

On preparcompactness in bitopological spaces

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Abstract

J. Dieudonne [8], introduced the notions of paracompactness and Martin M. K. [9] , introduced the notions of paracompactness in bitopological spaces and K. AL-Zoubi and S. AL-Ghour [10], introduced the notions of P3-paracompactness of topological space in terms of preopen sets .In this paper, we introduce paracompactness in bitopological spaces in terms of ij-preopen sets . We obtain various characterizations, properties of paracompactness and its relationships with other types of spaces.

Key words: ij-preparcompact, ij-precontinuous, separation axioms .

1. Introduction

The concepts of regular open , regular closed , semiopen , semiclosed , and preopen sets have been introduced by many authors in a topological space (cf. [1-4]). These concepts are extended to bitopological spaces by many authors (cf. [5-7]) .

Throughout the present paper (X , τ_1 , τ_2) and (Y , μ_1 , μ_2) (or simple X and Y) denote bitopological spaces . when A is a subset of a space X , we shall denote the closure of A and the interior of A in (X , τ_i) by $\tau_i -clA$ and $\tau_i -intA$, respectively, where $i= 1,2$, and $i,j = 1,2 ; i \neq j$.

A subset A of X is said to be ij- preopen (resp. ij-semiopen ,ij-regular open , ij-regular closed and ij-preclosed) if

$$A \subseteq \tau_i - int(\tau_j - clA) \text{ (resp. } A \subseteq \tau_i - cl(\tau_j - int A), A = \tau_i - int(\tau_j - clA) ,$$

$A = \tau_j - cl(\tau_i - int A)$, and $\tau_j - cl(\tau_i - int A) \subseteq A$) . The family of all ij-semiopen (resp. ij- regular open and ij- preopen) sets of X is denoted by ij-SO(X) (resp. ij-RO(X) and ij-PO(X)) . The intersection of all ij- preclosed sets which contain A is called the ij- preclosure of A and is denoted by ij-PclA . Obviously , ij-PclA is the smallest ij-preclosed set which contains A .

Definition 1.1 .A bitopological space (X , τ_1 , τ_2) is called ij-locally indiscrete if every $\tau_i -open$ subset of X is $\tau_j -closed$.

Definition 1.2 . A collection $\Xi = \{ F_\lambda : \lambda \in \Gamma \}$ of subsets of X is called ,

- (1) locally finite with respect to the topology τ_i (respectively , ij-strongly locally finite) , if for each $x \in X$, there exists $U_x \in \tau_i$ (respectively, $U_x \in ij - RO(X)$) containing x and U_x which intersects at most finitely many members of Ξ ;
- (2) ij-P-locally finite if for each $x \in X$, there exists a ij- preopen set U_x in X containing x and U_x which intersects at most finitely many members of Ξ .

Definition 1.3 . A bitopological space (X, τ_1, τ_2) is called $(\tau_i - \tau_j)$ paracompact with respect to the topology τ_i , if every τ_i -open cover of X has τ_j -open refinement which is locally finite with respect to the topology τ_i [9].

We obtain the following lemmas :

Lemma 1.4. For a bitopological space (X, τ_1, τ_2) , the followings are equivalent :

- (a) (X, τ_1, τ_2) is ij-locally indiscrete ;
- (b) Every subset of X is ij-preopen ;
- (c) Every τ_j -closed subset of X is ij-preopen .

Lemma 1.5 . If A is a ij-preopen subset of X , then τ_i -cl A is ij-regular closed .

Lemma 1.6 . Let A and B be subsets of a space X . Then

- (a) If $A \in ij-PO(X)$ and $B \in \tau_i$, then $(A \cap B) \in ij-PO(X)$.
- (b) If $A \in ij-PO(X)$ and $B \in ij-SO(X)$, then $(A \cap B) \in ij-PO(B, \tau_{1B}, \tau_{2B})$.
- (c) If $A \in ij-PO(B, \tau_{1B}, \tau_{2B})$ and $B \in ij-PO(X)$, then $A \in ij-PO(X)$.

Theorem 1.7 . Let $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ be a collection of ij-semiopen subsets of a bitopological space (X, τ_1, τ_2) , then Ξ is ij-P-locally finite if and only if it is ij-strongly locally finite .

Proof . (Necessity) . Since every ij-regular open subset of X is ij-preopen , so Ξ is ij-P-locally finite.

(Sufficiency) . Let $x \in X$ and W_x be a ij-preopen subset of X such that $x \in W_x$ and $W_x \cap F_{\lambda_i} \neq \Phi$ for each $i = 1, 2, \dots, n$. Put $R_x = \tau_i - \text{int}(\tau_j - clW_x)$, then R_x is a ij-regular open which contains x . We show for every $\lambda \in \Gamma / \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, $F_\lambda \cap R_x = \Phi$. For each $\lambda \in \Gamma$, choose $U_\lambda \in \tau_j$ such that $U_\lambda \subseteq P_\lambda \subseteq \tau_i - clU_\lambda$. Now, if $F_\lambda \cap R_x \neq \Phi$, then $\tau_i - clU_\lambda \cap R_x \neq \Phi$ and so $\tau_i - clU_\lambda \cap \tau_i - \text{int}(\tau_j - clW_x) \neq \Phi$, which implies that there exist $p \in \tau_i - clU_\lambda$ and $p \in \tau_i - \text{int}(\tau_j - clW_x)$. So for each τ_i -open set V_p containing p such that $V_p \cap U_\lambda \neq \Phi$, but $\tau_i - \text{int}(\tau_j - clW_x)$ is a τ_i -open containing p , which implies that $U_\lambda \cap \tau_i - \text{int}(\tau_j - clW_x) \neq \Phi$, since $\tau_i - \text{int}(\tau_j - clW_x) \subseteq \tau_j - clW_x$, so we get that $U_\lambda \cap \tau_j - clW_x \neq \Phi$, then there exists $q \in U_\lambda$ and $q \in \tau_j - clW_x$. Therefore for each τ_j -open sets H_q containing q , $H_q \cap W_x \neq \Phi$, but U_λ is τ_j -open containing q , so $U_\lambda \cap W_x \neq \Phi$, which implies that $\Phi \neq U_\lambda \cap W_x \subseteq P_\lambda \cap W_x$, which contradict the hypothesis . Thus $\lambda \in \Gamma / \{\lambda_1, \lambda_2, \dots, \lambda_n\}$.

Now using the above theorem and the fact that every ij-regular open is τ_i -open and every τ_j -open, ij-regular closed set is ij-semiopen set, we obtain the following corollaries :

Corollary 1.8 . Let $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ be a collection of τ_j -open (ij-regular closed) subset of a bitopological space (X, τ_1, τ_2) . If Ξ is ij-P-locally finite , then Ξ is locally finite with respect to the topology τ_i .

Now by using the above theorem , corollary 1.8 and the definition 1.3 , we obtain the following corollary :

Corollary 1.9 . A bitopological space (X, τ_1, τ_2) is $(\tau_i - \tau_j)$ paracompact with respect to the topology τ_i if and only if every τ_i -open cover of X has a τ_j -open refinement which is ij-p-locally finite .

Theorem 1.10 . Let $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ be a collection of subsets of a bitopological space (X, τ_1, τ_2) . Then

(a) Ξ is ij-P-locally finite if and only if $\{ij - pclF_\lambda : \lambda \in \Gamma\}$ is ij-P-locally finite ;

(b) If Ξ is ij-P-locally finite , then
$$\bigcup_{\lambda \in \Gamma} ij - pclF_\lambda = ij - pcl\left(\bigcup_{\lambda \in \Gamma} P_\lambda\right)$$
 ;

(c) Ξ is locally finite with respect to the topology τ_i if and only if the collection $\{ij - pclF_\lambda : \lambda \in \Gamma\}$ is locally finite with respect to the topology τ_i .

Proof . (a) Suppose that , Ξ is ij-P-locally finite . For each $x \in X$, there exists ij-preopen U_x containing x which meets only finitely many of the sets F_λ , say $F_{\lambda_1}, \dots, F_{\lambda_n}$. Since $F_{\lambda_k} \subseteq ij - pclF_{\lambda_k}$ for each $k=1,2, \dots, n$, thus U_x meets $ij - pclF_{\lambda_1}, \dots, ij - pclF_{\lambda_n}$. Therefore $\{ij - pclF_\lambda : \lambda \in \Gamma\}$ is ij-P-locally finite .

Conversely. Let $x \in X$, there exists ij-preopen U_x containing x which meets only finitely many of the sets $ij - pclF_\lambda$. Say $ij - pclF_{\lambda_1}, \dots, ij - pclF_{\lambda_n}$, we get that $U_x \cap ij - pclF_{\lambda_k} \neq \Phi$ for each $k = 1, \dots, n$. Let $q \in U_x$ and $q \in ij - pclF_{\lambda_k}$, which implies that , for every ij-preopen set V_q such that $V_q \cap F_{\lambda_k} \neq \Phi$, but U_x be ij-preopen containing q , so $U_x \cap F_{\lambda_k} \neq \Phi$ for each $k=1, \dots, n$. Thus Ξ is ij-P-locally finite.

(b) Suppose, Ξ is ij-P-locally finite , which can be easily got
$$\bigcup_{\lambda \in \Gamma} ij - pclF_\lambda \subseteq ij - pcl\left(\bigcup_{\lambda \in \Gamma} P_\lambda\right)$$
 . On the other hand , let $q \in ij - pcl\left(\bigcup_{\lambda \in \Gamma} F_\lambda\right)$, then

every ij-preopen V_q such that $V_q \cap \left(\bigcup_{\lambda \in \Gamma} F_\lambda\right) \neq \Phi$. By hypothesis , there exists ij-preopen U_q which meets only finitely many of the sets F_λ , say $F_{\lambda_1}, \dots, F_{\lambda_n}$. Thus for each ij-preopen set V_q containing q , $V_q \cap \left(\bigcup_{k=1}^n F_{\lambda_k}\right) \neq \Phi$, which implies that

$q \in ij - pcl(\bigcup_{k=1}^n F_{\lambda_k}) = \bigcup_{k=1}^n ij - pclF_{\lambda_k}$, there exists h such that $q \in ij - pclF_{\lambda_h}$. Therefore

we get that $q \in \bigcup_{\lambda \in \Gamma} ij - pclF_{\lambda}$ and hence $\bigcup_{\lambda \in \Gamma} ij - pclF_{\lambda} = ij - pcl(\bigcup_{\lambda \in \Gamma} P_{\lambda})$.

(c) Suppose, Ξ is locally finite with respect to the topology τ_i , which implies that for each $x \in X$, there exists τ_i -open set U_x which meets only finitely many of the sets F_{λ} , say $F_{\lambda_1}, \dots, F_{\lambda_n}$, but $F_{\lambda_k} \subseteq ij - pclF_{\lambda_k}$, we get that U_x which meets $ij - pclF_{\lambda_1}, \dots, ij - pclF_{\lambda_n}$. Thus $\{ij - pclF_{\lambda} : \lambda \in \Gamma\}$ is locally finite with respect to the topology τ_i .

Conversely, let $x \in X$, then there exists τ_i -open set U_x which meets only finitely many of the sets $ij - pclF_{\lambda}$'s, say $ij - pclF_{\lambda_1}, \dots, ij - pclF_{\lambda_n}$. Now choose a point $q \in U_x$ and $q \in ij - pclF_{\lambda_k}$. For each $k=1,2,\dots,n$, therefore for each ij -preopen set V_q containing q such that $V_q \cap F_{\lambda_k} \neq \Phi$, but $q \in U_x$, then we get U_x which meets only finitely many of the sets F_{λ} 's. Hence Ξ is locally finite with respect to the topology τ_i .

Definition 1.11 .A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called :

- (1) ij -precontinuous if for each $V \in ij - PO(Y)$, $f^{-1}(V) \in ij - PO(X)$;
- (2) ij -strongly precontinuous if for each $V \in ij - PO(Y)$, $f^{-1}(V) \in \tau_i$;
- (3) ij -preclosed if for each $V \in ij - PC(X)$, $f(V) \in ij - PC(Y)$;
- (4) ij -strongly preclosed if for each $V \in \tau_i - C(X)$, $f(V) \in ij - PC(Y)$;
- (5) ij -preopen if for each $V \in ij - PO(X)$, $f(V) \in ij - PO(Y)$;
- (6) $(\tau_i - \mu_i)$ -continuous ($(\tau_i - \mu_i)$ -open and $(\tau_i - \mu_i)$ -closed) if $f : (X, \tau_i) \rightarrow (Y, \mu_i)$ for $i=1,2$ are $(\tau_1 - \mu_1)$ -continuous and $(\tau_2 - \mu_2)$ -continuous ($(\tau_1 - \mu_1)$ -open, $(\tau_2 - \mu_2)$ -open and $(\tau_1 - \mu_1)$ -closed, $(\tau_2 - \mu_2)$ -closed).

Lemma 1.12 . Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be a function . Then

- (1) f is ij -preclosed if and only if for every $y \in Y$ and $P \in ij - PO(X)$ such that $f^{-1}(y) \subset P$, there is $V \in ij - PO(Y)$ such that $y \in V$ and $f^{-1}(V) \subset P$.
- (2) f ij -strongly preclosed if and only if for every $y \in Y$ and $P \in \tau_i$ which contains $f^{-1}(y)$, there is $V \in ij - PO(Y)$ such that $y \in V$ and $f^{-1}(V) \subset P$.
- (3) If f be $(\tau_i - \mu_i)$ -continuous, $(\tau_i - \mu_i)$ -open function, then f is ij -preopen function.

Proposition 1.13 .Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be a ij-precontinuous function . If $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ is a ij-P-locally finite collection in Y , then $f^{-1}(\Xi) = \{f^{-1}(F_\lambda) : \lambda \in \Gamma\}$ is a ij-P-locally finite collection in X .

Proposition 1.14 . Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be a ij-strongly precontinuous function . If $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ is a ij-P-locally finite collection in Y , then $f^{-1}(\Xi) = \{f^{-1}(F_\lambda) : \lambda \in \Gamma\}$ is a locally finite collection with respect to the topology τ_i .

Definition 1.15 .A subset A of a bitopological space (X, τ_1, τ_2) is called ij-strongly compact if for every cover of A by ij-preopen subsets of X has a finite subcover .

Proposition 1.16 . Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be a ij-strongly closed function such that $f^{-1}(y)$ is compact relative to the topology τ_i for every $y \in Y$. If $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ is a locally finite collection with respect to the topology τ_i , then $f(\Xi) = \{f(F_\lambda) : \lambda \in \Gamma\}$ is a ij-P-locally finite collection in Y .

Proposition 1.17 . Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be a ij-preclosed function such that $f^{-1}(y)$ is ij-strongly compact relative to X for every $y \in Y$. If $\Xi = \{F_\lambda : \lambda \in \Gamma\}$ is a ij-P-locally finite collection in X , then $f(\Xi) = \{f(F_\lambda) : \lambda \in \Gamma\}$ is a ij-P-locally finite collection in Y .

2 . ij-preparacompact spaces

In this section, we introduce the generalized paracompact and separation axioms using the notions of ij-preopen sets in bitopological spaces, and give some characterization of these types of spaces and study the relationships between them and other well known spaces .

Definition 2.1 . A bitopological space (X, τ_1, τ_2) is called ij-preparacompact if every τ_i -open cover of X has a ij-P-locally finite ij-preopen refinement .

Definition 2.2 . Let (X, τ_1, τ_2) be a bitopological space . The space X is said to be :

- (1) pair wise Hausdorff space if and only if for each distinct two points x, y in X , there exists two disjoint τ_i -open U and τ_j -open V such that $x \in U$ and $y \in V$ [11] ;
- (2) pair wise regular if and only if for each τ_i -closed set F and $x \in X, x \notin F$, there are two disjoint τ_i -open U and τ_j -open V such that $F \subset U, x \in V$ [11];
- (3) pair wise P-regular if and only if for each τ_i -closed set F and $x \in X, x \notin F$, there are two disjoint ij-preopen sets U and V , such that $x \in U$ and $F \subseteq V$;

- (4) pair wise P-normal if and only if whenever A and B are disjoint τ_i -closed sets in X, there are disjoint ij-preopen sets U and V with $A \subseteq U$ and $B \subseteq V$;
- (5) pairwise $P-T_1$ -space if for each two distincts points x and y in X, there are ij-preopen sets U and V in X such that $x \in U$ and $y \in V$;
- (6) pairwise T_1 -space if for each two distincts points x and y in X, there are τ_i -open U and τ_j -open V in X such that $x \in U$ and $y \in V$ [11];
- (7) pairwise $P-T_3$ -space if it's pairwise P-regular $P-T_1$ -space ;
- (8) pairwise $P-T_4$ -space if it's pairwise P-normal $P-T_1$ -space .

Lemma 2.3 .A bitopological space (X, τ_1, τ_2) is pairwise p-regular if and only if for each τ_i -open set U and $x \in U$, then there exists $P \in ij-PO(X)$ such that $x \in P \subseteq ij-pclP \subseteq U$.

Theorem 2.4 . Every ij-preparacompact pairwise Hausdorff bitopological space (X, τ_1, τ_2) is pair wise p-regular .

Proof : Let A be a τ_i -closed and let $x \notin A$. For each $y \in A$, choose an τ_i -open U_y and τ_i -open H_x such that $y \in U_y$, $x \in H_x$ and $U_y \cap H_x = \Phi$, so we get that $x \notin ij-pclU_y$. Therefore the family $U = \{U_y : y \in A\} \cup \{X - A\}$ is an τ_i -open cover of x and so it has a ij-P-locally finite ij-preopen refinement Π . Put $V = \{H \in \Pi : H \cap A \neq \Phi\}$, then V is a ij-preopen set containing A and $ij-pclV = \cup \{ij-pclH : H \in \Pi \text{ and } H \cap A \neq \Phi\}$ (Theorem 1.10 (b)). Therefore $U = X - ij-pclV$ is a ij-preopen set containing x such that U and V are disjoint subsets of X . Thus X is pairwise p-regular .

From the above theorem, the following corollaries are obtained :

Corollary 2.5 .Every ij-preparacompact pairwise Hausdorff bitopological space (X, τ_1, τ_2) is pair wise p-normal .

Corollary 2.6 .Every ij-preparacompact pairwise Hausdorff bitopological space (X, τ_1, τ_2) is

- (a) pairwise $p-T_3$ -space ;
- (b) pairwise $p-T_4$ -space .

Theorem 2.7 . Let (X, τ_1) and (X, τ_2) are regular spaces . Then (X, τ_1, τ_2) is ij-preparacompact if and only if every τ_i -open cover Ξ of X has a ij-P-locally finite ij-preclosed refinement Σ .

Proof : To prove necessity, let Ξ be τ_i -open cover of X . For each $x \in X$, we choose a member $U_x \in \Xi$ and by the regularity of (X, τ_1) and (X, τ_2) , an τ_i -open subsets V_x containing x such that $\tau_i-clV_x \subset U_x$. Therefore $\Psi = \{V_x : x \in X\}$ is an

τ_i -open cover of X and so by hypothesis it has a ij-P-locally finite ij-preopen refinement, say $\Omega = \{W_\lambda : \lambda \in \Gamma\}$. Now, consider the collection $ij-pcl\Omega = \{ij-pclW_\lambda : \lambda \in \Gamma\}$ as a ij-P-locally finite (Theorem 1.10 (a)) of ij-preclosed subsets of (X, τ_1, τ_2) . Since for every $\lambda \in \Gamma$, $ij-pclW_\lambda \subseteq ij-pclV_x \subseteq \tau_i-clV_x \subseteq U_x$ for some $U_x \in \Xi$, therefore $ij-pcl\Omega$ is a refinement of Ξ .

Conversely, let Ξ be an τ_i -open cover of X and Ψ be a ij-P-locally finite ij-preclosed refinement of Ξ . For each $x \in X$, choose $W_x \in ij-PO(X)$ such that $x \in W_x$ and W_x intersect at most finitely many members of Ψ . Let Σ be a ij-preclosed ij-P-locally finite refinement of $\Omega = \{W_x : x \in X\}$. For each $V \in \Psi$, let $V' = X/H$, where $H \in \Sigma$ and $H \cap V = \Phi$. Then $\{V' : V \in \Psi\}$ is a ij-preopen cover of X . Now, for each $V \in \Psi$, choose $U_V \in \Xi$ such that $V \subseteq U_V$, therefore the collection $\{U_V \cap V' : V \in \Psi\}$ is a ij-P-locally finite ij-preopen (Lemma 1.6 (a)) refinement of Ξ and thus (X, τ_1, τ_2) is ij-preparacompact.

Theorem 2.8 .Let A be a ij-regular closed subset of a ij-preparacompact bitopological space (X, τ_1, τ_2) . Then $(A, \tau_{1A}, \tau_{2A})$ is ij-preparacompact.

Proof : Let $\Psi = \{V_\lambda : \lambda \in \Gamma\}$ be an τ_i -open cover of A in $(A, \tau_{1A}, \tau_{2A})$. For each $\lambda \in \Gamma$, choose $U_\lambda \in \tau_i$ such that $V_\lambda = A \cap U_\lambda$, then the collection $\Xi = \{U_\lambda : \lambda \in \Gamma\} \cup \{X - A\}$ is an τ_i -open cover of the ij-preparacompact space X and so it has a ij-P-locally finite ij-preopen refinement. Say $\Psi = \{W_\delta : \delta \in \Delta\}$, then by Lemma 1.6(b) and since $ij-RO(X) \subseteq ij-SO(X)$, the collection $\{A \cap W_\delta : \delta \in \Delta\}$ is a ij-P-locally finite ij-preopen refinement of Ψ in $(A, \tau_{1A}, \tau_{2A})$. This completes the proof.

Theorem 2.9 .Let A and B be subsets of a bitopological space (X, τ_1, τ_2) such that $A \subseteq B$.

- (1) If $B \in ij-PO(X)$ and A is ij-preparacompact relative to $(B, \tau_{1B}, \tau_{2B})$, then A is ij-preparacompact relative to (X, τ_1, τ_2) .
- (2) If $B \in ij-SO(X)$ and A is ij-preparacompact relative to (X, τ_1, τ_2) , then A is ij-preparacompact relative to $(B, \tau_{1B}, \tau_{2B})$.

Proof : (1) Let $\Xi = \{U_\lambda : \lambda \in \Gamma\}$ be an τ_i -open cover of A in X . Then the collection $\Xi_B = \{B \cap U_\lambda : \lambda \in \Gamma\}$ is an τ_{iB} -open cover of A in $(B, \tau_{1B}, \tau_{2B})$. Since A is ij-preparacompact relative to $(B, \tau_{1B}, \tau_{2B})$. Hence Ξ_B has a ij-P-locally finite ij-preopen refinement Ω_B in $(B, \tau_{1B}, \tau_{2B})$. Since $B \in ij-PO(X)$ and $U_\lambda \in \tau_i, \forall \lambda \in \Gamma$, so by lemma 1.6 (a), $B \cap U_\lambda \in ij-PO(X), \forall \lambda \in \Gamma$. Then the

collection Ω_B is a ij-P-locally finite ij-preopen refinement of Ξ in (X, τ_1, τ_2) . Therefore A is a ij-preparacompact relative to (X, τ_1, τ_2) .

(2) Let $\Psi = \{V_\lambda : \lambda \in \Gamma\}$ be an τ_{iB} -open cover of A in $(B, \tau_{1B}, \tau_{2B})$. For every $\lambda \in \Gamma$, choose $U_\lambda \in \tau_i$ such that $V_\lambda = B \cap U_\lambda$. Then the collection $\Xi = \{U_\lambda : \lambda \in \Gamma\}$ is an τ_i -open cover of A relative to (X, τ_1, τ_2) and so it has a ij-P-locally finite ij-preopen refinement of Ξ in (X, τ_1, τ_2) , say Σ . Thus, by lemma 1.6 (b), the collection $\Sigma_B = \{P \cap B : P \in \Sigma\}$ is a ij-P-locally ij-preopen refinement of Ξ in $(B, \tau_{1B}, \tau_{2B})$.

Corollary 2.10. Let A be a subset of a bitopological space (X, τ_1, τ_2) .

- (1) If $A \in ij-PO(X)$ and the subspace $(A, \tau_{1A}, \tau_{2A})$ is ij-preparacompact, then A is ij-preparacompact relative to (X, τ_1, τ_2) .
- (2) If $A \in ij-SO(X)$ and A ij-preparacompact relative to (X, τ_1, τ_2) , then the subspace $(A, \tau_{1A}, \tau_{2A})$ is ij-preparacompact.

Theorem 2.11. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be $(\tau_i - \mu_i)$ -continuous, $(\tau_i - \mu_i)$ -open, ij-strongly ij-preopen and surjective function such that $f^{-1}(y)$ is ij-strongly compact relative to (X, τ_1, τ_2) for every $y \in Y$. If (X, τ_1, τ_2) is ij-preparacompact, then so is (Y, μ_1, μ_2) .

Proof : Let $\Psi = \{V_\lambda : \lambda \in \Gamma\}$ be an μ_i -open cover of (Y, μ_1, μ_2) . Since f is $(\tau_i - \mu_i)$ -continuous, the collection $\Xi = f^{-1}(\Psi) = \{f^{-1}(V_\lambda) : \lambda \in \Gamma\}$ is an τ_i -open cover of the ij-preparacompact (X, τ_1, τ_2) space and so it has a ij-P-locally finite ij-preopen refinement, say Ω . Since f is $(\tau_i - \mu_i)$ -continuous and $(\tau_i - \mu_i)$ -open, then by lemma 1.12 (3), f is ij-preopen function and so by Proposition 1.17, the collection $f(\Omega)$ is a ij-P-locally finite ij-preopen refinement of Ψ . The result is proved.

Theorem 2.12. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be a $(\tau_i - \mu_i)$ -closed ij-precontinuous surjective function such that $f^{-1}(y)$ is τ_i -compact in (X, τ_i) for every $y \in Y$. If (Y, μ_1, μ_2) is ij-preparacompact, then so is (X, τ_1, τ_2) .

Proof : Let $\Xi = \{U_\lambda : \lambda \in \Gamma\}$ be an τ_i -open cover of a bitopological space (X, τ_1, τ_2) . For each $y \in Y$, Ξ is an τ_i -open cover of the τ_i -compact subspace $f^{-1}(y)$ and so there exists a finite subcover Γ_y of Γ such that $f^{-1}(y) \subseteq \bigcup_{\lambda \in \Gamma_y} U_\lambda$. Put

$U_y = \bigcup_{\lambda \in \Gamma_y} U_\lambda$ is τ_i -open in (X, τ_i) . As f is a $(\tau_i - \mu_i)$ -closed function, for each

$y \in Y$, there exists μ_i -open subset V_y of Y such that $y \in V_y$ and $f^{-1}(V_y) \subseteq U_y$. Then the collection $\Psi = \{V_y : y \in Y\}$ is an μ_i -open cover of the ij-preparacompact

space (Y, μ_1, μ_2) and so it has a ij-P-locally finite ij-preopen refinement, say $\Omega = \{W_\gamma : \gamma \in \Delta\}$. Since f is ij-precontinuous, the collection $f^{-1}(\Omega) = \{f^{-1}(W_\gamma) : \gamma \in \Delta\}$ is a ij-preopen ij-P-locally finite cover of (X, τ_1, τ_2) such that for each $\gamma \in \Delta$, $f^{-1}(W_\gamma) \subseteq U_y$ for some $y \in Y$. Finally, the collection $\{f^{-1}(W_\gamma) \cap U_\lambda : \gamma \in \Delta, \lambda \in \Gamma_y\}$ is a ij-P-locally finite ij-preopen refinement of \mathfrak{E} . Thus (X, τ_1, τ_2) is ij-preparacompact.

3.Conclsions

We define the paracompact bitopological spaces using the ij-preopen sets. And proved some properties and relations with some other spaces.

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