NSS
Neutrosophic Sets and Systems, Vol. 51, 2022
University of New Mexico

The Neutrosophic Axial Set theory

Mustafa Hasan Hadi[1,*] and L. A.A. Al-Swidi[2]

[1] Affiliation 1; Department of Mathematics, University of Babylon, Iraq; pure.mustafa.hassan@uobabylon.edu.iq
[2] Affiliation 2; Department of Mathematics, University of Babylon, Iraq; pure.leal.abd@uobabylon.edu.iq

* Correspondence: pure.mustafa.hassan@uobabylon.edu.iq

Abstract: We presented in this paper a new concept of sets that we launched it neutrosophic axial sets . These sets are considered as generalization of neutrosophic sets . The union relationships , intersection, union , belonging and other concepts were built on these sets , then we created two different concepts of points . Also we studied many important properties and basic theories about axial sets theory.

Keywords: neutrosophic sets; fuzzy sets; ȘNĄ-points ; NĄ-sets ; union relationships.

1. Introduction

The neutrosophic sets [1] are the important and influential topic in human life in direct way. It's considered to be one of the applied and pure topics at the same time .

Also it contributes to quantum leaps in the field of electronics , software and other sciences as well as in various engineering branches . Where Salama, A., FlorentinSmarandache, and Valeri Kromov who were the researchers first to know these sets in [2,3]. Researchers and scientists have taken it upon them and solves to develop and work on it . On the other hand , these sets are considered to be a development to the second type of fuzzy sets , which the researcher Zadeh , L. A. know in 1965 [4] introduced. At the same time , the fuzzy sets are generalized into so-called the soft sets which were defined in [5- 7] and attributed to Molodtsov [8]. There are many researchers who worked in this field and remind them of [9- 12].

In 2019 Abdulsada, D.A., Al-Swidi, L.A.A. defined a new concept of sets and called it the center sets , for more information , you can review the papers [13- 15] , and the pillar of construction is proximity spaces by . A. Naimpally and . D. Warrack [16], where we combined the proximity space with the i-topological space by Al Talkany, A.Y.K.M., AL-Swidi, L.A.A [17] to produce the i-topological proximity space in 2020 [18, 19]. These ideas can be generalized on the topic of neutrosophic.

2. The Neutrosophic Axial Set theory

2.1. Definition Let X be any set , the set of the form $ꞐA̧A = \{< A, A_1, A_2 >;\ A \cap A_i = \emptyset, i = 1,2\}$ is called neutrosophic axial set , where A be any subset of X , and the sets A_1, A_2 are called the parts of $< A, A_1, A_2 >$ For example , if we take $X = R$ the real numbers ,then $ꞐA̧A = \{< (1,2), A1, A2 >;\ (1,2) \cap A_i = \emptyset, i = 1,2\}$ where

$$A_i = \begin{cases} \emptyset \ or\ discret\ set\ in\ {}^{R}/_{(1,2)} \\ [2, x) & for\ x \geq 2 \\ (y, 1] & for\ y \leq 1 \end{cases}$$

2.2. Definition

I- Let X be any non-empty set , the neutrosophic point (ꞐA̧-point) are of the forms $ꞐP_A^{\emptyset} = < A, \emptyset, \emptyset >$, $ꞐPA = < \emptyset, A, \emptyset >$ and
$ꞐPA = < \emptyset, \emptyset, A >$, for any proper non-empty subset A of .

II- The singular neutrosophic point (SꞐA̧-point) of any ꞐA̧-set $ꞐA̧A$ is denoted by $SA = < A, A_1, A_2 >$, where $A \cap A_i = \emptyset$ where $i = 1,2$, so $SA \in ꞐA̧A$ (where $\in$ is the notion of classical belongs to) .

So we can claim the number of ꞐA̧-points of any non-empty universal set X is $|X|.(|P(X)/\{\emptyset, X\}|$ where $|X|$ the number of elements of X.

Also from $|X|$ of above definition any ꞐA̧-set is the classical union of its SꞐA̧-points and any SꞐA̧-point of any ꞐA̧-set is neutrosophic set , but the converse is not true .

2.3. Definition

I- The empty ꞐA̧-set with respect to the subset A of X is denoted by $ꞐA_A^{\emptyset}$ is of the form
$ꞐA_A^{\emptyset} = \{< \emptyset, A_1, A_2 >;\ where\ A \cap A_i = \emptyset\ \ i = 1,2\}$.
For example , if $X = \{a, b, c\}$, then $ꞐA_{\{a,b\}}^{\emptyset} = \{< \emptyset, \emptyset, \emptyset >, < \emptyset, \{c\}, \{c\} >, < \emptyset, \{c\}, \emptyset >, < \emptyset, \emptyset, \{c\} >\}$.

II- The null ꞐA̧-set with respect to a subset A of , which denoted by $ꞐꞐA̧A$ is of form $ꞐꞐA̧A = \{< A, \emptyset, \emptyset >\}$.

2.4. Definition

I- The ꞐA̧-sum between two SꞐA̧-points SA and SB is denoted by the notion $\oplus_N$ which is defined by $S_A \oplus_N S_B = \{< A \cup B, C_1 \cup D_1, C_2 \cup D_2 >$, where $C_i \cap A = \emptyset\ \ and\ D_i \cap B = \emptyset, i = 1,2\}$. So from this definition we claim that every SꞐA̧-point is ꞐA̧-sum of two or more than two , but every ꞐA̧-point is SꞐA̧-point .

II- The $S\text{ЍĄ}$-point SA is called interlaced with respect to ЍĄ-set $\text{ЍĄ}B$, if $SA \in \text{ЍĄ}B$ there exist $SB \in \text{ЍĄ}B$ such that $A_i \sqsubseteq B_i$, $S_A =< A, A_1, A_2 >$, $S_B =< B, B_1, B_2 >$ $i = 1, 2$ .If any ЍĄ-point of the forms $\text{ЍP}_A^{\emptyset}$, $\text{Ѝ}PA$ and $\text{Ѝ}PA$ belong to ЍĄ-set $\text{ЍĄ}B$, if A is a part of some $S\text{ЍĄ}$-point of $\text{ЍĄ}B$ .So that we easily show that, $SB \in \text{ЍĄ}B$ iff $SB \in_{\text{Ѝ}} \text{ЍĄ}B$.

III- The ЍĄ-set $\text{ЍĄ}A$ is said to be interlaced set with one of the ЍĄ-set $\text{ЍĄ}B$ which is denoted by $\text{ЍĄ}A <_{\text{Ѝ}} \text{ЍĄ}B$ iff for each $S_A =< A, A_1, A_2 > \in \text{ЍĄ}B$ there exist $SB =< B, B_1, B_2 > \in \text{ЍĄ}B$ with the condition $A_i \sqsubseteq B_i, i = 1, 2$. Clearly every ЍĄ-set is interlaced set of $\text{ЍĄ}_{\emptyset}$, also $\text{ЍĄ}X$ is interlaced set of any ЍĄ-set.

2.5. Note

If $A \sqsubset B$, then $\text{ЍĄ}_B <_{\text{Ѝ}} \text{ЍĄ}_A$. Because, for every $S\text{ЍĄ}$-point$SB =< B, B_1, B_2 > \in \text{ЍĄ}B$ that is $B \cap B_i = \emptyset$ and $A \sqsubset B$ imply that $A \cap B_i = \emptyset$ for $i = 1, 2$, thus $SB \in \text{ЍĄ}A$, which satisfy the condition of interlaced set .Two ЍĄ-sets ЍĄ_A and $\text{ЍĄ}_B$are called intertwind sets which is denoted by $\text{ЍĄ}_A \approx_{\text{Ѝ}} \text{ЍĄ}_B$ iff $\text{ЍĄ}A <_{\text{Ѝ}} \text{ЍĄ}B$ and $\text{ЍĄ}B <_{\text{Ѝ}} \text{ЍĄ}A$.

2.6. Proposition

Let X be any sets and A, B are subsets of X. $A = B$ iff $\text{ЍĄ}_A \approx_{\text{Ѝ}} \text{ЍĄ}_B$.

Proof .

Assume that $= B$, so by (Note 2.5), we get that $\text{ЍĄ}_A \approx_{\text{Ѝ}} \text{ЍĄ}_B$. Conversely, if possible that $A \neq B$.

Case 1 . If $A \cap B = \emptyset$, then each $S_A =< A, A_1, A_2 > \in \text{ЍĄ}A$ and since each subset C of X with $C \cap B = \emptyset$ there is no $S\text{ЍĄ}$-points in ЍĄ_B which satisfy the condition of interlaced set, so $\text{ЍĄ}A$ is not interlaced set of ЍĄ_B . Similarly that ЍĄ_B is not interlaced set of $\text{ЍĄ}A$, which contradiction with $\text{ЍĄ}_A \approx_{\text{Ѝ}} \text{ЍĄ}_B$.

Case 2. If $A \cap B \neq \emptyset$, that is there exist a point x in A and not in B or the point y in B but not in A, so $SB =< B, \{x\}, \{x\} > \in \text{ЍĄ}B$ imply that no $S\text{ЍĄ}$-points in $\text{ЍĄ}A$ which satisfy the condition of interlaced set, hence $\text{ЍĄ}B$, similarly if we take $y \in B$ and y is not in A, which contradiction with $\text{ЍĄ}_A \approx_{\text{Ѝ}} \text{ЍĄ}_B$. Therefore we get $= B$.

2.7.Definition

The ЍĄ- complement of any ЍĄ-set ЍĄ_A which is denoted by $(\text{ЍĄ}_A)c$ and of the form $(\text{ЍĄ}_A)c = \text{ЍĄ}_{A^c}$. Now we give the notions of union, intersection of ЍĄ - sets .

2. 8. Definition

The $\mathcal{NA}$ – union of two $\mathcal{NA}$-sets $\mathcal{NA}_A$ and $\mathcal{NA}_B$, which is denoted by $\mathcal{NA}_A \cup_N \mathcal{NA}_B$ and is of the form $\mathcal{NA}_A \cup_N \mathcal{NA}_B = \{ < A \cup B, A_1 \cap B_1, A_2 \cap B_2 > ; \forall < A, A_1, A_2 > \in \mathcal{NA}_A, < B, B_1, B_2 > \in \mathcal{NA}_B \}$ .Also for the same away we defined that for any collection$\{\mathcal{NA}_{A_i}; i \in II\}$ of $\mathcal{NA}$ - sets , the $\mathcal{NA}$-union of this collection is of form $(\cup_{i\in II} A_i)_N = \{ < \cup_{i\in II} A_i, C_{j_1} \cap C_{j_2}, D_{j_1} \cap D_{j_2} >; \forall j_1, j_2 \in II\}$ where $S_{A_{j_1}} =< A_{j_1}, C_{j_1}, D_{j_1} >$and $S_{B_{j_2}} =< B_{j_2}, C_{j_2}, D_{j_2} >$.

The $\mathcal{NA}$ – intersection of two $\mathcal{NA}$-sets $\mathcal{NA}_A$ and $\mathcal{NA}_B$, which is denoted by $\mathcal{NA}_A \cap_N \mathcal{NA}_B$ and is of the form $\mathcal{NA}_A \cap_N \mathcal{NA}_B = \{ < A \cap B, A_1 \cap B_1, A_2 \cap B_2 > ; \forall < A, A_1, A_2 > \in \mathcal{NA}_A, < B, B_1, B_2 > \in \mathcal{NA}_B \}$. Also for the same away we defined that for any collection$\{\mathcal{NA}_{A_i}; i \in II\}$ of $\mathcal{NA}$ – sets , the $\mathcal{NA}$-intersection of this collection is of form $(\cap_{i\in II} A_i)_N = \{ < \cap_{i\in II} A_i, C_{i_1} \cap C_{i_2}, D_{i_1} \cap D_{i_2} >; \forall i_1, i_2 \in II\}$ where $S_{A_{i_1}} =< A_{i_1}, C_{i_1}, D_{i_1} >$and $S_{B_{i_2}} =< B_{i_2}, C_{i_2}, D_{i_2} >$.

The $\mathcal{NA}$ –participating of two $\mathcal{NA}$-sets $\mathcal{NA}_A$ and $\mathcal{NA}_B$, which is denoted by $\mathcal{NA}_A \Cap \mathcal{NA}_B = \mathcal{NA}_{A\cap B}$.

It is easy to show that the commutative and associative properties for the $\mathcal{NA}$ – union, $\mathcal{NA}$ – intersection and $\mathcal{NA}$ –participating are satisfied .

2.9. Proposition

For any set X and any subsets A, B , that is $\mathcal{NA}_A \cup_N \mathcal{NA}_B = \mathcal{NA}_{A\cup B}$.

Proof .

For any $S\mathcal{NA}$-point $SD \in \mathcal{NA}_A \cup_N \mathcal{NA}_B$, which of the form $SD =< A \cap B, A_1 \cap B_1, A_2 \cap B_2 >$, with $S_A =< A, A_1, A_2 > \in \mathcal{NA}_A$, $S_B =< B, B_1, B_2 > \in \mathcal{NA}_B$. Thus $SD \in \mathcal{NA}_{A\cup B}$, because that $(A \cup B) \cap (A_i \cap B_i) =(A \cap (A_i \cap B_i)) \cup(B \cap (A_i \cap B_i)) = \emptyset$.

Conversely , now let $S_{A\cup B} \in \mathcal{NA}_{A\cup B}$, if possible that $S_{A\cup B} \notin \mathcal{NA}_A \cup_N \mathcal{NA}_B$, since $S_{A\cup B} =< A \cup B, D1, D2 >$ with $(A \cup B) \cap Di = \emptyset$, $for\ i = 1, 2$. But $\emptyset =(A \cup B) \cap Di = (A \cap Di) \cup (B \cap Di)$, imply that $A \cap Di = \emptyset, B \cap Di = \emptyset$, so we have $< A, D1, D2 > \in \mathcal{NA}_A$ and $< B, D1, D2 > \in \mathcal{NA}_B$, also $S_{A\cup B} =< A \cup B, D1, D2 > = < A \cup B, D1 \cap D1, D2 \cap D2 > \in \mathcal{NA}_A \cup_N \mathcal{NA}_B$, which is a contradiction .

From this proposition we can prove easily the following corollary.

2.10 .Corollary

1. $\mathcal{NA}_A \cup_N \mathcal{NA}_{A^c} = \mathcal{NA}X$.
2. $\mathcal{NA}_A \cup_N \mathcal{NA}_A = \mathcal{NA}_A$.
3. $\mathcal{NA}_A \cup_N \mathcal{NA}X = \mathcal{NA}X$.

2.11. Remark

For any set X and subset A of X we have $\mathrm{NA_A} \cap_N \mathrm{NA_X} = \mathrm{NNA_A} = \{ \mathrm{NP_A^{\emptyset}} \}$, because $\mathrm{NA_A} \cap_N \mathrm{NA_X} = \{ < A \cap X, \emptyset \cap A_1 , \emptyset \cap A_2 >$; for each $< A, A_1, A_2 > \in \mathrm{NA_A} \} = \{ < A , \emptyset , \emptyset > \} = \{ \mathrm{N}P_A^{\emptyset} \} = \mathrm{NNA}_A$.

2.12. Remark

Let X be any set with $\mathrm{NA_A}$ and $\mathrm{NA_B}$ are NA – sets on X . If $\mathrm{NA_A} <_N \mathrm{NA_B}$, then $(\mathrm{NA}_A \cap_N \mathrm{NA}_B) <_N \mathrm{NA_A}$ also $(\mathrm{NA}_A \cap_N \mathrm{NA}_B) <_N \mathrm{NA}_B$ because for any $< A \cap B, C_1 \cap D_1, C_2 \cap D_2 > \in \mathrm{NA_A}$ from the fact $< A , C1 , C2 > \in \mathrm{NA}_A$ and $A \cap C_i \cap D_i = \emptyset$, for $i = 1 , 2$.from above (Remark 2.12.) and(Note 2.5.) we have the following proposition .

2. 13. proposition

1- $(\mathrm{NA_A} \cap_N \mathrm{NA_B}) <_N \mathrm{NA_{A \cap B}}$.

2- $(\mathrm{NA_A} \cap_N \mathrm{NA_B}) \approx_N \mathrm{NA_A}$.

3- If $\mathrm{NA_A} <_N \mathrm{NA_B}$ and $\mathrm{NA_B} <_N \mathrm{NA_C}$, then $\mathrm{NA_A} <_N \mathrm{NA_C}$.

4- If $\sqsubseteq B$, then $\mathrm{NA_A} \cap_N \mathrm{NA_B} \approx_N \mathrm{NA_B}$.

Proof (4).

By (Note 2 . 5) and Remark (2 .12) we have $(\mathrm{NA_A} \cap_N \mathrm{NA_B}) <_N \mathrm{NA_B}$. Now let us $< B, D_1, D_2 > \in \mathrm{NA_B}$, so $B \cap D_i = \emptyset$, but $A \sqsubseteq B$, then $A \cap D_i = \emptyset$ for $i = 1,2$, then $< A, D_1, D_2 > = < A \cap B, D_1, D_2 > \in \mathrm{NA_A} \cap_N \mathrm{NA_B}$. So we get the result .

2. 14. Proposition

For any NA – points $\mathrm{NP_D}, \mathrm{Np^D} \in_N \mathrm{NA_A}$ which satisfy that , there exist a part C of some $S\,\mathrm{NA}$ – point of $\mathrm{NA_B}$ such that $D \sqsubseteq C$ iff $\mathrm{NA_A} <_N \mathrm{NA_B}$.

Proof.

Let $SA = < A , C1 , C2 > \in \mathrm{NA}_A$, then NA – points $\mathrm{NP_{C_1}}$ and $\mathrm{NP^{C_2}}$ are in $\mathrm{NA_A}$, so by assumption , there exist parts $D1$, $D2$ of $S\,\mathrm{NA}$ – points in $\mathrm{NA_B}$ with $Ci \sqsubseteq Di$ and $B \cap D_i = \emptyset$, $i = 1 , 2$, so $< B, D_1, D_2 > \in \mathrm{NA_B}$, this imply that $\mathrm{NA_A} <_N \mathrm{NA_B}$.

Conversely , let $\mathrm{NA_A} <_N \mathrm{NA_B}$ and let $\mathrm{NP_D} \in_N \mathrm{NA_A}$, then the $S\,\mathrm{NA}$ – point $< A, D , \emptyset >$, $< A , \emptyset , D >$ or $< A , D , D >$ are in $\mathrm{NA_A}$, then there exist $< B , C1 , \emptyset >$. Such that $D \sqsubseteq C1$ with $\mathrm{NP_{C_1}} \in \mathrm{NA_B}$ or $< B, \emptyset , C2 >$ such that $D \sqsubseteq C2$ with $\mathrm{NP^{C_2}} \in \mathrm{NA_B}$ or $< B , H1 , H2 > \in \mathrm{NA}_B$ with $D \sqsubseteq Hi$, so $\mathrm{NP_{D_1}}$ and $\mathrm{NP^{D_2}} \in \mathrm{NA_B}$.

3. Conclusions

1. After an extensive study of these sets and spaces , we did this research establishing the basic structures for generalizing the neutrosophic sets , and under the name neutrosophicsets . Therefore , we can put the identification of the topological spaces on it , by taking a family of these ɈNĄ – sets that achieve the following ; $ɈNĄ_X$, $ɈNĄ_\emptyset$ belong it , second is closed under the finite ɈNĄ – intersection , finally is must be closed under ɈNĄ – union for any subfamily of it .

2. Also we can study their properties and characteristics , as well as define the functions on there to give as a good suggestions to work . Then , we can modify the various open sets and further study can be continued with this concept. For example , we can modify in the papers [20- 28] .

Acknowledgments: The authors remain thankful to the referee for his helpful suggestions and comments.

References

1. F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Re-search Press, Rehoboth, NM, **1999**.

2. Salama, A., FlorentinSmarandache, and Valeri Kromov. "Neutrosophic Closed Set and Neutrosophic Continuous Functions." Neutrosophic Sets and Systems, **2014**, 4.1.

3. Salama, A. A., Alblowi, S. A., Smarandache, F."Neutrosophic crisp open set and neutrosophic crisp continuity via neutrosophic crisp ideals". I.J. Information Engineering and Electronic Business, **2014**, vol. 3, pp.1 –8.

4. Zadeh , L. A. , Fuzzy sets , Inform. Control , **1965**, Vol. 8 , PP 338-353 .

5. Almohammed, R., AL-Swidi, L.A. , "New concepts of fuzzy local function " Baghdad Science Journal , **2020**, 17(2), pp. 515-522.

6. Al-Abbasi, H., Al-Swidi, L.A.A. , "On free-open sets " Journal of Advanced Research in Dynamical and Control Systems", **2020**, 12(7), pp. 782-787.

7. Al-Swidi, L.A., Al-Rubaye, M.S. , " New classes of separation axiom via special case of local function ," International Journal of Mathematical Analysis, **2014**, 8(21-24), pp. 1119-1131.

8. Molodtsov , D. , Soft set theory first results , Comput. Math. Appl., **1999**, Vol. 37 , PP 19-31.

9. Al-Swidi, L.A., Al-Ethary, M.A. , "Compactness with "Gem-Set" ," International Journal of Mathematical Analysis,**2014**, 8(21-24), pp. 1105-1117.

10. Al-Swidi, L.A., Awad, F.S.S. , "Analysis on the Soft Bench Points ," ICOASE 2018 - International Conference on Advanced Science and Engineering, **2018** , pp. 330-335 ,.

11. Al Razzaq, A.S.A., Al Swidi, L.A.A.H. , "Soft generalized vague sets: An application in medical diagnosis", Indian Journal of Public Health Research and Development, **2019** ,10(2), pp. 901-907.

12. Hadi, M.H., AL-Yaseen, M.A.A.-K., Al-Swidi, L.A. , "Forms weakly continuity using weak ω–open sets " Journal of Interdisciplinary Mathematics **2020**.

13. Abdulsada, D.A., Al-Swidi, L.A.A. , "Separation axioms of center topological space " Journal of Advanced Research in Dynamical and Control Systems , **2020**, 12(5), pp. 186-192.

14. Abdulsada, D.A., Al-Swidi, L.A.A. , " Some Properties of C-Topological Space " ,1st International Scientific Conference of Computer and Applied Sciences, CAS,**2019**, pp. 52-56.

15. Ali Abdulsada, D., Al-Swidi, L.A.A. " Compatibility of Center Ideals with Center Topology," IOP Conference Series: Materials Science and Engineering, **2020**, 928(4).

16. S. A. Naimpally and . D. Warrack , Proximity spaces , Cambridge Tracts No. 59, **1970** , Cambridge.

17. Mahdi Altalkany, Y.K., Al Swidi, L.A.A. , " Focal Function in i-Topological Spaces via Proximity Spaces " Journal of Physics: Conference Series, **2020**, 1591(1).

18. Al Talkany, A.Y.K.M., AL-Swidi, L.A.A. ," Ψ- operator proximity in i-Topological space ".Turkish Journal of Computer and Mathematics Education Research Article , **2021**, vol. 12, no. 1S, pp. 679-684.

19. Al Talkany, Y.K.M., AL-Swidi, L.A.A. ," New concepts of dense set in i-topological space and proximity space ". Turkish Journal of Computer and Mathematics Education Research Article, **2021**, vol. 12, no. 1S, pp. 685–690.

20. Abdulsada, D.A., Al-Swidi, L.A.A. , "Center Set Theory of Proximity Space" Journal of Physics: Conference Series, **2021**, 1804(1).

21. Al-Abbasi, H., Al-Swidi, L. , "Measurable SB-Functions Space and Confine MSB-Function Topology " Journal of Physics: Conference Series , **2021**, 1804(1).

22. Al-Abbasi, H., Al-Swidi, L.A.A., "Comparison between Confine MO-Connectedness and Connectedness, Confine MO-Countability and Countability" Journal of Physics: Conference Series , **2020**, 1591(1).

23. Al-Swidi, L.A., Mustafa, H.H. , "Characterizations of continuity and compactness with respect to weak forms of ω -Open sets ," European Journal of Scientific Research, , **2011**, 57(4), pp. 577-582.

24. Alswidi, L.A., Alhosaini, A.M.A. , " Weak forms of ω-open sets in bitopological spaces and connectedness ," European Journal of Scientific Research , **2011**, 52(2), pp. 204-212.

25. Abd Al-Haine Al-Swidi, L., Al-Fatlawe, I.J. , "On semi paracompactness and z-paracompactness in bitopological spaces ", European Journal of Scientific Research, **2010**, 47(4), pp. 554-561.

26. Al-Swidi, L.A.H. , "On preparacompactness in bitopological spaces ", European Journal of Scientific Research , **2010**, 46(2), pp. 165-171.

27. Hadi, M.H., Al-Yaseen, M.A.A.-K. , "Study of Hpre -open sets in topological spaces ," AIP Conference Proceedings 2292 , **2020**.

28. Al-Swidi, L.A., Awad, F.S.S. , " On soft turning points " Baghdad Science Journal, **2018** ,15(3), pp. 352-360.

Received: July 5, 2022. Accepted: September 19, 2022.