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Shadowing And Non-Shadowing In Dynamical Systems

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Abstract: This study was a survey article on shadowing property in a dynamical system given by a homeomorphism of a compact, metric space. We introduced beginners to the study of shadowing and some properties such that the cross product $\mathfrak{f} \times \mathfrak{f}$ and the composition $\mathfrak{f}^{k}$ had shadowing for every integer $k > 0$ if $\mathfrak{f}$ has the shadowing. And we showed the relationship with chaotic properties. We also gave an overview of the results in the topic with examples illustrating where it exists and does not exist. The self – homeomorphism map had non shadowing if it was distal on connected space or if it was on connected space with not point is minimal. Finally, every isometry of a compact Riemannian manifold of positive dimension had non shadowing.

INTRODUCTION

Shadowing is an important property in dynamical system in many branches : qualitative theory, chaotic behavior, ergodic theory, stability and global theory. Shadowing (or sometimes called pseudo orbit tracing property which is the same P.O.T.P.) It was established in 1967 by Anosov [7]. In 1975, it was used to prove $\omega -$ limit sets of diffeomorphism maps [22]. In the recent years, this property has been developed with many (stronger) types of it. In many litterateurs authors talked about shadowing and gave examples for functions which have this property and the other talked about the relations between it and the chaotic properties. In few litterateurs some authors studied the functions which did not have this property . We reviewed all of these in this study which is divided as follows, section two deals with the maps which have shadowing property with examples. In section three shows the relation between shadowing and some chaotic properties, and the final section gives examples of maps which does not have shadowing property.

RESEARCH METHOD

In the current study paper, we use space $\mathcal{X}$ is a compact and the metric distance $\mathfrak{D}$, with a map $\mathfrak{f}$ is a homeomorphism on $\mathcal{X}$. As it is known, a $\mathfrak{d}-$pseudo-orbit is a sequence of points $\{\varkappa_j\}_{j\in\mathbb{Z}}$ belong to $\mathcal{X}$ where $\mathfrak{d} > 0$ of $\mathfrak{f}$, for all $j \in \mathbb{Z}$, $\mathfrak{D}(\mathfrak{f}(\varkappa_j), \varkappa_{j+1}) < d$ .Then $\mathfrak{f}$ is said to have shadowing in discrete dynamical system if for every $\xi > 0$, there exists $\mathfrak{d} > 0$ such that, for every $\mathfrak{d}-$pseudo-orbit $\{\varkappa_j\}_{j\in\mathbb{Z}}$ of such that if $\sum_{j\mathbb{Z}} \mathfrak{d}(\mathfrak{f}(\varkappa_j), \varkappa_{j+1}) < d$, then $\sum_{j\mathbb{Z}} \mathfrak{d}(\mathfrak{f}^{j}(y), \varkappa_j) < \xi$ for some $y \in \mathcal{X}$ [15]. This property relates with some chaotic properties like topological transitive, chain transitive, mixing, chain mixing, expansive and strongly expansive as we see in section 4. An expansive map $\mathfrak{f}$ is defined as follows: $\varkappa = y$ for $\varkappa, y \in \mathcal{X}$, if there exists a constant $\mathfrak{c} > 0$ such that for all $j \in \mathbb{Z}$, $\mathfrak{d}(\mathfrak{f}^{j}(\varkappa), \mathfrak{f}^{j}(y)) \leq \mathfrak{c}$. It is called positively expansive if $j \geq 0$. The constant $\mathfrak{c}$ is called an expansive constant [13]. If $\mathfrak{b} < \infty$, a finite $\mathfrak{d}-$pseudo-orbit $\{\varkappa_j\}_{j=0}^{\mathfrak{b}}$ of $\mathfrak{f}$ is $\mathfrak{d}-$chain of $\mathfrak{f}$ from $\varkappa_0$ to $\varkappa_{\mathfrak{b}}$ of length $\mathfrak{b}$. If for any $\varkappa, y \in \mathcal{X}$ and any $\mathfrak{d} > 0$, the map $\mathfrak{f}$ is chain transitive if there is a $\mathfrak{d}-$chain from $\varkappa$ to y, and if there exists $\mathcal{N} > 0$ such that for every integer $n \geq \mathcal{N}$ there exists $\mathfrak{d} > 0$ from $\varkappa$ to y with length n [1]. A map $\mathfrak{f}$ is transitive if for any $\mathcal{U} \neq \emptyset$ and $\mathcal{V} \neq \emptyset$ open of , there exists $n \geq 0$ such that $\mathfrak{f}^{n}(\mathcal{U}) \cap \mathcal{V} \neq \emptyset$. Let $\mathcal{N} > 0$, the map $\mathfrak{f}$ is mixing if for all $n \geq \mathcal{N}, \mathfrak{f}^{n}(\mathcal{U}) \cap \mathcal{V} \neq \emptyset$. And $\mathfrak{f}$ is weakly mixing if $\mathfrak{f} \times \mathfrak{f}$ is transitive, and if iterates $\mathfrak{f}^{n}$ are transitive for all n then $\mathfrak{f}$ is totally transitive [8] . Finally we define distal and minimal dynamical systems

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as follows: $\mathfrak{f}$ is distal if $\inf_{n\in\mathbb{Z}}\mathfrak{d}(\mathfrak{f}^n(\varkappa),\mathfrak{f}^n(y)) = 0$ then $\varkappa = y$, and it is minimal if an $\mathfrak{f}$ −invariant closed set $\mathcal{K}$ is necessarily $\mathcal{K} = \emptyset$ or $\mathcal{K} = \mathcal{X}$ [19].

RESULTS AND ANALYSIS

In this section, we recalled the importatntresults of shadowing ,non shadowing and the relations between chaotic properties with shadowing

Shadowing Property

On a non-compact space with any metric, a dynamical system may have the shadowing concerning one metric yet not as for another that induces a similar topology. [25]Topological and metric shadowing are two concepts which are independent in general topological spaces but they are equivalent in a compact metric space. [26-29].

Proposition [8]

If a continuous map $\mathfrak{f}$ has the shadowing then so as $\mathfrak{f}\times\mathfrak{f}$.

Proposition [19]

If the map $\mathfrak{f}$ has shadowing, then $\mathfrak{f}^{k}$ has also shadowing for every integer $k > 0$.The next propositions give the equivalent condition to shadowing property.

Proposition [23][5]

The map $\mathfrak{f}:\mathcal{X}\to\mathcal{X}$ has shadowing property if and only if for every $\xi > 0$ there exists $\mathfrak{d} > 0$ such that every finite $\mathfrak{d}-$ pseudo-orbit is $\xi-$shadowed.The transversality condition which defined in [2] is equivalent to shadowing property in diffeomorphism maps:

Proposition [2]

A diffeomorphism map $\mathfrak{f}$ of a two-dimensional surface which satisfies Axiom A, is satisfying theC^0 transversality condition if and only if $\mathfrak{f}$ has the shadowing property.The restriction of continuous map has $D-$ shadowing property which is one of the kinds of shadowing property as we see in the next proposition. We recall the definition of $D-$ shadowing property:

Definition [11]

Let $D\subset Z(\mathcal{X})$. Then $\mathfrak{f}\in Z(\mathcal{X})$ has the $D-\xi>0$ there is $\mathfrak{d}>0$ such that if $\mathfrak{D}_0(\mathfrak{f},\mathfrak{g}) < d$ for $\mathfrak{g}\in D$, then any $\mathfrak{g}-$orbit is $\epsilon-$shadowed by a $\mathfrak{f}-$orbit : that means: for every $x\in X$, there exists $\varkappa_0\in\mathcal{X}$ such that $\mathfrak{D}(\mathfrak{f}^n(x_0),\mathfrak{g}^n(\varkappa))\leq\xi$ for all $n\in\mathbb{Z}$. If $\mathfrak{g}\in Z(\mathcal{X})$, then $\mathfrak{f}$ has the $H-$ shadowing property. If $Z(\mathcal{X}) = D$, then the $D-$shadowing and the $H-$shadowing are equivalent.

Proposition [11]

If the map $\mathfrak{f}$ has the $D-$shadowing then restriction $\vartheta_{|D}: D\to\mathcal{X}$ is continuous at $\mathfrak{f}$ .We turn now deal with shadowing in the set, sequences of maps and flow. In the $(\mathcal{X},\mathfrak{f})$, a set $\mathcal{Y}\subset\mathcal{X}$ has shadowing property if for each $\xi>0$ there is $\mathfrak{d}>0$ such that for a given $\mathfrak{d}-$ pseudo-orbit subset of $\mathcal{Y}$ there exists a point $y\in\mathcal{X}$ satisfying $\mathfrak{D}(\mathfrak{f}^{j}(y),\varkappa_j)<\xi$ for every $j\in\mathbb{Z}$. The next proposition says: If $\mathcal{X}$ has shadowing property then so as a subset $\mathcal{Y}$.

Proposition [3]

If $\mathfrak{f}$ has the finite shadowing on $\mathcal{Y} \subset \mathcal{X}$, then it has the shadowing on $\mathcal{Y}$.In addition, we define shadowing in sequences of maps $\mathfrak{f}_{1,\infty} = \{\mathfrak{f}_{j}\}_{j=1}^{\infty}$ in which for each $\mathfrak{f}_{j}: \mathcal{X} \to \mathcal{X}$is continuous. These sequences has this property if for each $\xi > 0$ there exists $\mathfrak{d} > 0$ such that for every $\mathfrak{d}-$ pseudo-orbit $\{\varkappa_0, \varkappa_1, \dots\}$, there is $y \in \mathcal{X}$ with $\mathfrak{D}(y, \varkappa_0) < \xi$ and $\mathrm{d}\left(\mathfrak{f}_1^{j}(y), \varkappa_{j}\right) < \epsilon$, for all $j \geq 1$.

Proposition [10]

The sequence $\mathfrak{f}_{1,\infty}$ has shadowing property if and only if for every $\xi > 0$ there exists $\mathfrak{d} > 0$ such that every finite $\mathfrak{d}$ −pseudo-orbit is ξ −shadowed.In [18] the authors define shadowing in flow as:

Definition [18]

A flow (continuous dynamical system) $(\mathcal{X}, \psi)$ has the weak (strong) shadowing if for any $\xi > 0$ there exist $\mathfrak{d}, \mathrm{T} > 0$ such that every $(\mathfrak{d}, \mathrm{T})$ −chain of ψ can be weakly ξ −traced (strongly ξ −traced) by some $\varkappa \in \mathcal{X}$. A flow$(\mathcal{X}, \psi)$ has the finite shadowing property if for each $\xi > 0$ there exists $\mathfrak{d}, \mathrm{T} > 0$ such that each finite $(\mathfrak{d}, \mathrm{T})$ −chain $\{\varkappa_{j};\ \mathrm{t}_{j}\}_0^{k} (0 \leq k < \infty)$ of ψ can be weakly ξ −traced by some $\varkappa \in \mathcal{X}$; i. e. there is $\mathfrak{g} \in \mathrm{Rep}$ with $\mathfrak{D}\left(\mathrm{x}_0 * \mathrm{t}, \varkappa.\mathfrak{g}(\mathrm{t})\right) \leqq \xi$ for $0 \leqq \mathrm{t} < \mathcal{S}_{k+1}$, (for definition of Rep and $\mathcal{S}_{j}$ see [18 definition 1].

We can see the shadowing property in the following properties

Proposition [18]

Let $\psi = \{\psi^{\mathrm{t}}\}_{\mathrm{t} \in \mathbb{R}}$ be a flow on $\mathcal{X}$ and Λ a basic set of ψ. Then (Λ, ψ) has the strong shadowing if$(\mathcal{X}, \psi)$ has this property.

Remark [18]

By definition shadowing property in the flow we can see the relations among the strong, weak, and the finite shadowing property as in the following figure 1:

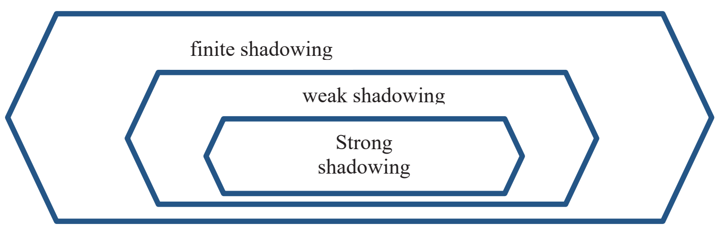

FIGURE 1. Relation between types of shadowing

There are many examples of maps which has shadowing property. In general, we have:

Examples

1. In a neighborhood of a hyberbolic set, a map has shadowing property if it is diffeomorphism [24].
2. Diffeomorphism $\mathfrak{f} : \mathrm{S}^1 \times \mathrm{S}^2 \to \mathrm{S}^1 \times \mathrm{S}^2$ has a shadowing property [2].
3. A compact manifold with dimension ≥ 2 has shadowing if the homeomorphism map is topologically stable [14].
4. If a smooth manifold $\mathcal{M}$ is compact space, and$\mathfrak{f}$ is a homeomorphism on $\mathcal{M}$ then has the shadowing and theH −shadowing property are equivalent [11].
5. Every (group) automorphism ϱ of a zero-dimensional compact metric group $\mathcal{X}$ has shadowing property [19].
6. The identity map on the union of Cantor set with some number of isolated points has shadowing [4].

7. A shift space has shadowing if and only if it is of finite type [4].
8. A diffeomorphism map has shadowing property on the whole manifold if it is structurally stable, but structural stability and shadowing are not equivalent [24]. The authors in [9] conjectured that the shadowing property implies C^1 −structurally stable if the map isC^1 −generic diffeomorphisms.
9. The cellular automaton has the shadowing property if any factorization of column is a sub-shift of finite type [20].
10. Every cellular automaton has the shadowing in equicontinuous dynamical system [20].
11. a Morse–Smale has the shadowing in a diffeomorphism [3].
12. The restriction flow to its nonwandering set has the strong shadowing property if the flow satisfies an Axiom A [18].
13. an irrational rotation of the circle dose not have had shadowing [5].
14. Anosov diffeomorphisms have shadowing property [6].

Now let us display some important examples with different spaces:

Example [23]

Let $\mathcal{X} = [0,1]$ and let $\mathfrak{f}_n$ be continuous map on $\mathcal{X}$. For any $n \geq 1$
$\mathfrak{f}_n = 1 - (|2n\varkappa - (2k - 1)|)^{\frac{1}{2n}}, \frac{(k-1)}{n} \leq \varkappa \leq \frac{k}{n}$, with,$k = 1,2, \dots, n$.
The map $\mathfrak{f}(\varkappa) = 0$ has shadowing property.

Example [4]

Let $\mathfrak{f}$ be any homeomorphism on $\mathcal{X}$, where $\mathcal{X} = [0,1] \cup \{-1/2^n : n \geq 1\}$. If $\mathfrak{f}(\varkappa) = \varkappa$ for $\varkappa = 1$ or $\varkappa \leq 0$ and $\mathfrak{f}(\varkappa) < \varkappa$ for $\varkappa \in (0,1)$, then this map has the shadowing.

Example [2]

If $\mathfrak{f} : S^1 \to S^1$ is a North Pole – South Pole mapping with a arepeller fixed point n(the North Pole) and an attractor fixed point s (the South Pole) then it has the shadowing.

Example [10]

Let $\mathcal{X} = \mathcal{A}^{\mathbb{N}}$ where $\mathcal{A}$ is the finite set of symbols and for $a_j \in \mathcal{A}$ the set of all infinite sequences $(a_1, a_2, \cdots)$ is denoted by $\mathcal{A}^{\mathbb{N}}$. And a metric $\mathfrak{D}$ on $\mathcal{X}$ is defined as : $\varkappa = (\varkappa_1, \varkappa_2, \cdots)$, $y = (y_1, y_2, \dots) \in \mathcal{X}$, $d(\varkappa, y) = \frac{1}{2^k}$ where k is the smallest positive integer for which $\varkappa_k \neq y_k$. The sequence $\sigma_j((\varkappa_1, \varkappa_2, \dots)) = (\varkappa_{i+1}, \varkappa_{i+2}, \dots)$ is sequence of shift maps on the compact metric space $(\mathcal{X}, \mathfrak{D})$. This map is continuous, and $\mathfrak{D}(\varkappa, y) = \frac{1}{2^j} \mathfrak{D}\left(\sigma_j(\varkappa), \sigma_j(y)\right)$, then the sequence $\{\sigma_j\}$ has shadowing property.

Example [18]

The geometric Lorenz attractor $(\widetilde{\mathcal{K}}_{\mathfrak{f}}, \widetilde{\varphi}_{\mathfrak{f}})$, with the return map $\mathfrak{f}$ from the interval $[0,1]$ to itself, has the finite shadowing if and only if $\mathfrak{f}(0) = 0$ and $\mathfrak{f}(1) = 1$.

Example [18]

If $\mathfrak{f}: [0,1] \to [0,1]$ has the finite shadowing property, then the geometric Lorenz semi flow $(\mathcal{K}_{\mathfrak{f}}, \varphi_{\mathfrak{f}})$ has the finite shadowing property.

Shadowing and Chaotic Properties

In this section we introduce the relation among shadowing and chaotic properties. The first four propositions give the conditions (expansivity, equicontinuoseand stability with chain transitive) on maps to have shadowing:

Proposition [13]

Let $\mathfrak{f}$ be a positively expansive map on $\mathcal{X}$. Then $\mathfrak{f}$ an open if and only if it has the shadowing.

Proposition [4]

If $\mathfrak{f}$ is a homeomorphism on $\mathcal{X}$ then it has shadowing and $\mathfrak{f}^{-1}$ is equicontinuous if and only if $\mathfrak{f}$ is equicontinuous and $\mathcal{X}$ is totally disconnected.

Proposition [3]

If $\mathfrak{f}$ is an Ω $-$stable system and chain transitive on $\mathcal{X}$ then it has the shadowing.

Proposition [16]

A strong expansive IFS (Iterated Function System) has shadowing if and only if it has the continuous shadowing.Furthermore, there are relations between types of shadowing property where the maps have chaotic properties as we show in next propositions:

Proposition [17]

1. if a set $\mathcal{A}$ is a topologically transitive and hyperbolic then $\mathcal{A}$ has strong shadowing property.
2. if a set $\mathcal{A}$ is a topologically transitive and has strong shadowing property then the periodic points are dense in $\mathcal{A}$.

Proposition [15]

If a homeomorphism f is an expansive then the following are equivalent:

1. The map f has the shadowing property,
2. The map f has the continuous shadowing property,
3. There is a compact space $\mathcal{X}$ with metric$\mathfrak{S}$ such that $\mathfrak{f}$,$\mathfrak{S}$.

Proposition [16]

Let $\mathcal{F}$ be an strongly expansive IFS on $\mathbb{Z}$. Then the shadowing and continuous shadowing areequivalent.

Proposition [16]

A parameterized IFS has the shadowing property if it is uniformly expanding or contracting.

Proposition [13]

let $\mathfrak{f}$ be a positively expansive map on $\mathcal{X}$. If $\mathfrak{f}$ expands small distances with constants $\mathfrak{d}_0 > 0$ and $\gamma > 1$ (with respect to $\mathfrak{D}$). Then $\mathfrak{f}$ is an open map if and only if it has the shadowing.

Proposition [13]

Let $\mathfrak{f}$ be as in the above proposition. Then $\mathfrak{f}$ is an open map if and only if it has the strong shadowing.
In the following propositions, the shadowing property is condition to get chaotic properties:

Proposition [21]

If a homeomorphism map $\mathfrak{f}$ has shadowing property then the following are equivalent:

1. There is an unstable chain recurrent point in $\mathrm{CR}(\mathfrak{f})$;
2. There is an infinite number of unstable chain recurrent points in $\mathrm{CR}(\mathfrak{f})$;
3. The map $\mathfrak{f}$ is chaotic.

Proposition [1]

If $\mathfrak{f}$ has the shadowing property then $\mathfrak{f}$ is topologically mixing if and only if it is chain mixing.

Proposition [8]

A transitive continuous map $\mathfrak{f}$ has dense small periodic sets if it has theshadowing.

Proposition [17]

For dynamical system $(\mathbb{R}^n, \mathfrak{f})$, the periodic points are dense in a set $\mathcal{A}$ if $\mathcal{A}$ is a topological transitive and has strong shadowing property.

Proposition [8]

If continuous map $\mathfrak{f}$ is a weakly mixing and has shadowing, then it is mixing.

Proposition [25]

If $\mathfrak{f}$ has the topological shadowing property on an uniform space $\mathcal{X}$, then $\mathfrak{f}$ is topologically chain mixing if and only if it is topologically mixing.

Proposition [8]

If a continuous map $\mathfrak{f}$ has shadowing property, then the following conditions are equivalent:
1. The map $\mathfrak{f}$ is totally transitive,
2. The map $\mathfrak{f}$ is topologically weakly mixing,
3. The map $\mathfrak{f}$ is topologically mixing.

Proposition [1]

If a continuous map$\mathfrak{f}$ is onto then it has shadowing and chain mixing if and only if it has shadowing and topologically mixing.

Proposition [3]

Let $\mathfrak{f}$ be an expansive homeomorphism map, then it has shadowing and chain mixing if and only if it has shadowing and topologically mixing.Let us recall the definition of continuum-wise expansive: if $\mathcal{C} \subset \mathcal{X}$ is compact and connected then it is called a continuum set. A homeomorphism $\mathfrak{f}$ is continuum-wise expansive if there is a constant $\iota > 0$ such that each non-trivial continuum set $\mathcal{C}$ satisfies $\sup_{n\in\mathbb{Z}} \text{diam}\mathfrak{f}^n(\mathcal{C}) > \iota$.

Proposition [14]

If a homeomorphism $\mathfrak{f}$ on a compact manifold $\mathcal{X}$of dimension ≥ 2 has shadowing, then expansive and continuum-wise expansive are equivalent.

Proposition [19]

Every zero-dimensional automorphism with zero topological entropy is distal and has shadowing property.

Proposition [4]

Every Expanding open map have shadowing property.

Proposition [3] [15]

Every expansive homeomorphism map $\mathfrak{f}$on $\mathcal{X}$, below
1. The map $\mathfrak{f}$ has shadowing,
2. The map $\mathfrak{f}$ has the strong shadowing,

3. The map $\mathfrak{f}$ has the continuous shadowing.

Proposition [12][15]

A homeomorphism $\mathfrak{f}$ is topologically stable if it has the continuous shadowing property on a compact manifold $\mathcal{M}$.

Proposition [25]

The topological shadowing together with chain transitivity implies topological ergodicity.Next propositions show that shadowing gives the stability.

Proposition [14]

If a homeomorphism $\mathfrak{f}$ on a compact connected $\mathcal{X}$ is continuum-wise expansive and shadowing, then it is topologically stable.

Non-Shadowing Property

In general, as we see in section two, not all maps have shadowing property, so as in section three that maps have this property with conditions related with chaotic properties. In this section we investigate the maps which has non shadowing property.

Proposition [18]

Let $\mathcal{M}$ be a compact 3-manifold and $\mathfrak{X}^2(\mathcal{M})$ the space of all$\mathrm{C}^2 -$ vector fields on $\mathcal{M}$ with the C^2 $-$topology. Then there is an open set $\mathcal{U} \subset \mathfrak{X}^2(\mathcal{M})$ such that each $\rho \in \mathcal{U}$ has the non strong shadowing.

Proposition [19]

Every distal homeomorphism of a compact connected metric space has non shadowing property.

Proposition [19]

Every isometry of a compact Riemannian manifold of positive dimension has non shadowing.

Proposition [1]

The map which has the usual specification property has non shadowing.

Example [2]

Let $\mathcal{X} = \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{\mathbb{Z}}$ be the Banach space with the norm which defined as : for $v \in \mathcal{X}$, where $v = (x, y, \{w_n\}_{n\in\mathbb{Z}})$, $|v| = \max(|x|, |y|, \sup_{n\in\mathbb{Z}}|w_n|)$. There is non shadowing in this diffeomorphism of a non-compact manifold.

Example [25]

Let $\mathcal{X} = \left\{x_k = \sum_{j=1}^{k} 1/_j \,; k \in \mathbb{N}\right\}$ be the subset of$\mathbb{R}$, with discrete uniformity. The identity map $\mathfrak{f}$ has the non metric shadowing property.

Example [25]

Let$\mathfrak{f}$: $\cup_{k\in\mathbb{Z}} \{k\} \times [0,1] \to\cup_{k\in\mathbb{Z}} \{k\} \times [0,1]$ be defined by $\mathfrak{g}(k, x) = (k + 1, x)$ with topology inherited from $\mathbb{R}^2$. Then there is nonshadowing.

Example [18]

Let $\psi = \{\psi^s\}_{s\in\mathbb{R}}$ be a flow on a compact manifold $\mathcal{M}$. If ψ has a basic set $\Upsilon \subset \mathcal{M}$ such that (Υ, ψ) is isomorphic to a geometric Lorenz attractor $(\widetilde{\mathcal{K}}_{\mathfrak{f}}, \widetilde{\varphi}_{\mathfrak{f}})$ with $\mathfrak{f}(0) \neq 0$ or $\mathfrak{f}(1) \neq 1$, then ψ has the non strong shadowing property.

Example [2]

As in example 2.16., let $\mathfrak{f} : S^1 \rightarrow S^1$ be a North Pole – South Pole mapping. Set $\mathcal{M} = S^1 \times \{0\} \cup S^1 \times \{1\}/(s, 0)$. Define the mapping $\mathfrak{f}$ on $\mathcal{M}$ as : $\mathfrak{f}(\varkappa, 0) = \mathfrak{f}\,(x)$ and $\mathfrak{f}(\varkappa, 1) = \mathfrak{f}\,(\varkappa)$. Then there is non shadowing.

Example [4]

Let $\mathcal{T}_2$be the full tent map with slop 2 and it is ball expanding. A tent map with gradient $\varsigma \in (1,2)$ whose critical point $\mathfrak{c}$ is not recurrent. This map will not have shadowing but for any closed set $\Upsilon \subseteq [0,1]$ for which $\mathfrak{c} \notin \Upsilon$, $\mathfrak{f}$ has shadowing on Υ .

Example [4]

The $\mathrm{LM}\mathfrak{g}_4(\varkappa) = 4\varkappa(1 - \varkappa)$ has shadowing, map $\mathcal{T}_2$ and this property is preserved between conjugate maps of a compact space.

CONCLUSION

In a compact metric space, the shadowing is important property in dynamical system. We see : (1) The cross product $\mathfrak{f} \times \mathfrak{f}$ and the composition $\mathfrak{f}^{\mathscr{k}}$ had shadowing for every integer $\mathscr{k} > 0$ if $\mathfrak{f}$ has the shadowing. (2) The self – homeomorphism map had non shadowing if it was distal on connected space or if it was on connected space with not point is minimal. (3) In the future work we will show the stronger property is called limit shadowing.

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