On New Concepts of Weakly Neutrosophic Continuous Functions

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Abstract

In this paper, we intend to utilize the ideas of Neu^{α}-open and $Neu^{S\alpha}$-open sets to distinguish several novel conceptions of weakly neutrosophic continuous functions, for instance; Neu^{α^*}-continuous, $Neu^{\alpha^{**}}$-continuous, $Neu^{S\alpha}$-continuous, $Neu^{S\alpha^*}$-continuous and $Neu^{S\alpha^{**}}$-continuous functions. Moreover, we will describe the interactions among these thoughts of weakly neutrosophic continuous functions.
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Keywords: $Neu^{S\alpha}$-open set, Neu^{α}-continuous, Neu^{α^*}-continuous, $Neu^{\alpha^{**}}$-continuous, $Neu^{S\alpha}$-continuous, $Neu^{S\alpha^*}$-continuous and $Neu^{S\alpha^{**}}$-continuous functions.

1. Introduction

F. Smarandache [3,4] initially presented the idea of a "neutrosophic set". A. A. Salama et al. [1] introduced the principles of neutrosophic topological space (fleetingly, Neu^{TS}). Q. H. Imran et al. [6] stated that a class of $Neu^{S\alpha}$-open sets in neutrosophic topological spaces. A. A. Salama et al. [2] submitted the notion of neutrosophic continuous mappings. Q. H. Imran et al. [7] established and examined the sense of continuity in neutrosophic generalized alpha generalized. The concentration of this article is to demonstrate pioneering perceptions of weakly neutrosophic continuous functions, for example; Neu^{α^*}-continuous, $Neu^{\alpha^{**}}$-continuous, $Neu^{S\alpha}$-continuous, $Neu^{S\alpha^*}$-continuous and $Neu^{S\alpha^{**}}$-continuous functions. Additionally, we have in mind to rationalize the associations among these thoughts of weakly neutrosophic continuous functions.

2. Preliminaries

In this article, $(\mathcal{M},\tau)$, $(\mathcal{N},\sigma)$, and $(\mathcal{O},\rho)$ (or simply $\mathcal{M},\mathcal{N}$, and $\mathcal{O}$) always mean Neu^{TSs}. A neutrosophic closed set (briefly Neu-closed set) in $(\mathcal{M},\tau)$ is a complement of a neutrosophic open set (briefly Neu-open set). Let neutrosophic set $\mathcal{D}$ be in a Neu^{TS} $(\mathcal{M},\tau)$, then the neutrosophic closure of $\mathcal{D}$, the neutrosophic interior of $\mathcal{D}$, and the neutrosophic complement of $\mathcal{D}$ symbolize by $NeuCl(\mathcal{D})$, $NeuInt(\mathcal{D})$ and $\mathcal{D}^c$, correspondingly.

Definition 2.1:
Assume $\mathcal{D}$ is a neutrosophic subset of a $Neu^{TS}(\mathcal{M},\tau)$, then it is named as:

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(1) A neutrosophic α-open set (in brief Neu^{α}-open set) [5] if $\mathcal{D} \subseteq NeuInt(NeuCl(NeuInt(\mathcal{D})))$. The collection of every Neu^{α}-open set of $\mathcal{M}$ is symbolized by $Neu^{\alpha}O(\mathcal{M})$. A neutrosophic α-closed set (in short Neu^{α}-closed set) is a complement of Neu^{α}-open set.
(2) A neutrosophic semi-α-open set (in short $Neu^{S\alpha}$-open set) [6] if for any a Neu^{α}-open set $\mathcal{P}$ in $\mathcal{M}$ where $\mathcal{P} \subseteq \mathcal{D} \subseteq NeuCl(\mathcal{P})$ or equivalently if $\mathcal{D} \subseteq NeuCl(NeuInt(NeuCl(NeuInt(\mathcal{D}))))$. The collection of every $Neu^{S\alpha}$-open set of $\mathcal{M}$ is symbolized by $Neu^{S\alpha}O(\mathcal{M})$. A neutrosophic semi-α-closed set (in short $Neu^{S\alpha}$-closed set) is a complement of $Neu^{S\alpha}$-open set.

Remark 2.2 [6]:
In a Neu^{TS} $(\mathcal{M}, \tau)$, then the succeeding arguments stand, and the opposite of every argument does not hold:
(1) Every Neu-open set is a Neu^{α}-open and $Neu^{S\alpha}$-open.
(2) Every Neu^{α}-open set is a $Neu^{S\alpha}$-open.

Definition 2.3 [2]:
Assume $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ is a function, then $\hbar$ is stated to be neutrosophic continuous (in short Neu-continuous) iff for each $\mathcal{D}$ Neu-open set in $\mathcal{N}$, then $\hbar^{-1}(\mathcal{D})$ is a Neu-open set in $\mathcal{M}$.

Definition 2.4 [5]:
Assume $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ is a function, then $\hbar$ is stated to be neutrosophic α-continuous (in short Neu^{α}-continuous) iff for each $\mathcal{D}$ Neu-open set in $\mathcal{N}$, then $\hbar^{-1}(\mathcal{D})$ is a Neu^{α}-open set in $\mathcal{M}$.

Theorem 2.5 [2]:
A function $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ is Neu-continuous iff $\hbar^{-1}(NeuInt(\mathcal{D})) \subseteq NeuInt(\hbar^{-1}(\mathcal{D}))$ for each $\mathcal{D} \subseteq \mathcal{N}$.

Definition 2.6 [2]:
Let $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ be a function, then $\hbar$ is stated to be neutrosophic open (in short Neu-open) iff for each $\mathcal{D}$ Neu-open subset in $\mathcal{M}$, then $\hbar(\mathcal{D})$ is a Neu-open subset in $\mathcal{N}$.

3. Concepts of Weakly Neutrosophic Continuous Functions

Definition 3.1:
Assume $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ is a function, then $\hbar$ is stated to be:
(1) Neutrosophic α*-continuous (in short Neu^{α^*}-continuous) iff for each $\mathcal{D}$ Neu^{α}-open subset in $\mathcal{N}$, then $\hbar^{-1}(\mathcal{D})$ is a Neu^{α}-open subset in $\mathcal{M}$.
(2) Neutrosophic α**-continuous (in short $Neu^{\alpha^{**}}$-continuous) iff for each $\mathcal{D}$ Neu^{α}-open subset in $\mathcal{N}$, then $\hbar^{-1}(\mathcal{D})$ is a Neu-open subset in $\mathcal{M}$.

Definition 3.2:
Assume $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ is a function, then $\hbar$ is stated to be:
(1) Neutrosophic semi-α-continuous (in short $Neu^{S\alpha}$-continuous) iff for each $\mathcal{D}$ Neu-open set in $\mathcal{N}$, then $\hbar^{-1}(\mathcal{D})$ is a $Neu^{S\alpha}$-open set in $\mathcal{M}$.
(2) Neutrosophic semi-α*-continuous (in short $Neu^{S\alpha^*}$-continuous) iff for each $\mathcal{D}$ $Neu^{S\alpha}$-open set in $\mathcal{N}$, then $\hbar^{-1}(\mathcal{D})$ is a $Neu^{S\alpha}$-open set in $\mathcal{M}$.
(3) Neutrosophic semi-α**-continuous (in short $Neu^{S\alpha^{**}}$-continuous) iff for each $\mathcal{D}$ $Neu^{S\alpha}$-open set in $\mathcal{N}$, then $\hbar^{-1}(\mathcal{D})$ is a Neu-open set in $\mathcal{M}$.

Theorem 3.3:
Assume $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ is a function. Then the succeeding arguments are the same:
(1) $\hbar$ is a $Neu^{S\alpha}$-continuous.
(2) The reverse image of each Neu-closed subset in $\mathcal{N}$ is $Neu^{S\alpha}$-closed subset in $\mathcal{M}$.
(3) $\hbar(NeuInt(NeuCl(NeuInt(NeuCl(\mathcal{C}))))) \subseteq NeuCl(\hbar(\mathcal{C}))$, for each $\mathcal{C} \in \mathcal{M}$.
(4) $NeuInt(NeuCl(NeuInt(NeuCl(\hbar^{-1}(\mathcal{D}))))) \subseteq \hbar^{-1}(NeuCl(\mathcal{D}))$, for every $\mathcal{D} \in \mathcal{N}$.

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Proof:

(1) $\Longrightarrow$ (2). Assume $\mathcal{D}$ is a Neu-closed set in $\mathcal{N}$. It indicates that $\mathcal{D}^c$ is a Neu-open set. Consequently, $\hbar^{-1}(\mathcal{D}^c)$ is a $Neu^{S\alpha}$-open set in $\mathcal{M}$. It means $(\hbar^{-1}(\mathcal{D}))^c$ is a $Neu^{S\alpha}$-open set in $\mathcal{M}$. So, $\hbar^{-1}(\mathcal{D})$ is a $Neu^{S\alpha}$-closed set in $\mathcal{M}$.

(2) $\Longrightarrow$ (3). Assume $\mathcal{C} \in \mathcal{M}$, and we have $NeuCl(\hbar(\mathcal{C}))$ is a Neu-closed subset in $\mathcal{N}$. Hence, $\hbar^{-1}(NeuCl(\hbar(\mathcal{C})))$ is $Neu^{S\alpha}$-closed set in $\mathcal{M}$.

Consequently, we have $\hbar^{-1}(NeuCl(\hbar(\mathcal{C}))) \supseteq NeuInt(NeuCl(NeuInt(NeuCl(\hbar^{-1}(NeuCl(\hbar(\mathcal{C}))))))) \supseteq NeuInt(NeuCl(NeuInt(NeuCl(\mathcal{C}))))$. Or $NeuCl(\hbar(\mathcal{C}))) \supseteq \hbar(NeuInt(NeuCl(NeuInt(NeuCl(\mathcal{C})))))$.

(3) $\Longrightarrow$ (4). Assume $\mathcal{D} \in \mathcal{N}$, $\hbar^{-1}(\mathcal{D}) \in \mathcal{M}$. Therefore, by the hypothesis, we obtain $NeuInt(NeuCl(NeuInt(NeuCl(\hbar^{-1}(\mathcal{D}))))) \subseteq NeuCl(\hbar(\hbar^{-1}(\mathcal{D}))) \subseteq NeuCl(\mathcal{D})$, that is $NeuInt(NeuCl(NeuInt(NeuCl(\hbar^{-1}(\mathcal{D}))))) \subseteq \hbar^{-1}(NeuCl(\mathcal{D}))$.

(4) $\Longrightarrow$ (1). Assume $\mathcal{C}$ is a Neu-open subset of $\mathcal{N}$. Suppose $\mathcal{D} = \mathcal{C}^c$ and $\mathcal{C} = \hbar^{-1}(\mathcal{D})$; so by (iii), we have $NeuInt(NeuCl(NeuInt(NeuCl(\hbar^{-1}(\mathcal{D}))))) \subseteq NeuCl(\mathcal{D}) = \mathcal{D}$. It means that the following fact holds $NeuInt(NeuCl(NeuInt(NeuCl(\hbar^{-1}(\mathcal{C}^c))))) \subseteq \hbar^{-1}(\mathcal{C}^c)$. Or $NeuInt(NeuCl(NeuInt(NeuCl(\hbar^{-1}(\mathcal{C}))))) \supseteq \hbar^{-1}(\mathcal{C})$. Consequently, $\hbar^{-1}(\mathcal{C})$ is a $Neu^{S\alpha}$-open set in $\mathcal{M}$ and therefore $\hbar$ is a $Neu^{S\alpha}$-continuous.

Proposition 3.4:

(1) Every Neu-continuous is a Neu^{α}-continuous; as a result, it is $Neu^{S\alpha}$-continuous. Nonetheless, the reverse does not stand.

(2) Every Neu^{α}-continuous is a $Neu^{S\alpha}$-continuous. Nevertheless, the contrary does not stand.

Proof:

(1) Let $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ be a Neu-continuous function and a Neu-open subset $\mathcal{D}$ be in $\mathcal{N}$. Then $\hbar^{-1}(\mathcal{D})$ is a Neu-open subset in $\mathcal{M}$. Since any Neu-open set is Neu^{α}-open ($Neu^{S\alpha}$-open), $\hbar^{-1}(\mathcal{D})$ is a Neu^{α}-open ($Neu^{S\alpha}$-open) set in $\mathcal{M}$. Thus $\hbar$ is a Neu^{α}-continuous ($Neu^{S\alpha}$-continuous).

(2) Let $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{N}, \sigma)$ be a Neu^{α}-continuous function and a Neu-open set $\mathcal{D}$ be in $\mathcal{N}$. Then $\hbar^{-1}(\mathcal{D})$ is a Neu^{α}-open set in $\mathcal{M}$. Since any Neu^{α}-open set is $Neu^{S\alpha}$-open, $\hbar^{-1}(\mathcal{D})$ is a $Neu^{S\alpha}$-open set in $\mathcal{M}$. Thus $\hbar$ is a $Neu^{S\alpha}$-continuous.

Example 3.5:

Let $\mathcal{M} = \{p, q\}$. Describe the neutrosophic subsets $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{D}$ in $\mathcal{M}$ in this manner:

$\mathcal{A} = \langle m, \left(\frac{p}{0.5}, \frac{q}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.7}\right)\rangle$, $\mathcal{B} = \langle m, \left(\frac{p}{0.5}, \frac{q}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}\right)\rangle$,

$\mathcal{C} = \langle m, \left(\frac{p}{0.6}, \frac{q}{0.3}\right), \left(\frac{p}{0.6}, \frac{q}{0.3}\right), \left(\frac{p}{0.4}, \frac{q}{0.7}\right)\rangle$ and $\mathcal{D} = \langle m, \left(\frac{p}{0.6}, \frac{q}{0.7}\right), \left(\frac{p}{0.6}, \frac{q}{0.7}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}\right)\rangle$.

Then the families $\tau = \{0_N, \mathcal{A}, 1_N\}$ and $\sigma = \{0_N, \mathcal{D}, 1_N\}$ are neutrosophic topologies on $\mathcal{M}$. Thus, $(\mathcal{M}, \tau)$ and $(\mathcal{M}, \sigma)$ are Neu^{TSs}. Define $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{M}, \sigma)$ as $\hbar(p) = p$, $\hbar(q) = q$. Then $\mathcal{D}$ is a Neu-open in $(\mathcal{M}, \sigma)$. But, $\hbar^{-1}(\mathcal{D})$ is not a Neu-open subset in $(\mathcal{M}, \tau)$ for $\mathcal{D} \in \sigma$. Hence, $\hbar$ is Neu^{α}-continuous ($Neu^{S\alpha}$-continuous); however, it is not a Neu-continuous.

Example 3.6:

Let $\mathcal{M} = \{q, r\}$. Describe the neutrosophic subsets $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{D}$ in $\mathcal{M}$ in this manner:

$\mathcal{A} = \langle m, \left(\frac{q}{0.5}, \frac{r}{0.3}\right), \left(\frac{q}{0.5}, \frac{r}{0.3}\right), \left(\frac{q}{0.5}, \frac{r}{0.7}\right)\rangle$, $\mathcal{B} = \langle m, \left(\frac{q}{0.5}, \frac{r}{0.6}\right), \left(\frac{q}{0.5}, \frac{r}{0.6}\right), \left(\frac{q}{0.5}, \frac{r}{0.4}\right)\rangle$,

$\mathcal{C} = \langle m, \left(\frac{q}{0.6}, \frac{r}{0.3}\right), \left(\frac{q}{0.6}, \frac{r}{0.3}\right), \left(\frac{q}{0.4}, \frac{r}{0.7}\right)\rangle$ and $\mathcal{D} = \langle m, \left(\frac{q}{0.6}, \frac{r}{0.7}\right), \left(\frac{q}{0.6}, \frac{r}{0.7}\right), \left(\frac{q}{0.4}, \frac{r}{0.3}\right)\rangle$.

Then the families $\tau = \{0_N, \mathcal{A}, 1_N\}$ and $\sigma = \{0_N, \mathcal{B}, 1_N\}$ are neutrosophic topologies on $\mathcal{M}$. Thus, $(\mathcal{M}, \tau)$ and $(\mathcal{M}, \sigma)$ are Neu^{TSs}. Define $\hbar: (\mathcal{M}, \tau) \longrightarrow (\mathcal{M}, \sigma)$ as $\hbar(q) = q$, $\hbar(r) = r$. Then $\mathcal{B}$ is a Neu-open in $(\mathcal{M}, \sigma)$. But, $\hbar^{-1}(\mathcal{B})$ is not a Neu^{α}-open subset in $(\mathcal{M}, \tau)$ for $\mathcal{B} \in \sigma$. Hence, $\hbar$ is a $Neu^{S\alpha}$-continuous; however, it is not a Neu^{α}-continuous.

Remark 3.7:

The ideas of Neu-continuous and Neu^{α^*}-continuous are independent.

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Theorem 3.8:

(1) If a function $\hbar:(\mathcal{M},\tau)\longrightarrow(\mathcal{N},\sigma)$ is Neu-open, Neu-continuous, and bijective, then $\hbar$ is a Neu^{α^*}-continuous.

(2) A function $\hbar:(\mathcal{M},\tau)\longrightarrow(\mathcal{N},\sigma)$ is Neu^{α^*}-continuous iff $\hbar:\left(\mathcal{M},Neu^{\alpha}O(\mathcal{M})\right)\longrightarrow\left(\mathcal{N},Neu^{\alpha}O(\mathcal{N})\right)$ is a Neu-continuous.

Proof:

(1) Assume that $\mathcal{D}\in Neu^{\alpha}O(\mathcal{N})$. To show that $\hbar^{-1}(\mathcal{D})\in Neu^{\alpha}O(\mathcal{M})$We have to prove that $\hbar^{-1}(\mathcal{D})\subseteq NeuInt(NeuCl(NeuInt(\hbar^{-1}(\mathcal{D}))))$.
Assume $r\in\hbar^{-1}(\mathcal{D})\Longrightarrow\hbar(r)\in\mathcal{D}$. Therefore, $\hbar(r)\in NeuInt(NeuCl(NeuInt(\mathcal{D})))$ (because $\mathcal{D}\in Neu^{\alpha}O(\mathcal{N})$). Consequently, for any Neu-open set $\mathcal{H}$ in $\mathcal{N}$ where $\hbar(r)\in\mathcal{H}\subseteq NeuCl(NeuInt(\mathcal{D}))$. Then $r\in\hbar^{-1}(\mathcal{H})\subseteq\hbar^{-1}(NeuCl(NeuInt(\mathcal{D})))$, but $\hbar^{-1}(NeuCl(NeuInt(\mathcal{D})))\subseteq NeuCl(\hbar^{-1}(NeuInt(\mathcal{D})))$ (because $\hbar^{-1}$ is a Neu-continuous, corresponding to $\hbar$ is a Neu-open and bijective). Next, $r\in\hbar^{-1}(\mathcal{H})\subseteq NeuCl(\hbar^{-1}(NeuInt(\mathcal{D})))$.
Therefore, $r\in\hbar^{-1}(\mathcal{H})\subseteq NeuCl(\hbar^{-1}(NeuInt(\mathcal{D})))\subseteq NeuCl(NeuInt(\hbar^{-1}(\mathcal{D})))$ (because $\hbar$ is a Neu-continuous). Consequently, $r\in\hbar^{-1}(\mathcal{H})\subseteq NeuCl(NeuInt(\hbar^{-1}(\mathcal{D})))$, but $\hbar^{-1}(\mathcal{H})$ is a Neu-open set in $\mathcal{M}$ (since $\hbar$ is a Neu-continuous). Therefore, $r\in NeuInt(NeuCl(NeuInt(\hbar^{-1}(\mathcal{D}))))$. Therefore, $\hbar^{-1}(\mathcal{D})\subseteq NeuInt(NeuCl(NeuInt(\hbar^{-1}(\mathcal{D}))))\Longrightarrow\hbar^{-1}(\mathcal{D})\in Neu^{\alpha}O(\mathcal{M})\Longrightarrow\hbar$ is a Neu^{α^*}-continuous.

(2) The proof of (2) is evident.

Theorem 3.9:

A function $\hbar:(\mathcal{M},\tau)\longrightarrow(\mathcal{N},\sigma)$ is a $Neu^{S\alpha^*}$-continuous iff $\hbar:\left(\mathcal{M},Neu^{S\alpha}O(\mathcal{M})\right)\longrightarrow\left(\mathcal{N},Neu^{S\alpha}O(\mathcal{N})\right)$ is a Neu-continuous.

Proof: Comprehensible.

Remark 3.10:

The ideas of Neu-continuous and $Neu^{S\alpha^*}$-continuous are autonomous.

Remark 3.11:

Every Neu^{α^*}-continuous is a Neu^{α}-continuous and $Neu^{S\alpha}$-continuous. Nevertheless, the contrary does not hold.

Remark 3.12:

The concepts of Neu^{α^*}-continuous and $Neu^{S\alpha^*}$-continuous are autonomous.

Theorem 3.13:

If a function $\hbar:(\mathcal{M},\tau)\longrightarrow(\mathcal{N},\sigma)$ is Neu^{α^*}-continuous, Neu-open, and bijective, then it is $Neu^{S\alpha^*}$-continuous.

Proof:

Assume that $\hbar:(\mathcal{M},\tau)\longrightarrow(\mathcal{N},\sigma)$ is a Neu^{α^*}-continuous, Neu-open, and bijective. Let $\mathcal{D}$ be a $Neu^{S\alpha}$-open set in $\mathcal{N}$. Then any Neu^{α}-open set say $\mathcal{P}$ where $\mathcal{P}\subseteq\mathcal{D}\subseteq NeuCl(\mathcal{P})$. Consequently, $\hbar^{-1}(\mathcal{P})\subseteq\hbar^{-1}(\mathcal{D})\subseteq\hbar^{-1}(NeuCl(\mathcal{P}))\subseteq NeuCl(\hbar^{-1}(\mathcal{P}))$ (because $\hbar$ is a Neu-open), but $\hbar^{-1}(\mathcal{P})\in Neu^{\alpha}O(\mathcal{M})$ (since $\hbar$ is a Neu^{α^*}-continuous). Later, $\hbar^{-1}(\mathcal{P})\subseteq\hbar^{-1}(\mathcal{D})\subseteq NeuCl(\hbar^{-1}(\mathcal{P}))$. Therefore, $\hbar^{-1}(\mathcal{D})\in Neu^{S\alpha}O(\mathcal{M})$. Thus, $\hbar$ is a $Neu^{S\alpha^*}$-continuous.

Remark 3.14:

Let $\hbar_1:(\mathcal{M},\tau)\longrightarrow(\mathcal{N},\sigma)$ and $\hbar_2:(\mathcal{N},\sigma)\longrightarrow(\mathcal{O},\rho)$ be two functions, then:

(1) If $\hbar_1$ and $\hbar_2$ are Neu^{α}-continuous, then $\hbar_2\circ\hbar_1:(\mathcal{M},\tau)\longrightarrow(\mathcal{O},\rho)$ need not to be a Neu^{α}-continuous.

(2) If $\hbar_1$ and $\hbar_2$ are $Neu^{S\alpha}$-continuous, then $\hbar_2\circ\hbar_1:(\mathcal{M},\tau)\longrightarrow(\mathcal{O},\rho)$ need not to be a $Neu^{S\alpha}$-continuous.

Theorem 3.15:

Assume $\hbar_1:(\mathcal{M},\tau)\longrightarrow(\mathcal{N},\sigma)$ and $\hbar_2:(\mathcal{N},\sigma)\longrightarrow(\mathcal{O},\rho)$ are two functions, then $\hbar_2\circ\hbar_1:(\mathcal{M},\tau)\longrightarrow(\mathcal{O},\rho)$ is

(1) a Neu^{α}-continuous if $\hbar_1$ is Neu^{α}-continuous and $\hbar_2$ is Neu-continuous.

(2) a Neu^{α}-continuous if $\hbar_1$ is Neu^{α^*}-continuous and $\hbar_2$ is Neu^{α}-continuous.

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(3) a Neu^{α^*}-continuous if $\hbar_1$ and $\hbar_2$ are Neu^{α^*}-continuous.
(4) a $Neu^{S\alpha^*}$-continuous if $\hbar_1$ and $\hbar_2$ are $Neu^{S\alpha^*}$-continuous.
(5) a $Neu^{\alpha^{**}}$-continuous if $\hbar_1$ and $\hbar_2$ are $Neu^{\alpha^{**}}$-continuous.
(6) a $Neu^{S\alpha^{**}}$-continuous if $\hbar_1$ and $\hbar_2$ are $Neu^{S\alpha^{**}}$-continuous.
(7) a $Neu^{\alpha^{**}}$-continuous if $\hbar_1$ is $Neu^{\alpha^{**}}$-continuous and $\hbar_2$ is Neu^{α^*}-continuous.
(8) a Neu-continuous if $\hbar_1$ is $Neu^{\alpha^{**}}$-continuous and $\hbar_2$ is Neu^{α}-continuous.
(9) a Neu^{α^*}-continuous if $\hbar_1$ is Neu^{α}-continuous and $\hbar_2$ is $Neu^{\alpha^{**}}$-continuous.
(10) a $Neu^{\alpha^{**}}$-continuous if $\hbar_1$ is Neu-continuous and $\hbar_2$ is $Neu^{\alpha^{**}}$-continuous.

Proof:

(1) Assume $\mathcal{F}$ is a Neu-open subset in $\mathcal{O}$. Subsequently, $\hbar_2$ is a Neu-continuous, ${\hbar_2}^{-1}(\mathcal{F})$ is a Neu-open subset in $\mathcal{N}$. Later, $\hbar_1$ is a Neu^{α}-continuous, ${\hbar_1}^{-1}({\hbar_2}^{-1}(\mathcal{F})) = (\hbar_2 \circ \hbar_1)^{-1}(\mathcal{F})$ is a Neu^{α}-open subset in $\mathcal{M}$. Therefore, $\hbar_2 \circ \hbar_1 : (\mathcal{M}, \tau) \longrightarrow (\mathcal{O}, \rho)$ is a Neu^{α}-continuous.

(2) Assume $\mathcal{F}$ is a Neu-open subset in $\mathcal{O}$. Subsequently, $\hbar_2$ is a Neu^{α}-continuous, ${\hbar_2}^{-1}(\mathcal{F})$ is a Neu^{α}-open subset in $\mathcal{N}$. Later, $\hbar_1$ is a Neu^{α^*}-continuous, ${\hbar_1}^{-1}({\hbar_2}^{-1}(\mathcal{F})) = (\hbar_2 \circ \hbar_1)^{-1}(\mathcal{F})$ is a Neu^{α}-open subset in $\mathcal{M}$. Therefore, $\hbar_2 \circ \hbar_1 : (\mathcal{M}, \tau) \longrightarrow (\mathcal{O}, \rho)$ is a Neu^{α}-continuous.

(3) Assume $\mathcal{F}$ is a Neu^{α}-open subset in $\mathcal{O}$. Subsequently, $\hbar_2$ is a Neu^{α^*}-continuous, ${\hbar_2}^{-1}(\mathcal{F})$ is a Neu^{α}-open subset in $\mathcal{N}$. Later, $\hbar_1$ is a Neu^{α^*}-continuous, ${\hbar_1}^{-1}({\hbar_2}^{-1}(\mathcal{F})) = (\hbar_2 \circ \hbar_1)^{-1}(\mathcal{F})$ is a Neu^{α}-open subset in $\mathcal{M}$. Therefore, $\hbar_2 \circ \hbar_1 : (\mathcal{M}, \tau) \longrightarrow (\mathcal{O}, \rho)$ is a Neu^{α^*}-continuous.

(4) Assume $\mathcal{F}$ be a $Neu^{S\alpha}$-open subset in $\mathcal{O}$. Since $\hbar_2$ is a $Neu^{S\alpha^*}$-continuous, ${\hbar_2}^{-1}(\mathcal{F})$ is a $Neu^{S\alpha}$-open subset in $\mathcal{N}$. Later, $\hbar_1$ is a $Neu^{S\alpha^*}$-continuous, ${\hbar_1}^{-1}({\hbar_2}^{-1}(\mathcal{F})) = (\hbar_2 \circ \hbar_1)^{-1}(\mathcal{F})$ is a $Neu^{S\alpha}$-open subset in $\mathcal{M}$. Therefore, $\hbar_2 \circ \hbar_1 : (\mathcal{M}, \tau) \longrightarrow (\mathcal{O}, \rho)$ is a $Neu^{S\alpha^*}$-continuous.

(5) Assume $\mathcal{F}$ be a Neu^{α}-open subset in $\mathcal{O}$. Subsequently, $\hbar_2$ is a $Neu^{\alpha^{**}}$-continuous, ${\hbar_2}^{-1}(\mathcal{F})$ is a Neu-open subset in $\mathcal{N}$. Later, any Neu-open subset is a Neu^{α}-open, ${\hbar_2}^{-1}(\mathcal{F})$ is a Neu^{α}-open subset in $\mathcal{N}$. Since $\hbar_1$ is a $Neu^{\alpha^{**}}$-continuous, ${\hbar_1}^{-1}({\hbar_2}^{-1}(\mathcal{F})) = (\hbar_2 \circ \hbar_1)^{-1}(\mathcal{F})$ is a Neu-open subset in $\mathcal{M}$. Therefore, $\hbar_2 \circ \hbar_1 : (\mathcal{M}, \tau) \longrightarrow (\mathcal{O}, \rho)$ is a $Neu^{\alpha^{**}}$-continuous. The proof is evident to others.

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Remark 3.16:
The later illustration clarifies the connection among various thoughts of weakly Neu-continuous functions:

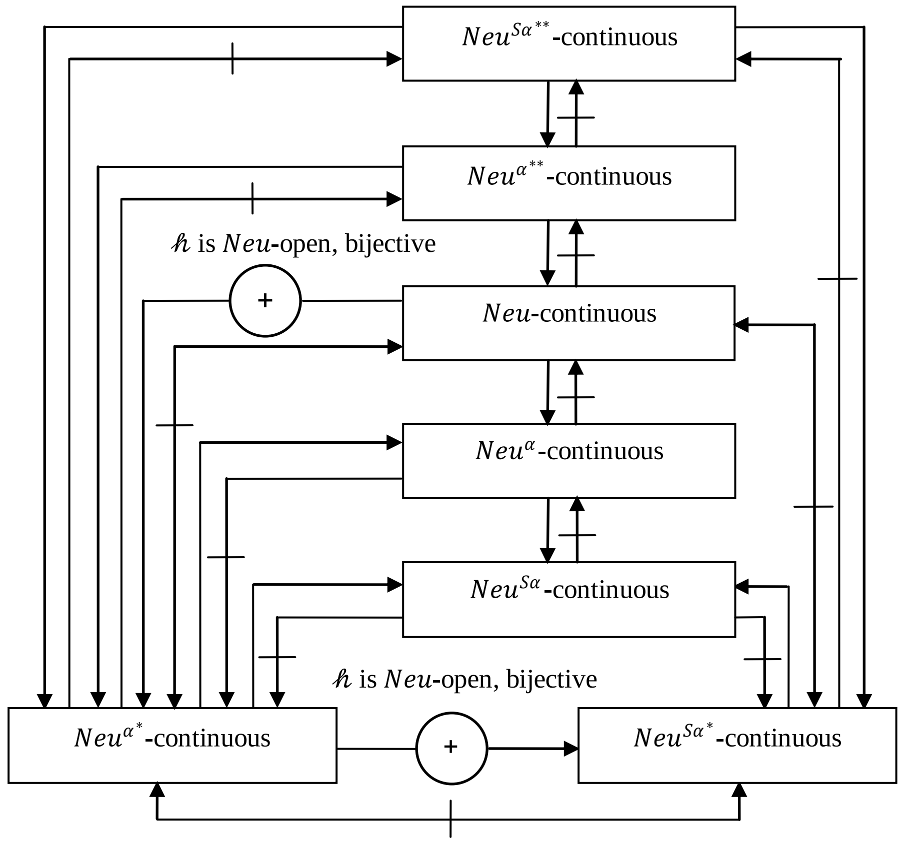

Fig. 1: The later

4. Conclusion

We intend to exercise the notions of Neu^{α}-open and $Neu^{S\alpha}$-open sets to outline some novel concepts of weakly Neu-continuous functions, for example; Neu^{α^*}-continuous, $Neu^{\alpha^{**}}$-continuous, $Neu^{S\alpha}$-continuous, $Neu^{S\alpha^*}$-continuous and $Neu^{S\alpha^{**}}$-continuous functions. The Neu^{α}-open and $Neu^{S\alpha}$-open sets can be employed to arise some novel notions of weakly Neu-open (Neu-closed) functions and Neu-separation axioms.

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