Author of reduction method	Reduced order model	Performance Index		
		ISE	IAE	ITAE
Safonov and Chiang (1989) [12]	$\frac{0.0961s + 0.0042}{s^2 + 0.1342s + 0.0046}$	2.3754	40.2052	2777.9
Gu (2005) [16]	$\frac{0.0492s + 0.0896}{s^2 + 0.9811s + 0.09526}$	2.4924	47.8283	2741.6
Singh et al. (2006) [17]	$\frac{5.6402s + 1}{87.3752s^2 + 15.9402s + 1}$	0.7712	18.1392	429.4817
Vishwakarma and Prasad (2008) [19]	$\frac{-0.5076s + 0.1209}{s^2 + 0.7377s + 0.1209}$	12.653	52.0168	689.369
Kumar et al. (2013) [21]	$\frac{1512s + 360}{2458s^2 + 2196s + 360}$	12.0636	49.9090	657.9140
Huang (2013) [23]	$\frac{0.0961s + 0.0046}{s^2 + 0.1342s + 0.0046}$	0.6091	20.4007	772.3096
Sikander and Prasad (2015) [25]	$\frac{8s + 1}{101.0101s^2 + 18.3s + 1}$	0.2506	10.4825	263.2632
Sikander and Prasad (2017) [26]	$\frac{0.0105s + 0.0404}{s^2 + 0.4266s + 0.0404}$	1.0487	17.5677	305.0096
Tiwari and Kaur (2018) [27]	$\frac{0.4809s + 0.6369}{s^2 + 6.1070s + 0.6393}$	0.2700	8.0013	136.8471
Prajapati and Prasad (2019) [28]	$\frac{0.0913s + 0.0209}{s^2 + 0.3066s + 0.0209}$	0.0210	2.2612	37.7357
Proposed method	$\frac{0.0448z - 0.03733}{z^2 - 1.822z + 0.8327}$	0.0094	2.6335	38.3224

Table 1, one can easily notice that the proposed reduction model could obtain the lowest ISE among the recent and well-known related works. With respect to IAE and ITAE, the comparisons illustrate competitive performance of the proposed model compared with that of the best related researches. The reason behind the superiority of low error indices among the related works is that the proposed model response is much closer to that of the original model compared to those of the other techniques.

Fig. 2 presents the step responses of the original system and the proposed model, and these responses show that they are approximately matched to each other. Fig. 3 shows the impulse response of the original system and its proposed model, and it also clearly shows that the impulse response of the reduced model is following the step response of the higher model. The closeness between the step response and impulse response of both reduced model and higher model demonstrates that the proposed reduction method is efficient to simplify and reflect the properties of the higher model.

To evaluate the performance of the proposed model against other recent related works, in terms of time response, a comparison between the proposed model and many other related methods is presented in Table 2. The comparison shows that the proposed model could produce reduced model with lowest over-shoot percentage (zero) compared with those of the recent related methods. The settling time and

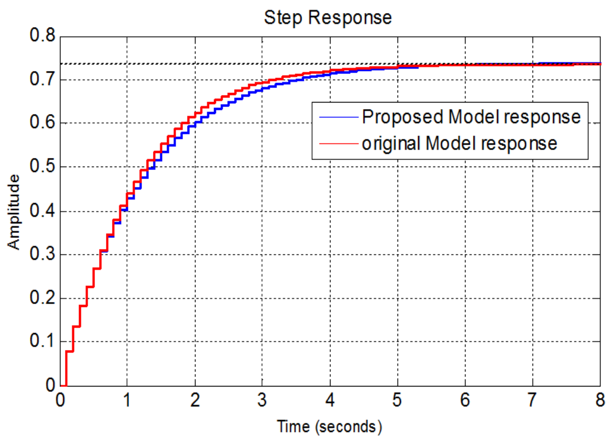

Figure. 2 Step response of the original system and the reduced model

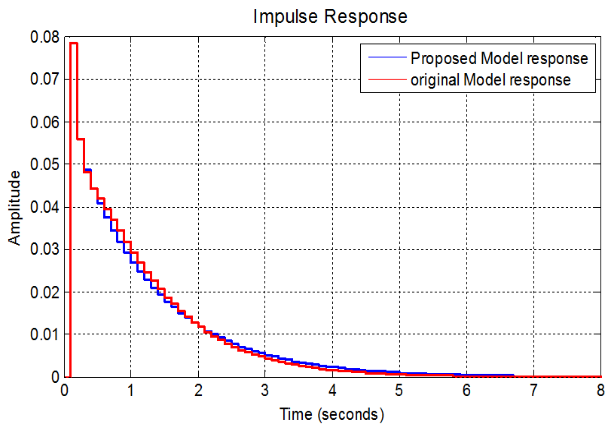

Figure. 3 Impulse response of the original system and its proposed model

Table 2. Quantitative comparisons of different methods in terms of overshoot percentage and time responses

Authors of reduction methods	Over-shoot %	Rise time(s)	Settling time (s)
Shamash [9]	10	0.206	0.892
Moore [11]	17.4	0.2116	0.8381
Gu [16]	16.5	0.217	0.869
Soloklo and Farsangi [22]	7.6	14.3	53
Huang et al. [23]	18	0.207	1.145
Sambariya & Manhar [24]	0	22.99	40.93
Sikander and Prasad [25]	7	2.3	3.41
Tiwari and Kaur [27]	0	3.3	4.04
Prajapati and Prasad [28]	5.6	0.203	0.933
Proposed method	0	2.8	4.4

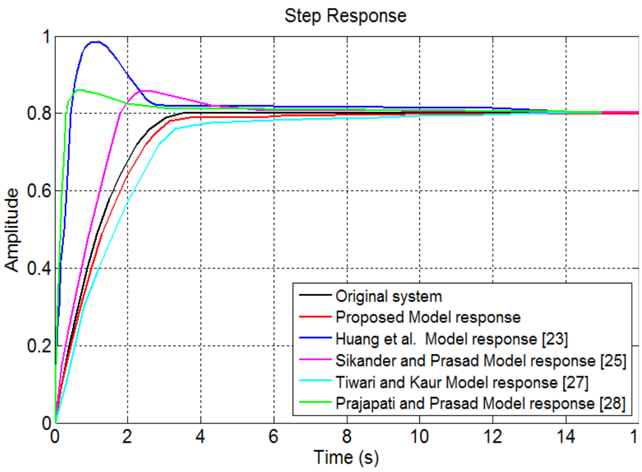

Figure 4: Comparisons of step responses between high-order model and low-order models for the proposed model and previous related works.

rise time are competitive compared with those who have obtained the lowest overshoot percentage.

It is obvious that the proposed method performance error indices is least for ISE and competitive for both IAE and ITAE. It is also clear that the proposed reduction approach obtains superior performance results compared to some robust model reduction methods. From table 1 and table 2, it can be noticed that it is not easy to find a method with superior performance for all performance criteria i.e. the time responses and the error indices. Therefore a compromise between assigned performance criteria should be considered in the evaluation of the model performance.

To demonstrate the goodness of the proposed reduction method, the step response of the proposed model is compared to those of many well-known related works as illustrated in Fig. 3. This figure clearly shows that the reduced model response is following the response of the original model. Looking into the response trajectories in Fig. 3, we can notice that even for those methods of fast rise time, they suffer of high over-shoot percentage which affects the settling time. From the comparisons in Table 2 and Fig. 4, it is observed that the proposed model performance outperforms the performance results of most recent related works. In addition, it ensures the preservation of initial few time moments of the complete order system in the reduced order model.

6. Conclusions

In this work, order reduction for high order sampled data systems is proposed. The analysis of the numerical example shows that the proposed method is an efficient approach for model order reduction. It is computationally simple and can ensure stable reduced model for the stable full order model. It has fast convergence and is very simple implementation compared with existing related works. From numerical example results of the proposed model, we can state the following properties: First, it preserves model stability. Second, it preserves the time domain characteristics. Third, the approximation error is small. Fourth, it is easily adaptable for digital computation. Extension of the proposed procedure to multivariable systems is possible.

Conflicts of Interest

The authors confirm that there is no conflict of interest in this paper.

Author Contributions

The conceptualization was from Ali Hasan, Hussain Jaafar, and Mahmoud Shaker. The methodology was organized by Hasan. The software, by Hasan and Jaafar. The validation, by Hasan and Shaker. The formal analysis, by Jaafar and Shaker. The resources, by Shaker. The data curation, by Hasan and Shaker. Writing—original draft preparation, by Jaafar. Writing—review and editing, by Shaker. Visualization, by Hasan and Shaker. Supervision, by Jaafar. Project administration, by Jaafar. funding acquisition, by Hasan, Jaafar, and Shaker.

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