Constructing a Membership Functions by Composite Way

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Abstract:

   In all fuzzy sets we use Membership Functions (MFs),these functions either Linear or Nonlinear. This paper percents a new idea through answering the question asking; What happen if the MF is a composite function of two or more MFs or with wavelet function? A gotten result(s) which also a functions has the properties of MFs that construct it, that effected by composite order of functions, these properties chosen that each MF must be normal (its maximum value is 1), and defined with positive values that in closed interval [0,1] which will preserve through composition .

الملخَّص:

   في كُلّ المجموعاتِ الضبابيةِ نَستعملُ دوال العضوية (MFs)، تلك الدوال أمَا خطيّة أو غيرخطّية. في هذا البحث تقدم الفكرة من خلال سُؤال يسألِ؛ ماالذي يَحْدَثُ إذا MF حصلت كدالة مركّبة لـ(إثنان أو أَكثر)من MFs او دالة مويجة ؟ النتيجة او النتائج االتي تم تحصيلها و التي هي أيضاً دوال رَبَّما تمتلك خواصُ دوال MFs التي شكلت ذلك التركيب، والتي تتأثر بترتيب دوال التركيب ، هذه الخواصُ إلمختارة لدوال MFs يَجِبُ أَنْ تَكُونَ الدالة قياسية (المدى الأقصى للقيم ١)، ومعرّفَة بالقيم الموجبةِ والتي تنتمي للفترةِ المُغلقةِ [١،٠] التي سَتَبقيَ خلال التركيبِ.

Introduction:

   Each work try to improve some thing ,and this work is also try to do that through introduce different idea on previous thoughts that work in the same side .Many thoughts try to improve fuzzy systems from these try to increase number of inputs or/and outputs variable as occurred in reference(Reznik 1997, Babuška, Leondes1998) or change the form of MF that used in rule base(for outputs to be linear or non linear) for system to get on other types as in (Duch 2004, Reznik 1997, Soto 2003) .This paper percent a new idea on modify MF but through that we will construct our MF through compose among MFs (for different types linear or nonlinear) ,many tables for dataset values for variable where presented supported by graphs that reflex the features and properties of resulted MFs that its standard for MFs that effected by composite order of functions, from these properties that MF is normal (its maximum value is 1), and that in interval [0,1] .High order of compose were use too ,that to enhance idea .

Choice and Compose Membership Function MF for Work

   The MF that were chosen for perform through composition way that used here to construct MF has active features in work in fuzzy systems and then practical applications the linear and nonlinear MF have been used in this paper .

   For let $f : R \rightarrow [0,1]$ supposed as Gauss MF (Reznik 1997):

$$\rho(x) = e^{-(x-t)^2/2s^2}$$
   ….. (1)

Where parameter t is the center of symmetry and s is the width of values distributed through domain.

   For $x_1 < t < x_2$, $/t$ -x_1 $/ < e$ and $/t$ - $x_2/ < ∂$,that is $/t$ -x_1 $/ \neq /t$ - $x_2/$, $\forall$ t , x_1 , x_2 in some set , and t is a center of values , e and $∂$ are constants , such that $e \neq ∂$, this reflected through MFs .

   This function is supposed with center at origin $t = 0$ and standard width at $s = 1$,which will put in compose function that will use in this paper .Table (1) will show the dataset for variable x that were in interval[5.75,-5.57] and the corresponding Gauss MF values .
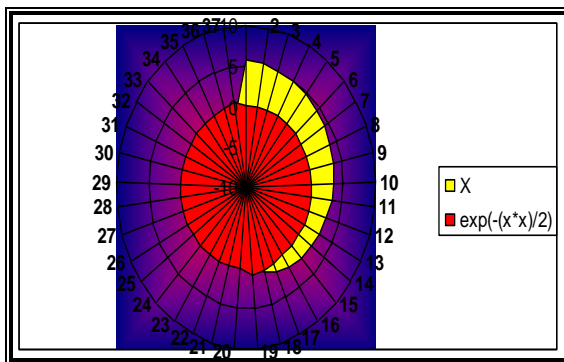
Graphics of this function were presented through different ways ;use linear ,polar linear and polar surface to study behavior of function as below :
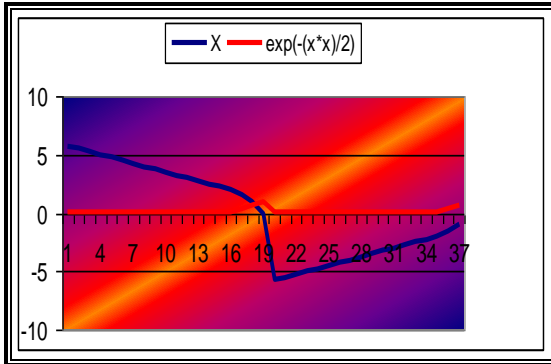
(a)
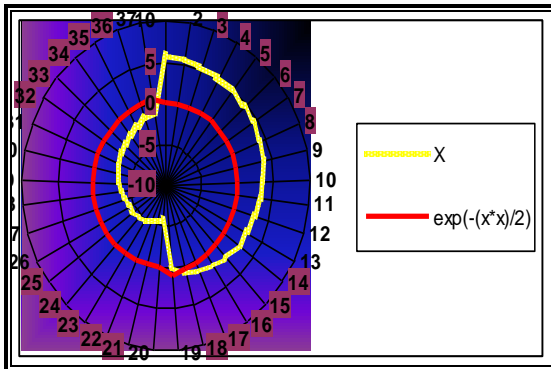
(b)

(c)

Figure (1): Drawing of Gaussian MF for -5.75 $\leq$ x $\leq$ 5.75

Figure (1) show the graphic in three ways note that all values are in [0,1] on dataset for table(1).

Table (1): Values of x and Gauss MF.

x	$e(-x^2/2)$
5.75	6.6156E-08
5.5	2.69958E-07
5.25	1.03485E-06
5	3.72665E-06
4.75	1.26071E-05
4.5	4.00653E-05
4.25	0.000119613
4	0.000335463
3.75	0.000883826
3.5	0.002187491
3.25	0.005086069
3	0.011108997
2.75	0.022794181
2.5	0.043936934
2.25	0.079559509
2	0.135335283
1.5	0.324652467
1	0.60653066
0	1
-5.75	6.6156E-08
-5.5	2.69958E-07
-5.25	1.03485E-06
-5	3.72665E-06
-4.75	1.26071E-05
-4.5	4.00653E-05
-4.25	0.000119613
-4	0.000335463
-3.75	0.000883826
-3.5	0.002187491
-3.25	0.005086069
-3	0.011108997
-2.75	0.022794181
-2.5	0.043936934
-2.25	0.079559509
-2	0.135335283
-1.5	0.324652467
-1	0.60653066

Composite and Composite with High Order:

The Composite MF supposed for same function that is for example Gauss MF with Gauss MF as: $\quad g \circ f = g(f(x)) \quad$ taking $\quad f = g$ (compose function with it self).

So: $(f \circ f)(x) = f(f(x)) = f(e^{-x^2/2})$

$$= e^{-(e^{-x^2/2})^2/2} \qquad\qquad ….. (2)$$

For triple composite or fourth composite, putting $h \circ g \circ f = h(g(f(x)))$ taking $f = g = h$ (compose function with it self),as:

$$(f \circ f \circ f)(x) = f(f(f(x))) = f(f(e^{-x^2/2})) = f(e^{-(e^{-x^2/2})^2/2}))$$

$$= e^{-[e^{-[e^{-x^2/2}]^2/2}]^2/2} \qquad = e^{-[e^{-[e^{-x^2/2}]_1]/2}} \quad ….. (3)$$

and $\quad (f \circ f \circ f \circ f)(x) = f(f(f(f(x)))) = f(f(f(e^{-x^2/2}))) \quad = e^{-[e^{-[e^{-[e^{-x^2/2}]_1]_1]/2}} \quad ….. (4)$

The following table will explain these compositions through dataset for x in interval [-1,1], and then the results also in [0,1] ;

Table (2): Values of x and Compose MFs for $-1 \leq x \leq 1$

x	$\rho(x)$	$\rho(\rho(x))$	$\rho(\rho(\rho(x)))$
0	1	0.60653066	0.831985954
0.1	0.995012479	0.609555719	0.830457033
0.2	0.980198673	0.618539194	0.825888612
0.3	0.955997482	0.633202127	0.818344022
0.4	0.923116346	0.653069388	0.807954275
0.5	0.882496903	0.677462965	0.79494843
0.6	0.835270211	0.705507298	0.779681112
0.7	0.782704538	0.736156021	0.762644833
0.8	0.726149037	0.768245288	0.744456832
0.9	0.666976811	0.800571819	0.725816814
1	0.60653066	0.831985954	0.707441517
-0.1	0.995012479	0.609555719	0.830457033
-0.2	0.980198673	0.618539194	0.825888612
-0.3	0.955997482	0.633202127	0.818344022
-0.4	0.923116346	0.653069388	0.807954275
-0.5	0.882496903	0.677462965	0.79494843
-0.6	0.835270211	0.705507298	0.779681112
-0.7	0.782704538	0.736156021	0.762644833
-0.8	0.726149037	0.768245288	0.744456832
-0.9	0.666976811	0.800571819	0.725816814
-1	0.60653066	0.831985954	0.707441517

Figures (2-5) showing the linear and polar graphic for dataset in table (2) ,note the graphs in polar how change around the polar .
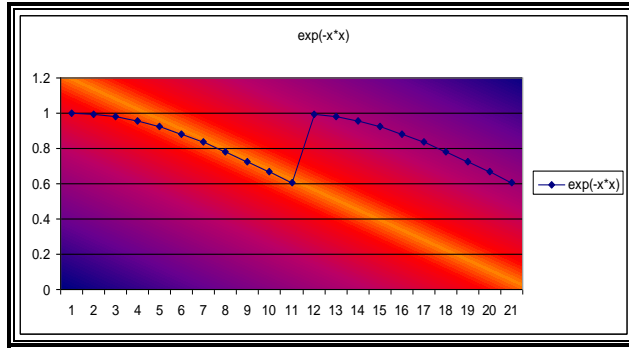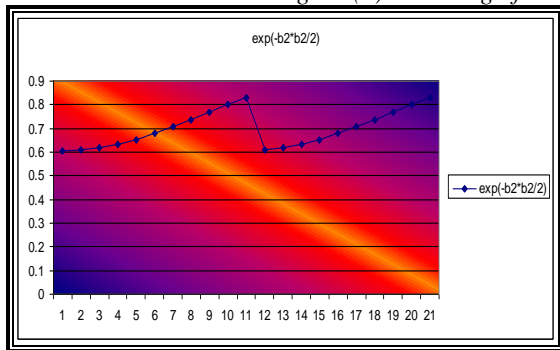
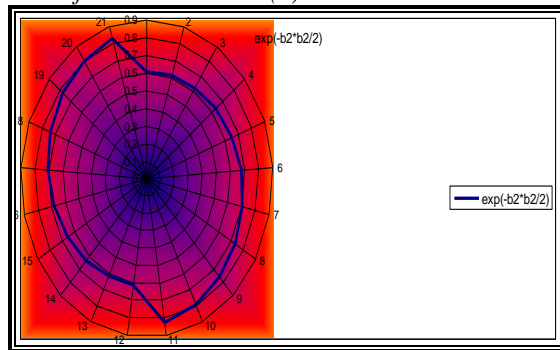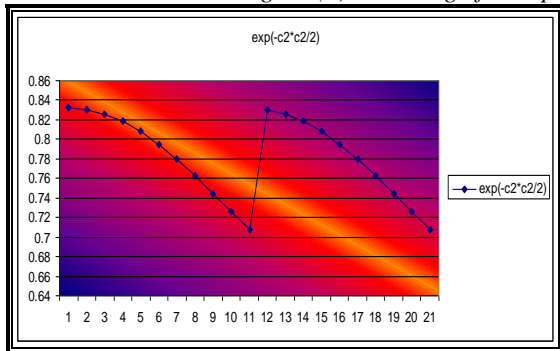*Figure (2): Drawing of Gaussian MFs for values in table(2)*

(a)   *(b)*
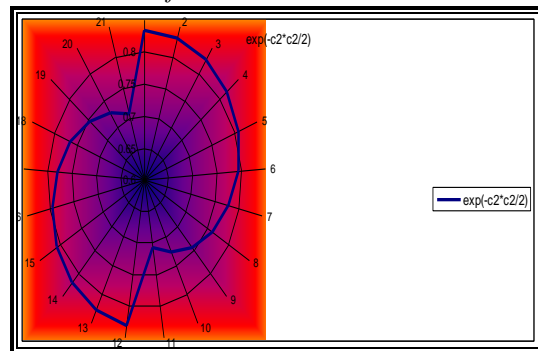
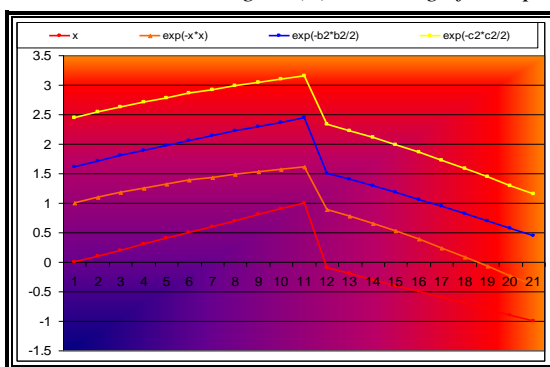*Figure (3): Drawing of Compose two Gaussian MFs for $-1 \leq x \leq 1$*

(a)   *(b)*
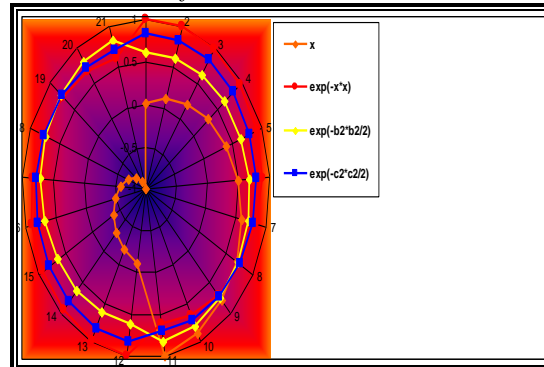
Figure (4): Drawing of Compose three Gaussian MFs for $-1 \leq x \leq 1$

(a)   *(b)*

Figure (5): General Drawing of the Compose Gaussian MFs for $-1 \leq x \leq 1$

The compose Gauss MF $\rho(x)$ with linear MFs such Triangular MF $\mu_1(x)$ (Triangular MF formula where occurred in reference Reznik 1997) as;

$$(\mu_1)(x) = \begin{cases} 0 & , & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & , & a_2 \le x \le a_3 \\ 0 & , & x > a_3 \end{cases} \qquad \text{..... (5)}$$

The form after compose on the intervals $[a_1, a_2]$ and $[a_2, a_3]$ as:

$$(\mu_1 \circ \rho)(x) = \begin{cases} \mu_1(e^{-(x)^2/2}) = 0 & , & x < a_1 \\ \mu_1(e^{-(x)^2/2}) = (\dfrac{(e^{-(x)^2/2}) - a_1}{a_2 - a_1}) & , & a_1 \le x \le a_2 \\ \mu_1(e^{-(x)^2/2}) = (\dfrac{a_3 - (e^{-(x)^2/2})}{a_3 - a_2}) & , & a_2 \le x \le a_3 \\ \mu_1(e^{-(x)^2/2}) = 0 & , & x > a_3 \end{cases} \qquad \text{.... (6)}$$

The chosen dataset for this compose and MF values $\mu_1(x)$ were presented in table(3) that explain this formula through choice values from -1 to 2 and values of intervals as [0,0.5] and [0.5,1] ,and then graphing this as shown in figures (6) and (7) which show that the graph is similar to triangular MF in some what .

Table (3): Values of x and Triangular MFs

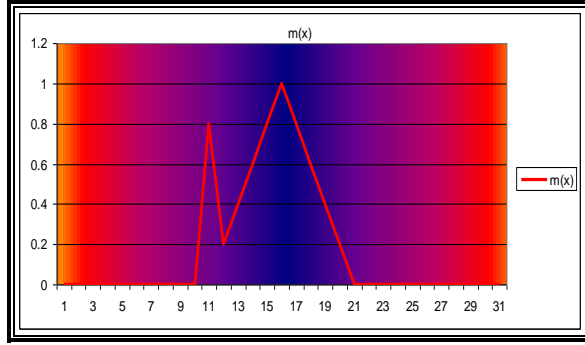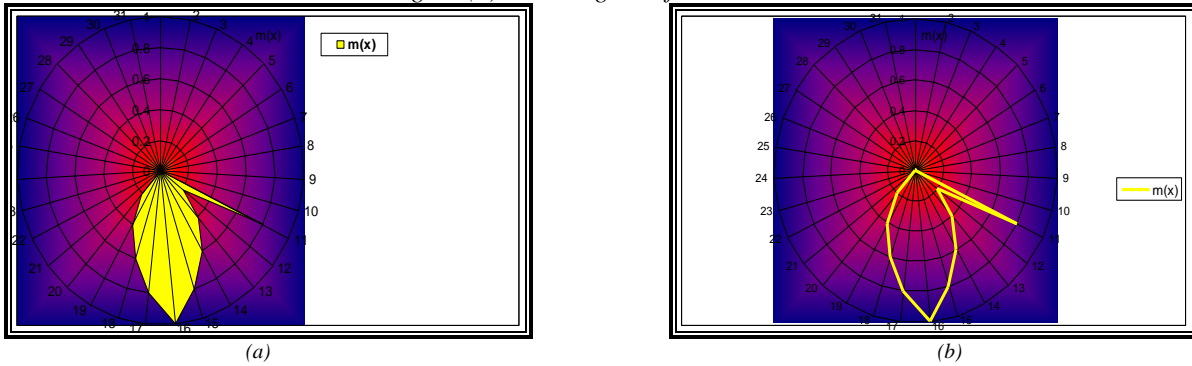| x	a1	a2	a3	$\mu_1(x)$
-1	0	0.5	1	0
-0.9	0	0.5	1	0
-0.8	0	0.5	1	0
-0.7	0	0.5	1	0
-0.6	0	0.5	1	0
-0.5	0	0.5	1	0
-0.4	0	0.5	1	0
-0.3	0	0.5	1	0
-0.2	0	0.5	1	0
-0.1	0	0.5	1	0
0	0	0.5	1	0.8
0.1	0	0.5	1	0.2
0.2	0	0.5	1	0.4
0.3	0	0.5	1	0.6
0.4	0	0.5	1	0.8
0.5	0	0.5	1	1
0.6	0	0.5	1	0.8
0.7	0	0.5	1	0.6
0.8	0	0.5	1	0.4
0.9	0	0.5	1	0.2
1	0	0.5	1	0
1.1	0	0.5	1	0
1.2	0	0.5	1	0
1.3	0	0.5	1	0
1.4	0	0.5	1	0
1.5	0	0.5	1	0
1.6	0	0.5	1	0
1.7	0	0.5	1	0
1.8	0	0.5	1	0
1.9	0	0.5	1	0
2	0	0.5	1	0

Figure (6): Drawing MF for -1≤ x ≤ 2

| (a) | (b) |

Figure (7): Drawing compose triangular and Gauss MFs for -1≤ x ≤ 2 .

While compose Gauss MF $\rho(x)$ with Trapezoidal MF $\mu_2(x)$ $\mu_1(x)$ (Trapezoidal MF formula where occurred in reference Reznik 1997) .The form after compose on the intervals $[a_1, a_2]$ and $[a_3, a_4]$ as:

$$(\mu_2 \circ \rho)(x) = \begin{cases} \mu_2(e^{-(x)^2/2}) = 0 & , \quad x < a_1 \\[2mm] \mu_2(e^{-(x)^2/2}) = (\dfrac{(e^{-(x)^2/2}) - a_1}{a_2 - a_1}) & , \quad a_1 \leq x \leq a_2 \\[2mm] \mu_2(e^{-(x)^2/2}) = 1 & , \quad a_2 \leq x \leq a_3 \\[2mm] \mu_2(e^{-(x)^2/2}) = (\dfrac{a_3 - (e^{-(x)^2/2})}{a_3 - a_4}) & , \quad a_3 \leq x \leq a_4 \\[2mm] \mu_2(e^{-(x)^2/2}) = 0 & , \quad x > a_4 \end{cases} \quad \dots \ (7)$$

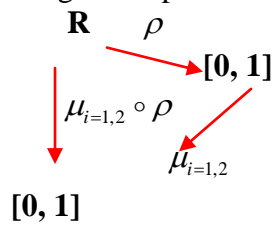This simple Flowchart diagram explain the compose between the MFs ;

Figure (8): Flowchart composition

Table (4): Values of x and Compose Triangular with Gauss MF   for -1≤ x ≤2

x	a1	a2	a3	a4	exp(-x*x/2)	μ (x)	Increasing order	μ (exp(-x*x)/2)
-1	0	0.333333	0.666666	1	0.60653066	-0.5	0.135335283	0.406006255
-0.9	0	0.333333	0.666666	1	0.666976811	0.6	0.164474457	0.493423864
-0.8	0	0.333333	0.666666	1	0.726149037	-0.6	0.197898699	0.593696691
-0.7	0	0.333333	0.666666	1	0.782704538	0.7	0.235746077	0.707238938
-0.6	0	0.333333	0.666666	1	0.835270211	-0.7	0.2780373	0.834112734
-0.5	0	0.333333	0.666666	1	0.882496903	0.8	0.324652467	0.973958375
-0.4	0	0.333333	0.666666	1	0.923116346	-0.8	0.375311099	1
-0.3	0	0.333333	0.666666	1	0.955997482	0.9	0.429557358	1
-0.2	0	0.333333	0.666666	1	0.980198673	-0.9	0.486752256	1
-0.1	0	0.333333	0.666666	1	0.995012479	1	0.546974427	1
0	0	0.333333	0.666666	1	1	-1	0.60653066	1
0.1	0	0.333333	0.666666	1	0.995012479	1	0.60653066	1
0.2	0	0.333333	0.666666	1	0.980198673	-0.9	0.666976811	0.000932431
0.3	0	0.333333	0.666666	1	0.955997482	0.9	0.666976811	0.000932431
0.4	0	0.333333	0.666666	1	0.923116346	-0.8	0.726149037	0.178448754
0.5	0	0.333333	0.666666	1	0.882496903	0.8	0.726149037	0.178448754
0.6	0	0.333333	0.666666	1	0.835270211	-0.7	0.782704538	0.348114918
0.7	0	0.333333	0.666666	1	0.782704538	0.7	0.782704538	0.348114918
0.8	0	0.333333	0.666666	1	0.726149037	-0.6	0.835270211	0.505811621
0.9	0	0.333333	0.666666	1	0.666976811	0.6	0.835270211	0.505811621
1	0	0.333333	0.666666	1	0.60653066	-0.5	0.882496903	0.647491414
1.1	0	0.333333	0.666666	1	0.546074427	0.5	0.882496903	0.647491414
1.2	0	0.333333	0.666666	1	0.486752256	-0.4	0.923116346	0.769349499
1.3	0	0.333333	0.666666	1	0.429557358	0.4	0.923116346	0.769349499
1.4	0	0.333333	0.666666	1	0.375311099	-0.3	0.955997482	0.86799271
1.5	0	0.333333	0.666666	1	0.324652467	0.3	0.955997482	0.86799271
1.6	0	0.333333	0.666666	1	0.2780373	-0.2	0.980198673	0.940596138
1.7	0	0.333333	0.666666	1	0.235746077	0.2	0.980198673	0.940596138
1.8	0	0.333333	0.666666	1	0.197898699	-0.1	0.995012479	0.985037467
1.9	0	0.333333	0.666666	1	0.164474457	0.1	0.995012479	0.985037467
2	0	0.333333	0.666666	1	0.135335283	0	1	1

The chosen dataset for compose and MF values $\mu_1(x)$ were presented in table(4) that explain this formula through choice values from -1 to 2 and values of intervals as [0,0.5] and [0.5,1] ,and then graphing this as shown in figures (6) and (7) which show that the graph is similar to triangular MF in some what .
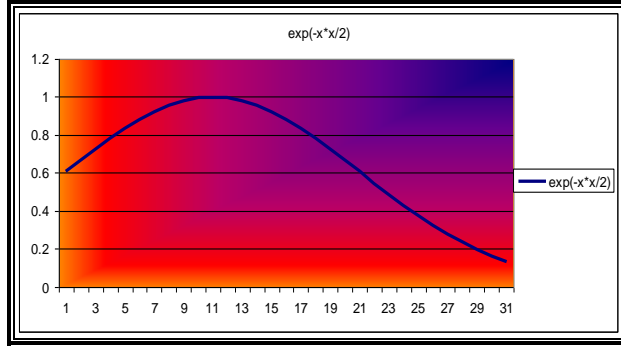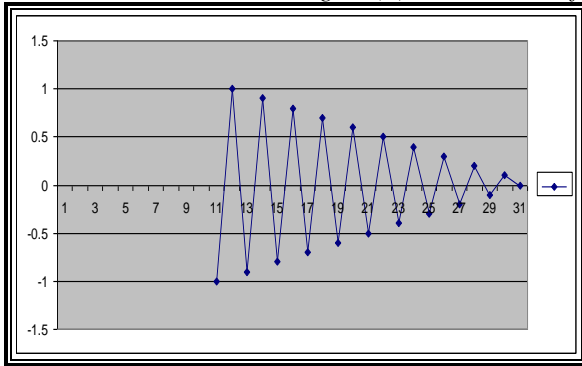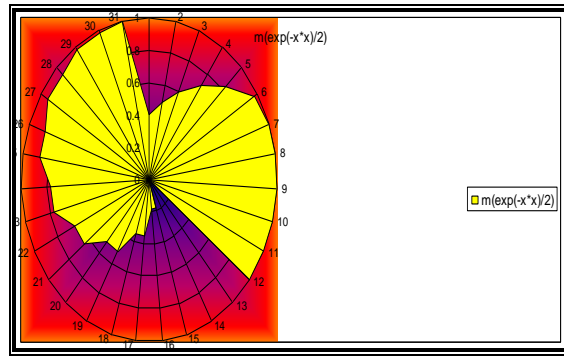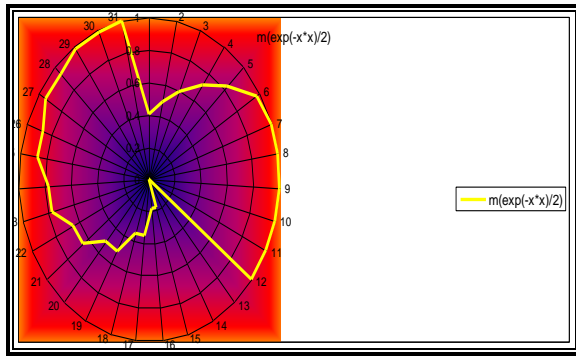
Figure (9): Distribution of values of x through Gauss MF

(a) *(b)*

Figure (10): (a)Distribution values through composed MFs
(b)Composed values of x and Trapezoidal composed Gauss MF

(a) *(b)*

Figure (11): Composed Trapezoidal composed Gauss MF

Composite with Identity Map i or Inclusion map e:

Identity map *i* were chosen here to benefit from its properties in transform features of functions from the domain into the range .If we suppose this map to has MF properties to be defined as ; $i:[0,1] \rightarrow [0,1]$,which will inserted in composite order with other MFs that if defined on R as ; $f:R \rightarrow [0,1]$ and $g:R \rightarrow [0,1]$ here either g or f must be inclusion map since $[0,1] \subset R$,so $e:[0,1] \subset R \rightarrow R$ in many cases we need not use inclusion map
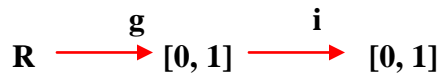
$$R \xrightarrow{\ \ g\ \ } [0, 1] \xrightarrow{\ \ i\ \ } [0, 1]$$

Figure (12): Flowchart

9

Driving MFs with Respect to x ***and*** s :

$$\frac{\mu'(x)}{dx} = e^{-(x-t)^2/2s^2}$$

….. (8)

$$= \frac{-x}{s^2} e^{-(x-t)^2/2s^2} = \frac{-x}{s^2} \mu(x)$$

Drive with respect to s as:

$$\frac{\mu'(x)}{ds} = e^{-(x-t)^2/2s^2}$$

….. (9)

$$= \frac{-1}{s} e^{-(x-t)^2/2s^2} = \frac{-1}{s} \mu(x)$$

Conclusion

Here discuss the usage of triangle and trapezoid MFs (linear functions) with Gauss MF. Both types of functions are reliable in practice .The main disadvantage using MFs through composite with other MFs that have applications in practice (real world data) .Membership degrees. In fact, for high membership degrees that gotten on still in interval [0,1] .The degree of enhancement is increased with degree of compose, that shown through different examples of values for every dataset of variable x and intervals of functions that used in formulas to get more reliable .By calculation or adaptation is still an open work for more functions **.** ***For*** more tries to update the composite we compose identity map i or inclusion map e with MFs also driving MFs with respect to x and parameter s .

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