On paracompact in bitopological spaces.

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SUMMARY.- *We modify the concept of paracompactness for spaces with two topologies and obtain several results concerning paracompact in bitopological spaces.*

1-Introduction

Bitopological space , initiated by Kelly [7], is by definition a set equipped with two non identical topologies , and it is denoted by (X, τ,μ) where τ and μ are two topologies defined on X .

A sub set F of a topological space (X , τ) is F_σ [11] if it is a countable union of τ-closed set . We will denote to such set by τ- F σ .

Let (X , τ) be a topological space . A cover (or covering) [3] of a space X is a collection $U = \{U_\lambda : \lambda \in \Delta\}$ of subset of X whose union is the whole X .

A sub cover of a cover U [3] is a sub collection v of u which is a cover .

An open cover of X [3] is a cover consisting of open sets , and other adjectives appling to subsets of X apply similarly to covers .

For an infinite cardinal number m , if the collection $U = \{U_\lambda : \lambda \in \Delta\}$ consists of at most m sub-sets, we say that it has cardinality $\leq m$ or simply card. $\leq m$. Some times this collection is denoted by $|U| \leq m (or) |\Delta| \leq m$.

If a sub set A of X is consisting of at most m elements we say that A has cardinality $\leq m$ (or with cardinality $\leq m$) , and is denoted by $|A| \leq m$. A bitopological space (X, τ, μ) is called (m) (τ-μ) compact if for every τ-open cover of X, (with cardinality $\leq m$), it has μ-open sub-covers . The function $f : (X, \tau, \mu, \rho) \to (Y, \tau`, \mu`, \rho`)$ is said to be $(\tau - \tau`) - close[(\tau - \tau`) continuous]$ function if the image [inverse image of each τ-closed[τ`-open] is τ`-closed [τ-open in X] in Y. .Let U={U_λ : λ∈Δ} and V={V_γ :γ∈Γ} be two coverings of X , V is said to be refine (or to be a refinement of) U , if for each V_γ there exists some U_λ with $V_\gamma \subset U_\lambda$.

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If W={W_δ :δ ∈ Ω} refine two covers U, V of X, then it is called common refinement [2] . A family U={ U_λ : λ∈ Δ}of sets in a space (X,τ) is called locally finite, if each point of X has a neighborhood V such that V∩U_λ≠ϕ for at most finitely many indices λ. In other word V∩U_λ= ϕ for all but a finite number of λ. A family U of set in a space (X,τ) is called σ-locally finite if

$$U = \bigcup_{n=1}^{\infty} U_n$$

where each U_n is a locally finite collection in X.

A bitopological space (X, τ ,μ) is called pairwise Hausdorff if for every two distinict points x and y of X, there exist τ -open set U and a μ -open set V such that x ∈ U, y ∈ V and U ∩V= ϕ.

A bitopological space (X, τ ,μ) is called (m)(τ, μ , μ)- regular if for every point x in X and every τ -closed set A with $|A| \leq m$ such that for x ∈ A, there exist two μ- open sets U, V such that x ∈ U, A ⊂ V, and U ∩V= ϕ.

Clearly every (τ, μ, μ)-regular space is m(τ,μ,μ)-regular space .

A bitopological space (X,τ,μ) is called (m-)(τ,μ,μ) –normal if for every pair disjoint τ-closed sets A,B of X,with $|A| \leq m, |B| \leq m$ there exist two μ-open sets U,V such that A ⊂U, B⊂V, and U∩V = ϕ.

Clearly every (τ,μ,μ) –normal space is m(τ,μ,μ) –normal.

A topological space (X , τ)is said to be :

1- m-paracompact [9], if every open cover of X with card .≤m has a locally finite open refinement.
2- paracompact[4], if every open cover of X has a locally finite open refinement.
3- (m-) semiparacompact, if every open cover of X (with card. ≤m) has a σ-locally finite open refinement .
4- (m-) a-paracompact[1] if every open cover of X with card. ≤m has a α-locally finite refinement not necessary either open or closed.

2-Main Results

2.1-Definition

A bitopological space (X , τ ,μ) is called (m-) (τ - μ) paracompact w.r.t μ ,if for every τ-open cover $U = \{U_\lambda : \lambda \in \Delta\}$of X (with card. ≤ m) has a μ-open refinement $V = \{V_\gamma : \gamma \in \Gamma\}$ which is locally finite w .r .t μ.

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2.2 - Proposition

Every (τ -μ)paracompact w .r .t.μ bitopological space (X, τ ,μ) is m (τ-μ)paracompact w .r .t μ .

2.3 - Definition

A bitopological space (X , τ ,μ) is called (m-) (τ - μ) semiparacompact w .r .t μ ,if every τ - open cover $U = \{U_\lambda : \lambda \in \Delta\}$ of X (with card. ≤ m) has a μ -open refinement $V = \{ V_\gamma | \gamma \in \Gamma \}$ which is σ-locally finite. w.r.t μ .

2.4 - Proposition

Every (τ -μ)semiparacompact w .r .t.μ bitopological space (X , τ ,μ) is m(τ -μ) semiparacompact w .r .t μ .

2.5 - Theorem

Every m(τ-μ)paracompact w .r .t. μ bitopological space (X , τ, μ) is m(τ -μ)semiparacompact w .r .t μ .

2.6 - Corollary

Every (τ-μ)paracompact w .r .t. μ bitopological space (X , τ ,μ) is (τ-μ) semiparacompact w .r .t. μ .

2.7 - Corollary

Every (τ-μ)paracompact w .r .t. μ bitopological space (X , τ ,μ) is m(τ -μ) semiparacompact w .r .t μ .

2.8 - Definition

A bitopological space (X,τ ,μ) is called (m-) (τ - μ) -a-paracompact w .r .t μ ,if for every τ - open cover $U = \{U_\lambda : \lambda \in \Delta\}$ of X (with card. ≤ m) has a refinement $V = \{V_\gamma : \gamma \in \Gamma\}$ of U not necessarily either μ-open or μ-closed which is locally finite . w .r .t. μ .

2.9 - Proposition

Every (τ-μ)-a-paracompact w .r .t. μ bitopological space (X , τ ,μ) is m (τ -μ) -a-paracompact w .r .t μ .

2.10 - Theorem

Every m(τ -μ)semiparacompact w .r .t.μ bitopological space (X , τ ,μ) is m (τ -μ) -a-paracompact w .r .t. μ .

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Proof

Suppose that (X, τ ,μ) be m (τ -μ)semiparacompact w .r .t.μ space. Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ -open cover of X with card ≤m ,then U has μ-open refinement V of U which is σ-locally finite w .r .t. μ , such that

$$V = \bigcup_{n=1}^{\infty} V_n$$

where each V_n is μ-open collection which is locally finite w.r.t.μ,say $V_n = \{V_{n\beta} : \beta \in \mathrm{B}\}$. For each n, let

$$\mathrm{W_n} = \bigcup_{\beta} \mathrm{V}_{n\beta}$$

then W_n is μ - open set. Since

$$\mathrm{X} = \bigcup_{\beta}\left(\bigcup_{n=1}^{\infty} V_{n\beta}\right) = \bigcup_{n=1}^{\infty}\left(\bigcup_{\beta} V_{n\beta}\right) = \bigcup_{n=1}^{\infty} W_n$$

Then the collection $W = \{W_n | n \in \mathrm{IN}\}$ is μ-open cover of X.

Define

$$A_i = W_i / \bigcup_{j\langle i} W_j \qquad \text{where } i=1,2,\ldots$$

then $A = \{A_n : n \in IN\}$ is a collection of sets that are not necessarily either μ-open or μ-closed.then A is cover of X, a refinement of Wand locally finite w .r .t. μ . Hence (X, τ ,μ) is m(τ-μ)-a- paracompact w.r.t μ

In the same way we can prove the following corollaries.

2.11 - Corollary

Every (τ-μ)semiparacompact w .r .t. μ bitopological space (X, τ ,μ) is (τ-μ)-a-paracompact w.r.t μ .

2.12 - Corollary

Every (τ-μ)semiparacompact w .r .t. μ bitopological space (X, τ ,μ) is m (τ-μ) -a-paracompact w .r .t μ .

2.13 - Corollary

Every m (τ-μ)paracompact w .r .t. μ bitopological space (X, τ ,μ) is m (τ-μ)- a-paracompact w .r .t. μ .

2.14 - Corollary

Every (τ-μ)paracompact w.r.t. μ bitopological space (X, τ ,μ) is (τ-μ)- a-paracompact w.r.t. μ .

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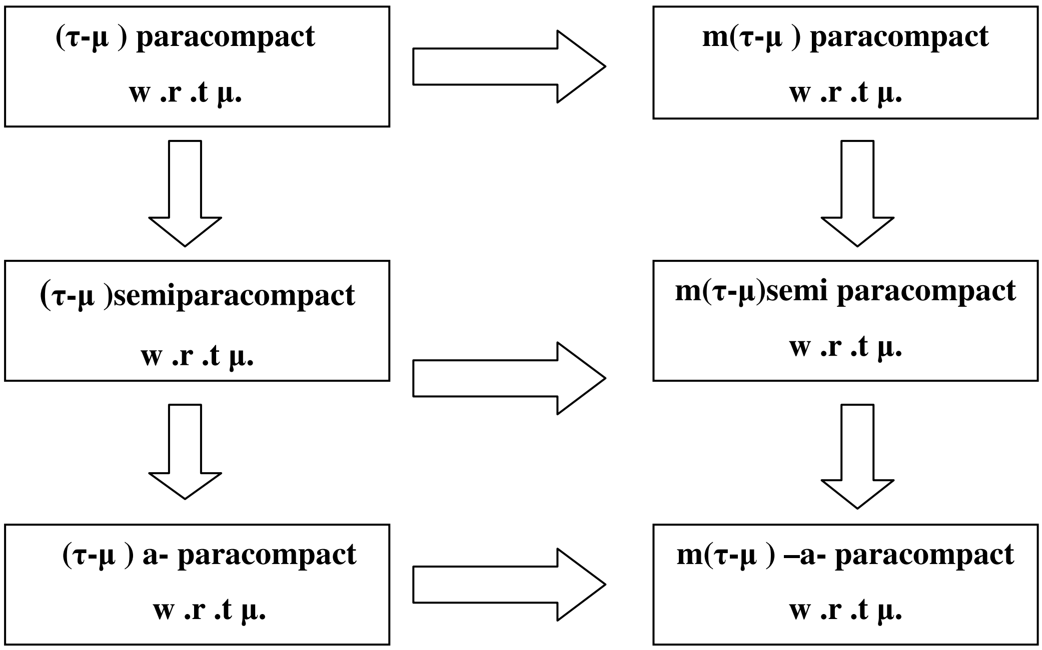
2.15 - Corollary

Every (τ-μ) paracompact w .r .t .μ bitopological space (X, τ ,μ) is m(τ-μ) -a-paracompact w .r .t .μ .

The following diagram show the relation a among the spaces which have been studied above



2.16 - Theorem

Let (X, τ ,μ) be an m(τ-μ)paracompact w .r .t μ. and pairwise Hausdroff space such that every τ- closed set in (X, τ ,μ) has card. ≤m, then (X, τ ,μ) is m(τ ,μ, μ)-regular space.

Proof

Suppose that. (X, τ ,μ) be an m(τ-μ) paracompact w .r .t μ space, A a τ- closed set in (X,τ,μ) having card. ≤m, and x Є X / A.

Since (X, τ ,μ) is pairwise Hausdorff, then for each yЄA, we can find a τ–open set V_y and a μ-open set U_y, such that $x \in U_y$,and $U_y \cap V_y = \phi$ the collection $\Pi = \{V_y : y \in A\} \cup \{X / A\}$ form a τ – open cover of X having card.≤m. and Π has a μ-open refinement $W = \{W_\gamma : \gamma \in \Gamma\}$ which is locally finite-w.r.t. μ.

Set

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$$V = \bigcup_{\gamma \in \Gamma} \{W_\gamma : W_\gamma \cap A \neq \phi\}$$

then V is μ-open set containing A.

Since the μ- open cover W is locally finite. w.r.t. μ,then x has a μ-neighborhood U* which meet only a finite number of $W\gamma_1, \ldots, W\gamma_n$ .If some $W\gamma_i$, i=1,2,…n meets A i.e $W_{\gamma i} \cap A \neq \phi$, then $W_{\gamma i} \subset X/A$ is impossible thus there exists $W_{\gamma i}$ .such that $W_{\gamma i} \subset V_{yi}$.

Set

$$U = U^* \cap \left(\bigcap_{i=1}^{n} W_{\gamma i} \right)$$

then x Є U and U is a μ- open set then $U \cap V = \phi$ .Therefore the bitopological space (X,τ,μ) is m(τ,μ,μ)- regular

2.17 - Corollary

If (X, τ ,μ) be a (τ-μ) paracompact w .r .t. μ ,and pairwise Hausdorff then (X, τ ,μ) is (τ,μ,μ) –regular.

2.18 - Theorem

If (X, τ,μ) is an m (τ -μ)paracompact w .r .t μ , and pairwise Hausdorff space, such that every τ-closed set in (X, τ,μ) has card.≤m , then (X, τ,μ) is m(τ,μ,μ)-normal .

proof

Suppose that (X, τ ,μ) be an m(τ -μ) paracompact w .r .t μ .Let A,and B be disjoint τ- closed sets in (X, τ ,μ)such that they has card. ≤m. Since (X, τ ,μ) is pairwise Hausdorff, then for each x∈A,y∈B we can find a τ–open set U_x and a μ-open set V_x , such that $x \in U_x, y \in V_x$,and $U_x \cap V_x = \phi$. Then $\Pi = \{U_x : x \in A\} \cup \{X/A\}$ form a τ-open cover of X having card.≤m. Then Π has a μ-open refinement $W = \{W_\gamma : \gamma \in \Gamma\}$ which is locally finite-w.r.t μ.

Set

$$U = \bigcup_{\gamma \in \Gamma} \{W_\gamma, W_\gamma \cap A \neq \phi\}.$$

Then U is μ-open set contains A.

For each $y \in B$,we can find μ-open nhd H_Y wich meets only a finite number of W_γ, say $W_{\gamma 1(y)},\ldots, W_{\gamma n(y)}$ (the value of n also depending on y) . Each $W_{\gamma i(y)}$ meeting A i.e $W_{\gamma i} \cap A \neq \phi$,then $W_{\gamma i} \subset X/A$ is impossible . Thus there exists $U_{x\ i}$ such that $W_{\gamma i(y)} \subset U_{x\ i}$ for $x_i \in A$.

Set $G_y = H_y \cap \left(\bigcap_{i=1}^{n} V_{Xi} \right)$

then G_y is a μ-open set which contains y but does not meet U

Let $V = \bigcup_{y \in B} G_y$.

Then V is a μ-open set, and $B \subset V$ and $U \cap V = \phi$. Therfore (X, τ ,μ) is m(τ ,μ , μ)-normal.

2.19 - Corollary

If (X, τ,μ) be a (τ -μ) paracompact w .r .t. μ ,and pairwise Hausdorff space then it is (τ, μ, μ) –normal .

2.20 - Theorem

Let (X, τ ,μ) be a bitopological space and let (Y, τ_Y, μ_Y) be a τ- closed subspace of (X ,τ, μ).If (X ,τ, μ) is m(τ-μ) paracompact w .r .t .μ , then (Y, τ_Y, μ_Y) is $m(\tau_Y-\mu_Y)$ paracompact w .r .t. μ_Y .

Proof

Suppose that (Y,τ_Y,μ_Y) be a τ- closed subspace of m(τ-μ) paracompact w .r .t.μ space (X,τ ,μ). Show that (Y,τ_Y,μ_Y) is $m(\tau_Y-\mu_Y)$paracompact w .r .t μ_Y.

Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ- open cover of Y with card. ≤m.

. Since U_λ is τ_Y-open subset of Y, there is τ- open subset V_λ of X such that each $U_\lambda = V_\lambda \cap Y$. The collection. $\prod = \{V_\lambda : \lambda \in \Delta\} \bigcup \{X/Y\}$ form a τ-open cover of`` X with card.≤m. By hypothesis $\prod$ has μ-open refinement $W = \{W_\gamma : \gamma \in \Gamma\}$ which is locally finite w.r.t. μ.

Now, let A ={ $W_\gamma \cap Y | \gamma \in \Gamma\}$,then A is a collection of μ_Y-open subset of Y , hence A is a cover Y and refine U locally finite w.r.t.μ. Therefore (X, τ_Y ,μ_Y) is $m(\tau_Y - \mu_Y)$ paracompact w .r .t. μ_Y.

2.21 - Corollary

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Let (X, τ ,μ) be a bitopological space and let (Y, τ_Y ,μ_Y) be a τ- closed subspace of (X ,τ, μ).If (X ,τ, μ) is (τ-μ) paracompact w .r .t μ ,then (Y, τ_Y ,μ_Y) is (τ_Y-μ_Y) paracompact w .r .t μ_Y .

2.22 - Theorem

Let (X, τ ,μ) be a bitopological space and let $\chi = \{ X_i : X_i \in \tau \cap \mu : i \in I\}$ be a partion of X. the space (X, τ ,μ) is m(τ-μ)paracompact w .r .t .μ iff (X_i , τ_i ,μ_i) is m(τ_i -μ_i) paracompact w .r .t .μ_i . for every i.

Proof

The "only if "part. Since $Xi = X/\bigcup_{j\neq i} X_j$ is τ- closed , then the subspace (X_i , τ_i ,μ_i) is m(τ_i - μ_i) paracompact w .r .t μ_i for every i

The "if" part . Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ- open cover of X with card. ≤m .The collection $\prod = \{U_\lambda \cap X_i : \lambda \in \Delta\}$ is a τ_i-open cover of X_i with card.≤m for every i. Since (Xi, τ_i ,μ_i) is m(τ_i -μ_i) paracompact w .r .t μ_i .$\forall i$, there exist a μ_i-open refinement $A_i = \{A_{i\lambda} : \lambda \in \Delta\}$ of $\prod$ which is locally finite. w.r.t .μ_i.

Let $W = \{ \bigcup_{i \in I} A_{i\lambda} | \lambda \in \Delta \}$.

Then W is μ- open cover of X refining U, and locally finite w.r.t. μ. Hence(X,τ,μ)is m(τ-μ)paracompact w.r.t .μ.

2.23 - Corollary

Let (X, τ ,μ) be a bitopological space, $\chi = \{ X_i : X_i \in \tau \cap \mu i \in I \}$ be a partition of X. The space (X, τ ,μ) is (τ -μ) paracompact w .r .t. μ iff the space (X_i , τ_i ,μ_i) is (τ_i -μ_i) paracompact w .r .t. μ_i for every i.

2.24 - Theorem

Let (X, τ ,μ) be a m(τ -μ) paracompact w.r.t. μ bitopological space and let (Y,τ_Y ,μ_Y) be a subspace of(X, τ ,μ). If Y is F_σ-set relative to τ then (Y,τ_Y ,μ_Y) is m(τ_Y-μ_Y) semiparacompact w.r.t. μ_Y.

Proof

Suppose Y is F_σ-set relative to τ . Then $Y = \bigcup Y_n$ where each Y_n,is τ- closed Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ_Y-open cover of Y with card. ≤ m.Since each U_λ is τ_Y- open

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subset of Y, we have $U_\lambda = V_\lambda \cap Y$, where V_λ is τ-open subset of X for each $\lambda \in \Delta$. For each fixed n , $E_n = \{V_\lambda : \lambda \in \Delta\} \cup \{X / Y\}$ form a τ-open cover of X with card. ≤ m.By hypothesis E_n has a μ-open refinement $W = \{W_{\lambda n} : (\lambda, n) \in \Delta \times IN\}$ which is locally finite .w.r.t.μ.For each n, let $B_n = \{W_{\lambda n} \cap Y : W_{\lambda n} \cap Y_n \neq \phi\}$.Let $B = \bigcup B_n$.then B is collection of μ_Y-open set,covers Y refines U and σ- locally finite w.r.t. μ_Y There for (X, τ ,μ) is (τ_Y -μ_Y) semiparacompact w.r.t. μ_Y.

2.25 - Corollary

Let (X, τ ,μ) be a (τ -μ) paracompact w.r.t. μ biological space and let (Y,τ_Y ,μ_Y) be a subspace of(X, τ ,μ). If Y is Fσ-set relative to τ then (Y,τ_Y ,μ_Y) is (τ_Y-μ_Y) semiparacompact w.r.t. μ_Y.

2.26 - Corollary

Let (X, τ ,μ) be a m (τ -μ) paracompact w.r.t. μ biological space and let (Y,τ_Y ,μ_Y) be a subspace of (X, τ ,μ) If Y is Fσ-set relative to τ then (Y,τ_Y ,μ_Y) is (τ_Y –μ_Y)-a-paracompact w.r.t. μ_Y.

2.27 - Corollary

Let (X, τ ,μ) be a (τ-μ) paracompact w.r.t. μ bitopological space and let (Y,τ_Y ,μ_Y) be a subspace of(X, τ ,μ) .If Y is F_σ-set relative to τ, then (Y,τ_Y ,μ_Y) is (τ_Y-μ_Y) semiparacompact w.r.t. μ_Y

2.28 - Theorem

let (X, τ ,μ) be a bitopological space and let (Y, τ_Y ,μ_Y) be a τ- closed subspace of (X,τ, μ).If (X ,τ, μ) is m(τ-μ)-a- paracompact w .r .t .μ , then (Y, τ_Y ,μ_Y) is m(τ_Y ,μ_Y)-a- paracompact w .r .t. μ_Y .

Proof

Suppose that. (Y, τ_Y ,μ_Y) be a τ- closed subspace of m(τ-μ)-a- paracompact w .r .t. μ space (X, τ ,μ). To show that (Y, τ_Y ,μ_Y) is m(τ_Y -μ_Y)-a- paracompact w .r .t. μ_Y.

Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ_Y- open cover of Y with card. ≤m. Since each U_λ is a τ_Y-open subset of Y, there is a τ- open subset V_λ of X such that each the collection

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$\prod = \{V_\lambda : \lambda \in \Lambda\} \bigcup \{X / Y\}$ form a τ -open cover of X with card. ≤m. By hypothesis $\prod$ has refinement $W = \{W_\gamma : \gamma \in \Gamma\}$ (not necessarily either μ-open or μ-closed) which is locally finite.w.r.t μ.

Now, let $A = \{W_\gamma \bigcap Y, \gamma \in \Gamma\}$,then A is a collection of subsets of Y(not necessarily either μ_Y-open or μ_Y-closed).Then A is a cover Y refines U and is locally finite . w.r.t .μ_Y. Therefore (X, τ_Y ,μ_Y) is m(τ_Y - μ_Y)-a- paracompact w .r .t. μ_Y.

2.29 - Corollary

Let (X, τ ,μ) be a bitopological space and let (Y, τ_Y ,μ_Y) be a τ- closed subspace of (X ,τ, μ).If (X ,τ, μ) is (τ -μ)-a- paracompact w .r .t. μ , then (Y, τ_Y ,μ_Y) is (τ_Y -μ_Y)-a- paracompact w .r .t .μ_Y .

2.30 - Theorem

Let (X,τ,μ)be a bitopological space and let $\chi = \{ X_i : X_i \in \tau \bigcap \mu : i \in I\}$ be a partition of X. The space (X, τ ,μ) is m(τ -μ)-a- paracompact w .r .t. μ ,iff (X_i ,τ_i,μ_i) is m(τ_i -μ_i)-a- paracompact w .r .t. μ_i for every i.

Proof

The "only if "part. Since

$$Xi = X / \bigcup_{j \neq i} X_j$$

is τ- closed , then the subspace (X_i,τ_i,μ_i) is m(τ_i-μ_i)-a-paracompact w .r .t. μ for every i

The "if" part .

Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ-open cover of X with card. ≤m .The collection $\prod = \{U_\lambda \bigcap X_i : \lambda \in \Delta\}$ is a τ_i- open cover of Xi with card. ≤m for every i. (X_i, τ_i ,μ_i) is m(τ_i -μ_i)-a- paracompact w .r .t. μ_i $\forall i$, there exist a refinement $Ai = \{A_{i\lambda} : \lambda \in \Delta\}$ of $\prod$ (not necessarily either μ_i-open or μ_i-closed) which is locally finite. w.r.t μ_i.

Let $W = \{ \bigcup_{i \in I} A_{i\lambda} | \lambda \in \Delta \}$.

Then W is a cover of X(not necessarily either μ-open or μ-closed) , refine U and is locally finite w.r.t .μ. hence W locally finite w.r.t μ . Hence (X,τ,μ) is a m(τ-μ) –a- paracompact w.r.t μ.

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2.31 - Corollary

Let (X,τ,μ) be a bitopological space and let $\chi = \{ X_i : X_i \in \tau \cap \mu \in I \}$ be a partition of X. The space (X, τ ,μ) is τ -μ)-a- paracompact w .r .t .μ iff the space (X_i, τ_i, μ_i) is (τ_i -μ_i) –a-paracompact w .r .t μi for every i.

2.32 - Theorem

If each τ-open set in an m(τ-μ) paracompact w .r .t μ bitopological space (X,τ,μ) is m(τ-μ) paracompact w .r .t. μ , then every subspace
(Y, τ_Y, μ_Y) is m(τ_Y-μ_Y) paracompact w .r .t .μ_Y .

Proof

Let $U = \{U_\lambda : \lambda \in \Delta\}$ is a τ_Y-open cover of Y with card.$\leq m$.Since each U_λ is τ_Y-open inY,we have $U_\lambda = V_\lambda \cap Y$ where V_λ is a τ-open subset of X,for every $\lambda \in \Delta$.Then $G = \underset{\lambda \in \Delta}{U} V_\lambda$ is a τ_Y-open set .Let $V = \{V_\lambda, \lambda \in \Delta\}$ be a τ_Y-open cover of G with card. $\leq m$.By hypothesis G is m(τ-μ) paracompact w .r .t. μ .Thus V has a μ-open refinement $A = \{A_\gamma, \gamma \in \Gamma\}$ which is locally finite w .r .t μ .

Set

$$B = \{B_\gamma, \gamma \in \Gamma\}, \qquad \text{where } B_\gamma = A_\gamma \cap Y .$$

then B is μ_Y-open cover of Y, refine U,and locally finite w .r .t μ_Y.

Therefore (Y, τ_Y, μ_Y) is m(τ_Y-μ_Y) paracompact w .r .t. μ_Y.

2.33 - Corollary

If each τ-open set in (τ - μ) paracompact w .r .t μ. the bitopological space is (τ - μ) paracompact w .r .t μ. Then every subspace (Y, τ_Y, μ_Y) is (τ_Y-μ_Y) para-compact w .r .t. μ_Y.

2.34 - Theorem

If *f* is (μ- τ`) closed , (μ-μ`) continuous mapping of a bitopological space (X,τ,μ) onto m(τ`-μ`) paracompact w.r.t.μ` bitopological space (Y,τ`,μ`) such that Z= $f^{-1}(y) : y \in Y$ is m(τ-μ) compact , then (X,τ,μ) is m(τ-μ) paracompact w. r. t. μ .

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Proof

Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ-open cover of X with card. $\leq m$. Then U cover of Z .Since Z is m(τ-μ) compact , there exists a finite subset γ of Δ such that $Z \subset \bigcup_{\lambda\in\gamma} U_\lambda$,where U_λ is a μ –open set for every $\lambda \in \gamma$.

Let Γ be the family of all finite sub set γ of Δ , then $|\Gamma| \leq m$.

Set $$V_\gamma = Y / f\left[X / \bigcup_{\lambda\in\gamma} U_\lambda\right].$$

Since $\bigcup_{\lambda\in\gamma} U_\lambda$ is a μ –open set , the set $X / \bigcup_{\lambda\in\gamma} U_\lambda$ is μ – closed , and since f is(μ-τ`) closed , then $f\left[X / \bigcup_{\lambda\in\gamma} U_\lambda\right]$ is τ`– closed in (Y,τ`,μ`) , hence V_γ is a τ`–open . Moreover $y \in V_\gamma$ and $f^{-1}[V_\gamma] \subset \bigcup_{\lambda\in\gamma} U_\lambda$ .Therefore $V = \{V_\gamma : \gamma \in \Gamma\}$is a τ`– open cover of Y with card. $\leq m$ .Since (Y,τ`,μ`) is m(τ`- μ`) paracompact w. r. t. μ` , then V has a μ` - open refinement $W = \{W_\delta : \delta \in \Omega\}$ which is locally finite w. r. t μ` .Set $\Pi = \{f^{-1}[W_\delta] \cap U_\lambda : (\delta, \lambda) \in \Omega \times \gamma_\delta\}$.then Π is a μ –open cover of X, refines U,and locally finite w. r. t μ. Therefore(X,τ,μ) is m(τ-μ) paracompact w. r. t μ.

2.35 - Corollary

If *f* is (μ- τ`) closed , (μ-μ`) continuous mapping of a bitopological space (X,τ,μ) onto (τ`-μ`) paracompact w.r.t.μ` bitopological space (Y,τ`,μ`) such that Z= $f^{-1}(y)$: $y \in Y$ is (τ-μ) compact , then (X,τ,μ) is
(τ-μ) paracompact w. r. t. μ .

2.36 - Theorem

If *f* is (μ- τ`) closed , (μ-μ`) continuous mapping of a bitopological space (X,τ,μ) onto m(τ`-μ`)semiparacompact w.r.t.μ` bitopological space (Y,τ`,μ`) such that Z= $f^{-1}(y)$: $y \in Y$ is m(τ-μ) compact , then (X,τ,μ) is m(τ-μ)semi-paracompact w. r. t. μ .

Proof

Let $U = \{U_\lambda : \lambda \in \Delta\}$ be a τ-open cover of X with card. $\leq m$ .Then U is a cover of Z. Since Z is m (τ-μ)compact,there exists afinite subset γ of Δ such

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That $Z \subset \bigcup_{\lambda \in \gamma} U_\lambda$, where U_λ is a μ-open set for every $\lambda \in \gamma$.Let Γ be the family of all finite subset γ of Δ, then $|\Gamma| \leq m$.

Set

$$V_\gamma = Y / f\left[X / \bigcup_{\lambda \in \gamma} U_\lambda\right].$$

Since $\bigcup_{\lambda \in \gamma} U_\lambda$ is μ-open set, the set $X / \bigcup_{\lambda \in \gamma} U_\lambda$ is μ-closed ,and since f is (μ- τ`)closed, then $f\left[X / \bigcup_{\lambda \in \gamma} U_\lambda\right]$ is τ`-closed in(Y,τ`,μ`) ,hence V_γ is τ` -open and $y \in V_\gamma$ and $f^{-1}[V_\gamma] \subset \bigcup_{\lambda \in \gamma} U_\lambda$.Therefore $V = \{V_\gamma : \gamma \in \Gamma\}$is a τ` -open cover of Y with card. $\leq m$.Since (Y,τ`,μ`) is m (τ`-μ`) semiparacompact w. r. t μ`,then V has a μ`-open refinement $W = \underset{n}{U} W_n$ where every W_n is locally finite w. r. t μ`.

Set

$$W_n = \{W_{n\delta} : \delta \in \Omega\}. \text{ Thus } \quad W = \underset{n}{U}\{W_{n\delta} : \delta \in \Omega\}.$$

Set $C = \underset{n}{U} C_n$, where $C_n = \left\{f^{-1}[W_{n\delta}] \cap U_\lambda : (\delta, \lambda) \in \Omega \times \gamma_\delta\right\}$. We claime that C_n is

(i) collection of μ-open sets;

(ii) locally finite w. r. t. μ;

Proof of (i)

Since $W_{n\delta}$ is a μ` -open $\forall \delta \in \Delta$ and $f$ is (μ-μ`) continuous, the set $\overset{-1}{f}[W_{n\delta}]$ is a μ-open $\forall \delta \in \Delta$,and since U_λ is a μ-open $\forall \lambda \in \gamma_\delta$,then $\overset{-1}{f}[W_\delta] \cap U_\lambda$ is a μ-open $\forall (\delta, \lambda) \in \Delta \times \gamma_\delta$.

Proof of (ii)

Let $x \in X \Rightarrow \exists y \in Y \ni y = f(x)$.Since W_n is locally finite w. r. t. μ $\Rightarrow \exists \mu_Y - nhd$ N of x such that $N \cap W_{n\delta} = \phi$ for all but finite number of δ$\Rightarrow f^{-1}[N] \cap \left(f^{-1}[W_{n\delta}] \cap U_\lambda\right) = \phi$ for all but finite number of (δ,λ) since f is (μ-μ`) continuous ,then $f^{-1}[N]$is a μ-nhd of x .Hence C_n is locally finite w. r. t μ. Its remains to show that C is:

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(i*) cover X ,and

(ii*) refine U

proof of (i*)

Let $x\in X \Rightarrow \exists U_\lambda \ni x\in U_\lambda$ and $\exists y\in Y \ni y=f(x) \Rightarrow \exists W_{n\delta} \ni y\in W_{n\delta}$ for some n,δ $\Rightarrow x\in \overset{-1}{f}[W_{n\delta}]$ for some n,δ $\Rightarrow x\in f^{-1}[W_{n\delta}]\cap U_\lambda$ for some(δ,λ).

Proof of (ii*)

Since $f^{-1}[W_{n\delta}]\cap U_\lambda \subset U_\lambda, \forall_{n,\delta} \Rightarrow \bigcup_{n=1}^{\infty}\left(f^{-1}[W_{n\delta}]\cap U_\lambda\right) \subset U_\lambda$

i.e Π refine U_λ. Therefore (X,τ,μ) is m(τ-μ) semiparacompact w. r. t μ.

2.37 - Corollary

If *f* is (μ- τ`) closed , (μ-μ`) continuous mapping of a bitopological space (X,τ,μ) onto (τ`-μ`)semiparacompact w.r.t.μ` bitopological space (Y,τ`,μ`) such that Z= $f^{-1}(y): y\in Y$ is (τ-μ) compact , then (X,τ,μ) is (τ-μ)semiparacompact w. r. t. μ .

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