Optimum Design of Rectangular Plate Heat Sink

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Abstract:

The trends in electronics are toward decreasing size and cost, while increasing speed. Performance and reliability is resulted by increasing heat dissipated from the operating devices. As heat loads increase, the thermal management to keep junction temperatures within safe operating limits is becoming more critical.

The heat sink optimization study allowed for a determination of the heat sink geometry, which would produce a minimum thermal resistance, while producing the highest heat dissipation.

The objective of this paper is to present a method for designing the optimum thermal performance of heat sinks with uniform cross section of plate by using the **Heat Capacity** for the material. The model was used to explore the optimal dimensions of heat sink that cooling natural convection. It is found from the results that the best design of a (50 mm x 50 mm x 5 mm) aluminum heat sink with a temperature input of (50 ^{o}C) consists of (7) fins with (0.5 mm) fin thickness. Also, (ANSYS 5.4) finite element code is used to analyze the thermal behavior of a heat sink for a given of optimum dimensions.

Keywords: Heat sinks, optimization, size and cost, thermal conductivity, temperature distribution, **Heat Capacity**.

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التصمــــيم الأمـــــثل لصفيحـــــة مغطـــس حـــراري مـــستطيل الشكل

الخلاصة:

إنّ الاتجاه الشائع في تصميم الإلكترونياتِ هو تقليل الحجمِ والكلفةِ مع زيادة سرعة الأداء والديمومـة، هـذا التصـميم يقدم زيـادة بالتشـتيت الحـراري أثنـاء الظروف التشغيلية لهذه الأجهزة. عند زيادة الأحمال الحراريةَ، ستصبح السيطرة الحرارية لإبْقاء معـدل درجـاتِ الحـرارة ضـمن حـدودِ الأمـان هـي الأكثـر اهتماما.

دراســـــــــــة الأمـــــثلـــــية في تصمـــــــــيم المـــــغاطـــس الحـــــــراريــــــــة تفسح المجال في تخمـين شكل المغطس الحـراريَ، التـي تُنتجُ أقل مقاومة حرارية مع أعلى تشتيت حراري. الغرض من هذا البحث هو ايجاد أفضل طريقة لتصميم معامل الأداء الحراري للمغاطس الحرارية لصفيحة منتظمة المقطع العرضي باستخدام طريقة السعة الحرارية (Heat Capacity). لقد أستخدم النموذج الحسابي للبحث عن الأبعاد المثالية لمغطس حراري مبرد بواسطة الحمل الطبيعي. وجد من خلال النتائج بأن أفضل تصميم لصفيحة مغطس حراري معدني من الألنيوم(50 mm x 50 mm x 5 mm) بدرجة حرارة (50 ^{o}C) تتكون من سبعة زعانف بسمك (0.5 mm). بالإضافة إلى ذلك، تم استخدام طريقة العناصر المحـددة برنـامج (ANSYS 5.4) لتحليل التصرف الحراري للمغطس الحراري النموذجي الذي تم الحصول عليه.

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1-Introduction:

Heat sinks are devices that enhance heat dissipation from a hot surface, usually the case of a heat generating component, to a cooler ambient, usually air. The air is assumed to be the cooling fluid. In most situations, heat transfer across the interface between the solid surface and the coolant air is the least efficient within the system, and the solid air interface represents the greatest barrier for heat dissipation. A heat sink lowers this barrier mainly by increasing the surface area that is in direct contact with the coolant. This allows more heat to be dissipated and/or lowers the device operating temperature. The primary purpose of a heat sink is to maintain the device temperature below the maximum allowable temperature specified by the device manufacturers.

Heat sinks offer a low cost, convenient method for lowering the film resistance and, in turn, maintaining junction operating temperatures at a safe level for long time, reliable operation.

Unfortunately, the selection of the most appropriate heat sink for a particular application can be very difficult, given the many design options available. Thermal analysis models, ranging from simple empirically-derived correlations to powerful numerical simulation codes, can be used to analyze the thermal performance of heat sinks for a given set of design conditions. However, analytical models, based on the fundamental formulations for conservation of mass, momentum and energy, provide distinct advantages over most other models. The advantages of analytical modeling include speed, accuracy, easy of use and simplicity of coding, especially for web based software tools.

Analytical model are presented for designing of a heat sink of a dimension ($L* W_{base}* H$), shown in Fig (1). When applied to a constant source temperature of (50 ^{o}C) in standard ambient air conditions, will dissipate the most heat for the least material cost.
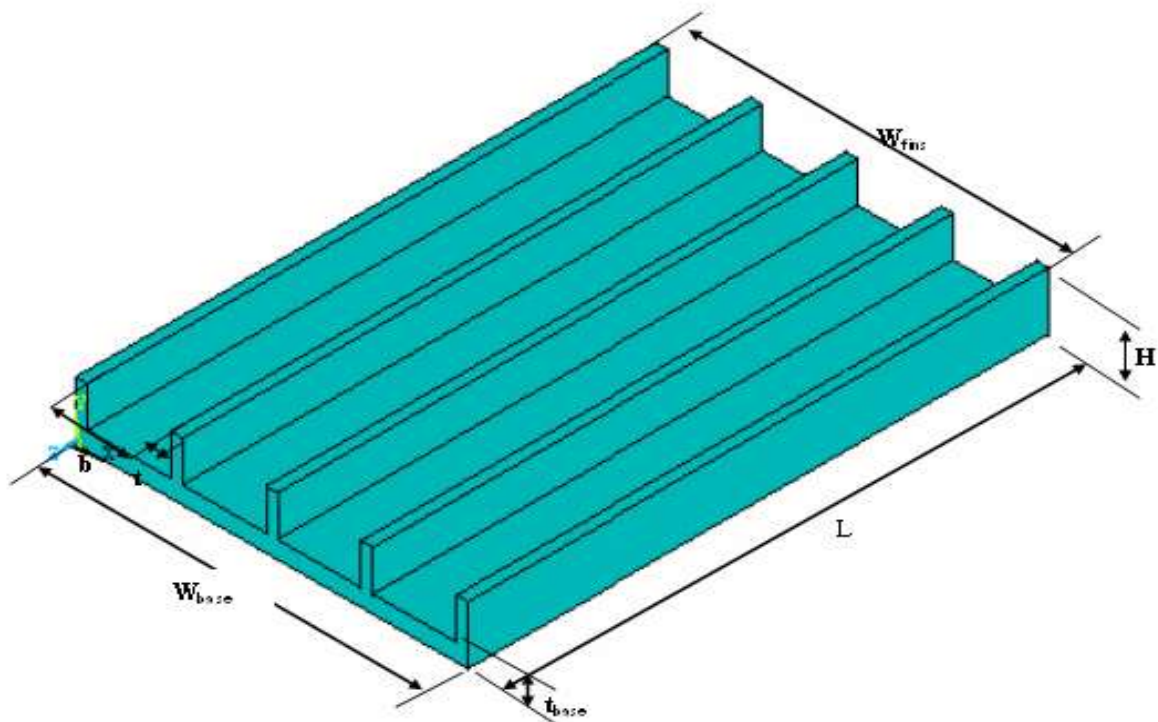

Fig (1): Geometry of a Heat Sink

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2-Optimization Overview:

In order to find the geometry which would result in the optimal heat sink, we first need to develop a system of governing equations which would allow us to accurately predict the convection and conduction occurring in a given geometry. Some preliminary researches such as **(Simons,2002).** Using these sources, it was found expressions that allowed us to calculate the average convection coefficient (h) of a heat sink and its thermal resistance when given the proper physical constants and the specific geometrical parameters described above. These equations will be presented and explained in more detail later.

The general equations were used to determine analytically the total amount of heat dissipated by the heat sink. The mass of the heat sink can be calculated from volume and density. Thus, the determination of all three factors (heat dissipated, mass, and temperature difference) that were used in the Heat Capacity of the heat sink. The ratio of the heat added to or removed from a substance to

the change in temperature produced is called the Heat Capacity (**H.C**) of the substance. With the ability to calculate the (H.C) of any given geometry, enable to run the calculations over a range of geometries, specifically different values of fin numbers (n) and fin thickness (t), and determine the combination of those two variables which would result in optimal heat sink performance.

3- Heat Sink Design:
3.1-Model Description:
This study is based on the following assumptions:
1. The fins are rectangular cross section area with adiabatic tip.
2. The convective heat transfer coefficient on the faces of the fin is constant and uniform over the entire surface of the fin.
3. The radiation heat transfer is negligible.
4. The fluid is considered incompressible with constant properties (air).
5. There is no contact resistance where the base of the fin joins the prime surface.
6. Heat source is assumed to be generating uniform temperature with time, T=50 ^{o}C.
7. Heat sink fins is oriented vertically to take full advantage of free air flow in natural convection.

The model specified calls for the design of a heat sink from a (50x50x5) mm, which when applied to a constant source temperature of (50 ^{o}C) in standard ambient air conditions, will dissipate the most heat for the least material cost. Also, three materials are tested in this paper, under same conditions for the purpose of comparison (Aluminum, Copper and Brass). The Heat Capacity (H.C) of the heat sink is given below;

$$H.C = \frac{Q}{m\ \Delta T}$$
(1)

Where (Q) is the heat dissipated (Watts), (ΔT) is the temperature difference (K) or (^{o}C) between the heat source and the ambient air, and m is the heat sink mass (kg). The goal is to achieve a heat sink within the given constraints that yield the highest (H.C) value, since a design that does this will be optimal within our specifications. In order to simple the testing process for many heat sink designs, we were given certain constraints about the base dimensions. For simplification, it was chose to rectangular plate fin pattern.

Where the (L) is the fixed length of the block, (W_{base}) is the fixed width of the block base, (W_{fin}) is the fixed total width of all fin space, (H) is fixed height of the block, (n) is the number of fins, (t_{base}) is the mandated thickness of the base, (t) is the thickness of each fin, and (b) is the band gap width between each fin. Table1 (A and B) shows physical properties and fixed parameters used during optimization.

Table (1A): Physical Properties**.**

Material	k[W/m K]	[kg/m^3] ρ	[J/kg K] C_p
Copper	385	8954	٣٨٧
Aluminum	204	٢٧٠٧	896
Brass(70%Cu,30%Zn)	١١١	٨٤٧٣	٣٨٥

Table (1B): Fixed parameters.

Dimension	Value
Length of the block (L)	50mm
Height of the block (H)	5mm
Width of the block (W_{base})	50mm
Fin Height (h)	3.5mm
Width of the Fin (W_{fin})	50mm

Although it would seen reasonable to assume that the more fins per unit area is the better. This is not always the case. Closely spaced fins will not be able to dispose of the heat well in a natural

convection heat sink(The benefits of natural convection include low cost of implementation, no need for fans, and the inherent reliability of the cooling process). This is partly because of the air flow that will actually be able to establish itself within a confined space, and partly because the fins will tend to radiate much of the heat to adjoining fins, which helps to stabilize the temperature, but does little to dispose of the heat. The band gap width between each fin is dependent on other parameters as follows;

$$W_{fin} = (n-1)\ b + n\ t$$

Then;

$$b = \frac{W_{fin} - nt}{n-1} \qquad (2)$$

Then, there are two remaining variables (n) and (t), and they will be the focus of our optimization.

Calculating the convection coefficient for a particular heat sink geometry and temperature conditions proves to be a difficult attempt. It was founded in the work of **Culham et. al.(2000)** an empirically validated method for our particular geometry. The method, which utilizes physical constants, heat sink geometry, and dimensionless numbers, is presented below. The convection coefficient (h) with the equation;

$$h = \frac{k_{air}.Nu}{b} \qquad (3)$$

Where k_{air}; is air thermal conductivity and (Nu) is the Nusselt number, defined as **Simons, R E.(2002):**

$$Nu = \frac{1}{24} Ra \left(1 - exp \left(-\frac{35}{Ra} \right) \right)^{\frac{3}{4}} \qquad (4)$$

And (Ra) is Rayleigh number, given by **Culham et. al** [2] as;

$$Ra = \frac{g\ \beta\ b^4\ \theta}{\alpha\ v_{air.}L} \qquad (5)$$

Where (θ) is temperature difference of the heat sink, and the remaining physical constants are the gravitational constant (g), air's coefficient of thermal expansion (β), air's thermal diffusivity (α), and air's kinematics viscosity (v_{air}) Physical constant values were found using **WPI's Air Property Calculator** [3].

3.2-Thermal Resistance Analysis:

Thermal Resistance can be obtained from a resistor network (Fig.2) formed between the heat source and the cooling medium.

$$R_{sink} = \frac{1}{\underbrace{n\left(\frac{1}{R_{fin}}\right) + (n-1)\left(\frac{1}{R_{channal}}\right)}_{R_{equivalent}}} + R_{base} \qquad (6)$$
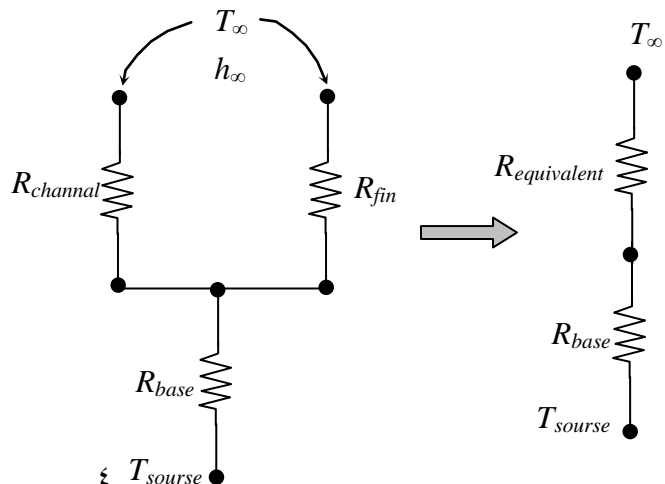

Fig (2): Thermal Resistance Representation.

Where;

$$R_{fin} = \frac{1}{\sqrt{h\,P\,k_m\,A_c}\,\tanh\!\big(m(H-t_{base})\big)} \qquad (7)$$

(A_c) is cross sectional area $(L*t)$ of the fin, (P) is the perimeter $(2L+2t)$, sub script (m) means metal, and the value of (m) is defined as;

$$m = \sqrt{\frac{h\,p}{k_m\,A_c}} \qquad (8)$$

And;

$$R_{channal} = \frac{1}{h\,b\,L} \qquad (9)$$

$$R_{base} = \frac{t_{base}}{k_m\,L\,W_{base}} \qquad (10)$$

By substituting Eq.(7), Eq.(9) and Eq.(10) into Eq.(6) and doing some rearrangements . The overall thermal resistance is;

$$R_{sink} = \frac{1}{\dfrac{n}{R_{fin}} + h(n-1)\,b\,L} + \frac{t_{base}}{k_m\,L\,W_{base}} \qquad (11)$$

It was note that the above expression makes qualitative sense, since it adds the resistances of fins and exposed base areas working in parallel. It was known that a parallel network's total resistance is reciprocal of the sum of all the reciprocals of each element's resistance. Thus, the (n/R_{fin}) expression describes sum of the reciprocals of each fin resistance element, while the $[h(n-1)b.L]$ expression gives the sum of the reciprocals of the resistance of each exposed area of the base. It was included the conductivity resistance part of the expression $(t_{base}/(k_m\,L\,W_{base}))$ for complete accuracy, though it will probably factor in very little since the thermal conductivity of copper or aluminum is so great.

After compute the total resistances of the heat sink, the total heat dissipated (Q) was determined using the following relationship:

$$Q = \frac{\theta}{R_{sink}} \qquad (12)$$

The mass of the heat sink can be determined from the following equation;

$$mass = \rho\big(t_{base}\,W_{base}\,L + n\,(H-t_{base})\,L\,t\big) \qquad (13)$$

Thus, the (H.C) of proposed heat sink geometry can be calculated.

3.2- Material Selection:

It is commonly believed that very high thermally conductive materials are the only types of material that can be used for heat sinks. Many designers will turn to copper as an alternative material to improve heat sink performance when an aluminum sink may not provide enough cooling. In some cases this switch is justified, in other cases it may not be. Following are few rules of thumb for when the extra cost and weight of copper makes sense;

1) Pure copper has about 2X the conductivity of extruded aluminum. This added conductivity helps reduce semiconductor temperature in heat spreading and fin efficiency.

2) The cost of an all copper heat sink is typically 3X the cost of an equivalent size(fin count, base thickness, fin height, etc.) aluminum part.

a- This cost increase is based on many factors. Copper cannot be extruded in the same manner as aluminum therefore it must be machined from flat plate with fins or other features brazed in place.
b- Due to density and its abrasive nature machining holes and other details in copper takes significantly longer time and wears tooling at a higher rate.

c- Cost of the base copper material is about the same as aluminum per pound but at 3X the density per cubic inch. This results in a raw material cost of 3X that of aluminum.

4-The Results and Discussion:

According to the Fig (1), heat sinks perform best when the fins are as thin as possible and few in number (between 2 and 20), which makes sense given that our **H.C** favors those with low mass. The highest (**H.C**) occurs when the heat sink is made of up (7) fins, each (0.5 mm) thick. At these dimensions, results of the calculations made according to the previous equations are presented in the table(2);

Table (2)

Material \ Optimization	Aluminum	Copper	Brass
Heat transfer coefficient(h)	6.618547 W/m^2	6.618547 W/m^2	6.618547 W/m^2
Heat dissipation (Q)	3.2 W	2.892 W	2.454484W
Mass(m)	14.28 g	49.2 g	39.41647
Heat Capacity(H.C)	16.4765 W/kg.k	3.9578 W/kg.k	5.842708
Thickness of Fin (t)	(0.5-5) mm	(0.5-5) mm	(0.5-5) mm
Number of Fins(n)	[2-20]	[2-20]	[2-20]

To further exploration, the performance of an entire family of heat sinks, the thermal resistance was analyzed by varying the fin thickness and number of fins keeping all other parameters fixed at specified values. The values used for these parameters are listed in Table (1A) along with the specified fin height, which also remains fixed. The thermal properties of the heat sink are based on aluminum and copper. The range considered for fin thickness and number of fins can also be found in Table (1B). The fin thickness was constrained to be between (0.5-5 mm) to prevent unrealistically small or negative values.

Thermal resistance (R_{sink}) defined by Eq. (11) is used as the performance metric. The thermal resistance was computed for each unique case and a surface plot of the results is shown in Figs (8) and (9) where a minimum occurs with a fin thickness of (0.5 mm) and (7) fins yielding an overall resistance of (6.5 kW). For heat sinks with (5) fins or greater, the thermal resistance could not be computed up to fin thicknesses of (5 mm) because the fin width was below the constraint of (1 mm).

Thermal resistance of heat sink was an energy balance for the convection flux and the over all surface area.

Then, the numbers of fins (*n*) increases for any given thickness (*t*), a drop in thermal resistance (R_{sink}) was observed for a little number of fins, see Eqs. (6 and 11). A minimum thermal resistance was reached before its increase with increasing number of fins; see Figs (3 and 4).

As mentioned in the above, when the number of the fins increased that lead to a drop in thermal resistance with increasing in the magnitude of heat dissipated for a small number of fins, see Eq. (12). A maximum heat dissipated was reached before the heat dissipated was reached before the heat dissipated decreasing with the increasing number of fins; see Figs (5 and 6).

Because of the amounts of the heat dissipated that calculated from Eq.(12) was governing with the H.C that calculated from Eq.(10), the behavior of heat dissipated was similar to the behavior of H.C ; see Figs (7 and 8).

The optimum number of fins is also dependent on the fin thickness. However, the high heat capacity of the heat sink material results in high fin efficiency for the fin thicknesses under consideration, as shown in Fig (9). Therefore, the thermal resistance of the heat sink increases for increasing fin thickness (i.e. decreasing channel width) due to the reduced boundary fluxes.

Fig (10 and 11) shows the thermal behavior for the aluminum and copper sinks respectively that obtained by using (ANSYS 5.4) finite element coder for a given optimum dimensions.
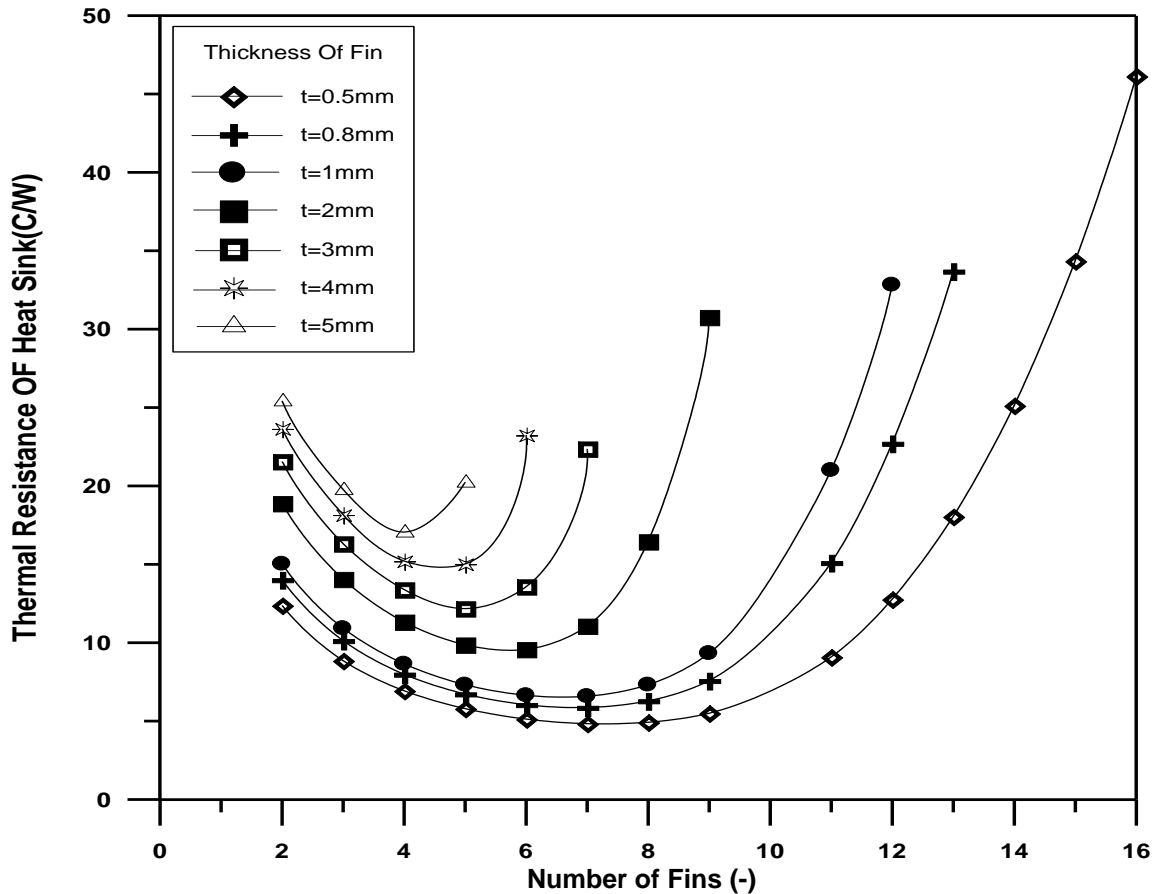
Fig (3): Thermal Resistance of Fins versus Number of Fins for various Fin Thickness of Aluminum.
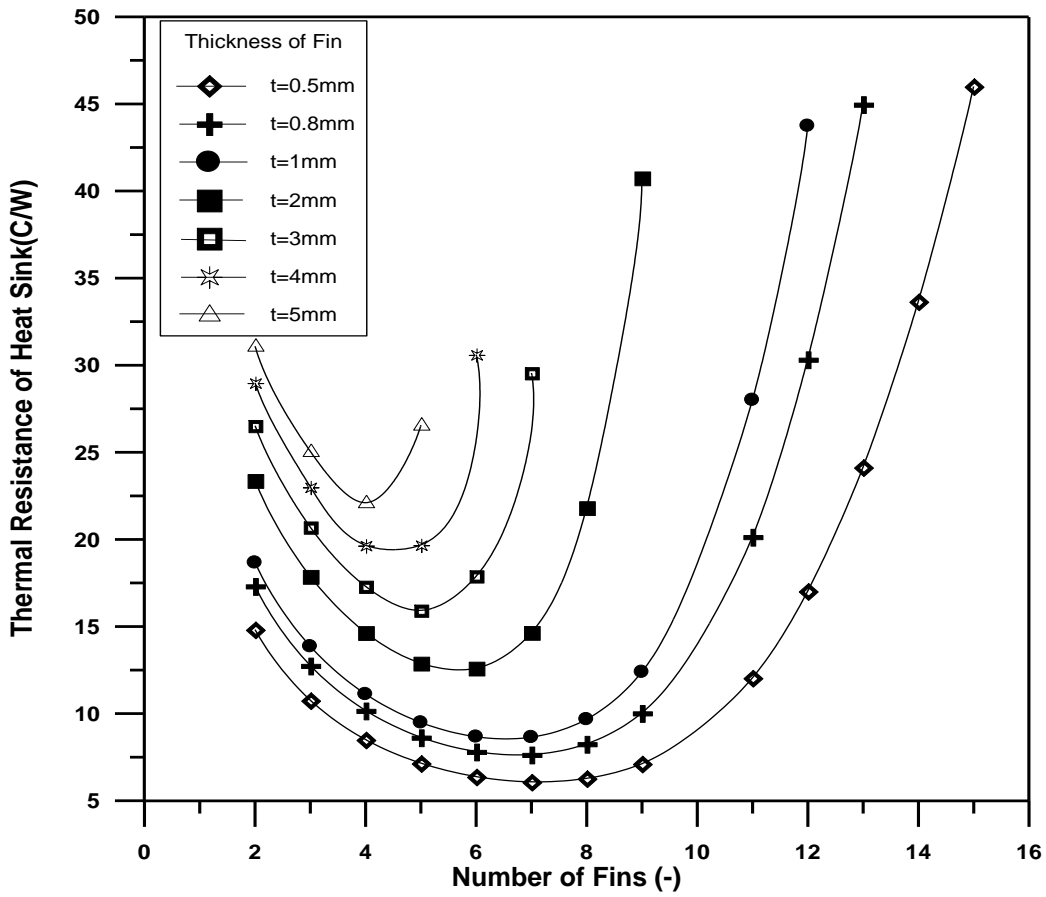
Fig (4): Thermal Resistance of Fins versus Number of Fins for Various Fin Thickness Copper.
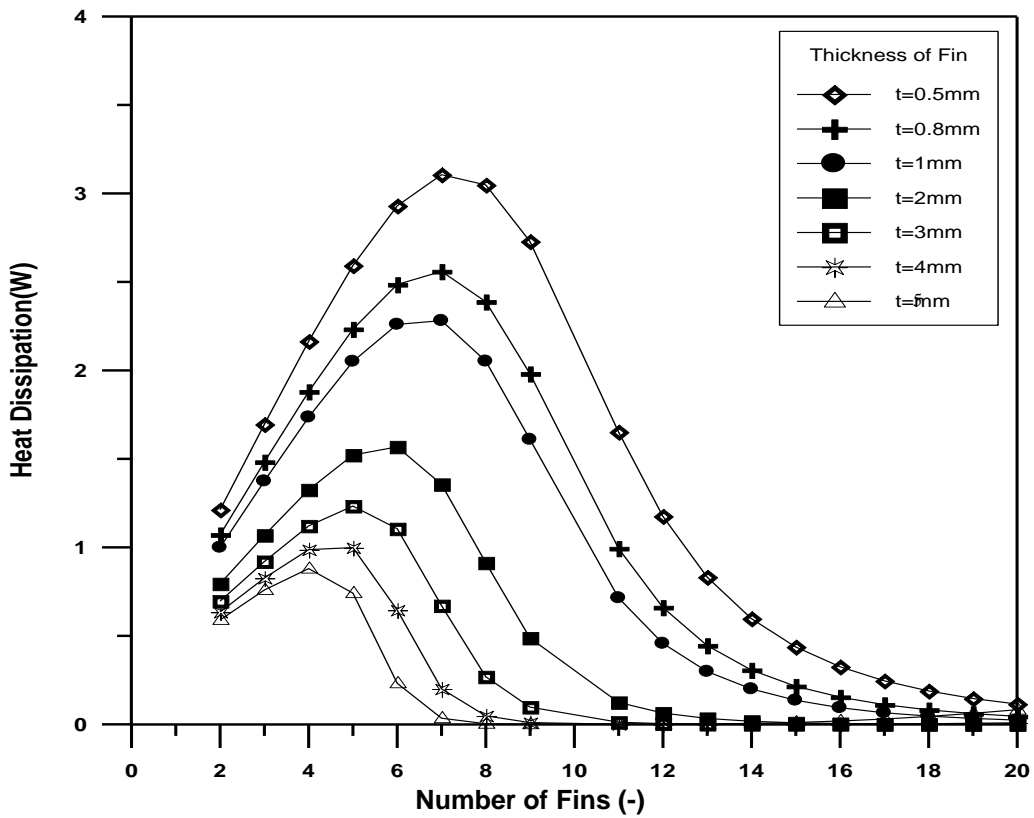
Fig (5): Heat Dissipation of Heat Sink versus Number of Fins for Various Fin Thickness Aluminum.
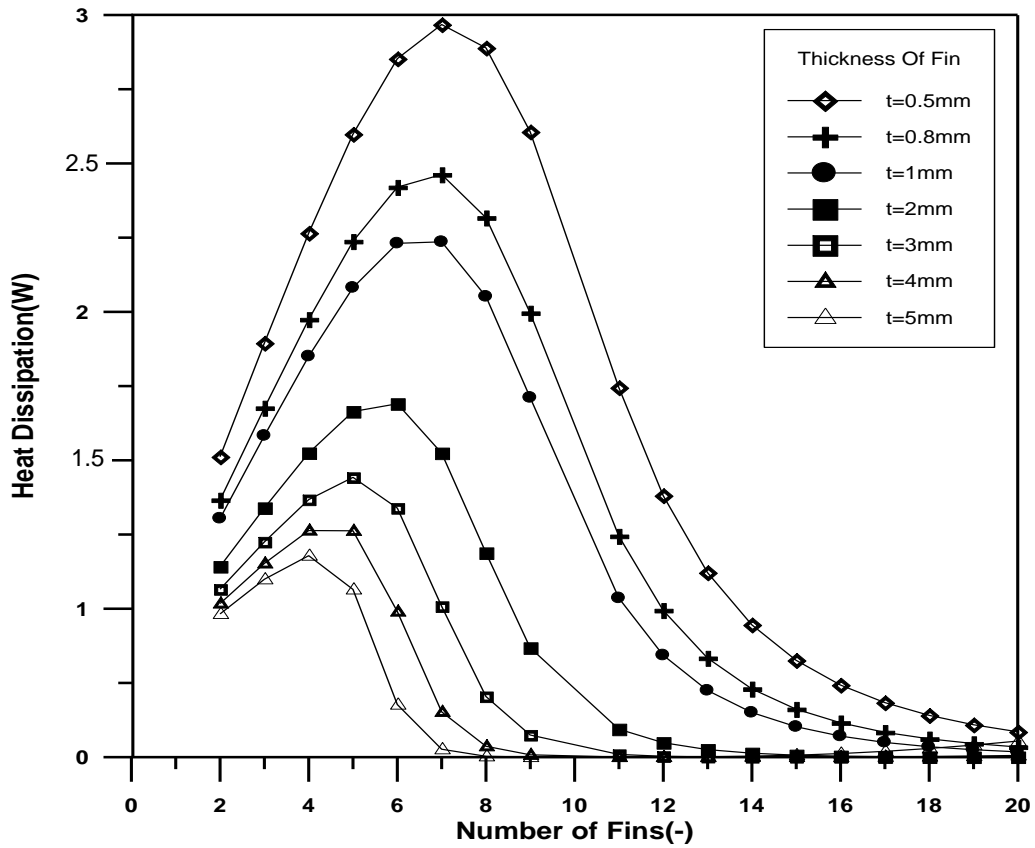
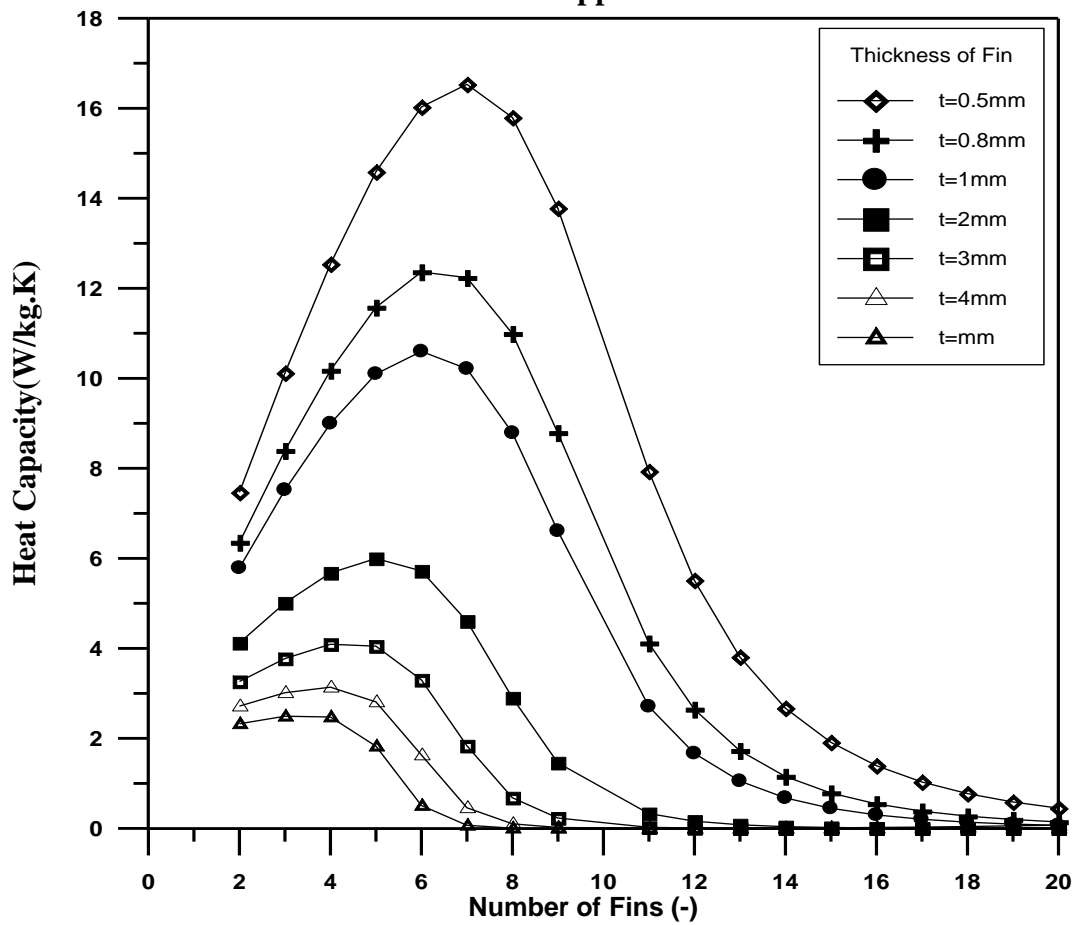**Fig (5): Heat Dissipation of Heat Sink versus Number of Fins for Various Fin Thickness Copper.**

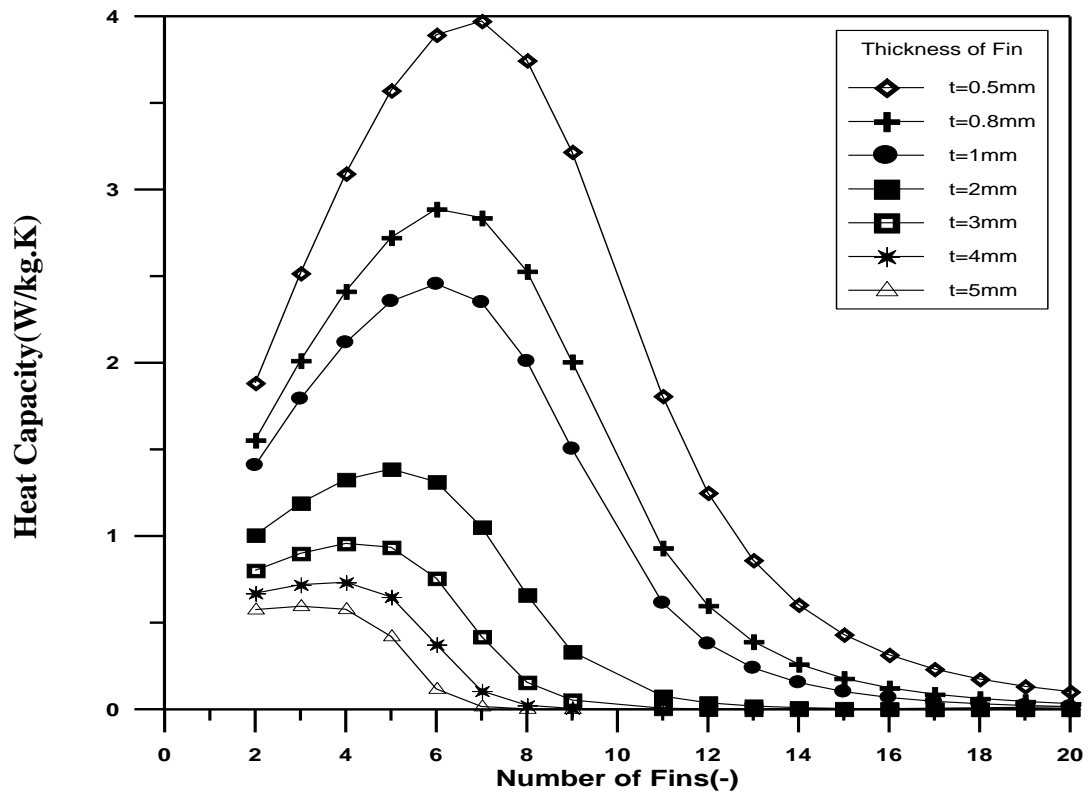**Fig (7): H.C versus Number of Fins for Difference Fin Thickness Aluminum.**

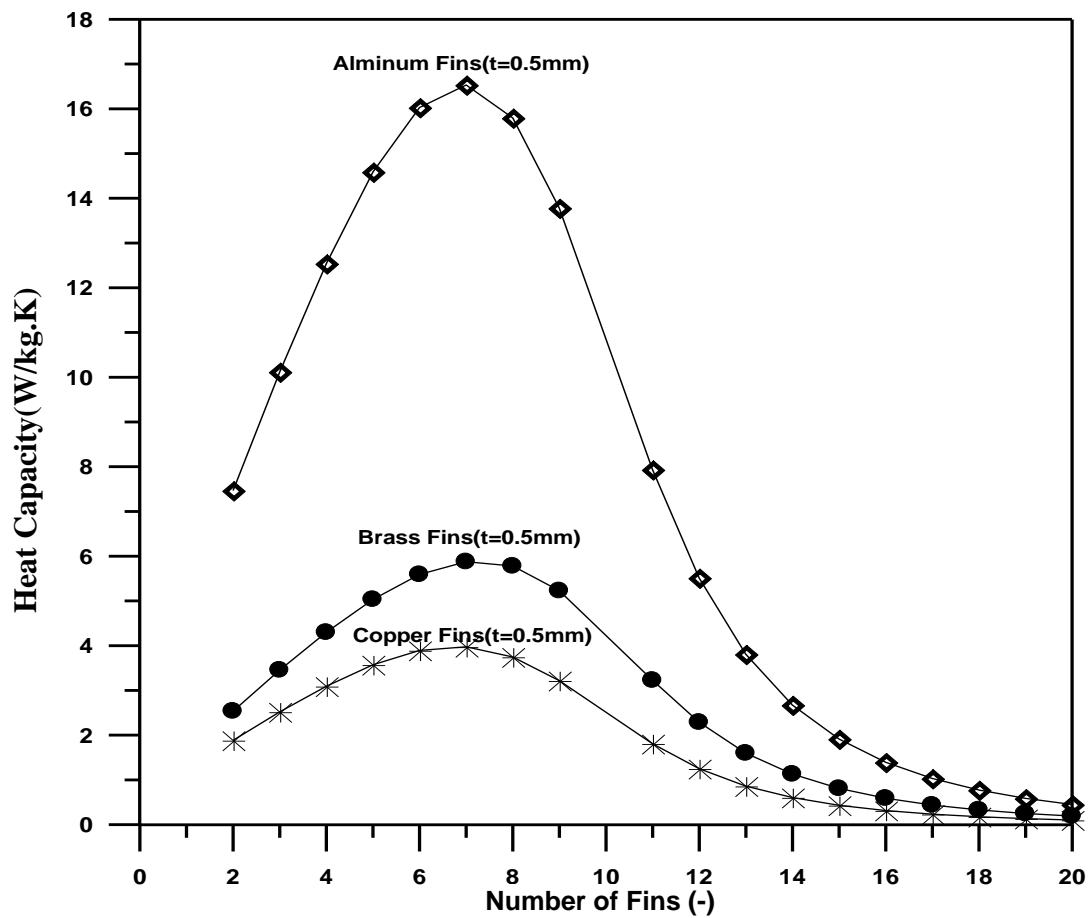**Fig (8): H.C versus Number of Fins for Difference Fin Thickness Copper.**

Fig (9): Heat Capacity of the Materials as Function of Number of Fins.

NODAL SOLUTION

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12:15:58

STEP=1
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RSYS=0
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SMX =50

MN

MX

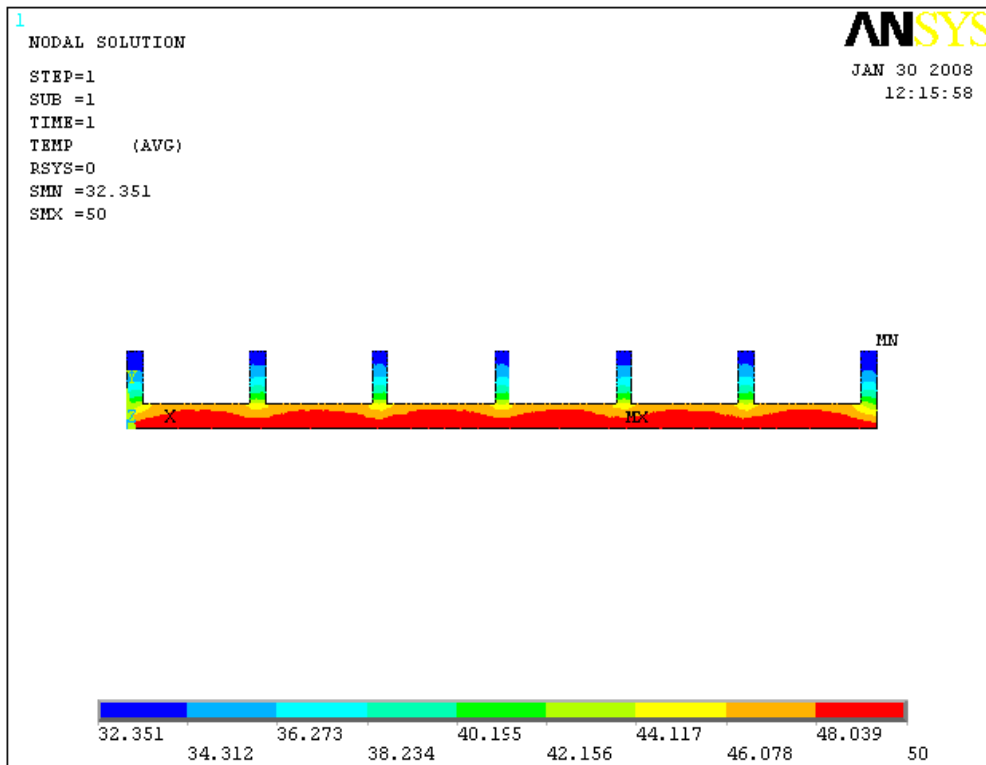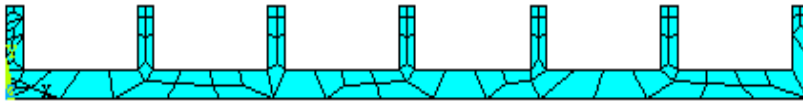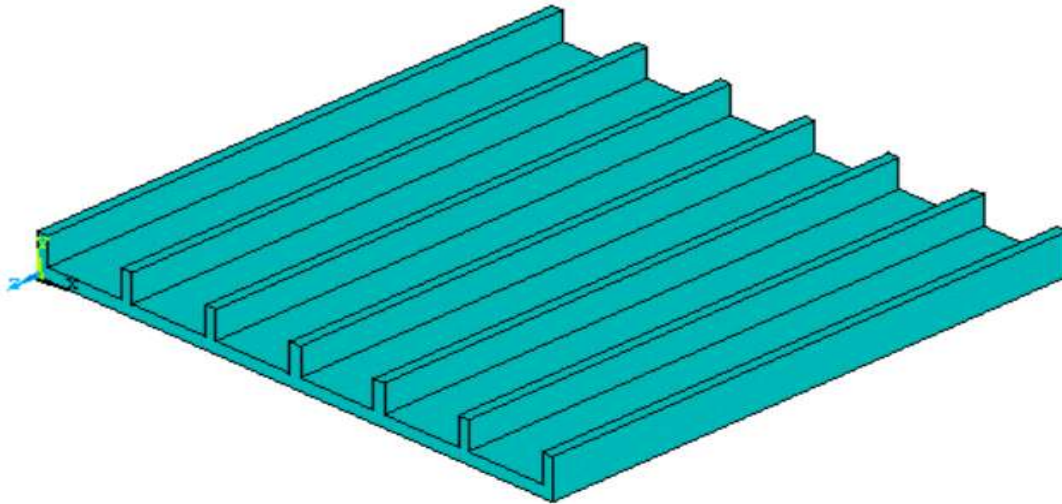| 32.351		36.273		40.195		44.117		48.039	
	34.312		38.234		42.156		46.078		50

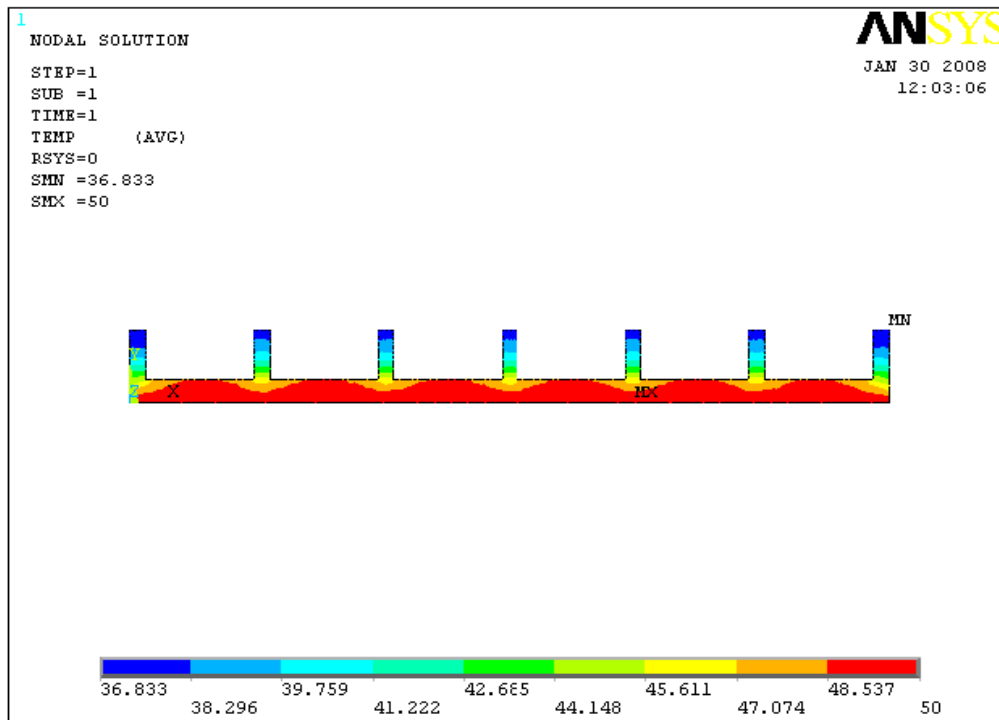**Fig (10): Steady State Thermal Behavior of Aluminum Heat Sink.**

Fig (11): Steady State Thermal Behavior of Copper Heat Sink.

5- CONCLUSION:

The objective of this paper is to present a method for designing the optimum thermal performance of heat sinks with uniform cross section of plate by using the **Heat Capacity** for the material.

The model was used to explore the optimal dimensions of heat sink that cooling natural convection. It is found from the results that the best design of a (50 x 50 x 5) mm aluminum heat sink with a temperature input of (50 ^{o}C) consists of (7) fins with (0.5 mm) fin widths.

The analytical model could be easily implemented to study the heat transfer in other geometries where convection is important. However, the numerical model by (ANSYS) package is currently formulated for rectangular, structured meshes and the formulation may require modification for other meshing geometries.

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