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OPTIMUM DESIGN OF HEAT SINK BY USING DIFFERENTIAL EVOLUTION AND SIMPLEX METHOD

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Abstract:

The optimum design of the heat sink by using differential evolution (DE) method is discussed in the present paper. The DE strategy (DE/ best/ 1/exp) is used here because this strategy is best strategy for heat transfer applications [1]. The main procedures for the heat sink optimization is found the minimum thermal resistance (maximize the heat transfer per unit volume) of the heat sink in order to reduce the cost of heat sink by reducing the heat sink material. The main design parameters (the fin diameter, d_f, the fin length, L_f, number of fins, N_f, the approach velocity, U_{app}, stream wise pitch, S_L, span wise pitch, S_T) assumed varied between lower and upper values during the present study to get the minimum thermal resistance. The overall dimension of the heat sink and the pressure drop across the heat sink are taken as deign constrains.
After applying the DE for the case study in the present paper, the optimum thermal resistance for maximize the heat transfer from inline fin arrangement heat sink is found (0.500467 °C/W) and for staggered fin arrangement heat sink is found (0.4021 °C/W). The effect of the constant parameters (the thickness, dimensions and material of the base plate) on the minimum thermal resistance is discussed.
Also, the effect and selections of the differential evolution parameters (crossover coefficient (CR) and scaling factor (F)) on the generation (iteration time) are examined. The optimum values of F & CR that minimize the generation for attaining the minimum thermal resistance are (F=0.9 & CR=0.8) . Also, the results of the DE are compared with Nelder Mead simplex method for same case study in order to check the accuracy and efficiency of the DE method. The DE was consumed less time than the simplex method for the same present case study.

وفاعلية طريقة ألـ(DE). طريقة ألـ(DE) احتاجت لوقت اقل لنفس الحالة الدراسية من الوقت الذي تحتاجه طريقة ألـ(Simplex Method).

Introduction:

The heat sink is the most common thermal management hardware used a micro and opto-electronics. It is improve the thermal control of electronic component, assemblies and modules by enhancing their surface area through the use of fins. Applications utilizing fin heat sinks for cooling of electronics have increasing significantly through the last few decades due to an increase in heat flux densities and product miniaturization. Optimization is a procedure of finding and comparing feasible solutions until better solution can be found [2].

Differential evolution (DE) is a population based search algorithm that comes under the category techniques. It is improved version of generic algorithm (GA), and is exceptional simple, significantly faster and robust at numerical optimization and is more likely to find a function's true global optimization [3] . the DE was introduced by Storn and Price in 1995, [1].

In the recent research, DE has been successfully used in different fields : digital filter design, [4], neural network learning, [5], Fuzzy-decision – making problems of fuel ethanol production, [6], design of fuzzy logic controllers, [7], batch fermentation process, [8] and [9], multi-sensor fussion, [10], dynamic optimization of continuous polymer reactor, [11]. DE can also be used for parameter estimation.

Babu and Sastry, [12] used DE for the estimation of effective heat transfer parameters in trickling bed reactors using radial temperature profile measurements. Babu and Munawar, [13], used DE for the shell and tube heat exchanger optimization but with only used (DE/ rad/1/bin) strategy. Babu and Rakesh Augira, [14], used the differential evolution strategies to optimize the water pumping system consisting of two parallel pumps drawing water from a lower reservoir and delivery it to another that is (40 m) higher. T. Rogalsky et al, [15], they compare the performance of some different differential evolution strategies when used by an aerodynamic shape optimization routine which design for blade shape.

W.A. Khan et al, [16], presented a mathematical model for determining the heat transfer from pin fin heat sink.

in our study , we found the optimum design of heat sink. Our optimization is considered by found the lower thermal resistance (maximum heat transfer) for the heat sink by using differential evolution method. The heat sink parameters in the present study were divided into two parts, the first part considered constant during the optimization like heat sink overall dimensions and materials and the second part is considered variable during the optimization like fin design parameters (length ,number, pitch and diameter of fin and the fluid velocity).

DE Technique

There are ten different working strategies proposed by Price and Storn [3]. In the present research, we used (DE/best/1/exp) strategy, because this strategy is best strategy for application of heat transfer according to the conclusion of Babu and Munawar, [1].

The procedures and applicable of the Differential Evolution method are mentioned in many modern literature and textbook, [1-10]. The scaling factor F that used in the DE is a assumed constant between ($0< F \leq 1.2$) in the present study, the optimal value of F for most of the functions lies in the range of 0.4 to 1.0, [3]. The crossover constant CR, that used here in the range $0 \leq CR \leq 1$. CR actually represents the probability that the child vector inherits the parameter values from the noisy random vector [3].

The code of DE used in the present study is given below:

- Firstly, we choose a number of the variable parameters that bounded between the upper and lower values and the constant parameter.
- Define the variable parameters, constant parameters and constrains (see table (1)).
- Initialize the values of D, NP, CR, F and MAXGEN (maximum generation) (D = 6 (number of variable parameters: d_f, L_f, Nf, Uapp, S_L and S_T), Np=10 (number of population vector in each generation), $0 \leq CR \leq 1$ (CR= 0.5), $0< F \leq 1.2$ (F= 0.8) and the MAXGEN=200).
- Initialize all the vectors of the first population randomly. The variables are normalized within the bounds (Upper bond (UP) and Lower bond (LP)). Hence generate a random number between 0 and 1 for all the design variables for initialization (example: for the diameter of the fin, the upper bond (UB = 3 mm) and the lower bound (LB= 1mm), then we generate ten values of fin diameter in the population vectors between 1mm to 3mm).

for j=1 to 10

for i = 1 to 6
*X (i,j) =LB+ RND * (UB -LB)*
next I
next j

- All the generated vectors should be satisfy the constraints (after completing the distribution of the population vector), then we must check the satisfy each vector with the constrains [$(S_L + S_P) * Np \leq W * W$, $d_f \leq S_L$, $df \leq S_T$ and $\Delta p \leq 250$ Pa] .
- Evaluate the thermal resistance of each population vector in each generation (determine the thermal resistance from eq.(6) for each population vector)

for i = 1 to 10
Rthi = Rth ( ) (from eq.(6))
next i

- Find out the vector which has a minimum thermal resistance value i.e. the best vector so far.

Rth min = Rth1 and best =1
for i = 2 to 10
if Rth i> Rth min then Rth min = Rth i and best = i
next i

- Perform mutation, crossover, selection and evaluation of the thermal resistance of the heat sink for each vector and for each generation.

If gen < MAXGEN
for i = 1 to 10

- For each vector Xi (target vector), select three distinct vectors Xr1, Xr2 and Xr3 (these vectors must be different) randomly from the current population other than the vector Xi.

*100 r1 = INT(random number * 10)*
*r2 = INT(random number * 10)*
*r3 = INT(random number * 10)*
if (r1=i) OR (r2=i) OR (r3=i) OR (r1=r2) OR (r2=r3) OR (r1=r3) then 100

- Perform crossover for each target vector Xi with its noisy vector Xn,i and create a trial vector, Xt,i. The noisy vector is created by performing mutation (see fig (A) for details).
- If CR = 0 inherit all the parameters from the target vector Xi, except one which should be from noisy vector Xn,i.
- For binomial crossover (see fig (A) for details, the crossover depend on the random number).

```
p = RND       "random number"
for n = 1 to 6
if  p=<CR  then
Xn,i = Xa,i + F ( X b,i - X c,i )
Xt,i = Xn,i
else
Xt,i = Xi,j
end if
next n
```

- Again, the NP (Np = 10) noisy random vectors that are generated should be satisfy the constraint [(S_L + S_P) * Np $\leq$ W * L , $d_f \leq S_L$, df $\leq S_T$ and $\Delta p \leq$ 250 Pa].
- Perform selection for each target vector, Xi by comparing its profit with that of the trial vector, Xt,i ; whichever has the minimum thermal resistance will survive for the next generation (see fig (A) for details).

```
Rth t,i = Rth ()
if (Rth t,i > Rth i ) then
for I =1 to 10
new Xi = Xt,I
next
Else
for I =1 to 10
new Xi = Xi
next
End if
```

- After generated a new generation vector, the same procedures are repeat to calculate the minimum thermal resistance for the heat sink. The program will stop if the number of generation reached to maximum number of generation or if we take the convergence criteria is the thermal resistance (when the difference in the thermal resistance between two previous generations should be less than (0.0001)), then the program will stop and print the results. The stopping criteria in the present study is the maximum number of generation (MAXGEN=200).

The schematic of the DE work for inline fin heat sink are mentioned below in the figure (A) for (MAXGEN=200, D=6, Np=10, CR=0.3 and F=0.8). This schematic (figure A) shows how to generate a new one vector in a new generation from the vectors of old generation.

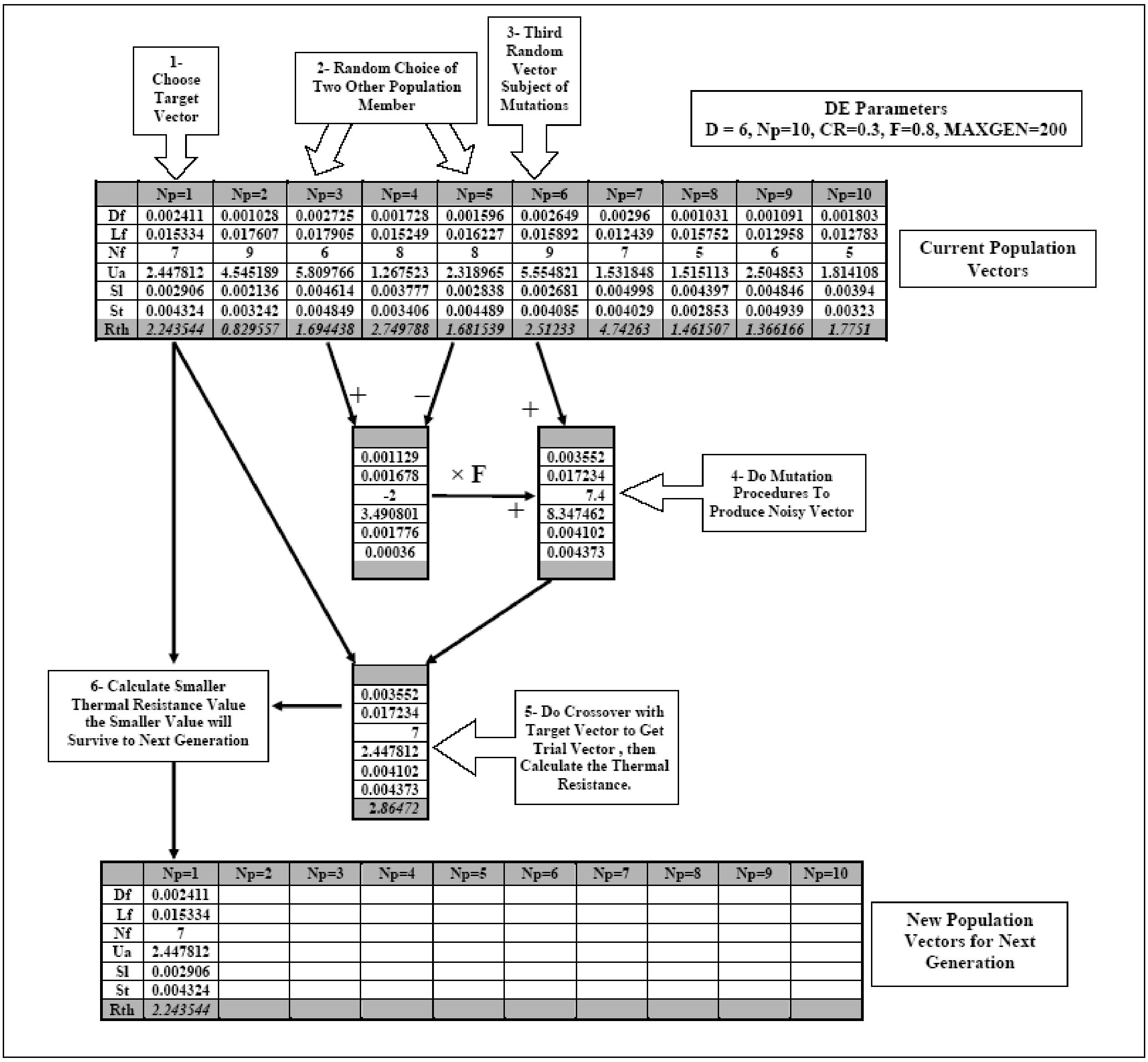

Figure (A): DE Procedures for Generating One Vector in New Population Heat Sink

Nelder-Mead Simplex Method

The local search method called the simplex method, this method is presented by Nelder and Mead, [16] is one of the most popular derivative-free nonlinear optimization methods. The formulation & procedures of this method is mentioned in many literature and textbook, [16] and [17]. In the present study, the four scalar parameters as following [17]; coefficients of reflection (**ρ=1**), expansion (**χ = 2**), contraction (**γ = 0.5**), and shrinkage (**σ =0.5**).

The procedures for evaluating the minimum thermal resistance of the heat sink by using simplex method in the present study is described in flow chart in appendix (1) .

Optimization Procedures/ Case Study

The objective of the following study is to be minimizing the thermal resistance of the heat sink. The design variables that taken in the present study were the fin diameter (d_f), the fin length (L_f), number of the fin (N_f), the approach velocity (U_{app}), stream wise pitch (S_L), span wise pitch (S_T). The assumptions of the case study are:

- Fins are plain and homogenous.
- Conduction heat transfer equal convection heat transfer at fin tip.
- Flow is steady and laminar.
- Fluid is Newtonian and incompressible.
- Radiation heat transfer is neglected.

The present study contains integer, discrete and continuous variables. The number of fin is integer variable and the diameter of the fin is continuous variable and the approach velocity may have discrete value according to the fan standard speed that using in the electronic package.

Minimize function = Rth(X)

The X denotes the vector of design variable, $X = [d_f, L_f, N_f, U_{app}, S_L, S_T]$

Subject to the constraints

$$N_f *(S_T+S_L) \leq W * W$$

$$d_f <= S_L \text{ and } d_f <= S_T$$

$$\Delta p \leq 250 \text{ Pa}$$

Mathematical Equations for Heat Sink

The fin heat sink that used in our study is a pin fin heat sink because this type of heat sink is best type, [18]. In our study we took inline and staggered fin arrangement, see fig (B).

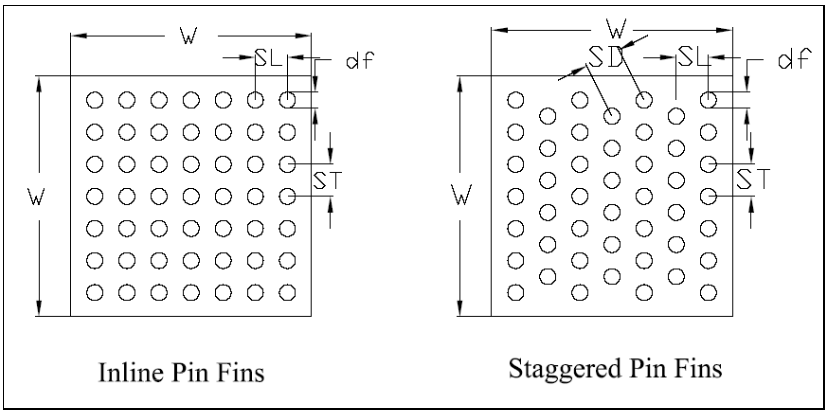

Figure (B): Heat Sink Types

Our work is considered to determine the minimum thermal resistance (maximum heat rate).

The following equations are used to calculate the total thermal resistance for the heat sink,[20]:

Total Heat Transfer from Heat Sink = Total Heat Transfer from Fins+ Total Heat Transfer from Bare Area

$$Q_T = Q_{FT} + Q_{UF} \quad \ldots(1)$$

$$Q_{FT} = N_f\, Q_f \quad \ldots(2)$$

$$Q_f = \eta_f\, h_f\, A_f\, (T_b - T_a) \quad \ldots(3)$$

$$\eta_f = \frac{\tanh(m l_c)}{m l_c} \quad \ldots(4)$$

Where $l_c = l_f + \frac{r}{2}$, $m = \sqrt{\frac{h_f p}{K A_c}}$, $p = 2\pi r$, $A_c = \pi r^2$, $A_f = p l_f = 2\pi r l_f$, $N_f = N_l * N_t$

$$Q_{UF} = h_{uf}\, A_{uf}\, (T_b - T_a) \quad \ldots(5)$$

Where $A_{uf} = (W * W - N_f\, \pi r^2)$

Assuming that the entire base plate is fully covered with electronic components and the fin are machined as in integral part of the base plate, the total resistance,[20]:

$$R_{th} = R_b + \frac{1}{\frac{1}{R_{ft}} + \frac{1}{R_{uf}}} \quad \ldots(6)$$

$$R_b = \frac{\Delta T_b}{Q_b} = \frac{b}{k_b WW} \quad \ldots(7)$$

$$R_{ft} = \frac{R_f}{N_f} = \frac{(T_b - T_a)}{N_f * Q_F} = \frac{1}{N_f \eta_f h_f\, 2\pi r l_f} \quad \ldots(8)$$

$$R_{uf} = \frac{(T_b - T_a)}{Q_{UF}} = \frac{1}{h_{uf}(WW - N_f \pi r^2)} \quad \ldots(9)$$

The mean heat transfer coefficient (h_f) and (h_{uf}) for the fin surface and un-fined area are obtain by Khan,[19], these equations are written as:

$$Nu_{uf} = 0.75\, C_1\, Re_d^{1/2}\, Pr^{1/3} \quad \ldots(10)$$

$$C_1 = \sqrt{\frac{(S_T / d_f) - 1}{N_L (S_T / d_f) * (S_L / d_f)}} \quad \ldots(11)$$

$$Nu_f = C_2\, Re_d^{1/2}\, Pr^{1/3} \quad \ldots(12)$$

$$C_2 = \begin{cases} [0.25+\exp(-0.55\,S_L/d_f)](S_T/d_f)^{0.285}(S_L/d_f)^{0.212} & \text{For Inline Arrangement} \\ \dfrac{0.61\,(S_T/d_f)^{0.091}(S_L/d_f)^{0.053}}{[1-2\,\exp(-1.09\,S_L/d_f)]} & \text{For Staggered Arrangement} \end{cases}$$

…….......(13)

$$Nu_{uf,f} = \frac{h_{uf,f}\,d_f}{k_a} \quad \text{……………………………………………………...….(14)}$$

$$Re = \frac{U_{max}\,d_f}{\nu} \quad \text{…………………………………………………………….(15)}$$

$$U_{max} = Max\left(\frac{S_T}{S_T-d_f}U_{app}\,,\,\frac{S_T}{S_D-d_f}U_{app}\right) \quad \text{……………….……….……………….….(16)}$$

Where,

$$S_D = \sqrt{S_L^2 + (S_T/2)^2} \quad \text{…………………………………………………………….….(17)}$$

The heat sink pressure drop (Δp), [18],

$$\Delta p = N_L\, f\left(\frac{1}{2}\rho_a U_{max}^2\right) \quad \text{………………………………………………….(18)}$$

$$f = \begin{cases} K_1[0.233+45.78/(S_T/d_f-1)^{1.1}Re_d] & \text{For Inline Arrangement} \\ K_1[\dfrac{378.6}{(S_T/d_f)^{\frac{13.1}{S_T}}}]/Re_d^{\frac{0.68}{ST^{1.29}}} & \text{For Staggered Arrangement} \end{cases} \quad \text{…(19)}$$

$$K_1 = \begin{cases} 1.009\left(\dfrac{S_T-d}{S_L-d}\right)^{\frac{1.09}{Re_d^{0.0553}}} & \text{For Inline Arrangement} \\ 1.175\left(\dfrac{S_L}{S_T\,Re_d^{0.3124}}\right)+0.5\,Re_d^{0.0807} & \text{For Staggered Arrangement} \end{cases} \quad \text{…..(20)}$$

Results and Discussions

The main objective of the present study is found the minimum thermal resistance (Rth) for the heat sinks (increase the rate of heat transfer removed from the heat sink). The new formulation of the DE method was discovered at 1997,[3]. At the last years, the DE is considered one of the best optimization method,[15], and it can be used widely in many different applications because it is simple and don't has derivative or any advance mathematics and the DE need a little time if it compared with other optimization methods,[1]. Then our results is mainly considered with applicable of DE for the heat sink optimization, then we used differential evolution (DE) for the following case study of heat sink, the variable and constants parameters for the case study mentioned in table (1).

In the present paper, we write a Q.BASIC computer program to calculate the optimum value of thermal resistance by Differential Evolution method.

In order to check the accuracy of our computer program and the advantage of the DE method, the resulting of the present program is compared with the resulting that getting by the Simplex method.

For the same case study (table 1) and for inline fin heat sink, the results of the minimum thermal resistance by using the Differential Evolution was (0.500467 ̊C/W) & this work consumed execution time about (8 second) to get the result, by applying the simplex method [17], for the same case study, the resulting of the minimum thermal resistance was (0.50048 ̊C/W) (this give indication about our computer program is OK) & this work consumed execution time about (20 second) (this give indication that the DE need less time than Simplex method) to get the result for same computer specification (PIII, 512 RAM, 1700MHZ CPU and 80GB HD), then we can notes the DE is faster then simplex method in execution to attain the optimum value.

Table 1: The Case Study Parameters

Variables	Constants	Constraints
$d_f = 1 – 3$ mm	$k_a = 0.026$ W/m.K	$[S_L + S_P] * N_p \leq W * W$
Lf = 10 -20 mm	$k_f = k_b = 203$ W/m.K	$d_f \leq S_L$
$N_f = N_l \times N_T$= 5×5 - 9×9 fins	ρ_a= 1.1614 kg/m^3	$d_f \leq S_T$
U_{app}= 1 -6 m/sec	$\nu = 1.58 * 10^{-5}$ m^2/Sec	$\Delta p \leq 250$ Pa
S_L= 2 – 5 mm	Cp =1.007 Kj/Kg.K	
S_T= 2 – 5 mm	Pr = 0.71	
	T_a = 300 K	
	T_b = 365 K	
	b = 2 mm	
	W = 25.4 mm	

After applying the DE for the case study, a sample of the population vectors for different generations and how the crossover is occur and also can see the best vector in each generation shows in appendix (2).

From the appendix (2), at last generation (generation =200), we can notes the best minimum values that reduce the heat transfer from the heat sink is (0.500467 C/W) and from these columns we can found the optimum values of d_f, L_f, N_f, U_{app}, S_l, and S_T that used to minimize the heat sink thermal resistance (maximize the heat transfer from fin). The same procedures were applied for the staggered heat sink and the best minimum resistance is (0.4021 ̊C/W).

From these procedures (appendix 2), we can notes the optimum value of thermal resistance (R_{th}) is converged during the tenth generations and become to nearest from the optimum values at (gneration70) (vector 6). At generation 120, the convergence become acceptable in more than one vector (column) and at generations from 140 to 200 all the columns become approximately convergence with the optimum value.

The effects of some constant parameters on the minimum thermal resistance are discussed. In figure (1), the effect of the overall heat sink dimension (W) on the thermal resistance of heat sink (for the same parameters in table (1)) is plotted, we can notes the increasing of the base plate width (W) will reduce the optimum thermal resistance because the bare area become larger with increasing of the (W) and the heat transfer will increase by increase the area with same temperature difference, then the optimum thermal resistance will decrease as the heat transfer increased. The difference between the initial and optimum value of (R_{th}) increase with decreasing of the (W) and in same time the generation to get the optimum value decreased because when the (W) is small the fin parameters plays important parameters in the increase or decrease the heat transfer compared with high value of (W). The effect of the base plate thickness on the optimum (R_{th}) & the generation time to get the optimum (R_{th}) are plotted in figure (2). The increasing of the base plate thickness will increase the (R_{th}) because this factor work to obstruct the heat transfer rate, then the (R_{th}) increase as heat transfer decrease. Figure (3), shows the effect of base plate material on the (R_{th}), we can notes the increase of the thermal conductivity of the base material is decreased the (R_{th}) . For the best minimum (R_{th}), the base plate material must be made from same fin material or from material has higher than fin material thermal conductivity.

The one of the most advantages of DE is consumed short time to get the optimum value and this advantage (time-generation) is effected strongly by the DE parameters (CR and F).

In order to check the effect of the (F & CR) on the MAXGEN to get the optimum value, we plotted these effects in figures (4 to 9) (we cannot plot all curves in same figure because this figure becomes very complicated). The values of CR in these figures were varied from (0 to 1) with step (0.1) and the values of F were varied from (0.5 to 1) with step (0.1).

From theses figures (4 to 9), we can notes the values of CR that reduce the generation & get the optimum value was approximately (CR= 0.8) and the lower generation may be take place at (F=0.9 & CR=0.8), the maximum generation in these figure is attained when the difference between the optimum values equal (0.001 °C/W) & the maximum allowable generation is (100).

Conclusions

The optimization of the heat sink by using DE achieved in the present study. The DE for the present case study was very efficient and too simple because it doesn't have any derivative or integration. The time to get the optimum value by using DE is low (minimum CPU-time) compared with simplex method optimization. Fin diameter, fin length, fin pitch and the approach fluid velocity taken as design variables, the overall dimensions of the heat sink and the pressure drop across the heat sink are taken as deign constrains.

The DE computer program for the present case study is very simple to modify for different values of heat sink parameters & for different case study. The inline fin arrangement gives higher heat sink thermal resistance compared with staggered fin arrangement, and then the cost of the staggered fin heat sink is lower than the inline fin heat sink. The optimum value of DE parameters (F and CR) for the case study are obtained

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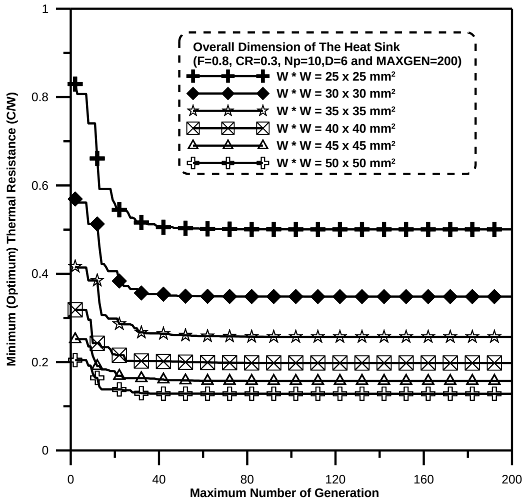
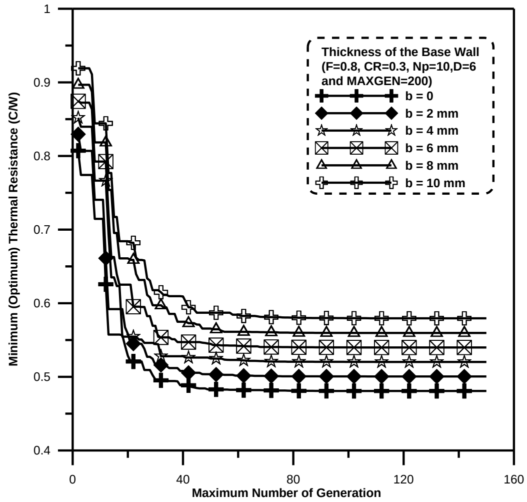
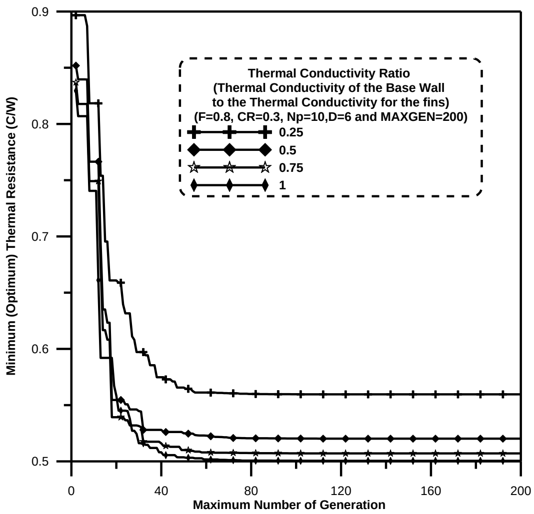


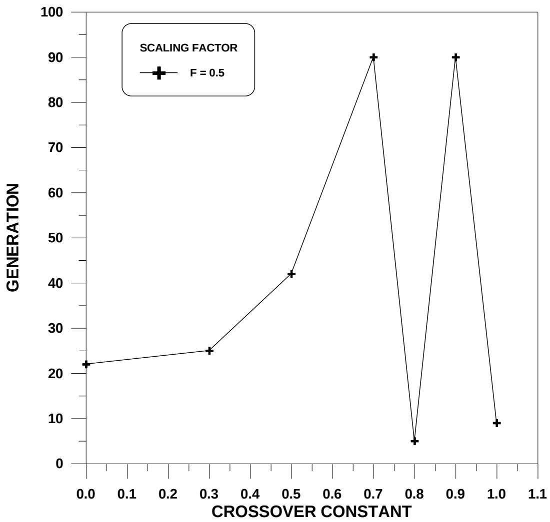
Fig(1): The effect of base wall dimensions (W) on optimum (R_{TH})



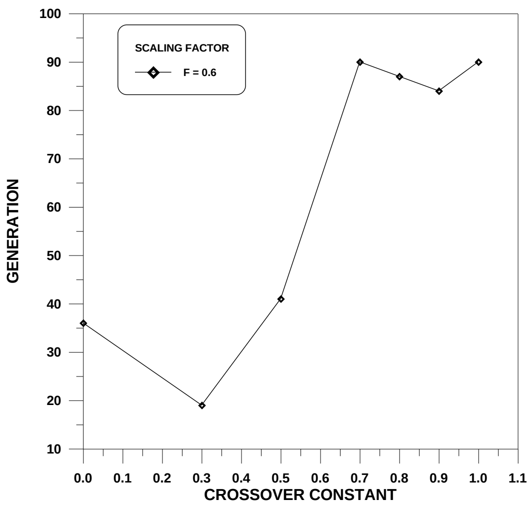
Fig(2): The effect of the base wall thickness (b) on optimum (R_{th})



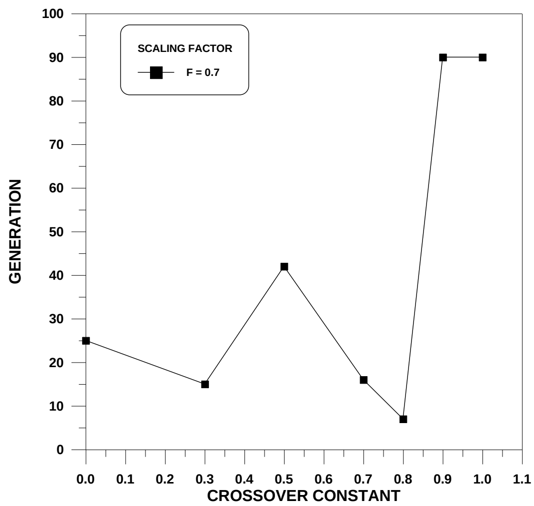
Fig(3): The effect of base wall material (k_a) on optimum (R_{th})



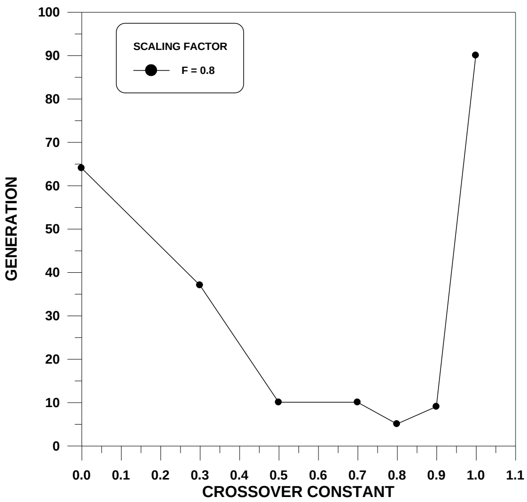
Fig(4): The variation of generation with CR at F=0.5



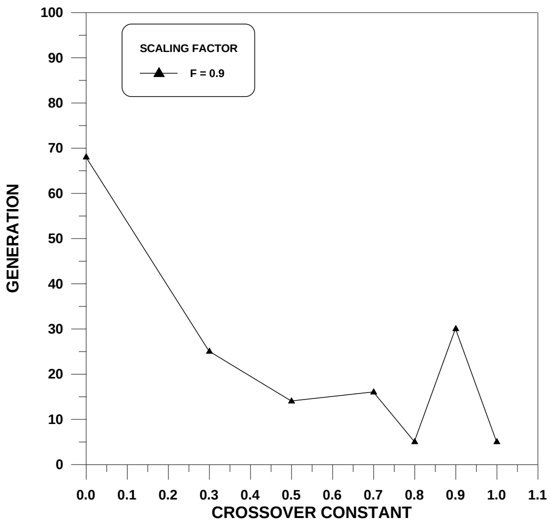
Fig(5): The variation of generation with CR at F=0.6



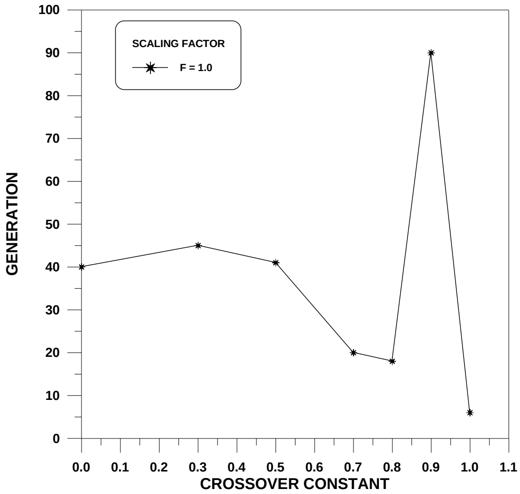
Fig(6): The variation of generation with CR at F=0.7



Fig(7): The variation of generation with CR at F=0.8

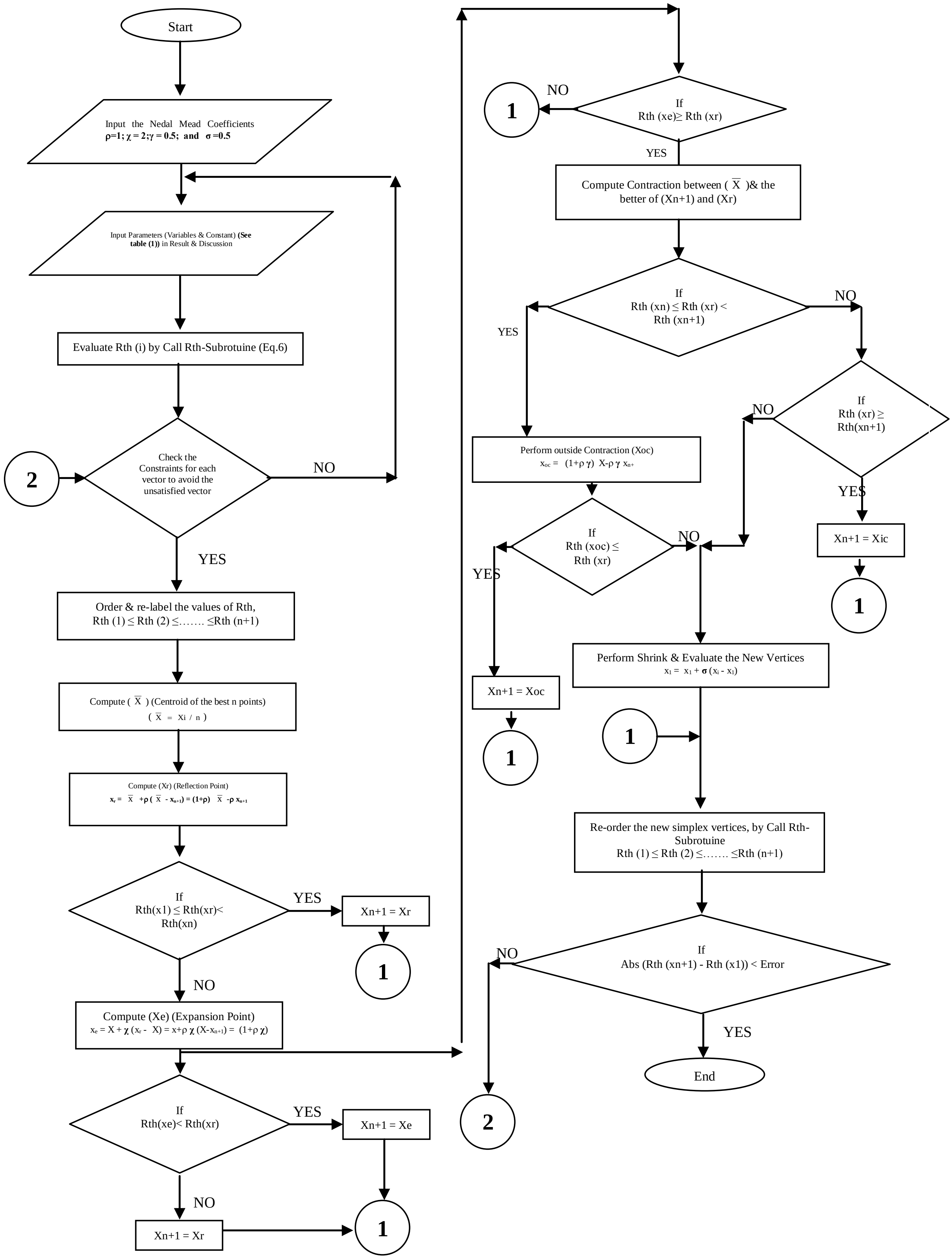


Fig(8): The variation of generation with CR at F=0.9



Fig(9): The variation of generation with CR at F=1

Appendix (1): Flow Chart for Simplex Method that Used in the Present Case Study


Start
Input the Nedal Mead Coefficients
ρ=1; χ = 2;γ = 0.5; and σ =0.5
Input Parameters (Variables & Constant) (See table (1)) in Result & Discussion
Evaluate Rth (i) by Call Rth-Subrotuine (Eq.6)
2
Check the Constraints for each vector to avoid the unsatisfied vector
NO
YES
Order & re-label the values of Rth,
Rth (1) ≤ Rth (2) ≤……. ≤Rth (n+1)
Compute (X̄) (Centroid of the best n points)
(X̄ = Xi / n)
Compute (Xr) (Reflection Point)
xr = X̄ +ρ (X̄ - xn+1) = (1+ρ) X̄ -ρ xn+1
If
Rth(x1) ≤ Rth(xr)<
Rth(xn)
YES
Xn+1 = Xr
1
NO
Compute (Xe) (Expansion Point)
xe = X + χ (xr - X) = x+ρ χ (X-xn+1) = (1+ρ χ)
If
Rth(xe)< Rth(xr)
YES
Xn+1 = Xe
1
NO
Xn+1 = Xr
1
NO
1
If
Rth (xe)≥ Rth (xr)
YES
Compute Contraction between (X̄)& the better of (Xn+1) and (Xr)
If
Rth (xn) ≤ Rth (xr) <
Rth (xn+1)
YES
NO
If
Rth (xr) ≥
Rth(xn+1)
NO
YES
Xn+1 = Xic
1
Perform outside Contraction (Xoc)
xoc = (1+ρ γ) X-ρ γ xn+
If
Rth (xoc) ≤
Rth (xr)
YES
NO
Xn+1 = Xoc
1
Perform Shrink & Evaluate the New Vertices
x1 = x1 + σ (xi - x1)
1
Re-order the new simplex vertices, by Call Rth-Subrotuine
Rth (1) ≤ Rth (2) ≤……. ≤Rth (n+1)
If
Abs (Rth (xn+1) - Rth (x1)) < Error
NO
2
YES
End

Appendix (2): Optimization Procedures for the Inline Fin Heat Sink for Following Data
MAXGEN=200, CR=0.5, F=0.8, Np=10, D=6 and the Constant, Variable and Constrains Data Mentioned in Table (1)

New Population at Generation =1										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.002411	0.001028	0.002725	0.001728	0.001596	0.002649	0.00296	0.001031	0.001091	0.001803
Lf	0.015334	0.017607	0.017905	0.015249	0.016227	0.015892	0.012439	0.015752	0.012958	0.012783
Nf	7	9	6	8	8	9	7	5	6	5
Ua	2.447812	4.545189	5.809766	1.267523	2.318965	5.554821	1.531848	1.515113	2.504853	1.814108
Sl	0.002906	0.002136	0.004614	0.003777	0.002838	0.002681	0.004998	0.004397	0.004846	0.00394
St	0.004324	0.003242	0.004849	0.003406	0.004489	0.004085	0.004029	0.002853	0.004939	0.00323
Rth	**2.243544**	**0.829557**	**1.694438**	**2.749788**	**1.681539**	**2.51233**	**4.74263**	**1.461507**	**1.366166**	**1.7751**
New Population at Generation =2										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.002411	0.001028	0.002725	0.001728	0.001596	0.002649	0.00296	0.001031	0.001091	0.00134
Lf	0.015334	0.017607	0.017905	0.015249	0.016227	0.019792	0.019792	0.015752	0.012958	0.010196
Nf	7	9	6	8	8	9	7	8	6	5
Ua	2.447812	4.545189	5.809766	1.267523	2.318965	5.554821	1.531848	4.962895	2.504853	1.814108
Sl	0.002906	0.002136	0.004614	0.002668	0.002838	0.002681	0.003301	0.004397	0.004846	0.00394
St	0.004324	0.003242	0.004849	0.003406	0.004489	0.004717	0.004717	0.004417	0.004939	0.004894
Rth	**2.243544**	**0.829557**	**1.694438**	**2.375866**	**1.681539**	**2.348106**	**3.801232**	**1.039502**	**1.366166**	**1.492911**
New Population at Generation =3										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.002411	0.001028	0.001082	0.001728	0.001596	0.002649	0.002386	0.001031	0.001091	0.00134
Lf	0.01839	0.017607	0.012032	0.015249	0.016227	0.019792	0.01619	0.015752	0.012958	0.010196
Nf	9	9	5	8	8	9	7	8	6	5
Ua	5.386343	4.545189	5.809766	1.267523	2.318965	5.554821	2.978393	4.962895	2.504853	1.814108
Sl	0.002272	0.002136	0.004614	0.002668	0.002838	0.002681	0.003473	0.004397	0.004846	0.00394
St	0.004324	0.003242	0.004849	0.003406	0.004489	0.004717	0.004717	0.004417	0.004939	0.004894
Rth	**1.900133**	**0.829557**	**0.806906**	**2.375866**	**1.681539**	**2.348106**	**2.147895**	**1.039502**	**1.366166**	**1.492911**
New Population at Generation =70										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.001001	0.001	0.001002	0.001002	0.001001	0.001	0.001002	0.001002	0.001001	0.001
Lf	0.019891	0.019705	0.01983	0.019839	0.01994	0.019766	0.019981	0.019814	0.019933	0.019679
Nf	5	5	5	5	5	5	5	5	5	5
Ua	5.991712	5.998633	5.995322	5.996275	5.998918	5.999243	5.996702	5.997541	5.993701	5.990473
Sl	0.002001	0.002002	0.002001	0.002001	0.002001	0.002001	0.002001	0.002001	0.002003	0.002001
St	0.003799	0.003649	0.003998	0.003979	0.004366	0.003614	0.004208	0.003878	0.004509	0.004359
Rth	**0.501543**	**0.501447**	**0.501757**	**0.501634**	**0.501132**	**0.501283**	**0.501335**	**0.501573**	**0.501405**	**0.501639**
New Population at Generation =100										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Lf	0.02	0.019995	0.019989	0.019934	0.019995	0.019985	0.019985	0.019971	0.019978	0.019991
Nf	5	5	5	5	5	5	5	5	5	5
Ua	5.999781	5.998073	5.999889	5.999352	5.999762	5.999968	5.99838	5.99942	5.999007	5.997329
Sl	0.002001	0.002	0.002001	0.002	0.002001	0.002001	0.002001	0.002001	0.002	0.002
St	0.004996	0.004773	0.004942	0.004714	0.004614	0.004974	0.004927	0.004793	0.004726	0.00494

Rth	**0.500558**	**0.500673**	**0.50064**	**0.50074**	**0.500763**	**0.500684**	**0.500694**	**0.500697**	**0.500682**	**0.500624**
New Population at Generation =120										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Lf	0.01999	0.019998	0.019986	0.019983	0.019975	0.019993	0.019988	0.019993	0.019998	0.019997
Nf	5	5	5	5	5	5	5	5	5	5
Ua	5.99951	5.999829	5.999501	5.999538	5.999815	5.999464	5.999454	5.999792	5.999949	5.99945
Sl	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
St	0.004996	0.004945	0.004896	0.004872	0.004904	0.004873	0.00488	0.004933	0.004823	0.004945
Rth	**0.500523**	**0.500496**	**0.500551**	**0.500557**	**0.500563**	**0.500536**	**0.500546**	**0.500513**	**0.500554**	**0.500517**
New Population at Generation =140										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Lf	0.019999	0.019998	0.019998	0.019999	0.019998	0.019999	0.019998	0.019999	0.019998	0.019998
Nf	5	5	5	5	5	5	5	5	5	5
Ua	5.999801	5.999864	5.99992	5.999887	5.999891	5.999958	5.999858	5.999948	5.999856	5.999799
Sl	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
St	0.004992	0.004968	0.004952	0.004997	0.004933	0.004968	0.004964	0.005	0.004984	0.004986
Rth	**0.500484**	**0.500494**	**0.500491**	**0.50048**	**0.500495**	**0.500485**	**0.500484**	**0.500481**	**0.500481**	**0.500483**
New Population at Generation =150										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Lf	0.02	0.019999	0.019999	0.019999	0.019998	0.019999	0.02	0.02	0.019999	0.019999
Nf	5	5	5	5	5	5	5	5	5	5
Ua	5.999985	5.999997	5.999988	5.999973	5.999922	5.999891	5.999918	5.999954	5.999991	5.99993
Sl	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
St	0.004992	0.004993	0.004972	0.004997	0.004995	0.004997	0.00499	0.004984	0.004999	0.004992
Rth	**0.500473**	**0.500475**	**0.500483**	**0.500475**	**0.500479**	**0.500476**	**0.500479**	**0.500478**	**0.500471**	**0.500476**
New Population at Generation =180										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Lf	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Nf	5	5	5	5	5	5	5	5	5	5
Ua	6	5.999997	6	5.999989	5.999998	5.999994	5.999994	6	5.999997	5.999999
Sl	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
St	0.004999	0.004999	0.004999	0.005	0.005	0.004999	0.004999	0.005	0.004999	0.004999
Rth	**0.500468**	**0.500468**	**0.500467**	**0.500468**	**0.500467**	**0.500468**	**0.500468**	**0.500467**	**0.500468**	**0.500468**
New Population at Generation =200										
	Np=1	**Np=2**	**Np=3**	**Np=4**	**Np=5**	**Np=6**	**Np=7**	**Np=8**	**Np=9**	**Np=10**
Df	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Lf	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Nf	5	5	5	5	5	5	5	5	5	5
Ua	6	6	6	6	6	6	6	6	6	6
Sl	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
St	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
Rth	**0.500467**	**0.50047**	**0.500467**	**0.500467**	**0.500467**	**0.500467**	**0.500467**	**0.500467**	**0.500467**	**0.500467**

NOMENCLATURES

A	Area (m^2)	p	Pressure (Pa)
A_c	Cross section area of the fin (m^2)	Pr	Prandtle number
b	Thickness of the base plate of Heat Sink (m)	Q_{FT}	Total heat transfer from fins (W)
Cp	Specific heat coefficient (Kj/Kg.K)	Q_T	Total heat transfer from heat sink (W)
CR	Crossover constant	Q_{UF}	Total heat transfer from bare area between fin (W)
D	Dimension of the problem (number of design variables)	r	Radius of the fin (m)
DE	Differential Evolution	Re_d	Reynold Number
d_f	Diameter of fin (m)	$R_{th}(i)$	Objective function thermal resistance (˚C/W)
F	Scaling factor	S_D	Digital pitch
f	Friction Coefficient	S_L	Stream wise pitch
h	Heat transfer coefficient ($W/m^2.K$)	S_T	Span wise pitch
k	Thermal conductivity (W/m.K)	T	Temperature (˚C)
Lc	Correct length of the fin (m)	U_{app}	Approach velocity of air
L_f	Length of fin (m)	UP	Upper bound
LP	Lower bound	W	Width of the heat sink (m)
MAXGEN	Maximum number of generation	X, Xc, Xa, Xb	Design variable
N_f	Total number of fins	Xe	Expansion point
N_L	Number of fin row in stream wise direction	Xic	Inside contraction point
Np	Population size	Xoc	Outside contraction point
N_T	Number of fin row in span wise direction	Xr	Reflection point
Nu	Nusselt number		

Greek

ρ	Reflection Coefficient – Simplex method	η_f	Fin efficiency
χ	Expansion Coefficient – Simplex method	θ_b	Temperature difference between the base and environment temperature (˚C)
γ	Contraction Coefficient – Simplex method	ν	Kinematics Viscosity (m^2/s)
σ	Shrinkage Coefficient – Simplex method	ρ_a	Density of the air (Kg/m^3)

Subscript

a	Environment (Air)	ft	Total number of fins
b	Base plate of heat sink	max	Maximum velocity of air
f	Fin	uf	Un-finned area